5th GACM Colloquium on Computational Mechanics

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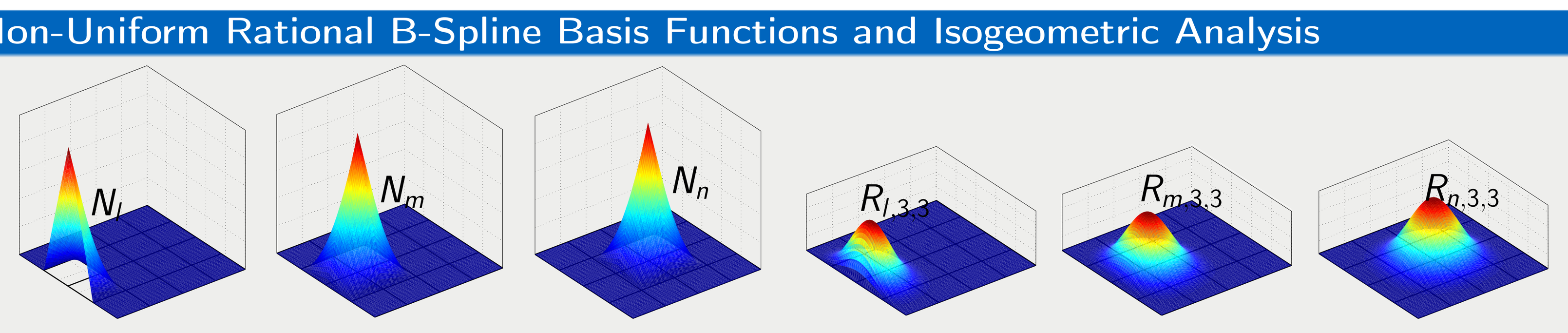
Non-Matching Grid Transfer Schemes for Partitioned Fluid-Structure Interaction Simulations using Isogeometric Analysis

Non-Uniform Rational B-Spline Basis Functions and Isogeometric Analysis



(a) Low order FEA basis functions (b) High order NURBS basis functions

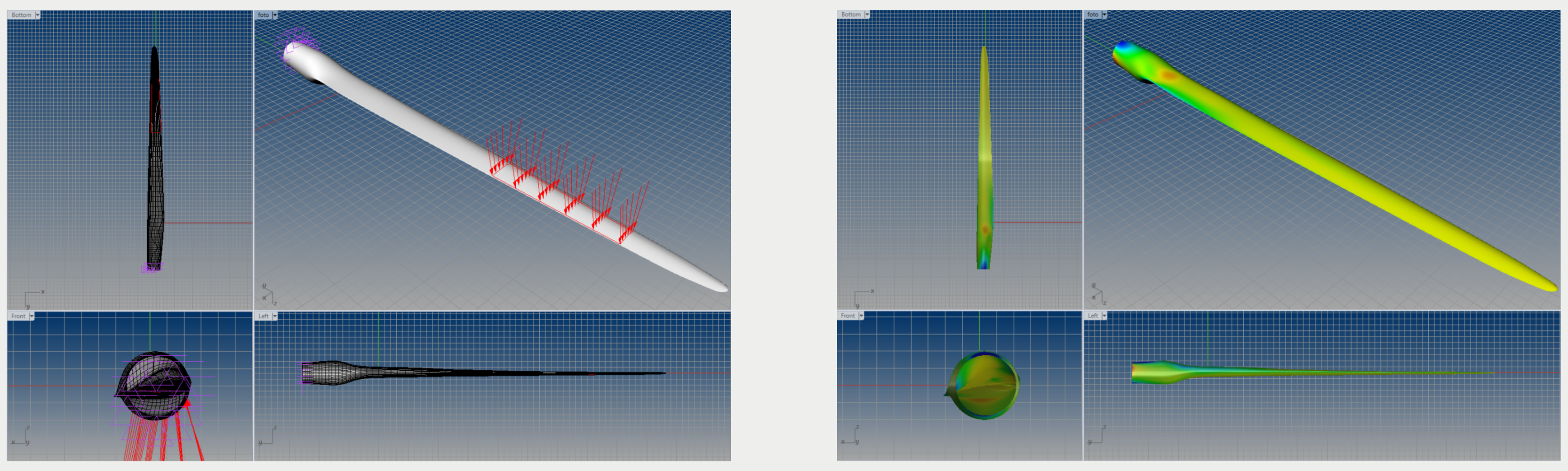
Figure 1: Piece-wise polynomial basis functions.

Classical Finite Element Analysis (FEA) uses typically C^0-continuous basis functions across the elements which also attain low polynomial order, see Figure 1(a), for numerically confronting Boundary Value Problems (BVPs). On the other hand, Isogeometric Analysis (IGA), proposed first in [1], makes use of high order basis functions the so-called Non-Uniform Rational B-Spline (NURBS) basis functions which in addition attain higher than C^0-continuity across the elements. Using the NURBS basis functions it can be also parametrized the low order basis functions. Then, the NURBS basis functions can be iteratively computed in 1D as:

$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi \in [\xi_i, \xi_{i+1}[\\ 0 & \text{elsewhere} \end{cases}, \quad \text{and } N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$

$$R_{i,p}(\xi) = \frac{N_{i,p}(\xi)}{\sum_{j=1}^{n} N_{j,p}(\xi)\, w_j}$$

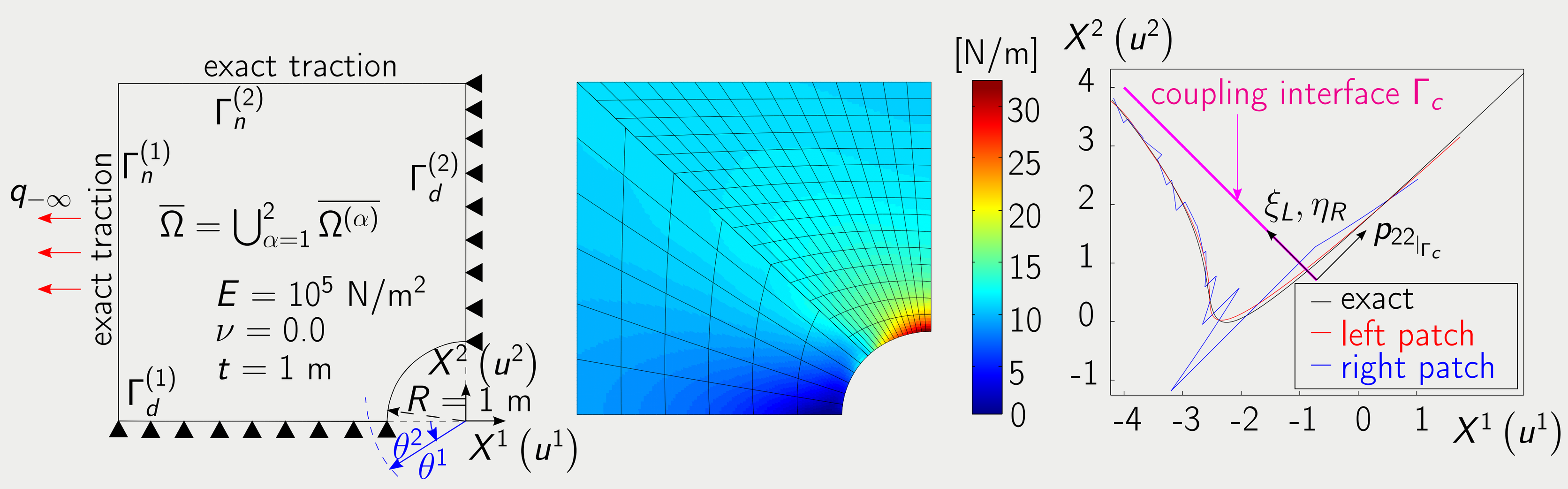
where $\xi \in \Xi$, Ξ denotes the so-called knot vector of the NURBS patch, $N_{i,p}$, $R_{i,p}$ and w_i are the are the B-Spline basis functions, the corresponding NURBS basis functions and their weights, respectively. In more than one dimensions, the NURBS basis functions are constructed as a tensor product of the 1D basis functions.



(a) NURBS model of a wind turbine blade (b) IGA applied on the wind turbine blade model

Figure 2: IGA performed on the wind turbine blade model.

The main feature of IGA is the employment of the geometry parametrization basis functions, namely the NURBS, for the interpolation of the physical field. Among others, significant implications of the method is the preservation of the geometrical model throughout the analysis, see Figure 2, and the high convergence rates, see [1].



(a) The problem placement (b) Visualization of the first principal force field (c) Normal force component n_{22} over the interface Γ_c

Figure 3: Infinite plate with circular hole subject to to constant pressure load at $X^1 = -\infty$.

Consider the two domain decomposed problem of an infinite plate with a circular hole subject to constant pressure load at $X^1 = -\infty$ solved with the Nitsche method [2] as depicted in Figure 3. Left patch is modelled using a $C^{2,3}$-continuous basis whereas right patch is modelled using a $C^{1,0}$-continuous basis. The results suggest that high order basis produce highly improved results compared to low order C^0-continuous basis, see Figure 3(c).

Partitioned Fluid-Structure Interaction

Assume the typical Fluid-Structure Interaction (FSI) scenario, as depicted in Figure 4, when using IGA for the structural domain. The interface conditions which ensure compatibility of the velocities and the tractions over the FSI boundary Γ_{FSI} are

$$\mathbf{u} - \dot{\mathbf{d}} = \mathbf{0}\,,$$
$$\mathbf{t}^{(S)} + \mathbf{t}^{(F)} = \mathbf{0}\,,$$

where $\mathbf{u}$ and $\mathbf{d}$ are the fluid velocities and the structural displacements, respectively. Assuming further that there exists a structural solver $\mathcal{S}\left(\mathbf{t}_h^{(S)}|_{\Gamma_{\mathrm{FSI}}}\right) = \mathbf{d}_h$, solving the weak equilibrium equation $W^{(S)}\left(\tilde{\mathbf{d}}_h, \mathbf{d}_h\right) = 0$ over the structural domain and one fluid solver $\mathcal{F}(\mathbf{d}_h) = \mathbf{t}_h^{(S)}|_{\Gamma_{\mathrm{FSI}}}$, solving the weak momentum balance and weak continuity equations $W^{(F)}(\tilde{\mathbf{u}}_h, \tilde{p}_h, \mathbf{u}_h, p_h) = 0$ over the fluid domain, where the subscript h and the tilde stand for the equivalent discretized variables and their corresponding variations. Then, the following Fixed-Point (FP) problem can be formulated:

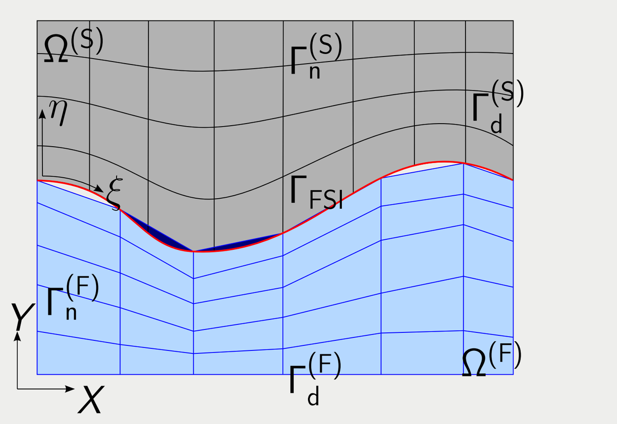


Figure 4: Fluid-Structure Interaction scenario using IGA

$$\mathcal{S} \circ \mathcal{F}(\mathbf{d}_h) = \mathbf{d}_h\,,$$

which can be solved iteratively with the partitioned Gauss-Seidel scheme depicted in Figure 5. Typically, the structural BVP in $\Omega^{(S)}$ is solved as a Neumann problem on the interface Γ_{FSI} meaning that it receives forces exerted by the fluid whereas the fluid BVP in $\Omega^{(F)}$ is solved as a Dirichlet problem on the interface Γ_{FSI}, that is, a mesh motion is prescribed on the fluid grid as a result of the structural displacement.

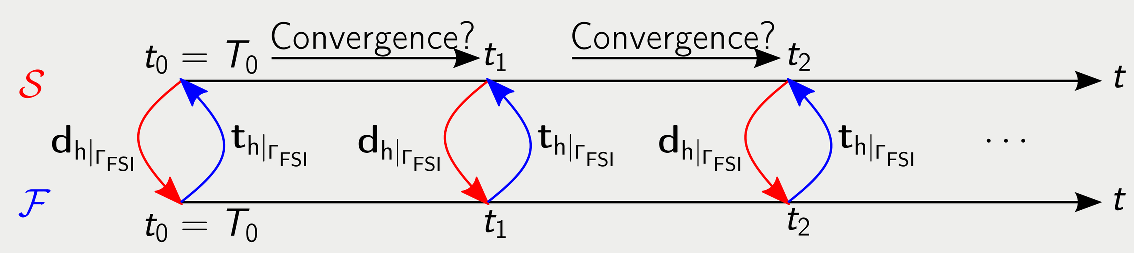


Figure 5: Partitioned Gauss-Seidel scheme

Additionally, an under-relaxation factor can be applied to the FP iterative scheme for the update of the state variables. A suitable estimation for the under-relaxation factor can be computed using the so-called Aitken relaxation method, see [3]. This factor accelerates and stabilizes the convergence of the FP iterations.

Non-Matching Grid Transfer using Isogeometric Analysis

The method of choice within this study is the so-called mortar method, which is based on the minimization of the gap function $\mathbf{d}_h^{(S)} - \mathbf{d}_h^{(F)}$ in the $L^2(\Gamma_{\mathrm{FSI}})$ space, namely:

$$\left\langle \mathbf{d}_h^{(S)} - \mathbf{d}_h^{(F)}, \boldsymbol{\mu} \right\rangle_{0,\Gamma_{\mathrm{FSI}}} = 0 \qquad \forall \boldsymbol{\mu} \in \left(L^2(\Gamma_{\mathrm{FSI}})\right)^3 .$$

The mortar-based method, in its discrete form, writes:

$$\hat{\mathbf{d}}^{(F)} = \left(\mathbf{C}^{(F)}\right)^{-1} \mathbf{C}^{(S)} \hat{\mathbf{d}}^{(S)}\,,$$

where the hat in the above vectors indicates that they contain the respective degrees of freedom. The coupling matrices are given by:

$$\mathbf{C}^{(S)} = \int_{\Gamma_{\mathrm{FSI}}} \left(\mathbf{N}^{\mu}\right)^{\mathsf{T}} \mathbf{R}\, \mathrm{d}\Gamma \quad \text{and} \quad \mathbf{C}^{(F)} = \int_{\Gamma_{\mathrm{FSI}}} \left(\mathbf{N}^{\mu}\right)^{\mathsf{T}} \mathbf{N}^{(F)}\, \mathrm{d}\Gamma\,,$$

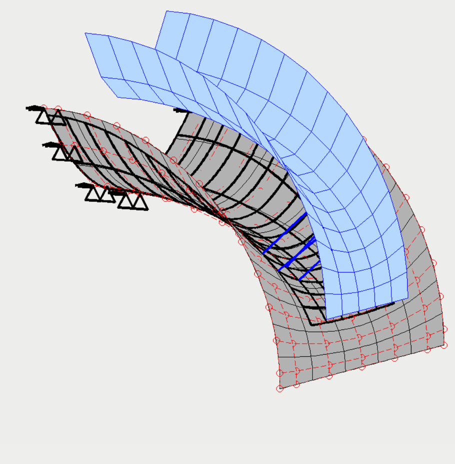


Figure 6: Projection phase

$\mathbf{N}^{\mu}$, $\mathbf{R}$ and $\mathbf{N}^{(F)}$ being the basis function matrices for the Lagrange multiplier field $\boldsymbol{\mu}$, the structural displacement and the fluid velocity field, respectively. It must be noted that within the mortar method it is chosen $\mathbf{N}^{\mu} = \mathbf{N}^{(F)}$ so that the transformation matrix $\mathbf{T} = \left(\mathbf{C}^{(F)}\right)^{-1} \mathbf{C}^{(S)}$ is symmetric, positive definite and diagonally dominant.

The typical scenario on the interface for an FSI problem is depicted in Figure 6, where the interface grids are non-matching. However, a unique interface must be identified so that the above integrals can be evaluated. For this reason each fluid node has to be projected onto the NURBS surface. This is done minimizing the distance of the fluid nodes from the NURBS surface in the Euclidean space, namely:

$$\mathbf{x}_0^{\mathrm{p}} = \arg\min_{\mathbf{x} \in \mathbf{S}} \|\mathbf{x} - \mathbf{x}_0\|_2 \Rightarrow \mathbf{r}(\xi, \eta) = \begin{bmatrix} (\mathbf{x} - \mathbf{x}_0) \cdot \mathbf{x}_{,\xi} \\ (\mathbf{x} - \mathbf{x}_0) \cdot \mathbf{x}_{,\eta} \end{bmatrix} = \mathbf{0}\,,$$

where $\mathbf{x}_0^{\mathrm{p}}$ denotes the projected fluid node onto the NURBS surface denoted by $\mathbf{S}$, see also Figure 6. The latter equation system is nonlinear, and provided that the surface is locally convex, it can be found its unique solution using the Newton-Raphson scheme.

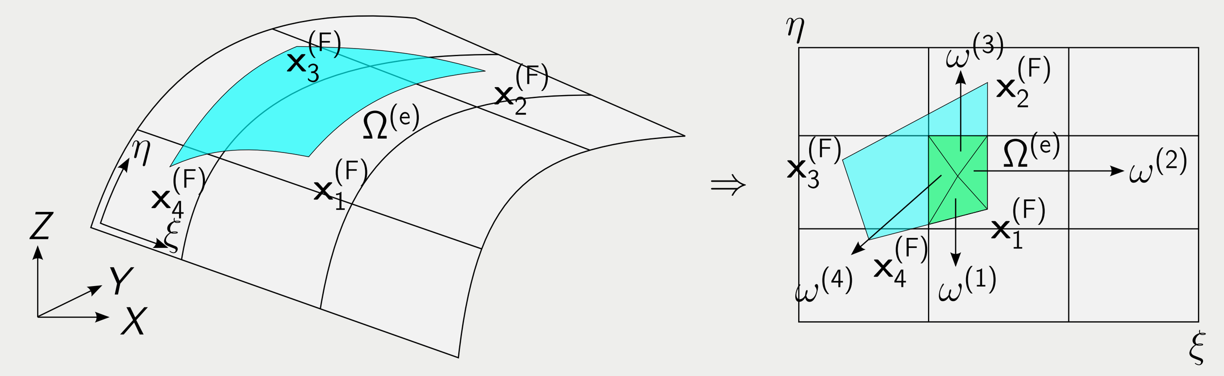
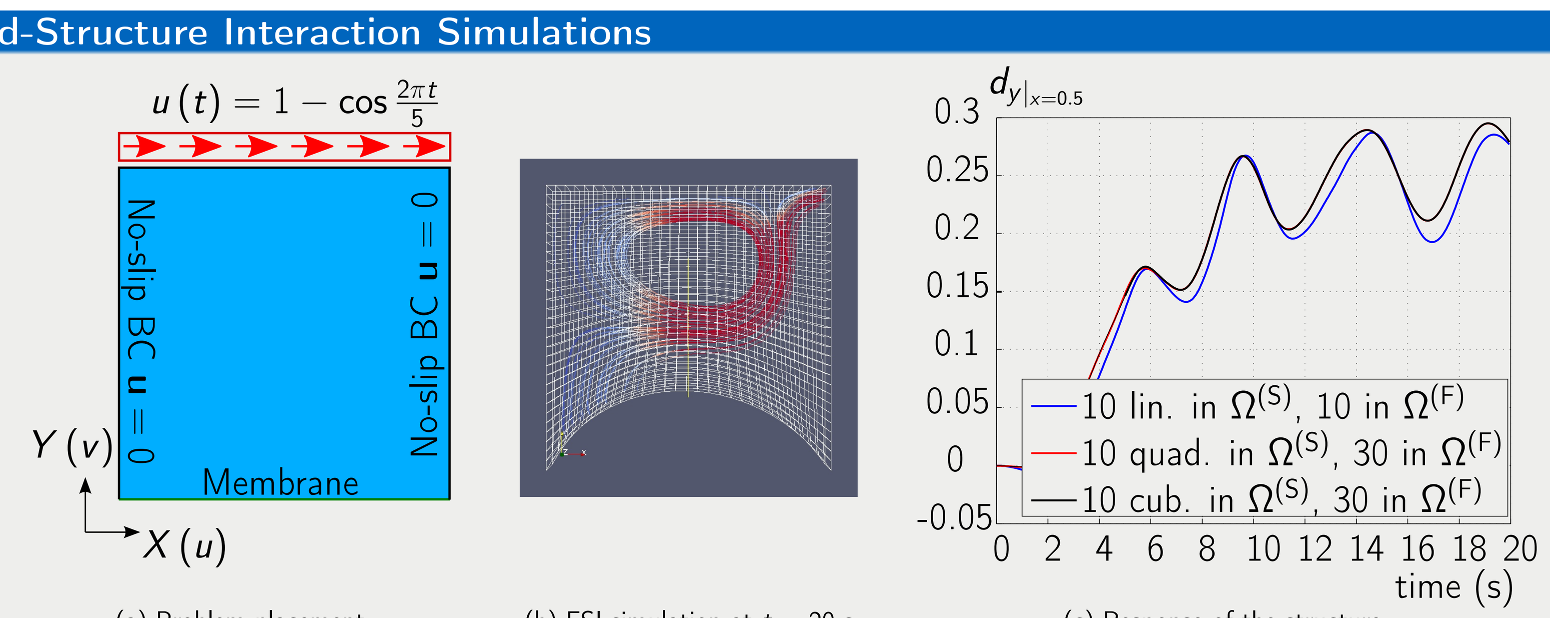


Figure 7: Clipping of the projected fluid elements

Then, once projection phase has been finalized integration must be carried through, which is performed at the sub-element level, see Figure 7. If the discrete virtual work over the interface is to be preserved, namely $\delta\mathcal{W}^{(S)} = \delta\mathcal{W}^{(F)}$ on Γ_{FSI}, then matrix $\mathbf{T}^{\mathsf{T}}$ can be used for the force transfer from the fluid to the structure.

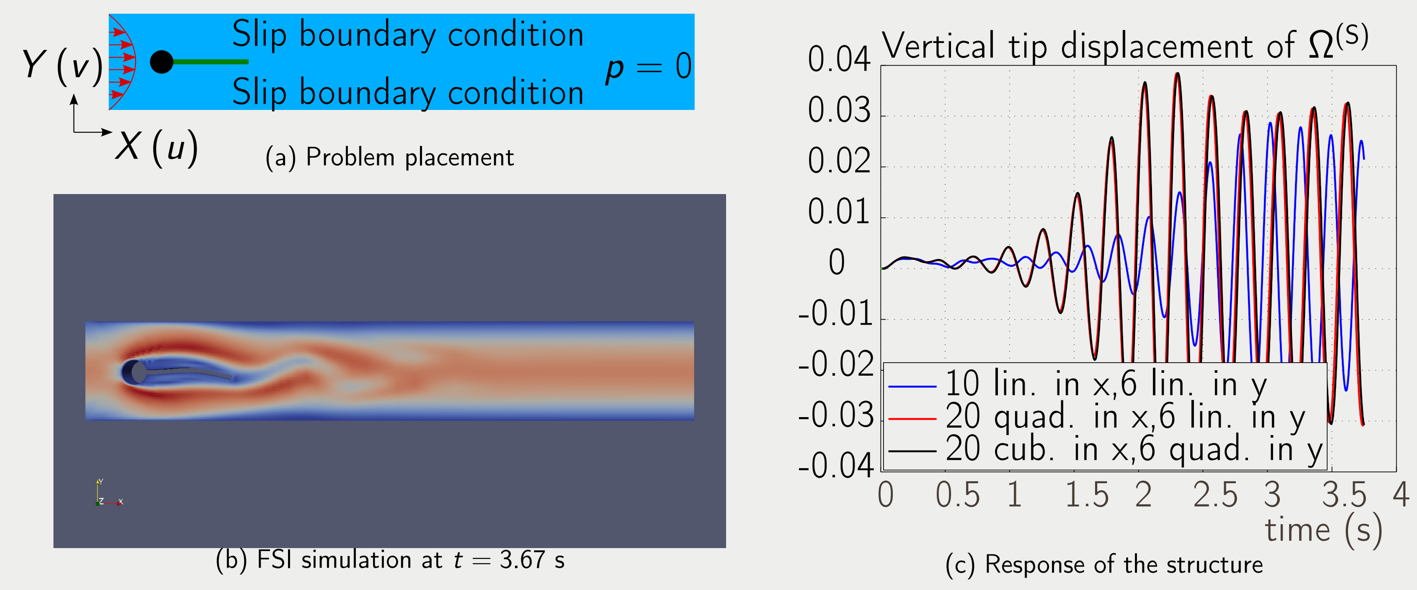
Fluid-Structure Interaction Simulations



(a) Problem placement (b) FSI simulation at $t = 20$ s (c) Response of the structure

Figure 8: Cavity flow with flexible membrane structure at its bottom

The cavity flow with a flexible bottom problem placement is shown in Figure 8(a). The fluid BVP is solved using the Finite Volume (FV) scheme within the openFOAM software. The flexible bottom is modelled with a membrane structure within the Carat++ software[†]. Then, the isogeometric coupling scheme has been implemented in the EMPIRE software[†]. Figure 8(b) shows the numerical result at the end time of the simulation, namely at $t = 20$ s. Different mesh grids and polynomial orders have been compared and shown in Figure 8(c).



(a) Problem placement (b) FSI simulation at $t = 3.67$ s (c) Response of the structure

Figure 9: Turek benchmark

The Turek benchmark problem setting, as proposed first in [4], is shown in Figure 9(a). In this case the structure is modelled with a NURBS-based linear solid element. The displaced structure and the magnitude of the fluid velocity field is then shown in Figure 9(b). Additionally, a comparative study is performed for different polynomial degrees of the structural model, see Figure 9(c), demonstrating robustness of the proposed method.

References

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