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Anomalous dimension of subleading-power N-jet operators

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Abstract: We begin a systematic investigation of the anomalous dimension of subleading power N-jet operators in view of resummation of logarithmically enhanced terms in partonic cross sections beyond leading power. We provide an explicit result at the one-loop order for fermion-number two N-jet operators at the second order in the power expansion parameter of soft-collinear effective theory.

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Contents

1 Introduction

The scattering amplitude of N well-separated, energetic, massless particles is one of the key quantities in gauge theories. Understanding its structure is of fundamental importance, both for its own reason by revealing mathematical structure that is not at all evident from the underlying Lagrangian and its Feynman rules, and for the phenomenology of high-energy scattering in QCD.

Of particular interest are the soft and collinear divergences, which exhibit a high degree of universality. Some form of analytic calculation is usually required in order to efficiently cancel the divergences between virtual and real emission effects in infrared-safe scattering cross sections. The infrared divergences of the virtual N-parton scattering amplitude are governed by the soft-collinear anomalous dimension, which up to the two-loop order has the very simple structure

$$\mathbf{\Gamma} = -\gamma_{\rm cusp}(\alpha_s) \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \ln\left(\frac{-s_{ij}}{\mu^2}\right) + \sum_i \gamma_i(\alpha_s) \qquad (1.1)$$

in colour-operator notation [1] and for all out-going momenta p_i with $s_{ij} = 2p_i \cdot p_j + i0$, $i, j = 1 \ldots N$. The soft [2] and collinear [3] contributions to $\mathbf{\Gamma}$ are known up to the three-loop order.[1] The above assumes that all scalar products s_{ij} are parametrically of the same order as the square of some hard scale Q. If the physical observable is sensitive to a smaller scale M generated by soft or collinear radiation, the anomalous dimension is a central object in the systematic all-order resummation of large logarithms $\ln Q/M$ in the expansion in the coupling α_s.

When this is the case the above anomalous dimension refers to the infrared singularities at leading order in the expansion in powers of M/Q ("leading power"). Given the advances in the understanding of multi-loop corrections to the leading power anomalous dimension, it is also timely to ask about the next, subleading power term in the M/Q expansion. It has been known for a long-time that single soft emission from an N-jet amplitude is described by a universal expression, the LBK amplitude, also at next-to-leading power [4, 5]. This result extends the eikonal formula and has recently attracted new interest in connection with a possible relation to an asymptotic symmetry at null infinity [6]. However, little is known about the structure of divergences of loops and the anomalous dimension at the subleading powers. The exponentiation of purely soft, "next-to-eikonal" effects has been discussed in refs. [7, 8]. However, a major complication at next-to-leading power arises from the interplay of soft and collinear radiation as can be seen, for example, from the failure (or rather — depending on the point of view — generalization) of the LBK formula for jet processes beyond the tree approximation [9, 10].

In this paper we begin with a systematic study of subleading power N-jet operators and their anomalous dimension with the ultimate goal of being able to sum logarithmically enhanced loop effects to all orders in perturbation theory. We base this study on soft-collinear effective theory (SCET) [11–14], which offers the advantage that the power counting required to identify all next-to-leading power terms is already built into the Lagrangian. While we will not solve the resummation problem here and do not even discuss logarithms for a physical process, our approach demonstrates a clear path how this could be done in principle and systematically. The structure of the anomalous dimension matrix of subleading-power N-jet operators will become apparent and we provide the first complete result for the class of fermion-number $F = 2$ operators to begin with. Previous work on anomalous dimensions at subleading power in SCET focused on specific cases, the heavy-to-light current [15, 16] (related to $J^{B1}_{\mathcal{A}\chi}$ in the operator basis defined below) in the position-space SCET formalism, and on power-suppressed tree-level currents relevant to $e^+e^- \to$ two jets in a different SCET framework [17, 18].

Several other works have recently addressed next-to-leading power (NLP) effects from a more practical perspective. In refs. [19–21] the threshold limit of the partonic Drell-Yan process has been investigated and all NLP terms of the next-to-next-to-leading order (NNLO) cross section have been successfully reproduced in a diagrammatic expansion analysis. Also a "radiative jet function" has been identified, related to collinear effects, which appear near threshold first at NLP. For colourless final states the interference of the NLP

[1] We refer to the above papers for a comprehensive list of references to relevant results at lower orders.

LBK amplitude with the tree process allows one to compute the NLP terms at NLO in the loop expansion [22]. Another recent development [23, 24] concerns the analytic computation of the leading NLP logarithm at NNLO in the separation parameter of the N-jettiness subtraction method [25, 26], making the cancellation of the dependence on the separation parameter in the full simulation of the process more efficient. All of these applications have in common that they refer at present to logarithms at fixed order in perturbation theory up to NNLO and to processes with only two collinear directions. The general approach outlined in the present paper, once developed, should allow the computation of further logarithms in these applications, and in particular their resummation to all orders. We finally take note that along a somewhat different direction a formula for fermion-mass suppressed double logarithms in the high-energy limit of certain fermion-scattering form factors has been derived [27, 28].

2 Subleading N-jet operator basis

It was noted in refs. [29, 30] that the infrared anomalous dimension (1.1) must correspond to the ultraviolet divergences of soft and collinear loops in SCET, if SCET is to be the correct effective field theory for jet processes. This observation also applies to subleading powers. The following analysis is based on the position-space field representation of SCET [13, 14]. The physical processes which are covered by this analysis are those for which the virtuality of collinear modes in any of the N jet directions is of the same order, and parametrically larger than the one of the soft mode. The power-counting parameter λ is set by the transverse momentum $p_{\perp i} \sim Q\lambda$ of collinear momenta with virtuality $\mathcal{O}(\lambda^2)$.[2] The components of soft momentum are all $\mathcal{O}(\lambda^2)$ and consequently soft virtuality scales as λ^4. Below, the term "NLP" refers to $\mathcal{O}(\lambda)$ and $\mathcal{O}(\lambda^2)$, since the first non-vanishing power correction to most physical processes of interest is $\mathcal{O}(\lambda^2)$.

Under these assumptions (often referred to as SCET_I) the SCET Lagrangian including all subleading power interactions to $\mathcal{O}(\lambda^2)$ was already given in ref. [14]. For N widely separated collinear directions, the Lagrangian

$$\mathcal{L}_{\text{SCET}} = \sum_{i=1}^{N} \mathcal{L}_i(\psi_i, \psi_s) + \mathcal{L}_s(\psi_s) \tag{2.1}$$

is the sum of N copies of collinear Lagrangians with N pairs of separate light-like reference vectors $n_{i\pm}$, $i = 1, \ldots, N$ satisfying $n_{i-} \cdot n_{j-} = \mathcal{O}(1)$. The collinear fields ψ_i all interact with the same soft field ψ_s but not among each other. The SCET Lagrangian is invariant under N separate collinear gauge transformations and a soft gauge transformation, see ref. [14].

We therefore proceed to the construction of a complete basis of subleading N-jet operators in SCET. The general structure

$$J = \int dt\, C(\{t_{i_k}\})\, J_s(0) \prod_{i=1}^{N} J_i(t_{i_1}, t_{i_2}, \ldots) \tag{2.2}$$

[2] Q denotes a generic large energy/hard scale, which we set to 1 in the following.

can be described by products of operators J_i associated to collinear directions n_{i+}, each of which is itself composed of a product of n_i gauge-invariant collinear "building blocks" ψ_{i_k} [31],

$$J_i(t_{i_1}, t_{i_2}, \dots) = \prod_{k=1}^{n_i} \psi_{i_k}(t_{i_k} n_{i+}), \tag{2.3}$$

and a soft operator J_s. In general, each of the collinear building blocks is integrated over the corresponding collinear direction in position space, where $C(\{t_{i_k}\})$ is a Wilson coefficient, and $dt = \prod_{ik} dt_{i_k}$. Apart from the displacement along each of the collinear directions, the operators are evaluated at position $X = 0$, corresponding to the location of the hard interaction.

The guiding principle for constructing building blocks is the requirement of collinear and soft gauge covariance. Because each collinear sector transforms under its own collinear gauge transformation, each collinear building block must be a collinear gauge singlet. However, the soft field may interact with different collinear sectors so we only need to assume that collinear building blocks transform covariantly under the soft gauge transformation. Note that, in general, the collinear building blocks may also contain multipole expanded soft fields. For a collinear block the transformation properties under collinear and soft gauge transformation may be summarized as follows

$$J_i(x) \xrightarrow{\text{coll.}} J_i(x), \qquad J_i(x) \xrightarrow{\text{soft}} U_s(x_{i-})J_i(x), \tag{2.4}$$

where $x_{i-}^\mu = (n_{i+}x)\, n_{i-}^\mu/2$ and U_s refers to the (not necessarily irreducible) colour representation of J_i. For the matrix adjoint representation we would have $J_i(x) \xrightarrow{\text{soft}} U_s(x_{i-})J_i(x)U_s^\dagger(x_{i-})$ with U_s in the fundamental representation.

The elementary collinear-gauge-invariant collinear building blocks are given by

$$\psi_i(t_i n_{i+}) \in \begin{cases} \chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i & \text{collinear quark} \\ \mathcal{A}_{\perp i}^\mu(t_i n_{i+}) \equiv W_i^\dagger [iD_{\perp i}^\mu W_i] & \text{collinear gluon} \end{cases} \tag{2.5}$$

for the collinear quark and gluon field in the i-th direction, respectively. W_i is the path-ordered exponential of $n_{i+}A_i$ ("i-collinear Wilson line") and the covariant derivative includes only the collinear gluon field. Both, the quark and gluon building blocks scale as $\mathcal{O}(\lambda)$ [13]. Objects containing $in_{i+}D_i$ or $in_{i+}\partial$ are redundant. The first can be reduced to the second with the help of $in_{i+}D_iW_i = W_i in_{i+}\partial$ and $W_i^\dagger in_{i+}D_i = in_{i+}\partial W_i^\dagger$.[3] The ordinary derivatives can be removed using $in_{i+}\partial\psi_{i_k}(t_{i_k}n_{i+}) = id\psi_{i_k}(t_{i_k}n_{i+})/dt_{i_k}$ followed by an integration by parts in the t_{i_k}-integral in eq. (2.2).

At leading power, only a single building block contributes for each direction, i.e. $n_i = 1$ for all $i = 1, \dots, N$, and the elementary building blocks are given by

$$J_i^{A0}(t_i) = \psi_i(t_i n_{i+}). \tag{2.6}$$

[3] Covariant derivatives acting on Wilson lines are understood as operators acting on functions to the right. When the derivative should be understood to operate only on the Wilson line, we add a square bracket as in eq. (2.5) for clarity. In all other cases the derivative is meant to act only on whatever is written explicitly to the right or within brackets.

The superscript in J_i^{A0} indicates the leading-power contribution, and the reason for this nomenclature will become clear in a moment. We are interested in N-jet operators that are suppressed by one or two powers of λ relative to the leading power. This suppression can arise in three ways:

(i) via higher-derivative operators, i.e. acting with either $i\partial_{\perp i}^{\mu} \sim \mathcal{O}(\lambda)$ or $in_{i-}D_s \equiv in_{i-}\partial + g_s n_{i-}A_s(x_{i-}) \sim \mathcal{O}(\lambda^2)$ on the elementary building blocks ψ_{i_k}. Here it is important to note that since the elementary building blocks transform under the soft gauge transformation with $U_s(x_{i-})$, the covariant soft derivative is the ordinary derivative for the transverse direction and $in_{i-}D_s$ for the n_{i-} projection. In other words, the soft covariant derivative *on collinear building blocks* is $iD_s^{\mu}(x) \equiv i\partial^{\mu} + g_s n_{i-}A_{\rm s}(x_{i-})\frac{n_{i+}^{\mu}}{2}$ due to the multipole expansion of the soft fields, which guarantees a homogeneous scaling in λ;

(ii) by adding more building blocks in a given direction, i.e. $n_i > 1$, since $\chi_i \sim \mathcal{O}(\lambda)$ and $\mathcal{A}_{\perp i}^{\mu} \sim \mathcal{O}(\lambda)$,

(iii) via new elementary building blocks that appear at subleading power, including purely soft building blocks in J_s.

In the following, we label operators that consist of a single building block by J_i^{An}, where $n = 1, 2$ indicates the relative power suppression due to additional derivatives. Using the equation of motion derived from the leading power collinear Lagrangian, it is possible to eliminate operators with $in_{i-}D_s$ derivatives (see below and appendix B), such that the operator basis consists of

$$J_i^{A1}(t_i) = i\partial_{\perp i}^{\nu} J_i^{A0} \qquad \mathcal{O}(\lambda)\,, \tag{2.7}$$

$$J_i^{A2}(t_i) = i\partial_{\perp i}^{\nu}\, i\partial_{\perp i}^{\rho} J_i^{A0} \quad \mathcal{O}(\lambda^2)\,. \tag{2.8}$$

Covariant derivative operators such as $(W_i^{\dagger} iD_{\perp i}^{\mu}\xi_i)(t_i n_{i+})$ and $(W_i^{\dagger} iD_{\perp i}^{\mu} iD_{\perp i}^{\nu} W_i)(t_i n_{i+})$ are special cases of $J_i^{A1}(t_i)$ and the $J_i^{B1}(t_{i_1}, t_{i_2})$ defined in the following with $t_{i_1} = t_{i_2}$. Hence all derivative basis operators are constructed from ordinary transverse derivatives acting on gauge-invariant collinear building blocks.

Operators with two collinear building blocks in the same direction i are suppressed at least by one power of λ with respect to the leading power, and we label them by J_i^{Bn}. At $\mathcal{O}(\lambda)$,

$$J_i^{B1}(t_{i_1}, t_{i_2}) = \psi_{i_1}(t_{i_1} n_{i+})\psi_{i_2}(t_{i_2} n_{i+}) \in \begin{cases} \mathcal{A}_{\perp i}^{\mu}(t_{i_1} n_{i+})\chi_i(t_{i_2} n_{i+}) \\ \chi_i(t_{i_1} n_{i+})\chi_i(t_{i_2} n_{i+}) \\ \mathcal{A}_{\perp i}^{\mu}(t_{i_1} n_{i+})\mathcal{A}_{\perp i}^{\nu}(t_{i_2} n_{i+}) \\ \chi_i(t_{i_1} n_{i+})\bar{\chi}_i(t_{i_2} n_{i+})\,. \end{cases} \tag{2.9}$$

The first operator has fermion-number one, the second two, and the last two have fermion-number zero. We do not list explicitly the conjugate operators with negative fermion-number.

At $\mathcal{O}(\lambda^2)$, the operators J_i^{B2} are obtained by acting with a $\partial^\mu_{\perp i}$ derivative on J_i^{B1}. We will use a basis where the derivative acts either on the second building block, or on both,

$$J_i^{B2}(t_{i_1}, t_{i_2}) \in \begin{cases} \psi_{i_1}(t_{i_1}n_{i+})i\partial^\mu_{\perp i}\psi_{i_2}(t_{i_2}n_{i+}) \\ i\partial^\mu_{\perp i}\big[\psi_{i_1}(t_{i_1}n_{i+})\psi_{i_2}(t_{i_2}n_{i+})\big] \,, \end{cases} \tag{2.10}$$

where $\psi_{i_1}\psi_{i_2}$ can be any combination from J_i^{B1}. Finally, at $\mathcal{O}(\lambda^2)$ it is possible to have operators composed of three elementary building blocks in a single direction, which we collectively call J_i^{C2},

$$J_i^{C2}(t_{i_1}, t_{i_2}, t_{i_3}) = \psi_{i_1}(t_{i_1}n_{i+})\psi_{i_2}(t_{i_2}n_{i+})\psi_{i_3}(t_{i_3}n_{i+}) \,. \tag{2.11}$$

This exhausts the options (i), (ii) from above at $\mathcal{O}(\lambda^2)$.

An example for a new building block that scales as order λ^2 and hence could be used to construct $\mathcal{O}(\lambda)$ suppressed operators is

$$n_{i-}\mathcal{A}_i \equiv W_i^\dagger in_{i-}D_iW_i - in_{i-}D_s = W_i^\dagger[in_{i-}D_iW_i] - g_sn_{i-}A_s \overset{\text{cLCG}}{=} g_sn_{i-}A_i \,, \tag{2.12}$$

where soft gauge covariance requires that $in_{i-}D_i$ includes the collinear gluon and the multipole-expanded soft gluon field. The subtraction term $-in_{i-}D_s$ in the second expression, which is also multipole expanded, is required to obtain a field rather than a differential operator, as is clear from the third expression, in which $in_{i-}D_i$ acts only within the square bracket.[4] The last expression shows that in collinear light-cone gauge $n_{i+}A_c = 0$ the new building block corresponds to the small component of the collinear gauge field. However, using the collinear-field equation of motion, we show in appendix B that $n_{i-}\mathcal{A}_i$ can be expressed in terms of the elementary building blocks with only $\partial_{\perp i}$ derivatives, hence $n_{i-}\mathcal{A}_i$ can be removed from the basis building blocks. As noted above for the transverse derivatives other possible placements of $in_{i-}D_i$ can always be reduced to (products of) existing objects. For example

$$W_i^\dagger in_{i-}D_i\xi_i = in_{i-}D_s\chi_i + n_{i-}\mathcal{A}_i\,\chi_i \,, \tag{2.13}$$

$$W_i^\dagger(iD^\mu_{\perp i}in_{i-}D_iW_i - iD^\mu_{\perp i}W_iin_{i-}D_s) = \mathcal{A}^\mu_{\perp i}\,n_{i-}\mathcal{A}_i \,. \tag{2.14}$$

As already mentioned we show in appendix B that the $in_{i-}D_s$ soft covariant derivative, which operates on the elementary collinear building blocks in the form

$$in_{i-}D_s\chi_i, \qquad [in_{i-}D_s, \mathcal{A}^\mu_{\perp i}], \tag{2.15}$$

can be eliminated by equation-of-motion operator identities in terms of the A2, B2 and C2 structures defined in eqs. (2.8), (2.10) and (2.11). This implies that $in_{i-}D_s$ can be eliminated from any collinear operator as

$$in_{i-}D_s(0)J_i(t_{i_1}, t_{i_2}, \dots) = \sum_{k=1}^{n_i}\psi_{i_1}(t_{i_1}n_{i+})\dots[in_{i-}D_s(0)\psi_{i_k}(t_{i_k}n_{i+})]\dots\psi_{i_{n_i}}(t_{i_{n_i}}n_{i+}) \,, \tag{2.16}$$

[4] Note that the collinear Wilson line transforms as $W_i \to U_c(x)W_i$ under collinear gauge transformations and $W_i \to U_s(x_-)W_iU_s(x_-)^\dagger$ under soft gauge transformations [14].

where the covariant derivative is understood in the colour representation of the object it operates on. Together with the above this implies that up to $\mathcal{O}\left(\lambda^2\right)$ we can use a basis of collinear building blocks that do not involve soft fields through covariant derivatives. It is constructed entirely from ordinary transverse derivatives and the elementary building block for the quark field and the transverse gluon field.

In addition to the collinear building blocks, the N-jet operator may also contain a pure soft building block J_s. The soft fields do not transform under the collinear gauge transformation, such that J_s is trivially a singlet under collinear gauge transformations. In the pure soft sector there is no need to perform the SCET multipole expansion of the soft fields and therefore the soft gauge transformation $U_s(x)$ in this case depends on x rather than on x_-. The soft transformation of J_s is

$$J_s(x) \xrightarrow{\text{coll.}} J_s(x), \qquad J_s(x) \xrightarrow{\text{soft}} U_s(x) J_s(x)\,, \tag{2.17}$$

with U_s taken in the appropriate representation. In the adjoint matrix representation we have $J_s(x) \xrightarrow{\text{soft}} U_s(x) J_s(x) U_s^\dagger(x)$ with $U_s(x)$ in the fundamental representation. The covariant pure soft building blocks start at $\mathcal{O}\left(\lambda^3\right)$, for example

$$q(x) \sim \lambda^3, \qquad F_s^{\mu\nu} \sim \lambda^4, \qquad iD_s^\mu q(x) \sim \lambda^5\,, \tag{2.18}$$

where *on soft building blocks* $iD_s^\mu(x) = i\partial^\mu + g_s A_s^\mu(x)$ and the soft field strength tensor is defined as $ig_s F_s^{\mu\nu} \equiv [iD_s^\mu, iD_s^\nu]$. We can therefore drop $J_s(0)$ in eq. (2.2) at $\mathcal{O}(\lambda^2)$. Therefore, soft fields enter neither via the soft nor via the collinear building blocks for our basis choice, up to $\mathcal{O}(\lambda^2)$. This implies that the emission of a soft gluon from the hard process, which generates the N-jet operator, is entirely accounted for by Lagrangian interactions.

The case of N-jet operators differs from that of heavy-to-light currents, which consist of one collinear direction and a soft heavy-quark field, whose decay is the source of large energy for the collinear final state. The basis of subleading SCET operators listed in ref. [31] does contain soft covariant derivatives at $\mathcal{O}(\lambda^2)$ due to the presence of the soft heavy-quark building block at leading power. The absence of soft building blocks in N-jet operators at $\mathcal{O}(\lambda^2)$ is also an important difference and simplification of the position-space vs. the label-field SCET formalism [11, 12], where soft fields must be included in the basis operators at $\mathcal{O}(\lambda^2)$ [10, 32]. The difference arises from a different split into collinear and soft, since in the label formalism only the large and transverse component of collinear momentum are treated as labels, while the residual spatial dependence of all fields, collinear and soft, is soft. The difference in the operator basis due to this is compensated by a corresponding difference in the soft-collinear interactions in the Lagrangian in the two formulations of SCET. In this respect it is important that the coefficients of operators with soft fields in the label formulation are related to those without by reparameterization invariance (RPI) [10], which suggests that one should combine in both formalisms all terms with hard coefficients related by RPI to demonstrate their equivalence.

It is useful to consider Fourier transformation with respect to the positions t_{i_k} in the collinear direction,

$$\begin{aligned}
J_i^{An}(P_i) &\equiv P_i \int dt_i \, e^{-it_iP_i} \, J^{An}(t_i) \,, \\
J_i^{Bn}(P_i, x_i) &\equiv P_i^2 \int dt_{i_1} dt_{i_2} \, e^{-i(t_{i_1}x_i + t_{i_2}\bar{x}_i)P_i} \, J^{Bn}(t_{i_1}, t_{i_2}) \,, \\
J_i^{Cn}(P_i, x_{i_1}, x_{i_2}) &\equiv P_i^3 \int dt_{i_1} dt_{i_2} dt_{i_3} \, e^{-i(t_{i_1}x_{i_1} + t_{i_2}x_{i_2} + t_{i_3}x_{i_3})P_i} \, J^{Cn}(t_{i_1}, t_{i_2}, t_{i_3})
\end{aligned} \tag{2.19}$$

for operators with one, two and three building blocks, respectively, where $\bar{x}_i = 1 - x_i$, $x_{i_3} = 1 - x_{i_1} - x_{i_2}$ and P_i is the total (outgoing) collinear momentum in direction i. Here we adopt the convention that $n_{i+}p_{i_k} = x_{i_k} P_i > 0$ for an outgoing momentum in direction i, such that from eq. (2.19) also $P_i > 0$ and $x_{i_k} \in (0, 1)$ for all outgoing momenta, which we shall assume in the following.[5] In general, the basis of N-jet operators can then be written in the form

$$J(\{P_i\}, \{x_{i_k}\}) = \prod_{i=1}^{N} J_i(P_i, x_{i_1}, x_{i_2}, \dots) \,, \tag{2.20}$$

where x_{i_k} are momentum fractions of the collinear momentum in direction i, carried by the k-th building block. The operators are given by $J_i \in \{J_i^{An}, J_i^{Bn}, J_i^{Cn}\}$, depending on the number of collinear building blocks and the order in λ. For each direction i one of the x_{i_k} can be eliminated using the constraint $\sum_k x_{i_k} = 1$, in accordance with the previous definitions. For brevity, we will omit the arguments P_i indicating the total collinear momentum in direction i if there is no danger of confusion, because it is conserved in all processes we consider.

The total power suppression of the N-jet operator is then obtained from adding up the suppression factors in λ from each direction. For example, at $\mathcal{O}(\lambda^2)$, it is possible to either have a J_i^{X2} operator (with $X = A, B, C$) in one direction and J_i^{A0} operators in the remaining $N - 1$ directions, or two operators $J_i^{X1} J_j^{Y1}$, with $X, Y = A, B$, and J_i^{A0} operators in the remaining $N - 2$ directions.

The infrared divergences of N-jet processes at NLP follow from the ultraviolet divergences of the matrix elements of the above operators computed with the SCET Lagrangian including NLP interactions. For the derivation of the anomalous dimension and renormalization group equation it is convenient to adopt the interaction picture and treat the subleading SCET Lagrangian as an interaction, such that all operator matrix elements are understood to be evaluated with the leading-power SCET Lagrangian. The basis of subleading power N-jet operators at a given order in λ then includes further "non-local" operators from the time-ordered products of the current operators J at lower order in λ

[5] Equivalently, one could assume $n_{i+}p > 0$ for ingoing momenta. It is possible to translate between both cases by flipping the signs $n_{i+} \to -n_{i+}$ and $n_{i-} \to -n_{i-}$ of all directions. This sign change can be compensated by substituting $t_{i_k} \to -t_{i_k}$, such that the form of the building blocks in position space is unchanged. The only difference is then the sign in the exponents in eq. (2.19), such that in collinear momentum space $P_i > 0$ for ingoing momenta in that case. We do not consider here the situation where some momenta are ingoing and others are outgoing.

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with the subleading terms in the SCET Lagrangian. The "local" (in reality, light-cone) currents do not mix into the non-local time-ordered product operators, but the latter can, in principle, mix into the former. The non-local operators mix into themselves but the corresponding matrix of renormalization factors is given by the one for the local currents of lower order in λ contained in the time-ordered product. The absence of further renormalization from the subleading soft-collinear interactions in the time-ordered product follows from the non-renormalization of the SCET Lagrangian to all orders in the strong coupling constant at any order in λ [13].

At $\mathcal{O}(\lambda)$ the time-ordered product operators are of the form

$$J_i^{T1}(t_i) = i\int d^4x\, T\left\{J_i^{A0}(t_i), \mathcal{L}_i^{(1)}(x)\right\}\,, \tag{2.21}$$

where $\mathcal{L}_i^{(1)} = \mathcal{L}_\xi^{(1)} + \mathcal{L}_{\xi q}^{(1)} + \mathcal{L}_{\rm YM}^{(1)}$ refers to the $\mathcal{O}(\lambda)$ suppressed terms in the SCET Lagrangian given in ref. [14]. It is understood that the collinear fields in these terms are those of direction i. The generalization to $\mathcal{O}(\lambda^2)$ should be evident.

In the following, we will focus on the case in which one of the collinear directions carries fermion-number $F = 2$. The simplification of this choice results from the absence of a leading-power operator J_i^{A0} (and consequently all J_i^{An}), since one needs two fermion fields in the same direction to begin with. Nevertheless, this simpler case allows us to display most of the features of the anomalous dimension at $\mathcal{O}(\lambda^2)$. The $F = 2$ operator basis at $\mathcal{O}(\lambda)$ consists of the single collinear operator

$$J^{B1}_{\chi_\alpha\chi_\beta}(t_{i_1}, t_{i_2}) = \chi_{i\alpha}(t_{i_1}n_{i+})\chi_{i\beta}(t_{i_2}n_{i+})\,. \tag{2.22}$$

We keep open the Dirac spinor indices α, β, because they will in general be contracted with components of the N-jet operator from the other collinear directions $j \neq i$. The same rule applies to Lorentz and colour indices, and we only assume that the total N-jet operator transforms as a colour singlet. At $\mathcal{O}(\lambda^2)$, we have

$$\begin{aligned}
J^{B2}_{\chi_\alpha\partial^\mu\chi_\beta}(t_{i_1}, t_{i_2}) &= \chi_{i\alpha}(t_{i_1}n_{i+})i\partial^\mu_{\perp i}\chi_{i\beta}(t_{i_2}n_{i+})\,,\\
J^{B2}_{\partial^\mu(\chi_\alpha\chi_\beta)}(t_{i_1}, t_{i_2}) &= i\partial^\mu_{\perp i}J^{B1}_{\chi_\alpha\chi_\beta}(t_{i_1}, t_{i_2})\,,\\
J^{C2}_{\mathcal{A}^\mu\chi_\alpha\chi_\beta}(t_{i_1}, t_{i_2}, t_{i_3}) &= \mathcal{A}^\mu_{\perp i}(t_{i_1}n_{i+})\chi_{i\alpha}(t_{i_2}n_{i+})\chi_{i\beta}(t_{i_3}n_{i+})\,.
\end{aligned} \tag{2.23}$$

We will omit the Dirac indices in the following for brevity and drop the direction index i in the notation for the operator unless ambiguities can arise. The time-ordered product operators at $\mathcal{O}\left(\lambda^2\right)$ are

$$\begin{aligned}
J^{T2}_{\chi\chi,\xi}(t_{i_1}, t_{i_2}) &= i\int d^4x\, T\left\{J^{B1}_{\chi\chi}(t_{i_1}, t_{i_2}), \mathcal{L}^{(1)}_\xi(x)\right\},\\
J^{T2}_{\chi\chi,\xi q}(t_{i_1}, t_{i_2}) &= i\int d^4x\, T\left\{J^{B1}_{\chi\chi}(t_{i_1}, t_{i_2}), \mathcal{L}^{(1)}_{\xi q}(x)\right\},\\
J^{T2}_{\chi\chi,\rm YM}(t_{i_1}, t_{i_2}) &= i\int d^4x\, T\left\{J^{B1}_{\chi\chi}(t_{i_1}, t_{i_2}), \mathcal{L}^{(1)}_{\rm YM}(x)\right\}.
\end{aligned} \tag{2.24}$$

The inclusion of these operators guarantees that the anomalous dimension matrix does not mix operators with different λ scaling. Note that in contrast to the local current operators, the time-ordered products always contain the soft fields.

3 Anomalous dimension

3.1 General structure

The operator renormalization in renormalized perturbation theory is given by

$$\langle \mathcal{O}_P(\{\phi_{\rm ren}\},\{g_{\rm ren}\})\rangle_{\rm ren} = \sum_Q Z_{PQ} \prod_{\phi\in Q} Z_\phi^{1/2} \prod_{g\in Q} Z_g \langle \mathcal{O}_{Q,{\rm bare}}(\{\phi_{\rm ren}\},\{g_{\rm ren}\})\rangle\,, \tag{3.1}$$

where P,Q label the N-jet operators as well as time-ordered products of N-jet operators with insertions of power-suppressed interactions $\mathcal{L}_{\rm SCET}$. The products run over all fields and couplings that enter $\langle \mathcal{O}_Q\rangle$, respectively. We omit the argument in the following for brevity. At one-loop, writing $Z_{PQ} = \delta_{PQ} + \delta Z_{PQ}$ and demanding that the left-hand side is finite implies

$$\text{finite} = \langle \mathcal{O}_{P,{\rm bare}}\rangle_{\text{1-loop}} + \sum_Q \left[\delta Z_{PQ} + \delta_{PQ}\left(\frac{1}{2}\sum_{\phi\in P}\delta Z_\phi + \sum_{g\in P}\delta Z_g\right)\right] \langle \mathcal{O}_{Q,{\rm bare}}\rangle_{\rm tree}\,. \tag{3.2}$$

For the operator basis we are interested in we need to consider also the continuous operator label $x=\{x_{i_k}\}$, and generalize the anomalous dimension to include integrations as well as summation over different types of operators

$$\begin{aligned}\text{finite} = {}& \langle J_P(x)\rangle_{\text{1-loop}} \\ &+ \sum_Q \int dy \left[\delta Z_{PQ}(x,y) + \delta_{PQ}\delta(x-y)\left(\frac{1}{2}\sum_{\phi\in P}\delta Z_\phi + \sum_{g\in P}\delta Z_g\right)\right]\langle J_Q(y)\rangle_{\rm tree}\,,\end{aligned} \tag{3.3}$$

where $\delta(x-y) \equiv \prod_i \prod_{k=2}^{n_i} \delta(x_{i_k} - y_{i_k})$ and accordingly $Z_{PQ}(x,y) = \delta_{PQ}\delta(x-y) + \delta Z_{PQ}(x,y)$. Note that for n_i collinear building blocks in one direction we need $n_i - 1$ integrals, because $\sum_k x_{i_k} = 1$. If there is only a single building block for a given direction i, then $x_{i_1} = 1$, and no integration over momentum fractions occurs. We use the convention that empty products are unity, so that the above equation covers also this case.

As discussed below, the soft loops within a single collinear direction vanish. Therefore, we split the renormalization constant to soft and collinear contributions via

$$\delta Z_{PQ}(x,y) = \sum_{i\neq j} \delta(x-y)\delta Z^{s,ij}_{PQ}(x) + \sum_i \delta^{[i]}(x-y)\delta Z^{c,i}_{PQ}(x,y)\,, \tag{3.4}$$

where we have used the fact that the soft loops are diagonal in x. The collinear loop along direction i is diagonal in the x_{j_k} for $j\neq i$, which is reflected by $\delta^{[i]}(x-y) \equiv \prod_{j\neq i}\prod_{k>1}\delta(x_{j_k} - y_{j_k})$. This gives the $\overline{\rm MS}$ scheme renormalization conditions

$$0 = \langle J_P(x)\rangle^{\text{soft,ij}}_{\text{1-loop, div.}} + \sum_Q \delta Z^{s,ij}_{PQ}(x)\langle J_Q(x)\rangle_{\rm tree}\,, \tag{3.5}$$

$$\begin{aligned}0 = {}& \langle J_P(x)\rangle^{\text{coll.,i}}_{\text{1-loop, div.}} + \sum_Q \int \prod_{k>1} dy_{i_k} \Bigg[\delta Z^{c,i}_{PQ}(x,y) \\ &+ \delta_{PQ}\prod_{k>1}\delta(x_{i_k} - y_{i_k})\left(\frac{1}{2}\sum_{\phi\in J_{Pi}}\delta Z_\phi + \sum_{g\in J_{Pi}}\delta Z_g\right)\Bigg]\langle J_Q(y)\rangle_{\rm tree}\,,\end{aligned} \tag{3.6}$$

where in the collinear part $x_{j_k} = y_{j_k}$ for $j \neq i$. In the last line we include only those field- and coupling renormalization factors that are associated to collinear building blocks of the direction i (that is, $\frac{1}{2}\delta Z_\chi = -\frac{\alpha_s C_F}{8\pi\epsilon}$ for each collinear fermion, and $\frac{1}{2}\delta Z_A + \delta Z_{g_s} = -\frac{\alpha_s C_A}{4\pi\epsilon}$ for each collinear gluon). We also use the notation $Z^{c,i}_{PQ}(x,y) = \delta_{PQ}\prod_{k>1}\delta(x_{i_k} - y_{i_k}) + \delta Z^{c,i}_{PQ}(x,y)$.

The anomalous dimension matrix is defined by

$$\mathbf{\Gamma} = -\mathbf{Z}^{-1}\frac{d}{d\ln\mu}\mathbf{Z}\,, \tag{3.7}$$

where we use matrix notation involving both discrete indices (P,Q) labelling the set of N-jet operators including open Lorentz, spinor and colour indices as well as continuous indices (x,y) for the collinear momentum fractions associated to each building block.

Before we proceed to discuss the details of each contribution, let us make a technical remark about the extraction of ultraviolet (UV) divergences. To compute the anomalous dimension we need to separate the UV and infrared (IR) poles of the amplitude. In our computation of the soft and collinear contributions we assume that the external states have small off-shellness $p^2_{i_k} \neq 0$. This choice regularizes the IR divergences of the amplitude and guarantees that all the $1/\epsilon^n$ divergences are related to UV poles of the SCET amplitude. At the end of the computation, the soft and collinear part are combined and only then the limit $p^2_{i_k} \to 0$ can be taken. The cancellation of the off-shell regulator dependence serves as an additional check of our computation.

3.2 Collinear part

The collinear contribution to the anomalous dimension can be extracted by computing one-loop matrix elements with a collinear loop. These loops do not contain soft fields, and therefore it is sufficient to concentrate on purely collinear interactions. In principle, there could be collinear one-loop diagrams with external soft gluons generated by the insertion of a power-suppressed Lagrangian interaction. The divergent part of any such diagram would correspond to the mixing of one of the time-ordered product operators into a current operator with a soft field. However, as shown in the previous section there are no such operators at $\mathcal{O}(\lambda^2)$ that cannot be removed by the field equations. It is therefore sufficient to focus on collinear loop amplitudes with external collinear lines only.

Since each collinear sector is interacting only with itself, collinear contributions factorize into individual contributions from each of the collinear directions n_{i+}, $i = 1,\ldots,N$, respectively. Therefore, it is sufficient to consider only the contribution J_i to the N-jet operator that contains collinear fields along the n_{i+}-direction, while the other contributions $J_{j\neq i}$ are irrelevant. Moreover, in the position-space SCET formulation there are no purely collinear power-suppressed interactions, so the power counting of the collinear loop is determined solely by the operator. We first consider the case of an $\mathcal{O}(\lambda)$ power suppressed operator J_i, and then turn to the more involved case of $\mathcal{O}(\lambda^2)$, where operator mixing occurs. We will often omit the label i of the collinear quantities in this section for brevity, since only a single collinear direction is involved.

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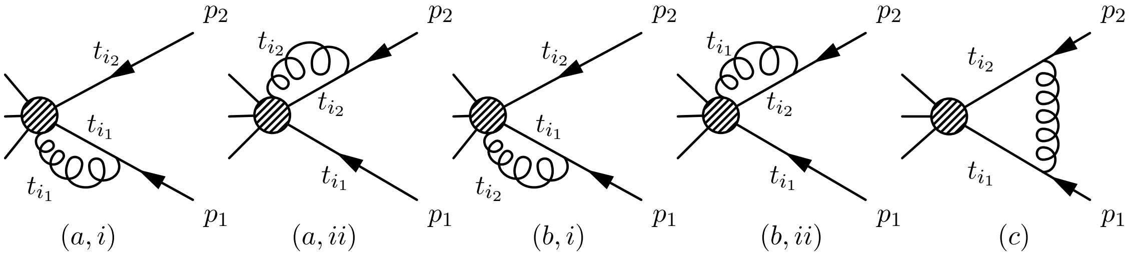

Figure 1. Collinear loops contributing to the anomalous dimension for two fermionic building blocks in the i-th direction. Arrows show the fermion flow for two outgoing antiquarks.

3.2.1 Order $\mathcal{O}(\lambda)$

In order to extract the anomalous dimension, we consider the matrix element of $J^{B1}_{\chi\chi}$ defined in eq. (2.22) with two external fermions with external momenta p_1 and p_2. To be specific, we take the two fermions to be distinguishable by their flavours, which we do not show explicitly. The extension to identical particles will be discussed below eq. (3.34). We show the collinear one-loop diagrams in figure 1. The labels t_{i_1} and t_{i_2} indicate whether the corresponding line is attached to the first or second building block of $J^{B1}_{\chi\chi}$.

For the first two diagrams, all internal lines contributing to the collinear loop are attached to a single building block. In the following, we refer to these contributions as type-(a) loops. Since effectively only a single building block is involved, type-(a) loops can be inferred from the leading-power result. In particular, collecting the sum of the two type-(a) one-loop diagrams, the tree-level diagram, and the contributions from wave-function renormalization from the right-hand side of eq. (3.6) for the two external building blocks of $J^{B1}_{\chi\chi}$ in a matrix element labelled with subscript (a), we find

$$\langle \bar{q}(p_1)\bar{q}(p_2)|J^{B1}_{\chi\chi}(t_{i_1},t_{i_2})|0\rangle_{(a)} = J_q(p_1^2)J_q(p_2^2)\langle \bar{q}(p_1)\bar{q}(p_2)|J^{B1}_{\chi\chi}(t_{i_1},t_{i_2})|0\rangle_{\text{tree}}\,, \tag{3.8}$$

where

$$J_q(p^2) = 1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-p^2}\right) + \frac{3}{2\epsilon}\right] + \mathcal{O}(\epsilon^0) \tag{3.9}$$

is the leading-power collinear contribution from a single fermionic building block [30, 33].

The third and fourth diagram in figure 1 appear similar to the first and second one at first sight, but differ in an important respect. Namely, the two internal lines of the collinear loop are attached to two *different* building blocks of $J^{B1}_{\chi\chi}$. As a consequence, the fractions of collinear momenta of the two lines attached to the operator will in general be different from the momentum fractions of the external lines. We consider the operator $J^{B1}_{\chi\chi}(x)$ in Fourier space with respect to the collinear direction, where x denotes the momentum fraction associated to the first building block, and correspondingly $\bar{x} = 1-x$ for the second building block. For the external momenta, we label the collinear momentum fractions by $y = n_{i+}p_1/(n_{i+}p_1 + n_{i+}p_2) = n_{i+}p_1/P$ and $\bar{y} = 1-y$. In this notation, the tree-level diagram is given by

$$\langle \bar{q}(p_1)\bar{q}(p_2)|J^{B1}_{\chi_\alpha\chi_\beta}(y)|0\rangle_{\text{tree}} = -P^2\delta(yP - n_+p_1)\delta(\bar{y}P - n_+p_2)v_\alpha(p_1)v_\beta(p_2)\,, \tag{3.10}$$

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where $v_\alpha(p)$ is the collinear spinor for the outgoing antiquark with momentum p and spinor index α. In order to compute the one-loop matrix element in collinear momentum space, we express loop integrals $d^d l = \frac{1}{2} dn_+ l \, dn_- l \, d^{d-2} l_\perp$ in light-cone coordinates, and first perform the $n_- l$ integration by closing the contour either in the upper or lower complex plane. Then the integration over $l_\perp$ can be performed by standard techniques, while the integration over $n_+ l$ is trivial and set by the fixed value of the momentum fraction x in collinear momentum space. Finally, we express the result in terms of the tree-level matrix element by first renaming $y \to y'$, inserting $1 = \int dy \delta(y - y')$, and using

$$\delta(yP - n_+ p_1)\delta(\bar{y}P - n_+ p_2) = \frac{1}{P}\delta(P - n_+(p_1 + p_2))\delta(y - n_+ p_1/P)\,. \tag{3.11}$$

For example, in position space we find for the contribution from diagram (b, i)

$$\begin{aligned}
\langle \bar{q}(p_1)\bar{q}(p_2)|J^{B1}_{\chi_\alpha\chi_\beta}(t_{i_1}, t_{i_2})|0\rangle_{(b,i)} &= \tilde{\mu}^{4-d}\int \frac{d^d l}{(2\pi)^d}\, e^{i(t_{i_1} n_+(p_1 - l) + t_{i_2} n_+(p_2 + l))} \\
&\times \left[\frac{i n_+(l - p_1)\frac{\not{n}_-}{2}}{(l - p_1)^2} i g_s n_-^\mu \frac{\not{n}_+}{2} t^a v_\alpha(p_1)\right] \frac{g_s n_{+\mu}}{n_+ l} t^a v_\beta(p_2)\, \frac{-i}{l^2} \\
&= \frac{\alpha_s e^{\gamma_E \epsilon}\Gamma(\epsilon)}{2\pi}\, [t^a v_\alpha(p_1)][t^a v_\beta(p_2)] \int_0^1 dz \left(\frac{\mu^2}{-p_1^2 z\bar{z}}\right)^\epsilon \frac{\bar{z}}{z}\, e^{i(t_{i_1}\bar{z} n_+ p_1 + t_{i_2}(n_+ p_2 + z n_+ p_1))}\,,
\end{aligned} \tag{3.12}$$

where $z = n_+ l / n_+ p_1$, $\bar{z} = 1 - z$, and l is the momentum of the gluon in the loop. Fourier transforming to collinear momentum space yields a delta function that allows to trivially evaluate the z integration. Following the steps described above we obtain

$$\begin{aligned}
&\langle \bar{q}(p_1)\bar{q}(p_2)|J^{B1}_{\chi_\alpha\chi_\beta}(x)|0\rangle_{(b,i)} \\
&= -\frac{\alpha_s e^{\gamma_E \epsilon}\Gamma(\epsilon)}{2\pi}\int_0^1 dy\, \theta(y - x)\left(\frac{\mu^2 y^2}{-p_1^2 x(y - x)}\right)^\epsilon \frac{x}{y(y - x)} \\
&\quad \times \mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}\langle \bar{q}(p_1)\bar{q}(p_2)|J^{B1}_{\chi_\alpha\chi_\beta}(y)|0\rangle_{\text{tree}} \\
&= -\frac{\alpha_s}{2\pi}\int_0^1 dy \left\{\frac{1}{\epsilon}\theta(y - x)\frac{x}{y(y - x)_+} - \delta(x - y)\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\ln\left(\frac{\mu^2 x}{-p_1^2 \bar{x}}\right)\right] + \mathcal{O}(\epsilon^0)\right\} \\
&\quad \times \mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}\langle \bar{q}(p_1)\bar{q}(p_2)|J^{B1}_{\chi_\alpha\chi_\beta}(y)|0\rangle_{\text{tree}}\,,
\end{aligned} \tag{3.13}$$

where we used colour-space operator notation for the generators, $[t^a v_\alpha(p_1)][t^a v_\beta(p_2)] \to \mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2} v_\alpha(p_1) v_\beta(p_2)$. Here $\mathbf{T}_{i_1}$ and $\mathbf{T}_{i_2}$ are understood to act on the fundamental colour index of the first and second building block of $J^{B1}_{\chi\chi}$, respectively. Diagram (b, ii) gives a similar result, that differs only by the replacement $x \leftrightarrow \bar{x}$, $y \leftrightarrow \bar{y}$ and $p_1^2 \leftrightarrow p_2^2$ outside of the matrix elements. For diagram (c) we find

$$\begin{aligned}
\langle \bar{q}(p_1)\bar{q}(p_2)|J^{B1}_{\chi_\alpha\chi_\beta}(x)|0\rangle_{(c)} &= -\frac{\alpha_s \mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{8\pi\epsilon}\int dy\left(\theta(x - y)\frac{\bar{x}}{\bar{y}} + \theta(y - x)\frac{x}{y}\right) \\
&\times \left(\gamma_\perp^\nu\gamma_\perp^\mu\right)_{\alpha\gamma}\left(\gamma_{\perp\nu}\gamma_{\perp\mu}\right)_{\beta\delta}\langle \bar{q}(p_1)\bar{q}(p_2)|J^{B1}_{\chi_\gamma\chi_\delta}(y)|0\rangle_{\text{tree}}\,.
\end{aligned} \tag{3.14}$$

Note that this contribution induces a spin-dependent structure, i.e. it is non-diagonal in Dirac indices. Collecting all results, we can read off the collinear contribution to the anomalous dimension using eq. (3.6). It has a diagonal part $\propto \delta(x - y)$ in collinear momentum

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space, and a non-diagonal part. Using (3.9) and $\mathbf{T}_{i_k}^2 = C_F$ for quarks, we can write the anomalous dimension in the form

$$\delta Z^{c,i}_{\chi_\alpha\chi_\beta,\chi_\gamma\chi_\delta}(x,y) = -\delta(x-y)\delta_{\alpha\gamma}\delta_{\beta\delta}X_{i_1i_2} + \frac{1}{\epsilon}\,\gamma^i_{\chi_\alpha\chi_\beta,\chi_\gamma\chi_\delta}(x,y)\,, \tag{3.15}$$

with

$$\begin{aligned} X_{i_1i_2} \equiv \frac{\alpha_s}{4\pi}\Bigg\{&\frac{2}{\epsilon^2}(\mathbf{T}_{i_1}+\mathbf{T}_{i_2})^2 + \frac{2}{\epsilon}(\mathbf{T}_{i_1}+\mathbf{T}_{i_2})\cdot\left[\mathbf{T}_{i_1}\ln\left(\frac{\mu^2}{-p_1^2}\right)\right. \\ &\left.+\mathbf{T}_{i_2}\ln\left(\frac{\mu^2}{-p_2^2}\right)\right] + \frac{1}{\epsilon}\left(\mathbf{T}^2_{i_1}c_{i_1} + \mathbf{T}^2_{i_2}c_{i_2}\right)\Bigg\}\,, \end{aligned} \tag{3.16}$$

and

$$\begin{aligned} \gamma^i_{\chi_\alpha\chi_\beta,\chi_\gamma\chi_\delta}(x,y) = \frac{\alpha_s\mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{2\pi}\Bigg\{&\delta_{\alpha\gamma}\delta_{\beta\delta}\Bigg(\theta(x-y)\left[\frac{1}{x-y}\right]_+ + \theta(y-x)\left[\frac{1}{y-x}\right]_+ \\ &-\theta(x-y)\frac{1-\frac{\bar{x}}{2}}{\bar{y}} - \theta(y-x)\frac{1-\frac{x}{2}}{y}\Bigg) \\ &-\frac{1}{4}\left(\sigma_\perp^{\nu\mu}\right)_{\alpha\gamma}\left(\sigma_{\perp\nu\mu}\right)_{\beta\delta}\left(\theta(x-y)\frac{\bar{x}}{\bar{y}} + \theta(y-x)\frac{x}{y}\right)\Bigg\}\,. \end{aligned} \tag{3.17}$$

Here we also expressed the Dirac gamma matrices in terms of $\sigma_\perp^{\mu\nu} \equiv \frac{i}{2}[\gamma_\perp^\mu, \gamma_\perp^\nu]$. Note that the contributions from wave-function renormalization in eq. (3.6) were already included in eq. (3.8), and are thus contained in the diagonal part, with $c_{i_1} = c_{i_2} = 3/2$ for quarks.

As mentioned above, in this work we restrict the discussion to the case of two-fermion operators. Detailed results for all possible contributions to the N-jet operator will be presented in a forthcoming paper.

3.2.2 Order $\mathcal{O}(\lambda^2)$

At $\mathcal{O}(\lambda^2)$ the three operators in eq. (2.23) contribute, and the anomalous dimension is correspondingly given by a 3×3 block matrix. We find the following structure at one-loop, that we will derive below:

$$\delta Z^c_{PQ} = \begin{array}{c||cc|c} & J^{B2}_{\chi\partial\chi} & J^{B2}_{\partial(\chi\chi)} & J^{C2}_{\mathcal{A}\chi\chi} \\ \hline J^{B2}_{\chi\partial\chi} & (3.19) & (3.20) & (3.23) \\ J^{B2}_{\partial(\chi\chi)} & 0 & (3.25) & 0 \\ \hline J^{C2}_{\mathcal{A}\chi\chi} & 0 & 0 & (3.29) \end{array} \tag{3.18}$$

The equation numbers point to the results for the non-zero entries. Note that the operators J_i^{T2} containing insertions of the power-suppressed SCET Lagrangian contain at least one soft field and therefore do not contribute in the purely collinear sector. We first discuss the first row $\delta Z^{c,i}_{\chi\partial\chi,Q}$, then the second row $\delta Z^{c,i}_{\partial(\chi\chi),Q}$, and finally the last row $\delta Z^{c,i}_{\mathcal{A}\chi\chi,Q}$, where $Q \in \{\chi\partial\chi, \partial(\chi\chi), \mathcal{A}\chi\chi\}$. The zero entries in the second row persist at higher orders in α_s (see below).

First row. The contributions $\delta Z^{c,i}_{\chi\partial\chi,\chi\partial\chi}$ and $\delta Z^{c,i}_{\chi\partial\chi,\partial(\chi\chi)}$ can be extracted by computing the matrix element $\langle\bar{q}(p_1)\bar{q}(p_2)|J^{B2}_{\chi\partial\chi}(x)|0\rangle$ at one-loop, involving diagrams as in figure 1. The additional $\partial_\perp$ derivative leads to an extra power of the loop momentum in the numerator, which yields a more involved structure of divergences compared to $\mathcal{O}(\lambda)$. The divergent part can be expressed in terms of the two tree-level contributions $\langle\bar{q}(p_1)\bar{q}(p_2)|J^{B2}_{\chi\partial\chi}(y)|0\rangle_{\rm tree}$ and $\langle\bar{q}(p_1)\bar{q}(p_2)|J^{B2}_{\partial(\chi\chi)}(y)|0\rangle_{\rm tree}$. The coefficients yield the corresponding anomalous dimensions, and we find

$$\delta Z^{c,i}_{\chi_\alpha\partial^\mu\chi_\beta,\chi_{\alpha'}\partial^\sigma\chi_{\beta'}}(x,y) = -\delta(x-y)\delta_{\alpha\alpha'}\delta_{\beta\beta'}\,g^{\mu\sigma}_\perp X_{i_1i_2} + \frac{1}{\epsilon}\,\gamma^i_{\chi_\alpha\partial^\mu\chi_\beta,\chi_{\alpha'}\partial^\sigma\chi_{\beta'}}(x,y)\,, \tag{3.19}$$

$$\delta Z^{c,i}_{\chi_\alpha\partial^\mu\chi_\beta,\partial^\sigma(\chi_{\alpha'}\chi_{\beta'})}(x,y) = \frac{1}{\epsilon}\,\gamma^i_{\chi_\alpha\partial^\mu\chi_\beta,\partial^\sigma(\chi_{\alpha'}\chi_{\beta'})}(x,y)\,, \tag{3.20}$$

with

$$\begin{aligned}
&\gamma^i_{\chi_\alpha\partial^\mu\chi_\beta,\chi_{\alpha'}\partial^\sigma\chi_{\beta'}}(x,y)\\
&\quad= \frac{\alpha_s\mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{2\pi}\Bigg\{\delta_{\alpha\alpha'}\delta_{\beta\beta'}g^{\mu\sigma}_\perp\Bigg(\theta(x-y)\left[\frac{1}{x-y}\right]_+ + \theta(y-x)\left[\frac{1}{y-x}\right]_+\\
&\qquad -\theta(x-y)\frac{\bar{x}+\bar{y}}{\bar{y}^2} - \theta(y-x)\frac{x+y}{y^2}\Bigg) + \frac{1}{4}M_{\chi_\alpha\partial^\mu\chi_\beta,\chi_{\alpha'}\partial^\sigma\chi_{\beta'}}(x,y)\Bigg\}\,,\\
&\gamma^i_{\chi_\alpha\partial^\mu\chi_\beta,\partial^\sigma(\chi_{\alpha'}\chi_{\beta'})}(x,y)\\
&\quad= \frac{\alpha_s\mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{2\pi}\left(\delta_{\alpha\alpha'}\delta_{\beta\beta'}g^{\mu\sigma}_\perp\theta(y-x)\frac{x}{y^2} + \frac{1}{4}M_{\chi_\alpha\partial^\mu\chi_\beta,\partial^\sigma(\chi_{\alpha'}\chi_{\beta'})}(x,y)\right)\,.
\end{aligned} \tag{3.21}$$

The last terms in each expression arise from diagram (c) and are given in appendix C.

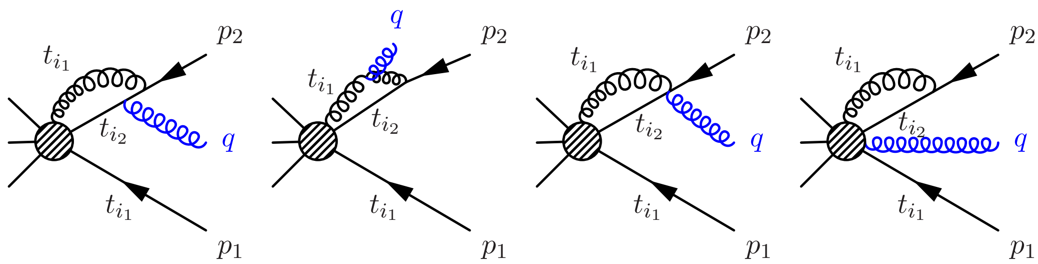
Let us now turn to $\delta Z^{c,i}_{\chi\partial\chi,\mathcal{A}\chi\chi}$, which describes the mixing of B- into C-type operators. To extract this contribution we compute the matrix element $\langle g(q)\bar{q}(p_1)\bar{q}(p_2)|J^{B2}_{\chi\partial^\mu\chi}|0\rangle$ at one-loop involving a gluon and two antiquarks. To determine the mixing with $J^{C2}_{\mathcal{A}\chi\chi}$ it is sufficient to consider a configuration where the gluon has only $\perp$ polarization, and the external momenta of all particles have vanishing $\perp$ component.

For loops that consist of two internal lines that are both attached to the same building block of the operator $J^{B2}_{\chi\partial^\mu\chi}$ (called type-(a) loops above) one of the collinear building blocks, that is not contributing to the loop, acts as a 'spectator', i.e. the matrix element factorizes,

$$\begin{aligned}
&\langle g_a(q)\bar{q}(p_1)\bar{q}(p_2)|J^{B2}_{\chi^\alpha\partial^\mu\chi^\beta}|0\rangle_{(a)}\\
&\quad= \langle g_a(q)\bar{q}(p_1)|\chi(t_{i_1}n_+)|0\rangle_{(a)}(-p^\mu_{2\perp})\langle\bar{q}(p_2)|\chi(t_{i_2}n_+)|0\rangle_{\rm tree}\\
&\qquad+\langle\bar{q}(p_1)|\chi(t_{i_1}n_+)|0\rangle_{\rm tree}(-p_2-q)^\mu_\perp\langle g_a(q)\bar{q}(p_2)|\chi(t_{i_2}n_+)|0\rangle_{(a)}\,,
\end{aligned} \tag{3.22}$$

where we have also used that the $\perp$ derivative acting on the second building block gives a simple factor of total momentum both at the tree- and loop-level. All contributions of type-(a) are therefore zero for vanishing external $\perp$ momenta.

Therefore, we can focus on loops that connect the two building blocks. They are obtained from the one-loop diagrams for a quark-quark matrix element shown in figure 1



$(b,i)_F$ $(b,i)_B$ $(b,i)_V$ $(b,i)_J$

Figure 2. Examples for the four possibilities of adding an extra collinear emission (indicated by the blue line) to a diagram with two fermion lines (chosen to be diagram (b,i) from figure 1. for illustration).

(specifically from diagrams (b,i), (b,ii) and (c)) with the additional emission of the gluon off either an internal fermion line (subscript F), internal boson (i.e. gluon) line (B), vertex (V), or directly from the operator (J). These four possibilities are illustrated in figure 2 for diagram (b,i). The case (J) is only possible if the gluon is attached to a Wilson line, and therefore this contribution vanishes for $\perp$ polarization. Analogous arguments hold for (b,ii) and (c). Similarly, the contributions $(b,i)_V$, $(b,ii)_V$ are zero, because the internal gluon is in this case connected to a Wilson line, and the four-point vertex connecting two collinear gluons and two collinear quarks vanishes when contracted with n_+^μ. Finally, there could be a contribution from one-particle reducible (1PR) diagrams for which the 1PR propagator is canceled by a corresponding momentum-squared suppression of the loop diagram. However, it turns out that there are no such contributions because the vertex for radiating off a $\perp$ polarized gluon from a quark line with momentum p vanishes for $p_\perp = 0$. In summary, all relevant loop diagrams are shown in figure 3.

The computation of the one-loop diagrams is straightforward and we find the result

$$\begin{aligned}
&\delta Z^{c,i}_{\chi^s_\alpha \partial^\mu \chi^t_\beta, \mathcal{A}^{\nu a} \chi^k_{\alpha'} \chi^l_{\beta'}}(x,y_1,y_2) \\
&= \frac{\alpha_s}{8\pi\epsilon}\Big\{ -i f^{abc} t^c_{sk} t^b_{tl} K^{\mu\nu}_{1,\alpha\alpha'\beta\beta'}(x,y_2,y_3) \\
&\quad + (t^a t^b)_{sk} t^b_{tl} K^{\mu\nu}_{2,\alpha\alpha'\beta\beta'}(x,y_1,y_2) - (t^a t^b)_{tl} t^b_{sk} K^{\mu\nu}_{2,\beta\beta'\alpha\alpha'}(\bar{x},y_1,y_3)\Big\} \\
&= \frac{1}{\epsilon}\,\gamma^i_{\chi^s_\alpha \partial^\mu \chi^t_\beta, \mathcal{A}^{\nu a} \chi^k_{\alpha'} \chi^l_{\beta'}}(x,y_1,y_2)\,,
\end{aligned} \tag{3.23}$$

where we made explicit colour indices for clarity. The y_k denote the collinear momentum fractions for $J^{C2}_{\mathcal{A}\chi\chi}$ with $y_1+y_2+y_3=1$, and y_1 corresponds to the gluonic building block $\mathcal{A}$. The kernels K are defined in appendix C. In colour-space notation

$$\begin{aligned}
\gamma^i_{\chi_\alpha \partial^\mu \chi_\beta, \mathcal{A}^\nu \chi_{\alpha'} \chi_{\beta'}}(x,y_1,y_2) &= \frac{\alpha_s}{8\pi}\Big\{ \mathbf{T}_{i_1} \times \mathbf{T}_{i_2} K^{\mu\nu}_{1,\alpha\alpha'\beta\beta'}(x,y_2,y_3) \\
&- \mathbf{T}_{i_1}(\mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}) K^{\mu\nu}_{2,\alpha\alpha'\beta\beta'}(x,y_1,y_2) + \mathbf{T}_{i_2}(\mathbf{T}_{i_2}\cdot\mathbf{T}_{i_1}) K^{\mu\nu}_{2,\beta\beta'\alpha\alpha'}(\bar{x},y_1,y_3)\Big\}\,,
\end{aligned} \tag{3.24}$$

where we defined a cross product via $(\mathbf{T}_{i_1} \times \mathbf{T}_{i_2})^a \equiv i f^{abc} \mathbf{T}^b_{i_1} \mathbf{T}^c_{i_2}$, and the subscripts refer to the first and second fermionic building block, respectively. In addition, we leave implicit

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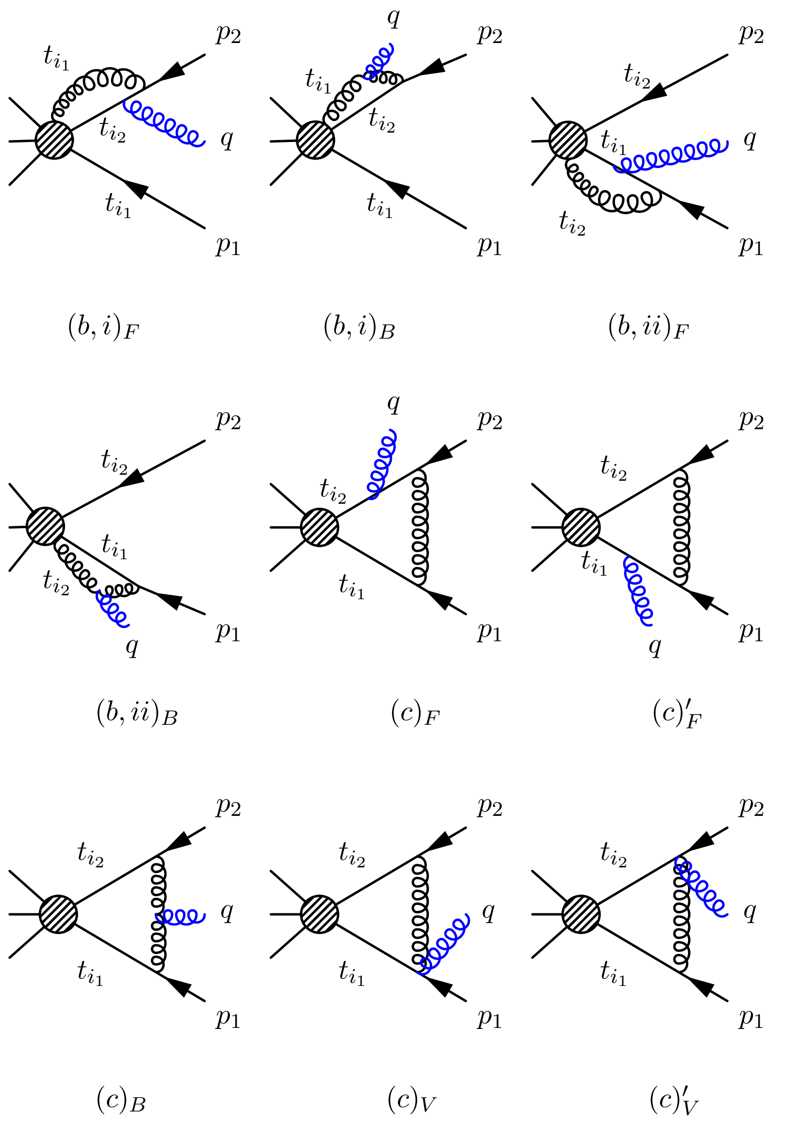

Figure 3. Collinear loops contributing to the anomalous dimension $\delta Z^{c,i}_{\chi\partial\chi,\mathcal{A}\chi\chi}$, that describes mixing of B- into C-type operators. Arrows show the fermion flow for two outgoing antiquarks.

the open adjoint index of the colour-space operators, which generates the additional colour label required for the gluonic building block of $\mathcal{A}\chi\chi$.

Second row. Matrix elements of the operator $J^{B2}_{\partial(\chi\chi)}$ can be trivially related to those of $J^{B2}_{\chi\chi}$, because the total derivative factors out of any loop diagram. Therefore, we can infer the corresponding entries in the anomalous dimension matrix from the $\mathcal{O}(\lambda)$ result,

$$\begin{aligned} \delta Z^{c,i}_{\partial^\mu(\chi_\alpha\chi_\beta),\partial^\nu(\chi_\gamma\chi_\delta)}(x,y) &= g_\perp^{\mu\nu}\delta Z^{c,i}_{\chi_\alpha\chi_\beta,\chi_\gamma\chi_\delta}(x,y), \\ \delta Z^{c,i}_{\partial(\chi\chi),Q} &= 0 \quad (Q = \chi\partial\chi, \mathcal{A}\chi\chi)\,. \end{aligned} \tag{3.25}$$

The last line follows from the equality

$$\langle g_a(q)\bar{q}(p_1)\bar{q}(p_2)|J^{B2}_{\partial^\mu(\chi\chi)}|0\rangle = -(q+p_1+p_2)^\mu_\perp\,\langle g_a(q)\bar{q}(p_1)\bar{q}(p_2)|J^{B1}_{\chi\chi}|0\rangle \tag{3.26}$$

at any loop order together with the first line of eq. (3.25). Since the $\mathcal{O}(\lambda)$ matrix element on the right-hand side is rendered finite by the $\delta Z^{c,i}_{\chi\chi,\chi\chi}$ counterterm, it is not necessary to introduce new counterterms to renormalize the left-hand side at $\mathcal{O}(\lambda^2)$. We have checked this explicitly by computing the left-hand side of eq. (3.26) at one loop.

Third row. For C-type operators with three collinear building blocks the one-loop anomalous dimension can be inferred from operators involving only two collinear building blocks. The reason is that at one-loop, at most two building blocks can be connected to the loop, while the third one acts as a spectator.

In particular, type-(a) loops operate on each building block separately, and therefore give the same result as at leading power (when including also coupling and wavefunction renormalization, as above)

$$\begin{aligned}&\langle g_a(p_1)\bar{q}(p_2)\bar{q}(p_3)|J^{C2}_{\mathcal{A}^\mu\chi\chi}(x_1,x_2)|0\rangle_{(a)}\\&= J_g(p_1^2)J_q(p_2^2)J_q(p_3^2)\langle g_a(p_1)\bar{q}(p_2)\bar{q}(p_3)|J^{C2}_{\mathcal{A}^\mu\chi\chi}(x_1,x_2)|0\rangle_{\rm tree}\,.\end{aligned}\tag{3.27}$$

The expression for $J_q(p^2)$ is given in eq. (3.9), and $J_g(p^2)$ is given by the same expression with $C_F\to C_A$, $3/(2\epsilon)\to 0$.

All other loops connect two building blocks. There are three possibilities to select a pair. For each pair, the computation is analogous to the corresponding case where the third collinear building block is absent. Therefore, we can obtain the anomalous dimension by rescaling the corresponding momentum fractions. For example, for the case where the loop connects the second and third building block (indicated by the subscript 23), the contribution to the anomalous dimension is related to the $\mathcal{O}(\lambda)$ result from eq. (3.15),

$$Z^{c,i}_{\mathcal{A}^\mu\chi\chi,\mathcal{A}^\rho\chi\chi}(x_1,x_2,y_1,y_2)\Big|_{23} = \frac{1}{1-y_1}\delta(x_1-y_1)g_\perp^{\mu\rho}Z^{c,i}_{\chi\chi,\chi\chi}(x,y)\,,\tag{3.28}$$

with $x = x_2/(x_2+x_3) = x_2/(1-x_1)$ and $y = y_2/(y_2+y_3) = y_2/(1-y_1)$. The momentum fractions in the first building block are not affected by the loop, and therefore identical, leading to the $\delta(x_1-y_1)$, and a similar argument applies to the Lorentz indices leading to $g_\perp^{\mu\rho}$. The prefactor is due to the Jacobian[6] $dy/dy_2 = 1/(1-y_1)$.

To obtain the full anomalous dimension we need to sum over the three pairs of collinear building blocks, 13, 23, 12. Note that the anomalous dimension on the right-hand side of eq. (3.28) captures also the contributions from type-(a) loops attached to either the second

[6]This can be seen by writing the corresponding delta functions in the tree-level matrix element in the form $\delta(y_1P-n_+p_1)\delta(y_2P-n_+p_2)\delta(y_3P-n_+p_3) = \delta(y_1P-n_+p_1)\delta(yP_{23}-n_+p_2)\delta(\bar{y}P_{23}-n_+p_3)$ where $P_{23}\equiv(1-y_1)P = (1-x_1)P$ is the collinear momentum of the two building blocks that are connected by the loop. Then the product $\delta(yP_{23}-n_+p_2)\delta(\bar{y}P_{23}-n_+p_3)$ has the same form as for the case with only two building blocks (except that $P\to P_{23}$). The remaining factor $\delta(y_1P-n_+p_1)$ is not affected by the loop integration, and therefore the same for the one-loop and tree-level matrix elements, leading to $\delta(x_1-y_1)$. Therefore the only re-scaling factor is the Jacobian obtained from the change of integration measure $Z^c_{\mathcal{A}^\mu\chi\chi,\mathcal{A}^\rho\chi\chi}(x_1,x_2,y_1,y_2)\big|_{23}\,dy_2dy_1 = \delta(x_1-y_1)g_\perp^{\mu\rho}dy_1\times Z^c_{\chi\chi,\chi\chi}(x,y)dy$. For example, the Jacobian ensures that the 'diagonal' contributions to $Z^c_{\chi\chi,\chi\chi}(x,y)$ have the correct normalization, because $\delta(x_1-y_1)\delta(x-y) = (1-y_1)\delta(x_2-y_2)\delta(x_1-y_1)$. Note also that the anomalous dimension does not explicitly depend on the total collinear momentum P in the direction n_{i+} under consideration.

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or the third building block. This will also be the case for the 23 and 12 contributions, such that the type-(a) loops are counted twice. We therefore need to subtract them once to obtain the correct result. In addition each term contains the tree-level contribution, which we need to subtract twice. Altogether,

$$\begin{aligned}
&Z^{c,i}_{\mathcal{A}^\mu\chi_\alpha\chi_\beta,\mathcal{A}^\nu\chi_{\alpha'}\chi_{\beta'}}(x_1,x_2,y_1,y_2)\\
&= \frac{1}{1-y_2}\delta(x_2-y_2)\delta_{\beta\beta'}Z^c_{\mathcal{A}^\mu\chi_\alpha,\mathcal{A}^\nu\chi_{\alpha'}}\left(\frac{x_1}{1-x_2},\frac{y_1}{1-y_2}\right)\\
&\quad+\frac{1}{1-y_1}\delta(x_1-y_1)g_\perp^{\mu\nu}Z^c_{\chi_\alpha\chi_\beta,\chi_{\alpha'}\chi_{\beta'}}\left(\frac{x_2}{1-x_1},\frac{y_2}{1-y_1}\right)\\
&\quad+\frac{1}{1-y_3}\delta(x_3-y_3)\delta_{\alpha\alpha'}Z^c_{\mathcal{A}^\mu\chi_\beta,\mathcal{A}^\nu\chi_{\beta'}}\left(\frac{x_1}{1-x_3},\frac{y_1}{1-y_3}\right)\\
&\quad-[1+J_g(p_1^2)^{-1}J_q(p_2^2)^{-1}J_q(p_3^2)^{-1}]\delta(x_1-y_1)\delta(x_2-y_2)\delta_{\alpha\alpha'}\delta_{\beta\beta'}g_\perp^{\mu\nu}\,. \qquad (3.29)
\end{aligned}$$

The last line contains the subtractions accounting for the over-counting (see Footnote 6 for the normalization). The anomalous dimension $Z^{c,i}_{\mathcal{A}\chi,\mathcal{A}\chi}$ is given in appendix C (see also refs. [15, 16]). Notice that the above equation is valid only up to one-loop. At higher loops, the three building blocks may be connected together. Eq. (3.29) can be brought into the form

$$\begin{aligned}
\delta Z^{c,i}_{\mathcal{A}^\mu\chi_\alpha\chi_\beta,\mathcal{A}^\nu\chi_{\alpha'}\chi_{\beta'}}(x_1,x_2,y_1,y_2) &= -\delta_{\alpha\alpha'}\delta_{\beta\beta'}g_\perp^{\mu\nu}\delta(x_1-y_1)\delta(x_2-y_2)X_{i_1i_2i_3}\\
&\quad+\frac{1}{\epsilon}\gamma^i_{\mathcal{A}^\mu\chi_\alpha\chi_\beta,\mathcal{A}^\nu\chi_{\alpha'}\chi_{\beta'}}\,, \qquad (3.30)
\end{aligned}$$

where

$$\begin{aligned}
X_{i_1i_2i_3} &= \frac{\alpha_s}{4\pi}\Bigg\{\frac{2}{\epsilon^2}(\mathbf{T}_{i_1}+\mathbf{T}_{i_2}+\mathbf{T}_{i_3})^2+\frac{2}{\epsilon}(\mathbf{T}_{i_1}+\mathbf{T}_{i_2}+\mathbf{T}_{i_3})\cdot\Bigg[\mathbf{T}_{i_1}\ln\left(\frac{\mu^2}{-p_1^2}\right)\\
&\quad+\mathbf{T}_{i_2}\ln\left(\frac{\mu^2}{-p_2^2}\right)+\mathbf{T}_{i_3}\ln\left(\frac{\mu^2}{-p_3^2}\right)\Bigg]+\frac{1}{\epsilon}\left(\mathbf{T}^2_{i_1}c_{i_1}+\mathbf{T}^2_{i_2}c_{i_2}+\mathbf{T}^2_{i_3}c_{i_3}\right)\Bigg\}
\end{aligned}$$

with $\mathbf{T}^2_{i_1}=C_A$ and $c_{i_1}=0$ for the gluonic building block and $\mathbf{T}^2_{i_2}=\mathbf{T}^2_{i_3}=C_F$, $c_{i_2}=c_{i_3}=3/2$ for the fermionic building blocks. The non-diagonal part is given by

$$\begin{aligned}
&\gamma^i_{\mathcal{A}^\mu\chi_\alpha\chi_\beta,\mathcal{A}^\nu\chi_{\alpha'}\chi_{\beta'}}(x_1,x_2,y_1,y_2)\\
&= \frac{1}{1-y_2}\delta(x_2-y_2)\delta_{\beta\beta'}\gamma^i_{\mathcal{A}^\mu\chi_\alpha,\mathcal{A}^\nu\chi_{\alpha'}}\left(\frac{x_1}{1-x_2},\frac{y_1}{1-y_2}\right)\\
&\quad+\frac{1}{1-y_1}\delta(x_1-y_1)g_\perp^{\mu\nu}\gamma^i_{\chi_\alpha\chi_\beta,\chi_{\alpha'}\chi_{\beta'}}\left(\frac{x_2}{1-x_1},\frac{y_2}{1-y_1}\right)\\
&\quad+\frac{1}{1-y_3}\delta(x_3-y_3)\delta_{\alpha\alpha'}\gamma^i_{\mathcal{A}^\mu\chi_\beta,\mathcal{A}^\nu\chi_{\beta'}}\left(\frac{x_1}{1-x_3},\frac{y_1}{1-y_3}\right). \qquad (3.31)
\end{aligned}$$

In addition, there is no mixing with operators with two building blocks, inherited from $\delta Z^{c,i}_{\mathcal{A}\chi,\partial\chi}=0$ at $\mathcal{O}(\lambda)$ (see appendix C), that is,

$$\delta Z^c_{\mathcal{A}\chi\chi,Q}=0 \quad (Q=\chi\partial\chi,\partial(\chi\chi))\,. \qquad (3.32)$$

3.2.3 General structure of the collinear anomalous dimension

The previous findings suggest a general structure for the collinear contributions to the anomalous dimension. We can write schematically for the contribution from collinear direction i with n_i building blocks ($n_i = 1, 2, 3$ for A-, B- ,C-type operators, respectively),

$$\delta Z^{c,i}_{PQ}(x, y) = -\delta_{PQ} \prod_k \delta(x_{i_k} - y_{i_k}) X_{i_1 \ldots i_{n_i}} + \frac{1}{\epsilon} \gamma^i_{PQ}(x, y) \,, \tag{3.33}$$

where the first term is the diagonal contribution, δ_{PQ} is non-zero for identical operators $P = Q$ and then stands for the product of $\delta_{\alpha\beta}$ for Dirac and $g^{\mu\nu}_{\perp}$ for Lorentz indices, x_{i_k} and y_{i_k} denote the collinear momentum fractions in direction i for the building blocks $k = 1, \ldots, n_i$, and $\gamma^i_{PQ}(x, y)$ encapsulates the non-diagonal contribution. Here x and y denote the vectors of momentum fractions as introduced above.

The non-diagonal contributions in general encapsulate rather lengthy results that depend on the Lorentz structure and on momentum fractions in a generic way. The diagonal contribution can be summarized in a universal way,

$$X_{i_1 \ldots i_{n_i}} = \frac{\alpha_s}{4\pi} \sum_{l,k=1}^{n_i} \mathbf{T}_{i_l} \cdot \mathbf{T}_{i_k} \left\{ \frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln \left(\frac{\mu^2}{-p^2_{i_k}} \right) + \delta_{lk} \frac{c_{i_k}}{\epsilon} \right\} , \tag{3.34}$$

where $c_{i_k} = 3/2$ for fermionic building blocks, and $c_{i_k} = 0$ for gluonic building blocks. For clarity we added an additional label to the off-shell regulator $p^2_{i_k}$ for the collinear direction it corresponds to.

So far we assumed that the two fermionic building blocks considered above have different flavours. It is straightforward to generalize the result in eq. (3.33) to the case of identical building blocks, which is relevant e.g. for quarks of identical flavour or when considering operators with more than one gluonic building block. For gluons (quarks), one has to symmetrize (anti-symmetrize) the anomalous dimension with respect to exchanging them (including a factor $1/N_s$ where N_s is the number of terms).[7] Moreover, if more than one $\perp$ derivative acts on the same building block at $O(\lambda^2)$, the corresponding Lorentz indices need to be symmetrized too.

The final result for the collinear contribution to the anomalous dimension is obtained by adding together the collinear contributions from all directions, which gives an additional sum over i,

$$\begin{aligned} \delta Z^c_{PQ}(x, y) &= \sum_{i=1}^{N} \delta^{[i]}(x - y) \delta Z^{c,i}_{PQ}(x, y) \\ &= -\delta_{PQ} \delta(x - y) \sum_i X_{i_1 \ldots i_{n_i}} + \sum_i \delta^{[i]}(x - y) \frac{\gamma^i_{PQ}(x, y)}{\epsilon} \,, \end{aligned} \tag{3.35}$$

where we used that $\delta^{[i]}(x - y) \prod_{k>1} \delta(x_{i_k} - y_{i_k}) = \delta(x - y)$ in the compact vector notation introduced above. This result is consistent with all individual results obtained above, for the fermion-number two case. We checked that the diagonal contributions are in accord with eq. (3.35) also for fermion-number one and zero up to $\mathcal{O}(\lambda^2)$.

[7] In this case, the association of the external momentum with the collinear building block is not unique; however, they appear then only in a symmetric form (e.g. $\ln(p^2_{i_1}) + \ln(p^2_{i_2})$) such that there is no ambiguity.

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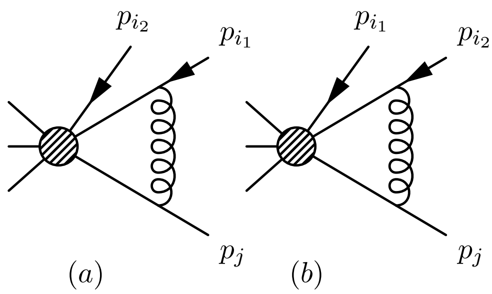

Figure 4. The leading power diagrams with a soft-gluon exchange. The j-direction parton is either a (anti)quark or a gluon created by either A0 or A1 current. In the two-fermion sector, the current can be either B1 or B2.

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3.3 Soft part

The soft fields mediate interactions between collinear fields in different directions. Here, we need to consider two types of contributions: first, soft loops with leading-power interactions, for which the power suppression arises purely from the N-jet operator, giving rise to current-current mixing. Second, soft loops containing insertions of the power-suppressed contributions to the SCET Lagrangian that describe subleading soft-collinear interactions. They give rise to operator mixing involving J_i^{T2} operators featuring time-ordered products, see eq. (2.24). This approach helps to keep the power-counting manifest and ensures that the anomalous dimension does not mix operators with different powers of λ. Because the leading two-fermion operator is $\mathcal{O}(\lambda)$, in this work we need to consider only a single insertion of the subleading interaction. The leading-power interaction between soft gluons and collinear particles can be used any number of times when constructing the amplitude.

3.3.1 Currents

For the current-current mixing, the soft loops within a single collinear sector vanish to all orders in α_s because the leading-power interaction contains only a single component of the soft field, $n_{i-}A_{\mathrm{s}}$. Hence, to determine the soft part of the anomalous dimension we only need to consider soft loops connecting different collinear sectors. At the one-loop level, only two different collinear directions can be connected by a soft loop. The result is then given as a sum of all possible pairings of fields belonging to different directions. For the two-fermion operator, the relevant diagrams are presented in figure 4. The parton belonging to the j direction can be either a (anti)quark or a gluon.

The divergent part of the diagrams shown in figure 4 with soft loops and leading power interaction is

$$\delta Z^{s,ij}_{PQ}(x) = -\delta_{PQ}\,\frac{\alpha_s}{4\pi}\sum_{l=1}^{n_i}\sum_{k=1}^{n_j}\frac{\mathbf{T}_{i_l}\cdot\mathbf{T}_{j_k}}{2}\left[\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\ln\left(\frac{-\mu^2 x_{i_l}x_{j_k}s_{ij}}{p^2_{i_l}p^2_{j_k}}\right)\right], \tag{3.36}$$

where $s_{ij}=\frac{1}{2}(n_{i-}\cdot n_{j-})P_iP_j$ depends only on the total collinear momentum in the directions connected by the soft loop.

The colour-space formalism reveals the universal form of the soft factor. The result in eq. (3.36) holds for gluons as well as for quarks. The soft factor depends only on the colour charge of the collinear particle but not on its spin. When there are identical building blocks within one collinear direction, the result should by symmetrized as in the collinear case.

The renormalization factor for the subleading currents with extra $\perp$ derivatives acting on the collinear fields is also given by eq. (3.36). In the soft-collinear vertices, only the n_- component of the momentum is conserved. The other components are conserved only within the collinear sector as dictated by the SCET multipole expansion of the soft fields. In the $\perp$ direction, the soft field wave-length is much larger than the size of typical fluctuations of the collinear field. As a result, the soft field is insensitive to the $\perp$ momentum of the collinear fields. Hence, the extra momentum factor in the N-jet operator Feynman rule that comes from the $\perp$ derivative does not affect the computation of the soft loop.

To summarize, the soft counterterm for the subleading local operators is universal, diagonal and given by eq. (3.36). This fact is easily understood by application of the soft decoupling transformation. The collinear fields can be redefined to remove the leading-power soft interactions from the SCET Lagrangian [12]. For example, for the fermion fields we define

$$\chi(n_{i+}t_{i_k}) = Y_i(0)\chi^{(0)}(n_{i+}t_{i_k}), \quad Y_i^\dagger(x) \equiv \mathbf{P}\exp\left[ig_s\int_0^\infty dsn_{i-}A_{\mathrm{s}}(x+n_{i-}s)\right]. \tag{3.37}$$

The fields building the N-jet operator are evaluated at $n_{i+}t_{i_k}$ so the decoupling transformation commutes with the derivative $\partial_{\perp i}$. The N-jet operator at $\mathcal{O}(\lambda)$ factorizes into a product of collinear fields $\chi^{(0)}$ that do not interact with the soft fields and a product of soft Wilson lines. Hence, the universality of the eq. (3.36) is a consequence of the standard eikonal approximation for the leading-power soft gluon coupling.

3.3.2 Time-ordered products

The decoupling transformation in eq. (3.37) does not remove the soft fields from the non-local time-ordered product operators. In this case, it is necessary to compute the soft loops explicitly. To obtain non-zero mixing into local operators we compute diagrams where the soft field from the Lagrangian insertion appears as an internal line. Non-zero mixing can occur only between operators with identical quantum numbers, and as the local currents do not contain soft fields, only these diagrams can induce mixing into local operators. Nevertheless, we checked that the one-loop amplitudes with one external soft gluon are indeed finite after combining the soft and collinear loop contributions.

Consider first the $J^{T2}_{\chi_\alpha\chi_\beta,\xi q}$ operator. Since there is no leading-power interaction between soft quarks and collinear partons it is impossible to form a soft loop and remove the soft quark field. Therefore, no mixing into any of the local operators is allowed for this operator.

The YM part in the operator $J^{T2}_{\chi_\alpha\chi_\beta,\mathrm{YM}}$ can form a non-vanishing Wick contraction with the gluon fields contained in the Wilson lines that accompany the quarks, or with the quarks directly. Choosing the light-cone gauge we immediately see that the former does not mix into any of the local operators. The latter also does not contribute after performing loop momentum integration due to the same reason as shown in the case of $\mathcal{L}_\xi^{(1)}$ below.

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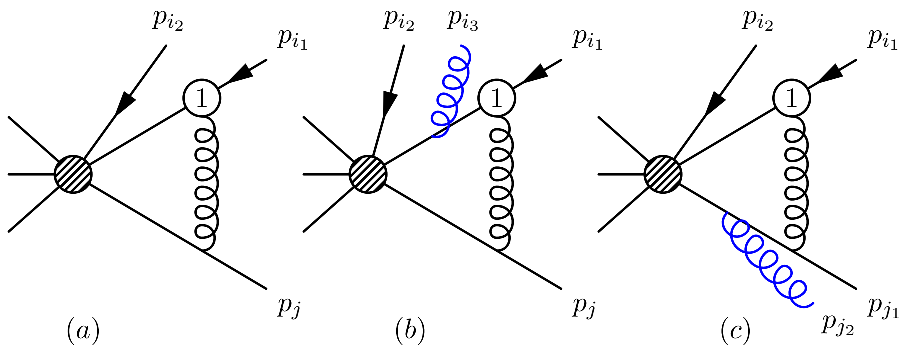

Figure 5. Sample diagrams contributing to mixing of time-ordered product into power-suppressed local operators. The circle denotes the $\mathcal{O}(\lambda)$ SCET Lagrangian insertion. Diagram (a) contributes to mixing into N-jet operator with B2-type currents; the diagram (b) can induce mixing into C2-type currents and the diagram (c) can generate mixing into an N-jet operator containing two different B1-type operators.

Finally, we investigate possible mixing of the time-ordered product containing $\mathcal{L}_\xi^{(1)}$. The $\mathcal{O}(\lambda)$ Lagrangian $\mathcal{L}_\xi^{(1)}$ contains interactions with the $\perp$ and n_- components of the soft field, thus it is not possible to form a contraction with the leading power soft-collinear interaction in the same collinear direction. Hence, just like in the case of local operators, the soft loops for the time-ordered product at $\mathcal{O}(\lambda)$ vanish within a single collinear sector. The soft loops connecting the time-ordered product with a different collinear direction are shown in figure 5. By explicit computation, we find that the operators containing $J^{T2}_{\chi_\alpha\chi_\beta,\xi}$ do not mix into any of the local operators. The diagrams containing a single time-ordered product of $\mathcal{L}_\xi^{(1)}$ and any type of the local current vanish at the one-loop level for external states without soft fields and any number of collinear fields. The reason is that the soft gluon field at $\mathcal{O}(\lambda)$ enters the Lagrangian only via the soft-field strength tensor with $\perp$ and n_- components, $x_\perp^\mu n_{i-}^\nu F_{\nu\mu_{\perp i}}$. Hence, we observe that in the Feynman gauge, a diagram with single $\mathcal{O}(\lambda)$ Lagrangian insertion always contains the factor

$$k_\alpha\left(g_{\perp i}^{\alpha\nu} n_{i-}^\mu - n_{i-}^\alpha g_{\perp i}^{\mu\nu}\right)(n_{j-})_\mu\,,$$

where k denotes the loop momentum and $(n_{j-})_\mu$ comes from the soft vertex on the j-collinear line. No further k-dependent terms appear in the numerator because only the n_- component of the soft momentum enters the collinear line and purely collinear interactions do not depend on the small component of the momentum. The one-loop soft loop integral depends on two vectors n_{i-} and n_{j-}, so any tensor integral can be reduced to a combination of these vectors and a metric tensor. After the tensor reduction of the loop integral, the numerator terms with $k \to n_{i-}$ vanish by definition of the light-cone coordinates. If $k \to n_{j-}$ then the total result is zero because of the anti-symmetric Feynman rule obtained from the soft gluon field-strength tensor.

In summary, the time-ordered product operators with $\mathcal{O}(\lambda)$ Lagrangians do not mix into local currents. The renormalization factor of the mixing of the time-ordered products containing the $\mathcal{O}(\lambda)$ Lagrangian with themselves is given by the Z-factor of its local component.

4 Combined result

In this section we discuss the combination of the collinear and soft contributions to the anomalous dimension. As concluded above, at fermion-number two we can focus on current-current contributions. We found that both the collinear and soft contributions can be summarized in a universal way, given by eq. (3.35) and eq. (3.36), respectively. In particular, the total soft contribution, summed over all pairs of collinear directions i,j with $i \neq j$, takes the form

$$\delta Z^s_{PQ}(x,y) = -\delta_{PQ}\delta(x-y)S \tag{4.1}$$

with

$$S = \frac{\alpha_s}{4\pi}\sum_{i,j=1}^{N}(1-\delta_{ij})\sum_{l=1}^{n_i}\sum_{k=1}^{n_j}\frac{\mathbf{T}_{i_l}\cdot\mathbf{T}_{j_k}}{2}\left\{\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\ln\left(\frac{-\mu^2 s_{ij}x_{i_l}x_{j_k}}{p^2_{i_l}p^2_{j_k}}\right)\right\}. \tag{4.2}$$

Notice that for identical building blocks, a symmetrization needs to be performed as discussed in the collinear case. We can write the logarithm as a sum of three terms involving $-s_{ij}x_{i_l}x_{j_k}/\mu^2$, $\mu^2/(-p^2_{i_l})$, and $\mu^2/(-p^2_{j_k})$, respectively. The last two terms are identical after renaming $i,l \leftrightarrow j,k$, thus we obtain

$$S = \frac{\alpha_s}{4\pi}\sum_{i,j}(1-\delta_{ij})\sum_{l,k}\mathbf{T}_{i_l}\cdot\mathbf{T}_{j_k}\left\{\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\left[\ln\left(\frac{-s_{ij}x_{i_l}x_{j_k}}{\mu^2}\right)+2\ln\left(\frac{\mu^2}{-p^2_{j_k}}\right)\right]\right\}. \tag{4.3}$$

Colour-neutrality of the entire N-jet operator implies $\sum_j\sum_k \mathbf{T}_{j_k} = 0$. We can use this to rewrite S as

$$S = \frac{\alpha_s}{4\pi}\sum_{i,j}\sum_{l,k}\mathbf{T}_{i_l}\cdot\mathbf{T}_{j_k}\left\{\frac{1}{\epsilon}\ln\left(\frac{-s_{ij}x_{i_l}x_{j_k}}{\mu^2}\right)(1-\delta_{ij})-\delta_{ij}\left[\frac{1}{\epsilon^2}+\frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-p^2_{j_k}}\right)\right]\right\}. \tag{4.4}$$

When combining this with the collinear result in eq. (3.35), we find that the regulator-dependent terms cancel, as expected. This is a consequence of the colour conservation and our assumption that the operator is a colour singlet. The cancellation serves as a consistency check proving that the N-jet operator matrix elements have the correct IR behaviour and no further basis operators, in particular with soft building blocks, are necessary. Therefore, all current-current contributions to the Z-factor can be summarized as

$$\begin{aligned}\delta Z_{PQ}(x,y) &= \delta Z^s_{PQ}(x,y)+\delta Z^c_{PQ}(x,y)\\ &= \delta_{PQ}\delta(x-y)\frac{\alpha_s}{4\pi}\sum_{i,j}\sum_{l,k}\mathbf{T}_{i_l}\cdot\mathbf{T}_{j_k}\left\{\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{\mu^2}{-s_{ij}x_{i_l}x_{j_k}}\right)\right](1-\delta_{ij})\right.\\ &\qquad\left.-\delta_{ij}\delta_{lk}\frac{c_{i_l}}{\epsilon}\right\}+\sum_i\delta^{[i]}(x-y)\frac{\gamma^i_{PQ}(x,y)}{\epsilon}.\end{aligned} \tag{4.5}$$

From this result we obtain the anomalous dimension matrix

$$\begin{aligned}\Gamma_{PQ}(x,y) &= \delta_{PQ}\delta(x-y)\left[-\gamma_{\rm cusp}(\alpha_s)\sum_{i<j}\sum_{l,k}\mathbf{T}_{i_l}\cdot\mathbf{T}_{j_k}\ln\left(\frac{-s_{ij}x_{i_l}x_{j_k}}{\mu^2}\right)+\sum_i\sum_l\gamma_{i_l}(\alpha_s)\right]\\ &\quad+2\sum_i\delta^{[i]}(x-y)\gamma^i_{PQ}(x,y),\end{aligned} \tag{4.6}$$

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where $\gamma_{\text{cusp}}(\alpha_s) = \frac{\alpha_s}{\pi}$, $\gamma_{i_l}(\alpha_s) \equiv -\frac{\alpha_s}{2\pi}\mathbf{T}_{i_l}^2 c_{i_l} = -\frac{3\alpha_s}{4\pi} C_F\,(0)$ for collinear quark (gluons), and the last line captures the off-diagonal contributions computed above.

This expression summarizes the main result of this work. We have checked that its form persists for all possible current-current contributions up to $\mathcal{O}\left(\lambda^2\right)$, beyond the $F = 2$ operators considered here. Operator mixing and non-diagonal contributions with respect to collinear momentum fractions always enter via the collinear contributions $\gamma^i_{PQ}(x, y)$.

As a cross-check, eq. (4.6) reduces to the leading-power result (1.1) when there is only a single building block in each collinear direction (i.e. $l, k = 1$, $x_{i_l}, x_{j_k} \to 1$), such that in the notation used above $\delta(x - y) \equiv \prod_i \prod_{k>1} \delta(x_{i_k} - y_{i_k}) \to 1$ is an empty product equal to unity. Furthermore, possibly non-diagonal contributions encapsulated in γ^i_{PQ} vanish at leading power.[8]

Note that the sets of labels collected in P and Q contain open Lorentz and spinor indices of the corresponding collinear building blocks, since we work with operators with completely uncontracted indices. Additional factors of order ϵ can arise when the open Lorentz indices of the total N-jet operator are contracted or the spinor indices projected onto a basis of Lorentz scalar operators, possibly including evanescent operators. This does not affect the anomalous dimension at the one-loop order in the conventional dimensional regularization (CDR) scheme, since the coefficient of the $1/\epsilon^2$ pole is diagonal in the Lorentz and Dirac indices. On the other hand, the $\mathcal{O}(\epsilon)$ terms which arise in the reduction of the non-diagonal single $1/\epsilon$ pole part do not contribute to the anomalous dimension. It is worth noting that the result above contains all information required to compute the anomalous dimension of evanescent operators in the CDR scheme. The statements above may not hold in dimensional regularization schemes that use explicitly four-dimensional quantities for internal lines. We refer to ref. [34] for a discussion of this issue in the SCET context.

In this work, we consider the case in which one of the collinear directions contains two fermionic building blocks (direction i, say). At $\mathcal{O}(\lambda)$, there is only a single type of operators of this kind, given by the product of $J_i = J^{B1}_{\chi\chi}(t_{i_1}, t_{i_2})$ defined in eq. (2.22) for the direction labelled by i and leading-power building blocks for all other $N-1$ directions $J_{j\neq i} = J^{A0}_j$. In this case, the anomalous dimension is off-diagonal in the collinear momentum fractions in direction i,

$$\sum_{j=1}^{N} \delta^{[j]}(x-y)\frac{\gamma^j_{PQ}(x,y)}{\epsilon} \to \frac{1}{\epsilon}\,\gamma^i_{\chi\chi,\chi\chi}(x_{i_1}, y_{i_1})\,, \tag{4.7}$$

where the right-hand side is given by eq. (3.17), and we have used $\gamma^j_{PQ}(x, y) = 0$ for all leading-power building blocks $j \neq i$. Furthermore the product of delta functions for the $N-1$ other directions $\delta^{[i]}(x - y) \equiv \prod_{j\neq i}\prod_{k>1} \delta(x_{j_k} - y_{j_k}) \to 1$ also collapses to unity.

At $\mathcal{O}(\lambda^2)$, there are two cases. Let us first consider the case that the direction i which we choose to carry fermion-number two encompasses itself the $\mathcal{O}(\lambda^2)$ suppression, i.e. it is represented by one of the three operators in eq. (2.23), $J_i \in \{J^{B2}_{\chi\partial\chi}, J^{B2}_{\partial(\chi\chi)}, J^{C2}_{\mathcal{A}\chi\chi}\}$. Then the other $N - 1$ directions have to contain leading-power building blocks, as before.

[8] Note that we use a different normalization for the gluonic building block compared to ref. [33], which affects $\gamma_{i_l}(\alpha_s)$. At leading power, it is easy to see that the results agree when taking the different convention into account.

The structure of the anomalous dimension follows directly from eq. (3.18), and leads to operator mixing,

$$\sum_j \delta^{[j]}(x-y)\frac{\gamma^j_{PQ}(x,y)}{\epsilon} \to \frac{1}{\epsilon}\begin{pmatrix} \gamma^i_{\chi\partial\chi,\chi\partial\chi} & \gamma^i_{\chi\partial\chi,\partial(\chi\chi)} & \gamma^i_{\chi\partial\chi,\mathcal{A}\chi\chi} \\ 0 & \gamma^i_{\partial(\chi\chi),\partial(\chi\chi)} & 0 \\ 0 & 0 & \gamma^i_{\mathcal{A}\chi\chi,\mathcal{A}\chi\chi} \end{pmatrix} \tag{4.8}$$

where the non-zero contributions are given in section 3.2.2 (specifically eqs. (3.21), (3.24) for the first and eq. (3.31) for the last row, and $\gamma^i_{\partial^\mu(\chi\chi),\partial^\nu(\chi\chi)} = g_\perp^{\mu\nu}\gamma^i_{\chi\chi,\chi\chi}$ is related to the $\mathcal{O}(\lambda)$ result in eq. (3.17)). The anomalous dimension is diagonal with respect to the other $N-1$ directions.

The second case that can occur at $\mathcal{O}(\lambda^2)$ is that direction i with $F=2$ is described by the $\mathcal{O}(\lambda)$ contribution $J_i = J^{B1}_{\chi\chi}(t_{i_1},t_{i_2})$, and one of the other $N-1$ directions, say direction i', contributes an additional $\mathcal{O}(\lambda)$ suppression. The remaining $N-2$ directions must then be represented by a leading-power building block. Since we do not require direction i' to have a definite fermion-number, there are more possibilities, in particular $J_{i'} \in \{J^{A1}_{\partial\chi}, J^{A1}_{\partial\mathcal{A}}, J^{B1}_{\mathcal{A}\chi}, J^{B1}_{\mathcal{A}\mathcal{A}}, J^{B1}_{\chi\chi}, J^{B1}_{\bar{\chi}\chi}\}$ (plus hermitian conjugated operators). In this case we need in addition the corresponding anomalous dimension matrices $\gamma^{i'}_{PQ}$ for these operators. They will be given in future work.

In summary, we have taken the first step in a systematic investigation of the anomalous dimension of subleading power N-jet operators in view of resummation of logarithmically enhanced terms in partonic cross sections beyond the leading power. We provide an explicit result at the one-loop order for fermion-number two N-jet operators. In a forthcoming paper we will present results at $\mathcal{O}(\lambda)$, $\mathcal{O}(\lambda^2)$ for general N-jet operators.

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A Conventions

- Collinear light-like reference vectors $n_{i\pm}$, $i=1,\ldots,N$ with $n_{i-}\cdot n_{i-} = n_{i+}\cdot n_{i+} = 0$, $n_{i-}\cdot n_{i+} = 2$. Any momentum can be decomposed as

$$p^\mu = \frac{1}{2}n_{i+}p\, n^\mu_{i-} + \frac{1}{2}n_{i-}p\, n^\mu_{i+} + p^\mu_{\perp i}\,. \tag{A.1}$$

- The different components of collinear momentum p_i scale as $(n_{i+}p_i, n_{i-}p_i, p^\mu_{i\perp i}) \sim (\lambda^0, \lambda^2, \lambda)$.
- n_i building blocks in direction i, labelled by i_k, $k=1,\ldots,n_i$.
- P_i is the total outgoing momentum in collinear direction i.
- Abbreviation $s_{ij} = \frac{1}{2}(n_{i-}\cdot n_{j-})P_iP_j$.

- Operators J^{An}, J^{Bn}, J^{Cn} with one, two, three building blocks, respectively, and power suppression $\mathcal{O}(\lambda^n)$. Here we count $J_\chi^{A0} = \chi_i = W_i^\dagger \xi_i$ and $J_\mathcal{A}^{A0} = \mathcal{A}_{\perp i}^\mu = W_i^\dagger [iD_{\perp i}^\mu W_i]$ as leading power ($n = 0$) for a collinear quark and gluon, respectively. The power suppression of all other operators is then counted relative to the leading power.

- Colour-space operator for parton labelled by i_k is $\mathbf{T}_{i_k}$ and colour conservation

$$\sum_{i=1}^{N} \sum_{k=1}^{n_i} \mathbf{T}_{i_k} = 0\,. \tag{A.2}$$

- We define $\alpha_s = \frac{g_s^2}{4\pi}$ and $\tilde{\mu}^2 = \mu^2 e^{\gamma_E}/(4\pi)$.

- Covariant derivatives

$$\begin{aligned} iD_{\perp i}^\mu &= i\partial_\perp^\mu + g_s A_{\perp i}^\mu(x), \\ in_{i+}D_i &= n_{i+}(i\partial + g_s A_i(x))\,, \\ in_{i-}D_i &= n_{i-}(i\partial + g_s A_i(x) + g_s A_s(x_{i-}))\,, \\ iD_s &= i\partial + g_s A_s(x) \quad \text{(on soft fields)}\,, \\ in_{i-}D_s &= n_{i-}(i\partial + g_s A_s(x_{i-})) \quad \text{(on collinear fields)}\,. \end{aligned} \tag{A.3}$$

B Redundant operators

B.1 Redundant collinear covariant derivative $in_{i-}D_i$

In this appendix, we show that the operator $n_{i-}\mathcal{A}_i = W_i^\dagger in_{i-}D_i W_i - in_{i-}D_s$, that could potentially contribute to the basis of collinear building blocks at (relative) $\mathcal{O}(\lambda)$, can be expressed in terms of the operator basis discussed in section 2, and is therefore redundant (see also ref. [35] for some closely related discussion).

The equation of motion for the collinear gauge field with respect to the i-th collinear direction derived from the leading-power collinear Lagrangian [14] reads

$$[iD_{\nu i}, G_i^{\mu\nu}] = ig_s t^a \bar{\xi}_i \left(n_{i-}^\mu t^a + \gamma_{\perp i}^\mu t^a \frac{1}{in_{i+}D_i} i\not{D}_{\perp i} + i\not{D}_{\perp i} \frac{1}{in_{i+}D_i} \gamma_{\perp i}^\mu t^a + \dots \right) \frac{\not{n}_{i+}}{2} \xi_i\,, \tag{B.1}$$

where $ig_s G_i^{\mu\nu} = [iD_i^\mu, iD_i^\nu]$ and the ellipsis stand for contributions involving n_{i+}^μ, that will drop out below. In the remainder of this appendix we will consistently omit the index i for the collinear direction i. The covariant derivative

$$iD^\mu(x) \equiv i\partial^\mu + g_s A^\mu(x) + g_s n_- A_\mathrm{s}(x_-) \frac{n_+^\mu}{2} \tag{B.2}$$

includes the multipole-expanded soft field in the n_- projection, in_-D. Contracting the equation of motion with $n_{+\mu}$ and multiplying with collinear Wilson lines from both sides gives,

$$W^\dagger [iD_\nu, [in_+D, iD^\nu]] W = -2g_s^2 W^\dagger t^a W\, \bar{\xi} t^a \frac{\not{n}_+}{2} \xi\,. \tag{B.3}$$

Next we use $\sum_a t^a_{ij} t^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}\right)$ to rewrite the colour ordering on the right-hand side (colour indices made explicit)

$$(W^\dagger[iD_\nu, [in_+D, iD^\nu]]W)_{ij} = -g_s^2 \left(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}\right) \overline{\chi}_k \frac{\not{n}_+}{2} \chi_l \,. \tag{B.4}$$

Writing the scalar product over ν on the left-hand side in terms of collinear basis vectors, and using $W^\dagger in_+ DW = in_+\partial$ to simplify gives

$$\begin{aligned}(in_+\partial)^2 (W^\dagger[in_- DW])_{ij} &= -2i\partial_{\perp\nu}(in_+\partial \mathcal{A}^\nu_\perp)_{ij} - 2[\mathcal{A}^\nu_\perp, (in_+\partial\mathcal{A}_{\perp\nu})]_{ij} \\ &\quad - 2g_s^2 \left(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}\right) \overline{\chi}_k \frac{\not{n}_+}{2} \chi_l \,. \end{aligned} \tag{B.5}$$

Next, we apply the inverse derivative operator formally given by $1/(in_+\partial)^2$. Note that $(in_+\partial)^2(W^\dagger[in_-DW])_{ij}$ transforms covariantly under the soft gauge symmetry, but $(W^\dagger[in_-DW])_{ij}$ does not, since the derivative acts only inside the bracket. However, on the left-hand side we can replace $W^\dagger[in_-DW] \to W^\dagger[in_-DW] - f(x_-)$ with an arbitrary function $f(x_-)$. This can also be seen as a freedom to add an integration constant when applying the inverse derivative operator. It can be fixed by the requirement of soft gauge covariance, and choosing $f(x_-) = g_s n_- A_{\mathrm{s}}(x_-)$ yields

$$\begin{aligned}(n_-\mathcal{A})_{ij} &= -\frac{2}{in_+\partial}(i\partial_{\perp\nu}\mathcal{A}^\nu_\perp)_{ij} - \frac{2}{(in_+\partial)^2}[\mathcal{A}^\nu_\perp, (in_+\partial\mathcal{A}_{\perp\nu})]_{ij} \\ &\quad - \frac{2g_s^2}{(in_+\partial)^2} \left(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}\right) \overline{\chi}_k \frac{\not{n}_+}{2} \chi_l \,, \end{aligned} \tag{B.6}$$

i.e. we can express the operator on the left-hand side in terms of other collinear building blocks. The previous equation receives corrections from the power-suppressed interactions in the SCET Lagrangian, which can be worked out in a similar manner. Leading-power redundant operators can always be removed iteratively from these further terms.

One peculiar property of this relation is that the soft field appears explicitly only on the left-hand side. We checked that the relation is indeed fulfilled in the matrix element with one soft and one collinear gluon. On the left-hand side, a 1PI diagram exists, where the soft gluon is attached directly to the operator. In addition, a 1PR diagram where the soft gluon is emitted from the collinear line contributes. On the right-hand side, only a 1PR diagram exists, that agrees with the sum of the 1PI and 1PR contribution from the left-hand side. We also checked explicitly that the identity holds in the matrix element with one and two collinear gluons with $\perp$ polarization.

B.2 Redundant soft covariant derivative $in_{i-}D_s$

We now show that the soft covariant derivative $in_{i-}D_s$ when operating on collinear fields can be removed using the collinear equations of motion. As before, we omit the label for the collinear direction in this section for brevity. Using the equation of motion for the collinear quark field we find

$$in_-D_s\chi = -\left[n_-\mathcal{A} + (i\not{\partial}_\perp + \not{\mathcal{A}}_\perp)\frac{1}{in_+\partial}(i\not{\partial}_\perp + \not{\mathcal{A}}_\perp)\right]\chi, \tag{B.7}$$

which yields an expression in terms of the operator basis discussed in section 2 after using the relation (B.6) for $n_-\mathcal{A}$. A computation similar to the one in section B.1, starting from the YM equation of motion (B.1) with open index μ projected in $\perp$ direction yields (with colour indices ij made explicit)

$$\begin{aligned}\left([in_-D_s,\mathcal{A}^\mu_\perp]\right)_{ij} &= \frac{1}{2}i\partial^\mu_\perp(n_-\mathcal{A})_{ij}+\frac{1}{2}\left([\mathcal{A}^\mu_\perp,n_-\mathcal{A}]\right)_{ij}+\frac{1}{2in_+\partial}\left([(in_+\partial\mathcal{A}^\mu_\perp),n_-\mathcal{A}]\right)_{ij}\\ &+\frac{1}{in_+\partial}\left([i\partial^\nu_\perp+\mathcal{A}^\nu_\perp,[i\partial^\mu_\perp+\mathcal{A}^\mu_\perp,i\partial_{\perp\nu}+\mathcal{A}_{\perp\nu}]]\right)_{ij}\\ &+\frac{g_s^2}{2in_+\partial}\left(\delta_{il}\delta_{jk}-\frac{1}{3}\delta_{ij}\delta_{kl}\right)\left(\bar\chi_k\gamma^\mu_\perp\frac{1}{in_+\partial}\left(\not{\mathcal{A}}_\perp\right)_{ll'}\frac{\not{n}_+}{2}\chi_{l'}\right.\\ &\left.+\bar\chi_{k'}\left(\not{\mathcal{A}}_\perp\right)_{k'k}\frac{1}{in_+\partial}\gamma^\mu_\perp\frac{\not{n}_+}{2}\chi_l+2\bar\chi_k\frac{i\partial^\mu_\perp}{in_+\partial}\frac{\not{n}_+}{2}\chi_l\right). \end{aligned}\tag{B.8}$$

C Auxiliary functions entering the anomalous dimension

For the anomalous dimension $Z^{c,i}_{\mathcal{A}\chi\chi,\mathcal{A}\chi\chi}$ at $\mathcal{O}(\lambda^2)$ we need also the anomalous dimension $Z^{c,i}_{\mathcal{A}\chi,\mathcal{A}\chi}$ at $\mathcal{O}(\lambda)$ as an input. It can be obtained by computing the one-loop matrix element $\langle g_a(q)\bar q(p)|J^{B1}_{\mathcal{A}^\mu\chi}(x)|0\rangle$ and we find

$$\delta Z^{c,i}_{\mathcal{A}^\mu\chi_\alpha,\mathcal{A}^\nu\chi_\beta}(x,y)=-g^{\mu\nu}_\perp\delta_{\alpha\beta}\delta(x-y)X_{i_1i_2}+\frac{1}{\epsilon}\gamma^i_{\mathcal{A}^\mu\chi_\alpha,\mathcal{A}^\nu\chi_\beta}(x,y)\,,\tag{C.1}$$

with $X_{i_1i_2}$ given by eq. (3.34) and

$$\begin{aligned}\gamma^i_{\mathcal{A}^\mu\chi_\alpha,\mathcal{A}^\nu\chi_\beta}(x,y) &= \frac{\alpha_s\mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{2\pi}\left\{g^{\mu\nu}_\perp\delta_{\alpha\beta}\left(\theta(x-y)\left[\frac{1}{x-y}\right]_++\theta(y-x)\left[\frac{1}{y-x}\right]_+\right.\right.\\ &\left.-\frac{\theta(x-y)}{\bar y}\left(1+\frac{\bar x(\bar x+\bar y)}{2x}\right)-\frac{\theta(y-x)}{2y}(\bar x+\bar y)\right)\\ &\left.+\frac{1}{4}([\gamma^\mu_\perp,\gamma^\nu_\perp])_{\alpha\beta}(x+y)\bar x\left(\frac{\theta(x-y)}{\bar y x}+\frac{\theta(y-x)}{y\bar x}\right)\right\}\\ &-\frac{\alpha_s(\mathbf{C_F}+\mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2})}{4\pi}\left\{g^{\mu\nu}_\perp\delta_{\alpha\beta}\left(\frac{\theta(x-\bar y)\bar x}{yx}(\bar x+\bar y)+\frac{\theta(\bar y-x)}{\bar y}(\bar x-y)\right)\right.\\ &\left.+\frac{1}{2}([\gamma^\mu_\perp,\gamma^\nu_\perp])_{\alpha\beta}\left(\frac{\theta(x-\bar y)\bar x}{yx}(\bar x-y-1)+\frac{\theta(\bar y-x)}{\bar y}(\bar x-y)\right)\right\}\\ &+\frac{\alpha_s\mathbf{C_F}}{4\pi}\,\bar x\,(\gamma^\mu_\perp\gamma^\nu_\perp)_{\alpha\beta}\,,\end{aligned}\tag{C.2}$$

where $\mathbf{C_F}\equiv\frac{1}{6}(1-3(\mathbf{T}_{i_1}+\mathbf{D}_{i_1})\cdot\mathbf{T}_{i_2})$ and we introduced the additional colour operator $\mathbf{D}^b|a\rangle=d^{abc}|c\rangle$ related to the symmetric d^{abc} symbol defined via $\{t^a,t^b\}=\frac{1}{3}\delta^{ab}+d^{abc}t^c$. We checked that our result agrees with refs. [15, 16] after subtracting the soft-loop contributions to the $\mathcal{O}(\lambda)$ heavy-to-light current from the anomalous dimension computed in these references. By computing the matrix element $\langle\bar q(p)|J^{B1}_{\mathcal{A}^\mu\chi}(x)|0\rangle$ we furthermore find

$$\delta Z^{c,i}_{\mathcal{A}^\mu\chi_\alpha,\partial^\nu\chi_\beta}(x,y)=0\,.\tag{C.3}$$

The functions entering $Z^{c,i}_{\chi\partial\chi,\chi\partial\chi}$ and $Z^{c,i}_{\chi\partial\chi,\partial(\chi\chi)}$ in eq. (3.21) are given by

$$
\begin{aligned}
&M_{\chi_\alpha\partial^\mu\chi_\beta,\chi_{\alpha'}\partial^\sigma\chi_{\beta'}}(x,y)\\
&= -\left(\theta(x-y)\frac{\bar{x}}{\bar{y}} + \theta(y-x)\frac{x}{y}\right) x\bar{x}\\
&\quad\times\Bigg[-\left(\frac{\gamma_\perp^\sigma\gamma_\perp^\nu}{x} + \frac{\gamma_\perp^\nu\gamma_\perp^\sigma}{y}\right)_{\alpha\alpha'}\left(\frac{\gamma_\perp^\mu\gamma_{\perp\nu}}{\bar{x}}\right)_{\beta\beta'} - \left(\frac{\gamma_\perp^\mu\gamma_{\perp\nu}}{x}\right)_{\alpha\alpha'}\left(\frac{\gamma_\perp^\sigma\gamma_\perp^\nu}{\bar{x}} + \frac{\gamma_\perp^\nu\gamma_\perp^\sigma}{\bar{y}}\right)_{\beta\beta'}\\
&\quad -2\delta_{\alpha\alpha'}\left(\frac{\gamma_\perp^\mu\gamma_\perp^\sigma}{\bar{x}\bar{y}}\right)_{\beta\beta'} - 2\left(\frac{\gamma_\perp^\mu\gamma_\perp^\sigma}{xy}\right)_{\alpha\alpha'}\delta_{\beta\beta'} - \frac{g_\perp^{\mu\sigma}}{x\bar{x}}\left(\gamma_\perp^\rho\gamma_\perp^\nu\right)_{\alpha\alpha'}\left(\gamma_{\perp\rho}\gamma_{\perp\nu}\right)_{\beta\beta'}\Bigg]\\
&\quad +\frac{1}{2}\left(\theta(y-x)\frac{x(x-2y)}{y^2} + \theta(x-y)\frac{\bar{x}(\bar{x}-2\bar{y})}{\bar{y}^2}\right)\times\Bigg[\left(\gamma_\perp^\sigma\gamma_\perp^\nu\right)_{\alpha\alpha'}\left(\gamma_\perp^\mu\gamma_{\perp\nu}\right)_{\beta\beta'}\\
&\quad +\left(\gamma_\perp^\mu\gamma_\perp^\nu\right)_{\alpha\alpha'}\left(\gamma_\perp^\sigma\gamma_{\perp\nu}\right)_{\beta\beta'} + g_\perp^{\mu\sigma}\left(\gamma_\perp^\rho\gamma_\perp^\nu\right)_{\alpha\alpha'}\left(\gamma_{\perp\rho}\gamma_{\perp\nu}\right)_{\beta\beta'}\Bigg],\\
&M_{\chi_\alpha\partial^\mu\chi_\beta,\partial^\sigma(\chi_{\alpha'}\chi_{\beta'})}(x,y)\\
&= -\left(\theta(x-y)\frac{\bar{x}}{\bar{y}} + \theta(y-x)\frac{x}{y}\right) x\bar{x}\\
&\quad\times\Bigg[\left(\frac{\gamma_\perp^\sigma\gamma_\perp^\nu}{x} + \frac{\gamma_\perp^\nu\gamma_\perp^\sigma}{y}\right)_{\alpha\alpha'}\left(\frac{\gamma_\perp^\mu\gamma_{\perp\nu}}{\bar{x}}\right)_{\beta\beta'} + 2\left(\frac{\gamma_\perp^\mu\gamma_\perp^\sigma}{xy}\right)_{\alpha\alpha'}\delta_{\beta\beta'}\Bigg]\\
&\quad +\frac{1}{2}\left(\theta(y-x)\frac{x(\bar{x}y+y-x)}{y^2} + \theta(x-y)\frac{\bar{x}^2}{\bar{y}}\right)\times\Bigg[\left(\gamma_\perp^\sigma\gamma_\perp^\nu\right)_{\alpha\alpha'}\left(\gamma_\perp^\mu\gamma_{\perp\nu}\right)_{\beta\beta'}\\
&\quad +\left(\gamma_\perp^\mu\gamma_\perp^\nu\right)_{\alpha\alpha'}\left(\gamma_\perp^\sigma\gamma_{\perp\nu}\right)_{\beta\beta'} + g_\perp^{\mu\sigma}\left(\gamma_\perp^\rho\gamma_\perp^\nu\right)_{\alpha\alpha'}\left(\gamma_{\perp\rho}\gamma_{\perp\nu}\right)_{\beta\beta'}\Bigg].
\end{aligned}
\tag{C.4}
$$

The functions entering $Z^{c,i}_{\chi\partial\chi,\mathcal{A}\chi\chi}$ are given by

$$
\begin{aligned}
K^{\mu\nu}_{1,\alpha\alpha'\beta\beta'}(x,y_2,y_3) &\equiv \delta_{\beta\beta'}G^{\mu\nu}_{\alpha\alpha'}(x,y_1,y_2) + \delta_{\alpha\alpha'}G^{\mu\nu}_{\beta\beta'}(\bar{x},y_1,y_3)\\
&\quad -H^{\mu\nu}_{1,\alpha\alpha'\beta\beta'}(x,y_1,y_2) - H^{\mu\nu}_{1,\beta\beta'\alpha\alpha'}(\bar{x},y_1,y_3) - J^{\mu\nu}_{\alpha\alpha'\beta\beta'}(x,y_2,y_3)\,,\\
K^{\mu\nu}_{2,\alpha\alpha'\beta\beta'}(x,y_1,y_2) &\equiv 2\delta_{\beta\beta'}F^{\mu\nu}_{\alpha\alpha'}(x,y_1,y_2) - H^{\mu\nu}_{1,\alpha\alpha'\beta\beta'}(x,y_1,y_2)\\
&\quad - H^{\mu\nu}_{2,\alpha\alpha'\beta\beta'}(x,y_1,y_2) + \frac{1}{2}I^{\mu\nu}_{\alpha\alpha'\beta\beta'}(x,y_1,y_2)\,,
\end{aligned}
\tag{C.5}
$$

where the contribution from diagram $(b,ii)_B$ and $(b,i)_B$ can be expressed in terms of

$$
\begin{aligned}
G^{\mu\nu}_{\alpha\alpha'}(x,y_1,y_2) \equiv{}& \frac{1}{1-y_3}\frac{1}{\bar{x}-y_3}\left(\theta(x-y_2)\theta(\bar{x}-y_3)\frac{\bar{x}-y_3}{y_1} + \theta(y_2-x)\frac{x}{y_2}\right)\\
&\times\left(-4xg_\perp^{\mu\nu} + (x-y_2+y_1)\gamma_\perp^\mu\gamma_\perp^\nu\right)_{\alpha\alpha'}\,.
\end{aligned}
\tag{C.6}
$$

The diagrams $(b,ii)_F$ and $(b,i)_F$ give

$$
\begin{aligned}
F^{\mu\nu}_{\alpha\alpha'}(x,y_1,y_2) \equiv{}& \frac{1}{1-y_3}\frac{1}{\bar{x}-y_3}\left(\theta(x-y_1)\theta(\bar{x}-y_3)\frac{\bar{x}-y_3}{y_2} + \theta(y_1-x)\frac{x}{y_1}\right)\\
&\times\left(2xg_\perp^{\mu\nu} - y_1\gamma_\perp^\mu\gamma_\perp^\nu\right)_{\alpha\alpha'}\,.
\end{aligned}
\tag{C.7}
$$

The diagrams $(c)_V$ and $(c)'_V$ give

$$
\begin{aligned}
H^{\mu\nu}_{1,\alpha\alpha'\beta\beta'}(x,y_1,y_2) \equiv\; & \left(\theta(x-y_1-y_2)\frac{\bar{x}}{y_3}+\theta(y_1+y_2-x)\frac{x}{y_1+y_2}\right) \\
& \times\left(\delta_{\beta\beta'}\frac{2\bar{x}}{y_1+y_2}(\gamma^\mu_\perp\gamma^\nu_\perp)_{\alpha\alpha'}+\frac{x}{y_1+y_2}(\gamma^\rho_\perp\gamma^\nu_\perp)_{\alpha\alpha'}(\gamma^\mu_\perp\gamma_{\perp\rho})_{\beta\beta'}\right) \\
H^{\mu\nu}_{2,\alpha\alpha'\beta\beta'}(x,y_1,y_2) \equiv\; & \left(\theta(x-y_1-y_2)\frac{\bar{x}}{y_3}+\theta(y_1+y_2-x)\frac{x}{y_1+y_2}\right)\frac{x}{x-y_1} \\
& \times(\gamma^\nu_\perp\gamma^\rho_\perp)_{\alpha\alpha'}(\gamma^\mu_\perp\gamma_{\perp\rho})_{\beta\beta'}\,.
\end{aligned}
\tag{C.8}
$$

The diagrams $(c)_F$ and $(c)'_F$ give

$$
\begin{aligned}
I^{\mu\nu}_{\alpha\alpha'\beta\beta'}(x,y_1,y_2) \equiv\; & \Bigg(-\theta(x-y_1)\theta(\bar{y}_3-x)\frac{x^2\bar{y}_1+\bar{x}^2\bar{y}_3-\bar{y}_1\bar{y}_3}{\bar{y}_1y_2\bar{y}_3} \\
& +\theta(y_1-x)\frac{x^2}{y_1\bar{y}_3}+\theta(x-\bar{y}_3)\frac{\bar{x}^2}{\bar{y}_1y_3}\Bigg)\Bigg\{\frac{x+y_1}{x-y_1}(\gamma^\nu_\perp\gamma^\rho_\perp)_{\alpha\alpha'}(\gamma^\mu_\perp\gamma_{\perp\rho})_{\beta\beta'} \\
& +g^{\mu\nu}_\perp(\gamma^\sigma_\perp\gamma^\rho_\perp)_{\alpha\alpha'}(\gamma_{\perp\sigma}\gamma_{\perp\rho})_{\beta\beta'}+(\gamma^\mu_\perp\gamma^\rho_\perp)_{\alpha\alpha'}(\gamma^\nu_\perp\gamma_{\perp\rho})_{\beta\beta'}\Bigg\}\,,
\end{aligned}
\tag{C.9}
$$

and the diagram $(c)_B$ yields

$$
\begin{aligned}
J^{\mu\nu}_{\alpha\alpha'\beta\beta'}(x,y_2,y_3) \equiv\; & \Bigg\{\frac{1}{2}\Bigg(-\theta(x-y_2)\theta(\bar{y}_3-x)\frac{x^2\bar{y}_2+\bar{x}^2\bar{y}_3-\bar{y}_2\bar{y}_3}{\bar{y}_2y_1\bar{y}_3} \\
& +\theta(y_2-x)\frac{x^2}{y_2\bar{y}_3}+\theta(x-\bar{y}_3)\frac{\bar{x}^2}{\bar{y}_2y_3}\Bigg)\Big[(\gamma^\nu_\perp\gamma^\rho_\perp)_{\alpha\alpha'}(\gamma^\mu_\perp\gamma_{\perp\rho})_{\beta\beta'} \\
& +g^{\mu\nu}_\perp(\gamma^\sigma_\perp\gamma^\rho_\perp)_{\alpha\alpha'}(\gamma_{\perp\sigma}\gamma_{\perp\rho})_{\beta\beta'}+(\gamma^\mu_\perp\gamma^\rho_\perp)_{\alpha\alpha'}(\gamma^\nu_\perp\gamma_{\perp\rho})_{\beta\beta'}\Big] \\
& +\delta_{\alpha\alpha'}(\gamma^\mu_\perp\gamma^\nu_\perp)_{\beta\beta'}\left(\theta(x-y_2)\theta(\bar{y}_3-x)\frac{\bar{x}-\bar{y}_2}{\bar{y}_2y_1}-\theta(x-\bar{y}_3)\frac{\bar{x}}{\bar{y}_2y_3}\right) \\
& +\delta_{\beta\beta'}(\gamma^\mu_\perp\gamma^\nu_\perp)_{\alpha\alpha'}\left(\theta(x-y_2)\theta(\bar{y}_3-x)\frac{x-\bar{y}_3}{\bar{y}_3y_1}-\theta(y_2-x)\frac{x}{y_2\bar{y}_3}\right)\Bigg\}\,.
\end{aligned}
\tag{C.10}
$$

For $0<y_i<1$ the functions $K_{1(2)}$ are regular for all $0<x<1$.[9]

[9] There are terms contributing to K_2 that can potentially be singular for $x\to y_1$, in particular

$$
\begin{aligned}
\frac{1}{2}I^{\mu\nu}_{\alpha\alpha'\beta\beta'}(x,y_1,y_2) \;\to\; & \frac{1}{2}\frac{x+y_1}{x-y_1}\left(-\theta(x-y_1)\theta(\bar{y}_3-x)\frac{x^2\bar{y}_1+\bar{x}^2\bar{y}_3-\bar{y}_1\bar{y}_3}{\bar{y}_1y_2\bar{y}_3}\right. \\
& \left.+\theta(y_1-x)\frac{x^2}{y_1\bar{y}_3}\right)(\gamma^\nu_\perp\gamma^\rho_\perp)_{\alpha\alpha'}(\gamma^\mu_\perp\gamma_{\perp\rho})_{\beta\beta'}\,, \\
-H^{\mu\nu}_{2,\alpha\alpha'\beta\beta'} \;\to\; & -\frac{x}{x-y_1}\theta(\bar{y}_3-x)\frac{x}{\bar{y}_3}(\gamma^\nu_\perp\gamma^\rho_\perp)_{\alpha\alpha'}(\gamma^\mu_\perp\gamma_{\perp\rho})_{\beta\beta'}\,.
\end{aligned}
\tag{C.11}
$$

One can check that the sum of both terms is regular for $x\to y_1$. (One can use that in this limit $\theta(\bar{y}_3-x)\to 1$ due to the assumption $y_2>0$. Then using $\frac{x^2\bar{y}_1+\bar{x}^2\bar{y}_3-\bar{y}_1\bar{y}_3}{\bar{y}_1y_2\bar{y}_3}\to-\frac{x}{\bar{y}_3}$ for $x\to y_1$, the two terms in the first and second line combine to cancel the pole in the third line.) Furthermore, there are additional occurrences of $1/(x-y_i)$, but one can check that the θ-functions multiplying them exclude the pole for $0<y_i<1$.

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