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On uncertain parameter identification from experimental data in structural dynamics and vibroacoustics

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ABSTRACT

Modeling of engineering problems in structural dynamics and vibroacoustics involves knowledge of uncertainty related with parameters, e.g. elastic and damping properties. The theoretical and numerical aspects of uncertainty quantification (UQ) in such parameters have been well developed over the past decades. The validation of results, however, remains as challenging issue due to the lack of coherent and consistent experimental data upon desired performance. This is particularly important when dealing with numerical FEM models with uncertain inputs. In this paper, the fundamental concept of experimentally UQ in structural dynamics and vibroacoustics will be discussed. The prompt will be on introducing a general framework by which UQ is performed using limited data available. Once the experimentally identified inputs are known, statistical properties of uncertain structural responses are updated using numerical FEM model.

Keywords: Uncertainty quantification, vibroacoustic, polynomial chaos

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1. INTRODUCTION

Uncertainty quantification (UQ) is initially an interdisciplinary analysis method for quantitative characterization and reduction of uncertainties. UQ deals with the identification of the uncertainty sources, modeling methods, propagation of uncertainty in the model and the prediction of the overall uncertainty in the system responses. Probabilistic modeling has been very popular over many years to incorporate uncertainty into the model calculations at all stages of the modeling. For complex physical systems, however, it is still a challenging research topic due to the computational cost of models performing UQ; particularly, in structural dynamics and vibroacoustic problems where one often deals with models including a large number of uncertain parameters. In addition to the spatial and the temporal domains in such problems, a new multi-dimensional random domain will be involved which is affected by the number of sources of uncertainty. Consequently, UQ can be more difficult in the case of applications involving large number of uncertain parameters, notably, for expensive deterministic computer simulations. Therefore, models having sparse random dimensions are highly effective and desirable due to the fact that in many cases a limited number of simulations may be available to construct reasonably accurate models for such systems.

Modeling of engineering problems in structural dynamics and vibroacoustics involves uncertainty related with parameters, e.g. material or geometric properties, that particularly leads to a large variety at the mid to high frequencies [1]. Vibroacoustic problems essentially as a kind of fluid-structure-interaction (FSI) problems have a very high complexity in the sense of inherent multi-physical and multi-scale phenomena. Such a complexity makes the results and predictions of the deterministic analysis and numerical simulation for the vibroacoustic problems insufficient accurate. The responses of vibroacoustic problems are very sensitive to manufacturing processes and small variabilities induced by any the design parameter, boundary conditions, material properties, etc. In fact, uncertainties are inherent in such a computational FSI model, even though, a sophisticated deterministic model is used. Accordingly, the demand for models which to capture uncertainties in various stages of modeling is increased. The efficiency and validation of the models, however, remain as challenging issues due to several points:

· such models include a large number of uncertain parameters

· numerical simulation models are very large and computationally very expensive

· lack of coherent and consistent experimental data upon desired performance to identify uncertainties

This is particularly important when dealing with numerical FEM models with uncertain inputs. The non-sampling based simulation methods provide the designer with such facility to overcome the above mentioned issues by modeling the uncertain inputs/outputs using a global estimation model independent of the complexity of the physical problem and, simultaneously, employing experimental data for the validation of the uncertainty. The methods have been applied to many structural dynamics and vibroacoustic problems, e.g. in applied acoustics [2, 3], structural dynamics [4, 5, 6, 7], vibration analysis [8, 9, 10, 11, 12] and uncertain parameter identification [13, 14]. Surveys on UQ based on stochastic modeling using polynomial chaos approximations are found in [15, 16].

Once the experimental data is available on uncertain parameters, the statistical properties are calculated and used to identify required probabilistic characteristics. The type of information needed to characterize the uncertainty in the input parameters may greatly differ from one application to another; however, a numerical module will be developed to calculate major properties such as the probability density function (PDF), the statistical moments or the joint PDF in the case of dependent parameters. Two classes of identification can be distinguished depending on the type of data available; direct and inverse identifications. In the first class, the data are available on the parameters and, accordingly, statistical properties of the data can be used directly to identify the uncertain parameters. If the data are available on the system responses, an inverse identification procedure is used in the second class to estimate the statistics of the parameters. I this paper we assume that one deals with the second case. For that, the statistical moments of the responses are used to identify the input uncertain parameters as is discussed in next sections.

This paper has been organized as follows: spectral representation of the parameters are given in the next section. Section 3 discusses the statistical moment based method for estimation of the uncertain input parameters from the response data. The application of the method is given in section 4. The conclusions are given in the last section of the paper.

2. SPECTRAL-BASED REPRESENTATION OF RANDOM PARAMETERS

Spectral-based methods provide a surrogate model for representation of the uncertain input/output of the system undertaken. The generalized Polynomial chaos (gPC) expansion, as a surrogate model, for uncertain parameters which behave as random variables exhibits constant deterministic coefficients. Let X be a $\mathbb{R}$–valued RV defined on a probability space $(\Omega, \mathcal{A}, P)$. The truncated gPC expansion for such RV is defined as

$$X(\boldsymbol{\xi}) = \sum_{i=0}^{N} a_i \Psi_i(\boldsymbol{\xi}) \tag{1}$$

The deterministic coefficients a_i are calculated as

$$a_i = \frac{1}{\left\langle \Psi_i^2 \right\rangle} \int_{\Omega} X(\boldsymbol{\xi}) \Psi_i(\boldsymbol{\xi}) f(\boldsymbol{\xi}) \, \mathrm{d}\boldsymbol{\xi} \tag{2}$$

in which f is the joint PDF of random vector $\boldsymbol{\xi}$ and for independent random variables ξ_i can be written as the multiplication of the individual PDF for each RV, i.e.

$$f(\boldsymbol{\xi}) d\boldsymbol{\xi} = f_1(\xi_1) f_2(\xi_2) \ldots f_n(\xi_n) \, \mathrm{d}\xi_1 \, \mathrm{d}\xi_2 \ldots \mathrm{d}\xi_n \tag{3}$$

For instance, assume that $X(\xi)$ is an uncertain parameter represented by the lognormal PDF, LN(μ, σ). Using random Hermite orthogonal polynomials, $H_i(\xi)$, the gPC expansion of $X(\xi)$ is given as

$$X(\xi) = a_0 + a_1 \xi + a_2(\xi^2 - 1) + a_3(\xi^3 - 3\xi) + a_4(\xi^4 - 6\xi^2 + 3) + \ldots \tag{4}$$

Knowing that $\left\langle H_i(\xi)^2 \right\rangle = i!$ and applying Eq. (2), the unknown coefficients are calculated as

$$a_i = \frac{1}{i!} \int_{-\infty}^{\infty} \exp\left(\mu + \sigma\xi\right) H_i(\xi) \, \frac{\exp(\frac{-\xi^2}{2})}{\sqrt{2\pi}} \, \mathrm{d}\xi \tag{5}$$

This leads to

$$a_i = \frac{\sigma^i}{i!}\exp(\mu + \frac{\sigma^2}{2}), \qquad i = 0, 1, 2, \ldots \tag{6}$$

The first 10 coefficients are shown in Fig. 1 for $\mu = 5$ and different values of σ. As demonstrated, while the first few coefficients of parameters with small σ seem

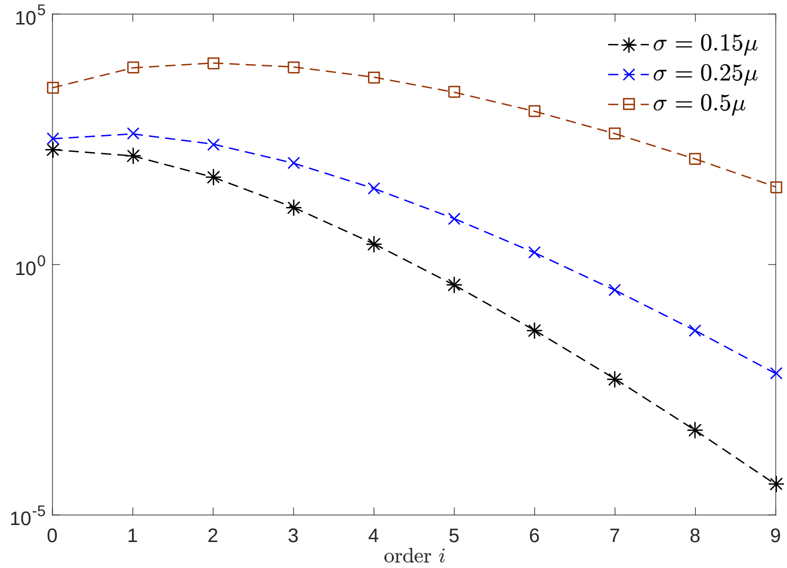

Figure 1: The first 10 coefficients of the gPC expansion of an uncertain parameter having the lognormal distributed $\mathrm{LN}(\mu, \sigma)$ *with* $\mu = 5$ *and different variance.*

to be enough for accurate gPC representation, for large value of uncertainty, the rate of convergence of the coefficients are low. This implies to the contribution of higher order gPC coefficients for an accurate representation of parameters having large uncertainty. Analogy to error minimization in deterministic FEM modeling, the error associated to the random space discretization by means of the gPC expansions has to be minimized to estimate the unknown coefficients of the responses. In a direct problem, the unknown deterministic coefficients of the responses have to calculated so that the error to be minimum. The unknown coefficients of the parameters are identified from the experimental tests available on the parameters, or on the responses via an inverse optimization problem. For that, the statistical properties of the parameters/responses are employed to define the objective function.

3. ESTIMATION OF THE COEFFICIENTS FROM EXPERIMENTAL DATA

Once the gPC expansion of an uncertain parameter is given, the statistical properties of the parameter such as distribution and moments can be calculated from the coefficients and applying the orthogonality property of the random basis. The k^{th}–order statistical moment of X from the gPc expansion given by

$$m_k = \mathrm{E}\left[X^k\right] = \int_\Omega \left[\sum_{i=0}^{\infty} a_i \Psi_i(\xi)\right]^k f(\xi)\,\mathrm{d}\xi \tag{7}$$

It is seen that the first 3 statistical moments of X are given by:

$$\mathrm{E}[X] = m_1 = \sum_{i_1=1}^{\infty} a_{i_1} \langle \Psi_0, \Psi_{i_1} \rangle = a_0 \tag{8}$$

$$\mathrm{E}[X^2] = m_2 = \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty} a_{i_1} a_{i_2} \langle \Psi_{i_1}, \Psi_{i_2} \rangle \tag{9}$$

$$\mathrm{E}[X^3] = m_3 = \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty}\sum_{i_3=1}^{\infty} a_{i_1} a_{i_2} a_{i_3} \langle \Psi_{i_1}, \Psi_{i_2}, \Psi_{i_3} \rangle \tag{10}$$

and accordingly for k^{th} moment

$$\mathrm{E}[X^k] = m_k = \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty}\ldots\sum_{i_k=1}^{\infty} a_{i_1} a_{i_2}\ldots a_{i_k} \langle \Psi_{i_1}, \Psi_{i_2}, \ldots, \Psi_{i_k} \rangle \tag{11}$$

The first 3 central statistical moments $\mu_i,\ i = 1$ to 4 are derived as

$$\mu_1 = 0 \tag{12}$$

$$\mu_2 = m_2 - m_1^2 \tag{13}$$

$$\mu_3 = m_3 - 3m_1 m_2 + 2m_2^2 \tag{14}$$

and so on, for the k^{th}–order central statistical moment ($m_0 = 1$)

$$\mu_k = \mathrm{E}\left[(X - \mathrm{E}[X])^k\right] = \sum_{i=0}^{k} \binom{k}{i} (-1)^{k-i} m_i m_1^{k-i}, \qquad k = 2, 3, \ldots \tag{15}$$

Similar expressions can be derived for the moments of RF and RP represented by the gPC expansion. The calculated moments form the gPC expansion can be compared to the corresponding values obtained from experimental data for an uncertain parameter.

In such a way, one can attempt to estimate the unknown coefficients from the available experimental data. An error function based on the least-square criterion corresponding to the difference between the theoretical and experimental estimation of statistical moments can be used to estimate the optimal coefficients a_i. This leads to a minimization problem as follows

$$\begin{aligned} &\underset{p_i}{\text{minimize}} \quad \sum_{n=1}^{k} f_n^2(a_i) \\ &\text{s.t.} \;\; f_0(a_i) = \mu_1^{\text{expri}} - \mu_1 = 0 \\ &\qquad f_n(a_i) = \mu_k^{\text{expri}} - \mu_k, \qquad n \geq 2 \end{aligned} \tag{16}$$

The first condition of the process denotes that the expected value of the data represents the first coefficient of the gPC expansion. Since the calculated moments for the gPC expansion are nonlinear functions of the coefficients, one has to employ nonlinear optimization procedure. The optimization leads to a unique solution under the convergence condition for coefficients of one-dimensional gPC, i.e. $\|a_{i+1}\| < \|a_i\|$. Accordingly, two type of objective functions can be defined, depending on information available:

1. forced-response based objective functions, where structural/vibroacoustic responses of the system due to deterministic/random applied force are known, or measured,

2. Modal-based objective functions, in which numerical/experimental modal data, e.g. natural frequencies, are available.

In the first case, the objective function can be characterized from the statistical data of the responses at a specific time/frequency and a node of the FEM mesh. More complicated functions ar defined over a time/frequency and spatial domains. For such cases, the function is prescribed from the statistical data of random fields/process. Having the modal data of the system under study such as natural frequencies, provides this facility to express the objective function employing the statistical moments of random variables.

4. APPLICATION TO VIBROACOUSTICS

To show the impact of uncertainty in vibroacoustics, consider a rectangular acoustic cavity that is bounded by rigid walls and en elastic plate on the left side having the dimensions of $(a, b, h) = (2, 0.5, 0.8)$ m, cf. Fig. 2. A harmonic force is applied on the

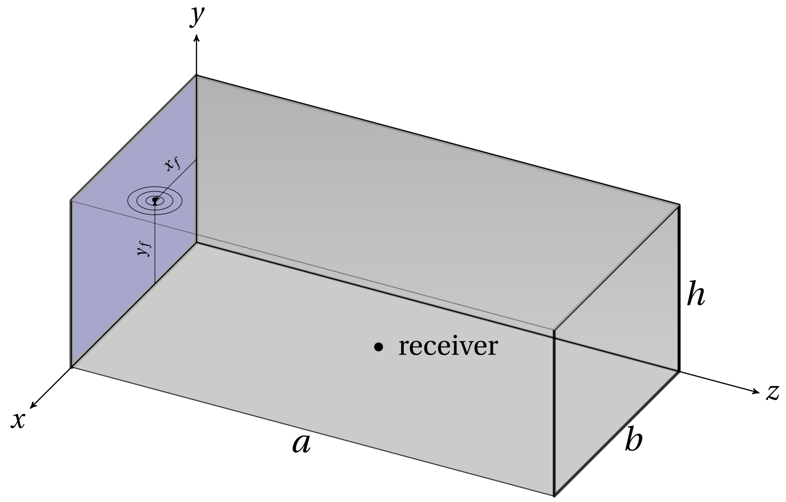

Figure 2: Cavity with elastic wall on the left side

plate at the position of $(x_f, y_f) = (0.1, 0.1)$ m for the frequency range of [60, 140] Hz. The elastic modulus of the plate is considered as uncertain parameter which is identified from the pressure response at the receiver position of $(0.13, 0.15, 0.88)$. The plate is assumed to be simply supported. The uncertain modulus is assumed to be lognormal distributed with $(\mu, \sigma) = (4.25, 0.03)$. Using the MC simulation, a large number samples of the parameter are generated and are used to realize the system responses. The FEM simulation of the problem constructed in ANSYS has been adopted as a black-box model to realize the responses. The random frequency responses function (FRF) of the pressure at the position of the receiver is given, as given in Fig. 3. As demonstrated, the uncertainty in the Young's modulus of the plate influences the FRF, particularly, yields in shifting in the position of the eigenfrequency as shown for the some sample realizations (dashed plots). The first natural frequency is considered as a random variables and is represented using the gPc expansion of third order. Accordingly, the statistical moments are calculated employing Eq. (15). The PDF of the first eigenfrequency and the first three central moments are shown in Fig. 4. A second order gPC expansion with unknown coefficients

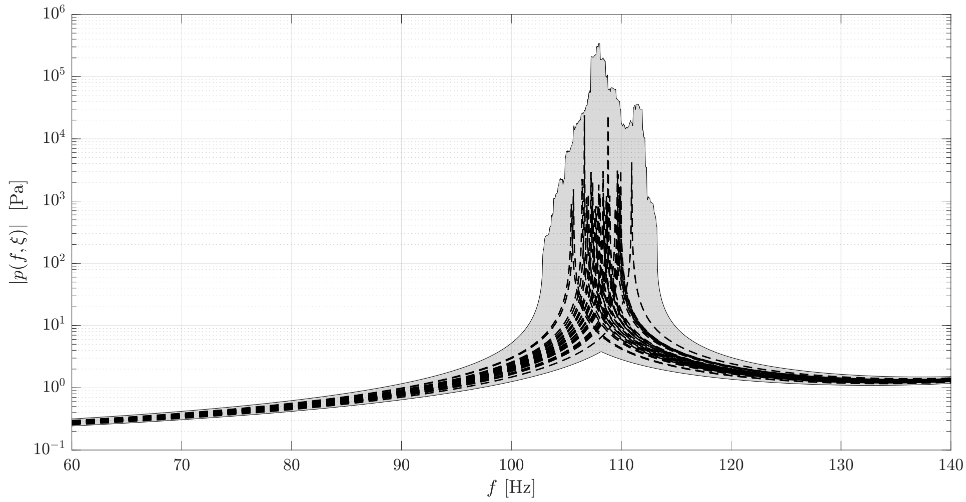

Figure 3: Bound of the random FRF at the receiver position due to uncertainty in Young's modulus of the plate (gray area). Dashed lines show samples of the pressure FRF at the receiver position.

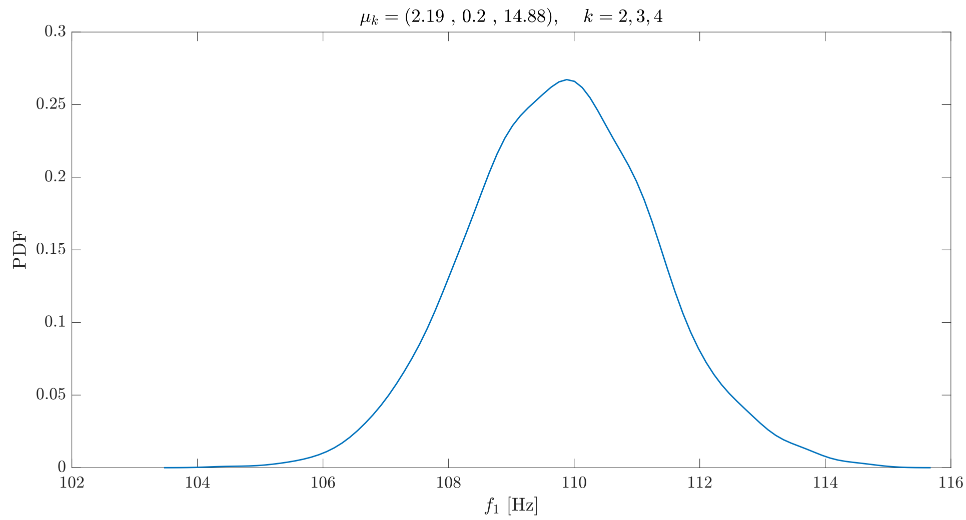

Figure 4: The PDF of the first eigenfrequency constructed from the third order gPC expansion. μ_k denotes the statistical central moments.

having Hermite orthogonal basis is used to approximate the uncertain modulus. The statistical moments of the gPC is derived as functions of the coefficients. The coefficients are then updated to achieve the moments of the frequency. The PDF of the identified parameter and the gPC coefficients are shown in Fig. 5. As shown, the second order gPC expansion has a high accuracy for representation of the uncertainty in the modulus comparing with the analytical PDF. The convergence rate of the gPC coefficients is very rapidly so that the higher order terms can be ignored without loss of the accuracy.

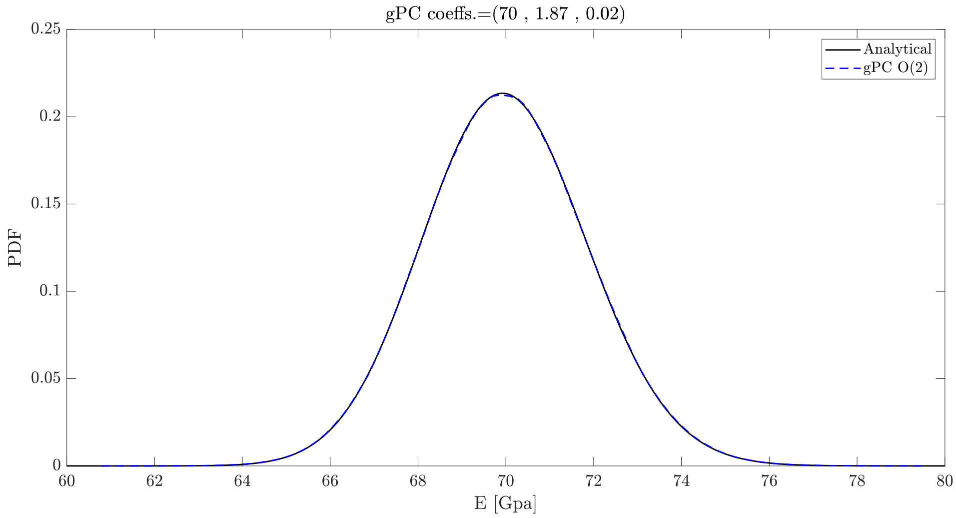

Figure 5: The identified PDF of uncertain Young's modulus and the gPC coefficients

5. CONCLUSIONS

Surrogate modeling provides the facility to use the available experimental data on the responses for identification of uncertain input parameters. This is independent of the model of the system undertaken which is considered as black-box solver. The gPC expansion as a classical surrogate model for representation of uncertain parameters has been used in this paper to quantify the uncertainty. For that, it has been assumed that the experimental data or limited realizations of the system responses are available, from which the input uncertain parameters are identified via an inverse optimization procedure. Statistical moments of the responses and uncertain parameters are employed to identify the gPC expansion. The application has been tested on a classical vibroacoustic problem with elastic wall having uncertain material parameter, Young's modulus. The parameter is identified from the harmonic response of the system at the first mode.

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