Shelf and Inventory Management with Space-Elastic Demand

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Abstract Managing limited shelf space is a core decision in retail as increasing product variety is in conflict with limited shelf space and operational replenishment costs. In addition to their classical supply function, shelf inventories have a demand generating function, as more facings lead to growing consumer demand. An efficient decision support model therefore needs to reflect space-elastic demand and logistical components of shelf replenishment. However shelf space management models have up to now assumed that replenishment systems are efficient and that replenishment costs are not decision relevant. But shelf space and inventory management are interdependent, e.g., low space allocation requires frequent restocking. We analyzed a multi-product shelf space and inventory management problem that integrates facing-dependent inventory holding and replenishment costs. Our numerical examples illustrate the benefits of an integrated decision model.

1 Introduction

Retailers need to manage complexity to match consumer demand with shelf supply by determining the interdependent problems of assortment size, shelf space assignments and shelf replenishments. Retail shelf space assignment has demand and inventory effects. The more space is assigned to products, the higher the demand (=space-elastic demand), and potentially the lower the replenishment frequency and vice versa. Traditional shelf space management models focus on space assignment and assume efficient inventory management systems. In other words, they decouple

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the shelf space assignment decision from replenishment. We therefore propose an extension to include restocking aspects in shelf space management. Our model ensures efficient and feasible shelf inventories, clarifies restocking requirements and enables the resolution of retail-specific problem sizes.

The remainder is organized as follows. We first provide a literature review, and then develop the model in section 3. Section 4 presents computational tests. Finally, section 5 discusses potential future areas of research.

2 Literature Review

Urban [9] takes into account inventory elastic demand, since sales before replenishment reduce the number of items displayed. A sequential model first optimizes shelf space assignment, and then restocking time. Hariga et al [3] determine assortment, replenishment, positioning and shelf space allocation under shelf and storage constraints for a limited four-item case. Abbott and Palekar [1] formulate an economic order quantity problem. The optimal replenishment time has an inverse relationship with initial space assignment and space elasticity. However, it requires an initial space assignment as input, and omits inventory holding costs. Yücel et al [10] analyze an assortment and inventory problem under consumer-driven demand substitution. They conclude that neglecting consumer substitution and space limitations has significant impact on the efficiency of assortments. Hübner and Kuhn [4] extend shelf space management models with out-of-assortment substitution and ensure appropriate service levels under limited replenishment capacities.

All models assume efficient restocking and omit restocking capacity constraints. They model instantaneous and individual restocking, i.e., the retailer immediately refills the empty shelf as soon as an item runs out of stock. A detailed discussion of shelf space models can be found in Hübner and Kuhn [6].

3 Model Formulation

The majority of consumers decide on their final purchases in store and make choices very quickly after minimal searches [2]. The demand for an item i is therefore a composite function of basic demand α_i, space elasticity β_i and out-of-assortment substitution μ_{ji} from j to i.

Items can be replaced immediately from backroom stock. Showroom inventory involves a set of products $i = 1, \ldots, I$ with display capacity S. Items are moved forward to the front row to avoid partial shelf-front depletion. Common retailer practice is to conduct joint replenishment of products, e.g., in the morning before the store opens [8, 7]. The costs of joint replenishment are assumed to be independent from the facing decision and non-decision relevant. To avoid lost sales, sales employees also refill any shelf gaps that arise between the basic refilling cycles. Accordingly, either demand d_{ik} of item i at facing level k exceeds supply q_{ik}, or vice versa. If demand exceeds supply, the insufficient basic supply requires refilling between periods t, or, if the opposite situation occurs, overstocked items increase capital employed. The third possibility, where demand matches supply exactly, is only theoretically possible, as supply is based on entire shelf slots. The following figure illustrates the development of shelf inventory levels and the associated refilling processes according to the demand-supply relationship. r_{ik} is the refilling volume and s_{ik} the overstocked volume.

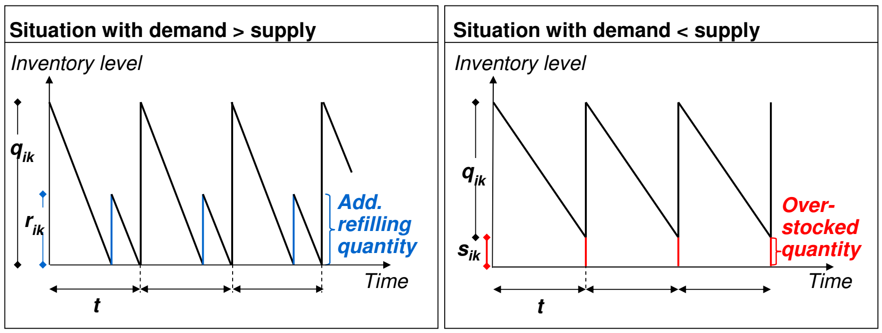

Fig. 1 Development of retail shelf inventory levels

The objective is to maximize product category profit. The *total profit* TP consists of TDP (total direct profit), TSP (total substitution profit), TCL (total listing costs), TCUS (total costs of undersupply) and TCOI (total costs of overstocked inventory).

$$\text{Max! TP} = \text{TDP} + \text{TSP} - \text{TCL} - \text{TCUS} - \text{TCOI} \tag{1}$$

The *total direct profit* covers the profit of items regardless of their relationship to the remaining assortment. d_{ik} is used to precalculate the demand for each item i and its associated facing level k, with $k = 0, 1, \ldots, K$. The preprocessing enables transfer of the non-linear demand function into a $0/1$ multi-choice knapsack problem where the binary variable y_{ik} selects the most profitable item-facing combination. p_i is the profit, b_i the breadth, and f_i the facing of the item i.

$$\text{TDP} = \sum_{i=1}^{I} \sum_{k=1}^{K} y_{ik} \cdot d_{ik} \cdot p_i \qquad \text{with} \quad d_{ik} = \alpha_i \cdot (b_i \cdot f_i)^{\beta_i} \tag{2}$$

The *total substitution profit* integrates profit from demand shifts for delisted items. The term $(\lambda_j \cdot d_{j1})$ symbolizes the latent demand for delisted items, with λ_j expressing a share, and d_{j1} the demand at one facing. The substitution matrix μ_{ji} integrates transition probabilities between items j and i. The binary variables y_{i0} and y_{j0} (i.e., $k = 0$) indicate whether an item is listed (set to 0) or delisted (set to 1).

$$\text{TSP} = \sum_{i=1}^{I} \sum_{\substack{j \neq i \\ j=1}}^{I} (\lambda_j \cdot d_{j1}) \cdot y_{j0} \cdot \mu_{ji} \cdot (1 - y_{i0}) \cdot p_i \tag{3}$$

Product listings induce fixed costs l_i for advertisement and layout changes.

$$\text{TCL} = \sum_{i=1}^{I} \sum_{k=1}^{K} y_{ik} \cdot l_i \tag{4}$$

The *total costs of undersupply* integrate the additional refilling requirements if demand is higher than supply, expressed by the refilling volume r_{ik}. The parameter h_i describes the capacity in units behind one facing, and RFC depicts refilling costs for one shelf slot.

$$\text{TCUS} = \sum_{i=1}^{I} \sum_{k=0}^{K} \frac{r_{ik}}{h_i} \cdot \text{RFC} \tag{5}$$

The *total costs of overstocked inventory* comprise capital costs for overstocked volume s_{ik}, i.e., where supply exceeds demand before the next basic shelf filling process. IR is an interest rate, and c_i stands for the product costs.

$$\text{TCOI} = \sum_{i=1}^{I} \sum_{k=1}^{K} s_{ik} \cdot c_i \cdot \text{IR} \tag{6}$$

The constraints are composed of hierarchical planning aspects and modeling requirements. (7) ensures that only the available space S can be distributed, with b_{ik} describing the facing-dependent breadth of item i. (8) and (9) define the volumes either for under- or oversupplied volumes. (10) allows only one facing level for each item.

$$\sum_{i=1}^{I} \sum_{k=1}^{K} y_{ik} \cdot b_{ik} \leq S \tag{7}$$

$$r_{ik} \geq y_{ik} \cdot (d_{ik} - q_{ik}) + \sum_{\substack{j \neq i \\ j=1}}^{I} \lambda_j \cdot d_{j1} \cdot y_{j0} \cdot \mu_{ji} \quad \forall i = 1,2,\ldots,I \quad \forall k = 0,1,\ldots,K \tag{8}$$

$$s_{ik} \geq y_{ik} \cdot (q_{ik} - d_{ik}) - \sum_{\substack{j \neq i \\ j=1}}^{I} \lambda_j \cdot d_{j1} \cdot y_{j0} \cdot \mu_{ji} \quad \forall i = 1,2,\ldots,I \quad \forall k = 0,1,\ldots,K \tag{9}$$

$$\sum_{k=0}^{K} y_{ik} = 1 \qquad \forall i = 1,2,\ldots,I \tag{10}$$

$$y_{ik} \in \{0;1\} \qquad \forall i = 1,2\ldots,I \quad \forall k = 0,1,\ldots,K \tag{11}$$

$$s_{ik}, r_{ik} \geq 0 \qquad \forall i = 1,2\ldots,I \quad \forall k = 0,1,\ldots,K \tag{12}$$

4 Numerical Examples

We apply test cases to assess the performance of our integrated model. We use data with a high profit-space correlation to evaluate the performance of a hard knapsack problem as in Hübner and Kuhn [5].

The test cases with 25 items demonstrate the profit impact over current industry practice, and the value of an integrated restocking and shelf space assignment model. Total profit increases by 17.5% compared to industry practice (lower bound), which could be quite substantial in low-margin industry retailing. Secondly, the example shows the value of an integrated restocking and shelf space assignment model. The model LS optimizes only for listing and spacing, i.e., supply costs are not part of the objective function, and are calculated a posteriori. Here, the test case reveals high undersupply costs leading to lower total profit. Disregarding supply costs results in frequent restocking needs.

Our tests also show that the solution structure (i.e., facing levels) changes significantly. 56% of the items receive different facing levels in the optimized LSR model compared to the lower bound.

Table 1 Numerical examples

Model [a]	**Lower bound**	**LS model**	**LSR model**
TDP [EUR]	343,275	404,177	408,669
TSP [EUR]	12,218	12,524	3,573
TCL [EUR]	2,600	3,400	4,000
TCUS [EUR]	4,850	20,396	10,159
TCOI [EUR]	9,376	11,731	162
Total [EUR]	338,667	381,175	397,921

[a] Lower bound: "space to sales" logic as in commercial software
LS: listing and spacing; LSR: listing, spacing and restocking

5 Conclusions and Future Research

Our model extends known shelf space models with replenishment costs and clarifies restocking requirements. It has been solved with CPLEX, and allows the computation of optimal results for category-specific problem sizes. The numerical example shows the benefits of an integrated model over current industry practice and aligns space and restocking requirements.

Areas of further research lie in investigation of the joint optimization of space assignment, instore replenishment cycles and order cycles for backroom replenishment. Additional possibilities are the integration of backroom capacities and inventory costs. Competitive scenarios and demand-influencing marketing effects could be part of an integrated analysis, as well as an extension to stochastic demand models.

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