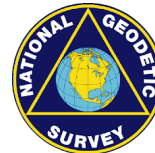
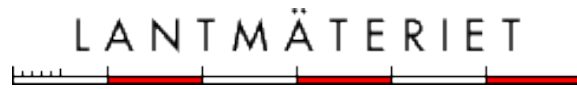


Advances in the realisation of the International Height Reference System

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Symposium SIRGAS2019
Rio de Janeiro, Brazil, Nov 11 - 14, 2019

Introduction

Results presented here are a joint effort of

- GGOS JWG: [Strategy for the realisation of the IHRS](#) (chair: L Sánchez)
- IAG JWG 2.2.2: [The 1 cm geoid experiment](#) (chair: YM Wang)
- IAG SC 2.2: [Methodology for geoid and physical height systems](#) (chair: J Ågren)
- ICCT JSG 0.15: [Regional geoid/quasi-geoid modelling - Theoretical framework for the sub-centimetre accuracy](#) (chair: J Huang)
- IAG JWG 2.1.1: [Establishment of a global absolute gravity reference system](#) (chair: H Wziontek)
- IAG regional sub-commissions for reference frames and geoid modelling
- IAG Commission 2 – Gravity Field (chair R Pail)
- International Gravity Field Service – IGFS (chair R Barzaghi)
- GGOS Bureaus of Networks and Observations – GGOS-BNO (chair: M Pearlman) and Products and Standards – GGOS-BPS (chair: D Angermann)

Introduction

A reference frame realises a reference system in two ways:

- physically, by a **solid materialisation of points** (or observing instruments),
- mathematically, by the **determination of coordinates** referring to that reference system.
- The coordinates of the points are computed from the measurements following the definition of the reference system.

In July 2015, the IAG released a resolution for the “**Definition and Realisation of a International Height Reference System (IHR)**”. During the last four years different actions have been conducted to

- Establish a global **reference network** for the IHR realisation: the International Height Reference Frame (IHRF)
- Evaluate different **strategies for the determination of reference coordinates** at the reference stations
- Identify **required standards, conventions and procedures** needed to ensure consistency between the definition (IHR) and the realisation (IHRF).

International Height Reference System (IHRIS)

IAG Resolution No. 1, Prague, July 2015

- 1) Vertical coordinates are **potential differences** with respect to a **conventionally fixed W_0** value:

$$C_P = C(P) = W_0 - W(P) = -\Delta W(P)$$

$$W_0 = \text{const.} = 62\,636\,853.4 \text{ m}^2\text{s}^{-2}$$

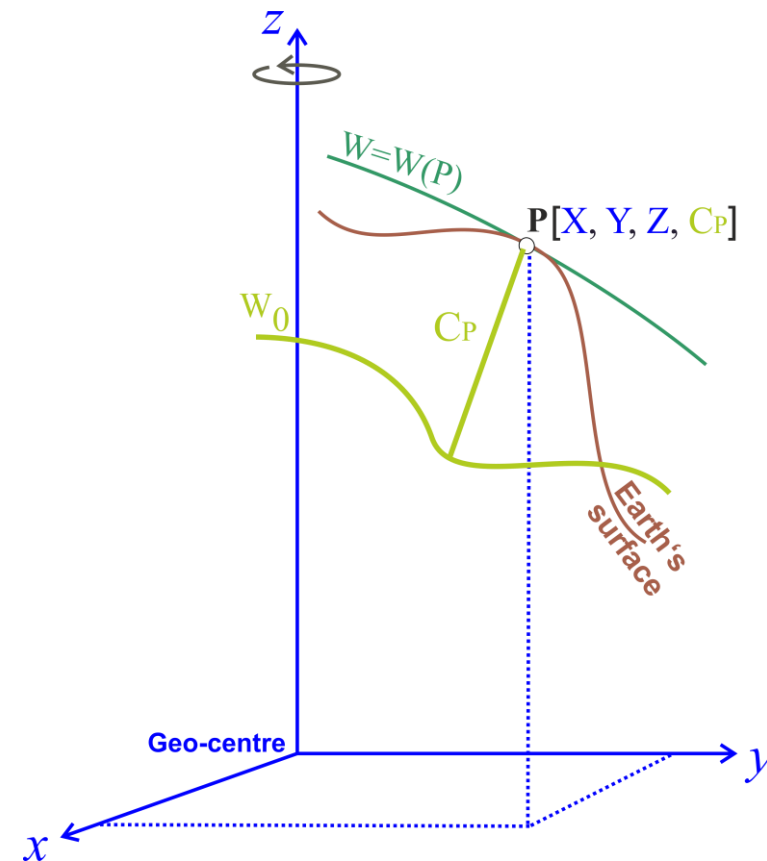
- 2) The position P is given in the ITRF

$$\mathbf{X}_P (X_P, Y_P, Z_P); \text{ i.e., } W(P) = W(\mathbf{X}_P)$$

- 3) The estimation of $\mathbf{X}(P)$, $W(P)$ (or $C(P)$) includes their variation with time; i.e., $\dot{\mathbf{X}}(P)$, $\dot{W}(P)$ (or $\dot{C}(P)$).

- 4) Coordinates are given in **mean-tide system / mean (zero) crust**.

- 5) The unit of length is the **meter** and the unit of time is the **second (SI)**.



See: Ihde J. et al.: *Definition and proposed realization of the International Height Reference System (IHRIS)*. *Surv Geophy* 38(3), 549-570, 10.1007/s10712-017-9409-3, 2017

Criteria for the IHRF reference network configuration

1) Hierarchy:

- A **global network** → worldwide distribution, including
- A **core network** → to ensure sustainability and long term stability
- **Regional and national densifications** → local accessibility

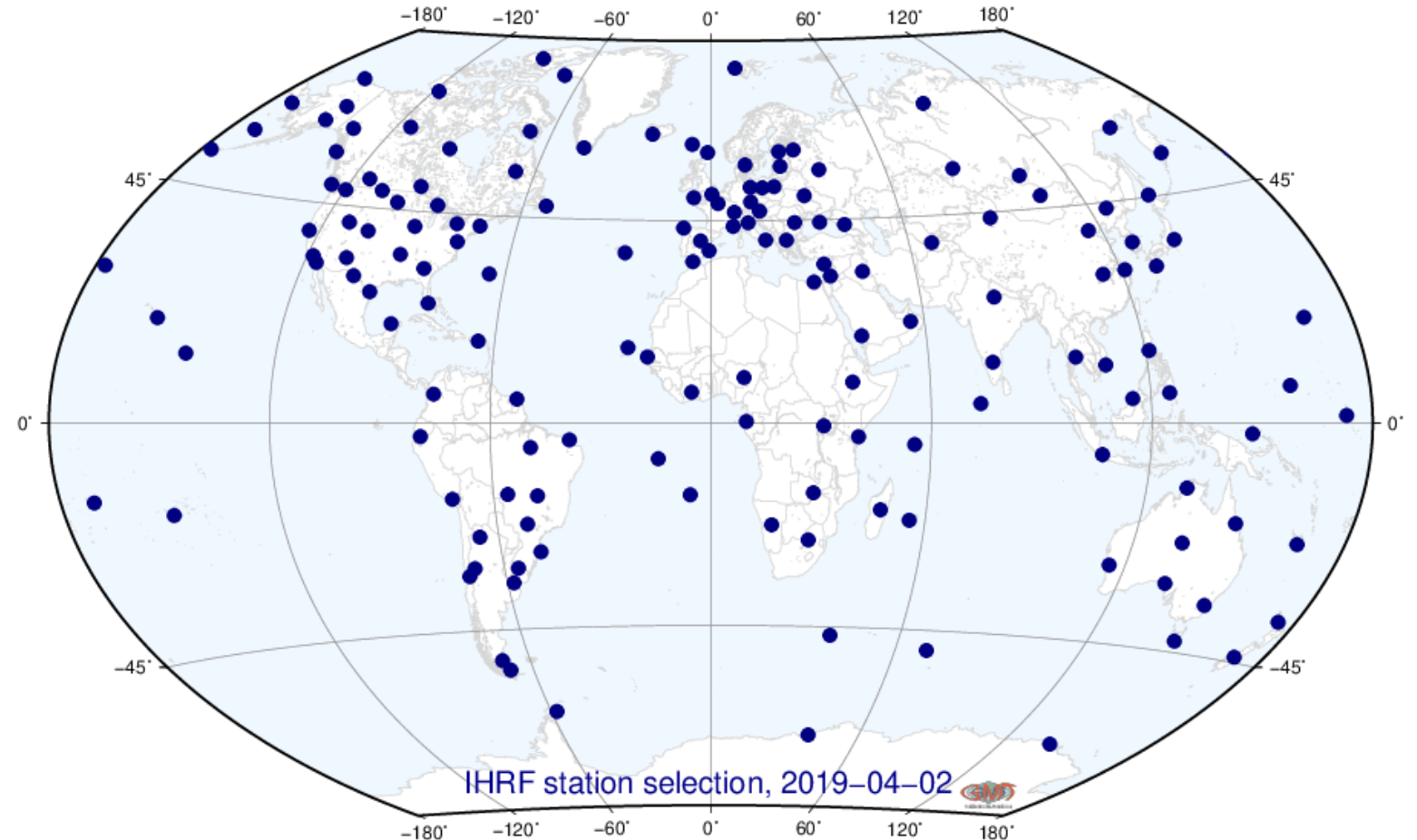
2) Collocated with:

- fundamental **geodetic observatories** → connection between \mathbf{X} , W , g and time realisation (reference clocks) → to support the GGRF;
- **continuously operating reference stations** → to detect deformations of the reference frame (preference for ITRF and regional reference stations, like SIRGAS, EPN, APREF, etc.);
- **reference tide gauges and national vertical networks** → to facilitate the vertical datum unification;
- reference stations of the new **International Gravity Reference Frame - IGRF** (see IAG Resolution 2, Prague 2015).

3) Main requirement: **availability of terrestrial gravity data around the IHRS reference stations for high-resolution gravity field modelling (i.e., precise estimation of W).**

First proposal for the IHRF core reference network (~170 stations)

- Station selection in agreement with the *GGOS Bureau of Networks and Observations*, the *Bureau Gravimétrique International* (absolute gravity stations), as well as with the IAG regional sub-commissions for reference frames and gravity field modelling.
- A *living network*: new stations and decommission of stations.
- To be *extended by regional/national densifications*.



Basic considerations on the IHRIS/IHRF coordinates

- 1) The IHRIS/IHRF is
 - a combination of a geometric component given by the *coordinate vector \mathbf{X} in the ITRS/IHRF* and
 - a physical component given by the determination of *potential values W at \mathbf{X}* .
- 2) The determination of \mathbf{X} follows the *IERS Conventions* and will not be further considered here.
- 3) According to the *GGOS Terms of Reference*, the expected accuracy of W is
 - Positions: $\approx \pm 3 \times 10^{-2} \text{ m}^2\text{s}^{-2}$ (about *3 mm*)
 - Velocities: $\approx \pm 3 \times 10^{-3} \text{ m}^2\text{s}^{-2}/\text{a}$ (about *0.3 mm/a*)
- 4) For the moment, our goal is $\pm 1 \times 10^{-1} \text{ m}^2\text{s}^{-2}$ (about *1 cm*)
- 5) The IHRIS/IHRF coordinates include the determination of time variations. For the moment, we consider *static coordinates only*.

Possibilities for the determination of potential values

1) Global Gravity Models of high degree (GGM-HD)

like the EGM2008 model (Pavlis et al., 2012, 2013) or the EIGEN-C series (e.g., Förste et al., 2012; 2014)

$$W(X, Y, Z) = \frac{GM}{r} \left[1 + \sum_{n=1}^{\infty} \left(\frac{a}{r} \right)^n \sum_{m=0}^n [C_{nm} \cos m\lambda + S_{nm} \sin m\lambda] P_{nm}(\cos\theta) \right] + \frac{1}{2} \omega^2 r^2 \cos(90^\circ - \theta)$$

- Expected accuracy (Rummel et al., 2014)
 - well surveyed regions: $\pm 0.4 \text{ m}^2\text{s}^{-2}$ to $\pm 0.6 \text{ m}^2\text{s}^{-2}$ (equivalent to $\pm 4 \text{ cm}$ to $\pm 6 \text{ cm}$)
 - sparsely surveyed regions: from $\pm 2 \text{ m}^2\text{s}^{-2}$... $\pm 4 \text{ m}^2\text{s}^{-2}$ ($\pm 20 \text{ cm}$ to $\pm 40 \text{ cm}$) to $\pm 10 \text{ m}^2\text{s}^{-2}$ ($\pm 1 \text{ m}$)
- This approach represents the “*ideal way*” to estimate potential values and hopefully, we will *get a better accuracy in the next future*. Ongoing studies with high expectation of improvement:
 - Combination of GGM with gravity effects of global topography (see Gruber’s presentation)
 - EGM2020

Possibilities for the determination of potential values

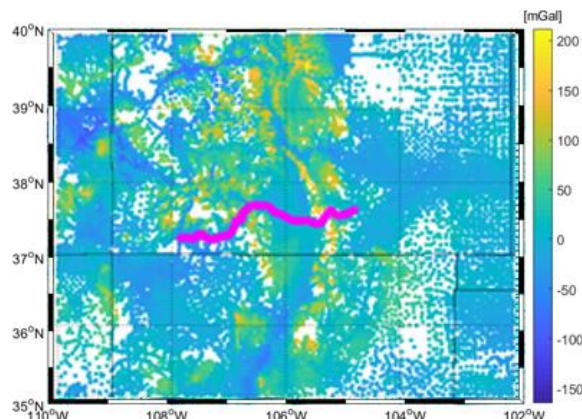
2) Disturbing potential

$$W_P = U_P + T_P \quad ; \quad T_P = T_{P,satellite-only} + T_{P,residual} + T_{P,terrain}$$

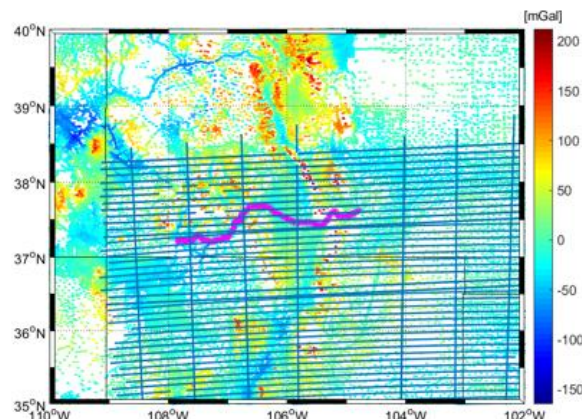
- GGM based on SLR, GRACE and GOCE are *very precise* ($\pm 1 \dots \pm 2$ cm @ 100 km)
- Mean omission error globally: $\approx \pm 45$ cm
- Goal is to *reduce these ± 45 cm to ± 1 cm* (only possible using terrestrial gravity data and considering topographic effects)
- The potential values realising the ITRS coordinates must be determined at the reference stations; i.e., at the Earth's surface and not at the geoid
 - With Molodensky's approach, the determination of W_P is straightforward
 - With Stokes' approach, the potential values should be 'moved' to the Earth's surface using the same hypotheses applied to reduce the observed gravity values to the geoid
- The determination of T_P demands a series of approximations, which influence the results; i.e., *different methodologies produce different potential values*

Comparison of computation methods

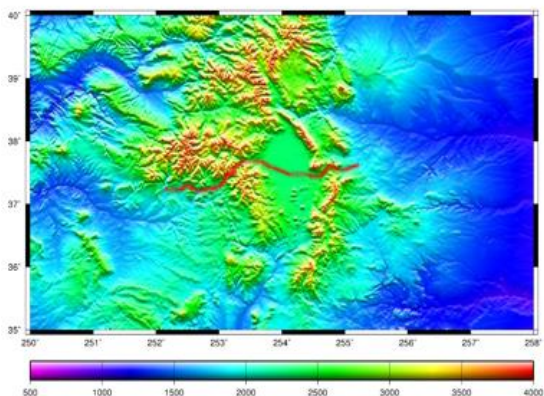
Colorado experiment: to compute *geoid*, *quasi-geoid* and *potential values* using exactly the same input data, a *set of basic standards*, and the *own methodologies* (software) of colleagues involved in the gravity field modelling.



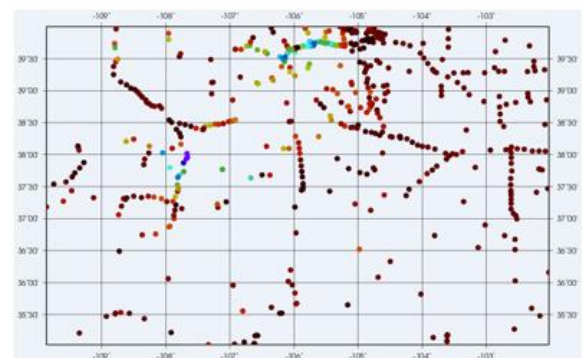
Surface gravity data (59,303 points)



Airborne gravity data
(41 lines E-W, 7 lines N-S)



Terrain model: SMRT V4.1



NGS *historical* GPS/levelling (510 points)

- Initiated in July 2017
- Data provided by US NGS
- Standards prepared by L Sánchez, J Ågren, J Huang, YM Wang, R Forsberg
- Three computations (two iterations) finished in June 2019
- Fifteen (final) contributing solutions
- Special Issue of the Journal of Geodesy with computation methods and comparison of geoid and quasi-geoid models (in preparation).

Colorado experiment: contributing solutions



Faculty of Engineering, Minia University, *Egypt*



İstanbul Teknik Üniversitesi, Istanbul, *Turkey*



Department of Geodesy and Surveying, Aristotle University of Thessaloniki, Thessaloniki, *Greece*



National Geodetic Survey, *USA*



Natural Resources Canada, *Canada*



Lantmäteriet, Swedish mapping, cadastral and land registration authority, *Sweden*



School of Earth and Planetary Sciences and The Institute for Geoscience Research, Curtin University, *Australia*



Escola Politécnica, Universidade de São Paulo; Centro de Estudos de Geodesia, *Brazil*



Deutsches Geodätisches Forschungsinstitut, Technische Universität München, *Germany*



Ingenieurinstitut für Astronomische und Physikalische Geodäsie, Technische Universität München, *Germany*



Chinese Academy of Surveying and Mapping, *China*



Politecnico de Milano, *Italy*



Faculty of Geodesy, University of Zagreb, *Croatia* - Research Institute of Geodesy, Topography and Cartography, *Czech Republic*



National Space Institute, Technical University of Denmark, *Denmark*



Geography and Crustal Dynamics Research Center, Geospatial Information Authority of Japan, *Japan*

Colorado experiment: summary of approaches and models

- Least squares modification of Stokes' formula with additive corrections (2)
- Least squares modification of Stokes' formula with additive corrections and biased Stokes' kernel modification
- Stokes' formula with Wong-Gore modification and 1D-FFT (2)
- Spherical radial basis functions
- Least squares collocation
- Fast collocation based on gravity gridded data
- Degree weighted Stokes' integral
- Modified degree-banded Stokes' kernel (2)
- Spherical FFT with modified Wong-Core Stokes' kernel
- UNB Stokes-Helmert scheme
- UNB Stokes-Helmert scheme with hybrid-Meissl-Molodensky modified spheroidal Stokes' kernel
- NGS Molodensky approach, Spherical Harmonics Analysis (SHA)
- GGMs: GOCO05s, XGM2016, XGM2018, xGEOID17B, EIGEN-6C4, EGM2008
- Topographic effects based on SRTM V4.1, EARTH2014, COLH19M05, ERTM2160
- 12 solutions based on height anomalies, 3 solutions based on geoid undulations

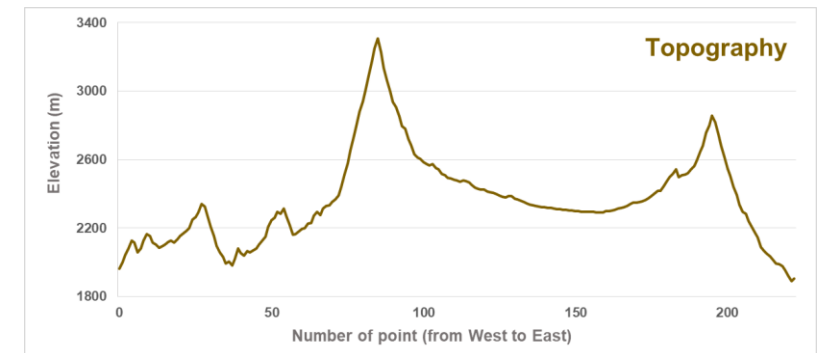
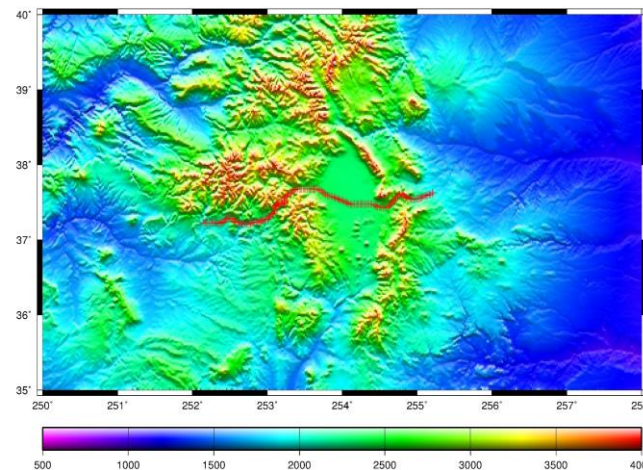
Colorado experiment: comparison of potential values

- 1) The comparison is carried out at 223 GSVS17 marks (Geoid Slope Validation Survey 2017) selected by NGS
- 2) Participants in the experiment got φ , λ , h ; levelling is not available (yet)
- 3) The potential values provided by the different solutions are converted to **geopotential numbers** with respect to the IHRM W_0 value

$$C(P) = W_0 - W(P) \quad ; \quad W_0 = 62\,636\,853.4 \text{ m}^2\text{s}^{-2}$$

- 4) and further transformed to **normal heights** (to see the differences in meters):

$$H^*(P) = C(P) / \gamma(P)$$



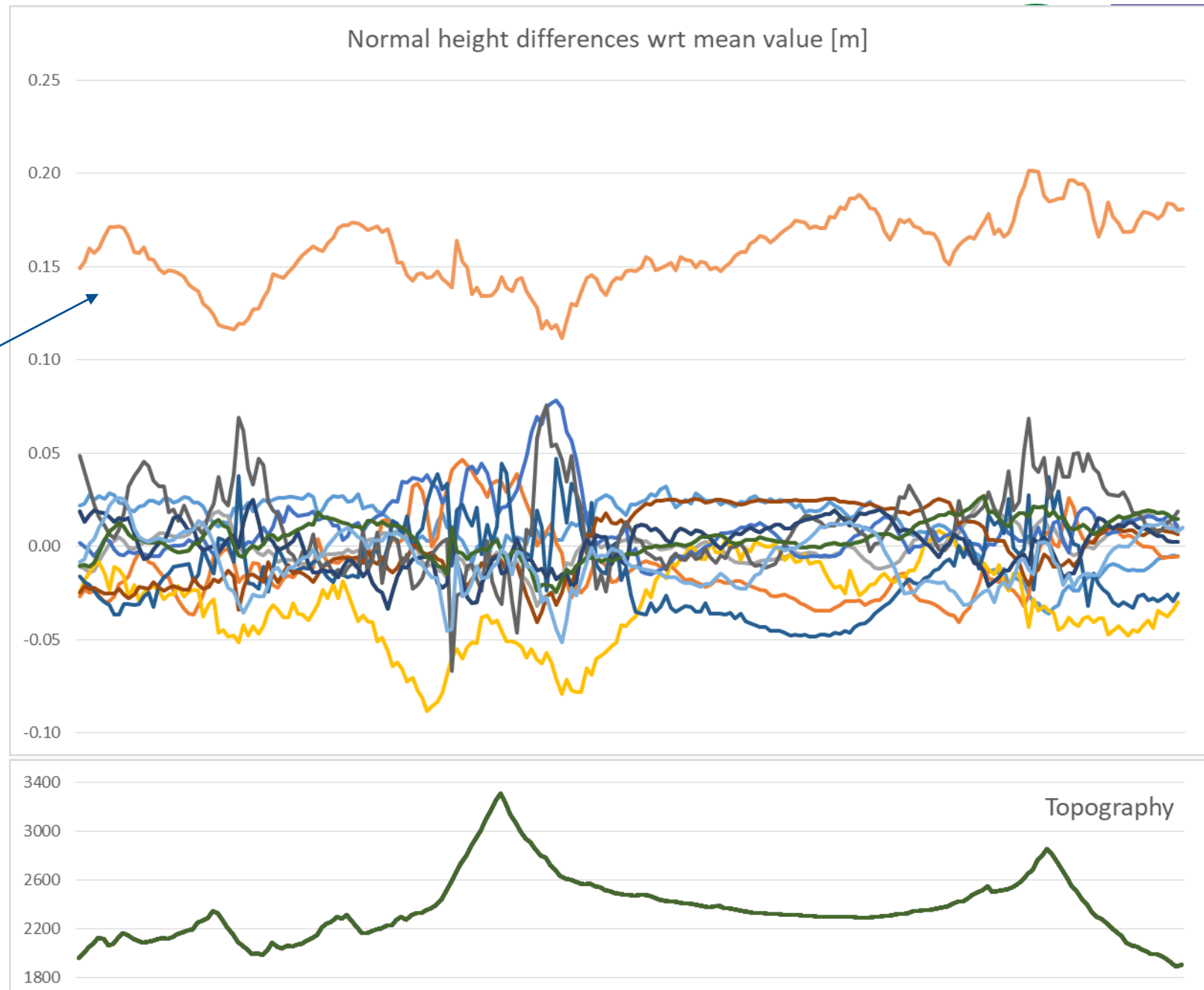
Colorado experiment: comparison of potential values

Normal height difference [cm]
(individual contribution – mean)

Outlier 1
Mean : 15.7 ± 1.9 cm
Range: 8.9 (11.2 ... 20.0)

Zero-degree term: 17.85 cm

$$\zeta_0 = \frac{(GM_{\text{GGM}} - GM_{\text{GRS80}})}{r_P \cdot \gamma_Q} - \frac{\Delta W_0}{\gamma_Q}$$



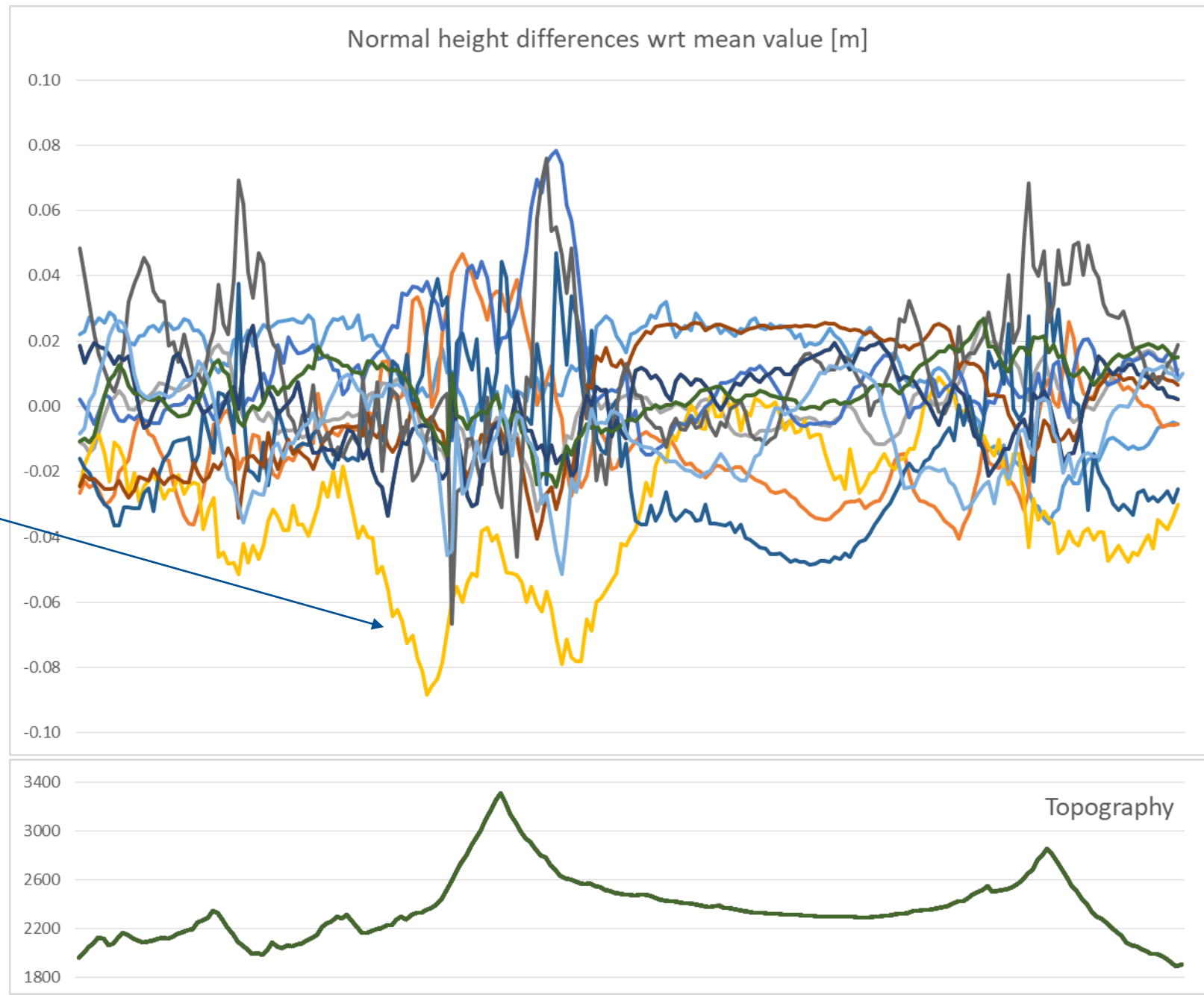
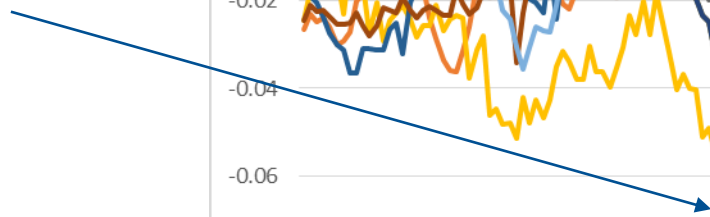
Colorado experiment: comparison of potential values

Normal height difference [cm]
(individual contribution – mean)

Outlier 2

Mean : -3.2 ± 2.1 cm

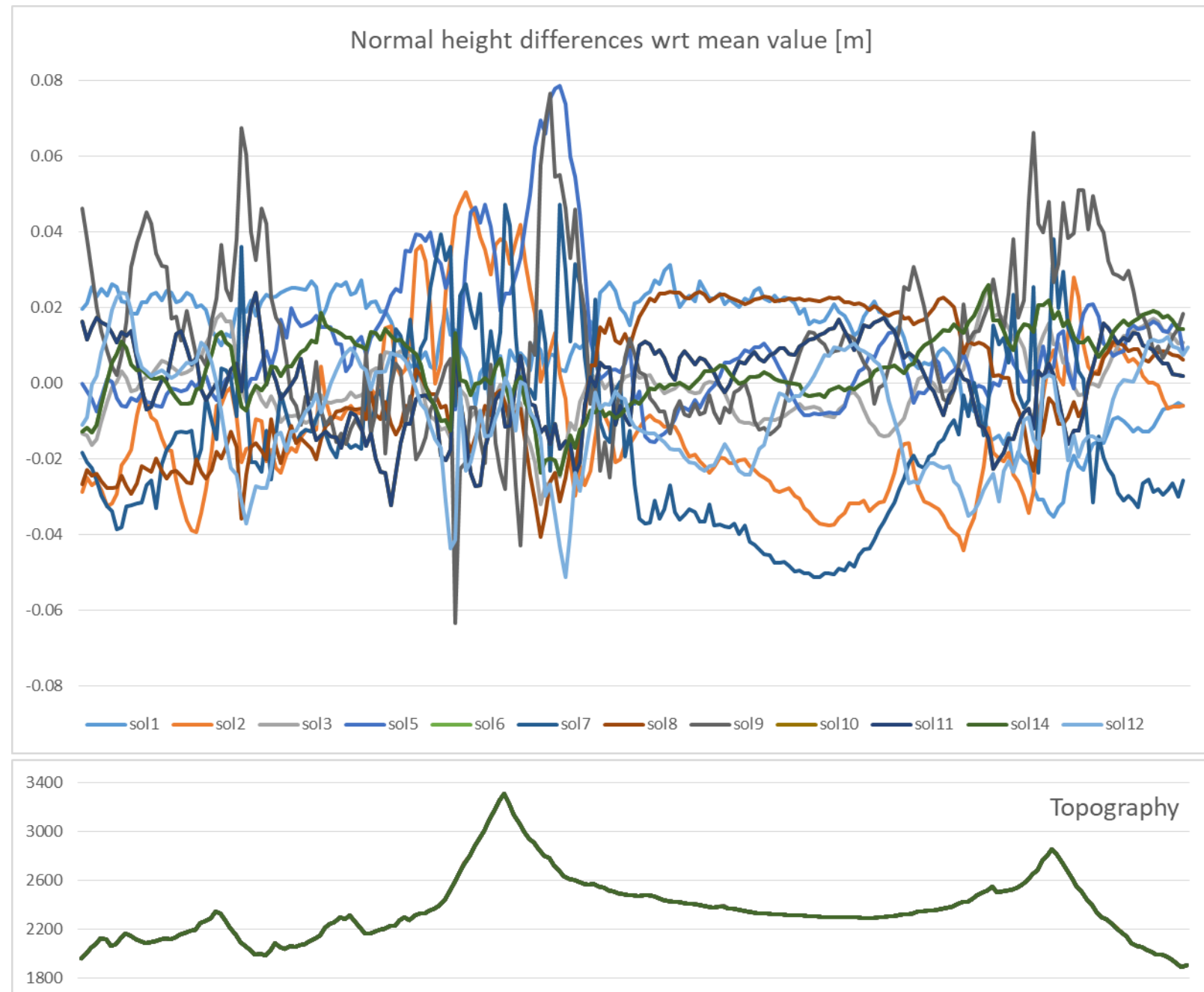
Range: 9.3 (-8.7 ... 0.6)



Colorado experiment: comparison of potential values

Normal height difference [cm]
(individual contribution – mean)

	Mean	±	STD	Range
sol1	1.0	±	1.6	6.6 (-3.5 ... 3.1)
sol2	-1.0	±	2.1	9.5 (-4.4 ... 5.1)
sol3	-0.1	±	1.0	5.8 (-3.2 ... 2.6)
sol5	1.0	±	1.8	9.4 (-1.6 ... 7.9)
sol6	0.4	±	1.0	5.3 (-2.7 ... 2.6)
sol7	-1.4	±	2.3	9.9 (-5.1 ... 4.7)
sol8	0.0	±	1.8	6.5 (-4.1 ... 2.4)
sol9	1.1	±	2.2	14.0 (-6.3 ... 7.7)
sol10	0.0	±	1.2	7.5 (-3.2 ... 4.3)
sol11	0.0	±	1.1	5.6 (-3.2 ... 2.4)
sol12	-0.9	±	1.4	7.5 (-5.1 ... 2.4)
sol14	0.4	±	1.0	5.0 (-2.4 ... 2.6)



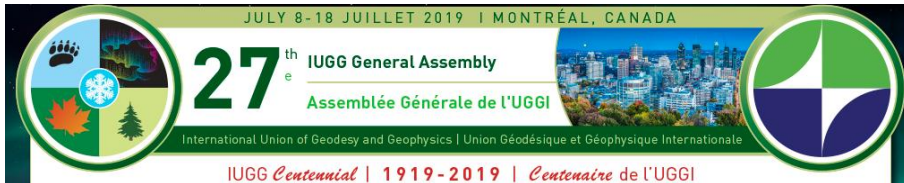
Learnings from the Colorado experiment

- 1) Twelve solutions agree within ± 1 cm to ± 2 cm *in terms of standard deviation with respect to the mean value.*
- 2) Results show the *discrepancies between computation methods.* Input data are assumed free of error. A *proper error propagation analysis* is still an open question.
- 3) We *are waiting for the levelling results* along the test profile to make comparisons with independent data.
- 4) Discrepancies between the different solutions are *highly correlated with the topography*
 - Handling of terrain gravity effects (model and strategy),
 - Degradation due to the conversion from geoid to quasi-geoid.
- 5) Difficulties reported by the colleagues contributing to the experiment
 - Processing of the *airborne gravity data*,
 - Handling of the *zero-degree term.*
- 6) A major confusion is the reference ellipsoid: which should be used *GRS80 or WGS84?*
 - *Are we needing a new reference ellipsoid?*

Outlook

- 1) Based on the contributing solutions to the Colorado experiment, to generate a detailed document with standards for the determination of potential values.
- 2) To investigate
 - better strategies for the modelling of topographic effects
 - the determination of potential changes with time \dot{W}
 - Reliable approaches for the accuracy assessment
- 3) To compute a first static solution for the IHRF core network to evaluate the achievable accuracy under the present conditions (data availability, computation methods, etc.) and to identify key actions to improve the determination of the IHRS/IHRF coordinates.
- 4) To extend the realisation of the IHRS to marine areas.
- 5) To explore the possibilities to establish an 'IHRS/IHRF element' within the IGFS to ensure the maintenance and availability of the IHRF:
 - Regular updates of the IHRF_{yyyy} to take account for:
 - new stations;
 - coordinate changes with time $\dot{\mathbf{X}}$, \dot{W} ;
 - improvements in the estimation of \mathbf{X} and W (more observations, better standards, better models, better computation algorithms, etc.)

IAG Resolution No. 3, 2019: Establishment of the IHRF



Resolution 3: Establishment of the International Height Reference Frame (IHRF)

The International Association of Geodesy,

Considering,

- The IAG Resolution for the Definition and Realization of an International Height Reference System (IHRF) released at the 26th IUGG General Assembly in July 2015;

Acknowledging,

- The achievements of
 - GGOS Focus Area “Unified Height System” and its JWG 0.1.2 “Strategy for the Realization of the International Height Reference System (IHRF)”,
 - IAG JWG 2.2.2 “The 1 cm geoid experiment”,
 - IAG SC 2.2 “Methodology for geoid and physical height systems”,
 - ICCT JSG 0.15 “Regional geoid/quasi-geoid modelling - Theoretical framework for the sub-centimetre accuracy”;
- in realizing this resolution;

Noting,

- The need of an operational infrastructure to ensure the determination, maintenance and availability of an International Height Reference Frame (IHRF) in the long-term basis;

Urges,

- All countries to engage with the IAG and concerned components, in particular the International Gravity Field Service (IGFS), in order to promote and support the implementation of the IHRF by
 - Installing IHRF reference stations at national level,
 - Conducting the necessary gravimetric surveys to guarantee the precise determination of potential values,
 - Making data available open access,
 - Contributing to the development of analysis strategies to improve the estimation of reference coordinates and modelling of the Earth’s gravity field,
 - Describing, archiving and providing geodetic products associated to the IHRF.