16. D-A-CH Tagung Erdbebeningenieurwesen & Baudynamik (D-A-CH 2019)
26. und 27. September 2019
Universität Innsbruck

MODELLING THE DYNAMIC SOIL-STRUCTURE INTERACTION FOR THE ROCKING OF RIGID FAÇADES

Francesca Taddei[1]*, Linda Giresini*[2] *and Gerhard Müller*[1]

[1] Lehrstuhl für Baumechanik, Technische Universität München, francesca.taddei@tum.de and gerhard.mueller@tum.de
[2] Department of Energy, Systems, Territory and Constructions Engineering, University of Pisa, linda.giresini@unipi.it

ABSTRACT
This contribution presents a numerical model for the simulation of the dynamic soil-structure interaction (SSI) for the seismic out-of-plane rocking behavior of rigid façades. The method already developed in previous works of the authors is extended including the horizontal foundation-soil flexibility due to dynamic interaction between soil, foundation and structure. The flexibility of the soil is simulated via lumped parameter models (LPM), which are tuned combinations of masses, springs and dashpots able to approximate the behavior of the actual soil-foundation system. This contribution presents the equations of motion of the coupled system façade-foundation-soil, and the most important mechanical parameters of the model are discussed.

Keywords: rocking, soil-structure interaction, lumped parameter models, out-of-plane

1. INTRODUCTION

Strong seismic events induce large structural lateral loads and therefore serious overturning moment. Many studies were recently performed on the rocking behavior of masonry walls considering several restraints, such as single or smeared horizontal and vertical restraints [8]. These works were inspired by the seminal Housner's work [9], which stated the equation of motion of the single degree of freedom system and the method to take into account the energy dissipation over motion. The dissipation of energy is considered through an analytical coefficient of restitution that reduces the velocity after each impact and is related to the slenderness ratio of the rocking wall.

Early studies on the rocking response of a rigid block supported on a base undergoing horizontal accelerated motion were presented by Housner [1]. He represented the oscillation of a rocking block subjected to root-point excitation through an inverse pendulum considering energy dissipation due to impacts, as shown in Figure 1. He showed that the rocking impacts reduce the velocity after each impact and this reduction is related to the slenderness ratio of the rocking wall. He also showed that there is a dependency between the frequency content of the root-point excitation (seismic pulse) and the slenderness of the block [2].

The observation that, beside the maximum seismic acceleration, also the duration of the seismic pulse plays an important role on the overturning behavior of a rigid block, indicates that the problem of the rocking of rigid blocks depends also on the characteristic frequencies of the vibrational behavior. It is well known that the presence of a flexible soil underneath a structure influences its frequency-dependent behavior and introduces a vibrational damping (called geometrical damping). Therefore, the soil-structure interaction can be very important for such a problem. In this contribution, we lay the foundation of the study of the dynamic soil-structure interaction effects for the rocking of rigid façades, by focusing on the horizontal flexibility of the soil-foundation system. This basic considerations can be used to extend the problem to all the degrees of freedom of the foundation and the coupling between them.

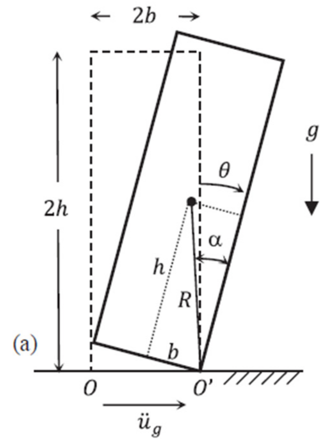

Figure 1: Housner model.

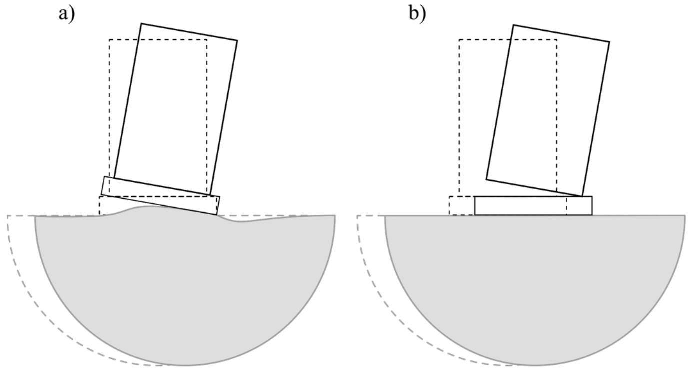

Figure 2: Possible uplift scenarios for a block-foundation-soil system subjected to an horizontal seismic action.

Soil-structure interaction (SSI) analyses assess the interplay between the structure and its underlying soil, usually connected through the foundation. SSI effects can be defined as the difference between the response of a building-soil system and the response of the building as rigidly constrained at its base. The contact stress distribution between structure and soil varies with the vibration frequency of the excitation. The motion of a body resting on the soil involves a mixed boundary conditions problem and rigorous solutions only exist for idealized cases. Besides the rigorous theoretical methods, it was found that the frequency-dependent behavior of the foundation-soil system can be approximated as a mass-spring-dashpot oscillator, also called lumped parameter models (LPM), with frequency-independent parameters. Thanks to the similarity of this model with the Housner model, the LPM can be easily coupled to the inverted pendulum model and it is used in this study to represent the foundation-soil subsystem.

There are mainly two approaches to deal with the rocking behavior of rigid blocks subjected to horizontal seismic actions including the foundation-soil subsystem: 1) it is assumed that the connection between the foundation and the structure is strong enough to cause the partial uplift at the soil-foundation interface and a plastic deformation of the soil surface (Figure 2a), 2) the connection between soil and foundation is strong enough to cause the uplift of the block at the block-foundation interface (Figure 2b). The first scenario is more representative for shallow foundations, while the second for embedded foundations or pile foundations.

The first scenario was investigated in several previous studies. Wolf [3] studied the rocking of a flexible single-degree-of-freedom (SDOF) system including SSI and uplift at the soil-foundation interface. The rocking resistance of the flexible SDOF systemwas characterized by its mass and stiffness while for a rigid block the resistance to the rocking is given by the restoring component of the weight. Wolf proposed a procedure to calibrate the soil reactions as a function of the reduced contact area between foundation and soil, which in turn depends on the rocking angle. The soil reactions under the conditions of detachment from the soil with subsequent uplifting of the foundation can be computed either with simplified LPM [3] or with rigorous analytical methods [4].

For the second scenario, Housner's theory can be applied considering that the rocking motion occurs on a foundation which moves with absolute displacement u, when subjected to the seismic input u_g. In this study we present the equations for this second scenario, adapting the method described in [5]. We assume that when the angle of rotation reverses, the rotation of the block continues smoothly from point O to O’ and that the impact force is concentrated as a point force which applies on the new pivot point O’.

2. LUMPED PARAMETER MODEL FOR THE SOIL-STRUCTURE INTERACTION

A LPM is a block of springs, dashpots and masses, able to reproduce the dynamic behavior of a soil-foundation system. Its real frequency-independent coefficients are found by approximating the dynamic stiffness, or its flexibility (or compliance), by a ratio of two polynomials. The optimal polynomial coefficients can be found using the least-squares method, which minimizes the error between the target compliance functions and the approximated ones. The obtained polynomial fraction is decomposed into simpler fractions through a partial fraction expansion. These minimum-order fractions can be associated with basic spring-dashpot elements, which are then connected in series or parallel. The general requirements for the successful application of LPMs are:

- the frequency-domain compliance or impedance functions of the foundation-soil system are known, either computed through rigorous simulation (for instance FEM/BEM coupling or comparable methods) or estimated through field measurements;
- the foundation can be considered stiff compared to the underlying soil;
- the coupling between vibrational modes of the foundation is negligible. For each direction (horizontal, vertical and rocking) an independent LPM is generated.

Simple LPM are already available in literature [6]. The accuracy of the model increases with the number of used parameters, and for the response of a rigid foundation resting on a homogenous half space the minimum number of parameters is two. The higher gets the number of parameters, the more complex becomes its response in the frequency domain and stability issues may arise [7].

For the sake of illustration, we assume in this study a prismatic foundation embedded into an (undamped) homogenous half space and we only consider its horizontal compliance, neglecting all the other degrees of freedom. In this case, the coefficients of the LPM for the horizontal translation of the foundation are already available in [6] and the LPM consists simply of a spring and a dashpot, therefore it involves only two parameters. The model represents the foundation-soil system accurately up to a circular frequencies equal to 5 c_s/l, where is c_s is the shear wave velocity of the soil and l is a reference length for the prismatic foundation.

The LPM for the horizontal oscillation of a massless prismatic rigid foundation:

$$k = \frac{Gd}{2-\nu}\left[6.8\left(\frac{l}{d}\right)^{0.65} + \frac{0.8l}{d} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{d}}\left(\frac{e}{d}\right)^{0.8}\right)\right]$$
$$c = \frac{d}{c_s}k\left[0.75 + 0.2\left(\frac{l}{d} - 1\right)\right] \quad (1)$$

where G is the shear modulus of the soil and ν its Poisson's ratio. d is the half-width of the foundation, l is the half-length in the in-plane direction of the block and e is the embedment of the foundation. The mass of the foundation is calculated as $m_f = 4ldt\rho_f$, where t is the thickness of the foundation and ρ_f the density. The LPM for other foundation-soil systems and other motions can be systematically constructed as explained in [8] and [7].

Depending on the type of parameters of the foundation-soil model, the seismic excitation is applied using different approaches. If the parameters of the LPM are only spring and damping coefficients, the seismic input can be directly applied to the far end of the LPM, resulting in the total displacement of the structure–soil system in one-step analysis. If mass parameters are added for an improved behavior, a two-step analysis is necessary, where firstly the seismic loads at the foundation level are computed using the LPM only, and secondly the obtained seismic loads are applied to the coupled system. This second procedure is described in Wolf [9] but it is not considered here, as the chosen LPM is built using springs and dashpots only. Therefore, the seismic excitation using the proposed LPM is directly applied at the far end of the LPM. Figure 3 shows the dynamic amplification of the LPM for an embedded prismatic foundation subjected to a seismic excitation at the far-end, for different values of the stiffness of the soil. The functions decrease at higher frequencies and for soft soils (e.g. G=10 MN/m^2) the amplification factor is reduced of more than 15% already from 10 Hz.

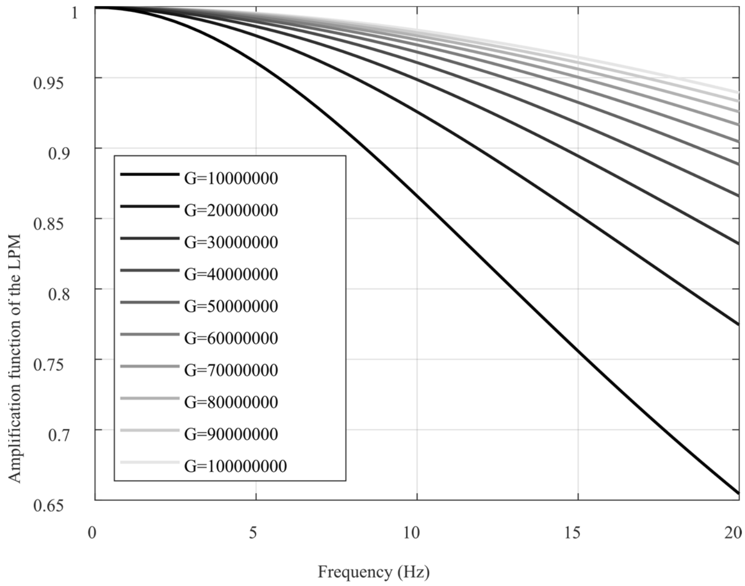

Figure 3: Dynamic amplification factor for base-excitation u_g for different G moduli given in N/m^2, for ν=0.33, d=1 , l=1, e=1.

3. EQUATIONS OF MOTION

The Housner model is adapted combining the original formulation with the additional effects due to the presence of the soil, as shown in Figure 4a. When the far end of the LPM is excited with a transient excitation ($\ddot{u}_g$, $\dot{u}_g$, u_g) in horizontal direction, the foundation moves with displacements $\dot{u}$, velocities $\dot{u}$ and accelerations $\ddot{u}$, as shown in Figure 4b.

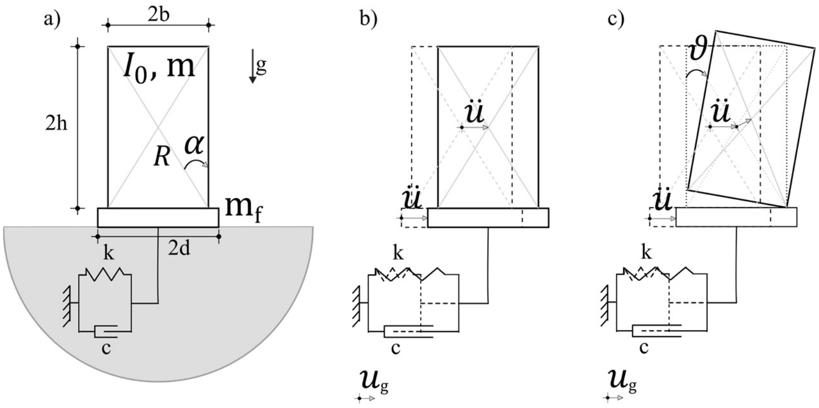

Figure 4: Housner model coupled to an horizontal LPM, representing the foundation-soil interaction.

The rocking of the block (Figure 4c) starts only under the following condition:

$$\ddot{u} > \frac{b}{h} g \tag{2}$$

For the purely translational behavior, the horizontal equilibrium reads:

$$\left(m + m_f\right)\ddot{u} + c(\dot{u} - \dot{u}_g) + k(u - u_g) = 0 \tag{3}$$

By bringing the known terms related to the seismic input on the right hand side, the equation becomes:

$$(m+m_f)\ddot{u}+c\dot{u}+ku=\underbrace{c\dot{u}_g+ku_g}_{P_{seism}} \tag{4}$$

where P_{seism} represents the equivalent seismic loads transferred to the block through the soil and the foundation.

For the combined rocking and translational behavior, additional terms related to centripetal force, inertial force and restoring moment are introduced and the equilibrium is written for the horizontal motion and for the rocking motion.

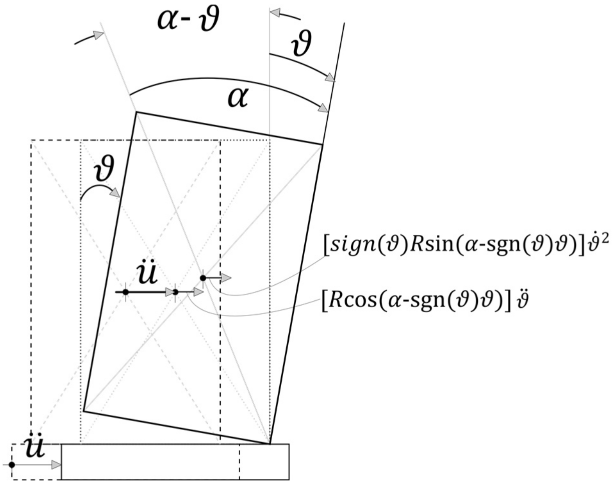

Figure 5: Rocking of the rigid block on horizontally flexible foundation-soil system.

In this case, the horizontal equilibrium becomes:

$$(m+m_f)\ddot{u}+c\dot{u}+ku+m[Rcos(\alpha-sgn(\vartheta)\vartheta]\ddot{\vartheta}+\ m[sgn(\vartheta)Rsin(\alpha-sgn(\vartheta)\vartheta]\dot{\vartheta}^2 \\ =P_{seism} \tag{5}$$

while the rocking equilibrium reads:

$$I_0\ddot{\vartheta}+\ mg[sgn(\vartheta)Rsin(\alpha-sgn(\vartheta)\vartheta]+m[Rcos(\alpha-sgn(\vartheta)\vartheta]\ddot{u}=0 \tag{6}$$

with $I_0=\frac{4}{3}m\,R^2$ being the moment of inertia of the block with respect to the corner. The problem leads to two explicit variables $\ddot{\vartheta}$ and $\ddot{u}$, and four state variables ϑ, u, $\dot{\vartheta}$ and $\dot{u}$.

Additionally, we introduce the definition of the impact event at the beginning of the rocking. Similarly to [2] and [10], we assume that the impact is perfectly inelastic and it occurs in an infinitesimally short time and the impulsive forces are much larger than the other forces in the system. We assume that the impact occurs only at the corner of the block and there is no sliding between block and foundation, but only rocking. Shortly before the impact, ϑ is zero and $\dot{\vartheta}^b$and $\dot{u}^b$are the velocities before the impact. During the impact between the rocking block and the foundation, the conservation of linear momentum is applied and the velocities right after the impact, $\dot{\vartheta}^a$and $\dot{u}^a$ are:

$$\dot{\vartheta}^a=e_1\dot{\vartheta}^b \tag{7}$$

$$\dot{u}^a=\dot{u}^b+e_2\dot{\vartheta}^b \tag{8}$$

where the two confidents of restitution, e_1 and e_2, are equal to:

$$e_1=\frac{4\bar{m}\lambda-2\bar{m}+\lambda-2}{4\bar{m}\lambda+4\bar{m}+\lambda+4}\ ;\quad e_2=\frac{6mhb^2}{4\bar{m}\lambda+4\bar{m}+\lambda+4} \tag{9}$$

with $\lambda=h/b$ and $\bar{m}=m_f/m$.

4. NUMERICAL EXAMPLE

Let us consider a rigid façade of thickness equal to 1.2 m with a rigid prismatic foundation resting on a homogenous half space. Table 1 and Table 2 contain all the parameters of soil, foundation and façade.

Table 1. Properties of the soil and foundation.

Soil	#1	#2
G [MN/m^2]	10	80
ρ [kg/m^3]	2000	2000
ν [-]	0.33	0.33
t [m]	2	2
$2d$ [m]	1	1
$2l$ [m]	1	1
m_f [-]	5000	5000

Table 2. Geometric parameters of the analyzed facade (symbols explained in Section 2; λ slenderness of the block = α^{-1}).

Block	#1	#2	#3
$2h$ [m]	12.00	20.00	10.00
$2b$ [m]	1.00	1.000	1.000
R[m]	6.03	10.018	5.036
α[rad]	0.0996	0.059	0.12
m[Kg]	3.23E+04	5.38E+05	2.69E+05
λ [-]	12	20	10
I_0 [N m s^2]	1.57E+06	5.36E+07	9.01E+07

4.1. Seismic input

The analysis was performed by considering a natural seismic input recorded on 2016, Oct. 26th during the 2016-2017 Central Italy Earthquake and scaled to a maximum acceleration of 0.5 g. More details about the selected accelerograms in terms of the most relevant Intensity Measures of the seismic shocks are specified in [12] . Figure 6 and Figure 7 shows the accelerations, the displacements and the spectrum of the displacements for the selected accelerograms.

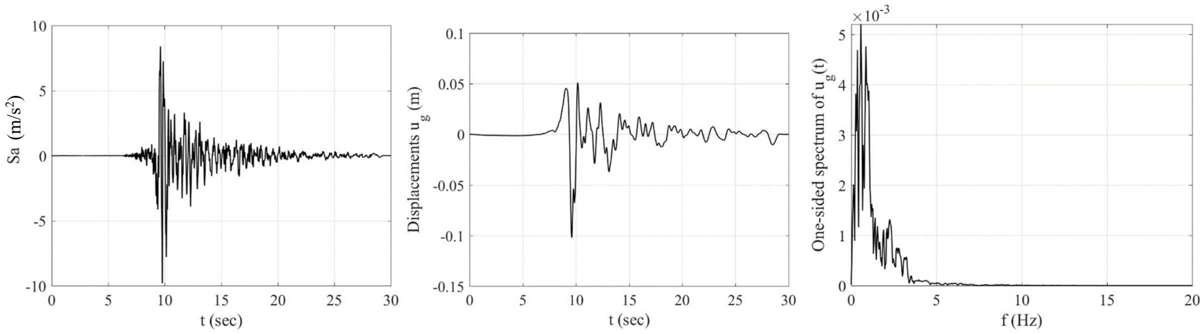

Figure 6: Input accelerations (left), input displacements (middle) and spectral values of the input displacements (right) for the station CMI.

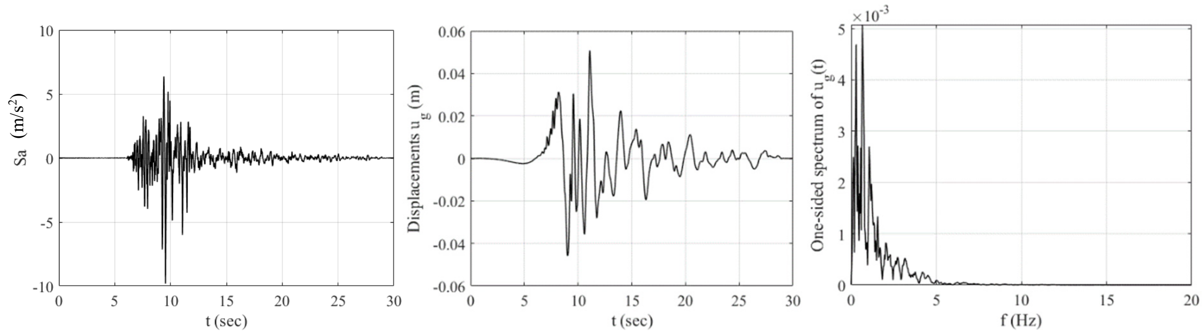

Figure 7: Input accelerations (left), input displacements (middle) and spectral values of the input displacements (right) for the station CNE.

4.2. Results

Figure 8 shows the rocking response in terms of θ/α over the time for the block#1 with and without consideration of the SSI using the soil#1, for the two different seismic events. For the seismic event

CNE (Figure 8a), the SSI leads to a general decrease of the rocking motions at the peak value, whereas for the seismic event CMI (Figure 8b) some of the peaks are amplified. For both events, the SSI leads to the lengthening of the oscillation period and an increase of the amplitudes at the tail of the oscillation. Figure 9a shows the comparison of the responses of block#1, block#2 and block #3 for both soil#1 and soil#2 for the event CMI. The rocking response is highly influenced by the slenderness of the façade and the relationship between slenderness and maximum rocking is not monotonic. Block#1 has an intermediate slenderness of 12 but reacts with the smallest rocking motions compared to block#2 (λ=20) and block#3 (λ=10) and this applies for both soil types. The SSI influences mainly the behavior after the main shock of the seismic events, leading to a difference in phase and amplitude. However, the influence for the main shock is limited, especially for block#2 and block#3. Figure 9b shows the relative displacement between foundation and far end of the LPM. For the relative displacements $(u - u_g)$, the relationship between maximum relative displacement and the slenderness is monotonic, the higher the slenderness the higher the foundation displacements. An obvious result (plots are not included here) is that the relative displacements $(u - u_g)$ are smaller for the stiffer soil (soil#2).

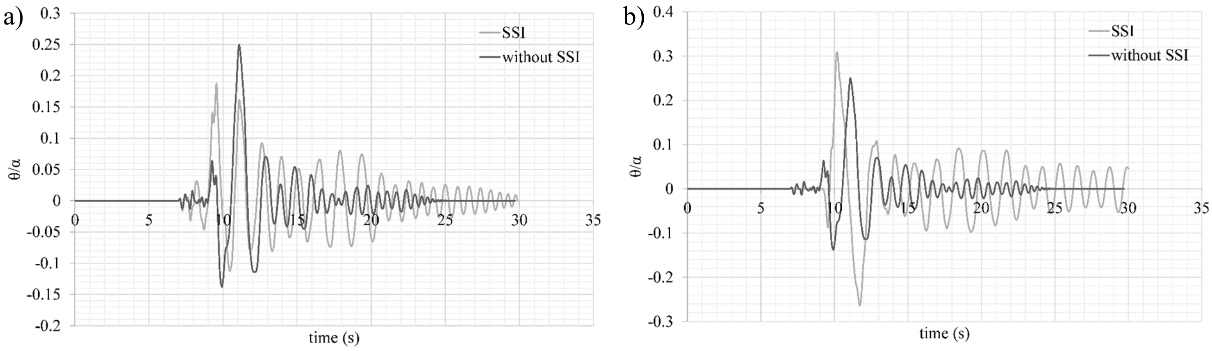

Figure 8: Rocking response block#1 with and without consideration of the SSI using the soil#1, for the two different seismic events: a) seismic event CNE and b) seismic event CMI.

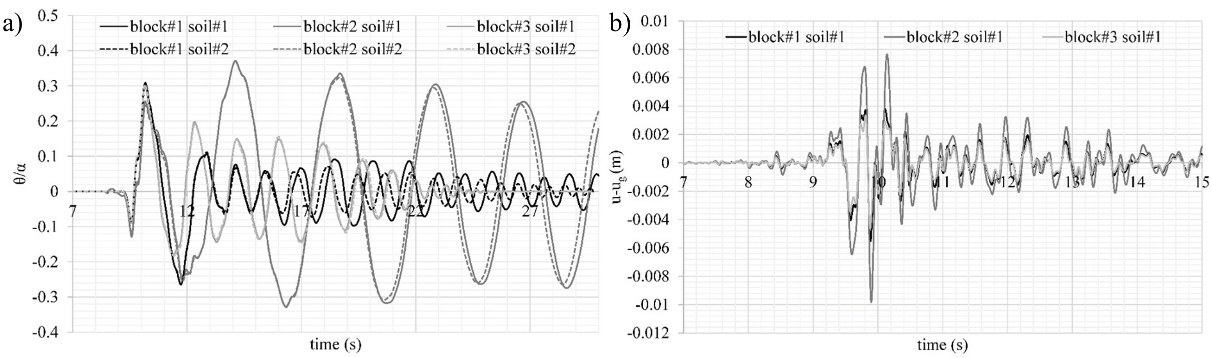

Figure 9: Responses of block#1, #2 and #3 for both soil#1 and #2 for the event CMI: a) rocking and b) relative displacements of the foundation (only for soil#1 for clarity).

5. CONCLUSIONS

In this contribution we presented an extension of the Housner model for the rocking of rigid façade which considers the soil-structure interaction. We derived the equations of motion and described the procedure for the application of the seismic excitation. The assumptions for the impacts between block and foundation are also described. For the sake of illustration, we showed an example for the interaction between a rigid façade and a rigid prismatic foundation resting on a homogenous soil subjected to a horizontal seismic excitation. A parametric study showed that the soil-structure interaction can both increase or decrease the rocking response of the façade, depending on the frequency content of the input. Further studies are necessary to investigate the effect of the additional degrees of freedom of the foundation-soil system and their coupling on the rocking response of the façade and the effect of complex foundation-soil geometries, such as layered soils and/or embedment. It is also important to investigate the problem through a probabilistic approach, varying the frequency content and the amplitude of the input.

ACKNOWLEDGMENTS

The authors wish to thank the TUM (Technische Universität München) University Foundation Fellowship that financed this research activity.

LITERATUR

[1] Housner, G. W. (1963) The behavior of inverted pendulum structures during earthquakes. *Bulletin of the Seismological Society of America*, 53(2), 403-417.

[2] Vassiliou, M. F., & Makris N. (2012) Analysis of the rocking response of rigid blocks standing free on a seismically isolated base. *Earthquake Engineering & Structural Dynamics*, 41(2), 177-196.

[3] Wolf, J. (1976) Soil-structure interaction with separation of base mat from soil (lifting-off). *Nuclear engineering and design*, 38(2), 357-384.

[4] Panagiotidou, A. I., Gazetas, G., & Gerolymos, N. (2012) Pushover and seismic response of foundations on stiff clay: Analysis with P-delta effects. Earthquake Spectra, 28(4), 1589-1618.

[5] Giresini, L., Fragiacomo, M., & Sassu, M. (2016) Rocking analysis of masonry walls interacting with roofs. *Engineering Structures*, 116, 107-120.

[6] Wolf, J. P. (1989) Soil-structure-interaction analysis in time domain. *Nuclear engineering and design*, 111(3), 381-393.

[7] Taddei, F., Schauer, M., & Meinerzhagen, L. (2017) A practical soil-structure interaction model for a wind turbine subjected to seismic loads and emergency shutdown. *Procedia engineering*, 199, 2433-2438.

[8] Wu, W. H., & Lee, W. H. (2002) Systematic lumped-parameter models for foundations based on polynomial-fraction approximation. *Earthquake Engineering & Structural Dynamics*, 31(7), 1383-1412.

[9] Wolf, J. P. (1994). Foundation vibration analysis using simple physical models. *Pearson Education*.

[10] Roussis, P., & Odysseos, S. (2012). Dynamic response of seismically isolated rigid blocks under near-fault ground motions. Paper presented at the *Proceedings of the 15th World Conference on Earthquake Engineering*.