Omega 80 (2018) 135–152

Contents lists available at ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

When does cross-space elasticity matter in shelf-space planning? A decision analytics approach☆

Kai Schaal[a], Alexander Hübner[b,*]

[a] Department of Operations, Catholic University Eichstaett-Ingolstadt, Auf der Schanz 49, Ingolstadt 85049, Germany
[b] Luxembourg Centre of Logistics and Supply Chain Management, The University of Luxembourg, 162a Avenue de la Faiencerie, Luxembourg 1511, Luxembourg

ARTICLE INFO

Article history:
Received 19 October 2016
Accepted 24 August 2017
Available online 28 September 2017

Keywords:
Shelf-space planning
Cross-space elasticity
Heuristic

ABSTRACT

Continuous product proliferation and scare shelf space require a thorough understanding of customer demand effects when planning product allocation to retail shelves. In this context, cross-space demand effects describe the impact of a change in the space assigned to one item, on the demand of other items. This effect is complex and costly to measure and it is complicated to integrate into decision modeling and solution approaches. The tremendous amount of possible product interlinks results in both a large number of possible combinations to be tested, and non-linear models. Nevertheless, there is a growing body of decision models that integrate cross-space effects. However, current research has not investigated whether cross-space elasticities have any impact at all on optimal shelf decisions. It is therefore unclear whether future research on the empirical measurement and the development of optimization models is economically meaningful and justified.

We approach this issue by conducting numerical studies and applying a stochastic shelf-space optimization model. Our results show that the impact of cross-space elasticities on shelf-space decisions and retail profit is very limited. This holds also true if elasticities exceed the values measured empirically thus far. Item characteristics, such as space elasticity, volatility and margin, dominate cross-space effects. The findings are relevant for the OR community, empirical researchers and retailers. Our findings help to streamline further research. First of all, further advances of shelf space models with cross-space elasticity should be based on our findings and have the caveat that they pay only off in extreme cases. Second, for empirical research we obtain guidelines as to when and how to test and esti-mate cross-space elasticity. As the empirical tests for this effect are very voluminous and costly, these findings serve as "guardrails" to define the scope of such empirical investigations.

Therefore, we demonstrate that the empirical measurement and optimization approaches for cross-space elasticities are of minor relevance for future research. We develop guidelines to help retailers identify the circumstances under which cross-space effects become important.

1. Introduction

Selecting and presenting products to customers on shelves is the *raison d'etre* of all merchandise retailing [1]. The increasing number of products competing for limited shelf space (cf. [2,3]) challenges retailers to maintain their space productivity (cf. [4]). Retailers must make the most efficient use of available shelf space by deciding how much space is allocated to which products. Since customer decisions frequently are made at the point of sale, it is essential for retailers to understand how these shelf-space decisions impact customer purchasing behavior, demand and ultimately retailer profit. The demand for an item increases when the item is more visible, e.g. thanks to a larger shelf quantity. This effect is called *space-elastic demand*. In this regard, the way a certain item is presented may also impact the demand for other items – just like the impact of cross-price elasticity in the case of price changes. This effect is called *cross-space elasticity (CSE)*, i.e. the increase or decrease in demand for a specific item if the space of another item is changed. For example, an increase in space for brand A may decrease demand for brands B and C.

Retailers therefore need to understand (1) how these space-dependent effects impact customer demand and (2) how they can optimize shelf space allocation to increase total profitability. Fig. 1

☆ This manuscript was processed by Associate Editor Prof. B. Lev.

* Corresponding author.
E-mail addresses: kai.schaal@ku.de (K. Schaal), alexander.huebner@uni.lu, alexander.huebner@ku.de (A. Hübner).

https://doi.org/10.1016/j.omega.2017.08.015
0305-0483/

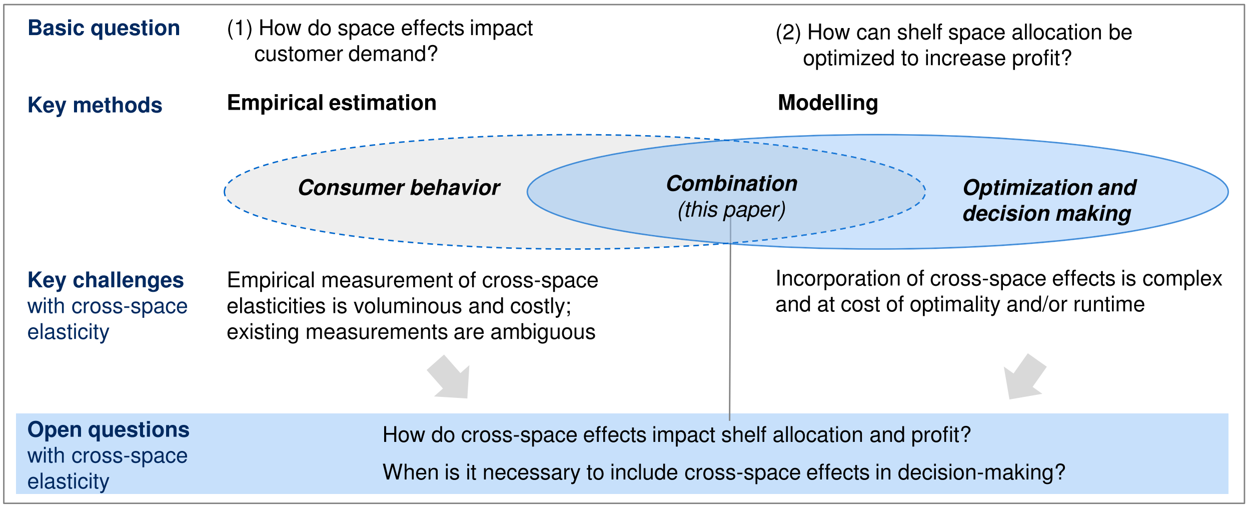

Fig. 1. Cross-space elasticities: Research streams, challenges and open questions.

illustrates the two basic questions and the approaches for answering these questions.

(1) The first question is addressed by consumer behavior studies (see e.g. [5–8]). Various empirical estimations have identified the space-elastic demand effect. Eisend [9] found 31 studies between 1960 and 2014 that analyzed space-elasticity effects, but only five studies that investigated CSE effects. The lack of CSE studies is largely due to challenges in measuring CSE effects caused by the variety of potential interdependencies between the items included in an assortment (cf. [10–12]) and the required data collection efforts. For instance, if a category contains 200 items, a total of $200 \cdot (200-1) = 39,800$ potential demand interlinks due to CSE effects would have to be measured, because each of the 200 items can be linked through CSE effects to each of the other $200-1=199$ items. (2) Regarding the second question, there is a growing stream of papers which present optimization models capable of accounting for space effects. In particular, there have been more than 15 publications during the last 20 years in well-known journals that included CSE in the decision model (cf. [13,14]). Here, the number of product interlinks due to CSE effects increases the complexity of the optimization models and requires more complex solution approaches. Therefore, corresponding optimization models can only be solved by extensive solution algorithms, which are limited in their solution quality, runtime and applicability to large-scale data sets.

Hence, two issues arise when it comes to the incorporation of CSE effects. First, it has been shown that empirical estimation procedures are very voluminous and costly. Second, incorporating CSE effects in optimization models is complex and comes at the expense of optimality and/or runtime. Despite these difficulties, no research to date has taken an integrated approach and answered the question of the relevance of CSE. Specifically, the impact of CSE on shelf-space allocation and profit is not fully clear. We address this research gap and analyze how and under which circumstances optimal shelf-space decisions and retail profits change if CSE effects are present. To do so, we will show when it becomes worthwhile for retailers and researchers to invest in CSE estimation procedures and advanced CSE modeling approaches. The remainder of this paper is structured as follows: Section 2 explains the conceptual background of our research and reviews the related literature. The optimization model and solution algorithm are explained in Section 3. Numerical results are presented in Section 4 and finally, Section 5 discusses our findings in light of extant literature, concludes and gives an outlook on future areas of research.

2. Planning problem, related demand effects and literature

This section explains the conceptual background of our paper. We first illustrate the shelf-space problem retailers face in Section 2.1, then describe how customer demand depends on the decision retailers make in shelf-space planning in Section 2.2, and finally we review the literature relevant to our topic in Section 2.3.

2.1. Decision problem and scope of analysis

Since assortment and shelf-space decisions are typically two sequential planning steps within the category planning process in retail practice [12,14–16], shelf-space planning assumes that an assortment consists of a given set of pre-selected items. Assortment planning is usually executed in an overarching planning step by the marketing department, whereas shelf planning is a subordinate planning problem and usually owned by the sales department. The retailer consequently needs to assign a set of items to a total shelf space of a given size such that total category profit is maximized. The profit of an item is determined by its profit margin and the realized demand. The demand is space- and cross-space elastic. This means the demand for an item grows if more space is assigned to it (=space-elastic demand), more space is assigned to complements and less space to substitutes (=cross-space elastic demand), so that the demand ultimately depends on the total space assigned to items. As customers can only recognize the facing of an item, i.e. the first visible unit in the front row on the shelf, it is sufficient to account for the number of facings in the optimization. Therefore, the retailer achieves profit maximization by determining the optimal number of facings for each item in the assortment. Since customer demand has a significant impact on shelf-space decisions, we investigate the relevant demand effects in detail in the following.

2.2. Related demand effects

Since we want to focus on investigating space effects, we do not consider further demand effects, such as the impact of unavailable items or vertical and horizontal shelf positions on customer demand (cf. [5]). Thus, there are two relevant demand effects for the scope of this study: (a) space elasticity and (b) cross-space elasticity.

Table 1
Cross-space elasticity matrix.

	1	2(C)	3	4(S)	5	…	N
1 (CI)	β_1	$\delta_{12} = 0.02$	…	…	…	…	…
2	…	β_2	…	…	…	…	…
3 (SI)	…	…	β_3	$\delta_{34} = -0.01$	…	…	…
4	…	…	…	β_4	…	…	…
5	…	…	…	…	β_5	…	…
…	…	…	…	…	…	…	…
N	…	…	…	…	…	…	β_N

(a) Space-elastic demand. Customers frontally observe a retail shelf and item facings. The number of facings is one of the most important in-store factors impacting customer demand (cf. e.g. [5,8,17]). The more facings an item is assigned, the higher its visibility on the shelf and the higher its demand. Thus, the demand for an item grows with an increasing number of facings. The magnitude of this demand increase depends on the item's space-elasticity factor, which indicates the percentage increase in demand of an item every time the number of facings increases by a given amount. Various empirical studies include tests that quantify space-elasticity effects (cf. [5,11,18–20]). Recently, Eisend [9] found in a meta-analysis comprising 1268 space-elasticity estimates that the average effect was 17%. For a detailed discussion of empirical evidence on space-elasticity effects, we refer to [9] and [21].

(b) Cross-space-elastic demand. Demand for an item can also depend on the number of facings of other items, which is referred to as "cross-space elasticity" (CSE).

There are two ways in which the number of facings of an item can impact the demand of another item:

- *Complements and complemented items.* If items are linked to one another by complementary CSE effects, the demand for the complemented item (CI) increases when the number of facings of the complement (C) increases. The CSE factor is positive in this case. Spaghetti and pasta sauce are examples of items with complementary CSE links.
- *Substitutes and substituted items.* If items are linked to one another by substitute CSE effects, the demand for the substituted item (SI) decreases when the number of facings of the substitute (S) increases. In this case, the CSE factor is negative. Coke and Pepsi are examples of items with substitution CSE links.

The magnitude of the demand change for item i due to a facing change of another item j is represented by the CSE factor between items i and j, δ_{ij}, which implies that every time the number of facings of item j changes by a given amount, demand for item i changes by the magnitude of δ_{ij}. Technically, the CSE links between items can be illustrated by a CSE matrix. Table 1 shows an example. Item 1 is a complemented item (CI) with complement item 2 (C). Every time the number of facings of item 2 increases, the demand for item 1 increases by the magnitude of $\delta_{12} = 0.02$, since δ_{12} is positive. Item 3 is a substituted item (SI) and gets substituted with item 4 (S). $\delta_{34} = -0.01$ indicates that every time the number of facings of item 4 increases, demand for item 3 decreases. Note that CSE matrices are not necessarily symmetric, i.e. δ_{ij} is not necessarily equal to δ_{ji} (see e.g. [22]). The diagonal elements in the CSE matrix correspond to the item space elasticities β_i.

A few empirical analyses measure CSE effects: Eisend [9]'s meta-analysis yields an average CSE of −1.6%. Corstjens and Doyle [23] use regression analyses across 140 stores to analyze 5 impulse-buy categories (chocolate confectionary, toffee, hard-boiled candy, greetings cards and ice cream). They find that CSE effects are lower than space-elasticities, statistically significant and identify substitution and complementary links ranging between −14% (chocolate confectionary and toffee) and 3% (chocolate confectionary and greetings cards). The average CSE effect is −2.8%. Brown and Lee [6] conduct tests on orange and refrigerated juice and find that the difference of the CSE effects from zero is not statistically significant. Campo et al. [24] use asymmetric attraction models to quantify CSE effects. They find evidence for substitution and complementary effects: Fish weakly complements other fresh products like fruit, vegetables, dairy and fine meat. Indoor leisure items are complemented by groceries and staples. Substitution interlinks are found between fruit/vegetables and clothing. The CSE effects identified are statistically significant but generally weak. Flynn [7] investigates items from the ambient assortment (beans and noodles) and finds weak substitution links with CSE ranging between −2.0 and −0.15%. These contributions show that CSE effects are ambiguous. Zufryden [10] argues that considering CSE at an individual level would be impossible in practice due to the overwhelming number of cross-elasticity terms that would need to be estimated. For this reason, Desmet and Renaudin [11] exclude CSE from their consideration. Kök et al. [12] come to a similar conclusion and find no empirical evidence that product-level demand can be modeled with CSE. After having defined the scope of the investigation and the related demand effects, we review the associated literature on existing shelf-space optimization models below.

2.3. Related literature on shelf-space optimization models

Literature review. Because we seek to answer the research question as to how CSE effects impact optimal shelf layouts by means of a shelf-space optimization model, the focus of this literature review is on shelf-space optimization models that assume limited shelf space, use the number of facings as the key decision variable, and take into account space- and cross-space elasticity effects. We discuss the contributions related to these criteria in the following. We refer to [12–14] for comprehensive overviews of research into other shelf-space problems.

One of the first contributions is based on the work of Corstjens and Doyle [23], who propose a shelf-space model. Geometrical programming is applied to solve the model, which is limited in its capability of solving large-scale problem instances.

Borin et al. [25] propose a model based on the demand function of [23]. They also account for substitution and apply simulated annealing to solve the model for up to six items. Yang [26] simplifies the polynomial space elasticity form and assumes a linear function within a constrained number of facings. A multi-knapsack heuristic is proposed for the resulting model and tested on instances of up to 10 items.

Hariga et al. [27] integrate inventory control and replenishment aspects and considers backroom space availability. A standard solver is applied to solve small problem instances with only four items. Hwang et al. [28] develop an optimization model and solve it through a genetic algorithm for a problem instance with four items.

Gajjar and Adil [29] reformulate a shelf-space model through piecewise linearization and additionally develop a local-search heuristic to solve the model on data sets with up to 200 items. Hansen et al. [22] compare a meta-heuristic with different versions of the heuristic developed by Yang [26]. Numerical tests encompass up to 100 items. Irion et al. [30] develop a non-linear model that is solved by piecewise linear approximation, which supports the handling of large data sets of up to 50 items. Zhao et al. [31] integrate shelf space, space location and replenishment decisions. Their model is solved through a multi-stage simulated annealing hyperheuristic and applied to data sets of 100 items and 20 shelves. Recently, Hübner and Schaal [21] proposed the first stochastic shelf-space optimization model, which is solved by a specialized heuristic for up to 200 items. They show the necessity of properly accounting for demand volatility by modeling stochastic demand. Otherwise, suboptimal facing decisions and lower profit levels result. Frontoni et al. [32] present a linear integer shelf space model focusing on the minimization of out-of-stock events, for which they obtain real time data using sensor network technology.

To sum up, shelf-space management literature typically assumes deterministic demand to factor in space-and cross-space elasticity [12,13]. Most of the existing contributions test the respective solution approaches only on small-scale problem instances. Furthermore, many simplify and consider the non-linear demand to be linear, or apply linearization techniques.

Discussion and derivation of research question. Our literature review shows that major effort has gone into modeling and developing solution approaches for non-linear demand functions with CSE effects. However, none of the existing contributions systematically analyze whether CSE effects are relevant at all, i.e. to what extent they impact space decisions and retail profits. Furthermore, the empirical measurement of CSE effects is complex and costly and, from a decision analytics perspective, no guidelines exist thus far on how strong CSE effects must theoretically be to justify expensive, precision measurement. It is therefore important for retailers to understand the extent to which CSE impacts optimal shelf layouts. Despite the availability of shelf-space optimization models, this research question has not been given sufficient attention and current literature lacks an in-depth analysis of large-scale and realistic problem instances. It is important to point out the circumstances under which CSE effects matter and how retailers should change facing decisions if CSE effects are present. We contribute to the research by answering the research question:

"How does CSE impact shelf-space planning and to what extent is it worthwhile to empirically test it?"

3. Optimization model and solution approach

Among others, Agrawal and Smith [33], Kök and Fisher [34] and Hübner and Schaal [35] have shown that stochastic demand needs to be included in decision models due to the existing demand volatility in retailing. Hübner and Schaal [21] show that if stochastic demand is not properly taken into account, it will result in suboptimal shelf configurations (with up to 70% of the items with incorrect facings and up to 5% lower profits). Properly accounting for stochastic demand is therefore of fundamental importance for retailers in order to maximize profits. We therefore apply a modified version of the stochastic model of [21]. It is an appropriate starting point for this investigation, because it is the only model that considers the aforementioned demand characteristics, namely stochastic as well as space- and cross-space elastic demand, and it is scalable and applicable to large categories containing $>$ 100 items. Moreover, it has been shown that the solution approach is efficient in terms of solution quality and runtime.

To analyze the impact of CSE on retail profits and facing decisions, we apply a version of the **S**tochastic **C**apacitated **S**helf-space **P**roblem with **c**ross-space- and **sp**ace-elastic demand, abbreviated $SCSP_{csp}$. First, Section 3.1 discusses the assumed demand model. Section 3.2 then describes the resulting optimization model $SCSP_{csp}$, which is solved through reformulation into a binary-integer problem and a specialized heuristic explained in Section 3.3.

3.1. Modeling the demand function

This subsection introduces the demand model and explains how space- and cross-space elasticities can be accounted for in a stochastic environment.

Space-elastic demand. According to [36], the general relationship between the total space-elastic demand $D_i^{sp}(k_i)$ for an item i, $i \in \mathbb{N}$, its minimum demand D_i^{min}, its space elasticity factor β_i and the number of facings k_i allocated to the item i can be calculated as $D_i^{sp}(k_i) = D_i^{min} \cdot k_i^{\beta_i}$. The minimum demand D_i^{min} corresponds to the demand for an item i if it were represented with one facing ($k_i = 1$). If more than one facing is allocated to an item, customer demand will increase. The higher the space elasticity β_i, the more the total space-elastic demand D_i^{sp} increases per additional facing. We assume that the probability density function $f_{D_i^{min}}$ for the minimum demand for item i, $i \in \mathbb{N}$, is exogenously known and does not include any space- or cross-space elasticity, i.e. does not depend on the number of facings. The corresponding density function that accounts for space-elastic demand but ignores CSE effects is denoted by $f_{D_i^{sp}}(k_i)$.

Space- and cross-space-elastic demand. In line with [23] and [30], we use Eq. (1) to adapt the demand function and incorporate CSE effects. The space- and cross-space-elastic demand for an item i, $D_i^{csp}(\bar{k})$, no longer depends exclusively on the number of facings of the item i (k_i), but also on the number of facings of all other items ($\bar{k}$), with $j \neq i$, where $\bar{k}$ denotes the respective vector for the facings of all items.

$$D_i^{csp}(\bar{k}) = D_i^{min} \cdot (k_i)^{\beta_i} \cdot \prod_{j \in \mathbb{N}, j \neq i} (k_j)^{\delta_{ij}} \quad (1)$$

We denote the corresponding (space- and cross-space-elastic) density function for item i by $f_{D_i^{csp}}(\bar{k})$. Note that any demand distribution can be assumed for the total demand density function (c.f. [21]).

3.2. Optimization model: $SCSP_{csp}$

Calculation of single-item profits. Since the aim of our paper is to analyze the impact of CSE effects on retail profits, we formulate an optimization model that maximizes retail profits by selecting the optimal number of facings per item. Thus, the number of facings per item is the decision variable and, as shown in the previous section, determines the expected customer demand from space- and cross-space effects. To streamline our analysis, we use a simplified version of the model of [21], because we are focusing here on CSE and do not account for other effects.

The retailer maximizes profit across a set of items $\mathbb{N}$ within a category of perishable and/or non-perishable products, which must be assigned to a shelf with limited space by selecting the number of facings k_i for each item i. The set of items is exogenously given, where $\mathbb{N}$ denotes the entire set of items and $N = |\mathbb{N}|$. We assume that the assortment was determined in a previous planning step, such that each item must be listed and therefore receives at least one facing, i.e. $k_i \geq 1$. Furthermore, all items must fit onto the available shelf space. The item-specific profits $\pi_i(k_i, x_i)$ are determined by the number of facings k_i and total shelf quantities x_i. While the number of facings k_i determines customer demand (see Section 3.1), the total shelf quantity of an item is used to fulfill customer demand. Behind each facing, there is a fixed stock of units of the respective item. Multiplying the number of facings by the stock per facing results in the total shelf quantity of an item that is available to satisfy customer demand. The total shelf quantity again is uniquely defined by the number of facings k_i and the stock per facing g_i. Thus, we calculate the shelf quantity x_i as an auxiliary variable by $x_i = k_i \cdot g_i$, where g_i corresponds to the stock behind each facing. Finally, the item-specific profit function π_i consists of four major elements:

$$\pi_i(\bar{k}, x_i|_{x_i=k_i \cdot g_i}) = -c_i \cdot x_i + r_i \cdot \int_0^{x_i} y f_i^* dy + r_i \cdot \int_{x_i}^{\infty} x_i f_i^* dy + v_i \cdot \int_0^{x_i} (x_i - y) f_i^* dy - s_i \cdot \int_{x_i}^{\infty} (y - x_i) f_i^* dy \quad (2)$$

The first term calculates the *total purchasing cost* that occurs for every unit put on the shelf. Processing costs (e.g. for replenishment) are incorporated in the unit cost c_i. The second and third terms calculate the *expected revenues*, assuming that each unit can be sold for a sales price r_i. If excess (unsold) items remain at the end of the period, a *salvage cost*, calculated by the fourth term, is generated. Excess items are disposed of at a salvage value v_i and the retailer incurs a loss of $(c_i - v_i)$ on each item i, assuming that $c_i > v_i$. Note that besides this interpretation of v_i as the salvage value in the case of perishable products, it can also be interpreted as the remaining value after the inventory carrying cost has been subtracted from the unit purchasing cost c_i in the case of non-perishable items. The items do not perish after the sales period and the retailer pays inventory holding costs for items not sold at the end of the period. The model as such can consequently also be applied to non-perishable items (cf. [34]). The last term calculates the *penalty cost*, which occurs if the expected demand D_i for an item i is greater than its shelf quantity x_i. Excess demand is lost and the retailer suffers the shortage cost s_i per unsold unit. The profit calculation therefore corresponds to the Newsvendor setting (see also [37,38]). The probability density function f_i^* in Eq. (2) accounts for the relevant demand distribution, which must be quantified in accordance with assumed customer behavior (i.e. with or without space and cross-space elasticity). Depending on which demand density function described in Section 3.1 is assumed for f_i^* in Eq. (2) (i.e. $f_{D_i^{\min}}$, $f_{D_i^{sp}}(k_i)$ and $f_{D_i^{csp}}(\bar{k})$), different optimization models result. Since we assume stochastic demand with space and cross-space elasticities, we set $f_i^* = f_{D_i^{csp}}(\bar{k})$ and calculate the corresponding profits $\pi_i(\bar{k})$, with $\bar{k}$ indicating that the profit of an item i depends on the number of its own facings and the facings of all other items.

Optimization model SCSP$_{csp}$. After having derived the profit per item, we define the retailer's decision problem as follows: Eq. (3) is the objective function and maximizes the total profit calculated as the sum of the single item profits. Eq. (4) is the shelf-space restriction and ensures that the available shelf space is not exceeded. Shelf space corresponds to the one-dimensional length of the shelf (e.g. measured in meters) and is consumed through the placement of facings (k_i), whereas each facing has the length b_i. Finally, Eq. (5) define that facings are positive integers.

$$\max! \quad \Pi(\bar{k}) = \sum_{i=1}^{\mathbb{N}} \pi_i(\bar{k}) \quad (3)$$

$$\sum_{i=1}^{\mathbb{N}} k_i \cdot b_i \le S \quad (4)$$

$$k_i \in \mathbb{Z}^+ \qquad i \in \mathbb{N} \quad (5)$$

Model complexity. The SCSP$_{csp}$ is an NP-hard knapsack problem (cf. e.g. [39]). The number of possible combinations Y for allocating N items to a shelf of size S is given by Eq. (6). For instance, with $N = 30$ and $S = 50$, $Y = 2.8 \cdot 10^{13}$ possible combinations result.

$$Y(N, S) = \binom{S-1}{N-1} \quad (6)$$

If items are linked through CSE effects, each of these Y combinations corresponds to a unique demand setting, since each combination implies a different number of facings across the products. An optimal approach for solving SCSP$_{csp}$ would be to fully enumerate the problem by calculating the resulting profit for each of the Y combinations. The example above shows that the numerical complexity only allows this for small-scale problems. For larger problems, SCSP$_{csp}$ must be solved through a heuristic, which we present in the following section.

3.3. Solution approach using binary-integer model SCSP$^{BIP}_{csp}$ and specialized heuristic

The central element of the solution approach is the reformulation of SCSP$_{csp}$ into a binary-integer version, denoted as SCSP$^{BIP}_{csp}$ in the following (see also [21]). This version of the model accounts for the fact that retailers do not assign more than a predefined number of facings (K, e.g. 15 or 20) to a single item. This upper limit allows for the precalculation of item-specific profits (π_{ik}) for all possible facing values, i.e. from 1 to K. The optimization variables γ_{ik} then determine whether item i gets k facings.

$$\max! \quad \Pi(\bar{\gamma}) = \sum_{i=1}^{\mathbb{N}} \sum_{i=1}^{K} \pi_{ik} \cdot \gamma_{ik} \quad (7)$$

subject to

$$\sum_{i=1}^{\mathbb{N}} \sum_{k=1}^{K} k \cdot b_i \cdot \gamma_{ik} \le S \quad (8)$$

$$\sum_{k=1}^{K} \gamma_{ik} = 1 \qquad i \in \mathbb{N} \quad (9)$$

$$\gamma_{ik} \in \{0, 1\} \qquad i \in \mathbb{N}, k \in 1, \ldots, K \quad (10)$$

Eq. (7) is the objective function corresponding to Eq. (3). Eq. (8) is the shelf-space restriction. Eq. (9) ensure that each item is assigned exactly one facing number. Eq. (10) express that the decision variables γ_{ik} are binary.

The isolated precalculation of item- and facing-specific profits in step 1 does not yet account for potential item interlinks due to CSE effects. In other words, we calculate $\pi_i(k_i)$ instead of $\pi_i(\bar{k})$ and thus ignore the dependence of an item's profit on the number of facings of the other items. To account for the demand interlinks due to CSE effects, we then iteratively update the demand in step 2 and solve SCSP$^{BIP}_{csp}$ through the following specialized heuristic (see Fig. 2):

For initialization, we assume at iteration $\ell = 1$ for each item that its demand density is equal to the demand density without CSE, $f_i^{*,\ell} = f_{D_i^{sp}}(k_i)$ (Step 1.2), and then precalculate the corresponding profits for all items and the range $1, \ldots, K$. Using these profits, we solve the corresponding BIP (Step 1.3). The results of this iteration ($\bar{k}^{\ell}$) are used to update the total demand density function (now accounting for space and cross-space elasticities) for all items in Step 1.4.

In each step of the following iterations ℓ, we first assume that the total demand density function for an item is equal to the density function obtained in the previous iteration $\ell - 1$. We then use this information to solve the corresponding BIP, store the results and use them to again update the total demand density functions (Steps 2.1 – 2.4). The procedure stops when the number of facings no longer changes from one iteration to the next (Step 2.5). Note that because we iteratively update the demand, cross-space effects are – apart from the first iteration – correctly accounted for in each iteration. Hübner and Schaal [21] explain why the results may deviate from the optimum and prove that the iterative approach yields fast and near-optimal results (average solution quality > 99%) in an efficient manner, even for large problem instances of up to 200 items.

Initialization: solving the BIP without cross-space elasticity effects
Step 1.1: Set $\ell = 1$
Step 1.2: For $i \in \mathbb{N}$:
    For all possible combinations of $k_i \in [1; K]$:
        Calculate π_{ik} with Eq. (2) using $f_i^{*,\ell} = f_{D_i^{sp}}(k_i)$
    end for
end for
Step 1.3: Solve the BIP using Eq. (7) to (10) and store $\bar{k}^\ell$
Step 1.4: For $i \in \mathbb{N}$:
        Update demand with $f_i^{*,\ell} = f_{D_i^{csp}}(\bar{k}^\ell)$
end for

Iterations: solving the BIP with cross-space elasticity effects
Step 2.1: Set $\ell = \ell + 1$
Step 2.2: For $i \in \mathbb{N}$:
    For all possible combinations of $k_i \in [1; K]$:
        Calculate π_{ik} with Eq. (2) using $f_i^{*,\ell} = f_i^{*,\ell-1}$
    end for
end for
Step 2.3: Solve the BIP using Eq. (7) to (10) and store $\bar{k}^\ell$
Step 2.4: For $i \in \mathbb{N}$:
        Update demand with $f_i^{*,\ell} = f_{D_i^{csp}}(\bar{k}^\ell)$
end for
Step 2.5: If $\bar{k}^\ell = \bar{k}^{\ell-1}$ stop, elso go to step 2.1

Fig. 2. Specialized heuristic for solving $SCSP_{csp}$, cf. Hübner and Schaal [21].

4. Numerical results

In this section, we conduct numerical tests to investigate the impact of CSE on optimal facing decisions and retail profits. Section 4.1 explains the test setting and data used, and introduces the key statistics to evaluate the impact of CSE. Section 4.2 then investigates the general impact of CSE. Section 4.3 analyzes the interlinks between CSE and key differentiating item characteristics, such as space elasticity or item demand volatility. Section 4.4 investigates whether CSE matters more when shelf space is scarce, and finally, Section 4.5 summarizes the findings from the numerical tests, develops managerial insights and discusses the contribution to theory.

4.1. Test setting, data applied and key statistics for evaluating the impact of cross-space elasticity

Test setting and data applied. Because measuring CSE effects for categories of practice-relevant size would result in an enormous data estimation effort, we use simulated data here and randomly generate a large set of problem instances with realistic model parameters. This enables us to draw general conclusions based on different data settings instead of only one specific case study.

If not stated otherwise, the parameters are generated as follows: For sales prices, unit cost, salvage values and penalty cost, we assume that the following inequalities hold true for all items $i \in \mathbb{N}$: $r_i > c_i > v_i > s_i$ and that the corresponding parameters lie within the following ranges: $r_i \in [10, 14]$, $c_i \in [7, 9]$, $v_i \in [3, 6]$, $s_i \in [1, 2]$. Customer demand is normally distributed with an average demand of $\mu_i \in [7, 10]$ and a corresponding coefficient of variation $CV_i \in [1\%, 50\%]$. By modeling demand volatility with CV_i, we ensure that negative demand cannot occur. To focus on the core effects, we use identical item widths with $b_i = 1$ and stocks per facing of $g_i = 1$, $\forall i \in \mathbb{N}$. Space elasticity β_i is assumed to vary between $0 \le \beta_i \le 0.35$ (see [9], who identified on average $\beta_i = 0.17$).

The applied test data are similar in terms of problem sizes, demand distribution, margins etc. to the ones used in other numerical studies, see e.g. [34,40–42]. To understand the general impact of CSE on profits and solution structures, we start with two general analyses in Section 4.2. Here we differentiate between one- and multi-directional CSE links (see Fig. 3).

Multi-directional CSE links imply that one item can be linked to more than one other item. Furthermore, an item can simultaneously be a substitute for some items and a complement to others, and at the same time be complemented by again other and substituted by even other items. In this standard case, this means $\delta_{ij} \neq 0$. The analysis with this assumption is completed in Section 4.2.1.

One-directional CSE links only allow for connections between items in one direction. For instance, an item A can complement (or substitute) another item B, but at the same time, B cannot complement (or substitute) A. Furthermore, we assume that one item can be linked to at most one other item. In the example, A and B cannot have any link to a third item C, if there is already a connection between A and B. We do this to avoid any mixed effects and identify patterns, since demand for item B would be impacted by the facings of items A (its complement or substitute) and C (another complement or substitute) simultaneously. The analysis with the special case of one-directional CSE links is completed in Section 4.2.2 to better identify different patterns.

The assortment consists of N items, each of which belong to one of three groups: (1) Items with complementary CSE links (complemented items (CI) i and complements (C) j with $\delta_{ij} > 0$), (2) Items with substitution CSE links (substituted items (SI) i and substitutes (S) j with $\delta_{ij} < 0$), and (3) Neutral items without any CSE links ($\delta_{ij} = \delta_{ji} = 0$). Note that because from Section 4.2.2 on we assume one-directional links, one item belongs to only one of these groups. For example, if item B is a complement for item A, item B falls into to the group of complements (C) and item A belongs to the group of complemented items (CI). In Section 4.3, we get more specific and assume that the items are not all similar, as

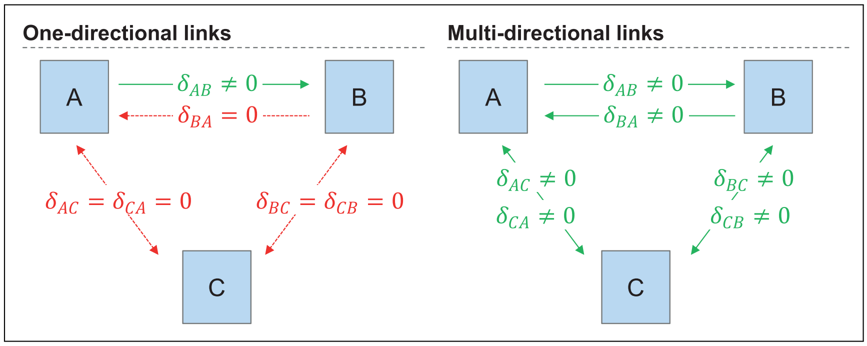

Fig. 3. One- vs. multi-directional CSE links between items.

in the data generation process described above, but that they differ in one of three key item characteristics: Space elasticity, demand volatility and item margin.

Each test instance consists of 100 randomly generated examples. For all examples of an instance, the assortment size N and shelf size S are assumed to be identical. All numerical tests were conducted on a Windows 7 32-bit Intel Core i5-2520 with 2.5GHz and 4GB memory. The tests were implemented in VB.net (Visual Studio 2013) and GAMS 24.1 to use the CPLEX solver.

Key statistics for evaluating the impact of CSE. To assess the effectiveness and necessity of integrating CSE, we must compare the results for which CSE is correctly and directly integrated into the decision model, with those for which CSE is ignored in the decision model and added “a posteriori”. The effect must be evaluated in two dimensions:

(a) To evaluate the financial impact and the impact on the *objective function* of correctly accounting for CSE effects, we measure the profit advantage a retailer has when he takes CSE effects into account instead of ignoring them:
  - To calculate the *profit advantage*, we run the $SCSP_{csp}$ assuming that all δ_{ij} are 0 and then evaluate the resulting facings with the “a posteriori” model $SCSP^{*}_{csp}$ assuming non-zero CSE effects. The resulting profit is then compared to the profit obtained when correctly accounting for CSE effects in the $SCSP_{csp}$. The profit advantage is calculated as ($SCSP_{csp}$ profit / $SCSP^{*}_{csp}$ profit - 1).

(b) To evaluate the impact on *solution structures*, we calculate the following measures:
  - *Overall share of shelf space for an item group with and without CSE.* To understand the share of total shelf space allocated to the three different item groups (C/CI, S/SI and neutrals), we calculate the *item group shelf space share* as (sum of facings for all items within group / total shelf space). The statistic is calculated for two scenarios, with and without CSE effects. For example, if all items with substitution CSE links (S/SI) get a total shelf space of 100 facings, and total shelf space is $S = 1000$, the share of shelf space for the SI/S group corresponds to 10%.
  - *Change in relative item group shelf space share.* To understand the relative change in shelf space, we calculate the *change in relative item group shelf space share* as (total item group shelf space with CSE effects / total item group shelf space without CSE effects - 1). For example, if all C/CI get a total shelf space of 120 facings with CSE effects and 100 facings without CSE effects, the change in the relative share of shelf space for an item group corresponds to 20%.
  - *Share of items that get a different number of facings.* To evaluate how many items get a different number of facings due to CSE effects, we calculate the *share of items which get a different number of facings* as (Number of items with facing changes within a respective group / total number of items within a respective group).
  - *Increase and decrease in the number of facings of each individual item.* To not only investigate whether facings change but also understand the magnitude of these changes, we calculate the average increase (decrease) in the number of facings for the items that are given a different number of facings, e.g. if the average SI gets 10 facings without and 9 with CSE effects, the decrease is 1 facing.

Overview of numerical tests. Table 2 provides an overview of the numerical tests conducted below.

4.2. General impact of cross-space elasticities on profits and facing decisions

In this section, we analyze the general impact of CSE on profits and shelf-space decisions.

4.2.1. Multi-directional CSE links

We start with a general analysis, where each item can have multiple CSE links. To account for the fact that, in reality, an item does not have CSE effects relating to all other items, we limit the maximum share of other items to which an item can have CSE links to 20%, e.g. if the category contains $N = 50$ items, an item can have CSE links to up to 10 other items. We randomly choose CSE effects $\delta_{ij} \in [-3\%, 3\%]$, which corresponds to approximately twice the absolute amount of the average CSE effects of −1.6% measured empirically thus far (cf. Eisend [9]).

Table 3 shows the results of an “a posteriori” analysis, where CSE effects have been ignored, facings chosen accordingly and profits then evaluated assuming positive CSE effects. The results show that CSE effects have a very limited impact. Even though on average about one-third of the items get a facing change, the average facing change is marginal (0.22 facings). Furthermore, the profit is only about 1% higher on average if CSE is correctly accounted for compared to a scenario where CSE is ignored.

4.2.2. One-directional CSE links

Because in Section 4.2.1 an item could have multiple CSE links to other items, the measured effects could be mixed. To avoid this and understand the general impact of CSE on profits and facing decisions, we assume from here on out that CSE effects are one-directional. Furthermore, each item is clearly assigned to one of five groups: substitutes, substituted items, complements, complemented items and neutral items. To get a clear idea of the directional impact CSE effects have, we start with CSE values of up to

Table 2
Overview of numerical tests.

Section	Purpose	δ_{ij}	Further parameters varied/key item characteristics
4.2	Analyzes the general impact of CSE effects on profits and facing decisions		
4.2.1	Multi-directional CSE links	$\in[-3, 3\%]$	–
4.2.2	One-directional CSE links	$\in[-50, 50\%]$	–
4.3	Analyzes the interlink between CSE and three key differentiating item characteristics:	$\in[-10, 10\%]$	
	• Space elasticity		β_i: weak: 0–15%; strong: 20–35%
	• Demand volatility		CV: low: 1–40%; high: 50–90%
	• Item margin		Low: $r_i \in [17, 20]$, $c_i \in [13, 16]$; high: $r_i \in [17, 20]$, $c_i \in [8, 11]$
4.4	Analyzes the extent to which the scarcity of shelf space changes the impact of CSE on optimal profits and facing decisions	$\in[-10, 10\%]$	–

Table 3
Generalized results with multi-directional CSE links: Impact of CSE on profit and solution structure.

	Profit advantage (%)	Share of items with facing changes (%)	Change in the no. of facings
Min.	0.42	18	0.14
Avg.	1.04	31	0.22
Max.	2.66	44	0.44

$N = 50$, $S = 300$, $K = 30$, averages across 100 randomly generated data sets.

+/−50%, which differ significantly from the empirically measured average of −1.6% (cf. [9]). Table 4 shows the impact of CSE on facing decisions: First, the group of complemented items (CI) receives more, and the group of substituted items (SI) less shelf space. If CSE effects are strong ($\delta_{ij} \in |11.0 - 50.0\%|$), the increase (decrease) is strongest for CI (SI) and corresponds to approx. +39% for CI and −24% for SI. For moderate CSE links ($\delta_{ij} \in |1.1 - 10.0\%|$), the relative changes are smaller (CI:+6%, SI: −7%). Weakly complemented items receive even less shelf space (−0.9%), since shelf space is allocated to strongly complemented items. Up to approx. 15% of the items receive facing changes, whereas if a CI receives more facings, the increase corresponds to 8.64 more facings, and if it receives less facings, the decrease corresponds to 2.02 facings (in the case of strong CSE effects). For SI, the decrease is 4.80 facings and the increase 0.28 facings. We see that complementary CSE effects tend to have a stronger impact than substitution CSE effects.

In summary, even if CSE significantly deviates from the empirically measured effects, the impact on solution structures is only moderate. Please note that we report only the share of CI and SI items, because we are concentrating on the effects of these items. Complements (C) and substitutes (S) are shown in the Appendix. We also refer to Section 4.1, where the statistics shown in Table 4 are explained in detail.

4.3. Relationship between cross-space elasticity and item characteristics

After having investigated the general impact of CSE effects on facing decisions and profit, we now assume CSE values that are closer to the empirically measured values ($\delta_{ij} \in [-10, 10\%]$) and investigate the relationship between CSE and three key differentiating item characteristics: item space elasticity, demand volatility and item margin. To do so, we again assume a category with three subgroups (CI/C, SI/S, neutrals) and additionally assume that the items differ from one another in terms of a respective item characteristic (weak vs. strong space elasticity, low vs. high demand volatility, low vs. high margin). For each numerical test, we assume two scenarios to account for the fact that CSE links can exist *between two similar items* (e.g. low margin item complements low margin item) or *between two different items* (e.g. high margin item complements low margin item). Section 4.3.1 shows the impact on objective value and Section 4.3.2 investigates in detail the impact on facing decisions for each item characteristic.

4.3.1. Impact on objective value

Table 5 shows the results of an "a posteriori" analysis. In other words, we calculate the profit resulting from the facings based on the incorrect assumption of zero CSE effects, when in reality nonzero CSE effects are present. The profit advantage retailers have when correctly accounting for CSE is significantly lower than 0.5%,

Table 4
General impact of cross-space elasticity: Changes in facing decisions.

Item group[a]	CSE effects[b]	Total item group share (%)		Relative change in item group share (%)	Share of items w/ facing changes [%]	Avg. change in no. of facings	
		$\delta = 0$	$\delta \neq 0$			+[c]	−[c]
CI	Weak	5.6	5.6	−0.9	3.8	0.31	0.76
	Moderate	5.9	6.2	+5.7	13.7	1.62	0.72
	Strong	5.6	7.8	+38.9	15.3	8.64	2.02
SI	Weak	5.5	5.5	−1.0	4.1	0.13	−0.49
	Moderate	5.2	4.8	−6.9	12.0	0.25	−1.54
	Strong	5.4	4.1	−24.1	14.9	0.28	−4.80

$N = 180$ (60 complements, 60 substitutes, 60 neutrals), $S = 1000$, $K = 30$, averages across 100 randomly generated data sets, see Appendix Tables 11 and 12 for details.
[a] CI = complemented item, SI = substituted item; complements (C) and substitutes (S) shown in Appendix
[b] Weak: 0.1–1.0%; moderate: 1.1–10.0%; strong: 11.0–50.0%
[c] Average change in facings of all items with more (+) or less (−) facings

Table 5
Impact on financial performance: Profit advantage if CSE is correctly accounted for $N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets.

	CSE effects between items with ...	
Item characteristic	...similar characteristic (%)	...different characteristic (%)
Space elasticity[a]	0.24	0.16
Demand volatility[b]	0.44	0.14
Item margin[c]	0.39	0.02

[a] weak: 0–15%; strong: 20–35%.
[b] Low: 1–40%; high: 50–90%.
[c] Low: $r_i \in [17, 20]$, $c_i \in [13, 16]$; high: $r_i \in [17, 20]$, $c_i \in [8, 11]$.

i.e. there is virtually no impact on the objective value. We further see that the profit advantage is consistently higher if CSE effects only exist between items which do not differ in the key characteristic. As soon as CSE effects exist between items which differ in the respective characteristic, the profit advantages diminish. This is due to opposing effects, which can be explained by the analysis of the impacts on facing decisions below.

4.3.2. Impact on facing decisions

This section investigates the impact of CSE effects on facing decisions for each of the three item characteristics.

Relationship between cross-space elasticity and item space elasticity. Table 6 shows that the observations made in Section 4.2 apply here as well: The impact of CSE on facing decisions is highest when CSE effects are strong. This applies to CI and SI, whereas again complementary CSE effects tend to be stronger than substitution CSE effects. If we now consider the additional item characteristic (space elasticity), we see that if CSE effects exist between similar items, CSE effects and space elasticity mutually reinforce each other: Items that have high space elasticity and get strongly complemented by a highly space-elastic complement, get an average of 2.86 facings more (or < 0.1% of total shelf space), whereas weakly space-elastic items strongly complemented by a weakly space-elastic complement get only an average of 1.33 more facings. If CSE effects exist between two items with different space elasticity, the impact on facing decisions is ambiguous: In the case of strong CSE effects, a highly space-elastic item complemented by a weakly space-elastic complement gets an average of 1.72 more facings.

The reason for this ambiguity is opposing effects: Because of its weak space elasticity, the complement itself receives only few facings (cf. [21]). This results in only a small demand push for the complemented item (cf. Eq. (1), low k_j). On the other hand, the complemented item has high space elasticity, which again pushes its demand. The analogous logic applies to the opposite case, where a highly space-elastic item complements a weakly space-elastic item. In this case, the CI gets an average of 1.18 more facings. We see that the space elasticity of a complemented item is still a more dominant determinant than the space elasticity of its complements. The same observations apply to the group of substitutes and substituted items.

Relationship between cross-space elasticity and demand volatility. If we assume that items differ in their demand volatility, we see that similar observations apply (cf. Table 7): If items linked through CSE effects are similar (i.e. have a similar demand volatility), the impact of CSE on facing decisions is unambiguous and low demand volatility and high CSE effects reinforce each other. If two linked items differ in their volatility, the impact of CSE on facing decisions is less clear. For instance, a complement with high demand volatility receives few facings, which in turn only slightly pushes demand for the complemented item through CSE effects. If the complemented item has low volatility, demand again is pushed such that

Table 6
CSE and space elasticity: Changes in facing decisions.

	Item group[a]	CSE effects[b]	Space elasticity[c]	Total item group share (%) $\delta = 0$	Total item group share (%) $\delta \neq 0$	Relative change in item group share (%)	Share of items w/ facing changes (%)	Avg. change in no. of facings +[d]	Avg. change in no. of facings −[d]
Items with similar space elasticity	CI	Weak	Weak	3.1	3.1	+0.2	1.3	0.18	0.06
			Strong	3.9	3.9	+0.5	1.6	0.36	0.07
		Moderate	Weak	3.1	3.2	–	3.4	0.49	0.14
			Strong	4.0	4.0	+1.0	4.7	0.63	0.41
		Strong	weak	3.2	3.4	+7.3	12.5	1.33	0.02
			Strong	4.3	4.4	+8.2	8.1	2.86	0.31
	SI	Weak	Weak	3.1	3.1	−0.2	0.8	0.03	0.13
			Strong	4.2	4.1	−0.4	2.1	0.16	0.35
		Moderate	Weak	3.0	3.0	−1.5	3.7	0.04	0.56
			Strong	3.7	3.7	−1.5	4.5	0.07	0.64
		Strong	Weak	3.1	2.9	−5.7	12.2	0.08	1.09
			Strong	3.8	3.6	−5.8	7.3	0.49	2.02
Items with different space elasticity	CI	Weak	Strong → weak	3.9	3.9	+0.5	3.3	0.27	0.03
			Weak → strong	3.1	3.1	+0.4	2.5	0.19	0.06
		Moderate	Strong → weak	4.0	4.1	+1.4	10.0	0.71	0.23
			Weak → strong	3.1	3.2	+1.0	7.0	0.49	0.14
		Strong	Strong → weak	4.0	4.3	+6.3	20.9	1.72	0.29
			Weak → strong	3.1	3.3	+5.6	20.8	1.18	0.08
	SI	Weak	Strong → weak	4.2	4.2	–	2.0	0.15	0.15
			Weak → strong	3.1	3.1	–	0.9	0.04	0.05
		Moderate	Strong → weak	3.7	3.7	−1.8	11.2	0.07	0.72
			Weak → strong	3.0	3.0	−1.5	6.9		0.51
		Strong	Strong → weak	3.8	3.6	−6.0	19.9	0.32	1.82
			Weak → strong	3.1	3.0	−5.0	20.5	0.08	1.12

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets, see Appendix Tables 13–16 for details.
[a] CI = complemented item, SI = substituted item; complements (C) and substitutes (S) shown in Appendix
[b] Weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%
[c] Weak: 0–15%; strong: 20–35%
[d] Average change in facings of all items with more (+) or less (−) facings

Table 7
CSE and demand volatility: Changes in facing decisions.

	Item group[a]	CSE effects[b]	Demand volatility[c]	Total item group share (%)		Relative change in item group share (%)	Share of items w/ facing changes (%)	Avg. change in no. of facings	
				$\delta = 0$	$\delta \neq 0$			+[d]	-[d]
Items with similar volatility	CI	Weak	Low	5.1	5.1	+0.4	2.2	0.33	0.07
			High	2.1	2.1	–	0.8	0.09	0.09
		Moderate	Low	4.9	5.0	+2.3	9.1	0.89	0.16
			High	2.1	2.1	+0.6	1.8	0.24	0.12
		Strong	Low	5.2	5.8	+12.0	22.4	2.13	0.15
			High	2.3	2.3	+2.6	4.6	0.95	0.37
	SI	Weak	Low	4.8	4.8	−0.8	2.8	0.02	0.45
			High	2.1	2.1	−0.6	0.7	–	0.20
		Moderate	Low	5.0	4.8	−2.2	8.2	0.01	0.84
			High	2.1	2.1	−1.0	2.4	0.13	0.38
		Strong	Low	4.9	4.4	−9.9	21.6	0.07	1.70
			High	2.1	2.1	−4.0	4.6	0.01	0.94
Items with different volatility	CI	Weak	High → low	2.1	2.1	+0.2	1.5	0.13	0.08
			Low → high	5.1	5.1	+0.2	1.4	0.21	0.10
		Moderate	High → low	2.1	2.1	+1.2	3.5	0.42	0.17
			Low → high	4.9	5.0	+0.6	5.5	0.53	0.29
		Strong	High → low	2.3	2.4	+6.8	12.2	1.25	0.32
			Low → high	5.2	5.2	+4.2	13.3	1.46	0.11
	SI	Weak	High → low	2.1	2.1	−0.2	0.8	0.08	0.13
			Low → high	4.8	4.8	−0.3	1.5	0.04	0.22
		Moderate	High → low	2.1	2.1	−1.6	4.6	0.19	0.53
			Low → high	5.0	4.9	−1.0	4.9	0.02	0.59
		Strong	High → low	2.1	2.0	−6.9	11.2	0.07	1.09
			Low → high	4.9	4.7	−3.3	10.8	0.21	1.24

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets, see Appendix Tables 17–20 for details.
[a] CI = complemented item, SI = substituted item; complements (C) and substitutes (S) shown in Appendix
[b] Weak: 0.1–0.5%; moderate:0.6–2.0%; strong: 2.1–10.0%
[c] Low: 1–40%; high: 50–90%
[d] Average change in facings of all items with more (+) or less (−) facings

Table 8
CSE and item margin: Changes in facing decisions.

	Item group[a]	CSE effects[b]	Item margin[c]	Total item group share (%)		Relative change in item group share (%)	Share of items w/ facing changes (%)	Avg. change in no. of facings	
				$\delta = 0$	$\delta \neq 0$			+[d]	−[d]
Items with similar margin	CI	Weak	Low	1.0	1.0	+0.1	0.4	0.03	0.03
			High	6.3	6.3	+0.7	3.5	0.46	0.03
		Moderate	Low	0.9	1.0	+1.4	0.4	0.24	0.04
			High	6.3	6.4	+2.7	14.0	0.97	0.18
		Strong	Low	1.0	0.9	−0.4	0.7	0.12	0.19
			High	6.3	7.0	+15.4	36.9	1.93	0.17
	SI	Weak	Low	1.1	1.0	−1.1	0.5	0.11	0.28
			High	6.3	6.2	−0.9	3.9	0.03	0.60
		Moderate	Low	1.0	1.0	−0.4	0.3	0.01	0.07
			High	6.1	5.9	−3.1	14.0		0.99
		Strong	Low	1.0	1.0	−0.1	0.7	0.14	0.13
			High	6.1	5.3	−13.7	37.6	0.19	1.71
Items with different margin	CI	Weak	High → low	6.3	6.3	–	1.9	0.10	0.09
			Low → high	1.0	1.0	+0.3	0.5	0.05	–
		Moderate	High → low	6.3	6.3	+0.2	2.4	0.20	0.02
			Low → high	0.9	0.9	+1.1	1.8	0.16	0.02
		Strong	High → low	6.1	6.1	+0.8	7.0	0.69	0.08
			Low → high	1.0	1.0	+4.1	5.1	0.61	0.10
	SI	Weak	High → low	6.3	6.3	+0.1	2.3	0.21	0.14
			Low → high	1.1	1.0	−0.1	0.2	–	0.02
		Moderate	High → low	6.1	6.1	−0.3	3.5	0.05	0.27
			Low → high	1.0	1.0	−1.3	1.9	–	0.18
		Strong	High → low	6.1	6.1	−0.6	6.7	0.23	0.53
			Low → high	1.0	1.0	−3.8	6.3	0.17	0.58

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets, see Appendix Tables 21–24 for details.
[a] CI = complemented item, SI = substituted item; complements (C) and substitutes (S) shown in Appendix.
[b] Weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%.
[c] Low: $r_i \in [17, 20]$, $c_i \in [13, 16]$; high: $r_i \in [17, 20]$, $c_i \in [8, 11]$.
[d] Average change in facings of all items with more (+) or less (−) facings.

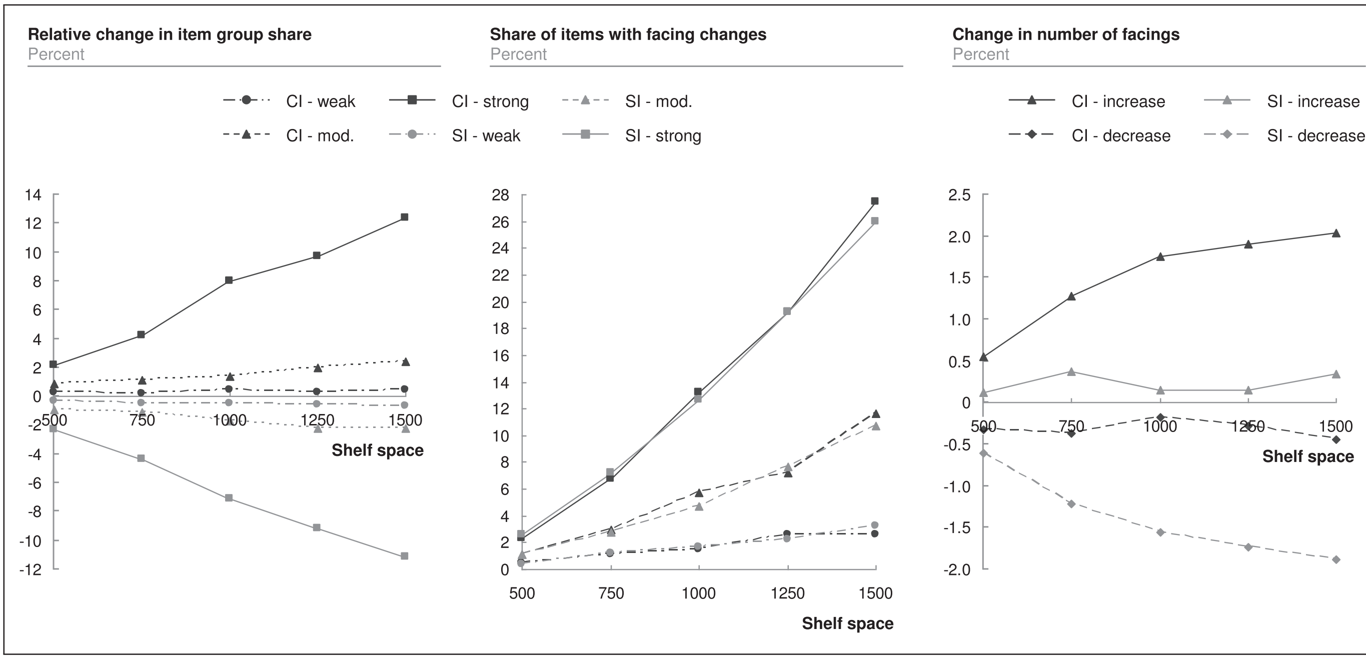

Fig. 4. Impact of CSE and shelf space on solution structure: Change in facing decisions if CSE is correctly accounted for, $N = 180$ (60 CI/C, 60 SI/S, 60 neutrals), $K = 30$, averages across 100 randomly generated data sets.

Table 9
Impact of CSE and shelf space on objective value: Profit advantage if CSE is correctly accounted for $N = 180$ (60 CI/C, 60 SI/S, 60 neutrals), $K = 30$, averages across 100 randomly generated data sets.

Shelf space S	500	750	1000	1250	1500
Profit advantage(%)	0.10	0.14	0.21	0.28	0.36

the overall impact on the complemented item is mixed. Again, we see that an item's own demand volatility is a stronger determinant than the demand volatility of complements and substitutes.

Relationship between cross-space elasticity and item margin. Table 8 shows the results of testing CSE and the third, key differentiating item characteristic, item margin. We see that item margin is an even stronger determinant for facing decisions. High margin items get the majority of shelf space, while low margin items receive few facings. If two items with a similar margin have strong CSE links, CSE and item margin reinforce one another (see e.g. 37% of strongly complemented items with high margin get an average of 1.93 more facings). If items with a different margin have CSE links, we see that the item's own margin is significantly more dominant than the margin of the complement or substitute. For instance, if a low margin complement is strongly complemented by a high margin item, it still receives an average of only 0.61 more facings.

4.4. Cross-space elasticity and scarcity of shelf-space

In this section, we investigate whether the scarcity of shelf space impacts the magnitude of CSE effects on retail profits and facing decisions. Table 9 shows the profit advantage when correctly accounting for CSE as a function of available shelf space. As before, the impact of CSE effects on profits is limited, with an advantage of up to 0.36% in the case of $S = 1500$. Furthermore, we see that the more limited the shelf space, the smaller the advantage of correctly accounting for CSE, i.e. the less relevant is CSE.

Fig. 4 shows the impact on facing decisions, which is – in line with the impact on profits – higher when shelf space is less limited. For instance, at $S = 500$, around 2% of all items get facing changes if strong complementary or substitution CSE effects are in place. At $S = 1500$, this value corresponds to 26–28%. Similarly, the average change in the number of facings is 0.5 (-0.6) for strong complements (substitutes) at $S = 500$ and 2.0 (-1.9) at $S = 1500$. The explanation is intuitive: The more limited the shelf space, the more retailers are required to focus on key item characteristics (e.g. item margin and demand volatility). If space becomes less limited, retailers can afford to focus on less relevant effects, such as CSE.

4.5. Summary of findings, discussion of managerial insights and implications for research

Summary of numerical findings. We have compared the effect of CSE on solution structures (i.e. number of facings) and objective values (i.e. total profit) by comparing the results of the $SCSP_{csp}$, where CSE is correctly taken into account, with those of the $SCSP^*_{csp}$, where CSE is integrated only "a posteriori". Our analysis reveals several general insights into the impact of CSE on shelf layouts, which pave the way for managerial insights and future research areas:

1. *The impact of cross-space elasticity on facing decisions and retail profits is limited.* Facing decisions and retail profits are only affected substantially if CSE effects differ significantly (i.e. greater than +/−10%) from the empirically measured average (i.e. −1.6%). The profit advantage of correctly accounting for CSE effects typically is less than 1% and even if up to a third of items can get a different number of facings, the average increase (decrease) in the number of facings is incremental and less than 2 units.
2. *Complemented items (positive CSE effects) receive more, substituted items (negative CSE effects) less shelf space.* In terms of magnitude, complementary CSE effects tend to have a slightly stronger impact than substitution CSE effects.

Table 10
Guidelines for retailers on handling CSE.

(a) If no CSE data available or no indication that CSE significantly differs from ±10%		Focus shelf-space decisions on key item characteristics, e.g. margin, space elasticity and volatility
(b) If CSE data available	CSE ∈ [−10%; 10%]	CSE has negligible impact, *see above*
	CSE < −10% and/or > 10%	If shelf space is more scarce, focus on key item characteristics; otherwise, develop a thorough understanding of CSE magnitude and evaluate impact on shelf decisions using appropriate decision tools

3. *Item characteristics like space elasticity, volatility and margin dominate CSE effects.* These characteristics have a much stronger impact on decision-making and objective value. CSE and item characteristics like space elasticity, demand volatility and item margin reinforce one another if CSE effects exist between similar items. If different items are linked through CSE effects, the impact of CSE is ambiguous. For the facing decision of an item, its own characteristics are more dominant determinants than the characteristics of its complements or substitutes. This especially applies to margins.
4. *The more limited the shelf space, the more retailers should focus on key item characteristics.* Only if shelf space is sufficiently available can retailers afford to pay attention to less relevant effects like CSE.
   These findings regarding a negligible impact of cross-space effects on planograms comply with general intuition to a certain extent: Previous empirical studies found that cross-space effects have a *minor impact on customer demand* in the first place. Since customer demand is one of the key determinants of planograms, a similarly *insignificant impact of cross-space effects on planograms* is a logical consequence. If one furthermore considers the *integer nature of facings*, it becomes even more obvious that decimal changes in demand (induced by cross-space effects) are unlikely to induce major changes of decision variables. Finally, the *scarcity of shelf space* further diminishes these effects, since other parameters (like margin) have a more significant and direct impact than the indirect and minor link cross-space effects have on decision variables through customer demand.

Guidelines for retailers. Generally speaking, if the magnitude of CSE effects corresponds to the empirically measured values thus far, there is no need for retailers to invest in expensive decision tools that account for CSE effects. Even if CSE effects should exceed the empirically measured values, it is not necessary to account for CSE until a threshold of at least ±10% has been reached. Shelf space decisions may be influenced only if CSE significantly deviates from this threshold. In this case, retailers should only invest in understanding CSE in more detail if shelf space is sufficiently available, and otherwise focus on key item characteristics, such as margin, demand volatility and space elasticity. Table 10 summarizes these guidelines.

Discussion in light of the literature. Our findings have implications for the *empirical research*. Because of the complexity of measuring CSE effects, only a few empirical studies have analyzed this phenomena. They found that CSE effects exist (cf. [9]). Using these insights, we assumed CSE effects in the magnitude of the empirically measured values and found that CSE effects have a minor impact on profits and solution structures. This shows that the significant effort associated with the empirical measurement of CSE effects is not worthwhile.

Our findings further reveal insights for the literature on *decision modeling*: A few shelf-space optimization models exist that account for CSE effects. Due to the combinatorial complexity arising from CSE effects, the majority of these are constrained in their applicability to instances of practice-relevant size, runtime or solution quality. Our findings show that CSE effects have a minor impact on retail decisions. Therefore, complex solution approaches (such as heuristics and piecewise linearization), which suffer from either runtime efficiency or low solution quality problems, are unnecessary. Research should focus instead on optimization models that account for space elasticity or other demand effects apart from CSE.

5. Conclusion and future areas of research

In this paper, we numerically investigated the impact of CSE on shelf-space planning, optimal facing decisions and retail profits. In general, we can conclude that the impact is very limited and that CSE is not the main determinant on which retailers should focus when assigning shelf space to items. We found clear evidence that CSE effects have zero to very low impact on profits and shelf layouts. We also investigated the interplay between CSE and key item characteristics and found that retailers should focus on understanding, collecting data on and modeling an item's own characteristics rather than on the characteristics of potential substitutes or complements.

Future research could extend the analysis and investigate the extent to which assortments and pricing are impacted by CSE. For this purpose, an integrated assortment, pricing and shelf-space model would need to be applied that optimizes assortments, pricing and facings, while accounting for all relevant demand effects – which in this case would include substitution demand for potentially delisted items and cross-price elasticity (cf. e.g. [41,43,44]). Although we have shown for general cases that CSE effects do not significantly impact facing decisions, special cases may exist in which this does not apply, including, for instance, specific product groups or categories, or categories comprising items from different CPGs. In this context, it is also interesting to note that our optimization model takes the perspective of a retailer who wants to optimize category profit. In contrast, a manufacturer pursues the objective of brand profit optimization, which raises the topic of "*category captainship*" (cf. e.g. [45,46]). A comprehensive study would have to address all the relevant subjects of negotiation between manufacturers and retailers, such as assortment, price and shelf space. It would also be of interest to investigate the relationship between CSE and other relevant demand effects, such as the horizontal and vertical position of items on a shelf (cf. e.g. [21,47]). Furthermore, items are often assigned according to merchandising rules accounting for product family characteristics, which might interact with CSE effects (c.f. [42]). A specific focus could be put on impulse-purchase items. It was shown that space elasticity is particularly high for these items (cf. [20]), and this may well apply also for cross-space elasticities. To investigate these special cases, further empirical analysis would be required to measure the CSE effects. The insights from such analysis would then serve as input for decision support systems suitable for daily use by a retailer (cf. e.g. [48]). This paper and the applied model serve as basis for the proposed areas of future research.

Appendix A

A1. General impact of cross-space elasticities on facing decision

Table 11
Changes in total item group shelf space.

Item group	CSE effects[a]	Avg. total item group share (%)		Avg. relative change in item group share (%)
		CSE = 0	CSE ≠ 0	
Complements	Weak	5.2	5.2	−1.4
	Moderate	5.6	5.5	−0.9
	Strong	5.7	5.6	−1.5
Complemented	Weak	5.6	5.6	−0.9
	Moderate	5.9	6.2	+5.7
	Strong	5.6	7.8	+38.9
Substitutes	Weak	5.6	5.6	−1.3
	Moderate	5.7	5.7	−1.2
	Strong	5.5	5.5	−0.8
Substituted	Weak	5.5	5.5	−1.0
	Moderate	5.2	4.8	−6.9
	Strong	5.4	4.1	−24.1
Neutrals		33.4	33.1	−1.1

$N = 180$ (60 CI/C, 60 SI/S, 60 neutrals), $S = 1,000$, $K = 30$, averages across 100 randomly generated data sets.
[a] weak: 0.1–1.0%; moderate: 1.1–10.0%; strong: 11.0–50.0%.

Table 12
Changes in facing decisions.

Item group	CSE effects[a]	Avg. share of items with facing changes (%)	Avg. change in no. of facings	
			Increase	Decrease
Complements	Weak	2.9	+0.12	−0.57
	Moderate	3.1	+0.28	−0.53
	Strong	2.8	+0.05	−0.57
Complemented	Weak	3.8	+0.31	−0.76
	Moderate	13.7	+1.62	−0.72
	Strong	15.3	+8.64	−2.02
Substitutes	Weak	2.8	+0.15	−0.80
	Moderate	3.0	+0.14	−0.50
	Strong	2.6	+0.09	−0.43
Substituted	Weak	4.1	+0.13	−0.49
	Moderate	12.0	+0.25	−1.54
	Strong	14.9	+0.28	−4.80
Neutrals		6.7	+0.69	−1.49

$N = 180$ (60 CI/C, 60 SI/S, 60 neutrals), $S = 1,000$, $K = 30$, averages across 100 randomly generated data sets.
[a] weak: 0.1–1.0%; moderate: 1.1–10.0%; strong: 11.0–50.0%.

A2. Interlink between cross-space elasticity and item space elasticity

A2.1. Cross-space elasticity between items of equally strong space elasticity

Table 13
CSE and space elasticity: Changes in total item group shelf space (CSE effects between equally space-elastic items).

Item group	CSE effects[a]	Space elasticity[b]	Avg. total item group share (%)		Avg. relative change in item group share (%)
			CSE = 0	CSE ≠ 0	
Complements	Weak	Weak	3.2	3.1	−0.5
		Strong	4.0	4.0	−0.3
	Moderate	Weak	3.0	3.0	−0.1
		Strong	4.1	4.1	−0.1
	Strong	Weak	3.0	3.0	−0.2
		Strong	4.2	4.2	–
Complemented	Weak	Weak	3.1	3.1	+0.2
		Strong	3.9	3.9	+0.5
	Moderate	Weak	3.1	3.2	–
		Strong	4.0	4.0	+1.0
	Strong	Weak	3.2	3.4	+7.3
		Strong	4.3	4.4	+8.2
Substitutes	Weak	Weak	3.1	3.2	−0.2
		Strong	4.3	4.3	−0.2
	Moderate	Weak	3.1	3.1	–
		Strong	4.3	4.3	−0.5
	Strong	Weak	3.0	3.0	–
		Strong	4.1	4.1	−0.1
Substituted	Weak	Weak	3.1	3.1	−0.2
		Strong	4.2	4.1	−0.4
	Moderate	Weak	3.0	3.0	−1.5
		Strong	3.7	3.7	−1.5
	Strong	Weak	3.1	2.9	−5.7
		Strong	3.8	3.6	−5.8
Neutrals		Weak	6.1	6.1	−0.2
		Strong	8.3	8.3	−0.3

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets.
[a] Weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%.
[b] Weak: 0–15%; strong: 20–35%.

A2.2. Cross-space elasticity between items of differently strong space elasticity

Table 14
CSE and space elasticity: Changes in facing decisions (CSE effects between equally space-elastic items).

Item group	CSE effects[a]	Space elasticity[b]	Avg. share of items with facing changes (%)	Avg. change in no. of facings	
				Increase	Decrease
Complements	Weak	Weak	0.9	+0.06	−0.31
		Strong	0.5	+0.02	−0.18
	Moderate	Weak	0.5	+0.04	−0.09
		Strong	1.1	+0.23	−0.31
	Strong	Weak	0.6	+0.04	−0.15
		Strong	0.8	+0.08	−0.10
Complemented	Weak	Weak	1.3	+0.18	−0.06
		Strong	1.6	+0.36	−0.07
	Moderate	Weak	3.4	+0.49	−0.14
		Strong	4.7	+0.63	−0.41
	Strong	Weak	12.5	+1.33	−0.02
		Strong	8.1	+2.86	−0.31
Substitutes	Weak	Weak	0.6	+0.02	−0.12
		Strong	0.8	+0.03	−0.17
	Moderate	Weak	0.5	+0.13	−0.12
		Strong	0.6	+0.06	−0.36
	Strong	Weak	0.7	+0.10	−0.12
		Strong	0.9	+0.04	−0.11
Substituted	Weak	Weak	0.8	+0.03	−0.13
		Strong	2.1	+0.16	−0.35
	Moderate	Weak	3.7	+0.04	−0.56
		Strong	4.5	+0.07	−0.64
	Strong	Weak	12.2	+0.08	−1.09
		Strong	7.3	+0.49	−2.02
Neutrals		Weak	1.6	+0.20	−0.34
		Strong	1.4	+0.09	−0.40

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets.
[a] weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%.
[b] Weak: 0–15%; strong: 20–35%.

A3. Interlink between cross-space elasticity and demand volatility

A3.1. Cross-space elasticity between items of equally high demand volatility

A3.2. Cross-space elasticity between items of differently high demand volatility

Table 15
CSE and space elasticity: Changes in total item group shelf space (CSE effects between differently space-elastic items).

Item group	CSE effects[a]	Space elasticity[b]	Avg. total item group share (%) CSE=0	CSE≠0	Avg. relative change in item group share (%)
Complements	Weak	Strong → weak	4.0	4.0	−0.1
		Weak → strong	3.2	3.1	−0.2
	Moderate	Strong → weak	4.1	4.1	−0.1
		Weak → strong	3.0	3.0	−0.1
	Strong	Strong → weak	4.2	4.2	–
		Weak → strong	3.0	3.0	−0.1
Complemented	Weak	Strong → weak	3.9	3.9	+0.5
		Weak → strong	3.1	3.1	+0.4
	Moderate	Strong → weak	4.0	4.1	+1.4
		Weak → strong	3.1	3.2	+1.0
	Strong	Strong → weak	4.0	4.3	+6.3
		Weak → strong	3.1	3.3	+5.6
Substitutes	Weak	Strong → weak	4.3	4.3	–
		Weak → strong	3.2	3.2	–
	Moderate	Strong → weak	4.3	4.3	–
		Weak → strong	3.1	3.1	–
	Strong	Strong → weak	4.1	4.1	–
		Weak → strong	3.0	3.0	−0.1
Substituted	Weak	strong → weak	4.2	4.2	–
		Weak → strong	3.1	3.1	–
	Moderate	Strong → weak	3.7	3.7	−1.8
		Weak → strong	3.0	3.0	−1.5
	Strong	Strong → weak	3.8	3.6	−6.0
		Weak → strong	3.1	3.0	−5.0
Neutrals	Strong	Strong	8.3	8.3	−0.2
		Weak	6.1	6.1	−0.2

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets
[a] Weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%
[b] Weak: 0–15%; strong: 20–35%

A4. Interlink between cross-space elasticity and item margin

A4.1. Cross-space elasticity between items of equally high margin

A4.2. Cross-space elasticity between items of differently high margin

Table 16
CSE and space elasticity: Changes in facing decisions (CSE effects between differently space-elastic items).

Item group	CSE effects[a]	Space elasticity[b]	Avg. share of items with facing changes (%)	Avg. change in no. of facings Increase	Decrease
Complements	Weak	Strong → weak	1.1	+0.08	−0.15
		Weak → strong	1.1	+0.06	−0.15
	Moderate	Strong → weak	1.8	+0.23	−0.28
		Weak → strong	1.0	+0.03	−0.09
	Strong	Strong → weak	1.1	+0.07	−0.04
		Weak → strong	1.2	+0.06	−0.11
Complemented	Weak	Strong → weak	3.3	+0.27	−0.03
		Weak → strong	2.5	+0.19	−0.06
	Moderate	Strong → weak	10.0	+0.71	−0.23
		Weak → strong	7.0	+0.49	−0.14
	Strong	Strong → weak	20.9	+1.72	−0.29
		Weak → strong	20.8	+1.18	−0.08
Substitutes	Weak	Strong → weak	0.9	+0.05	−0.04
		Weak → strong	0.8	+0.06	−0.05
	Moderate	Strong → weak	1.6	+0.12	−0.15
		Weak → strong	1.1	+0.11	−0.12
	Strong	Strong → weak	1.0	+0.05	−0.05
		Weak → strong	1.2	+0.06	−0.12
Substituted	Weak	Strong → weak	2.0	+0.15	−0.15
		Weak → strong	0.9	+0.04	−0.05
	Moderate	Strong → weak	11.2	+0.07	−0.72
		Weak → strong	6.9	–	−0.51
	Strong	Strong → weak	19.9	+0.32	−1.82
		Weak → strong	20.5	+0.08	−1.12
Neutrals	Strong	Strong	2.5	+0.07	−0.21
		Weak	2.5	+0.17	−0.23

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets
[a] Weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%
[b] Weak: 0–15%; strong: 20–35%

A5. Interlink between cross-space elasticity and scarcity of shelf-space

Table 17
CSE and demand volatility: Changes in total item group shelf space (CSE effects between equally volatile items).

Item group	CSE effects[a]	Demand volatility[b]	Avg. total item group share (%) CSE=0	CSE≠0	Avg. relative change in item group share (%)
Complements	Weak	Low	4.9	4.9	−0.03
		High	2.3	2.3	+0.27
	Moderate	Low	5.2	5.2	+0.05
		High	2.3	2.2	−0.41
	Strong	Low	5.0	5.0	−0.21
		High	2.2	2.2	−0.27
Complemented	Weak	Low	5.1	5.1	+0.42
		High	2.1	2.1	–
	Moderate	Low	4.9	5.0	+2.26
		High	2.1	2.1	+0.58
	Strong	Low	5.2	5.8	+11.95
		High	2.3	2.3	+2.59
Substitutes	Weak	Low	5.0	5.0	−0.05
		High	2.1	2.1	−0.19
	Moderate	Low	5.0	5.0	−0.21
		High	2.0	2.0	−0.83
	Strong	Low	5.0	5.0	−0.16
		High	2.2	2.2	−0.28
Substituted	Weak	Low	4.8	4.8	−0.76
		High	2.1	2.1	−0.63
	Moderate	Low	5.0	4.8	−2.23
		High	2.1	2.1	−0.97
	Strong	Low	4.9	4.4	−9.87
		High	2.1	2.1	−4.04
		High	2.1	2.1	−4.04
Neutrals		Low	9.9	9.9	−0.01
		High	4.4	4.4	−0.14

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets
[a] weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%
[b] Low: 1–40%; high: 50–90%

Table 18
CSE and demand volatility: Changes in facing decisions (CSE effects between equally volatile items).

Item group	CSE effects[a]	Demand volatility[b]	Avg. share of items with facing changes (%)	Avg. change in no. of facings: Increase	Decrease
Complements	Weak	low	0.7	+0.06	−0.08
		high	0.6	+0.17	−0.08
	Moderate	low	0.5	+0.12	−0.08
		high	1.2	+0.04	−0.19
	Strong	low	0.7	+0.04	−0.20
		high	0.6	+0.03	−0.12
Complemented	Weak	low	2.2	+0.33	−0.07
		high	0.8	+0.09	−0.09
	Moderate	low	9.1	+0.89	−0.16
		high	1.8	+0.24	−0.12
	Strong	low	22.4	+2.13	−0.15
		high	4.6	+0.95	−0.37
Substitutes	Weak	low	0.8	+0.06	−0.10
		high	0.6	+0.06	−0.15
	Moderate	low	0.8	+0.14	−0.30
		high	0.8	+0.08	−0.33
	Strong	low	0.4	+0.02	−0.13
		high	0.6	+0.04	−0.12
Substituted	Weak	low	2.8	+0.02	−0.45
		high	0.7	0.00	−0.20
	Moderate	low	8.2	+0.01	−0.84
		high	2.4	+0.13	−0.38
	Strong	low	21.6	+0.07	−1.70
		high	4.6	+0.01	−0.94
Neutrals		low	2.0	+0.28	−0.26
		high	1.4	+0.07	−0.20

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets.
[a] weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%.
[b] Low: 1–40%; high: 50–90%.

Table 19
CSE and demand volatility: Changes in total item group shelf space (CSE effects between differently volatile items).

Item group	CSE effects[a]	Demand volatility[b]	Avg. total item group share (%): CSE=0	CSE≠0	Avg. relative change in item group share (%)
Complements	Weak	High → low	2.3	2.3	+0.35
		Low → high	4.9	4.9	−0.05
	Moderate	High → low	2.3	2.3	−0.12
		Low → high	5.2	5.2	+0.21
	Strong	High → low	2.2	2.2	−0.42
		Low → high	5.0	5.0	−0.05
Complemented	Weak	High → low	2.1	2.1	+0.22
		Low → high	5.1	5.1	+0.18
	Moderate	High → low	2.1	2.1	+1.22
		Low → high	4.9	5.0	+0.58
	Strong	High → low	2.3	2.4	+6.80
		Low → high	5.2	5.2	+4.15
Substitutes	Weak	High → low	2.1	2.1	+0.26
		Low → high	5.0	5.0	−0.01
	Moderate	High → low	2.0	2.0	−0.17
		Low → high	5.0	5.0	−0.15
	Strong	High → low	2.2	2.2	−0.18
		Low → high	5.0	5.0	−0.15
Substituted	Weak	High → low	2.1	2.1	−0.19
		Low → high	4.8	4.8	−0.26
	Moderate	High → low	2.1	2.1	−1.60
		Low → high	5.0	4.9	−0.97
	Strong	High → low	2.1	2.0	−6.87
		Low → high	4.9	4.7	−3.33
Neutrals	Strong	High	4.4	4.1	−0.12
		Low	9.9	9.9	−0.07

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets.
[a] Weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%.
[b] Low: 1–40%; high: 50–90%.

Table 20
CSE and demand volatility: Changes in facing decisions (CSE effects between differently volatile items).

Item group	CSE effects[a]	Demand volatility[b]	Avg. share of items with facing changes (%)	Avg. change in no. of facings: Increase	Decrease
Complements	Weak	High → low	0.6	+0.18	−0.06
		Low → high	0.6	+0.02	−0.06
	Moderate	High → low	1.2	+0.06	−0.10
		Low → high	0.8	+0.20	−0.05
	Strong	High → low	0.9	+0.02	−0.16
		Low → high	0.5	+0.02	−0.06
Complemented	Weak	High → low	1.5	+0.13	−0.08
		Low → high	1.4	+0.21	−0.10
	Moderate	High → low	3.5	+0.42	−0.17
		Low → high	5.5	+0.53	−0.29
	Strong	High → low	12.2	+1.25	−0.32
		Low → high	13.3	+1.46	−0.11
Substitutes	Weak	High → low	0.7	+0.15	−0.07
		Low → high	0.9	+0.05	−0.07
	Moderate	High → low	0.8	+0.07	−0.12
		Low → high	0.4	+0.04	−0.15
	Strong	High → low	1.1	+0.03	−0.09
		Low → high	0.7	+0.02	−0.13
Substituted	Weak	High → low	0.8	+0.08	−0.13
		Low → high	1.5	+0.04	−0.22
	Moderate	High → low	4.6	+0.19	−0.53
		Low → high	4.9	+0.02	−0.59
	Strong	High → low	11.2	+0.07	−1.09
		Low → high	10.8	+0.21	−1.24
Neutrals	Strong	High	1.8	+0.09	−0.19
		Low	1.8	+0.11	−0.20

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets.
[a] weak: 0.1–0.5%; moderate: 0.6–2.0%.; strong: 2.1–10.0%
[b] Low: 1–40%; high: 50–90%.

Table 21
CSE and item margin: Changes in total item group shelf space (CSE effects between items with similarly high margin).

Item group	CSE effects[a]	Item margin[b]	Avg. total item group share (%): CSE=0	CSE≠0	Avg. relative change in item group share (%)
Complements	Weak	L:ow	1.0	1.0	+0.20
		High	6.3	6.3	−0.01
	Moderate	Low	0.9	0.9	−0.42
		High	6.2	6.2	−0.16
	Strong	Low	1.0	1.0	−0.61
		High	6.0	6.0	−0.10
Complemented	Weak	Low	1.0	1.0	+0.07
		High	6.3	6.3	+0.68
	Moderate	Low	0.9	1.0	+1.42
		High	6.3	6.4	+2.74
	Strong	Low	1.0	0.9	−0.35
		High	6.1	7.0	+15.40
Substitutes	Weak	Low	1.0	1.0	+0.07
		High	6.0	6.0	−0.09
	Moderate	Low	1.0	1.0	–
		High	6.2	6.2	−0.03
	Strong	Low	1.0	1.0	+0.07
		High	6.2	6.2	−0.13
Substituted	Weak	Low	1.1	1.0	−1.08
		High	6.3	6.2	−0.88
	Moderate	Low	1.0	1.0	−0.40
		High	6.1	5.9	−3.11
	Strong	Low	1.0	1.0	−0.13
		High	6.1	5.3	−13.66
Neutrals		Low	1.9	1.9	+0.42
		High	12.2	12.2	−0.22

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets.
[a] weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%.
[b] Low: $r_i \in [17, 20]$, $c_i \in [13, 16]$; high: $r_i \in [17, 20]$, $c_i \in [8, 11]$.

Table 22
CSE and item margin: Changes in facing decisions (CSE effects between items with similarly high margin).

Item group	CSE effects[a]	Item margin[b]	Avg. share of items with facing changes (%)	Avg. change in no. of facings: Increase	Decrease
Complements	Weak	Low	0.3	+0.07	−0.04
		High	1.5	+0.15	−0.11
	Moderate	Low	0.3	+0.01	−0.07
		High	2.1	+0.11	−0.25
	Strong	Low	0.4	+0.09	−0.18
		High	1.3	+0.07	−0.14
Complemented	Weak	Low	0.4	+0.03	−0.03
		High	3.5	+0.46	−0.03
	Moderate	Low	0.4	+0.24	−0.04
		High	14.0	+0.97	−0.18
	Strong	Low	0.7	+0.12	−0.19
		High	36.9	+1.93	−0.17
Substitutes	Weak	Low	0.3	+0.07	−0.06
		High	1.2	+0.07	−0.13
	Moderate	Low	0.2	+0.02	−0.02
		High	1.5	+0.13	−0.13
	Strong	Low	0.2	+0.03	−0.02
		High	1.0	+0.05	−0.16
Substituted	Weak	Low	0.5	+0.11	−0.28
		High	3.9	+0.03	−0.60
	Moderate	Low	0.3	+0.01	−0.07
		High	14.0	–	−0.99
	Strong	Low	0.7	+0.14	−0.13
		High	37.6	+0.19	−1.71
Neutrals		Low	0.3	+0.15	−0.03
		High	2.8	+0.16	−0.46

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets.
[a] weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%.
[b] Low: $r_i \in [17, 20]$, $c_i \in [13, 16]$; high: $r_i \in [17, 20]$, $c_i \in [8, 11]$.

Table 23
CSE and item margin: Changes in total item group shelf space (CSE effects between items with differently high margin).

Item group	CSE effects[a]	Item margin[b]	Avg. total item group share (%): CSE=0	CSE≠0	Avg. relative change in item group share(%)
Complements	Weak	High → low	6.3	6.3	−0.06
		Low → high	1.0	1.0	+0.27
	Moderate	High → low	6.2	6.2	+0.05
		Low → high	0.9	0.9	−0.07
	Strong	High → low	6.0	6.0	−0.07
		Low → high	1.0	1.0	−0.20
Complemented	Weak	High → low	6.3	6.3	+0.01
		Low → high	1.0	1.0	+0.33
	Moderate	High → low	6.3	6.3	+0.21
		Low → high	0.9	0.9	+1.07
	Strong	High → low	6.1	6.1	+0.84
		Low → high	1.0	1.0	+4.06
Substitutes	Weak	High → low	6.0	6.1	−0.06
		Low → high	1.0	1.0	+0.33
	Moderate	High → low	6.2	6.2	+0.05
		Low → high	1.0	1.0	−0.33
	Strong	High → low	6.2	6.2	−0.01
		Low → high	1.0	1.0	+0.07
Substituted	Weak	High → low	6.3	6.3	+0.06
		Low → high	1.1	1.0	−0.13
	Moderate	High → low	6.1	6.1	−0.27
		Low → high	1.0	1.0	−1.27
	Strong	high → low	6.1	6.1	−0.56
		low → high	1.0	1.0	−3.82
Neutrals	Strong	high	12.2	12.2	−0.09
		low	1.9	1.9	−0.07

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets.
[a] weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%.
[b] Low: $r_i \in [17, 20]$, $c_i \in [13, 16]$; high: $r_i \in [17, 20]$, $c_i \in [8, 11]$.

Table 24
CSE and item margin: Changes in facing decisions (CSE effects between items with differently high margin).

Item group	CSE effects[a]	Item margin[b]	Avg. share of items with facing changes (%)	Avg. change in no. of facings: Increase	Decrease
Complements	Weak	High → low	1.1	+0.03	−0.08
		Low → high	0.2	+0.05	−0.01
	Moderate	High → low	1.7	+0.11	−0.16
		Low → high	0.1	0.00	−0.01
	Strong	High → low	1.4	+0.03	−0.08
		Low → high	0.7	+0.02	−0.05
Complemented	Weak	High → low	1.9	+0.10	−0.09
		Low → high	0.5	+0.05	–
	Moderate	high → low	2.4	+0.20	−0.02
		Low → high	1.8	+0.16	−0.02
	Strong	High → low	7.0	+0.69	−0.08
		Low → high	5.1	+0.61	−0.10
Substitutes	Weak	High → low	0.7	+0.01	−0.06
		Low → high	0.5	+0.06	−0.01
	Moderate	High → low	1.1	+0.07	−0.03
		Low → high	0.3	+0.10	−0.15
	Strong	High → low	2.1	+0.08	−0.10
		Low → high	0.1	+0.01	–
Substituted	Weak	High → low	2.3	+0.21	−0.14
		Low → high	0.2	0.00	−0.02
	Moderate	High → low	3.5	+0.05	−0.27
		Low → high	1.9	–	−0.18
	Strong	High → low	6.7	+0.23	−0.53
		Low → high	6.3	+0.17	−0.58
Neutrals	Strong	High	2.9	+0.09	−0.22
		Low	0.4	+0.01	−0.03

$N = 280$ (120 CI/C, 120 SI/S, 40 neutrals), $S = 1,500$, $K = 30$, averages across 100 randomly generated data sets.
[a] weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%.
[b] Low: $r_i \in [17, 20]$, $c_i \in [13, 16]$; high: $r_i \in [17, 20]$, $c_i \in [8, 11]$.

Table 25
CSE and space scarcity: Changes in total item group shelf space, avg. total item group share (%).

Item group[a]	CSE effects[b]	S = 500		S = 750		S = 1000		S = 1250		S = 1500	
		CSE=0	CSE ≠ 0	CSE=0	CSE ≠ 0	CSE=0	CSE ≠ 0	CSE=0	CSE ≠ 0	CSE = 0	CSE ≠ 0
C	Weak	5.8	5.8	5.6	5.6	5.6	5.6	11.2	11.2	11.2	11.2
	Mod.	5.7	5.7	5.9	5.9	5.7	5.7	11.1	11.2	11.1	11.2
	Strong	5.3	5.2	5.5	5.5	5.5	5.5	11.2	11.7	11.3	12.0
CI	Weak	5.7	5.8	5.7	5.7	5.7	5.7	11.0	11.0	11.1	11.0
	Mod.	5.3	5.4	5.5	5.5	5.5	5.6	11.2	11.1	11.1	10.9
	Strong	5.7	5.8	5.5	5.7	5.5	6.0	11.3	10.8	11.2	10.6
S	Weak	5.7	5.7	5.7	5.6	5.6	11.3	11.2	11.2	11.2	11.2
	Mod.	5.7	5.7	5.6	5.6	5.7	11.3	11.1	11.2	11.1	11.2
	Strong	5.7	5.8	5.8	5.8	5.9	11.5	11.2	11.7	11.3	12.0
SI	Weak	5.5	5.5	5.5	5.5	5.4	11.3	11.2	11.2	11.2	11.2
	Mod.	5.5	5.5	5.4	5.4	5.4	11.3	11.1	11.2	11.1	11.2
	Strong	5.3	5.2	5.2	5.0	5.4	11.5	11.2	11.7	11.3	12.0
Neutr.		33.1	33.1	33.1	33.2	33.0	33.0	33.1	33.0	33.0	33.0

$N = 180$ (60 CI/C, 60 SI/S, 60 neutrals), $K = 30$, averages across 100 randomly generated data sets.
[a] C - complements, CI - complemented items, S - substitutes, SI - substituted items.
[b] Weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%.

Table 26
CSE and space scarcity: Changes in total item group shelf space, avg. relative change in item group share (%).

Item group[a]	CSE effects[b]	S = 500	S = 750	S = 1000	S = 1250	S = 1500
C	Weak	–	+0.2	−0.2	+0.3	–
	Moderate	−0.3	+0.1	+0.2	−0.2	−0.3
	Strong	−0.9	−0.2	−0.1	+0.1	−0.1
CI	Weak	+0.3	+0.2	+0.4	+0.3	+0.4
	Moderate	+0.9	+1.1	+1.4	+2.0	+2.4
	Strong	+2.2	+4.2	+8.0	+9.7	+12.4
S	Weak	+0.1	−0.3	–	−0.1	−0.2
	Moderate	+0.1	–	+0.1	–	−0.2
	Strong	+0.1	−0.2	−0.1	–	–
SI	Weak	−0.3	−0.5	−0.5	−0.6	−0.7
	Moderate	−0.9	−1.1	−1.7	−2.2	−2.2
	Strong	−2.3	−4.4	−7.1	−9.2	−11.1
Neutr.		+0.1	+0.1	−0.1	−0.1	−0.1

$N = 180$ (60 CI/C, 60 SI/S, 60 neutrals), $K = 30$, averages across 100 randomly generated data sets.
[a] C - complements, CI - complemented items, S - substitutes, SI - substituted items.
[b] Weak: 0.1–0.5%; moderate: 0.6–2.0%.; strong:2.1–10.0%

Table 27
CSE and space scarcity: Changes in facing decisions: share of items with facing changes (%).

Item group[a]	CSE effects[b]	S = 500	S = 750	S = 1000	S = 1250	S = 1500
C	Weak	0.1	0.5	0.8	1.3	1.1
	Moderate	0.2	0.4	0.7	0.7	1.5
	Strong	0.5	0.8	0.8	0.7	1.2
CI	Weak	0.5	1.1	1.5	2.6	2.6
	Moderate	1.1	3.0	5.7	7.2	11.6
	Strong	2.3	6.7	13.2	19.2	27.4
S	Weak	0.4	0.6	0.8	0.9	1.5
	Moderate	0.2	0.7	1.1	1.0	1.2
	Strong	0.5	0.8	1.0	1.3	0.9
SI	Weak	0.4	1.2	1.7	2.2	3.2
	Moderate	1.1	2.8	4.7	7.7	10.7
	Strong	2.6	7.2	12.7	19.2	26.0
Neutr.		0.6	1.4	1.6	1.9	2.7

$N = 180$ (60 CI/C, 60 SI/S, 60 neutrals), $K = 30$, averages across 100 randomly generated data sets.
[a] C - complements, CI - complemented items, S - substitutes, SI - substituted items.
[b] weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%.

Table 28
CSE and space scarcity: changes in facing decisions: avg. change in no. of facings.

Item group [a]	CSE effects[b]	$S = 500$ +	$S = 500$ −	$S = 750$ +	$S = 750$ −	$S = 1000$ +	$S = 1000$ −	$S = 1250$ +	$S = 1250$ −	$S = 1500$ +	$S = 1500$ −
C	Weak	+0.13	−0.03	+0.11	−0.04	+0.03	−0.10	+0.44	−0.20	+0.10	−0.10
	Mod.	+0.24	−0.10	+0.08	−0.03	+0.18	−0.09	+0.05	−0.14	+0.07	−0.25
	Strong	+0.55	−0.32	+0.05	−0.11	+0.05	−0.11	+0.23	−0.16	+0.08	−0.15
CI	Weak	+0.13	−0.03	+0.22	−0.14	+0.21	−0.06	+0.38	−0.20	+0.36	−0.10
	Mod.	+0.24	−0.10	+0.42	−0.11	+0.66	−0.26	+0.81	−0.04	+0.93	−0.29
	Strong	+0.55	−0.32	+1.28	−0.37	+1.76	−0.18	+1.90	−0.28	+2.04	−0.44
S	Weak	+0.06	−0.10	+0.02	−0.16	+0.10	−0.07	+0.06	−0.10	+0.10	−0.21
	Mod.	+0.04	−0.24	+0.11	−0.10	+0.19	−0.17	+0.09	−0.13	+0.09	−0.26
	Strong	+0.12	−0.60	+0.10	−0.19	+0.10	−0.15	+0.12	−0.17	+0.12	−0.10
SI	Weak	+0.06	−0.10	+0.08	−0.31	+0.16	−0.38	+0.04	−0.38	+0.05	−0.43
	Mod.	+0.04	−0.24	+0.12	−0.45	+0.02	−0.68	+0.03	−0.85	+0.28	−0.91
	Strong	+0.12	−0.60	+0.38	−1.22	+0.15	−1.55	+0.16	−1.73	+0.35	−1.88
Neutr.		+0.38	−0.21	+0.84	−0.50	+0.51	−0.79	+0.50	−0.52	+0.65	-0.84

$N = 180$ (60 CI/C, 60 SI/S, 60 neutrals), $K = 30$, averages across 100 randomly generated data sets.
[a] C - complements, CI - complemented items, S - substitutes, SI - substituted items.
[b] Weak: 0.1–0.5%; moderate: 0.6–2.0%; strong: 2.1–10.0%.

References

[1] Fisher ML, Raman A. The new science of retailing: How analytics are transforming the supply chain and improving performance. Boston: Harvard Business School Publishing; 2010.
[2] Carlotti SJ, Coe ME, Perrey J. Designing and managing winning brand portfolios: profiting from proliferation. McKinsey Quart 2006.
[3] Cologne, EHI. Retail data 2014: Structure, key figures and profiles of international retailing. 2014.
[4] Gutgeld Y, Sauer S, Wachinger T. Growth - but how? Akzente 2009;3(3):14–19.
[5] Drèze X, Hoch SJ, Purk ME. Shelf management and space elasticity. J Retail 1994;70(4):301–26.
[6] Brown MG, Lee JY. Allocation of shelf space: a case study of refridgerated juice products in grocery stores. Agribusiness 1996;12(2):113–21.
[7] Flynn D. Category management: Models and a decision support tool. School of Marketing, Curtin University of Technology, Australia; 2004. Honors Dissertation.
[8] Chandon P, Hutchinson WJ, Bradlow ET, Young SH. Does in-store marketing work? Effects of the number and position of shelf facings on brand attention and evaluation at the point of purchase. J Mark 2009;73:1–17.
[9] Eisend M. Shelf space elasticity: a meta-analysis. J Retail 2014;90:168–81.
[10] Zufryden FS. A dynamic programming approach for product selection and supermarket shelf-space allocation. J Oper Res Soc 1986;37(4):413–22.
[11] Desmet P, Renaudin V. Estimation of product category sales responsiveness to allocated shelf space. Int J Res Mark 1998;15(5):443–57.
[12] Kök G, Fisher ML, Vaidyanathan R. Assortment planning: review of literature and industry practice. In: Agrawal N, Smith SA, editors. Retail supply chain management. International Series in Operations Research & Management Science, vol. 223. Springer US; 2015. p. 175–236. ISBN 978-1-4899-7561-4.
[13] Hübner AH, Kuhn H. Retail category management: a state-of-the-art review of quantitative research and software applications in assortment and shelf space management. Omega 2012;40(2):199–209.
[14] Bianchi-Aguiar T., Hübner A., Carravilla M.A., Oliveira J.F.. Retail shelf space planning problems: a comprehensive review and classification framework. Working Paper University Porto, 2016a;1–23.
[15] Hübner AH, Kuhn H, Sternbeck MG. Demand and supply chain planning in grocery retail: an operations planning framework. Int J Retail Distrib Manag 2013;41(7):512–30.
[16] Hübner A. A decision support system for retail assortment planning. Int J Retail Distrib Manag 2017;45(7/8):808–25.
[17] Inman JJ, Winter RS, Ferraro R. The interplay among category characteristics, customer characteristics, and customer activities on in-store decision making. J Mark 2009;73(September):19–29.
[18] Cox K. The responsiveness of food sales to shelf space changes in supermarkets. J Mark Res 1964;1(2):63–7.
[19] Frank RE, Massy WF. Shelf position and space effects on sales. J Mark Res 1970;7(1):59–66.
[20] Curhan RC. The relationship between shelf space and unit sales in supermarkets. J Mark Res 1972;9(4):406–12.
[21] Hübner A, Schaal K. A shelf-space optimization model when demand is stochastic and space-elastic. Omega 2017(68):139–54.
[22] Hansen JM, Raut S, Swami S. Retail shelf allocation: a comparative analysis of heuristic and meta-heuristic approaches. J Retail 2010;86(1):94–105.
[23] Corstjens M, Doyle P. A model for optimizing retail space allocations. Manag Sci 1981;27(7):822–33.
[24] Campo K, Gijsbrechts E, Goossens T, Verhetsel A. The impact of location factors on the attractiveness and optimal space shares of product categories. Intl J Res Mark 2000;17(4):255–79.
[25] Borin N, Farris P, Freeland J. A model for determining retail product category assortment and shelf space allocation. Decis Sci 1994;25(3):359–84.
[26] Yang M-H. An efficient algorithm to allocate shelf space. Eur J Oper Res 2001;131(1):107–18.
[27] Hariga MA, Al-Ahmari A, Mohamed A-RA. A joint optimisation model for inventory replenishment, product assortment, shelf space and display area allocation decisions. Eur J Oper Res 2007;181(1):239–51.
[28] Hwang H, Choi B, Lee G. A genetic algorithm approach to an integrated problem of shelf space design and item allocation. Comput Ind Eng 2009;56(3):809–20. doi:10.1016/j.cie.2008.09.012.
[29] Gajjar HK, Adil GK. A piecewise linearization for retail shelf space allocation problem and a local search heuristic. Ann Oper Res 2010;179(1):149–67.
[30] Irion J, Lu J-C, Al-Khayyal FA, Tsao Y-C. A piecewise linearization framework for retail shelf space management models. Eur J of Oper Res 2012;222(1):122–36.
[31] Zhao J, Zhou Y-W, Wahab M. Joint optimization models for shelf display and inventory control considering the impact of spatial relationship on demand. Eur J Oper Res 2016;255(3):797–808.
[32] Frontoni E, Marinelli F, Rosetti R, Zingaretti P. Shelf space re-allocation for out of stock reduction. Comput Ind Eng 2017;106:32–40.
[33] Agrawal N, Smith S. Estimating negative binomial demand for retail inventory management with unobservable lost sales. Nav Res Logist 1996;43(6):839–61.
[34] Kök GA, Fisher ML. Demand estimation and assortment optimization under substitution: Methodology and application. Oper Res 2007;55(6):1001–21.
[35] Hübner A, Schaal K. An integrated assortment and shelf-space optimization model with demand substitution and space-elasticity effects. Eur J Oper Res 2017b;261(1):302–16.
[36] Hansen P, Heinsbroek H. Product selection and space allocation in supermarkets. Eur J Oper Res 1979;3(6):474–84.
[37] Baker RC, Urban TL. A deterministic inventory system with an inventory-level-dependent demand rate. J Oper Res Soc 1988:823–31.
[38] Khouja M. The single-period (news-vendor) problem: literature review and suggestions for future research. Omega 1999;27(5):537–53.
[39] Pisinger D. Where are the hard knapsack problems? Comput Oper Res 2005;32(9):2271–84.
[40] Smith SA, Agrawal N. Management of multi-item retail inventory systems with demand substitution. Oper Res 2000;48(1):50–64.
[41] Hübner A, Kühn S, Kuhn H. An efficient algorithm for capacitated assortment planning with stochastic demand and substitution. Eur J Oper Res 2016;250(2):505–20.
[42] Bianchi-Aguiar T, Silva E, Guimarães L, Carravilla MA, Oliveira JF. Allocating products on shelves under merchandising rules: multi-level product families with display directions. Omega 2017. doi:10.1016/j.omega.2017.04.002. Forthcoming
[43] Katsifou A, Seifert RW, Tancrez J-S. Joint product assortment, inventory and price optimization to attract loyal and non-loyal customers. Omega 2014;46(0):36–50.
[44] Ghoniem A, Maddah B, Ibrahim A. Optimizing assortment and pricing of multiple retail categories with cross-selling. J Glob Optim 2016;66(2):291–309.
[45] Kurtulus M, Toktay BL. Category captainship vs. retailer category management and limited retail shelf space. Prod Oper Manag 2011;20(1):47–56.
[46] Martínez-de Albéniz V, Roels G. Competing for shelf space. Prod Oper Manag 2011;20(1):32–46.
[47] Lim A, Rodrigues B, Zhang X. Metaheuristics with local search techniques for retail shelf-space optimization. Manag Sci 2004;50(1):117–31.
[48] Bianchi-Aguiar T, Silva E, Guimarães L, Carravilla MA, Oliveira JF, Amaral JG, et al. Using analytics to enhance shelf-space management in a food retailer. Interfaces 2017;46(5):424–44.