On the variability of the creep coefficient of structural concrete

D. Diamantidis

Dipl.-Ing., Research Assistant, Institut für Massivbau, Technical University of Munich, Munich, W. Germany.

H. O. Madsen

Visiting professor, Institut für Massivbau, Technical University of Munich, Munich, W. Germany. Permanent address: Danish Engeneering Academy, Building 373, DK2800 Lyngby, Denmark.

R. Rackwitz

Dr.-Ing., Research Associate, Institut für Massivbau, Technical University of Munich, Munich, W. Germany.

Experimental data from many creep test series are compared with predictions from various formulas in international codes. A mathematical simple creep formula is proposed and its accuracy is determined from comparison with the experimental data. Based hereon, a probabilistic model for the creep coefficient is proposed and the different sources of uncertainty are quantified. The proposed model is well suited for hand calculations.

1. INTRODUCTION

When deterministically predicted concrete creep effects are compared with actually measured effects, large discrepancies can be observed. This may be ascribed partly to a lack of knowledge about the creep processes and partly to unknown variations in the parameters influencing the creep processes. A third source of uncertainty arising from inherent uncertainties in the microscopic creep processes showing up macroscopically may also be of importance. Whether or not this third source of uncertainty can be ignored is still an open question – see [1], [2], [3]. In this paper such uncertainties are dealt with together with the model uncertainty, i. e. the aforementioned uncertainty due to the limited knowledge of the creep processes. The other source of uncertainty is referred to as parameter uncertainty.

The present paper is a "background" paper for the Basic Note R-02 on Concrete Strength [4]. The aim of this paper is to formulate a simple probability-based uncertainty model for creep effects applicable to the design of codes or to practical calculations in situations where creep effects are of major importance as e.g. to slender columns and prestressed members. The paper also attempts to quantify and weigh the different uncertainties. The paper does not, however, try to give a new physically based creep model.

In the first part of the paper a large number of test results is compared with values predicted according to present deterministic codes. Based on this comparison a simple mathematical formula for the creep coefficient is chosen. The formula contains a model uncertainty factor and factors depending on the external parameters such as concrete composition and environmental conditions. The different contribution to the total uncertainty arising from these external parameters then is analysed.

2. MODEL UNCERTAINTY. COMPARISON OF PREDICTION FORMULAS

2.1. Prediction formulas

The total stress dependent strain at an instant t under constant stress σ_0 applied at time τ is

$$\varepsilon = \frac{\sigma_0}{E_C(\tau)} + \frac{\sigma_0}{E_C(\tau)}\varphi(t,\tau) \qquad (2.1)$$

where $E_C(\tau)$ is the modulus of elasticity at the time of loading while $\varphi(t,\tau)$ is the *creep coefficient.* The first term in (2.1) is the elastic strain and the second term the creep strain. The creep coefficient is thus the ratio of creep strain to elastic strain. The creep strain is often further divided e.g. into delayed elastic and flow strain.

Different prediction formulas for the creep coefficient have been proposed in recent codes and are here compared with test results. The prediction formulas for the following 7 codes were used.

a) CEB/FIP recommendation 1966, (C66), [5];
b) CEB/FIP recommendation 1970, (C70), [6];
c) CEB/FIP recommendation 1978, (C78), [7];
d) German reinforced concrete code DIN 1045 1975, (DIN_1), [8];
e) German reinforced concrete code DIN 1045, 1981, [9] and German prestressed concrete code DIN 4277, 1981, (DIN_2), [10];
f) ACI Committee 209, (ACI), [11];
g) British Concrete Society, (BCS), [12].

Only codified formulas have been taken here, but similar comparisons also with other suggested formulas can be found in [13], [14].

2.2 Prediction errors

Creep tests with normal weight concretes with various concrete compositions and in various environmental conditions have been reported by many researchers, [15-27]. These test results have been compared with results predicted by the afore-mentioned formulas. The comparisons were either made at the values at the end of each test or in some cases at an equivalent final value, which has been extrapolated in the used literature by the method of Ross [28]. Although some investigators proposed in their rheological models that creep does not approach a final value [13] the validity of Ross' extrapolation hyperbola is assumed throughout this paper since for the purpose of this investigation this point is only of minor significance. The final value was chosen since the long term creep effects are particularly relevant for structural applications.

For each prediction formula and each test series the ratio between the predicted and the observed creep coefficient was calculated

$$m_{ij}=\frac{\text{predicted } \varphi}{\text{observed } \varphi} \text{ with } j=1,\ 2,\ \ldots,\ n_i,\quad i=1,\ 2,\ \ldots,\ k \tag{2.2}$$

where n_i is the number of reported tests in the i'th test series and k is the number of test series being analysed. Sample statistics for the mean value $\bar{m}_i$ and coefficient of variation v_i for the i'th test series were then determined by

$$\bar{m}_i=\frac{1}{n_i}\sum_{j=1}^{n_i} m_{ij} \tag{2.3}$$

$$v_i=\sqrt{\frac{1}{n_i-1}\sum_{j=1}^{n_i}(m_{ij}-\bar{m}_i)^2}\Big/\bar{m}_i \tag{2.4}$$

For each prediction formula the sample statistics $\bar{m}$, $\bar{v}$ and v_v for all test series together were calculated as

$$\bar{m}=\sum_{i=1}^{k} n_i\bar{m}_i\Big/\sum_{i=1}^{k} n_i \tag{2.5}$$

$$\bar{v}=\sum_{i=1}^{k} n_i v_i\Big/\sum_{i=1}^{k} n_i \tag{2.6}$$

$$v_v=\sqrt{\frac{1}{\sum_{i=1}^{k} n_i-1}\sum_{i=1}^{k} n_i(v_i-\bar{v})^2}\Big/\bar{v} \tag{2.7}$$

Corresponding sample statistics are shown in Table I. It is noted that several prediction formulas are considerably biased as the mean value $\bar{m}$ differs substantially from 1. It is further noted that the average coefficient of variation $\bar{v}$ varies between values as large as 25 and 40%. The largest coefficients of variation v_i were observed for data from experiments carried out before 1940. Apparently the laboratory conditions could not be well controlled and the measuring techniques were less advanced. A large additional measuring uncertainty therefore appears to be present in those test results.

2.3. Suggested prediction formula

A prediction formula similar to the C78 prediction formula is chosen. The C78 prediction formula appears to be a reasonable compromise between accuracy and simplicity. The formula is further well suited for a probabilistic analysis. The creep coefficient is expressed as

$$\varphi(t,\ \tau)=\beta_a(\tau)+\varphi_f\,\beta_f(t,\ \tau)+\varphi_d\,\beta_d(t,\ \tau) \tag{2.8}$$

where $\beta_a(\tau)$ represents the irreversible part of the creep strain which develops during the first few days after the load is imposed, $\varphi_f\,\beta_f(t,\ \tau)$ represents the flow creep strain and $\varphi_d\,\beta_{d(t,\ \tau)}$ the delayed elastic creep strain. The constants φ_d and φ_f are taken as

$$\varphi_d=0.4 \tag{2.9}$$

$$\varphi_f=\varphi_{f1}\,\varphi_{f2}\,\varphi_{f3} \tag{2.10}$$

where

$$\varphi_{f1}=4.5-\frac{1}{28}w \tag{2.11}$$

$$\varphi_{f2}=1.2+\exp(-0.14\,r^{2/3}) \tag{2.12}$$

$$\varphi_{f3}=\begin{cases}0.8 & \text{for } s\leqq 30\\ 0.8+\dfrac{0.8}{30}(s-30) & \text{for } 30<s\leqq 50\end{cases} \tag{2.13}$$

w is the relative humidity in percent of the environment; r is the notional thickness (in units of cm) defined as $2A_c/p_c$ where A_c is the area and p_c is the perimeter of the concrete section in contact with the atmosphere. s is the slump measure (in units of cm) measured as described in [9]. The functions $\beta_a(\tau)$, $\beta_d(t,\ \tau)$ and $\beta_f(t,\ \tau)$ are

$$\beta_a(\tau)=0.8\exp(-0.2\sqrt{\tau}) \tag{2.14}$$

$$\beta_d(t,\ \tau)=1-\exp(-0.02(t-\tau)) \tag{2.15}$$

$$\beta_f(t,\ \tau)=\left(\frac{t}{t+\psi(r)}\right)^{1/3}-\left(\frac{\tau}{\tau+\psi(r)}\right)^{1/3} \tag{2.16}$$

where

$$\psi(r)=\begin{cases}20r+200 & \text{for } r\leqq 40\\ 15r+400 & \text{for } r>40\end{cases} \tag{2.17}$$

The times t and τ are measured in units of days. t is the effective age of the concrete at the time of observation and τ is the effective age of concrete at the time of loading. The effective age t is determined as

$$t=\alpha\int_0^{t_r}\frac{T(s)+10}{30}\,ds \tag{2.18}$$

where t_r is the true age of concrete in units of days, $T(s)$ is the ambient temperature in units of °C and α is a coefficient related to the type of cement

$$\alpha=\begin{cases}1 & \text{for normal and slowly hardening cement}\\ 2 & \text{for rapid hardening cement}\\ 3 & \text{for rapid hardening high strength cement}\end{cases} \tag{2.19}$$

The proposed formula (2.8) was compared with the test results and the ratio of the calculated to the observed values had a sample mean of 0.95 and a sample coefficient of variation of 0.165. The sample consisted of the data from tests in [15], [18], [19] since sufficient informations on parameters were only available for these tests.

3. The probabilistic model

Analogous with the deterministic formula (2.8) the following probabilistic formula for the random creep coefficient $\Phi(t,\ \tau)$ is suggested

$$\Phi(t,\ \tau)=J(\beta_a(\tau)+\Phi_f\,\beta_f(t,\ \tau)+\varphi_d\,\beta_d(t,\ \tau)) \tag{3.1}$$

where J is a random variable taking into account model uncertainty while Φ_f is a random variable taking into account parameter uncertainty. Φ_f is as in (2.10)-(2.13) but now the concrete composition expressed by the slump measure is random as well as the environmental humidity. The notional thickness r and delayed modulus of elasticity φ_d are taken non-random while the effective times t and τ are random due to random temperature variation. If the random variables J and Φ_f are assumed uncorrelated and if the randomness in the temperature variation is ignored, the mean value and covariance function of $\Phi(t,\ \tau)$ are, [29],

$$E[\Phi(t,\ \tau)]=E[J](\beta_a(\tau)+E[\Phi_f]\,\beta_f(t,\ \tau)+\varphi_d\,\beta_d(t,\ \tau)) \tag{3.2}$$

$$\begin{aligned}\mathrm{Cov}[\Phi(t_1,\ \tau_1),\ \Phi(t_2,\ \tau_2)]&=(E[J]^2+\mathrm{Var}[J])\,\mathrm{Var}[\Phi_f]\,\beta_f(t_1,\ \tau_1)\,\beta_f(t_2,\ \tau_2)\\&+\mathrm{Var}[J]\,(\beta_a(\tau_1)+E[\Phi_f]\,\beta_f(t_1,\ \tau_1)\\&+\varphi_d\,\beta_d(t_1,\ \tau_1))\,(\beta_a(\tau_2)+E[\Phi_f]\,\beta_f(t_2,\ \tau_2)\\&+\varphi_d\,\beta_d(t_2,\ \tau_2))\end{aligned} \tag{3.3}$$

The mean value and variance of Φ_f are from (2.10), [29],

$$E[\Phi_f]=E[\Phi_{f1}]\,\varphi_{f2}\,E[\Phi_{f3}] \tag{3.4}$$

TABLE I

COMPARISON OF PREDICTED VALUES WITH TEST DATA

Lit.	n_i		C 66	C 70	C 78	DIN_1	DIN_2	ACI	BCS	
[15]	48 (*)	m	0.970	1.095	0.884	1.057	0.826	0.978	0.769	(24) (**)
		v%	22.1	46.1	19.2	51.0	19.6	64.9	17.9	
[16]	8 (*)	m	1.165	1.234	1.034	0.764	0.920	0.615	0.956	(8) (**)
		v%	13.7	13.7	23.4	24.6	24.6	24.6	24.6	
[17]	3 (*)	m	0.983	1.035	1.215	0.903	1.088	0.730	0.936	(3) (**)
		v%	8.2	8.3	33.6	33.6	33.6	33.6	33.6	
[18]	12 (*)	m	1.056	1.085	0.808	0.609	0.764	0.603	0.777	(8) (**)
		v%	19.5	20.9	20.2	14.7	16.3	15.4	16.4	
[19]	15	m	1.541	1.221	1.368	1.328	1.210	0.937		
		v%	15.3	27.3	18.8	26.8	20.2	15.0		
[20]	23	m	0.999	0.788	0.680	0.833	0.441	0.786		
		v%	17.7	37.9	19.3	26.4	19.8	21.9		
[21]	15	m	1.864	2.083	1.224	1.722	0.923	1.047		
		v%	58.1	48.2	42.6	51.8	36.9	53.2		
[22]	4	m	1.071	1.143	1.417	1.249	1.229	0.856	1.396	(4) (**)
		v%	17.8	16.0	11.7	11.7	12.1	11.7	20.0	
[23]	10 (*)	m			0.707		0.707			
		v%			20.5		20.5			
[24]	27	m	1.705	1.550	0.919	0.753	0.777	0.716		
		v%	47.7	42.8	42.9	42.3	45.2	46.3		
[25]	17 (*)	m	0.986	0.894	1.261	0.856	1.096	0.876	1.025	(12) (**)
		v%	10.6	17.4	19.6	22.2	19.0	24.9	23.7	
[26]	25 (*)	m	1.145	1.096	0.929	0.859	0.829	0.709		
		v%	20.8	21.7	26.8	23.4	29.1	34.7		
	12 (*)	m	0.956	1.000	0.894	0.697	0.788	0.583		
		v%	27.0	18.5	22.8	21.6	22.8	27.8		
[27]		$\bar{m}$	1.209	1.183	0.966	0.964	0.834	0.821	0.899	
	219	$\bar{v}$	25.46	32.93	25.13	34.06	25.31	38.50	19.02	
		v_v	54.20	39.17	36.37	38.57	36.39	47.10	24.09	

(*) Comparison with the final creep value.
(**) Number of experiments which have been calculated by BCS method.

$$\mathrm{Var}[\Phi_f] = \varphi_{f2}^2 (E[\Phi_{f1}]^2 \,\mathrm{Var}[\Phi_{f3}] + \mathrm{Var}[\Phi_{f1}]\, E[\Phi_{f3}]^2 + \mathrm{Var}[\Phi_{f1}]\,\mathrm{Var}[\Phi_{f3}]) \quad (3.5)$$

The uncertainties in the different parameters are treated next in detail.

Model uncertainty

The comparison between test results and results predicted by (2.8) suggests an expected value of $1/J$ of 0.95 with a corresponding coefficient of variation of 0.165. A histogram of sample points m_{ij} shows that $1/J$ is well described by a log-normal distribution. J can therefore be taken lognormally distributed with mean value 1.05 and coefficient of variation 0.165. Due to uncertainty in the laboratory conditions during the tests, the coefficient of variation can be somewhat reduced. Based on data in [30] it is suggested to use values

$$\left.\begin{array}{l} E[J] = 1.05 \\ V_J = 0.13 \end{array}\right\} \quad (3.6)$$

Uncertainty in concrete composition

The randomness in concrete composition is taken into account by the random slump measure which is assumed constant across the structure. Thereby it is assumed that within structure variations generally average out. The mean value and in particular the variance of the slump measure depend on the knowledge of the concrete composition. If no detailed knowledge is available, the following values are suggested for the random variable Φ_{f3}

$$E[\Phi_{f3}] = \left\{\begin{array}{l} 1.30 \text{ wet concrete} \\ 1.05 \text{ normal concrete} \\ 0.80 \text{ dry concrete} \end{array}\right\} \quad (3.7)$$

$$V_{\Phi_{f3}} = 0.10$$

If the slump is measured, the uncertainty can be ignored.

Influence of random temperature

The random temperature is taken into account by using an effective time similarly to (2.18). The relation between the effective age t and the true age t_r is taken as

$$t = \alpha \int_0^{t_r} \left(\frac{T(s)+10}{30}\right)^p ds \quad \text{for } T(s) > -10 \quad (3.8)$$

where α is given by (2.19), $T(s)$ is the temperature measured in units °C, and where the exponent p is taken equal to 1 in C78 while it is suggested to take p equal to 1.7 in [31].

The temperature variation is suggested to be modelled by one of the following four formulas ordered according to increasing complexity

$$T(t) = E[T] \quad (3.9)$$

$$T(t) = T \quad (3.10)$$

$$T(t) = T + A_1 \cos(2\pi f_1 + \theta_1) \quad (3.11)$$

$$T(t) = T + A_1 \cos(2\pi f_1 t + \theta_1) + A_2 \cos(2\pi f_2 t + \theta_2) \quad (3.12)$$

where $1/f_1 = 1$ year and $1/f_2 = 1$ day. T is the mean yearly temperature and A_1 and A_2 are independent mean zero random variables describing the amplitudes in the yearly and daily temperature variations.

When the effective ages are random, also the function β_a, β_d and β_f (2.14)-(2.16) are random. The function β_f in (2.16) is thus a random function $B_f(t, \tau)$

$$B_f(t,\tau) = \left(\frac{\alpha \int_0^t ((T(s)+10)/30)^p ds}{\alpha \int_0^t ((T(s)+10)/30)^p ds + \psi(r)}\right)^{1/3} - \left(\frac{\alpha \int_0^\tau ((T(s)+10)/30)^p ds}{\alpha \int_0^\tau ((T(s)+10)/30)^p ds + \psi(r)}\right)^{1/3} \quad (3.13)$$

With suitable assumptions about the distribution types of T, A_1 and A_2 it is possible to calculate the mean value and covariance function for $B_f(t, \tau)$ from (3.13). Numerical calculations do, however, show that the uncertainty in the functions β_a, β_d and β_f are minor compared to the other uncertainties. In most cases it is therefore sufficient to use the simple model for $T(t)$ in (3.9). In that case the functions β_a, β_d and β_f are non-random.

Environmental humidity

In all reported test series the relative air humidity was kept constant. The drying creep processes are in reality influenced by some vapor pressure in the pores and for constant air humidity a state of equilibrium is developed for this pressure. The pressure can therefore be measured in terms of this equivalent relative air humidity and its influence on the creep process be determined as in (2.11).

When the relative air humidity is time varying, also the vapor pressure in the pores varies with time and so does the equivalent relative air humidity. The purpose is now to determine the variation in the equivalent relative air humidity from the variation in the relative air humidity.

Measurements in W. Germany have shown that a sufficient realistic model for the relative air humidity variation is

$$W(t) = W_0 + A \cos(2\pi f_0 t + \theta) \quad (3.14)$$

where $1/f_0 = 1$ year. W_0 is the mean relative humidity at the location $-W_0$ may be known deterministically or be a random variable. $A \cos(2\pi f_0 t + \theta)$ is the variation around the mean value. The variation is taken as a periodic process. The value of A was determined

TABLE II. — RELATIVE AIR HUMIDITY $w(t)$ MEASURED AT THREE DIFFERENT LOCATIONS IN WEST GERMANY, $w(t) = w_0 + A \cos(2\pi ft + \theta)$.

Location	w_0	A
Munich	78%	8%
Bochum	82%	7%
Rheinhausen	74%	8%

for 3 locations in Germany—see Table II. Figure 1 illustrates the effect on the creep development of varying W_0 and of the periodic term $A \cos(2\pi f_0 t + \theta)$.

The variation of the equivalent relative air humidity is found from a very simple model since only little knowledge about the diffusion processes is available. The variation is calculated for an infinitely long plane wall of thickness $2d$—Figure 2. Only a stationary situation is considered and the air humidity is assumed to vary as $e^{i2\pi ft}$. The wall is assumed homogeneous and a pore pressure $p(x, t)$ found as the solution of the diffusion equation

$$\frac{\partial p}{\partial t} = D \frac{\partial^2 p}{\partial x^2} \quad (3.15)$$

with the boundary conditions

$$p(-d, t) = p(d, t) = e^{i2\pi ft} \quad (3.16)$$

and D = the diffusion coefficient which is assumed constant. The equivalent relative air humidity $r(t)$ is found by averaging $p(x, t)$ over the thickness of the wall

$$r(t) = \frac{1}{d}\int_0^d p(x, t)\, dt \quad (3.17)$$

The solution to (3.15) is written as

$$p(x, t) = H(x, f)\, e^{i2\pi ft} \quad (3.18)$$

where $H(x, f)$ is found as

$$H(x, f) = c \cos\left[(1-i)\sqrt{\frac{\pi f}{D}}\, x\right] \quad (3.19)$$

and the constant c is found by introducing the boundary conditions (3.16)

$$H(x, f) = \frac{\cos[(1-i)\sqrt{\pi f/D}\, x]}{\cos[(1-i)\sqrt{\pi f/D}\, d]} \quad (3.20)$$

For $r(t)$ the solution becomes

$$r(t) = H(f)\, e^{i2\pi ft} = \frac{\tan(1-i)\xi}{(1-i)\xi} e^{i2\pi ft} \quad (3.21)$$

where

$$\xi = \sqrt{\frac{\pi f d^2}{D}}$$

The amplitude in the variation of the equivalent relative air humidity is therefore $|H(f)|$ times the amplitude in the variation of the environmental air humidity. From (3.21) $|H(f)|^2$ is found as

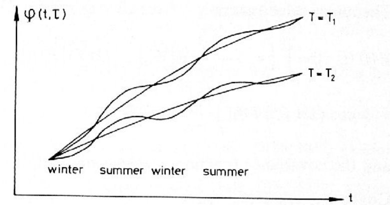

Fig. 1. — **Sketch of seasonal variation of the creep coefficient for different mean yearly relative air humidities.**

$$|H(f)|^2 = \frac{1}{2\xi^2}\,\frac{\cos h2\xi - \cos 2\xi}{\cos h2\xi + \cos 2\xi} \quad (3.23)$$

The equivalent air humidity is thus modelled as

$$R(t) = W_0 + |H(f)|\, A \cos(2\pi f_0 t + \theta) \quad (3.24)$$

The second moment representation of $U(t, \tau) = \Phi_{f1}\,\beta_f(t, \tau)$ can now be calculated. When the air humidity varies, the terms should be written as

$$U(t, \tau) = \int_\tau^t \Phi_{f1}(u) \frac{\partial \beta_f}{\partial u}(u, \tau)\, du \quad (3.25)$$

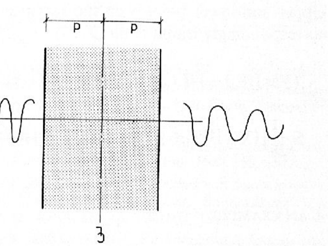

Fig. 2. — **Plane concrete wall. Time varying relative air humidity.**

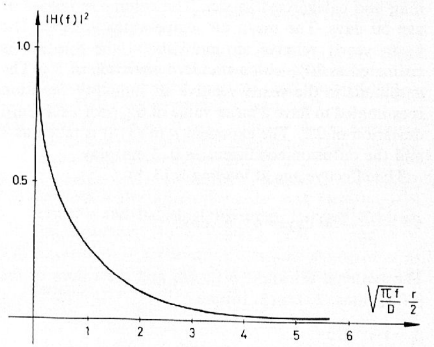

Fig. 3. — **Amplitude reduction factor** $|H(f)|^2$.

The mean value function is from (2.11) and (3.25)

$$E[U(t, \tau)]=\int_{\tau}^{t}\left(4.5-\frac{1}{28}(E[W_0]+|H(f)|\,E[A]\right.$$
$$\left.\times\cos(2\pi f_0 u+\theta)\right)\frac{\partial\beta_f}{\partial u}(u, \tau)\,du \qquad (3.26)$$

and the covariance function is correspondingly

$$\mathrm{Cov}[U(t_1, \tau_1), U(t_2, \tau_2)]=$$
$$\int_{\tau_1}^{t_1}\int_{\tau_2}^{t_2}\frac{1}{28^2}\ Var[W_0]+|H(f)|^2\ Var[A]$$
$$\times\cos(2\pi f_0 u_1+\theta)\times\cos(2\pi f_0 u_2+\theta))$$
$$\times\frac{\partial\beta_f}{\partial u}(u_1, \tau_1)\times\frac{\partial\beta_f}{\partial u}(u_2, \tau_2)\,du_1\,du_2 \quad (3.27)$$

The variance function can be easily obtained by taking $t_1=t_2$ and $\tau_1=\tau_2$.

It follows from (3.26) that upper and lower bounds on $E[U(t, \tau)]$ are

$$\left(4.5-\frac{1}{28}(E[W_0]+|H(f_0)|\,E[A])\right)\beta_f(t, \tau)\leqq E[U(t, \tau)]$$
$$\leqq\left(4.5-\frac{1}{28}(E[W_0]-|H(f_0)|\,E[A])\right)\beta_f(t, \tau). \ (3.28)$$

Upper and lower bounds on the variance function are correspondingly found from (3.27)

$$\frac{\ }{28^2}(\mathrm{Var}[W_0]-|H(f_0)|^2\ \mathrm{Var}[A])\ \beta_f(t, \tau)^2\leqq\mathrm{Var}\ [U(t, \tau)]$$
$$\leqq\frac{1}{28^2}(\mathrm{Var}[W_0]+|H(f_0)|^2\,\mathrm{Var}[A])\ \beta_f(t, \tau)^2 \quad (3.29)$$

4. AN EXAMPLE

The mean value and the coefficient of variation of the final creep coefficient for a 50 cm x 50 cm concrete column are estimated. The concrete is normally hardening and categorized as wet. The column is loaded at age 50 days. The mean air temperature is 15°C. The mean yearly relative air humidity at the location is estimated as 80% with a standard deviation of 3%. The amplitude in the yearly relative air humidity variation is estimated to have a mean value of 6% with a standard deviation of 2%. The exponent p in (3.8) is taken as 1 and the diffusion coefficient as 0.3 cm²/day.

The effective age at loading is (3.8)

$$\tau=\frac{15+10}{30}\,50\text{ days}=41.67\text{ days}$$

The notional thickness is 25 cm and the values of the β-functions (2.14)-(2.16) are

$$\beta_a(\tau)=0.220$$
$$\beta_d(\infty, \tau)=1$$
$$\beta_f(\infty, \tau)=0.617$$

The value of ξ is calculated from (3.22)

$$\xi=\sqrt{\frac{\pi}{365\text{ days }0.3^2\text{ cm/day}}}\ \frac{25\text{ cm}}{2}=2.117$$

and the variance reduction factor $|H(f_0)|$ follows from (3.22)

$$|H(f_0)|=0.338$$

The φ-factors have mean values and standard deviations from (3.28), (3.29) and (3.7)

$$1.57=4.5-\frac{1}{28}(80+0.338.6)\leqq E[\varphi_{f1}]$$
$$\leqq 4.5-\frac{1}{28}(80-0.338.6)=1.72$$
$$0.104=\frac{1}{28}\sqrt{3^2-0.338^2.2^2}\leqq D[\varphi_{f1}]$$
$$\leqq\frac{1}{28}\sqrt{3^2+0.338^2.2^2}=0.110$$
$$E[\varphi_{f3}]=1.30\ D[\varphi_{f3}]=0.13$$

Using the values for the model uncertainty in (3.6) the mean value and standard deviation of the final creep coefficient φ_∞ follows from (3.2)-(3.5)

$$2.64\leqq E[\varphi_\infty]\leqq 2.83$$
$$0.42\leqq D[\varphi_\infty]\leqq 0.45$$

The calculation of several examples has indicated that the coefficient of variation of φ_∞ usually may be expected between 15% and 20%. These calculations also show that the model uncertainties generally are the dominating sources of uncertainty when predicting creep coefficients.

5. SUMMARY

Various formulas suggested in national and international codes for prediction of the creep coefficient have been compared with experimental data. Some formulas are found to give rather biased results. The formula suggested by CEP/FIP in the 1978 Model Code appears to be the best of the formulas. A slightly modified version of this formula is given in the paper in a simple mathematical form. Based on this formula a probabilistic model for determination of the creep coefficient is suggested. Mean values and variances for the random concrete and environmental parameters in the model are given next. The uncertainty in the relative air humidity and its influence on the creep coefficient is studied in some detail using a simple diffusion model. It is finally demonstrated that the model is well suited for hand calculations and the mean value and variance of the creep coefficient is calculated in a specific case.

NOTATION

The following symbols are used in this paper:

$E[\ ]$,	expected value;
Cov [,],	covariance
Var [ ],	variance;
D[ ],	standard deviation;
m,	sample mean;
v,	sample coefficient of variation;
n,	sample size;
ε,	strain;
σ_0,	stress;
$E_c(\ )$,	modulus of elasticity;
φ, $\Phi(\ ,\ )$,	creep coefficient;
φ_∞,	final creep coefficient;
β_a, $B_a(\ )$,	irreversible creep strain function;
β_f, $B_f(\ ,\ )$,	flow creep strain function;
β_d, $B_d(\ ,\ )$,	delayed elastic creep strain function;
φ_d, φ_f, Φ_f, φ_{f1}, Φ_{f1}, φ_{f2}, φ_{f3}, Φ_{f3},	constants depending on concrete and environmental parameters;
t, τ,	time;
w, W,	relative air humidity;
T,	temperature;
r,	notional thickness;
A_c,	area;
p_c,	perimeter;
s, S,	slump measure;
α,	coefficient related to type of cement;
J,	model uncertainty random variable;
$p(\ ,\ )$,	pore pressure;
D,	diffusion coefficient;
$r(\ )$,	equivalent relative air humidity;
$H(\ )$,	transfer function;
i,	$\sqrt{-1}$;
f,	frequency;

REFERENCES

[1] CINLAR E., BAŽANT, OSMAN E. — *Stochastic process for extrapolating concrete creep*, Journal of the Engineering Mechanics Division, Vol. 103, No. EM6, December 1977, pp. 1069-1088.

[2] JORDAAN J., *Discussion of* [1], Closure by E. Cinlar, Z. Bažant, E. Osman. — Journal of the Engineering Mechanics Division, Vol. 105, No. EM3, June 1979, pp. 485-489.

[3] DITLEVSEN O., MADSEN H. O., *On the uncertainty of concrete creep*, In *DIALOG* 1-80, Department of Civil Engineering, Danish Engineering Academy, Lyngby, Denmark, 1980, pp. 127-172.

[4] Joint Committee on Structural Safety, CEB-CECM-CIB-IABSE-RILEM: *Basic notes on resistances*, *R*-02 *concrete properties*, Draft, 1981.

[5] CEB-*Recommandations pratique pour le calcul et l'exécution des ouvrages en béton précontraint*, Bulletin d'Information No. 54, Paris, 1966.

[6] CEB-FIP-*International recommendations for the design and construction of concrete structures*, Vol. 1, Information Bulletin No. 72, Paris, 1970.

[7] CEB-FIP-*Model Code for concrete structures*, CEB Bulletin No. 124/125-E, Paris 1978.

[8] *German reinforced concrete code* DIN 1045, Edition 1975.

[9] *German reinforced concrete code* DIN 1045, Edition 1981.

[10] *German prestressed concrete code* DIN 4277, Edition 1981.

[11] ACI Committee 209/II. — *Prediction of creep, shrinkage and temperature effects in concrete structures*, ACI-SP27, American Concrete Institute, Detroit, 1971, pp. 51-93.

[12] British Concrete Society. — *A simplified method for estimating the elastic modulus and creep of normal weight concrete*, Cement and Concrete Association Training Centre, TDH 7376, London, June 1978.

[13] BAŽANT Z. and PANULA L. — *Creep and shrinkage characterization for analyzing prestressed concrete structures*, PCI Journal, Prestressed Concrete Institute, Vol. 25, No. 3, May/June 1980.

[14] HILSDORF H. K. and MÜLLER H. S. — *Comparison of methods to predict time dependent strains of concrete*, Institut für Baustofftechnologie, Universität Karlsruhe, October 1979.

[15] WESCHE K., SCHRAGE I. and von BERG W. — *Versuche zum Einfluss des Belastungsalters auf das Kriechen von Beton*, Deutscher Ausschuss für Stahlbeton, Heft 295, 1978.

[16] RÜSCH H., KORDINA K. and HILSDORF H. — *Der Einfluss des Mineralogischen Charakters der Zuschläge auf das Kriechen von Beton*, Deutscher Ausschuss für Stahlbeton, Heft 146, 1962.

[17] HUMMEL A., WESCHE W. and BRAND W. — *Der Einfluss der Zementart, des Wasser-Zement-Verhältnisses und des Belastungsalters auf das Kriechen von Beton*, Deutscher Ausschuss für Stahlbeton, Heft 146, 1962.

[18] KORDINA K. — *Langzeitversuche an Stahlbetonstützen*, Deutscher Ausschuss für Stahlbeton, Heft 250, 1975.

[19] WISCHERS G. and DAHMS J. — *Kriechen von frühbelasteten Beton mit hoher Anfangsfestigskeit*, Sonderdruck aus Beton H2 pp. 69-74, H3 pp. 104-108, 1977.

[20] WITTMANN F. and LUCAS J. — *Rechnerische Erfassung von Kriechen und Schwinden des Betons* (Zwischenbericht), Laboratorium für Werkstoffphysik der Technische Universität München, July, 1974.

[21] DAVIS R. — *Flow of concrete under sustained compressive stress*, Proceedings American Concrete Institute, Vol. 24, 1928, pp. 303-335.

[22] ROSTASY F. S., TEICHEN K.-Th. and ENGELKE H. — *Beitrag zur Klärung des Zusammenhanges von Kriechen und Relaxation bei Normalbeton*, Otto-Graf-Institut Schriftenreihe, Heft 57, Universität Stuttgart, 1972.

[23] TROST H., CORDES H. and GÜNTHER A. — *Kriech-und Relaxations-versuche an sehr altem Beton*, Deutscher Ausschuss für Stahlbeton, Heft 295, 1978.

[24] GLANVILLE W. H. — *Studies in reinforced concrete III, the creep or flow of concrete under load*, Department of Scientific and Industrial Research, Building Research Technical Paper, No. 12, London, 1930.

[25] BOLOMEY J. — *Déformation élastiques plastiques et de retrait de quelques bétons*, Bulletin Technique de la Suisse romande, vol. 68, No. 15, 1942.

[26] DUTRON R. — *Déformations lentes du béton et du béton armé sous l'action de charges permanentes*, Annales de Travaux Publics de Belgique, Num. 37 de la 2. série, 1936/1937.

[27] WEIL G. — *Influence des dimensions et des contraintes sur le retrait et le fluage du béton*, RILEM Bulletin, No. 3, July 1959.

[28] ROSS A. D. — *Concrete creep data*, Structural engineer, Vol. 15, No. 8, 1937.

[29] DITLEVSEN O. — *Uncertainly Modeling*, McGraw-Hill, 1981.

[30] PAT M. G. M. and REINHARDT H. W. — *Variability of creep of concrete-Analysis of the results*, Delft University of Technology, Department of Civil Engineering, Report 5-80-8, Research No. 7802, December 1980.

[31] NIELSEN L. F. — *On the prediction of rheological parameters for concrete*, in DIALOG 1-80, Department of Civil Engineering, Danish Engineering Academy, Lyngby, Denmark, 1980, pp. 81-118.

RÉSUMÉ

De la variabilité du coefficient de fluage du béton structural. — *On a comparé plusieurs formules proposées dans les codes nationaux et internationaux pour la prévision du coefficient de fluage. On a trouvé que certaines formules donnaient des erreurs systématiques. La formule proposée par le CEB et la FIP dans le Code Modèle de 1978 semble la meilleure. On donne une version légèrement modifiée de cette formule sous une forme mathématique simple. On propose, en s'appuyant sur cette formule, un modèle probabiliste de détermination du coefficient de fluage. On donne ensuite des valeurs moyennes et des variances pour le béton quelconque et les variables du milieu. On étudie de façon détaillée l'influence des variations d'humidité de l'air sur le coefficient de fluage à l'aide d'un modèle de diffusion simple. On démontre enfin que le modèle est bien adapté pour les calculs manuels et on calcule la valeur moyenne et la variation du coefficient de fluage sur un cas particulier.*