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Optimizing systematically renewed structures

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Abstract

Based on earlier work, a renewal model for structural failures with subsequent systematic reconstruction is developed. It is shown that simple objective functions can be obtained by making use of some asymptotic results of renewal theory. Complete objective functions must include benefits and expected failure cost which both must be discounted appropriately. Some techniques in optimization for locally stationary and locally non-stationary failure models are developed. Those are illustrated at simple examples. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Rosenblueth and Mendoza [22], Hasofer [5] and Rosenblueth [24] proposed a renewal model for structures. In particular, two replacement strategies are considered, i.e. one where the structure is given up after failure or after service and the other where the structure is systematically rebuilt upon failure or obsolescence. The latter strategy appears suitable for most building facilities, especially for infrastructure buildings like bridges. It is assumed that time-variant loads and/or deteriorating system properties cause failure. Rosenblueth and Mendoza [22] already pointed out that such a model is most appropriate in the context of optimization. Rackwitz [20] reviewed those early results and studied various additional aspects of the renewal model and its application to reliability-oriented cost-benefit analysis. The most important conclusion in the context discussed in this paper is that the likelihood of failure should be measured in terms of failure rates or renewal densities instead of failure probabilities related to certain reference times. In this paper some more results are presented partially based on Hasofer and Rackwitz [6]. The theory of the renewal model is briefly reviewed. It is then applied to structural optimization, which includes expected failure cost. If the factor time is involved in risk–benefit analysis cost occurring at a later time than the decision point must be discounted. Some discussion is provided on how to choose appropriate discount rates. As other technical objects, structural facilities also involve risks to human life and limb. Therefore, emphasis is then given to the question of risk acceptability in the context of optimization of structural facilities subject to various hazards. A new approach to define acceptability criteria derived from the so-called life quality index as proposed by Nathwani et al. [15] is used. It turns out that risk acceptability is essentially a matter of efficiency of investments into life saving measures.

In Kuschel and Rackwitz [10], a first proposal is made to perform reliability-oriented optimization if a stationary Poissonian process generates structural failures. An efficient one-level scheme is proposed which is refined below. The non-stationary case, i.e. when non-stationary loads act on the structure or some deterioration occurs in time, has not yet been studied previously. It is substantially more difficult and, at the moment, can be solved only approximately by asymptotic concepts. More specifically, the asymptotic renewal density, a quantity not varying in time, must be used. The practical computation of the asymptotic renewal density for non-Poissonian failure models was first studied in Rackwitz and Balaji Rao [18]. In this paper reliability-oriented optimization using the objective function proposed in Rackwitz [20] for Poissonian and non-Poissonian failure models is further developed in part making use of ideas put forward in Kuschel and Rackwitz [11,12] and including a criterion defining acceptable risks for human life and limb.

2. Objective Functions

Assume that the objective function of a structural component is

$$Z(p) = B(p) - C(p) - D(p) \tag{1}$$

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$B(p)$ is the benefit derived from the existence of the structure, $C(p)$ are the construction costs, $D(p)$ the damage cost and p generally is a design parameter vector. Without loss of generality all quantities will be measured in monetary units. Statistical decision theory dictates that the expected values for $B(p)$, $C(p)$ and $D(p)$ have to be taken. $B(p)$ will be unaffected or slightly decrease with each component of p. For simplicity, $B = B(p)$ is assumed. $C(p)$ increases with each component of p under normal circumstances. The magnitude of $C(p)$, in general, is most easy to assess. $D(p)$ decreases with p in some fashion assuming that the probability of failure is a decreasing function of p. For each involved party, i.e. the builder, the user and the society, $Z(p)$ should be positive. Otherwise one should not undertake the realization of the structure. This is illustrated in Fig. 1. Benefits, cost and damages are not necessarily the same for all involved parties. Therefore, the intersection of the domains where $Z(p)$ is positive is the domain of p, which makes sense for all parties.

Furthermore, the decision about p has to be made at $t = 0$. This requires capitalization of all cost. A continuous capitalization function is used.

$$\delta(t) = \exp(-\gamma t) \tag{2}$$

where γ the interest rate and t time in suitable units. As will be seen below the usual discrete discounting function is not suitable for our purposes. If a yearly discount rate γ' for discontinuous discounting is given, it is related to the rate for continuous discounting by $\gamma = \ln(1 + \gamma')$. γ is corrected for de- and inflation and averaged over sufficiently long periods to account for fluctuations in time. It is also assumed that the time for construction is negligible short as compared to the average lifetime of structures. Although the formula's for failure upon construction can also be given [20] only the case of failures due to random loads and/or deterioration of system properties is considered in the following.

3. The renewal model for structural failures with subsequent systematic reconstruction

Assume that the sequence of structural failures (and subsequent reconstruction) can be described by a renewal process whose most important characteristic is the density $g(t)$ of the time between failures. Let

$$g_n(t) = \int_0^t g_{n-1}(t - \tau) g(\tau) \mathrm{d}\tau; \qquad n = 2, 3, \ldots \tag{3}$$

be the density function of the time to the n-th renewal written as a convolution integral. Slightly more general it is assumed that the density of time to first failure is different from the one for subsequent failures. The latter density is denoted by $g_1(t)$. The corresponding renewal process will be termed modified renewal process. Define by

$$g_1^*(\theta) = \int_0^\infty \exp[-\theta t] g_1(t) \mathrm{d}t$$

$$g^*(\theta) = \int_0^\infty \exp[-\theta t] g(t) \mathrm{d}t \tag{4}$$

the Laplace transforms of $g(t)$ and $g_1(t)$, respectively. For the important stationary Poisson process with intensity λ it is simply

$$g_1^*(\theta) = g^*(\theta) = \int_0^\infty \exp[-\theta t] \lambda \exp[-\lambda t] \mathrm{d}t = \frac{\lambda}{\theta + \lambda} \tag{5}$$

For convolutions we have

$$g_n^*(\theta) = g_1^*(\theta) g_{n-1}^*(\theta) = g_1^*(\theta)[g^*(\theta)]^{n-1} \tag{6}$$

The Laplace transform of the renewal density defined by

$$h(t) = \sum_{i=1}^{\infty} g_i(t)$$

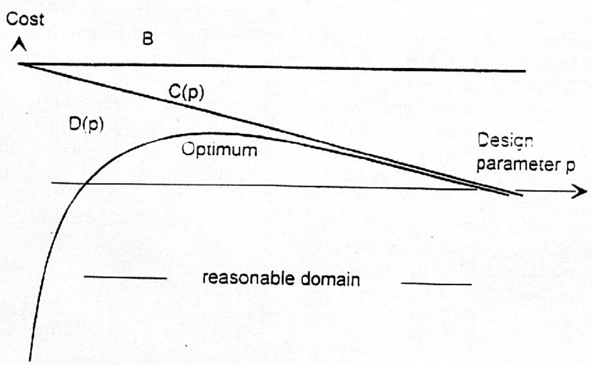

Fig. 1. Cost and benefit over design parameter p(after Rosenblueth and Esteva, 1972)

then is

$$h_1^*(\theta) = \sum_{n=1}^{\infty} g_n^*(\theta) = \sum_{n=1}^{\infty} g_1^*(\theta)[g^*(\theta)]^{n-1} = \frac{g_1^*(\theta)}{1 - g^*(\theta)} \tag{7}$$

for modified renewal processes and

$$h^*(\theta) = \sum_{n=1}^{\infty} g_n^*(\theta) = \sum_{n=1}^{\infty} g^*(\theta)[g^*(\theta)]^{n-1} = \frac{g^*(\theta)}{1 - g^*(\theta)} \tag{8}$$

for ordinary renewal processes.

It is now assumed that if structures fail systematic reconstruction is chosen. The renewal model requires independence of the failure times. After first failure the failure times are identically distributed. These conditions are assured by assuming that the loads between different times to failure are independent and structures are realized according to the same rules with random and independent properties at each reconstruction.The discounted total benefit B for constant benefit b per unit time then simply is:

$$B = b \int_0^\infty \exp[-\gamma t] \mathrm{d}t = \frac{b}{\gamma} \tag{9}$$

The present value of the expected failure cost for systematic reconstruction after failure is in using $\theta = \gamma$ for modified renewal processes:

$$\begin{aligned} D(p) &= (C(p) + H) \sum_{n=1}^{\infty} \int_0^\infty \delta(t) g_n(t, p) \mathrm{d}t \\ &= (C(p) + H) \sum_{n=1}^{\infty} \int_0^\infty \exp[-\gamma t] g_n(t, p) \mathrm{d}t) \\ &= (C(p) + H) \sum_{n=1}^{\infty} g^*(\gamma, p)^n = (C(p) + H) \frac{g_1^*(\gamma, p)}{1 - g^*(\gamma, p)} \\ &= (C(p) + H) h_1^*(\gamma, p) \end{aligned} \tag{10}$$

Therefore,

$$Z(p) = \frac{b}{\gamma} - C(p) - (C(p) + H) h_1^*(\gamma, p) \tag{11}$$

For ordinary renewal processes $h^*{}_1(\gamma,p)$ is replaced by $h^*(\gamma,p)$. For stationary Poissonian failure processes with rate $\lambda(p)$ we simply have

$$Z(p) = \frac{b}{\gamma} - C(p) - (C(p) + H) \frac{\lambda(p)}{\gamma} \tag{12}$$

$\lambda(p)$ may be identified as the stationary crossing rate $\nu^+(p)$ of a random load process out of the safe domain of structural states.

If one assumes a renewal process for certain disturbances, for example earthquakes, with inter-arrival densities $f_1(t)$ and $f(t)$, respectively, and in which failure can occur independently with probability $P_f(p)$ one can show that [6]

$$Z(p) = \frac{b}{\gamma} - C(p) - (C(p) + H) \frac{P_f(p) f_1^*(\gamma)}{1 - (1 - P_f(p)) f^*(\gamma)} \tag{13}$$

For Poissonian disturbances with intensity λ this reduces to

$$Z(p) = \frac{b}{\gamma} - C(p) - (C(p) + H) \frac{P_f(p)\lambda}{\gamma + P_f(p)\lambda} \tag{14}$$

The Laplace transform of the renewal density is analytic only in a few cases. The general case is difficult and needs to be determined numerically. Yet more difficult is the inverse transform, which is necessary when some limit for the renewal density or failure rate has been set. The renewal density looks like a damped oscillation with period around the mean time between failures. The magnitude of the oscillations is larger for smaller coefficients of variation of failure times. It overshoots the constant asymptotic value by roughly a factor of 2–4 [20] for failure times with small coefficient of variation. However, there is an important asymptotic result that can be used without much error. The asymptotic renewal density is [1]

$$\lim_{t \to \infty} h(t) = \frac{1}{\mu} \quad \text{provided that} \quad f(t) \to 0 \quad \text{for} \quad t \to \infty \tag{15}$$

with $\mu = E[T]$ the mean time between failures. It is valid for both ordinary and modified renewal processes. The corresponding asymptotic Laplace transform is:

$$\lim_{\gamma \to 0} \gamma h(\gamma) = \frac{1}{\mu} \tag{16}$$

Therefore, for the case of systematic reconstruction the objective function is:

$$Z(p) \approx \frac{b}{\gamma} - C(p) - (C(p) + H) \frac{1}{\gamma \mu(p)} \tag{17}$$

It is worth noting that this is also the result for the stationary Poissonian failure process to which any non-Poissonian failure model obviously converges.

4. Discussion and risk acceptability

$C(p)$ as well as $\mu(p)$ usually are reasonably assumed to increase in each component of p. Therefore, $Z(p)$ has an optimum for some p. Apparently, a maximum interest rate $\gamma_{\max}$ exists beyond which $Z(p)$ will be negative for any p.

$$\gamma_{\max} = \frac{\mu(p) b - (C(p) + H)}{\mu(p) C(p)} \tag{18}$$

This implies that for a reasonable undertaking we must have $\mu(p) b > (C(p) + H)$, i.e. the benefit received during the average time between failures must exceed the sum of direct reconstruction cost and damage cost. Eq. (17) shows that the interest rate must be non-zero, i.e. $\gamma > 0$. Otherwise the expected damage cost would grow to infinity. For very small interest rate we see from Eq. (17) that the benefit term and the damage term dominate over the construction cost term. Hence, Eq. (17) is simplified to

$$Z(p) = b - \frac{(C(p) + H)}{\mu(p)} \tag{19}$$

which is independent of the interest rate. It is observed that the same condition $\mu(p)b > (C(p) + H)$ applies here, too. $(C(p) + H)$ usually grows weakly in p while $\mu(p)$ is expected to grow strongly in p. Hence, the optimum may be found for $\mu(p) \to \infty$, which corresponds to absolutely safe facilities and those do not exist. It follows that there must be $0 < \gamma \leq \gamma_{max}$.

Hasofer and Rackwitz [6] also considered the case of time-dependent benefit $b(t)$. Assume that at each renewal the benefit function (per unit time) starts at the same value of $b(0)$. Then, due to

$$T_n = \sum_{j=1}^{n} \tau_j$$

and in noting that T_{n-1} is independent of τ_n it is

$$\begin{aligned} B_T &= \int_0^{\tau_1} \exp(-\gamma t) b(t)\mathrm{d}t + \sum_{i=2}^{\infty} \int_0^{\tau_n} \exp[-\gamma(T_{n-1} + t)] b(t)\mathrm{d}t \\ &= \int_0^{\tau_1} \exp(-\gamma t) b(t)\mathrm{d}t + \sum_{i=2}^{\infty} \exp[-\gamma T_{n-1}] \\ &\quad \times \int_0^{\tau_n} \exp[-\gamma t] b(t)\mathrm{d}t \end{aligned} \tag{20}$$

With

$$B_D(t) = \int_0^{t} \exp[-\gamma t] b(t)\mathrm{d}t \tag{21}$$

we have

$$\begin{aligned} B &= \int_0^{\infty} B_D(t) f_1(t)\mathrm{d}t + \left(\sum_{n=2}^{\infty} \int_0^{\infty} \exp(-\gamma t) f_{n-1}(t)\mathrm{d}t \right) \\ &\quad \times \int_0^{\infty} B_D(t) f(t)\mathrm{d}t \\ &= \int_0^{\infty} B_D(t) f_1(t)\mathrm{d}t + \left(\sum_{n=2}^{\infty} f_1^*(\gamma)[f^*(\gamma)]^{n-2} \right) \int_0^{\infty} B_D(t) f(t)\mathrm{d}t \\ &= \int_0^{\infty} B_D(t) f_1(t)\mathrm{d}t + \frac{f_1^*(\gamma)}{1 - f^*(\gamma)} \int_0^{\infty} B_D(t) f(t)\mathrm{d}t \end{aligned} \tag{22}$$

For the stationary Poissonian failure process this simplifies to:

$$B = \left(1 + \frac{\lambda}{\gamma}\right) \int_0^{\infty} B_D(t) \lambda \exp[-\lambda t]\mathrm{d}t \tag{23}$$

For example, if $b(t) = b_0(1 + b_1 t^m)$ one can integrate and obtain:

$$B = b_0\left(1 + \frac{\lambda}{\gamma}\right)\left(\frac{1}{\gamma + \lambda} + \frac{b_1 m!}{(\gamma + \lambda)^{m+1}}\right) \tag{24}$$

b_1 may be taken as $-t_s^{-m}$ with t_s a pre-specified service time, for example. $_m > 2$ can model the fact that $b(t)$ remains almost constant over a large period of time but decays at the end of the intended service time. For $\lambda \to 0$ one can show that

$$B \to \frac{b_0}{\gamma}\left(1 + \frac{b_1 m!}{\gamma^m}\right)$$

whereas for $\lambda \to \infty$ it is $B \to b_0/\gamma$ as in Eq. (9). B now depends also on λ but, since usually $\lambda \ll \gamma$, the dependence is only weak and one simply may take $B = b_0/\gamma$. Clearly, one also must have $b_0/\gamma > 1$. A time-dependent $b(t)$ may be assumed if there are maintenance cost increasing with time.

It is necessary to discuss the damage term H. This is subject to a failure consequence investigation. Decision theory dictates that expected values must be taken. Under realistic conditions direct failure cost are between 1 and 10 times of the reconstruction cost in most cases. Quite frequently, this term also involves some quantity measuring the efforts for saving life and limb of persons. Its value, no doubt, is a highly sensitive matter. Traditionally, such values, or more precisely acceptable risks, have been set on a widely empirical basis, for example, by looking at other partly non-technical involuntary life risks or by comparison with technical risks which obviously have been commonly accepted in the past. This is all but satisfactory. One of the most promising proposals has recently been made by Nathwani et al. [15]. Nathwani et al. define a social index, the so-called life quality index (LQI). Other such indices exist, for example, the human development index (HDI) used by the United Nations to judge and control the state of development of a country. The LQI measures the quality of life by the product of the yearly GNP (gross national product) as a function of the fraction of time spent in economical activities times the life expectancy being part of which is the time available for the enjoyment of life. In particular, the LQI is defined by

$$L = g^w e^{1-w} \tag{25}$$

where g is the average individual contribution to the yearly GNP, possibly reduced by the amount spent for health care, w is the average fraction of time necessary to raise this amount, e the quality-adjusted life expectancy at birth of a population in years, i.e. times in poor health or spent in hospital are subtracted form e. Each variable g and e is raised to the power of the associated proportion of time. The LQI is an index by which a project or undertaking involving risks can be judged. It is especially suited to judge and compare public expenditures into public health, into road traffic safety, into all other life saving public regulations and, thus, also into structural safety by the relevant codes. The project, regulation or undertaking is acceptable

as long as the LQI remains positive. For example, improving safety results in a positive change in e while the cost of the intervention reduce g. Conversely, a negative change in e can be acceptable if there is sufficient compensation in g. The LQI and its implications are thoroughly discussed in Nathwani et al. [15], especially why the three parameters g, e and w, play such an outstanding role and are sufficient for the purpose. It is surprisingly simple and, as will be seen below, has dramatic, unexpected consequences. Its primary value is not so much that it generates hard numerical values for applications. But it identifies the main factors and opens them up to public discussion and judgement. There are, of course, values such as cultural values, religion or environmental ideals, which hardly can be covered by the LQI. For simplicity, those are not included in this discussion.

Using relative changes in L, e and g leads to a general acceptance criterion:

$$\frac{dg}{de} \geq -\frac{g}{e}\frac{1 - w}{w} \tag{26}$$

The mentioned reductions in g and e are about the same percentage so that they approximately cancel in Eq. (26). By some further appealing and subtle considerations, which cannot be given here, the investments into structural safety should be [13,21]

$$\frac{\mathrm{d}C}{\mathrm{d}P_f} \geq -\frac{g(1 - w)\bar{e}}{we} N_F = K N_F \tag{27}$$

$\bar{e}$ is the mean number of life years lost in the event of structural failure usually taken as one half of e and N_F the expected number of persons killed in the event. Taking $g = 18{,}000$ US\$ per year and person, $w = 1/8$, $e = 78$ years and $\bar{e} = 39$ years one arrives at 63,000 US\$ per year and person or 2,500,000 US\$ per person for lost life years. In Eq. (27), P_f is a yearly failure rate. The constant K in Eq. (27) has the same dimension as g. More data including historical aspects can be found in Appendix B and in Nathwani et al. [15]. Therefore, H may be decomposed into the direct monetary losses H_D in case of failure including demolition cost and loss of opportunity during the time of reconstruction and into a term covering the expenses to save life years equal to:

$$H_F = \frac{g(1 - w)\bar{e}^2}{we} N_F \tag{28}$$

Remember that this quantity is not an indication for the magnitude of a possible monetary compensation of victims in the event. It is a number, which the society should be willing to pay for structural safety via structural codes, quality assurance procedures or other regulations. It enters into optimization as a fictitious number at the decision point. If the legal conditions for compensation of victims are valid, it is assumed that this compensation is covered by insurance for which the premium reduces the benefit.

According to Pate-Cornell [16] and Lind [13] direct failure cost and investments into saving human lives should be discounted at the same rate. Otherwise inconsistencies occur which cannot be accepted. Since the time horizon for structural facilities is in the order of 100 years the interest rate for investments into structural safety may be hardly larger than the long term growth rate of the GNP [21]. According to Steckel and Floud [27] this was 2.86% per year during 1848–1960 for the US, 2.3% per year during 1756–1980 for the UK, 1.3% per year during 1820–1990 for France and 1.8% during 1820–1965 for Sweden. The United Nations Human Development Report [28] gives values between 1.2 and 1.9% for industrialized countries during 1975–1998. Skjong and Ronold [25] computed an interest rate from the growth in the LQI for some industrialized countries during 1984–1994 and found values between 1.0 and 4.2% with an average at about 2%. A computation for the years 1850–1998 based on the data in Steckel and Floud [27], Maddison [14] and United Nations [28] of the capitalization of the constant K for $N_F = 1$ for several industrialized countries and taking into account the changes in life expectancy, fraction of life expectancy devoted to work and GNP yields 1.8–3.1% with an average of 2.4%. Communities and governments frequently use an interest rate of about 2% for investments into the infrastructure. This number appears to be appropriate but further research is necessary. It may be slightly different for various countries even within the same development category.

Can optimization automatically fulfil criterion (27)? Rewrite Eq. (27) as

$$\| \nabla_p C(p) \| \geq \frac{g(1 - w)e_m N_F}{we} \| \nabla_p \nu^+(p) \| \tag{29}$$

for the stationary case and

$$\| \nabla_p C(p) \| \geq \frac{g(1 - w)e_m N_F}{we} \left\| \nabla_p \frac{1}{E[T(p)]} \right\| \tag{30}$$

for the non-stationary case. Here, P_f is replaced by the outcrossing rate in the stationary case and by the asymptotic renewal density in the non-stationary case, respectively. Checking this condition at the optimal solution p^* we recognize that cost efficiency of safety related measures is also required. A facility should not be built if those conditions are not fulfilled. $\| \nabla_p C(p) \|$ usually increases more than proportionally in $\| p \|$ whereas $\| \nabla_p \nu^+(p) \|$ or $\| \nabla_p 1/E[T(p)] \|$ decrease strongly for larger $\| p \|$. Therefore, for high reliability structures conditions (29) or (30) can eventually be fulfilled. When optimizing structures by Eqs. (11) or (13), Eqs. (29) or (30) have to be added as a constraint. It turns out that this constraint is active quite frequently. It is also interesting to note that Eqs. (29) or (30) are differential criteria and do not contain interest rates. There is no direct limit on the failure rate (asymptotic renewal rate) but a limit defining the cost efficiency of life saving measures.

5. Optimization

5.1. Stationary case

In general, optimization of structures is difficult and not always unique. A structure may fail in different failure modes and the failure modes may be associated with different mean failure times and damage cost. In the following we will discuss the case of one failure mode only and ignore the fact that many structures are replaced after they have become obsolete. Some concepts on those topics are outlined in Rackwitz [20]. Also, the material presented in Eqs. (20)–(24) may be of use. Further, we will assume that $C(p)$ is differentiable as well as the mean time between failures. Structural states will be described by a at least twice-differentiable state function $g(x(t),p)$ where $x(t)$ is a vector of random variables including process variables for the loads. Failure states are defined for $g(x(t),p) < 0$.The process variables have properties such that the process of downcrossings of $g(x(t),p)$ below zero is a regular point process so that a crossing rate is well defined [2]. It is also assumed that FORM/SORM techniques in so-called standard space are applied (see Ref. [9] and the rich literature on reliability of structures under random process loading). In the following vectorial rectangular wave renewal processes and vectorial differentiable processes and combinations thereof are considered only. The optimization problem for the stationary case can be formulated as follows:

minimise:

$$-Z(p) = -\frac{b}{\gamma} + C(p) + (C(p) + H_D + H_F)\frac{\nu^+(p)}{\gamma}$$

subject to:

$$g(u,p) \le 0$$

$$u_i \parallel \nabla_u g(u,p) \parallel + \nabla_u g(u,p)_i \parallel u \parallel = 0; \quad i = 1,7\ldots,n-1$$

$$\beta(p) \ge \beta_r$$

$$h_j(p) \le 0; \qquad j = 1,\ldots,k$$

$$\parallel \nabla_p C(p) \parallel \ge \frac{g(1-w)e_m N_F}{we} \parallel \nabla_p \nu^+(p) \parallel \tag{31}$$

with β_r the solution of

$$\left(\sum_{i=1}^{n_j} \lambda_i \Phi(-\beta_r) + \frac{\omega_0}{\sqrt{2\pi}}\varphi(\beta_r)\right) C_{SORM} - \nu_{admissible} = 0 \tag{32}$$

If the case treated in Eq. (14) is of interest $\nu^+(p)/\gamma$ must be replaced by $\lambda P_f(p)/[\gamma + \lambda P_f(p)]$. In Eq. (31) the first two conditions are the usual Kuhn–Tucker conditions for a valid β-point [9]. The third condition is a restriction on the downcrossing rate that can also be absent. The forth conditions are possible restrictions for the feasibility of the design parameters. Simple bounds on p may also be added. Finally, the last condition is condition (29) for saving human lives efficiently. It needs to be added only if $H_F \neq 0$. Note that $\mu(p) = 1/\nu^+(p)$ where $\nu^+(p)$ is the downcrossing rate independent of t due to stationarity. In Eq. (32), the first term corresponds to the downcrossings due to a stationary rectangular wave renewal vector process. The jump rates of the components of this process are denoted by λ_i. The second term corresponds to the downcrossings due to a stationary differentiable translation vector process which are generated from a normal process by a one-to-one transformation $x(t) = f(u(t))$. ω_0 is the central cycling frequency of the process. λ_i and ω_0 are independent of p because they are generally uncontrollable loading characteristics. Regularity assures that the corresponding rates can be added. $\Phi(.)$ and $\varphi(.)$ define the standard normal integral and the standard normal density, respectively. $C_{SORM} \approx 1$ is a second-order correction factor. After transforming arbitrarily distributed random variables into standard normal variables by $u = T(x)$ [3,4,7], $\beta(p)$ is defined as the solution of the following optimization problem:

$$\beta(p) = \min\{ \parallel u \parallel \} \quad \text{for} \quad \{u : g(u,p) \le 0\} \tag{33}$$

Since the Kuhn–Tucker conditions for this problem are added to the general optimization problem in Eq. (31), an explicit solution of Eq. (33) is not necessary.

The advantage of this formulation is that it is a true one-level optimization [10], i.e. the optimization in p is performed simultaneously with the optimization to find the β-point in the reliability problem. Of course, it must be assumed that a unique β-point exists. The disadvantage clearly is that second-order derivatives of $g(u,p)$ are required at each iteration step for gradient-based search algorithms like sequential quadratic programming (SQP). Second-order derivatives of $g(u,p)$ with respect to u are also required for the determination of C_{SORM}. Therefore, it is proposed to solve the optimization task iteratively. Initially, $C_{SORM} = 1$ and the Hessian $\nabla_u^2 g(u,p)$ is set equal to a zero matrix corresponding to a linear failure surface. A first approximation can then be found by solving Eq. (31). At the solution $p^{(i)}$ the Hessian $\nabla_u^2 g(u,p^{(i)})$ and with it C_{SORM} and, possibly, a new $\beta_r^{(i)}$ are determined. The optimization problem can now be solved again with a constant approximation for the Hessian $\nabla_u^2 g(u,p^{(i)})$ when determining the gradient of the gradient condition in Eq. (31). This scheme is repeated until convergence is reached. Experience with example problems indicates that convergence usually is reached after at most three iterations.The stationary time-variant optimization problem has already been solved Kuschel and Rackwitz [10] making use of the full Hessian of $g(u,p)$ in each iteration step and simple FORM. For high-dimensional problems this results in very large computational effort. Moreover, numerical computation of the Hessian can cause instabilities of the algorithm. The iteration scheme proposed above is a substantial improvement as it starts with a zero Hessian and, then, needs its computation only once at each reiteration. Simultaneously, it updates the outcrossing

rate according to higher-order theories (SORM and/or Monte Carlo simulation) at each reiteration. A similar updating procedure has been proposed by Polak et al. [29].

The same algorithm can also be applied for time-invariant problems after appropriate modifications.The human safety condition is difficult to fulfil because it is a differential criterion. The gradient of $C(p)$ can usually be determined numerically. For the determination of the gradient of $\nu^+(p)$ some useful approximations are given in Appendix A.

5.2. Locally non-stationary case

The optimization problem can be given as follows:

minimize:

$$-Z(p) = -\frac{b}{\gamma} + C(p) + (C(p) + H_D + H_F)\frac{1}{\gamma E[T(p)]}$$

subject to:

$$\frac{1}{E[T(p)]} \le \nu_{admissible}$$

$$h_j(p) \le 0; \qquad j = 1,\ldots,k$$

$$\parallel \nabla_p C(p) \parallel \ge \frac{g(1-w)e_m N_F}{we} \left\| \nabla_p \frac{1}{E[T(p)]} \right\| \tag{34}$$

The reliability condition now is modified according to Eq. (15). $E[T(p)]$ must be determined numerically. Some more details are given below. The reliability condition may also be omitted. It is added formally as in the stationary case. The second condition defines the feasible parameter space. The third condition is the condition for saving human lives efficiently. It needs to be added only if $H_F \neq 0$. If $H_F \neq 0$, there normally is no need to include the first condition.

According to Rackwitz and Balaji Rao [18] the following two approaches for the determination of $E[T(p)]$ can be used. If, due to some deterioration, a monotonically decreasing smooth state function $g(x,p,\tau)$ can be assumed and, thus, the first-passage time from the safe domain into the failure domain is $P_f(t,p) = P(g,(X,p,t) \le 0)$, we have in using the well-known mean value formula for positive random variables

$$\mu = \int_0^\infty (1 - F(t))\mathrm{d}t$$

$$E[T(p)] = \int_0^\infty (1 - P(g(X,p,\tau) \le 0))\mathrm{d}\tau \tag{35}$$

where X is the vector of basic variables. If, on the other hand, the failure probability must be determined by the downcrossing approach it is:

$$E[T(p)] = \int_0^\infty \exp[-\int_0^\tau \nu^+(p,\theta)\mathrm{d}\theta]\mathrm{d}\tau$$

$$\le \int_0^\infty \left(1 - P_f(0,p) - \int_0^\tau \nu^+(p,\theta)\mathrm{d}\theta\right)\mathrm{d}\tau \tag{36}$$

The first formula is a well-known asymptotic result. The second upper bound result is valid under all conditions. Frequently, the term $P_f(0,p)$, i.e. the probability that there is failure already at time $t = 0$, is neglected.

Integration can be performed by one of the simpler integration formulas. In both cases one can conveniently make use of modern FORM/SORM computation tools for $P(g(X,p,\tau) \le 0)$ [9] or for

$$\int_0^\tau \nu^+(p,\theta)\mathrm{d}\theta$$

[19]. For example, in the context of FORM, Eq. (35) reduces to

$$E[T(p)] = \int_0^\infty \Phi(\beta(t))\mathrm{d}t \tag{37}$$

Here, $\beta(t)$ is the time-dependent reliability index.

Similarly, if non-stationary downcrossing rates must be determined it is possible to approximately perform the necessary inner time-integration in Eq. (36) also using FORM/SORM technology. Then, the mean number of downcrossings can be written as

$$E[N^+(p,t)] = \int_0^t \nu^+(p,\tau)\mathrm{d}\tau$$

$$= \left(\sum_{i=1}^{n_j} \lambda_i \Phi(-\beta(p,\tau^*)) + \frac{\omega_0}{\sqrt{2\pi}}\varphi(\beta(p,\tau^*))\right) \times C_{SORM} C_{TIME} \tag{38}$$

with

$$\beta(p,\tau^*) = \min\{ \parallel u \parallel \} \quad \text{for} \quad \{u : g(u,p,\tau) \le 0\} \tag{39}$$

C_{TIME} is a factor taking into account the correction for the time integration effects on the critical downcrossing rate at τ^* given in the first parenthesis in Eq. (38). More details about the techniques to compute outcrossing rates are given in Rackwitz [19]. Unfortunately, most of these results are of approximate and/or asymptotic nature.

The non-stationary case is difficult and numerically involved. A two-level optimization scheme must be employed. The most difficult part is the determination of the mean time between failures as it may require many reliability analyses for every given p. The corresponding integral must be determined numerically. Some clever device is needed in order to concentrate reliability analyses in the region where reliability (as a function of time) effectively decays and to avoid reliability analyses in regions where it is either close to one or close to zero. The case of smooth decreasing state functions as in Eq. (35) usually is considerably less time-consuming than when the mean failure time has to be determined by the downcrossing approach.

Here again, the human safety condition is difficult to fulfil. For the determination of the gradient of $1/E[T(p)]$ some approximations are given in Appendix A.

6. Examples

6.1. Example 1: Short reinforced concrete column (stationary failure phenomenon)

Consider a short quadratic reinforced concrete column with state function

$$g(x(t), p) = (d^2 F_c + A_s F_s) - M_L (L_1(t) + L_2(t)) \tag{40}$$

where the basic random variables F_c, F_s, and M_L are concrete strength, yield stress, model uncertainty, respectively. d (m) is side length, A_s (m^2) the steel area. $L_1(t)$ is a stationary rectangular wave renewal load with (shifted) Rayleigh-distributed amplitudes, mean equal to 2, standard deviation 0.5 and jump rate $\lambda = 10$ [1/year]. $L_2(t)$ is a log-normal differentiable process with mean 2, standard deviation 0.5, auto-correlation function $\exp[-a\tau^2]$, $a = 0.1$. $L_1(t)$ and $L_2(t)$ are independent processes.. The other variables are log-normally distributed with parameters given in the following table

Variable	Mean	Stand. Dev.
Concrete strength (MPa)	35	6
Yield stress (MPa)	460	30
Model uncertainty	1	0.1

The objective function is

$$-Z(d, A_s) = -\frac{b}{\gamma} + C_0 + C_1 d^2 + C_2 A_s + (H + C_0 + C_1 d^2 + C_2 A_s)\frac{\nu^+(d, A_s)}{\gamma} \tag{41}$$

with $b = 0.1$, $\gamma = 0.05$, $C_0 = 1$, $C_1 = 0.5$, $C_2 = 10$, $H = 10$. The reinforcement is between 0.5 and 8% of d^2. The reliability constraint in Eq. (31) is set as 10^{-3} [1/year] and no human life is endangered. The first optimum solution using FORM-results is $(d, A_s) = (0.59, 0.0015)$. For this solution one determines a downcrossing rate by SORM of 2.06×10^{-5} that is only some 20% larger than the one by FORM. Therefore, a second iteration using the Hessian at the solution point in fact reproduces the first result. Also, the reliability constraint is fulfilled.

6.2. Example 2: Short reinforced concrete column (non-stationary failure phenomenon)

The same slightly modified example is used to demonstrate a locally non-stationary case with monotonically decreasing state function. Here, strength reduction may be due to some deterioration. The state function is

$$g(x) = (d^2 F_c + A_s F_s)(1 + at) - M_L L \tag{42}$$

where the basic random variables F_c, F_s, M_L and L are concrete strength, yield stress, model uncertainty and load, respectively. d (m) is side length, A_s (m^2) the steel area, $a = -0.001$ [1/years] a constant and t the parameter time measured in years. The lognormal load has mean 3.5 and standard deviation 0.35. The other basic variables are as in example 1. The objective function is:

$$-Z(d, A_s) = -\frac{b}{\gamma} + C_0 + C_1 d^2 + C_2 A_s + (H + C_0 + C_1 d^2 + C_2 A_s)\frac{1}{\gamma E[T(d, A_s)]} \tag{43}$$

The same constants as in example 1 are chosen. The reliability constraint in Eq. (34) is set as 10^{-3} (1/year). No human life is endangered. The solution is $(d, A_s) = (0.55, 0.0015)$ using the gradient-free algorithm COBYLA [17] and FORM for the time-dependent failure probabilities. The algorithm searches only for side length d while the minimum reinforcement is kept. The corresponding mean time to failure is 2176 (years) implying an (asymptotic) failure rate of 0.00046 (1/year) so that the reliability constraint is fulfilled. The minimum should be negative and is, in fact, negative at a value of −0.736. Therefore, the design is acceptable. The same result is obtained with the gradient-based algorithm NLPQL [26] where Eq. (A2) in the Appendix must be applied.

6.3. Example 3: Inclusion of cost for saving human lives

A final simple example is included illustrating the effect of investments for saving lives. Here, formula (14) divided by C_0 is used. The cost for human lives is included and the efficiency criterion Eq. (29) is added as a constraint.

$$-\frac{Z(p)}{C_0} = -\frac{b}{\gamma C_0} + (1 + \frac{C_1}{C_0}p^a) + \left(\left(1 + \frac{C_1}{C_0}p^a\right) + \frac{H_D}{C_0} + \frac{H_F}{C_0}\right)\frac{P_f(p)\lambda}{\gamma + P_f(p)\lambda}$$

subject to:

$$\frac{d}{dp}\left(1 + \frac{C_1}{C_0}p^a\right) \geq -\frac{g(1-w)e_m N_F}{we}\frac{d}{dp}(P_f(p)\lambda) \tag{44}$$

with

$$P_f(p) = \Phi\left(-\frac{\ln\left(p\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right)}{\sqrt{\ln((1+V_S^2)(1+V_R^2))}}\right)$$

being one of the very few closed-form exact results in

structural reliability for a failure function of the form $g(x) = r - s$. R and S are both log-normally distributed with separation of the means equal to the design parameter p (which equals the so-called central safety factor) and coefficients of variation V_R and V_S, respectively. In particular, assume $C_0 = 10^6$, $C_1 = 10^5$, $a = 1.25$, $H_D = 3\ 10^6$, H_F as in Eq. (28) with $g = 18000$, $e = 70$, $e_m = 35$, $w = 0.125$ and $N_F = 1$, $\lambda = 1$ (1/year), $V_R = 0.2$ and $V_S = 0.3$, respectively. Further, it is $b = 0.05$ (1/year) and $\gamma = 0.02$ (1/year). All monetary values are given in US$. Performing the optimization task (44) including the term H_F yields $p^* = 3.488$ corresponding to a yearly failure rate of 1.627×10^{-4} while the safety condition Eq. (29) requires $p_{\text{limit}} = 5.214$ corresponding to a failure rate 1.125×10^{-6}. p_{limit} is the solution to the equality of the constraint in Eq. (44). In this case, optimization including the term H_F in Eq. (28) does not satisfy the safety criterion and the admissible design parameter is p_{limit}. The failure rate corresponding to p_{limit} decreases roughly inversely proportional with N_F.

7. Discussion

The above simple examples illustrate how to formulate the concepts outlined in the theoretical part. None of them claims to be already complicated as in realistic practical applications. In fact, many numerical and technical aspects are still under study and some results will be presented in a subsequent paper. Most interesting is the last example although optimization is performed only in one dimension and with a relatively simple stochastic model. The optimum failure rate from a purely economic optimization including the cost for saving lives H_F suggests that this is in the order of magnitude of what is considered acceptable in present practice, at least for some outstanding structural facilities like offshore platforms, long tunnels and long-span suspension bridges. For other buildings like normal houses and office buildings one must expect smaller values because the fictitious cost H_F for saving lives dominate and a higher total damage cost H drags the optimum towards smaller optimum failure rates. The LQI-criterion, however, independent of an arbitrary interest rate and, to a certain extent, arbitrary stochastic models for loads and resistances yields an acceptable failure rate of two orders of magnitude smaller. Similar findings have also been found for other stochastic models and slightly more complicated failure phenomena.

8. Summary

The renewal model proposed in Hasofer and Rackwitz [6] and earlier is reviewed and a suitable objective function is derived. The properties of the objective function are discussed. Special emphasis is given to the damage term if human lives are endangered. A criterion is derived based on a special social index. The additional gradient condition, expressing efficiency of safety related measures, is new. It is based on the life quality index proposed by Nathwani et al. [15].

Differentiability of the objective function as well as of the reliability measures has been assumed. Clearly, many technical problems exist where reliability related measures are not differentiable. Then, other optimization procedures have to be applied.

Several results of the theory outlined before are conceptually important. Firstly, if a reliability constraint is set this constraint should be formulated in terms of a failure rate (yearly failure probability, renewal density, and failure intensity) in contrast to failure probabilities related to some intended service time. Such constraints, however, are strictly not compatible with an overall optimization approach where expected failure cost including the cost for saving human life and limb are considered. The risk associated with the optimum is a function of the benefit derived from the facility, the construction cost, the damage cost and the interest rate with which benefit and expected damage are capitalized. Secondly, the risk acceptability criterion is no more a criterion on failure probabilities or failure rates. The new criterion is formulated in terms of increments in the failure rate and the investment cost, respectively. Finally and thirdly, if it is accepted that investments into human safety have a higher ethical value than purely economic investments the new risk acceptability criterion usually dominates an optimal solution of a project.

Appendix A

The derivatives of $\nu^+(p)$, $\lambda P_f(p)$ or $1/E[T(p)]$ with respect to p are required if human lives are endangered and condition (29) or (30) must be fulfilled. To first order there is:

$$\frac{\partial}{\partial p_i}\nu^+(p) = \frac{\partial}{\partial p_i}\left(\sum_{j=1}^{n_J}\lambda_j\Phi(-\beta(p)) + \frac{\omega_0}{\sqrt{2\pi}}\varphi(\beta(p))\right) = -\left(\sum_{j=1}^{n_J}\lambda_j\varphi(\beta(p)) + \frac{\omega_0}{\sqrt{2\pi}}\beta(p)\varphi(\beta(p))\right)\frac{\partial\beta(p)}{\partial p_i} \tag{A1}$$

$$\frac{\partial}{\partial p_i}(\lambda P_f(p)) = \frac{\partial}{\partial p_i}(\lambda\Phi(-\beta(p))) = -\lambda\varphi(\beta(p))\frac{\partial\beta(p)}{\partial p_i} \tag{A2}$$

$$\frac{\partial}{\partial p_i}\frac{1}{E[T(p)]} = \frac{\partial}{\partial p_i}\frac{1}{\int_0^\infty\Phi(\beta(t,p))dt} = -\frac{\int_0^\infty\frac{\partial}{\partial p_i}\Phi(\beta(t,p))dt}{[\int_0^\infty\Phi(\beta(t,p))dt]^2} = -\frac{\int_0^\infty\varphi(\beta(t,p))\frac{\partial\beta(t,p)}{\partial p_i}dt}{[\int_0^\infty\Phi(\beta(t,p))dt]^2} \tag{A3}$$

For the more complicated case of Eq. (36) we assume

$P_f(0,p) \approx 0$. Then,

$$\frac{\partial}{\partial p_i}\frac{1}{E[T(p)]} = \frac{\partial}{\partial p_i}\frac{1}{\int_0^\infty\{1-\int_0^t \nu^+(\vartheta,p)\mathrm{d}\vartheta\}\mathrm{d}t} = -\frac{\int_0^\infty\int_0^t\left(\sum_{i=1}^{n_J}\lambda_i + \frac{\omega_0}{\sqrt{2\pi}}\beta(\vartheta,p)\right)\varphi(\beta(\vartheta,p))\frac{\partial\beta(\vartheta,p)}{\partial p_i}\mathrm{d}\vartheta\,\mathrm{d}t}{\left[\int_0^\infty\left\{1-\int_0^t\left(\sum_{i=1}^{n_J}\lambda_i\Phi(-\beta(\vartheta,p)) + \frac{\omega_0}{\sqrt{2\pi}}\varphi(\beta(\vartheta,p))\right)\mathrm{d}\vartheta\right\}\mathrm{d}t\right]^2} \quad \text{(A4)}$$

The sensitivities of $g(u,p,t)$ must be determined for every time instant considered during integration. The upper limit of the outer integration is whenever

$$\int_0^t \nu^+(\vartheta,p)\mathrm{d}\vartheta \geq 1$$

beyond which the derivative of $1/E[T(p)]$ is zero. Two integrals need to be evaluated for every iteration in p. The results in Eqs. (A1)–(A4) may further be used if gradient-based algorithms have to be applied. The gradient of $C(p)$ must be determined numerically. The elements of the gradient of $\| \nu^+(p) \|$, which then are necessary in Eq. (29), are

$$\frac{\partial}{\partial p_i}\| \nabla_p\nu^+(p) \| = \frac{\nabla_p\nu^+(p)^{\mathrm{T}}\left(\frac{\partial}{\partial p_i}\nabla_p\nu^+(p)\right)}{\| \nabla_p\nu^+(p) \|} \quad \text{(A5)}$$

and, similarly, for Eq. (30).

The second derivatives of Eqs. (A1)–(A3) with respect to p are lengthy. The second derivative of $1/E[T(p)]$ in Eq. (A4) is not given herein

$$\frac{\partial^2\nu^+(p)}{\partial p_i\partial p_j} = \left\{\left(\sum_{i=1}^{n_J}\lambda_i\beta(p) + \frac{\omega_0}{\sqrt{2\pi}}(\beta(p)^2-1)\right)\frac{\partial\beta(p)}{\partial p_i}\frac{\partial\beta(p)}{\partial p_j} - \left(\sum_{i=1}^{n_J}\lambda_i + \frac{\omega_0}{\sqrt{2\pi}}\beta(p)\right)\frac{\partial^2\beta(p)}{\partial p_i\partial p_j}\right\}\varphi(\beta(p)) \quad \text{(A6)}$$

$$\frac{\partial^2(\lambda P_f(p))}{\partial p_i\partial p_j} = \lambda\varphi(\beta(p))\left(\beta(p)\frac{\partial\beta(p)}{\partial p_i}\frac{\partial\beta(p)}{\partial p_j} - \frac{\partial^2\beta(p)}{\partial p_i\partial p_j}\right) \quad \text{(A7)}$$

$$\frac{\partial^2}{\partial p_i\partial p_j}\left(\frac{1}{E[T(p)]}\right) = \frac{\partial}{\partial p_j}\left(\frac{\partial}{\partial p_i}\left(\frac{1}{E[T(p)]}\right)\right)$$
$$= \frac{\partial}{\partial p_j}\left(-\frac{N(p)}{D(p)}\right) = -\frac{1}{D(p)}\frac{\partial}{\partial p_j}N(p) + \frac{N(p)}{D(p)^2}\frac{\partial}{\partial p_j}D(p)$$

where $N(p)$ is the numerator in Eq. (A3) and $D(p)$ the denominator. Further it is:

$$\frac{\partial N(p)}{\partial p_j} = \int_0^\infty\left(-\beta(t,p)\frac{\partial\beta(t,p)}{\partial p_i}\frac{\partial\beta(t,p)}{\partial p_i} + \frac{\partial^2\beta(t,p)}{\partial p_i\partial p_j}\right)\varphi(\beta(t,p))\mathrm{d}t$$

$$\frac{\partial D(p)}{\partial p_j} = \frac{\partial}{\partial p_j}\left(\int_0^\infty\Phi(\beta(t,p))\mathrm{d}t\right)^2$$
$$= 2\int_0^\infty\Phi(\beta(t,p))\mathrm{d}t\int_0^\infty\varphi(\beta(t,p))\frac{\partial\beta(t,p)}{\partial p_j}\mathrm{d}t$$

and, thus:

$$\frac{\partial^2}{\partial p_i\partial p_j}\left(\frac{1}{E[T(p)]}\right)$$
$$= \frac{2\int_0^\infty\varphi(\beta(t,p))\frac{\partial\beta(t,p)}{\partial p_i}\mathrm{d}t\int_0^\infty\varphi(\beta(t,p))\frac{\partial\beta(t,p)}{\partial p_j}\mathrm{d}t}{\left[\int_0^\infty\Phi(\beta(t,p))\mathrm{d}t\right]^3}$$
$$+ \frac{\int_0^\infty\varphi(\beta(t,p))\left(\beta(t,p)\frac{\partial\beta(t,p)}{\partial p_i}\frac{\partial\beta(t,p)}{\partial p_j} - \frac{\partial^2\beta(t,p)}{\partial p_i\partial p_j}\right)\mathrm{d}t}{\left[\int_0^\infty\Phi(\beta(t,p))\mathrm{d}t\right]^2} \quad \text{(A8)}$$

In these equations a convenient asymptotic result by Hohenbichler and Rackwitz [8] for $\partial\beta(t,p)/\partial p_i$ can be used:

$$\frac{\partial\beta(p)}{\partial p_i} \approx \frac{1}{\| \nabla_u g(u,p) \|}\frac{\partial g(u,p)}{\partial p_i} \quad \text{(A9)}$$

The second derivative is also computed from this approximation:

$$\frac{\partial^2\beta(p)}{\partial p_i\partial p_j} \approx \frac{1}{\| \nabla_u g(u,p) \|}\frac{\partial^2 g(u,p)}{\partial p_i\partial p_j} - \frac{1}{\| \nabla_u g(u,p) \|^3}\frac{\partial g(u,p)}{\partial p_i} \times\left(\sum_{k=1}^{n}\frac{\partial g(u,p)}{\partial u_k}\frac{\partial^2 g(u,p)}{\partial u_k\partial p_j}\right) \quad \text{(A10)}$$

The Hessians required in this equation have to be determined numerically. The Hessians may be set to zero initially and have to be determined at least once before a new cycle starts.

Appendix B

In Table B1, a few data are collected for some industrial countries. All data are from Steckel and Floud [27] except those for 1998, which are taken from United Nations [28]. All monetary units are in US$ in 1985. The 1998 US$ is assumed to have only 62.5% of its value in 1985 due to inflation. In order to obtain GNP and K in US$ of 1998 all given values should be multiplied by 1.6. The value of K is computed as in Eq. (27) with estimated $w = 0.125$ for 1998, $w = 0.15$ for 1950, $w = 0.188$ for 1900 and $w = 0.25$ for 1850, respectively. No differentiation of w for the different countries is made. This overestimates the K-value for Japan by about 10% in 1998. Less developed countries with lower GNP and LE but generally larger w have substantially smaller K-value and thus less severe safety criteria.

Table B1
Gross national product and life expectancy for some industrialized countries [GNP = gross national product; LE = life expectancy; K = constant in Eq. (27)]

Country	1850			1900			1950			1998		
	GNP	LE	K	GNP	LE	K	GNP	LE	K	GNP	LE	K
United Kingdom	1943	39.5	2915	3792	48	8216	5628	69	15,950	12,650	77.4	44,280
United States	1179	29.5	1769	3824	47.8	8285	8588	68.2	24,330	18,550	78.1	64,930
France	1150	40	1725	2250	46.8	4875	4149	66.8	11,760	17,490	78.6	61,210
Netherlands	1551	37.3	2327	2842	49	6158	4706	71.3	13,330	17,590	77	61,580
Sweden	871	43.9	1307	1895	52.9	4106	5834	74.4	16,530	17,320	79.3	60,620
Germany	875	37.1	1313	1743	44.4	3777	2554	66.5	72,36	19,460	77.1	68,120
Australia	2517	46	3776	4100	55	8883	5931	69.5	16,800	13,680	80	47,860
Japan	606	38	909	947	44	2052	1563	58	4429	26,300	80.1	92,059
Average	1336	41.3	2005	2674	49	5794	4869	63.5	13,796	17,880	78.5	62,580

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