STRUCTURAL SAFETY

ELSEVIER

Structural Safety 25 (2003) 201–225

www.elsevier.com/locate/strusafe

Aspects of parallel wire cable reliability

M.H. Faber[a],*, S. Engelund[b], R. Rackwitz[c]

[a] *Swiss Federal Institute of Technology, ETH Hönggerberg, HIL E32.3, CH-8093 Zürich, Switzerland*

[b] *RAMBØLL, Kjærulfsgade 2, DK-9400 Nørresundby, Denmark*

[c] *Technische Universitaet Muenchen, Theresienstrasse 90, D-80290 Munich, Germany*

Received 27 February 2002; accepted 19 August 2002

Abstract

Recently developed models for the assessment of the strength and fatigue life of cables are presented and illustrated. The illustrations are based on experimental data and experience gained from the design and assessment of several major cable supported bridges. It is shown how inspection results may be used to update the reliability of cables. The procedure is illustrated using results of Ultra Sonic inspections. Finally, aspects of design and assessment of stay cables are considered. Special emphasis is given to the effect of design philosophy and corrosion protection on the resistance factors for the stay cable resistance.
© 2002 Published by Elsevier Science Ltd.

Keywords: Cable; Parallel wires; Reliability; Static strength; Fatigue; Inspection

1. Introduction

The stock of aging cable supported bridges is steadily increasing and the effect of degradation processes such as fatigue and corrosion has become an evident problem for the cables of several of these bridges, see e.g. Haight et al. [1] and Fu et al. [2]. It is well recognised, that cables in cable supported structures are highly redundant and that cables are relatively robust with regard to degradation. However, there still remains to establish a rationale for the assessment of the strength of cables subject to degradation as well as a consistent methodology for the utilisation of inspections and in situ test results for condition control and reliability updating.

Reliability analysis of wires and wire bundles is traditionally based on parametric statistical models for the strength and fatigue characteristics of individual wires. It has been widely accepted that the static wire strength must be modelled by a random variable. Its variability can be decomposed into at least two parts, one which models the variations between wires and one

* Corresponding author. Tel.: +41-1633-3117; fax: +41-1633-1064.
E-mail address: faber@ibk.baug.ethz.ch (M.H. Faber).

0167-4730/02/$ - see front matter
PII: S0167-4730(02)00057-7

which models the fluctuations along the wire. The fluctuations along the wire are responsible for a pronounced weakest-link effect, i.e. a reduction of strength as the wire length is increased. Parallel wire cables are characterized by the fact that there is only little mechanical interaction among wires between their anchorages. Wire length also determines stress–strain behaviour. In short wires tested according to standard procedures one observes almost perfect elastic behaviour until plastification occurs at usually only one randomly located weakest section. Further increase of the loading then will lead to a certain strain hardening in this section, allowing for further load increase, and almost all deformations concentrate in the resulting constriction until ultimate strength is reached. This gives the well-known elastic–plastic behaviour of steel. The precise characteristics, of course, depend on how the wire is manufactured. Long wires behave differently. The same phenomena will be observed until yielding of the weakest section. However, as the load is increased the elastic deformations in the not-yielding sections dominate those in the constriction zone. As a result a long wire shows almost perfect elastic behaviour. Perfect parallel systems show a high degree of redundancy. Further, the forces along a cable vary very little. These almost ideal situations have lead to some theoretical models rather early. The primary models are named after Weibull [3] to take account of the effect of length and after Daniels [4] to take account of the effect of redundancy. The most important results are of asymptotic nature. Both models will be discussed below in some detail. The experimental verification of the length effect has been performed by Fernandes-Cantelli et al. [5], Castillio and Fernandes-Canteli [6] and elsewhere. Less experimental work is available to verify the Daniels effect (see [7,8]).

In part alternative models for the assessment of the static strength of parallel wire cables such as cables in suspension bridges and cable stayed bridges have been proposed by Matteo et al. [9], Haight et al. [1] and recently by Fu et al. [2], apparently without knowledge of the work previously mentioned. The models proposed by Matteo et al. [9] and Haight et al. [1] take into account the effect of the wire length as well as brittle and ductile wire behaviour but they both omit the so-called Daniels effect. This is also reflected in their observation that their models overestimate the cable ultimate strength by up to 10%. This effect is, however, included indirectly by Fu et al. [2] where the capacity of parallel wire cables is assessed by simulation of the stress strain relationship for the cable.

Less work has so far been presented regarding the assessment of the fatigue life of parallel wire cables. The fatigue life of short and long wires is again well modelled by a Weibull distribution (see e.g. [10,11]) supported by experiments (for example, [8,12]). Using parametric models for the fatigue lives of individual wires, models for the fatigue life of parallel wire cables can be derived. Such models are reviewed in Rackwitz and Faber [13]. Coleman [14] established the first theoretical model for parallel wire cables. It was later extended by Phoenix [15].

The present paper attempts to give an overview of reliability based assessment of parallel wire cables, both for design and for assessment of cables in existing structures.

First the basic theory and statistical models for the ultimate and fatigue strength of parallel wire cables is given and it is shown how test results from both new wires and wires from existing cables may be utilised to determine the parameters of the statistical models.

Thereafter, it is described how the reliability of a parallel wire cable may be updated on the basis of inspections of the number of broken wires in the cable. For the special case of Ultra

Sonic inspections of wire ruptures close to the sockets of stay cables, it is demonstrated how the so-called probability of detection of defects can be evaluated. Also, the effect of the inspections on the reliability of the stay cable is illustrated.

Finally, aspects of design and assessment of stay cables are considered. Special emphasis is given to the effect of design philosophy and corrosion protection on the cable resistance safety factors.

2. Static cable strength

A probabilistic model for the assessment of the static strength of the individual cables should be able to incorporate all available information regarding wire material characteristics and the number of corroded and damaged wires.

In the following such a model is presented. The model includes a reduction of the ultimate stay tensile capacity due to the so-called length effect as well as a reduction due to the large number of individual wires working in parallel—the so-called Daniels effect.

2.1. *The strength of individual wires*

Individual wires may be considered as being a weakest link structural system (a series system). The number of elements in the system depends on the length of the wire and on the statistical characteristics of the material parameters together with—especially for deteriorated wires—the defects (cracks, corrosion pits, etc.) in the wires.

The material parameters and the defects in the wires may conveniently be described in terms of

- the statistical characteristics of the ultimate stress for the wires as obtained from tensile tests
- the correlation length L_ρ, i.e. the length over which the material parameters and/or the defects in the wires may be assumed to be correlated.

The correlation length may be defined as

$$L_\rho = \int_0^\infty |\rho_{ZZ}(\tau)| \mathrm{d}\tau \tag{1}$$

where $\rho_{zz}(\tau)$ is the auto-correlation function for the wire strength. Some times the correlation length is referred to indirectly as the Weibull size effect. If the correlation length of the wire is short compared with the length of the wire, the number of elements in the series system will be large. If the correlation length of the wire is long compared with the length of the wire, the number of elements in the series system will be small.

Failures of individual wires may be ductile or brittle depending on the wire material characteristics. However, in case of a ductile wire failure the length of the plastic zone will be in the order of 1–3 times the diameter of the wire. As the wires usually are very long in comparison to their diameter it is easily realised that the elastic strains at failure will always dominate why in general a wire failure will have a brittle characteristic.

The strength of the individual elements in the weakest link system consisting of m elements may be considered as a realisation of a set of m independent random variables Z_i with identical distribution functions $F_{Zi}(z_i)$. See Bolotin [11]. As the strength of the weakest link system Z is determined by the element with the lowest strength, i.e.

$$Z = \min(Z_1, Z_2, ...Z_m) \tag{2}$$

it is seen that the strength of the system corresponds to the smallest realisation of the element strength random variable Z_i in m trials. It is obvious that it is more likely to achieve a low realisation of Z if m is large and the strength of the wire will therefore decrease for increasing wire length and for decreasing correlation length.

The distribution function $F_Z(z)$ for the strength z of a wire of length L may appropriately be given by a Weibull distribution with parameters λ, u and k as

$$F_Z(z) = 1 - \exp\left[-\lambda\left(\frac{z}{u}\right)^k\right] \tag{3}$$

with mean value

$$E[Z] = u\ \lambda^{-1/k}\ \Gamma(1 + 1/k) \tag{4}$$

and variation

$$V[Z] = u^2\lambda^{-2/k}\left[\Gamma(1 + 2/k) - \Gamma^2(1 + 1/k)\right] \tag{5}$$

The scale factor λ is given by

$$\lambda = \frac{L}{lL_0} \tag{6}$$

where L_0 is the length of the reference (test) wire specimen and $L_\rho = l \cdot L_0$ is the correlation length of the material parameters and/or the defects in the wire.

For new and undamaged wires it has been estimated that the correlation length is in the same order of magnitude or even larger than the length of the considered wire e.g. $L = 1000$ m. For old or damaged wires the correlation length may be reduced to a length in the order of the diameter of the wire e.g. ~5–7 mm.

The scale factor, λ, together with the parameters k and u may be estimated from ultimate capacity tests by the Maximum Likelihood Method. Having observed from n_e experiments the ultimate capacities $\underline{x}$ the parameters k, u and l may be estimated from

$$\max_{u,k,\lambda}(\mathbf{L}(u, k, \lambda)) \tag{7}$$

where the likelihood function $\mathbf{L}(u, k, \lambda)$ is given by

$$\mathbf{L}(u, k, \lambda) = \prod_{i=1}^{n_e} f_X(\underline{x}_i, u, k, \lambda) \tag{8}$$

$$f_{\underline{X}}(\underline{x}_i, u, k, \lambda) = \frac{\lambda k}{u}\left(\frac{x_i}{u}\right)^{k-1}\exp\left[-\lambda\left(\frac{x_i}{u}\right)^k\right] \tag{9}$$

As is seen from Eq. (9), λ and u are represented in the problem only in terms of a product between λ and u^{-k}. This functional relationship poses a problem when the maximisation according to Eq. (7) is performed. There is no unique solution for λ and u but rather for λu^{-k}. However, as the correlation length (the product between l and L_0) may be assumed to be constant for test specimens of different length L_0 this problem can be overcome if wire specimens of different length are tested. Naturally, some of these different lengths must be larger than the correlation length.

The maximum likelihood method also yields an estimate of the statistical uncertainty of the estimated parameters. The parameters are asymptotically unbiased and normally distributed. The covariance matrix can be determined on the basis of the Hessian matrix of the maximum likelihood function at the optimum, see e.g. Lindley [16], and allows quantifying the statistical error of the estimates.

Finally, by the estimation of the model parameters it should be investigated whether all wires in a given cable originate from the same batch, e.g. by variance analysis. The variation of the mean tensile strength in different batches may be larger than the variation of the tensile strength between different wires from the same batch. Therefore, if wires from different batches are used in a given cable this may have to be included in the model. However, variance analysis may also lead to the result that the wires cannot belong to the same population of wires, which could e.g. be the case if the wires have been subjected to corrosion affecting the individual wires differently. In this case a grouping of the wires should be made based on the results of the variance analysis. How these are then treated for the assessment of the statistical characteristics of bundles of wires is considered in Section 2.2.1.

As an example consider a 100 m long parallel wire cable with 200 wires. The diameter of the wires is 7 mm. For the evaluation of the strength of the cable 30 tensile tests of wires from the same batch as the wires in the cable have been performed. In Table 1 the results of the tensile strength tests of the wires are given. All test items were $L_0 = 500$ mm long. Therefore, it is not possible on the basis of the tests to determine the correlation length of the tensile strength.

Using maximum likelihood estimation the model parameters are found to be $u = 1788.7$ $k = 72.62$ where the scale factor, λ, has been set to one because all test items have the same length. The standard deviations of the parameters u and k are 4.77 and 9.89, respectively. The parameters are correlated with the correlation coefficient 0.332.

In Fig. 1 the strength of a 100 m long wire is shown as a function of the scale factor. If the results given in Fig. 1 are assumed valid for an undamaged cable the correlation length will be large in the order of one third of the length of the cable i.e. $\lambda = 3$. This implies that the strength of the wire in mean is equal to 1748 MPa. On the other hand if the data are assumed valid for a corroded wire the correlation length of the cable will probably not be larger than the length of the test specimens, i.e. $l = 1.0$ and consequently $\lambda = 200$. On this basis the mean strength of the wire can be found to be equal to 1650 MPa. It is important note that the strength reduction is not due to a reduction of the cross-section but alone due to the effect of minor surface damages reducing the correlation length of the wire. In the present case the reduction of the strength of the wire of 5.6% is observed, however, in general reductions of up to 10% can be found.

Table 1
Results of tensile strength tests

Test	Strength (kN)	Test	Strength (kN)	Test	Strength (kN)
1	69.0	11	67.0	21	68.1
2	67.3	12	69.7	22	66.9
3	67.5	13	70.3	23	66.5
4	68.5	14	68.7	24	68.0
5	67.2	15	69.0	25	67.2
6	68.7	16	67.5	26	68.5
7	68.0	17	68.7	27	67.2
8	69.0	18	68.9	28	68.6
9	70.1	19	69.8	29	69.7
10	68.7	20	66.9	30	68.2

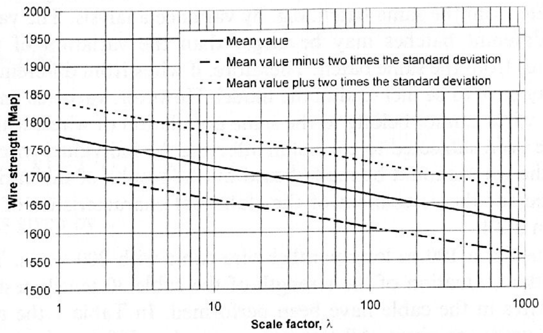

Fig. 1. Strength of a 100 m long wire as function of the scale factor.

2.2. The strength of parallel wire cables

The strength of a bundle of parallel wires may be assessed by modelling the cable as a parallel system, see Fig. 2.

2.2.1. The Daniel's model for the strength of a parallel wire cable

The strength of a parallel system with n components may, if n is large enough ($n > 150$), be shown (see [4,17,18]) to be normal distributed with mean value

$$E_n = nx_0(1 - F_Z(x_0)) + c_n \tag{10}$$

and standard deviation

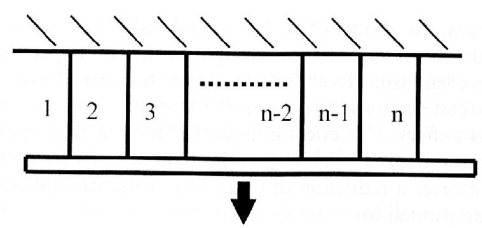

Fig. 2. Parallel system for the modelling of bundles of parallel wires.

$$D_n = x_0[nF_Z(x_0)(1 - F_Z(x_0))]^{1/2} \tag{11}$$

where c_n may be assessed from

$$c_n = 0.966n^{1/3}a \tag{12}$$

and

$$a^3 = \frac{f_Z^2(x_0)x_0^4}{(2f_Z(x_0) + x_0 f'_Z(x_0))} \tag{13}$$

$f_Z(x_0)$ is the density function for the wire strength. The parameter x_0 is the solution of

$$x_0 = \max\{x(1 - F_Z(x)\} \tag{14}$$

provided that there is $f_Z(z) = 0$ and $(1 - F_Z(z)) \rightarrow 0$. A correction for the standard deviation can also be given but is usually not important. In particular, if Z is Weibull-distributed the parameter x_0 may be determined from

$$x_0 = \left[l\frac{L_0}{Lk}\right]^{1/k} u \tag{15}$$

c_n may be considered as a correction term to the asymptotic solution (which is valid for large n) in cases where n is below say 150.

The free length i.e. L in Eq. (15) should be assessed by considering the length over which a ruptured wire will be bonded by friction to the adjacent wires and thus regain its load carrying ability. The so-called bond length is mainly influenced by e.g. clamps on main suspension cables, however, also the wire wrapping typically installed for the purpose of protecting the cable from degradation will have a positive effect on the bond length. Usually the bond length is taken as the length between the clamps for main suspension cables or as the length between the sockets for stay cables.

In Fig. 3 the mean strength of a cable (per wire) is illustrated as a function of the number of wires for the parameters $u = 1788.7$, $k = 72.62$ and $\lambda = 3$. In the considered case it is seen that the strength reduction amounts to about 6.5%. In general reductions of up to about 8% may be achieved. However, as the mean value of the strength of the cable (per wire) decreases, the standard deviation decreases even faster. Even for a moderate number of wires ($n > 100$) the strength of a parallel wire bundle can be considered to be deterministic. Together the two effects are often referred to as the Daniel's effect. This effect implies that for practical applications the uncertainties influencing the failure probability for static failure modes of wire bundles may be attributed to the loading alone. However a reduction of up to 8% of the strength as measured on the individual wires should be accounted for.

In Fig. 4 the strength of the 100 m long cable with 200 wires is shown as a function of the scale factor.

For the undamaged cable, i.e. $\lambda = 3.0$, the strength of the cable is 12.7 MN. For the corroded cable, i.e. $\lambda = 200$, the strength is 11.9 MN. Again it is important to note that the strength reduction is not due to a reduction of the cross-section of the individual wires but alone due to a reduction of the correlation length for the strength of the wires.

In Fig. 5 it is seen that the difference between the mean value and the mean value plus/minus two standard deviations is small. This demonstrates that the (mean) strength of the cable is subject to very little uncertainty. In fact, the coefficient of variation of the strength of the cable is less than 1.0%.

2.2.2. *Stress–strain relationship for parallel wires*

As mentioned previously the conditions may not be fulfilled, under which the statistical characteristics [Eqs. (10) and (11)] of the ultimate capacity of a parallel wire cable have been derived. This is e.g. the case if the wires in the considered cable have been subject to corrosion degradation or if the cable is subject to bending. In the first case the individual wires may no

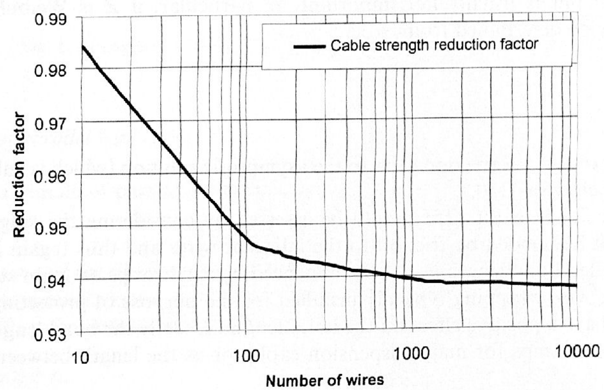

Fig. 3. Illustration of the Daniel's effect, i.e. a systematic reduction of the cable strength (per wire) as function of the number of wires in the cable.

4.1. *Inspection of the number of broken wires*

The inspection of the number of broken wires can be performed by means of various Non Destructive Evaluation methods (NDE), techniques such as e.g. electromagnetic inspections, X-ray and Ultra Sonic inspection or by wedging the cables open and inspecting the wires visually. The different methods each have their strong points and the reader is referred to e.g. Stahl and Gagnon [20] for further details.

The important point in the context of utilising inspection results for the purpose of reliability updating is that the quality of the inspection method applied is appropriately modelled. This means that the inspection reliability in regard to finding defects as well as finding defects which are not there must be quantified. Furthermore given that a defect is found also the precision of the sizing of the defect must be quantified. In this context the different inspection methods may be treated along the same principles.

In the following a procedure is outlined by consideration of Ultra Sonic inspections applied for detecting ruptured wires in stay cables in the immediate vicinity of the cable sockets.

The Ultra Sonic inspection of the individual wires is performed through the bottom and the top anchor sockets of the stays. By the Ultra Sonic inspection a sound wave is transmitted through each of the wires and the reflection of the sound wave is measured. On basis of the measured reflection it can be determined if the wire is ruptured and if it is ruptured also the location of the rupture may be determined.

4.1.1. *Probability of detection*

Any inspection method including Ultra Sonic inspection and hands-on inspection is not able to detect all ruptured wires with probability one. Furthermore, the inspection may indicate that intact wires are ruptured.

To evaluate the reliability of an inspection method it is necessary to determine the probability of detecting a defect given that it exists as well as the probability of detecting a defect given that it does not exist.

The probability of detecting a defect, i.e. a ruptured wire, given it exists is denoted F_D. By the considered Ultra Sonic inspection it may be assumed that F_D depends on the distance from the socket to the location of the fracture. By a visual inspection F_D may be assumed to be a constant.

The probability of detecting a wire rupture by Ultra Sonic (US) inspection given it exists is $F_D(d, \mathbf{p})$ where d is the distance from the socket to the location of the fracture and $\mathbf{p}$ are the parameters to be estimated.

The problem is to estimate the parameters of $F_D(d, \mathbf{p})$ to experimental inspections with observations of the type: detection/no detection. The maximum likelihood method (see e.g. Lindley [16]) is applied for this purpose since it provides the joint probability density $f_{\mathbf{p}}(\mathbf{p})$ i.e. the full information about the statistical uncertainty. Assuming that the individual inspection trials are independent, the likelihood function $L(\mathbf{p})$ has the following form, corresponding to N experimental inspections performed with the distances to the defect, d_i^e, $i = 1, 2, \ldots N$

$$L(\mathbf{p}) = \prod_{i=1}^{N} P_i(\mathbf{p}) \tag{23}$$

where

$$P_i(\mathbf{p}) = \begin{cases} F_D(d_i^e|\mathbf{p}) & \text{if detection} \\ 1 - F_D(d_i^e|\mathbf{p}) & \text{if no detection} \end{cases} \tag{24}$$

The maximum likelihood estimates, $\mathbf{p}^*$, are obtained by solving the optimisation problem

$$\min_{\mathbf{p}} L(\mathbf{p}) \tag{25}$$

For large sample sizes the joint distribution function of the parameters, $\mathbf{p}$, tends to a Normal distribution with expected values $\mu_{\mathbf{p}} = \mathbf{p}^*$ and covariance matrix $\mathbf{C}_{\mathbf{pp}}$ given by (see e.g. [16])

$$\mathbf{C}_{\mathbf{pp}} = [-\mathbf{H}]^{-1} \tag{26}$$

where the elements of the matrix $\mathbf{H}$ are given by

$$H_{ij} = \frac{\partial^2 \ln L(\mathbf{p}^*)}{\partial p_i \partial p_j} \tag{27}$$

To evaluate the goodness of fit of the selected function, $F_D(d, \mathbf{p})$, the robustness of the estimates, $\mathbf{p}^*$ should be examined. As the maximum likelihood estimates are statistically robust the estimates $\mathbf{p}^*$ should at least be insensitive to augmenting the experimental inspection trial sample with one additional sample. If this is not the case, the selected function is not suitable. Otherwise the function $F_D(d, \mathbf{p})$ can now be used for reliability analysis and/or for inspection and maintenance planning.

By a visual inspection of the wires the probability of detection does not depend on the distance from the socket to the location of the rupture, i.e. the probability of detection is constant. Hence, the estimation of the probability of detection can be performed by Bayesian analysis of a Binomial distribution, see e.g. Box and Tiao [21]. It can be shown that the probability of detection follows a β-distribution with density function

$$f_{F_D}(F_D) = \frac{\Gamma(N+1)}{\Gamma(n_d+0.5)\Gamma(N-n_d+0.5)} F_D^{n_d-0.5}(1-F_D)^{N-n_d-0.5} \tag{28}$$

if a nearly non-informative prior distribution is used, i.e.

$$f'_{F_D}(F_D) \propto [F_D(1-F_D)]^{-1/2} \quad 0 < F_D < 1 \tag{29}$$

and where it has been assumed that the individual trials are independent and where N is the number of broken wires and n_d is the number of broken wires detected by the inspection.

Wire fractures will often tend to occur at about the same distance from the socket. This implies that the probability of detecting a wire fracture by a US-inspection may be assumed to be constant. In that case the probability of detecting a defect given that it exists can be determined on the basis of Eq. (28) where N is replaced by the number of wires and n_d by the number of wires which were found to be broken.

5. Condition updating

On the basis of the information obtained from the inspection it is possible to update the distribution of the number of ruptured wires.

Prior to an inspection the probability that a given wire in the cable is ruptured is p' and the probability that a given number of wires is ruptured is given by the Binomial distribution. Let N be a stochastic variable describing the number of ruptured wires and let m be the number of wires in the cable. The probability that the number of ruptured wires is n is given by

$$P(N=n) = \binom{m}{n} p'^n (1-p')^{m-n} \tag{30}$$

The results of the inspection may be used to update the probability that a wire is ruptured. The updating is performed by Bayesian analysis.

The information consists of the number of ruptured wires, which were detected by the inspection. If the probability of detection is constant, i.e. independent of the distance to the fracture, the probability of detecting a given number of ruptured wires by the inspection is also given by a Binomial distribution. This implies that the probability of detecting n_{d} ruptured wires for a given value of F_D in a population of m wires is given by

$$P(N_{\mathrm{d}} = n_{\mathrm{d}}) = \binom{m}{n_{\mathrm{d}}} (F_D p)^{n_{\mathrm{d}}} (1 - F_D p)^{m-n_{\mathrm{d}}} \tag{31}$$

where p denotes the true but unknown probability that a wire is ruptured.

Using Bayes rules the posterior distribution of p for a given F_{D} can be determined as

$$f''_{\mathrm{P}}(p) = \frac{L(n_{\mathrm{d}}|p) f'_{\mathrm{P}}(p)}{\int_0^1 L(n_{\mathrm{d}}|p) f'_{\mathrm{P}}(p) \mathrm{d}p} \tag{32}$$

where $L(n_{\mathrm{d}}|p)$ is the likelihood of observing n_{d} ruptured wires for the given value of p, i.e. the likelihood given in Eq. (31), and where $f'_{\mathrm{P}}(p)$ is the prior distribution of p.

Unless numerical solutions to Eq. (32) are pursued two problems must be addressed. First an appropriate prior probability density function for the probability of wire rupture p must be identified. Secondly the integration of the nominator has to be performed. Starting with the problem of the choice of the prior distribution of p this might be chosen as

$$f'_{\mathrm{P}}(p) \propto [F_D p(1 - F_D p)]^{-1/2} \quad 0 \leqslant p \leqslant 1 \tag{33}$$

which, however, is realized not to be a proper prior in the sense that it does not integrate to one. Further, it is not even a non-informative prior. However, this prior leads to a simple approximate solution for the posterior distribution of p. If the number of wires is large and the number of observed ruptures is small, the influence of the prior is negligible and the prior given in Eq. (33) can be used to determine the distribution of p.

Naturally p cannot be larger than 1. However, if the integration in Eq. (32) is performed from 0 to $\frac{1}{F_D}$ an analytical solution can be determined. By this analytical solution it is found that the posterior of $F_D p$ follows a β-distribution given by

$$f''(F_D p) = \frac{\Gamma(m+1)}{\Gamma(n_\mathrm{d}+0.5)\Gamma(m-n_\mathrm{d}+0.5)}(F_D p)^{n_\mathrm{d}-0.5}(1-F_D p)^{m-n_\mathrm{d}-0.5} \qquad (34)$$

For a given value of F_D the posterior distribution of p can now be evaluated by

$$f''_\mathrm{P}(p) = F_D f''(pF_D) \qquad (35)$$

It is important to notice that the expressions given in Eqs. (34)–(35) are approximations. The distribution of $F_D p$ is defined in the interval $0 \leqslant F_D p \leqslant 1$ implying that there is a probability of obtaining outcomes of p larger than 1. However, if the number of tested wires is large and the unknown probability that a wire is ruptured is small in comparison with F_D the probability of obtaining outcomes of p larger than one is negligible and the approximation outlined above in Eqs. (34) and (35) may be used. If these conditions do not hold the evaluation of the posterior distribution of p [Eq. (32)] may have to be performed by numerical integration.

Once the posterior of p is determined the distribution of the number of ruptured wires can be determined on the basis of the Binomial distribution. However, the distribution must be truncated because it is known that at least n_d wires are ruptured. On this basis the distribution of the number of broken wires in the cable can be determined for given values of p and F_D as

$$P(N = n | N \geqslant n_\mathrm{d}) = \frac{\binom{m}{n} p^n (1-p)^{m-n}}{1 - \sum_{i=0}^{n_\mathrm{d}-1} \binom{m}{i} p^i (1-p)^{m-i}} \qquad n \geqslant n_\mathrm{d} \qquad (36)$$

The uncertainty related to p and F_D may taken into account by integrating Eq. (36) over all possible outcomes of p and F_D.

As an example consider a cable with 200 wires where all the wires are tested by an UT-inspection. It is assumed that all fractures are located roughly at the same distance from the socket, implying that the probability of detection may be assumed to be constant. The inspection reveals that 10 wires are ruptured. Further, it is known that the probability of detecting a ruptured wire by the inspection is 0.867.

The posterior distribution of $F_D p$ may be determined by Eq. (34). Further, the posterior distribution of p is given by

$$f''_\mathrm{P}(p) = 0.867 f''(0.867p) \qquad (37)$$

The posterior distribution of the probability that a given wire in the cable is ruptured, p, is also shown in Fig. 6.

In Fig. 6 it is seen that the probability of obtaining outcomes of p larger than 1 is small, i.e. less than 10^{-100}. In this case, the error made by the approximation is negligible.

In Fig. 7 the posterior distribution of the number of ruptured wires is shown.

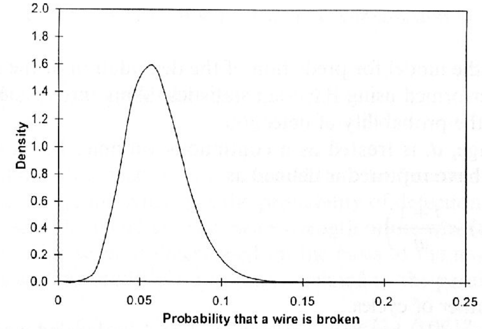

Fig. 6. The probability that a given wire in the cable is ruptured.

The mean number of ruptured wires is 13.68 and the standard deviation of the number of ruptured wires is 3.60. This implies that the coefficient of variation of the number of ruptured wires is 26%. On the other hand the mean number of intact wires is 186.3 and the standard deviation of the number of intact wires is also 3.60. The coefficient of variation of the number of intact wires is 1.93%. This demonstrates that very accurate information about the load-carrying capacity may be obtained on the basis of an inspection. This conclusion is also valid if the uncertainty related to the probability of detection is taken into account. Naturally, if the number of ruptured wires detected by the inspection is large compared to the total number of wires the uncertainty related to the number of intact wires is large. However, in practice cables with a large number of broken wires will be replaced immediately.

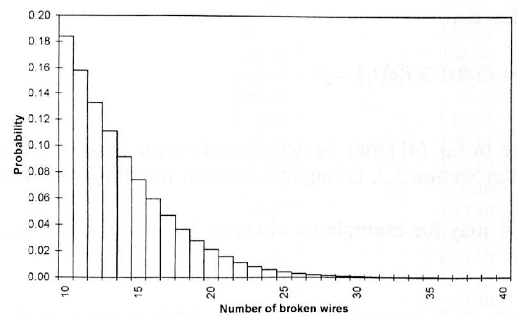

Fig. 7. Distribution of the number of ruptures.

6. Reliability updating

Based on inspections the model for prediction of the degradation of the cable may be updated. The updating can be performed using Bayesian statistics taking into account the accuracy of the inspection method, i.e. the probability of detection.

In Eq. (21) the damage, d, is treated as a continuous variable. For a cable with n wires the probability that i wires have ruptured is defined as

$$P(d(t) = i) = P\left(\frac{i}{n} \leqslant d(t) < \frac{i+1}{n}\right) \tag{38}$$

where t denotes the number of cycles.

If an inspection of the cables has been performed after t_1 cycles and a given observation, $\{O\}$, has been made at this inspection the probability of a given damage occurring at $t > t_1$ may be updated. The observation, $\{O\}$, in this case consists of an inspection of the cable at the time t_1 and the detection of a given number of ruptured wires. The probability that the number of ruptured wires at the time t exceeds the number i may be determined by

$$P(d(t) \geqslant i|O) = \sum_{j=1}^{n} P(d(t) \geqslant i|d(t_1) = j, O)P(d(t_1) = j|O) \tag{39}$$

where the probability that j wires are damaged after t_1 cycles given the observation, O, is given by

$$P(d(t_1) = j|O) = \frac{P(O|d(t_1) = j)P(d(t_1) = j)}{\sum_{k=1}^{n} P(O|d(t_1) = k)P(d(t_1) = k)} \tag{40}$$

Further, the probability $P(d(t) \geqslant i|d(t_1) = j, O)$ must be independent of the observation, $\{O\}$. When the damage at the time t_1 is known, the observation performed at the time t_1 gives no additional information on the basis of which the distribution of the damage at the time $t > t_1$ can be updated. This implies that

$$(d(t) \geqslant i|d(t_1) = j, O) = P(d(t) \geqslant i|d(t_1) = j) \tag{41}$$

The probability given in Eq. (41) may be determined on the basis of the model of the degradation of the cable given in Section 3.2, taking into account the statistical uncertainty related to the model parameters.

The observation, $\{O\}$ may for example be obtained by an US-inspection of the wires. In this case the observation is

$$\{O\} = \{\text{observation of } k \text{ ruptured wires by an US-inspection at the time } t_1\}$$

The probability of observing k ruptured wires by the US-inspection at t_1 given that j wires have ruptured is

$$P(O = k|d(t_1) = j) = \binom{j}{k} P_D^k (1 - P_D)^{j-k} \tag{42}$$

where P_D is the probability of detection. It is assumed that the fractures are located at about the same distance from the socket implying that the probability of detection is constant.

Again consider a cable with 200 wires. The mean strength of the wires is 1789 MPa, see Section 2.2. The deterioration of the wires is determined on the basis of the model given in Section 3.2. The model parameters and the statistical uncertainty related to the parameters are

$$\begin{array}{l} m = 1.50 \\ \alpha = 2.76 \\ K_0 = 1.19 \cdot 10^{10} \end{array} \quad \mathbf{C} = \begin{bmatrix} C_{mm} & C_{m\alpha} & C_{mK_0} \\ C_{m\alpha} & C_{\alpha\alpha} & C_{\alpha K_0} \\ C_{mK_0} & C_{\alpha K_0} & C_{K_0K_0} \end{bmatrix} = \begin{bmatrix} 0.0773 & 0.0718 & 0.416 \\ 0.0718 & 0.0894 & 0.538 \\ 0.416 & 0.538 & 3.25 \end{bmatrix}$$

The cable is subject to $1.5 \cdot 10^6$ stress cycles of magnitude 30 Mpa each year. On the basis of the estimated model parameters the deterioration of the cable can be predicted. After a period of one year a US-inspection of the cable is performed. At the US-inspection no ruptured wires are detected. The probability of detecting a ruptured wire is modelled by a β-distribution with mean 0.865 and standard deviation 0.0196.

Based on the information that no ruptured wires have been detected the probability that a given number of ruptured wires exists after the US-inspection can now be determined according to Eq. (36). The updated probability of a given number of ruptured wires after t_1 cycles is given in Table 2. The prior probability, $P(d(t_1) = j)$, also given in Table 2 has been determined for $t_1 = 1.5 \cdot 10^6$ (1 year) by Monte Carlo simulation taking the statistical uncertainty related to the model parameters.

Table 2
Probability of a given damage after US-inspection

Number of broken wires after t_1 cycles, j	$P(O\|d(t_1) = j)$	$P(d(t_1) = j)$	$P(d(t_1) = j\|O)$
0	1.0	$2.75 \cdot 10^{-2}$	$8.80 \cdot 10^{-1}$
1	$1.35 \cdot 10^{-1}$	$2.46 \cdot 10^{-2}$	$1.06 \cdot 10^{-1}$
2	$1.85 \cdot 10^{-2}$	$2.08 \cdot 10^{-2}$	$1.23 \cdot 10^{-2}$
3	$2.59 \cdot 10^{-3}$	$1.73 \cdot 10^{-2}$	$1.43 \cdot 10^{-3}$
4	$3.71 \cdot 10^{-4}$	$1.63 \cdot 10^{-2}$	$1.93 \cdot 10^{-4}$
5	$5.41 \cdot 10^{-5}$	$1.50 \cdot 10^{-2}$	$2.60 \cdot 10^{-5}$
6	$8.05 \cdot 10^{-6}$	$1.32 \cdot 10^{-2}$	$3.49 \cdot 10^{-6}$
7	$1.21 \cdot 10^{-6}$	$1.20 \cdot 10^{-2}$	$4.65 \cdot 10^{-7}$
8	$1.88 \cdot 10^{-7}$	$1.17 \cdot 10^{-2}$	$7.04 \cdot 10^{-8}$
9	$2.95 \cdot 10^{-8}$	$1.09 \cdot 10^{-2}$	$1.03 \cdot 10^{-8}$
10	$4.71 \cdot 10^{-9}$	$9.26 \cdot 10^{-3}$	$1.40 \cdot 10^{-9}$
11	$7.65 \cdot 10^{-10}$	$8.89 \cdot 10^{-3}$	$2.18 \cdot 10^{-10}$
12	$1.26 \cdot 10^{-10}$	$9.23 \cdot 10^{-3}$	$3.72 \cdot 10^{-11}$
13	$2.12 \cdot 10^{-11}$	$8.28 \cdot 10^{-3}$	$5.62 \cdot 10^{-12}$

Now the probability that a given number of wires are ruptured 6 months after the US-inspection can be determined. In Table 3 the conditional probability, $P(d(t) \geq i | d(t_1) = j)$ is given for $t = 2.25 \cdot 10^6$ ($1\frac{1}{2}$ years). The value of i has been chosen as $i = 20$, i.e. the relative damage is 0.10. These probabilities are determined in the same way as the probabilities $P(d(t_i) = i)$ above.

Using Eq. (36) together with the results in Tables 2 and 3 the probability that the damage is larger than 0.10 6 months after the last US-inspection can now be found to be $3.78 \cdot 10^{-8}$.

7. Issues relating to design and assessment

For parallel wire cables, such as cables in suspension bridges and cable stayed bridges, a format for design and assessment taking into account deterioration may differentiate between the assumed deterioration mechanism, preventive/mitigating measures, maintenance strategies, years in service and service life in addition to the usual classifications.

7.1. Safety of cables assuming no deterioration

The static strength of cables prior to deterioration R_0 can be shown to exhibit a very small or even negligible uncertainty due to the normally large number of wires in the cables. This is discussed in Section 2.2 where it is shown that the coefficient of variation of the breaking strength of parallel wire cables is normally well below 1%. However, due to the length effect and due to the Daniels effect (wires in parallel) the mean breaking stress of a cable is reduced deterministically by a factor 0.9 and 0.92, respectively, as compared to the mean breaking strength of the individual wires as assessed by testing of standard specimens.

The safety factor for the cable strength, assuming no deterioration, may be evaluated by

$$\gamma_r = \frac{r_k}{0.9 \times 0.92 \times \mu_r \exp(\alpha \ \beta \ V_r)} \tag{43}$$

where r_k is the characteristic or nominal wire breaking strength, μ_r is the mean wire breaking strength, α is the sensitivity factor for the resistance, β is the target safety index and V_r is the coefficient of variation of the wire breaking strength related to the between batch variability of the mean wire breaking strength.

Table 3
Conditional probability of obtaining a given damage

Number of ruptured wires after t_1 cycles, j	$P(d(t) \geq 20 \| d(t_1) = j)$
5	$6.94 \cdot 10^{-5}$
6	$1.52 \cdot 10^{-3}$
7	$1.83 \cdot 10^{-2}$
8	$1.94 \cdot 10^{-1}$
9	$8.46 \cdot 10^{-1}$
10	1.0

As an example consider a case where the mean breaking strength μ_r is 1850 MPa and the coefficient of variation is 2%. If β is set to 3.4, α conservatively assumed to be equal to 1 and the characteristic value r_k defined as the 5% percentile value of the probability distribution function of the wire breaking strength, the cable strength safety factor is found to be equal to 1.25. This is significantly below the commonly assumed values around 1.7–2.0. The difference in strength safety factors allows for deterioration corresponding to 27% of the cable cross section.

7.2. Safety of cables subject to deterioration

The partial safety factor for cables subject to deterioration must be determined such that the cables have a sufficient level of safety even when some damage has occurred. Let the damage which occurs in the period of time from $t = 0$ to $t = t$ be denoted $d(t)$.

For moderate degradation or large cable cross sections the partial safety factor given by

$$\gamma = \frac{\gamma_0}{1 - d(t)} \tag{44}$$

ensures a sufficient level of safety in the considered period of time if γ_0 ensures a sufficient level of safety at the time when no damage has occurred.

Normally the governing deterioration mechanisms for cables are fatigue and corrosion or the combined effect of these.

Following the model given in Section 3 the evaluation of the service life or residual service life of parallel wire cables takes basis in the estimation of the probability distribution function of the lifetime of a single wire. The effect of corrosion on the fatigue lives may be included by fitting the model parameters on the basis of experiments performed on wire specimens, which have been subjected to corrosion.

The degradation function $g(t)$ for the cable strength subject to fatigue deterioration may thus be written as

$$g(t) = 1 - d(n(t)) \tag{45}$$

If the static loads acting on the cable and the yield stress of the cable is known it is possible to determine the probability that the cable fails in a given period of time, $t \in [0, T]$ where t denotes time and T the length of the considered period

$$P_f(T) = 1 - P(R_0 \times g(n(t)) \leq S(t) \ \forall \ t \in [0; T]) \tag{46}$$

where R_0 is the load bearing capacity of the undamaged cable and $S(t)$ is the time-variant load acting on the cable.

The degradation functions corresponding to the two different situations are shown in Figs. 8 and 9 for typical load conditions for main cables and hanger and stay cables, respectively. The degradation functions shown in Figs. 8 and 9 are determined on the basis of parameters estimated by Rackwitz and Faber [13], see also Section 3.2.

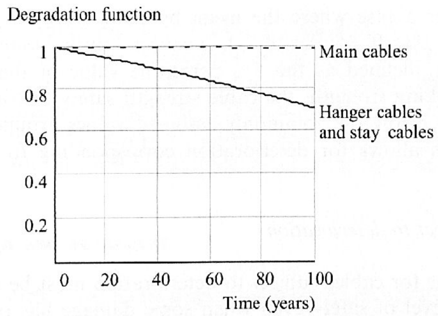

Fig. 8. Typical degradation functions for main cables, hanger cables and stay cables for non-corroded wires.

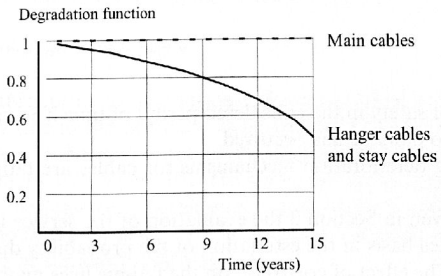

Fig. 9. Typical degradation functions for main cables, hanger cables and stay cables for corroded wires.

For cables subjected to live loads inducing stress ranges and extreme stresses below the endurance limit of $r_0 = 200$ MPa fatigue will in general not take place. A condition for an endurance limit is, however, that damages of the wire surfaces, due to corrosion or handling of the wires are avoided.

In order to investigate the effect of fatigue deterioration on the required cable resistance a series of reliability analysis have been performed for cables subjected to the combined effect of live loads L and dead loads D. It is assumed that the endurance limit has been diminished due to e.g. initiating fatigue or excessive loads and as a consequence fatigue deterioration is initiated. The analysis have been performed for three different ratios between the characteristic values of L and D, namely $L_k/D_k = 0.2$, 0.25 and 0.3 where the characteristic value of the dead load and the live load are defined as the 50 and the 98% fractile, respectively. This covers most situations encountered for cables in cable stayed bridges and suspension bridges. Furthermore, the analyses have been performed for different ratio's κ between the fatigue loading D_S and the mean live load, namely 0.05 and 0.10. In the case of $\kappa = 0.05$ which resembles the typical fatigue loading situation for main cables on suspension bridges it has further been assumed that the mean number of stress cycles is $5 \cdot 10^5$ cycles per year. For $\kappa = 0.1$, corresponding to the stress situation for hanger cables on suspension bridges and stay cables on cable stayed bridges, the mean number of stress cycles has been set to $5 \cdot 10^6$ per year. It has been assumed that the service life of the cables is 100 years and the service life target safety index is $\beta = 3.4$.

The results of the analysis are shown in Fig. 11 where the factor by which the resistance corresponding to an optimal design taking into account the effect of fatigue deterioration may be reduced if by some means the deterioration can be avoided.

From Fig. 10 it is also seen that rather significant reductions of the design cable cross section can be justified if it is ensured that deterioration is avoided. For situations resembling hanger and stay cables reductions of the design cable cross section of up to 40% cable can be justified. It is, however, also seen that for situations resembling main cables, no reduction is justified. This is due to the fact that even if fatigue is initiated, the stress situation is of such a nature that the fatigue deterioration will be very modest within the design service life of the cable.

It is interesting to notice that the reduction factor for typical cases of $\rho = 0.25$–0.3 for hanger and stay cables is in the range of 20–40%, i.e. in the same range as the deterioration of 27% implicitly allowed for in current cable design practice.

In Fig. 11 the effect of the design service life is illustrated for the case of $\kappa = 0.1$.

As a consequence of the results shown in Figs. 10 and 11, a differentiation of the required resistance safety factor may be justified for hanger and stay cables depending on their design

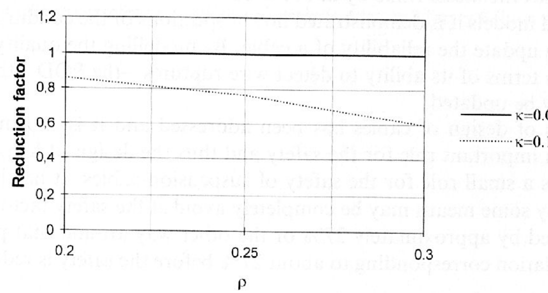

Fig. 10. Reduction factor which may be applied on the design resistance if deterioration can be avoided.

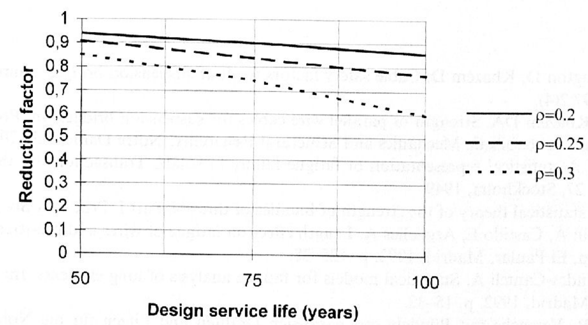

Fig. 11. Reduction factor for different design service lives and ratio of live to dead load, for the case $\kappa = 0.1$.

service life and whether or not deterioration will be avoided. The differentiation furthermore shall depend on the ratio between the live and the dead load component as well as the ratio between the fatigue load to mean extreme live load.

8. Conclusions

The theoretical basis for assessing the safety of parallel wire cables has been given considering the safety in regard to both the ultimate capacity as well as the effects of fatigue and corrosion. It has been shown how the presented probabilistic models may be adapted to experiments results obtained by testing standard wire specimens in the laboratory.

Based on the presented models it is found that the strength of a parallel wire cable may effectively be treated as deterministic, if the statistical uncertainty of the model parameters used for describing the model is small. This may be achieved if a sufficiently large number of tests are performed.

The effect of the length of the considered cable on the safety of the cable can be appropriately accounted for by a 10% reduction of the mean strength established by testing of a standard wire test specimen. The Daniels effect, i.e. the effect of having a large number of wires working in parallel further reduces the mean value by about 8%.

Using the presented models it is demonstrated how inspections of the condition of the wires of a cable may be used to update the reliability of a cable. By modelling the quality of the performed inspection method in terms of its ability to detect wire ruptures—the POD—the reliability of the cable as a whole may be updated.

Finally the aspects of design of cables has been addressed and it is shown that the effect of degradation plays an important role for the safety and thus the design of hanger and stay cables whereas it only plays a small role for the safety of suspension cables. It has been demonstrated that if degradation by some means may be completely avoided the safety factors for parallel wire cables may be reduced by approximately 27% or the other way around that parallel wire cables may sustain a degradation corresponding to about 27% before the safety is reduced below what is normally accepted.

References

[1] Haight RO, Billington D, Khazem D. Cable safety factors for four suspension bridges. Journal of Bridge Engineering, ASCE 1997;2(4).

[2] Fu G, Moses F, Khazem DA. Strength of parallel wire cables for suspension bridges. In: Proc. 8th ASCE Speciality Conference on Probabilistic Mechanics and Structural Reliability, Notre Dame, July 2000.

[3] Weibull Wallodi, A statistical representation of fatigue failure in solids. Transactions on the Royal Institute of Technology, No. 27, Stockholm, 1949.

[4] Daniels HE. The statistical theory of the strength of bundles of threads, Part I. Proc Roy Soc, A 1945;183:405–35.

[5] Fernandes-Cantelli A, Castillo E, Argüelles A. Length effect on fatigue of wires and pre-stressing steels. In: Proc. IABSE Workshop, El Paular, Madrid, 1992. p. 125–38.

[6] Castillo E, Fernandes-Canteli A. Statistical models for fatigue analysis of long elements. In: Proc. IABSE Workshop, El Paular, Madrid, 1992. p. 15–32.

[7] Andrä W, Saul R. Versuche mit Bündeln aus parallelen Drähten und Litzen für die Nordbrücke Mannheim-Ludwigshafen und das Zeltdach in München. Bautechnik 1974;289–97, 332–40, 371–3.

[8] Castillo E, Fernandes-Canteli A, Esslinger V, Thürlimann B. Statistical model for fatigue analysis of wires, strands and cables. Int. Assoc. for Bridge and Struc. Engrg. (IABSE). Proc., P. 82/85, 1985.

[9] Matteo J, Deodatis G, Billington DP. Safety analysis of suspension-bridge cables: Williamsburg Bridge. Journal of Structural Engineering 1994;120(11):3197–210.

[10] Freudenthal A, Gumbel EJ. On the statistical interpretation of fatigue tests. Proceedings of the Royal Society of London UK, series A 1953;216:309–31.

[11] Bolotin VV. Initial flaws distribution and size effect of fatigue fracture. Length Effect on Fatigue of Wires and Strands, IABSE Workshop, El Paular, Madrid, Spain, 1992. p. 75–88.

[12] Matsukawa A, Kamei M, Mizoguchi T, Sasaki Y. Fatigue resistance analysis of parallel wire strand cables based on statistical theory of extremes. Stahlbau 1988;57(7):205–10.

[13] Rackwitz R, Faber MH. Reliability of parallel wire cable under fatigue. Proceedings of the ICASP6 Conference, Mexico 1991;1:166–75.

[14] Colemann BD. On the strength of classical fibres and fibre bundles. J Mech Phys Solids 1958;7:60.

[15] Phoenix SL. Stochastic strength and fatigue of fiber bundles. Int J Fracture 1978;14(3):327–44.

[16] Lindley DV. Introduction to probability and statistics from a Bayesian viewpoint, vols. 1 and 2. Cambridge: Cambridge University Press; 1976.

[17] Daniels HE. The maximum size of a closed epidemic. Adv Appl Prob 1974;6:607–21.

[18] Gollwitzer S, Rackwitz R. On the reliability of Daniels systems. Structural Safety 1990;7:229–43.

[19] Guers F, Rackwitz R. Time-variant reliability of structural systems subject to fatigue. Proc ICASP 5, Vancouver, May 1987;vol. I:497–505.

[20] Stahl FL, Gagnon CC. Cable corrosion in bridges and other structures. ASCE Press; 1995.

[21] Box GEP, Tiao GC. Bayesian inference in statistical analysis. Classics Library Edition. Wiley; 1992.