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PERMAS-RA/STRUREL system of programs for probabilistic reliability analysis

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Abstract

Programs for reliability analysis of structural, operational and other systems based on first- and second-order reliability concepts were made available as early as 1976 by the Technical University of Munich. In the meantime and since 1987 by RCP Consult GmbH (RCP) the programs have experienced many revisions, improvements and additional developments. The programs now cover the preparatory steps as well as the computational tasks in technical reliability, decision making under uncertainty and in statistical analysis. Important modules of STRUREL have also been embedded in the finite element program PERMAS developed and maintained by INTES GmbH (INTES). The programs have been used in structural engineering and code making, in hydrology, operations research, financial planning and mathematical statistics and, in particular, in the nuclear power plant, offshore, ship, automotive and aerospace industry.

Keywords: Structural reliability; Reliability; Probability; Risk analysis; Optimization; Statistical analysis

1. Introduction

STRUREL is a set of programs for the reliability analysis of structural, operational and other systems employing state-of-the-art techniques:

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- *Comrel* – a program for time-invariant and time-variant component reliability analysis,
- *Sysrel* – a program for system reliability analysis including reliability updating,
- *Costrel* – a program for reliability oriented optimization,
- *Statrel* – a program for reliability oriented statistical analysis and simulation.

All programs share a rich set of distribution functions (at present 44 different stochastic models) and provide means to model correlations and also arbitrary dependence structures between the basic random variables. Alternatively and less generally, the so-called Nataf and Hermite model can be used where stochastic dependencies are specified by marginal distributions and correlation matrices.

All programs are available for the Windows platform as 32-Bit applications with a Graphical User Interface (GUI) providing user friendliness, graphical pre- and post-processing and rich online help.

- PERMAS is a general purpose FE-Program (INTES) with *Comrel* and *Costrel* embedded within the program module PERMAS-RA.

PERMAS and PERMAS-RA are available for many Unix platforms and for Linux as well as for the Windows platform. With PERMAS-FEPRE a GUI is available that supports interactive stochastic modeling on top of the imported finite element model.

2. The programs

2.1. Comrel

Comrel [1] comprises the modules *Comrel-TI* for time-invariant and *Comrel-TV* for time-variant reliability analysis of individual failure modes based on advanced FORM/SORM methodology and Monte Carlo techniques (FORM/SORM: First/Second Order Reliability Method [2]). The computational operations are performed in the so called standard space (U-space), the space of independent and standard normal variables. The necessary transformation from basic space (X-space) $\mathbf{U} = \mathbf{T}(\mathbf{X})$ and back $\mathbf{X} = \mathbf{T}^{-1}(\mathbf{U})$ is done internally so that the interface to the state functions $g_l(\mathbf{X})$ always is in the space of basic random variables. Deterministic parameters may also included in $g_l(\mathbf{X})$. By convention the failure domain is defined as the set $\{F\} = \{g(\mathbf{X}) \leqslant 0\}$. The limit state surface $g(\mathbf{X}) = 0$ separates failure and survival domain.

For the *time-invariant case* several algorithms on different theoretical bases to find the most likely failure point (β-point) are implemented including a gradient free algorithm for non-differentiable failure criteria (state functions). Complementary or alternative computational options are mean value first order (MVFO), crude Monte Carlo simulation, adaptive sampling, spherical sampling and several importance sampling schemes. FORM/SORM techniques allow to compute a rich set of sensitivity measures showing the impact on reliability of individual basic random variables, of distribution parameters and of other constant parameters. Provided characteristic values are specified partial safety factors for all basic variables are another straightforward result.

Time-variant reliability is computed by the outcrossing approach also based on FORM/SORM methodology for stationary or non-stationary cases [3]. Available random process models are regular or intermittent rectangular wave processes, differentiable Gaussian and non-Gaussian translation processes (Hermite or Nataf processes). The models can be scalar processes and vector processes. All random process types can be combined with each other including the possibility to combine intermittent and non-intermittent processes [4]. Sensitivity measures and partial safety factors are provided similar to time-invariant analysis. In addition, various other exceedance measures like excursion time, hazard rate, point-in-time non-availability are evaluated.

2.2. Sysrel

Sysrel [1] covers time-invariant system reliability evaluation including event updating. It offers the same features of stochastic modeling as Comrel. The theoretical basis is in turn given in [2]. In the GUI of Sysrel the logical model is connected with the failure criteria and the stochastic model (basic random variables) for fully interactive control. System modeling includes not only the representation by a (minimal) set of parallel systems in series as well as pure series and parallel systems but also the important case of conditional events including criteria given as equality constraints (observations, event updating).

For the FORM/SORM methods Sysrel is based on several efficient and reliable algorithms searching for the β-point(s) (multi-constraint optimization) with special solution strategies. An alternative computational option is crude Monte Carlo simulation.

As Comrel also Sysrel works in the standard space (U-space) with built-in **U** to **X** transformation for the interface to the state functions. Sysrel contains an efficient computation scheme for the multinormal integral needed to evaluate the probability content of intersections and unions of failure events.

2.3. Costrel

Costrel [1] is the first integrated solution for reliability-oriented optimization. It combines the rich features of stochastic modeling (Comrel, Sysrel) with two alternative one-level gradient based optimizers [5]. Both performance- (cost-, weight-, volume-) optimization under a reliability constraint and reliability optimization under a performance constraint are possible. For both types of problems an arbitrary number of additional constraints (stress or deformation constraints, etc.) can be included. The approach is based on FORM/SORM techniques providing a rich set of sensitivity measures for better interpretation of results. At present time-invariant components and series systems can be dealt with.

Costrel has the same interface as Comrel and Sysrel to the state functions enhanced by the vector of design variables (cost variables) **p**, so we have $g_l(\mathbf{X}, \mathbf{p})$. In addition there is an interface to the initial and failure cost functions $C_i(\mathbf{p})$ and $C_f(\mathbf{p})$ as well as to additional design constraints $h_k(\mathbf{p})$.

2.4. Two versions of Comrel, Sysrel and Costrel

For these Windows based programs two versions with identical capabilities are available, one for the professional user requiring the state functions to be written in Fortran 90 and subsequent

(GUI driven) compiling and linking, and the other having a powerful *Symbolic Processor* with many predefined functions.

The *Professional* version may be necessary for complicated iterative state functions or when external programs have to be linked to the state functions. Application of the *Symbolic* version is very easy. State functions can be specified in normal mathematical notation. Names for variables and parameters can be chosen freely and are automatically transferred into the stochastic model and vice versa. Important constants are predefined. Built-in functions include all elementary, trigonometric, hyperbolic, logarithmic and some special functions like the Gaussian distribution function and its inverse, Bessel and Gamma functions. Several alternatives for numerical integration, differentiation and root finding are available as well as comparative operators and test functions. Auxiliary user defined functions and reference functions can be defined and used similar to subroutines in the professional version.

2.5. Statrel

For all models included in other STRUREL modules Statrel performs parameter estimation by different methods, confidence interval and quantile estimation as well as hypothesis testing including tests for sample validity, distribution functions and parameters. Simple analyses of variance and regression are also included. Several Bayesian procedures like posterior parameter estimation are implemented.

Results are made visible in terms of numerous graphical representations such as histograms, cumulative frequencies, bivariate plots, bar charts, probability paper plots. Import of data from spreadsheet programs like Excel is possible.

As an add-on Statrel includes schemes for simulation of random numbers, random vectors and random time series (stationary, non-stationary, Gaussian, Hermite) to allow numerical experiments. Different generation techniques such as ARMA, simple or fast Fourier transforms can be used. Many predefined spectra can be selected and various forms of non-stationarity can be defined. The implemented features for time series analysis provide the necessary tools to set up and to test models. The features include graphical representations of the time series such as histograms, probability paper plots, scatter diagrams, moving averages, Husid functions, mean value crossings and periodograms. Transformation and filter techniques including trend removals, smoothing, differentiation and integration, amplitude and frequency modulation are provided. Non-parametric spectral estimation and ARMA modeling as well as amplitude and frequency trend estimation are available.

2.6. Permas

2.6.1. General

PERMAS [6,7] is a general purpose finite element program maintained and distributed by INTES. It offers analysis capabilities for

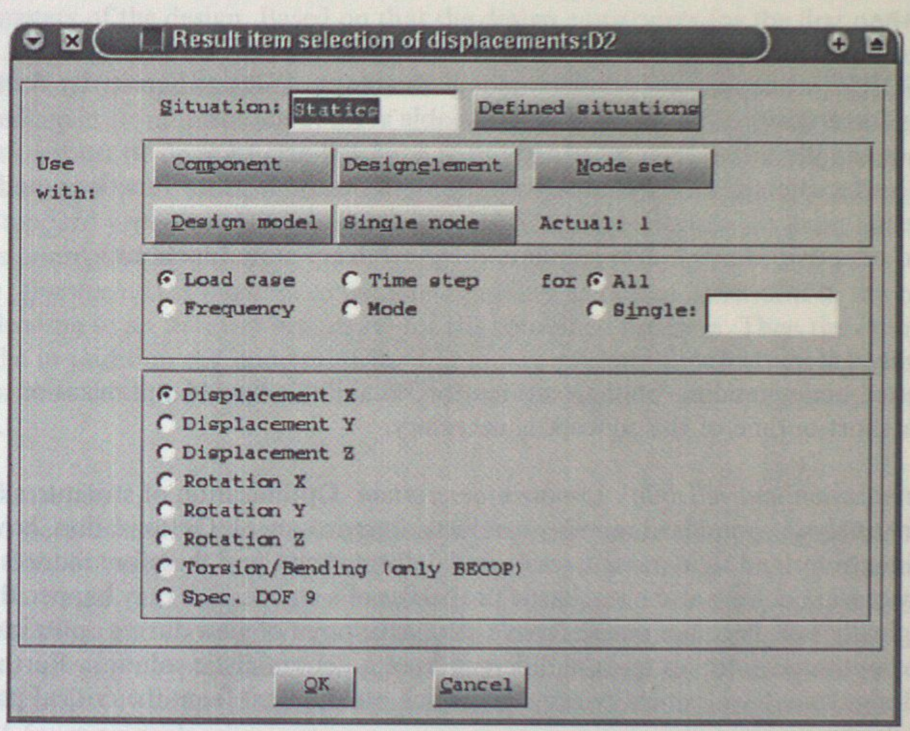
- statics (linear/nonlinear(geometrical and material)/contact),
- dynamics (Eigenvalues, time and frequency domain),
- fluid-structure acoustics, optimization (sizing/shape/topology),
- heat transfer,

- electromagnetics,
- laminate analysis.

Some features are highlighted below. A more comprehensive description is available with the PERMAS short description at www.intes.de.

This program was coupled to STRUREL programs in several steps, the last being a joint EU-project in which the computational kernels of *Comrel* and *Costrel* were fully integrated into PERMAS-RA (which computes sensitivities). This is a unique approach to change from a deterministic to a stochastic model within the finite element code.

Furthermore, a GUI-support for the assignment of the stochastic model (basic random variables) to random properties in the FE-model was developed. The functionality also covers the selection of responses as shown in the subsequent dialog for displacement quantities.



The GUI also supports display of results from the reliability analysis as well as e.g. history plots from reliability-oriented optimization or from Monte Carlo simulation. So, a visual feedback for the definition of variables, as well as for responses enables a thorough checking process. This feature is of specific value, if complex numerical models are being addressed.

The following parameters of the finite element model can be linked to the stochastic model:

- design variables for element properties, coordinates and material data,
- load factors,

- parameters of the state function, e.g. limit values for stresses, etc.,
- parameters of the probability distribution function of another basic variable to model arbitrary dependence structures.

In the state functions one has access to:

- all kind of results from the FE-analysis like displacements, stresses, etc.,
- basic random variables and constant parameters,
- predefined functions thereof.

2.6.2. Reliability analysis

For reliability analysis the following methods from Comrel (time-invariant case) are integrated into PERMAS-RA:

- FORM/SORM with a special optimizer taking advantage of Design Sensitivity Analysis (DSA) in PERMAS for gradient evaluation of differentiable state functions,
- importance sampling on top of FORM/SORM to check accuracy,
- response surface method (RSM) to cover non-differentiable state functions and non-linear FE-analysis,
- crude and adaptive Monte Carlo simulation for arbitrary state functions.

2.6.3. Optimization and reliability

For a better understanding of the features for reliability-oriented optimization available in PERMAS-RA a short outline of the concept is necessary.

2.6.3.1. Optimization and reliability: problem description. Optimization of structures by means of finite element analysis is standard in today's industrial environment. Optimization, however, tends to reduce dimensions and to increase stresses and deformations, and therefore reduces safety margins in many cases. Looking at the stochastic properties of a structure, it may happen that a design, which is originally safe, becomes unsafe from a stochastic point of view during optimization. A reliability-based optimum solution may be different from a deterministic solution. Furthermore, the optimized design may have critical parameters which are different from the critical parameters of the original design.

2.6.3.2. Optimization and reliability: design state and failure state. For the design optimization, a design model is established which defines the design variables and their relation to properties of the model, the objective function and the design and side constraints. The values of the design variables define the design state of the structure. The optimization process leads to a continuous sequence of design states where the last one fulfills all constraints and minimizes or maximizes the design objective. Due to the stochastic properties of the model, to each design state there exist a corresponding failure state, the state of the structure with the highest probability of failure. To determine the probability of failure, a stochastic model is needed which defines the stochastic properties of the basic variables and

the failure condition. Design model and stochastic model are not independent from each other. For example, the optimization process modifies the thickness of a plate, the probability distribution of the corresponding basic variable is shifted, i.e., the mean value of the basic variable is modified by the optimization whereas standard deviation, etc., can be kept constant.

2.6.3.3. Optimization and reliability: solution methods. Combination of optimization and reliability analysis may be regarded as an optimization with reliability as design constraint. The determination of the reliability, however, is an expensive task by its own, and it is rather difficult to include it into the standard optimization process. In PERMAS-RA, there are two different approaches available which are called *Two Step Approach* and *One Step Approach*.

- *Two Step Approach*: This approach uses separate procedures for design optimization and reliability analysis. The reliability analysis of the basic design delivers some information about critical parameters of the design. Based on that the design constraints for the first optimization are selected. After optimization, the probability of failure is determined for the optimized structure. If not sufficient, the optimization is repeated with modified constraints where the modifications are derived again from the importance of the stochastic properties for the failure behavior. The modification must be done by hand and the combination of one optimization and one reliability analysis per step is repeated until the optimized design has a sufficiently low probability of failure. Standard procedures for optimization and reliability analysis are used, and the iterative process is controlled by the user.
- *One Step Approach*: It is possible to include reliability as design constraint in the optimization process, leading to an iterative procedure for the combined problem. Then the optimized structure fulfills in addition the constraint for the failure probability. Such an optimization algorithm has been developed by RCP and TUM and the method was integrated in PERMAS-RA.

2.6.4. Parallelization

A general parallelization approach enables high performance computing of all time critical operations. This is based on a state-of-the-art *fine grain parallelization* that is efficient especially for derivative based reliability analysis methods. The demand on computing power for these methods is in about the same order as for a deterministic structural optimization.

Moreover, *coarse grain parallelization* (distributed computing) has been developed in the scope of a research project to support the handling of independent calculations for Monte Carlo methods or RSM function evaluations.

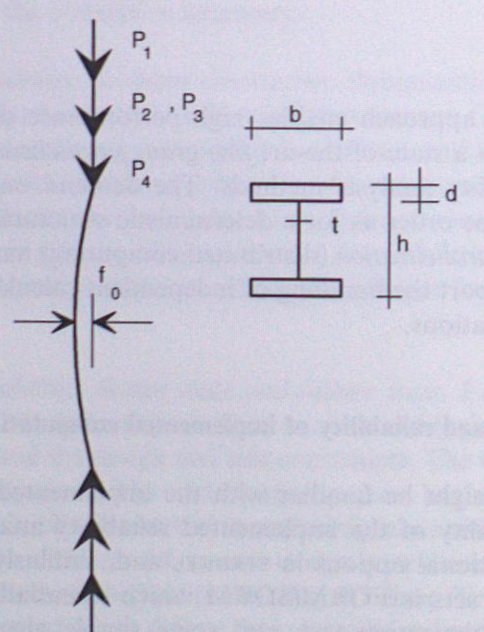
3. Remarks about accuracy and reliability of implemented computational methods

Although most readers might be familiar with the implemented methods some few remarks on the accuracy and reliability of the implemented reliability analysis tools may motivate the presence of many computational options in STRUREL and, similarly, in PERMAS-RA. The theoretical basis of both program sets is FORM/SORM which essentially reduces the task of probability integration to an optimization task and some simple algebra. FORM/SORM provide approximate but usually accurate probability estimates – SORM being even asymptotically exact [2] – given that unique most likely failure point(s) has (have) been found by optimization. But it is well known that many optimization algorithms exist, all with their merits and disadvantages with respect to reliability, efficiency and robustness. Therefore, several alternative algorithms are provided which should enable the user to find a solution if one or the other algorithm fails. In order to check accuracy several Monte Carlo methods which in the limit for an infinite sample size deliver exact results are also implemented. Crude Monte Carlo – in difference to adaptive sampling schemes – even works in case of multiple most likely failure points but, usually, is far too expensive.

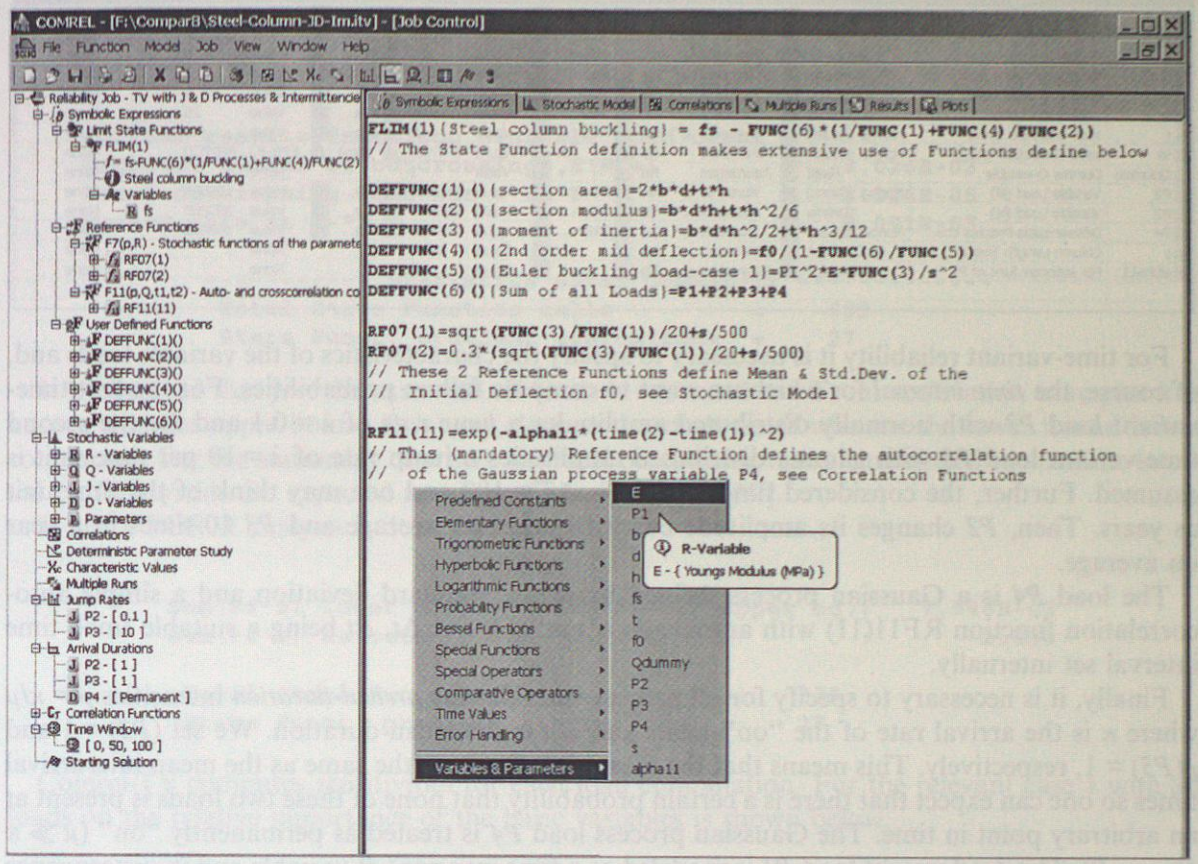
4. Examples

4.1. Instability of a steel column (time-variant analysis)

As an example for *Comrel-TV* the computation of the probability of instability of a steel column with I-shaped cross-section is demonstrated. Out of the four loads the dead weight *P1* is time-invariant, two loads (*P2*, *P3*) are intermittent rectangular wave renewal processes (*J-Variables*) and the load *P4* is modeled as a differentiable (Gaussian) process (*D-Variable*). The problem is stationary, i.e., all uncertain resistance variables are time-invariant and the state function does not explicitly depend on time.



4.1.1. State function

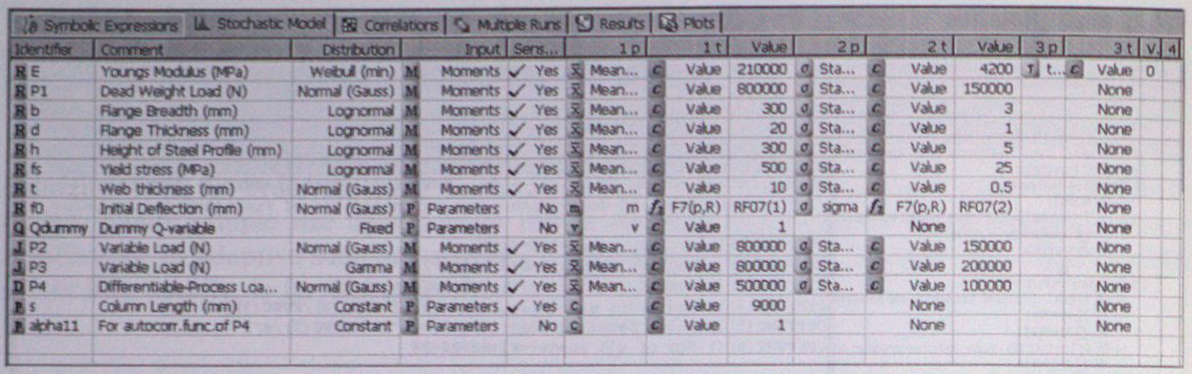


The state function definition in the symbolic notion of Comrel is in terms of **keywords** and *names* of the basic variables and deterministic parameters. The *names* are the link to the stochastic model where the properties (distribution type, mean and standard deviation, etc.) are defined. User defined functions allow a structured programming. The reference functions defined here are used in the stochastic model. The screenshot above also shows access to the variables and parameters (and to built in functions).

This example is a rather critical one because its state function has a singularity for very large loads and/or large column length (see the denominator in user defined function 4).

4.1.2. Stochastic model

The following table exported from Comrel provides an overview. The normal distribution for f_0 is specified by assigning reference functions (RF07(1) and RF07(2)) to parameter 1 (*m*) and parameter 2 (*sigma*). The shift parameter (τ) for the Weibull distributed modulus E is set to 0. All variables are independent.



Symbolic Expressions | Stochastic Model | Correlations | Multiple Runs | Results | Plots

Identifier	Comment	Distribution	Input	Sens...	1 p	1 t	Value	2 p	2 t	Value	3 p	3 t	V	4
R E	Youngs Modulus (MPa)	Weibull (min)	Moments	✓ Yes	Mean...	Value	210000	Sta...	Value	4200	t...	Value	0	
R P1	Dead Weight Load (N)	Normal (Gauss)	Moments	✓ Yes	Mean...	Value	800000	Sta...	Value	150000		None		
R b	Flange Breadth (mm)	Lognormal	Moments	✓ Yes	Mean...	Value	300	Sta...	Value	3		None		
R d	Flange Thickness (mm)	Lognormal	Moments	✓ Yes	Mean...	Value	20	Sta...	Value	1		None		
R h	Height of Steel Profile (mm)	Lognormal	Moments	✓ Yes	Mean...	Value	300	Sta...	Value	5		None		
R fs	Yield stress (MPa)	Lognormal	Moments	✓ Yes	Mean...	Value	500	Sta...	Value	25		None		
R t	Web thickness (mm)	Normal (Gauss)	Moments	✓ Yes	Mean...	Value	10	Sta...	Value	0.5		None		
R f0	Initial Deflection (mm)	Normal (Gauss)	Parameters	No	m	F7(p,R)	RF07(1)	sigma	F7(p,R)	RF07(2)		None		
Q Qdummy	Dummy Q-variable	Fixed	Parameters	No	v	Value	1		None			None		
J P2	Variable Load (N)	Normal (Gauss)	Moments	✓ Yes	Mean...	Value	800000	Sta...	Value	150000		None		
J P3	Variable Load (N)	Gamma	Moments	✓ Yes	Mean...	Value	800000	Sta...	Value	200000		None		
D P4	Differentiable-Process Loa...	Normal (Gauss)	Moments	✓ Yes	Mean...	Value	500000	Sta...	Value	100000		None		
P s	Column Length (mm)	Constant	Parameters	✓ Yes		Value	9000		None			None		
P alpha11	For autocorr.func.of P4	Constant	Parameters	No		Value	1		None			None		

For time-variant reliability it is necessary to specify the characteristics of the variable loads and, of course, the *time interval* for which we want to compute failure probabilities. For the first time-variant load *P2* with normally distributed amplitudes a *jump rate* of $\lambda = 0.1$ and for the second time-variant load *P3* with gamma distributed amplitudes a jump rate of $\lambda = 10$ per time unit is assumed. Further, the considered time interval is $\Delta T = 100$ and one may think of the time unit as years. Then, *P2* changes its amplitude every 10 years on average and *P3* 10 times in a year on average.

The load *P4* is a Gaussian process defined by mean, standard deviation and a simple autocorrelation function RF11(11) with arguments t_1 and $t_2 = t_1 + \Delta t$, Δt being a suitable small time interval set internally.

Finally, it is necessary to specify for all process variables the *arrival-duration* intensities $\rho = \kappa/\mu$ where κ is the arrival rate of the "on"-times and $1/\mu$ is the mean duration. We set $(P2) = 1$ and $\rho(P3) = 1$, respectively. This means that the mean durations are the same as the mean interarrival times so one can expect that there is a certain probability that none of these two loads is present at an arbitrary point in time. The Gaussian process load *P4* is treated as permanently "on" ($\mu \gg \kappa$ or $\rho \to 0$). The dead weight load *P1* is modeled as a time invariant *R*-variable and in consequence has no jump rate λ and no intensity ρ.

4.1.3. Some results

For the case at hand we have 4 possible load combinations with incidences for *P2*, *P3*, *P4*: (1,1,1), (1,0,1), (0,1,1), (0,0,1). For each combination a FORM/SORM analysis is performed and the results are summed up weighted by the probabilities of the combinations (which all are $P = 0.25$ here).

```
---------------------------------------------------------------------------
   Job name ........... : Steel-Column-JD-Im
   Failure criterion no. :       1
   Comment : Steel column buckling
   Transformation type    : Rosenblatt
   Optimization algorithm: RFLS-T
   COMRELTV, (Version 8), (c) Copyright: (RCP GmbH 1998-2003)
---------------------------------------------------------------------------
 Case    1; loads on:  3 with Load-case Pf=  8.033E-03; Sum(Pf's)=  8.033E-03
 Case    2; loads on:  2 with Load-case Pf=  9.018E-11; Sum(Pf's)=  8.033E-03
```



```
 Case    3; loads on:  2 with Load-case Pf=  4.539E-07; Sum(Pf's)=  8.033E-03
 Case    4; loads on:  1 with Load-case Pf=  1.713E-25; Sum(Pf's)=  8.033E-03

           Sum(Pf's) lower boud:    6.705E-06; upper bd.:    8.033E-03
           sum(Pf's) as beta     :       4.353;          :       2.407

    Further results from loop over all load cases :
    Expected number of outcrossings E[N_+]             =    8.026E-03
    Local outcrossing rate nue_+ at T-*                =    8.026E-05
    Hazard rate at T-*                                 =    8.091E-05

         ----------- Statistics after COMREL-TV -------------
         Total State Function calls               =    699
         State Funct. gradient evaluations        =    27
```

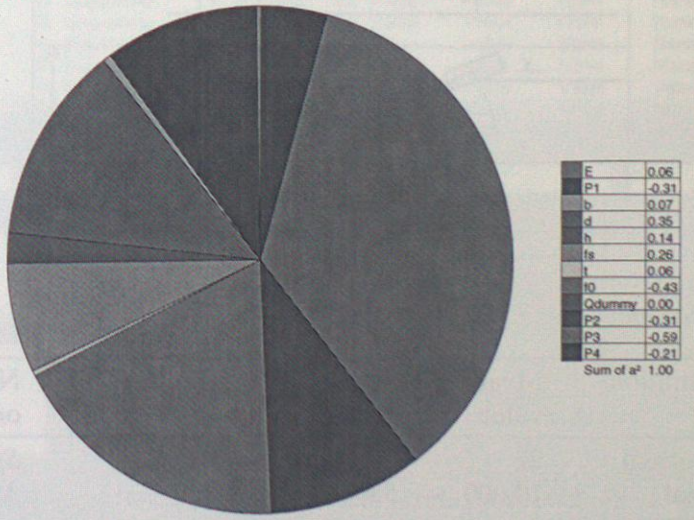
It is observed that just the case with all loads is relevant here. From verification examples it is known that the upper bound P_f is very close to the exact result whereas the lower bound is less convincing. For the stationary case here the local outcrossing rate v^+ multiplied by the time interval $\{0,100\}$ yields the expected number of outcrossings $E[N^+]$.

If just a FORM analysis is done one obtains:

```
           Sum(Pf's) lower bound:    6.295E-06; upper bd.:    7.546E-03
           sum(Pf's) as beta     :       4.367;          :       2.430
                                   . . .
            Total State Function calls              =    339
            State Funct. gradient evaluations       =    27
```

Comrel-TV computes sensitivities for each load combination. For the relevant case 1 with all loads on the relative importance of the basic variables is shown below.

Representative Alphas of Variables FLIM(1), Steel-Column-JD-Im.ptv



Load-case Pf: 7.55e-003; Active J & D-variables: P2, P3, P4

4.2. Several events must happen to cause failure (parallel system)

This example demonstrates one of the important features of *Sysrel*: The computation of the failure probability of a parallel system. Sysrel deals with multiple failure modes:

in a series system (union of failure events) $P_{\text{f,series}} = P\left\{\bigcup_{(i)}[F_i]\right\} = P\left\{\bigcup_{(i)}[g_i(\mathbf{x} \leqslant 0)]\right\},$

in a parallel system (intersection of events) $P_{\text{f,parallel}} = P\left\{\bigcap_{(i)}[F_i]\right\} = P\left\{\bigcap_{(i)}[g_i(\mathbf{x} \leqslant 0)]\right\},$

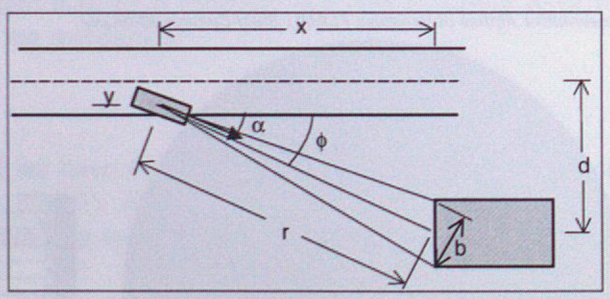
and in a (minimal) cut − set $P_{\text{f,system}} = P\left\{\bigcup_{(i)}\left[\bigcap_{(j)} F_{i,j}\right]\right\}.$

The failure probability of a structural element under truck impact can be determined by making use of the following simplified model. The mechanical and probabilistic models are taken from an analysis about vehicle impact by Vrouwenvelder. The impact force, based on energy balance, is

$$F_i = \sqrt{k \cdot \kappa \cdot m\left(v_0^2 - \frac{2 \cdot a \cdot d}{\sin(\phi)}\right)} \quad \text{for } v_0^2 > \frac{2 \cdot a \cdot d}{\sin(\phi)},$$

where v_0 is the initial velocity; k, the equivalent stiffness; m, the total mass; a, the deceleration; κ, the pay load factor ($0 \leqslant \kappa \leqslant 1$); ϕ, the angle between collision course and track direction; d, distance from the structural element to the road ($d \leqslant \frac{v_0^2 \sin(\phi)}{2a}$).

The geometrical settings are illustrated in the figure below.



4.2.1. The stochastic model

Basic variable	Distribution	Mean value	Standard deviation	C.o.V. (%)	Name of variable or constant
V_0 (m/s)	Lognormal	22	2.2	10	*Speed0*
m (kg)	Normal	10,000	5500	55	*Mass*

(*continued on next page*)

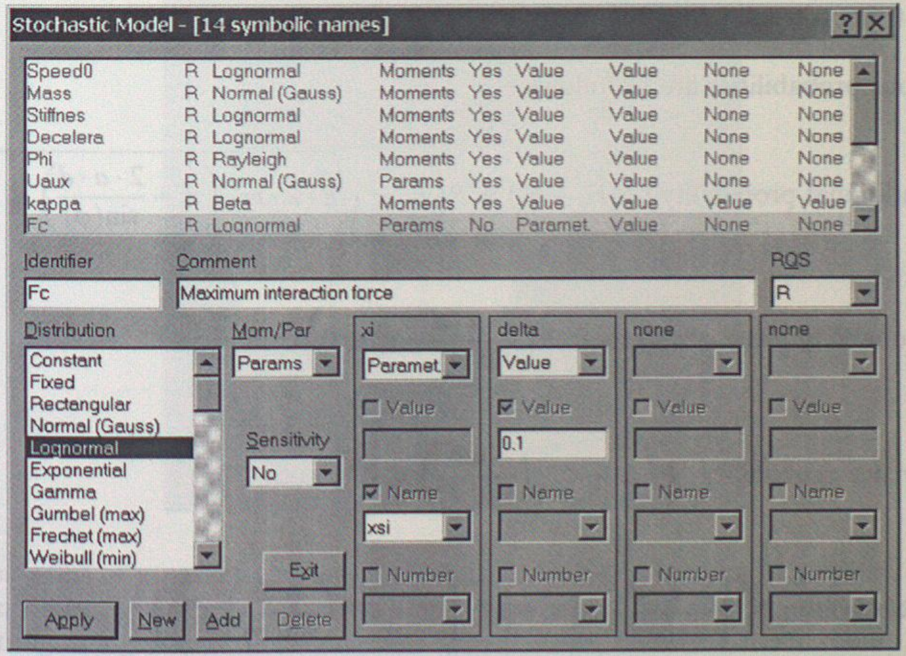
Basic variable	Distribution	Mean value	Standard deviation	C.o.V. (%)	Name of variable or constant
K (N/m)	Lognormal	300,000	60,000	20	*Stiffness*
A (m/s^2)	Lognormal	4	1.3	33	*Decelera*
ϕ (°)	Rayleigh	10	5.21	52	*phi*
κ (–)	Beta $0.3 \leqslant \kappa \leqslant 1.0$	0.7	0.1	14	*kappa*
F_c (N)	Lognormal	$2.010E+6$	$2.015E+5$	10	*Fc*
d (m)	Constant	4.5	–	–	*Dist*

Note: The lognormal resistance (maximal interaction force) F_c is defined by distribution parameters median $\xi = 2.0E+6$ and $\delta = 0.100$. See also the screenshot of the stochastic model below.

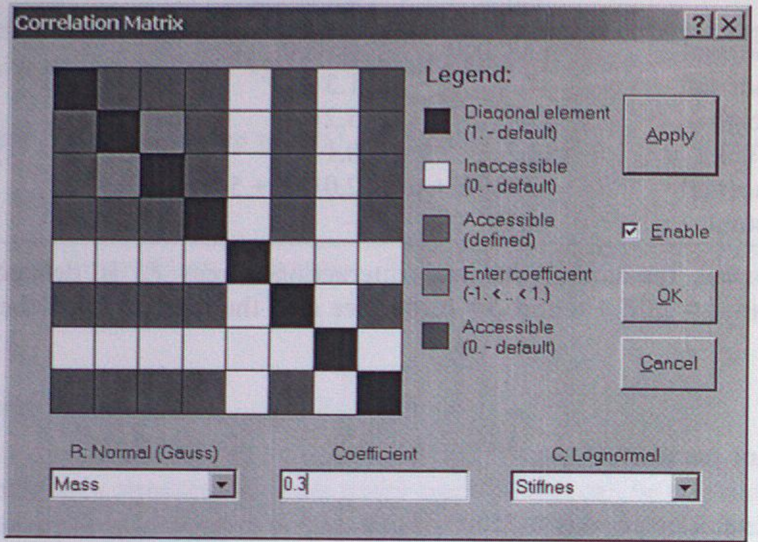
Further constant parameters are:

n = number of vehicles per day = 2500;
t = reference time in days = 36,500 days;
λ = rate for leaving the track = 10^{-10};
b = width of truck (or obstacle) = 2.5 m.

The stochastic model in *Sysrel* (ξ is assigned to the constant parameter named 'xsi'):



The basic variables *Mass* and *Stiffness* of the vehicle are positively correlated:



4.2.2. State functions and logical model

There are three events to consider:

1. The event of failure of the structural element given that the truck reaches the structural element.
2. The event that the truck does not come to a stop before hitting the target.
3. The event that a truck leaves the road.

The various probabilities are as follows:

$$\text{Unconditional probability}: P(F_c - F_i) = P\left(F_c - \sqrt{k \cdot \kappa \cdot m \cdot \left(v_0^2 - \frac{2 \cdot a \cdot d}{\sin(\phi)}\right)} \leqslant 0\right),$$

$$\text{Hit probability given occurrence}: P\left(\frac{2 \cdot a \cdot d}{\sin(\phi)} - v_0^2 \leqslant 0\right),$$

$$\text{Occurrence probability}: P\left(U - \Phi^{-1}\left(\frac{\lambda ntb}{\sin(\phi)}\right) \leqslant 0\right),$$

$$\text{Hit probability}: P\left(\left\{\frac{2 \cdot a \cdot d}{\sin(\phi)} - v_0^2 \leqslant 0\right\} \cap \left\{U - \Phi^{-1}\left(\frac{\lambda ntb}{\sin(\phi)}\right) \leqslant 0\right\}\right),$$

$$\text{Total probability}: P\left(\left\{F_c - \sqrt{k \cdot \kappa \cdot m \cdot \left(v_0^2 - \frac{2 \cdot a \cdot d}{\sin(\phi)}\right)} \leqslant 0\right\} \cap \left\{\frac{2 \cdot a \cdot d}{\sin(\phi)} - v_0^2 \leqslant 0\right\} \cap \left\{U - \Phi^{-1}\left(\frac{\lambda ntb}{\sin(\phi)}\right) \leqslant 0\right\}\right),$$

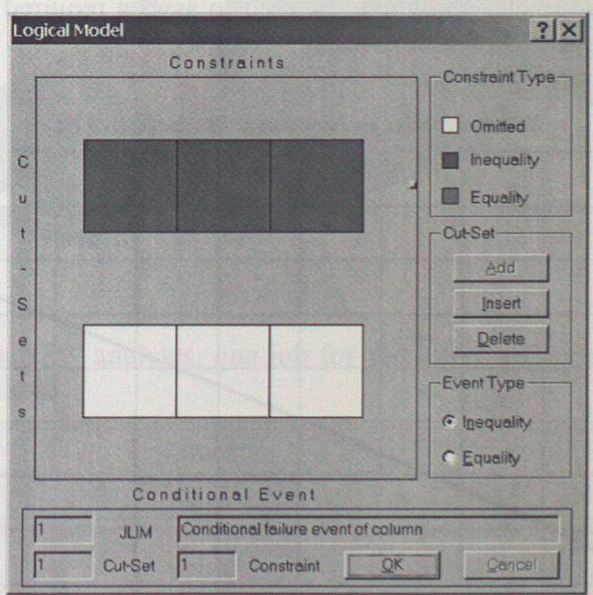
where U = auxiliary standard normal variable.

The third event is introduced to model the discrete occurrence event. It depends on the random hit angle. The formulation of the failure functions is as follows.

```
FLIM(1){crash}=1 - kappa*Mass*Stiffnes*
        (Speed0^2-2.*Decelera*Dist/SIN(Phi*PI/180.)) / (Fc*Fc)
FLIM(2){cond.}=2.*Decelera*Dist/SIN(Phi*PI/180.)-Speed0^2
FLIM(3)(auxiliary limit for event probabilities)=
        Uaux-ICPHI(t*n*lamda*b/SIN(Phi*PI/180.))
```

Note the special formulation for the first failure function which numerically is much mor venient than the direct formulation. It avoids not only negative roots but is also well scaled the second term is a number around unity.

The logical model in *Sysrel* is provided as an incidence matrix. The 3 events in the intersection fo row in this matrix. A union over several intersections (i.e., a cut-set) would be defined by several

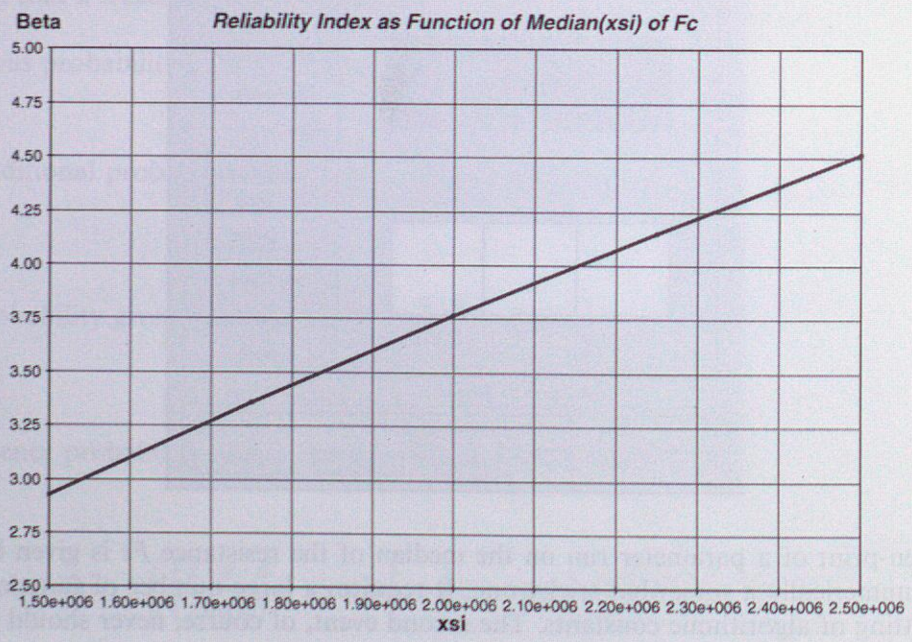


The screen-print of a parameter run on the median of the resistance *Fc* is given below. This example is numerically a somewhat tricky one. It requires a large number of function calls and a careful setting of algorithmic constants. The second event, of course, never should become active, which is maintained in the results below – the Multi-Normal Integral (MNI) is of dimension 2 with components 1 and 3.

```
------------------------------------------------------------------------
    Job name ........... : CRASH
    Transformation type    : Rosenblatt
    Optimization algorithm: NLPQL
------------------------------------------------------------------------
    *SYSREL*: Linearizing C U T - S E T  No.  1
   M.N.I. with dimension  2

      Lower Bound Pf, L.B.-beta  ,Upper Bound Pf, U.B.-beta  ,      IER
          .8221E-04      3.768        .8221E-04      3.768            0


  -------------- Parameter study for Parameter: xsi     ------------
    Par.Value ,  Lower Bound Pf, L.B.-beta , Upper Bound Pf, U.B.-beta
       .1500E+07     .1718E-02     2.926          .1718E-02    2.926
  -------------------------------------------------------------------
       .1750E+07     .3905E-03     3.359          .3905E-03    3.359
  -------------------------------------------------------------------
       .2000E+07     .8221E-04     3.768          .8221E-04    3.768
  -------------------------------------------------------------------
       .2250E+07     .1644E-04     4.153          .1644E-04    4.153
  -------------------------------------------------------------------
       .2500E+07     .3159E-05     4.516          .3159E-05    4.516
  -------------------------------------------------------------------

     Statistics in SYSREL:
     No.of state-function calls= 1102,  gradient calls= 120
```

The result from this parameter study may be used to asses a required mean value (median) of the resistance *Fc* for a specified reliability level.

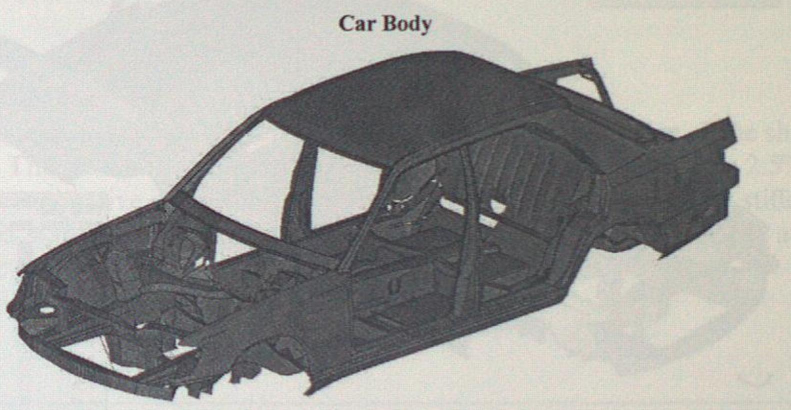
A plot generated in *Sysrel* looks as follows:



4.3. Reliability analysis with PERMAS-RA: Car body

A car body with about 400,000 degrees of freedom was investigated with frequency limits. Whereas the finite element model is a real-life model, the stochastic properties of the model are estimated. It was assumed, that the thicknesses of all sheets are uncertain due to tolerances and a normal distribution with a standard deviation of 5% was selected for each thickness. For a sheet with a thickness of 2.0 mm, e.g. it means that 95.4% of all samples lie between 1.8 and 2.2 mm. Since there are 73 independent sheet thicknesses, the stochastic model contains 73 basic variables. All other possible sources for stochastic behavior like spot welds, etc., are treated as deterministic in this case.

During car development, the basic Eigen frequencies of the body-in-white (i.e., the pure car body without any attachment parts, etc.) must be positioned between certain limits to separate the Eigen frequencies of the final vehicle. In the following it was assumed, that the Eigen frequency f_{T1} of the first global torsion (twisting of the car body around the longitudinal axis) should lie in the range $f_{T1} \pm 1.0$ [cps], where f_{T1} is the Eigen frequency calculated with nominal values.



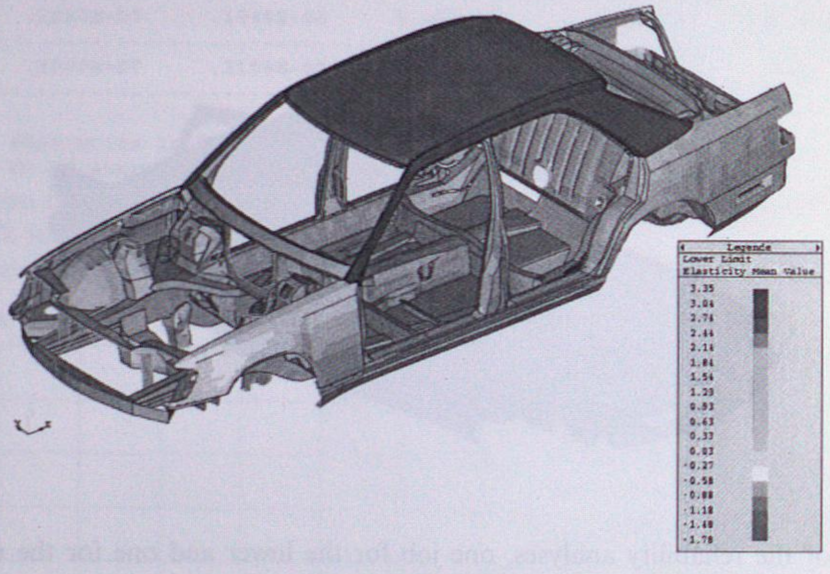
The results of the reliability analyses, one job for the lower and one for the upper limit are shown in the following table:

Limit	Lower		Upper	
Importance sampling	No	Yes	No	Yes
Analyses	30	82	12	81
$P(\lvert f_T - f_{T1}\rvert > 1.0)$	$5.77E-5$	$1.21E-4$	$2.69E-5$	$2.16E-5$
Reliability index $\beta = -\Phi^{-1}$ (P)	3.856	3.670	4.039	4.090
Correction factor		2.103		0.805
Corr. coeff. of variation (%)		30.5		14.3
Failure rate $1/P_f \approx$	17,000	8000	37,000	46,000

Importance sampling was used to improve the results of the FORM method. In general, importance sampling checks the difference between the true failure surface and the first order approximation, and calculates a correction factor for the probability of failure (i.e., the ratio between the true probability of failure and the first order approximation) which is 2.103 in this example for the lower limit case. Resulting from sampling, the correction factor is an uncertain quantity, and therefore a coefficient of variation (normalized standard deviation) can be calculated which is equal 30.5% in this case, a rather high value which could be reduced if necessary by increasing the number of samples.

The critical case is given by the lower limit. According to the results after importance sampling one of about 8000 car bodies will have a first global torsion mode below the lower frequency limit. To improve the design, the elasticities of the mean values and standard deviations of the basic variables are investigated. As an example these elasticities are shown for the sheet thicknesses in the following figures.

Elasticity (Dimensionless Sensitivity) of
Mean Values of Sheet Thickness at Lower Limit

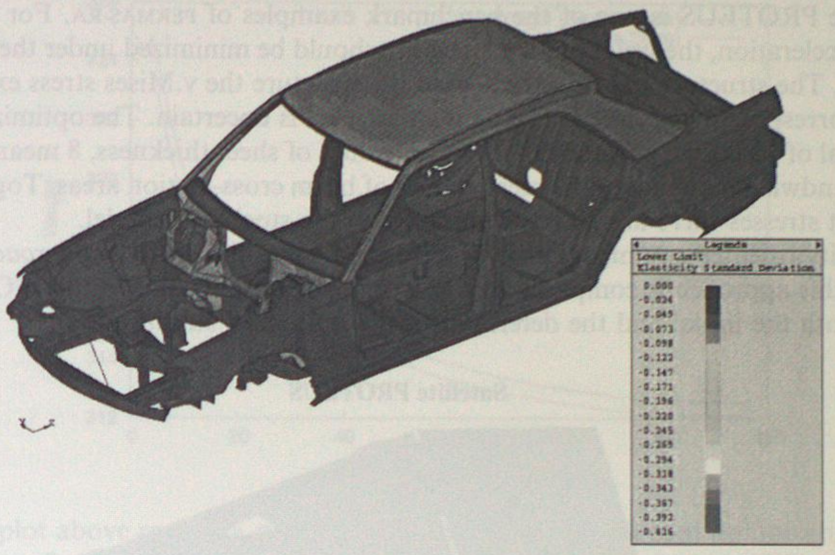


The dimensionless elasticities are derived from sensitivities, i.e., the derivatives of the reliability index with respect to mean value and standard deviation of the basic variables. Let τ be an arbitrary parameter (also mean, standard deviation) of a probability distribution of a basic variable. Then the elasticity is defined as:

$$e_\tau = \frac{\partial \beta}{\partial \tau}\frac{\tau}{|\beta|} \quad |\beta \neq 0;\ \tau \neq 0.$$

The sensitivities depend on the gradient in basic space (X-space) provided by PERMAS-RA and on derivatives of the transformation $\mathbf{X} = \mathbf{T}^{-1}(\mathbf{U})$ computed internally.

Elasticity (Dimensionless Sensitivity) of
Standard Deviations of Sheet Thicknesses at Lower Limit



Since the nominal frequency should remain unchanged, the mean values of the sheet thic are kept constant. The standard deviation, however, was reduced from 5% to 2.5% for tl parts with the largest negative elasticity, i.e., for the roof, the roof frame and a stiffener wi A-pillar. A reduced standard deviation may be achieved simply by reducing the allowabl ances for the sheet thickness.

The following table shows the improved behavior:

Limit	Lower		Upper	
Importance sampling	No	Yes	No	Yes
Analyses	30	82	12	81
$P\ (\lvert f_T - f_{T1}\rvert > 1.0)$	$5.83E-9$	$2.57E-8$	$2.08E-9$	$1.71E-9$
Reliability index $\beta = -\Phi^{-1}$ (P)	5.705	5.447	5.878	5.911
Correction factor		4.411		0.819
Corr. coeff. of variation (%)		42.9		27.3

It can be seen that just by reducing the standard deviation of a few important parts, the probability of exceeding the frequency window can be reduced by more than three orders of magnitude. Again the coefficient of variation of the correction factor on P is rather high (42.9%), but due to the low probability, the given accuracy seems to be sufficient.

4.4. Weight optimization with PERMAS-RA: Satellite PROTEUS

The satellite PROTEUS is one of the benchmark examples of PERMAS-RA. For a given inertia load due to acceleration, the weight of the structure should be minimized under the condition that $P_f \leqslant 1.0E-6$. The structure fails if anywhere in the structure the v.Mises stress exceeds the limit stress of the corresponding material. The limit stress itself is uncertain. The optimization problem contains a total of 28 design parameters, 17 mean values of sheet thickness, 8 mean values of skin thickness of sandwich materials and 3 mean values of beam cross-section areas. Together with two uncertain limit stresses there are 30 basic variables in the stochastic model.

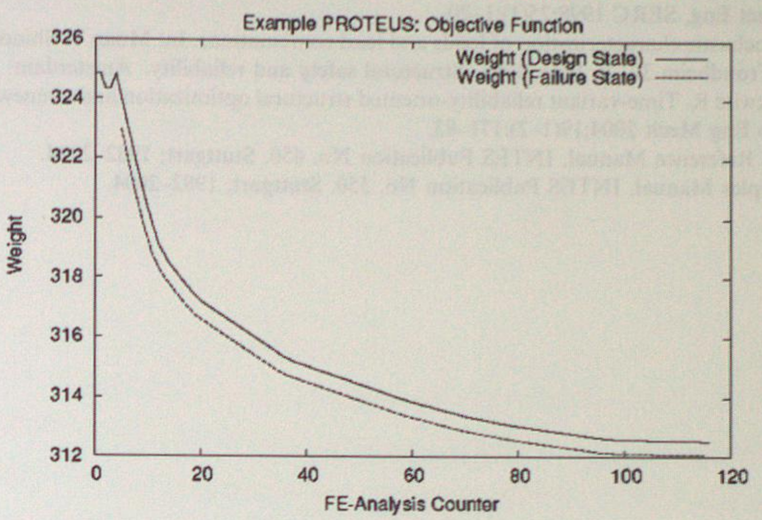
The reliability-oriented optimization was performed using the *one step approach* in that case. The result of this approach is compared to a deterministic optimization with a FORM reliability analysis for both the initial and the deterministically optimized structure.

Satellite PROTEUS

Results from deterministic and probabilistic optimization

	Basic	Optimized	One step
Mass	324.8	308.9	312.5
Maximum stress	$9.6E+7$	$1.2E+8$	$5.5E+6$
Failure probability P_f	$8.2E-7$	$4.2E-5$	$1.0E-6$
$1/P_f$	1.2E + 6	23 809	$1.0E+6$
Total no. of jobs	1	2	1
Total no. of FE analyses	23	14	117

Weight History of Optimization with Reliability Constraint



The history plot above reveals a peculiarity of the reliability-oriented optimization which may not be obvious at first sight. For each major iteration step (new design) *two* FE-analyses have to be performed: One for the *design state* (with the random variables **X** set to their means or the values of the design parameters **p** associated with the means) and one analysis for the *failure state* (with $\mathbf{X} = \mathbf{x}^*$ such that $g(\mathbf{x}^*) = 0$ at the most likely failure point or β-point).

It is demonstrated that a deterministically optimized structure tends to become unsafe due to tolerances and scattering of dimensions, loads, etc. Those uncertainties, however, are taken into account by the reliability analysis, which delivers reasonable information about the failure behavior of the structure. Using the combination of optimization and reliability analysis, either user controlled or fully integrated, it is possible to eliminate weak points of the design in order to achieve a robust design that is less sensitive to tolerances and scattering.

The combination of finite element method, high performance computing and advanced solution algorithms enables an economical usage of the provided methods.

Availability

- COMREL, SYSREL, COSTREL, STATREL: RCP Consult GmbH, Barerstr. 50, D-80799 Munich, Germany (Stephan Gollwitzer) website: www.strurel.de, e-mail: gollwitzer@strurel.de
- PERMAS, PERMAS-RA: INTES GmbH, Schulze-Delitzsch-Str. 16, D-70565 Stuttgart, Germany (Rolf Fischer) website: www.intes.de, e-mail: fischer@intes.de

References

[1] COMREL & SYSREL & COSTREL Users Manual. RCP Consult. Munich; 1987–2004.
[2] Hohenbichler M, Gollwitzer S, Kruse W, Rackwitz R. New light on first- and second-order reliability methods. Struct Saf 1987;4(4):267–84.

[3] Rackwitz R. Computational techniques in stationary and non-stationary load combination – a review and some extensions. J Struct Eng, SERC 1998;25(1):1–20.
[4] Shinozuka M. Stochastic characterization of loads and load combinations. In: Moan T, Shinozuka M, editors. Proc 3rd ICOSSAR, Trondheim 32–25 June, 1981. Structural safety and reliability. Amsterdam: Elsevier; 1981.
[5] Streicher H, Rackwitz R. Time-variant reliability-oriented structural optimization and a renewal model for life-cycle costing. J Probab Eng Mech 2004;19(1–2):171–83.
[6] PERMAS: Users Reference Manual. INTES Publication No. 450. Stuttgart; 1982–2004.
[7] PERMAS: Examples Manual. INTES Publication No. 550. Stuttgart; 1982–2004.