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TECHNISCHE UNIVERSITÄT MÜNCHEN

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zur
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CROSSING RATE BASED FORMULATIONS IN FATIGUE RELIABILITY

F. Guers, R. Rackwitz

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Munich, June 1986 The authors

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On a reliability formulation for structural components subjected to strength deterioration

F. Guers and R. Rackwitz

1. INTRODUCTION

The study of the reliability of structural components subjected to load-induced strength deterioration has produced only partial solutions as of yet. The analytical approaches available are of a highly phenomenological nature. In this study, some of these formulations are used in an attempt to formulate the overall reliability problem in view of the fact that structural components usually fail under "large" loads meeting either an essentially unchanged strength (this is called an extreme value failure) or a possibly substantially reduced strength, if fatigue has developed under cyclic loading.

Most fatigue phenomena are associated with the initiation and the propagation of cracks. The typical fatigue failure occurs if the growth becomes unstable, i.e. if the energy provided by the stresses at the zone around the crack tip can no more be balanced by the amount of energy dissipated into heat, surface energy, etc. This physical concept at least applies to most of the structural materials of brittle nature.

If the materials considered perform essentially ductile, the effect of crack growth is to reduce the section of the component. Then, it is proposed to take yielding in the net cross section as a failure criterion.

The intermediate case where the crack develops in semi-ductile materials without affecting too much the carrying section will be studied in some detail herein. The exceedance of a given crack size not leading to rupture but, for example, to leakage which alternatively can be defined as a failure criterion is not considered herein.

In a separate paper it will be shown that the formulation pursued in the following is particularly useful in the reliability analysis of redundant structural systems.

2. FORMULATION OF THE RELIABILITY PROBLEM

The life time of a structural component under random loading can be subdivided into three phases.

The manufacture and the various operations of installation usally are responsible for the formation of a multiplicity of microscopic cracks, nucleations and other types of discontinuities which, under sufficiently low constant loading, would have a certain chance to never grow and merge. If this happens, one speaks of an endurance limit. Under random virtually unlimited loading such a limit does not exist, and one can include this first time interval in the so called initiation phase.

This phase is the one in which the microscopic initial defects grow to form cracks of visible size and during which the strength against extreme loading is unaffected.

If there are initial defects of visible size such as notches, flaws and the like, this first phase is, of course, absent.

The second phase is denoted as the phase of stable crack propagation; the strength against overloading will be considered as only insignificantly affected by the presence of cracks, because their size remains small. If failure occurs, it will be an extreme load type failure as in the previous phase, but at a somewhat reduced strength level.

The third and last phase is characterised by rapid crack growth and crack instability. The brittle behavior of the material then is the primary source of failure rather than the decreasing strength due to the diminuation of the net cross section of the component.

This last phase usually is relatively short as compared to the previous phases; in many cases, the majority of the life-time is spend in the first phase. And it is worth mentioning that protective actions, i.e. repair after inspection, can only be undertaken in the last two phases.

A representation of the different phases is made in figure 1. Let t be the intended time of use of the structure and its structural components. Denote by T_e the random time to classical first-passage failure over the assumed constant threshold. T_i is the random time of crack initiation. T_p is the random time of crack propagation following T_i and ending at the time instant where the residual "strength" represented by the threshold of crack instability becomes smaller than the strength relevant for extreme load failure.

Figure 1: Life phases and failure modes of a component

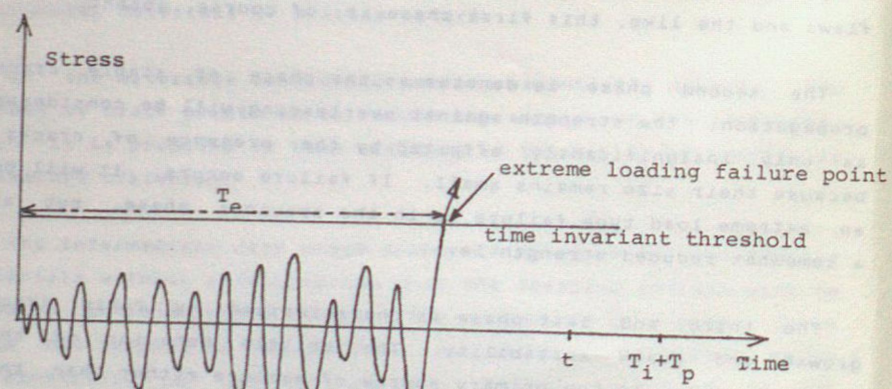

Figure 1.a: Extrem loading failure ($t<T_i+T_p$)

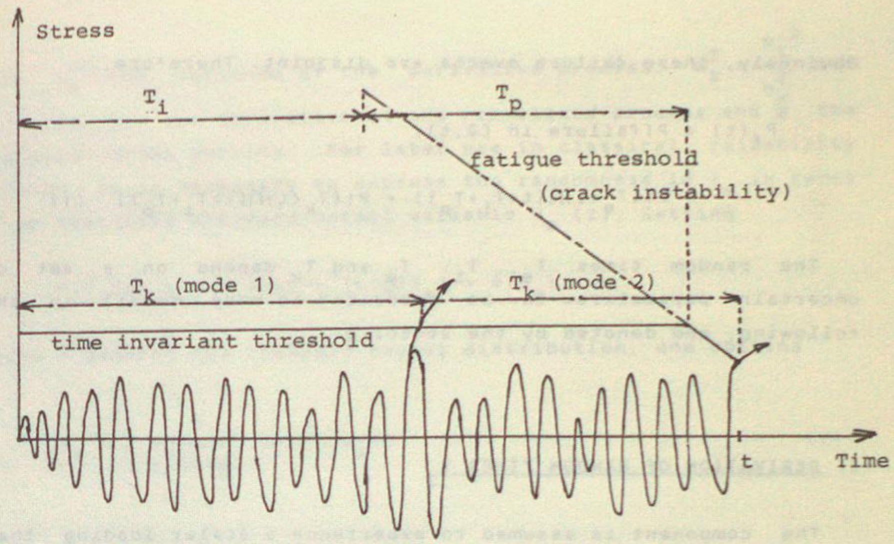

Figure 1.b: Failure modes for $t>T_i+T_p$

In figure 1.a, it is assumed that $t<T_i+T_p$. The threshold remains constant up to time t. Hence, only extreme value failure can be possible at random time T_e during [0,t]. In the case shown in figure 1.b, i.e. for $t>T_i+T_p$, either an extreme value upcrossing has happened before T_i+T_p (mode 1) or the stress process upcrosses the decreasing fatigue threshold between T_i+T_p and t (mode 2 is the realization of a typical fatigue failure).

In this study the concept of independent barrier crossings is applied throughout the different phases irrespective of the failure mode. Consequently, if fatigue failure is possible, the random time T_k leading to failure contains both failure modes whereas the random time T_e only refers to mode 1.

In summary, failure occurs before t whenever T_e is smaller than t and $t<T_i+T_p$ or for $t>T_i+T_p$ whenever T_k is smaller than t.

Obviously, these failure events are disjoint. Therefore,

$$P_f(t) = P(\text{failure in } [0,t])$$

$$= P(\{T_e<t\}\cap\{t<T_i+T_p\}) + P(\{T_k<t\}\cap\{t>T_i+T_p\}) \quad (1)$$

The random times T_e, T_i, T_p and T_k depend on a set of uncertain parameters to be described in more detail in the following, and denoted by the vector $\underline{\theta}$.

3. DERIVATION OF RANDOM TIMES T_j

The component is assumed to experience a scalar loading that causes far field stresses representable by the stationary Gaussian stochastic process X(t) with mean value m_X and variance σ_X^2. Further properties of this process will be given when necessary.

3.1 TIME TO FIRST EXTREME LOAD FAILURE T_e

Define the variable L as the possibly uncertain time-invariant failure threshold and assume a sufficiently mixing loading process. Then, for sufficiently high threshold, the time to first failure is given by [1]

$$F_{T_e}(t) = 1 - \exp[-\nu^+(L|\underline{\theta})t]$$

where $\nu^+(L|\underline{\theta})$ is the upcrossing rate of X(t) for level L possibly depending on further parameters of $\underline{\theta}$. For Gaussian load processes it is [1]

$$\nu^+(L|\underline{\theta}) = \frac{\omega_0}{\sqrt{2\pi}}\,\varphi\left(\frac{L-m_X}{\sigma_X}\right)$$

with $\sigma_{\dot{X}}^2$ the variance of the derivative process, $\omega_0^2 = \frac{\sigma_{\dot{X}}^2}{\sigma_X^2}$ the variance of the derivative of the normalized process and φ the standard normal density. For later use in classical reliability methods it is necessary to express the randomness in T_e in terms of an auxiliary standard normal variable U_e [2]. Setting

$$F(T_e|\underline{\theta}) = 1 - \exp[-\nu^+(L|\underline{\theta})T_e] = \Phi(U_e),$$

where Φ denotes the standard normal distribution, one obtains

$$T_e = -\frac{1}{\nu^+(L|\underline{\theta})}\,\ln(\Phi(-U_e)) \quad (2)$$

3.2 CRACK INITIATION TIME T_i

No realistic physical models appear to exist for the description of this phase. Therefore, it is assumed that S-N-tests are available where N(S) denotes the number of cycles of constant stress ranges S at which the first macrocrack is initiated, i.e. the first visible crack is observed. The S-N-curves usually are well approximated by

$$N(S)S^B = A = D\varepsilon \quad (3)$$

where B and D are random variables expressing the statistical uncertainity implied by a limited number of tests and ε the random residual of this relationship. In the sequel, D and ε are represented by a single variable A.

For a stress history composed of n_j cycles of stress ranges S_j in each of p possible stress states, Palmgren-Miners damage accumulation rule can be written as:

$$\sum_{j=1}^{p} \frac{n_j}{N(S_j)} = 1$$

where $N(S_j)$ is as in eq. (3).

For random stresses, this damage accumulation hypothesis may also be adopted. A visible crack appears whenever:

$$\int_{S=0}^{\infty} \frac{dn}{N(S)} = 1 \qquad (4)$$

Let $N(T_i)$ be the number of load cycles in $[0, T_i]$. For $N(T_i)$ being large, one can use

$$dn \approx N(T_i)\, f(S)dS$$

where f is the density function of the stress ranges. Eq. (4) becomes

$$\frac{N(T_i)}{A} \int_0^{\infty} S^B\, f(S)dS = 1 \qquad (5)$$

For a narrow-band stress process, one can write

$$N(T_i) = \frac{\omega_0}{2\pi} T_i = \nu_0^+ T_i$$

where ν_0^+ is the mean value upcrossing frequency. Therefore, eq. (5) leads to

$$T_i = \frac{A}{\nu_0^+ E[S^B]} \qquad (6)$$

with $E[S^B] = \int_0^{\infty} S^B\, f(S)dS$

For a narrow-band Gaussian process X(t), the stress ranges S are even Rayleigh distributed and according to [3] or [4], one has:

$$E[S^B] = 2^B E[R^B] = (2\sqrt{2})^B \sigma^B\, \Gamma(1+\frac{B}{2})$$

Formula (6) shows that T_i is a random variable, because A and B but possibly also other parameters determining the stochastic nature of X(t) and S(t) are random.

3.3 CRACK PROPAGATION TIME T_p

One of the simplest crack propagation laws is due to Paris Erdogan

$$\frac{da}{dn} = c(\Delta K)^m \qquad (7)$$

where a denotes crack length, $\Delta K = Y(a)\, S\sqrt{\pi a}$ the stress intensity factor. c and m are material properties which may be taken as random. The geometry factor $Y(a) = Y$ is considered as constant. Eq. (7) is separable and, here, can be integrated (see [5]) analytically leading for m>2 to

$$a(t) = a_0 \left(1 - K_2 \sum_{j=1}^{n(t)} S_j^m\right)^{\frac{2}{2-m}}$$

where a_0 is the initial crack length and n(t) the number of cycles experienced up to t from the beginning of crack propagation. The crack is unstable for $K = Y\sqrt{\pi a}\, X = K_c$. This defines the brittle failure threshold

$$\xi(t) = K_1(1-K_2 \sum_{j=1}^{n(t)} S_j^m)^{\frac{1}{m-2}}$$

with $K_1 = \frac{K_c}{Y\sqrt{\pi a_0}}$ and $K_2 = \frac{m-2}{2} c(Y\sqrt{\pi})^m a_0^{\frac{m-2}{2}}$

It has been shown in [5], that the dispersion of the threshold is negligible and, therefore, one approximates $\sum_{j=1}^{n} S_j^m$ by its mean value.

The stable propagation phase is defined to end when the risk of instability becomes larger than the risk for extreme value failure for the constant threshold L, i.e. when $\xi(t) \leqslant L$. Therefore, T_p is given by:

$$K_1(1-K_2 E[\sum_{j=1}^{N(T_p)} S_j^m])^{\frac{1}{m-2}} = L$$

For a narrow-band process X(t) as before it is

$$E[\sum_{j=1}^{N(T_p)} S_j^m] = \nu_0^+ T_p E[S^m]$$

and by solving for T_p:

$$T_p = \frac{1-(\frac{L}{K_1})^{m-2}}{K_2 \nu_0^+ E[S^m]} \qquad (8)$$

Therefore, T_p is also a random variable depending on K_1, K_2, L and as T_i on the parameters determining the process X(t).

3.4 TIME T_k TO POSSIBLE FATIGUE FAILURE ($t \geqslant T_i+T_p$)

The time the component can spend in the "fatigue failure phase" is short and one can approximate the threshold by a linear threshold function which is exact in the case m=3 for a narrow band process (see figure 1.b). The failure threshold in [0,t] then is decomposed in a constant barrier from 0 to T_i+T_p and in a linearly decreasing function during the "fatigue failure phase".

The time T_k leading to failure can be handled similarly as T_e with the help of the upcrossing concept provided that X(t) is a sufficiently mixing process. Then,

$$F_{T_k}(t) = 1 - \exp(-\int_0^t \nu^+(\zeta|\underline{\theta})d\zeta)$$

where $\nu^+(\zeta|\underline{\theta})$ is the upcrossing rate of the threshold relevant at time ζ.

Define $I(t) = \int_0^t \nu^+(\zeta|\underline{\theta})d\zeta$, which is an increasing and consequently invertable function.

For a convenient reliability computation, T_k is again expressed by an auxiliary standard normal variable U_k, in setting

$$F(T_k|\underline{\theta}) = 1 - \exp(-I(T_k|\underline{\theta})) = \Phi(U_k)$$

gives

$$T_k = I^{-1}[-Ln(\Phi(-U_k))|\underline{\theta}]$$

The integral can be calculated as follows:

$$I(t) = \int_0^{T_i+T_p} \nu^+(\zeta|\underline{\theta})d\zeta + \int_{T_i+T_p}^{t} \nu^+(\zeta|\underline{\theta})d\zeta$$

resulting in

$$I(t) = (T_i+T_p)\, v^+(L|\underline{\theta}) + \int_0^{t-(T_i+T_p)} v^+(\zeta_1|\underline{\theta})d\zeta_1$$

If the fatigue threshold is a linear decreasing function which can be written as $\xi(\zeta_1) = L-\alpha\zeta_1$ with $\alpha = K_1K_2\, v_0^+\, E[S^m]$, one can write the upcrossing rate:

$$v^+(\zeta_1|\underline{\theta}) = \omega_0\, \varphi\left(\frac{L-m_X}{\sigma_X} - \frac{\alpha}{\sigma_X}\zeta_1\right) \Psi\left(-\frac{\alpha}{v_0^+\sigma_X}\right)$$

where $\Psi(x) = \varphi(x) - x\Phi(-x)$, φ and Φ being the density and the distribution function, respectively, of a standard normal variable.

Then, the integral can be given explicitely.

$$\int_0^{t-(\zeta_i+\zeta_p)} v^+(\zeta_1|\underline{\theta})d\zeta_1 =$$

$$\frac{v_0^+\sigma_X}{\alpha}\Psi\left(-\frac{\alpha}{v_0^+\sigma_X}\right)\left\{\Phi\left(\frac{L-m_X}{\sigma_X}\right) - \Phi\left(\frac{L-m_X}{\sigma_X} - \frac{\alpha}{\sigma_X}[t-(T_i+T_p)]\right)\right\}$$

It is now also possible to explicitely write the transformation of $T_k|\underline{\theta}$ in U_k, leading to:

$$T_k = T_i+T_p+$$

$$+\frac{\sigma_X}{\alpha}\left\{\frac{L-m_X}{\sigma_X} - \Phi^{-1}\left[\Phi\left(\frac{L-m_X}{\sigma_X}\right) + \frac{\frac{\omega_0}{\sqrt{2\pi}}(T_i+T_p)\varphi\left(\frac{L-m_X}{\sigma_X}\right)+Ln(\Phi(-U_k))}{\frac{v_0^+\sigma_X}{\alpha}\Psi\left(-\frac{\alpha}{v_0^+\sigma_X}\right)}\right]\right\} \quad (9)$$

By this formula T_k is seen to be random as well. T_k depends on

T_i, T_p and the other uncertain parameters of $\underline{\theta}$.
The randomness of the vector $\underline{\theta} = (\theta_1,\ldots,\theta_l)$ can also be expressed by the Rosenblatt transformation in terms of a set of auxiliary independent standard normal variables $(U_1,\ldots,U_l)$ by setting [2]

$$F(\theta_1) = \Phi(U_1)$$
$$\vdots$$
$$F(\theta_j|\theta_1,\ldots,\theta_{j-1}) = \Phi(U_j)$$
$$\vdots$$
$$F(\theta_l|\theta_1,\ldots,\theta_{l-1}) = \Phi(U_l)$$

4. NUMERICAL EXAMPLE OF COMPONENTAL RELIABILITY

The purpose of this section is to show the efficiency of the proposed formulation in a numerical example and the principal influence of some parameters. Therefore, the assumed values and types of distribution for the uncertain parameters are somewhat simplified but they allow a qualitative interpretation of the results.

4.1 CHARACTERISTICS OF THE COMPONENT

In the following, numerical data are given for lengths measured in mm and stresses in N/mm^2. The distribution assumption for the basic variables are given below:

$L \sim N(230,\ 10)$

$A \sim N(\exp(30),\ 0.2 \times \exp(30))$

$C \sim N(5\ 10^{-14},\ 0.3 \times 5\ 10^{-14})$

$a_0 \sim N(2,\ 0.5)$

$$Y \sim N(1,\ 0.05)$$

$$K_{IC} \sim LN(2250,\ 200)$$

where $N(\alpha,\beta)$ (resp. $LN(\alpha,\beta)$) is the Normal (resp. Lognormal) distribution with mean value α and standard deviation β.
The exponent in the S-N-relations is assumed to be equal to the exponent in the Paris law. Both values are set to be $m=B=3$.

4.2 CHARACTERISTICS OF THE LOAD PROCESS

$X(t)$ is assumed to be a stationary, sufficiently mixing, narrow band Gaussian process with mean value $m_X \sim N(50;5)$ and standard deviation $\sigma_X = 25\ X_\sigma$, $X_\sigma \sim N(1,\ 0.05)$. The mean value upcrossing frequency is fixed at $\nu_0^+ = 0{,}1\ [s^{-1}]$ (corresponding to a mean period of 10 s). This frequency is only a scaling factor and it will be interesting to consider the mean number $\nu_0^+ t$ of load cycles experienced by the component at the time t.

In view of the reliability computation by FORM or SORM [6], the basic variables $\underline{\Theta} = (L, A, C, a_0, Y, K_{IC}, m_X, \sigma_X)$ are transformed into standardized and independent normal variables U_1 to U_8 via the Rosenblatt transformation.

$$L = 230 + 10\ U_1$$

$$A = e^{30}(1 + 0{,}2\ U_2)$$

$$C = 5\ 10^{-14}(1 + 0{,}3\ U_3)$$

$$a_0 = 2 + 0{,}5\ U_4$$

$$Y = 1 + 0{,}05\ U_5$$

$$K_{IC} = 2243\ \exp(0{,}085\ U_6)$$

$$m_X = 50 + 5\ U_7$$

$$\sigma_X = 25(1 + 0{,}05\ U_8)$$

Therefore, the random time T_e, T_i, T_p and T_k defined in eqs. (2), (6), (8) and (9) are expressed as functions of the standardized independent normal variables U_1 to U_{10}, where $U_9 = U_e$ and $U_{10} = U_k$. The computation of eq. (1) is carried out with the program SYSREL [6].

4.3 RESULTS

The reliability results are presented in terms of the safety index β according to $\beta(t) = \Phi^{-1}[P_f(t)]$ with $P_f(t)$ as in eq. (1). At first, we investigate the influence of the different events in formula (1) on the final β. Figure 2 shows the different terms as a function of time.

One can see that during the early phases of the life of the component the global β (curve 1) is identical with the extreme value β (curve 3). Fatigue failure is represented by curve 4. One recognizes that it has more and more effect on the global β (curve 1) up to a certain time beyond which the global β is determined almost entirely by the fatigue criterion. Curve 3 represents the evolution of β when there is no strength deterioration of the component.

Figure 3 shows the influence of the value of the initial threshold L on $\beta(t)$. For high value L and constant fracture mechanical characteristics, T_p is reduced and, consequently, fatigue effects occur at earlier times. But even at times where fatigue is remarkable, e.g. around $2\ 10^7$ sec, the differences between the 3 curves are considerable and disappear only for larger times.

In figure 4. the relative influence of A, which is an important parameter for T_i, and of C, which determines the inclination of the instability threshold and, therefore, the value of T_p is studied. In this example considerable differences in reliability can be observed for not too large variations of A and/or C. One has to keep in mind, however, that the adopted numerical values are crucial for the length of the different phases and, thus, also for the resulting failure probabilities. In the example, the propagation phase is approximately one half of the initiation phase and this partly explains why the parameter C seems to have less influence on $\beta(t)$ than A.

5. ADDITIONAL REMARKS

There are a number of assumptions made previously which need some discussion. First of all, most of them are made purely for analytical convenience and may be abandoned at the expense of increased numerical effort. In this sense the normality of the stress process is not an essential requirement for the approach to work. The upcrossing rate of a normal stress process and the moments of the stress ranges can only be calculated relativly easy. But the process must always have certain mixing properties.

For wide-band stress processes one has to choose a proper stress cycle counting method. Fortunately, it has been shown that a Rayleigh-distribution, with the variance of the stress process as parameter, for the stress ranges between positive zero crossings together with a certain widely empirical correction factor yields relatively good results. In any case, it is computationally comfortable if $E[\Sigma S^m]$ is linear in the time t.

Extreme value failures have been treated for a time-invariant threshold - even in the propagation phase. It is, however, possible to make the calculations more realistic. For example,

for very ductile materials the constant threshold L should be replaced by

$$L(t) = L \frac{A(a(t))}{A_0}$$

where A_0 is the initial section area and $A(a(t))$ the net section area at time t. a(t) is determined according to an appropriate crack growth relationship. In this case an upcrossing of the decreasing threshold L(t) can also occur. Then failure is caused by yielding in a reduced cross-section rather than by instability of the crack. Numerical integration and inversion of eq. (3.4) might be necessary in most applications.

In any case the initial or the constant threshold must be chosen carefully with due consideration of the behaviour of the material during the very first loading cycles. Other systematic time-dependent influences on the threshold can be treated similarly.

The Paris-Erdogan crack propagation formula is simple but not always sufficiently realistic. It can be replaced by other crack propagation formulae but attention should be given to separability and integrability in order to render further manipulations analytical as far as possible. In some cases one might wish to also include the spatial variability of material properties which can be done without serious complication, for example according to [7].

6. SUMMARY AND CONCLUSIONS

Rupture of structural components subject to repeated random loading either occurs under extreme load conditions or under less extreme or even normal load conditions but at fatigue deteriorated strength caused by the development of cracks. A complete reliability formulation for these two types of failure modes is given in the context of the so-called upcrossing approach. The formulation is adapted to the use of FORM or SORM in order to facilitate numerical calculations.

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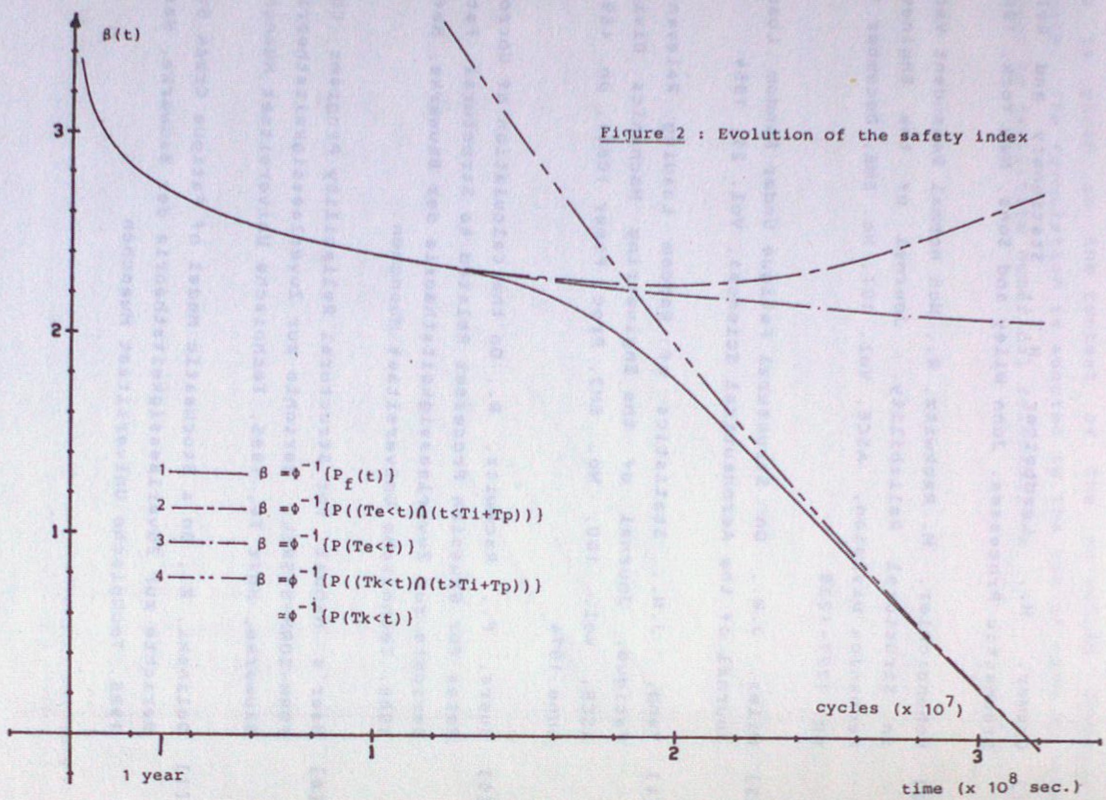

Figure 2 : Evolution of the safety index

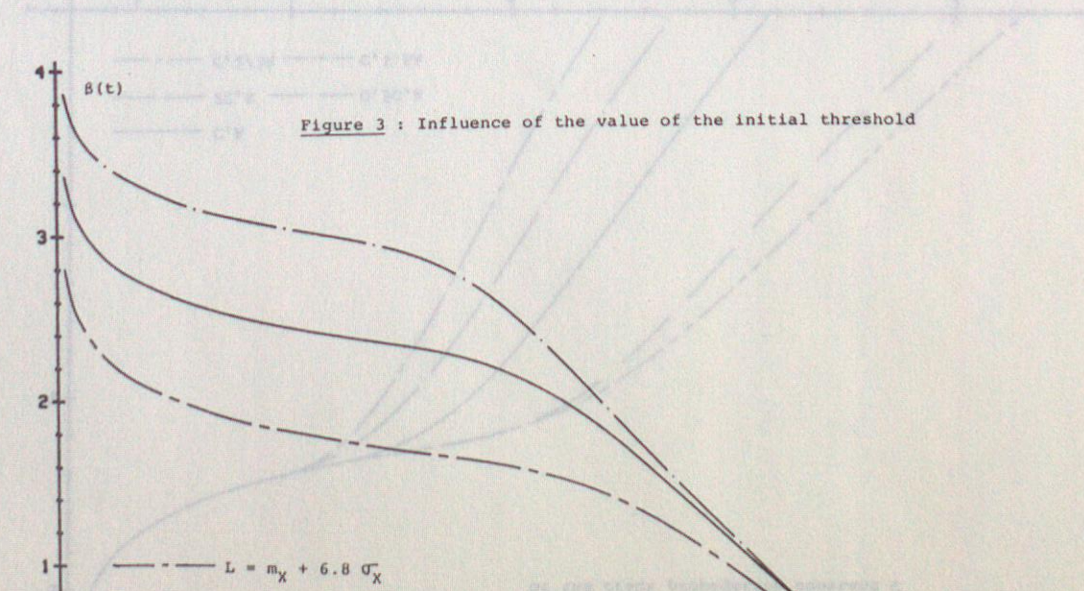

Figure 3 : Influence of the value of the initial threshold

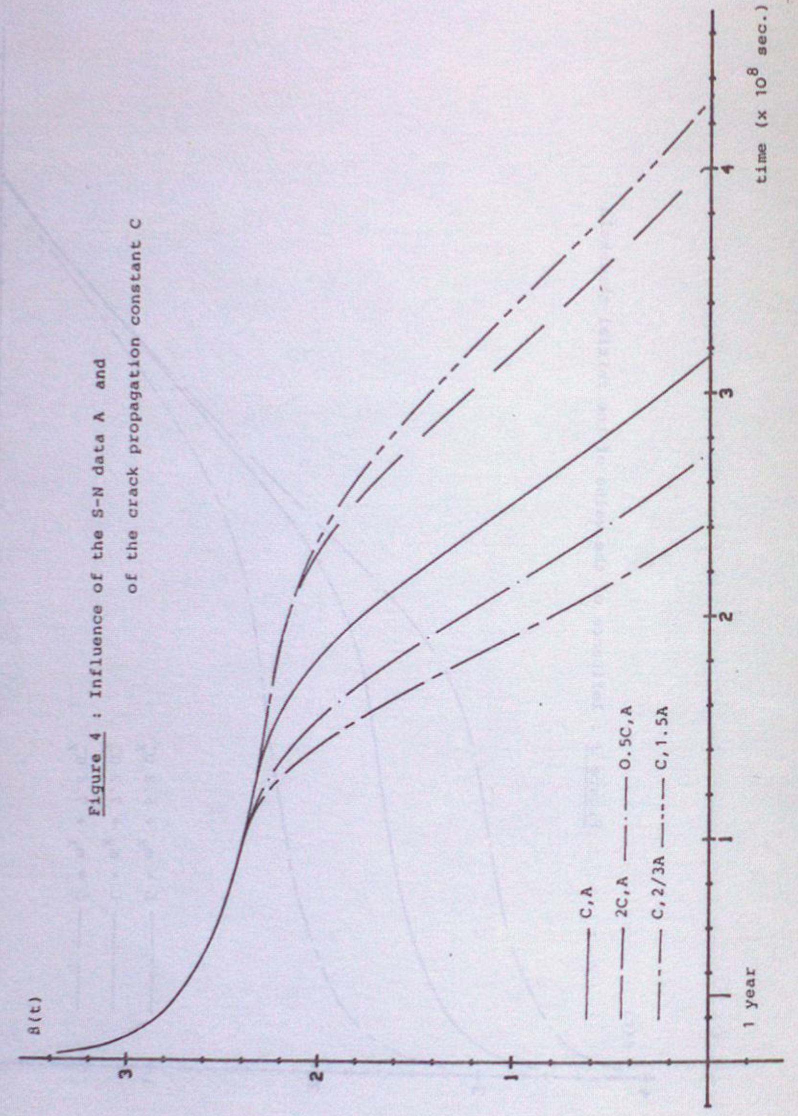

Figure 4 : Influence of the S-N data A and of the crack propagation constant C

On the Calculation of Upcrossing Rates for Narrow-Band Gaussian Processes Related to Structural Fatigue

F. Guers and R. Rackwitz

1. INTRODUCTION

Structures that are subject to cyclic loading caused, for example, by gusty wind, earthquakes or seawaves, are prone to fatigue failure. The evolution of the componential fatigue state(s) is generally described by a cumulative damage law like the one proposed by Palmgren-Miner in connection withthe well-known S-N-curves. It is, however, recognized that the crack propagation phase represents an important part of the lifetime in some cases. Then,a fracture mechanics approach to structural fatigue is more appropriate. It directly describes the crack growth phenomena. Herein, we consider a structural component subject to cyclic loading represented by a narrow-band scalar process $X(t)$. Failure is considered as the first excursion above a stress threshold $\xi(t)$ determined by fracture mechanics. The upcrossing rate is a basic information for the computation of the failure probability at a given time. This rate can be calculated for the initial process $X(t)$ or for its envelope process $R(t)$ and for a threshold that is either a deterministic function or a stochastic process [1,2]. Because little attention has been paid to the last case we propose in this paper an analytical expression when the threshold, the residual "strength", is governed by fatigue as well as two other solutions under the more general assumption of a Gaussian threshold function. A comparison with the case of a deterministic threshold is made. This leads to important conclusions for practical applications.

2. CALCULATION OF UPCROSSING RATES

Consider a Gaussian narrow-band stress process $X(t)$ with mean $m(t)$ and variance $\sigma^2(t)$. For computational convenience use is made ofthe normalized process $Y(t) = \dfrac{X(t)-m(t)}{\sigma(t)}$ whose time derivative has variance $\omega_0^2(t)$. The results to be given imply weak stationarity of $X(t)$ but they are also valid for arbitrary $X(t)$, except the

formulae (2.2.1) and (2.4.2) which at least require stationarity in the variance of $X(t)$. For easy reference and for later comparisons some well-known classical results are given first.

2.1 Upcrossing of a deterministic threshold by a Gaussian process

Consider the normalized Gaussian process $Y(t)$. The associated threshold is obtained in a similar manner as $\eta(t) = \dfrac{\xi(t)-m(t)}{\sigma(t)}$

The transformation of $X(t)$ into $Y(t)$ has made $Y(t)$ and $\dot{Y}(t)$ independent. The upcrossing rate then is [1]:

$$\nu_\xi^X(t) = \nu_\eta^Y(t) = \omega_0 \varphi(\eta) \psi(\dot{\eta}/\omega_0) \qquad (2.1.1)$$

$$\text{with} \quad \psi(x) = \varphi(x) - x\,\phi(-x)$$

where φ and ϕ are the density and the distribution function of the standard normal variable, respectively.

2.2 Upcrossing of a Gaussian threshold process by a Gaussian process

The threshold is considered as a Gaussian process. Following Bolotin [2], we form the difference $X(t)-\xi(t)$ and compute the positive zero-crossing rate. If $V(t)=Y(t)-\eta(t)$ and $\mathrm{cov}(V(t),\dot{V}(t))=0$ (this requires $\sigma(t)$ to be constant), then, it is:

$$\nu_\eta^Y(t) = \nu_0^V(t) = \frac{\sigma_{\dot{V}}}{\sigma_V} \varphi\left[\frac{m_V}{\sigma_V}\right] \psi\left[-\frac{m_{\dot{V}}}{\sigma_{\dot{V}}}\right]$$

In noting that $m_V=-m_\eta$, $m_{\dot{V}}=-m_{\dot{\eta}}$, $\sigma_V^2=1+\sigma_\eta^2$ and $\sigma_{\dot{V}}{}^2=\omega_0^2+\sigma_{\dot{\eta}}{}^2$, we arrive at:

$$\nu_{\xi}^{X}(t) = \omega_0 \left[\frac{1+\sigma_{\dot\eta}^2/\omega_0^2}{1+\sigma_\eta^2}\right]^{1/2} \varphi\left[m_\eta \frac{1}{\sqrt{1+\sigma_\eta^2}}\right] \psi\left[\frac{m_{\dot\eta}}{\omega_0} \frac{1}{\sqrt{1+\sigma_{\dot\eta}^2/\omega_0^2}}\right] \tag{2.2.1}$$

2.3 Upcrossing of a deterministic threshold by the envelope process

$Y(t)$ and $\dot Y(t)/\omega_0(t)$ are uncorrelated and normalized. The two-dimensional process has representation (in polar coordinates):

$$Y(t) = R(t) \sin\theta(t)$$

$$\frac{\dot Y(t)}{\omega_0(t)} = R(t) \cos\theta(t)$$

where $\theta(t)$ is the phase process. $R(t) = \left[Y^2(t) + \frac{\dot Y^2(t)}{\omega_0^2(t)}\right]^{1/2}$ is called the envelope process. The time derivative $\dot R(t)$ is Gaussian distributed with variance ω_R^2 which can be computed from the spectral moments $\lambda_j = \int_0^\infty \omega^j G_X(\omega)d\omega$ of $X(t)$. The following formula $\omega_R^2 = \omega_0^2 \frac{1-\alpha^2}{2\alpha^2}$, where $\alpha = \frac{\lambda_2}{\sqrt{\lambda_0\lambda_4}}$ is the regularity factor, is well known.

We also can apply another definition for $R(t)$ when $X(t)$ is a stationary process; one defines $R(t) = (Y^2(t)+\hat Y^2(t))^{1/2}$, $\hat Y(t)$ being the Hilbert transform of $Y(t)$:

$$\hat Y(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{Y(s)}{t-s} ds.$$

In contrast to $Y(t)$ and $\dot Y(t)/\omega_0(t)$, $Y(t)$ and $\hat Y(t)$ are identically distributed. Therefore, the last definition of $R(t)$ is smoother than the first one. In that case, $\omega_R^2 = \omega_0^2(1-\delta^2)$ where $\delta = \frac{\lambda_1}{\sqrt{\lambda_0\lambda_2}}$.

Independently of these envelope definitions, the upcrossing rate is [1]:

$$\nu_\eta^R(t) = \sqrt{2\pi}\, \omega_R\, \eta\, \varphi(\eta)\psi(\dot\eta/\omega_R) \tag{2.3.1}$$

2.4 Upcrossing of a Gaussian threshold process by an envelope process

For deterministic thresholds, the upcrossing rates can be calculated from Rice's formula:

$$\nu_\eta^R(t) = \int_{\dot\eta}^{\infty} f_{R\dot R}(\eta,\dot r)(\dot r-\dot\eta)\, d\dot r$$

Consider here a Gaussian threshold process independent of R. Then:

$$\nu_\eta^R(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{\dot\eta}^{+\infty} f_{R\dot R H\dot H}(\eta,\dot r,\eta,\dot\eta)\, d\dot r\, d\dot\eta\, d\eta \tag{2.4.1}$$

Remark: The integration over $\dot\eta$ should be done for the domain $]-\infty,0]$ because, by assumption, the threshold is non-increasing; the integration over η should carried out in $[0,\infty[$ because the envelope is always positive. However, the errors made by taking $-\infty$ and $+\infty$ as bounds in these integrals can be neglected.

In order to arrive at an analytical result we further make the following assumptions:

- $f_{R\dot R H\dot H} \cong f_{R\dot R}\, f_{H\dot H}$ (which can be criticised because $\dot\eta = f(r)$ for fatigue application)
- $\text{cov}(\eta(t),\dot\eta(t)) \cong 0$ (which can be verified asymptotically for fatigue applications in the case of stationarity in the variance of $X(t)$). In this case, it is: $f_{H\dot H} \cong f_H f_{\dot H}$.

For the envelope process, we have

$$f_{R\dot{R}}(r,\dot{r}) = \frac{1}{\omega_R} f_{Ray}(r)\, \varphi(\dot{r}/\omega_R)$$

where f_{Ray} is the probability density of the Rayleigh-distribution. $\eta(t)$ and $\dot{\eta}(t)$ are Gaussian distributed. Hence, eq.(2.4.1) can be expressed as:

$$\nu_\eta^R(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{\eta}^{+\infty} \frac{1}{\omega_R} f_{Ray}(\eta)\, \varphi\,(\dot{r}/\omega_R)$$

$$\times \frac{1}{\sigma_\eta}\, \varphi \left[\frac{\eta - m_\eta}{\sigma_\eta}\right] \frac{1}{\sigma_{\dot{\eta}}}\, \varphi \left[\frac{\dot{\eta} - m_{\dot{\eta}}}{\sigma_{\dot{\eta}}}\right] (\dot{r}-\dot{\eta})\; d\dot{r}\; d\dot{\eta}\; d\eta$$

After integration one obtains:

$$\nu_\eta^R(t) = \sqrt{2\pi}\;\omega_R \frac{\sqrt{1+\sigma_{\dot{\eta}}^2/\omega_R^2}}{1+\sigma_\eta^2} \frac{m_\eta}{\sqrt{1+\sigma_\eta^2}}\, \varphi \left[\frac{m_\eta}{\sqrt{1+\sigma_\eta^2}}\right] \psi \left[\frac{m_{\dot{\eta}}}{\omega_R} \frac{1}{\sqrt{1+\sigma_{\dot{\eta}}^2/\omega_R^2}}\right] \qquad (2.4.2)$$

2.5 Discussion

The improvement of 2.3.1 (resp. 2.4.2) as compared with 2.1.1 (resp. 2.2.1) depends on the values of σ_η and $\sigma_{\dot{\eta}}$ and should be verified in applications. The upcrossing rates for the envelope process and for the Gaussian process differ by:

$$\frac{\nu_\eta^R(t)}{\nu_\eta^Y(t)} = \sqrt{2\pi}\;\frac{\omega_R}{\omega_0} \left[\frac{1+\sigma_{\dot{\eta}}^2/\omega_R^2}{1+\sigma_{\dot{\eta}}^2/\omega_0^2}\right]^{1/2} \frac{m_\eta}{1+\sigma_\eta^2}\; \frac{\psi\left[\frac{m_{\dot{\eta}}}{\omega_R}(1+\sigma_{\dot{\eta}}^2/\omega_R^2)^{-1/2}\right]}{\psi\left[\frac{m_{\dot{\eta}}}{\omega_0}(1+\sigma_{\dot{\eta}}^2/\omega_R^2)^{-1/2}\right]}$$

It is seen that the envelope has more upcrossings at high threshold values than the process itself. A semi-empirical correction

(interpolation) formula due to Vanmarcke [3] might then be used with advantage.

For fatigue failure application, it will be seen from the numerical computation that $m_{\dot{\eta}}$ and $\sigma_{\dot{\eta}}$ are very small as compared with one; therefore, we can replace $\psi(x)$ by $(2\pi)^{-1/2}$ and neglect $\sigma_{\dot{\eta}}$. The improvement of eq.(2.4.2) as compared with eq.(2.2.1) can be approximated in that case by:

$$\frac{\nu_\eta^R\text{sto}}{\nu_\eta^R\text{det}} = \frac{1}{(1+\sigma_\eta^2)^{3/2}} \exp\left[\frac{\sigma_\eta^2}{2(1+\sigma_\eta^2)}\, m_\eta^2\right]$$

For small values of σ_η and $m_\eta \geq \sqrt{3(1+\sigma_\eta^2)}$, this ratio is larger than one and increasing in σ_η.

3. THRESHOLD CHARACTERISTICS FOR FATIGUE APPLICATIONS

3.1 Crack propagation model

The evolution of a crack of length a during dn cycles is assumed to follow Paris-Erdogan's law [4]:

$$\frac{da}{dn} = c\;\Delta K^m \qquad (3.1.1)$$

where c and m are material parameters. The variation of the intensity factor K during the tensile phase of the cycle is: $\Delta K = Y(a)\sqrt{\pi}\; S\; a^{1/2}$ where S is the stress range. If the geometry factor Y(a) is taken as constant, we can simplify to:

$$\Delta K = c_1 a^{1/2} S \qquad (3.1.2)$$

$C_1 = \sqrt{\pi}$

Substitution of eq.(3.1.2) in eq.(3.1.1) and integration yields for $m \neq 2$

$$a(t) = a_0 \left[1 - \frac{\frac{m-2}{2} c_1^m c}{a_0^{1-m/2}} \sum_{i=1}^{n} S_i^m \right]^{\frac{2}{2-m}} \qquad (3.1.3)$$

where a_0 is the initial crack length and n the number of cycles counted from the beginning of crack propagation.

Failure occurs if the crack becomes unstable, i.e. for $K = Y(a)\ X\ \sqrt{\pi a} = K_{ic}$ where X is the far field stress. The brittle failure threshold can then be defined as:

$$\xi(t) = \frac{K_{ic}}{c_1 \sqrt{a}} \qquad (3.1.4)$$

Inserting eq.(3.1.3) for m>2 yields

$$\xi(t) = K_1 \left[1 - K_2 \sum_{i=1}^{n} S_i^m \right]^{\frac{1}{m-2}} \qquad (3.1.5)$$

with $K_1 = \frac{K_{ic}}{c_1 \sqrt{a_0}}$ and $K_2 = \frac{m-2}{2} c_1 c\ a_0^{m/2-1}$

In order to proceed further the threshold process $\xi(t)$ is expanded such that $\xi(t)$ becomes approximately a Gaussian process for large t.

3.2 Expansion of $\xi(t)$

The threshold process $\xi(t) = K_1 \left[1 - K_2 \sum_{i=1}^{n} S_i^m \right]^{\frac{1}{m-2}}$ with argument $\sum_{i=1}^{n} S_i^m$ is expanded in the neighbourhood of $E\left[\sum_{i=1}^{n} S_i^m \right]$. One obtains:

$$\xi(t) \cong K_1 \left[1 - K_2 E\left[\sum_{i=1}^{n} S_i^m \right] \right]^{\frac{1}{m-2}}$$

$$- \frac{K_1 K_2}{m-2} \left[1 - K_2 E\left[\sum_{i=1}^{n} S_i^m \right] \right]^{\frac{1}{m-2} - 1} \left[\sum_{i=1}^{n} S_i^m - E\left[\sum_{i=1}^{n} S_i^m \right] \right]$$

The S_i's are Rayleigh distributed and for large n, $\sum_{i=1}^{n} S_i^m - E\left[\sum_{i=1}^{n} S_i^m \right]$ tends to be Gaussian distributed as $N(0, \sigma_{S_i^m} \sqrt{n})$ if the S_i's fulfill the conditions for the central limit theorem.

The second term of the development is small compared to the first one and, therefore, the threshold is "almost" deterministic for small n. For large n the second term becomes a Gaussian process as mentioned before and, hence, the threshold becomes a Gaussian process.

<u>Remark:</u> The use of eq.(3.2.1) is valid as long as

$$K_2 \left[\sum_{i=1}^{n} S_i^m - E\left[\sum_{i=1}^{n} S_i^m \right] \right]$$

is small compared to one. This assumption which will be verified numerically lateron.

3.3 Deterministic threshold

If one considers only the mean value of $\zeta(t)$ in eq.(3.2.1) the deterministic function is:

$$\zeta(t) = K_1\left[1-K_2E\left[\sum_{i=1}^{n} S_i^m\right]\right]^{\frac{1}{m-2}}$$

Because of the narrow band character of $X(t)$ one can write:

$$E\left[\sum_{i=1}^{n} S_i^m\right] \cong n\,E(S^m)$$

and, therefore,

$$\zeta(t) = K_1\left[1-K_2E(S^m)n\right]^{\frac{1}{m-2}} \qquad (3.3.1)$$

The deterministic and the process threshold differ by:

$$\frac{K_1K_2}{m-2}\left[1-K_2E\left[\sum_{i=1}^{n} S_i^m\right]\right]^{\frac{1}{m-2}-1}\left[\sum_{i=1}^{n} S_i^m - E\left[\sum_{i=1}^{n} S_i^m\right]\right]$$

This last term approximately increases with $\sqrt{n}$ but considering the remark made in section 3.2 it will be small as compared with to the deterministic term; its influence on the rate is given by eq.(2.2.1) and eq.(2.5.2).

3.4 Time derivative of the threshold

With eq.(3.1.5) $\dot\zeta(t)$ can be expressed as follows:

$$\dot\zeta(t) = -\frac{K_1K_2}{m-2}\left[1-K_2\sum_{i=1}^{n} S_i^m\right]^{\frac{1}{m-2}-1}\frac{d}{dt}\left[\sum_{i=1}^{n} S_i^m\right] \qquad (3.4.1)$$

Because each S_i represents a stress range during the tensile phase of each cycle, we assume that $\sum_{i=1}^{n} S_i^m$ is constant, and consequently $\dot\zeta(t)=0$, during each compression phase. The time interval between a minimum and the following maximum - compression phase - is almost constant and equal to $\frac{\pi}{\omega_0} = \frac{T}{2}$. Therefore, we assume

$$\frac{d}{dt}\left[\sum_{i=1}^{n} S_i^m\right] \cong \frac{\sum_{i=1}^{n} S_i^m - \sum_{i=1}^{n-1} S_i^m}{T/2} = \frac{\omega_0}{\pi} S_n^m$$

Replacing the first term of e.q. (3.4.1) by its mean value, one obtains:

$$\dot\zeta(t) = \frac{K_1K_2}{m-2}\frac{\omega_0}{\pi}\left[1-K_2E\left[\sum_{i=1}^{n} S_i^m\right]\right]^{\frac{1}{m-2}-1} S_n^m$$

$$= g(S_n,t) = c(t)\,S_n^m \qquad (3.4.2)$$

4. UPCROSSINGS OF THE FATIGUE THRESHOLD

We first reformulate eq.(2.4.1) for the special application to brittle fatigue failure.

4.1 Upcrossings of the fatigue threshold by the envelope process

It has been outlined - see eq.(2.4.2) - that $\dot\eta(t)$ depends explicitely on $R = \frac{S}{2}$. At an upcrossing point we have $r = \eta$ and, therefore, the upcrossing rate can be written as:

$$\nu_\eta^R(t) = \int_{-\infty}^{+\infty}\int_{g(\eta,t)}^{+\infty} f_{R\dot{R}H}(\eta,\dot r,\eta)\,(\dot r - g(\eta,t))\,d\dot r\,d\eta$$

Analogously to the above section the integration over η has been extended from the interval $[0,\infty[$ to $]-\infty,+\infty[$ in good approximation. Introducing $f_{RRH}(r,\dot{r},\eta) = \frac{1}{\omega_R} f_{Ray}(r)\, \varphi(\dot{r}/\omega_R)\, \frac{1}{\sigma_\eta}\, \varphi\left[\frac{\eta-m_\eta}{\sigma_\eta}\right]$ leads to:

$$\nu_\eta^R(t) = \frac{\omega_R}{\sigma_\eta}\int_{-\infty}^{+\infty} f_{Ray}(\eta)\, \varphi\left[\frac{\eta-m_\eta}{\sigma_\eta}\right] \varphi\left[\frac{g(\eta,t)}{\omega_R}\right] d\eta \qquad (4.1.1)$$

For the η values for which $f_{Ray}(\eta)\, \varphi\left[\frac{\eta-m_\eta}{\sigma_\eta}\right]$ contributes most to the integral, $g(\eta,t)$ remains small and we can adopt the expansion $\varphi(x) \cong \frac{1}{\sqrt{2\pi}} - \frac{x}{2}$; then

$$\nu_\eta^R(t) \cong \frac{\omega_R}{\sigma_\eta}\left[\int_{-\infty}^{+\infty} \eta\, \varphi(\eta)\, \varphi\left[-\frac{m_\eta}{\sigma_\eta} + \frac{1}{\sigma_\eta}\eta\right] d\eta\right.$$

$$\left. - \sqrt{2\pi}\,\frac{c(t)}{\omega_R}\int_{-\infty}^{+\infty} \eta^{m+1} \varphi(\eta)\, \varphi\left[-\frac{m_\eta}{\sigma_\eta} + \frac{1}{\sigma_\eta}\eta\right] d\eta\right]$$

in which

$$\frac{\omega_R}{\sigma_\eta}\int_{-\infty}^{+\infty} \eta\, \varphi(\eta)\, \varphi\left[-\frac{m_\eta}{\sigma_\eta} + \frac{1}{\sigma_\eta}\eta\right] d\eta = \frac{\omega_R}{1+\sigma_\eta^2}\, \frac{m_\eta}{\sqrt{1+\sigma_\eta^2}}\, \varphi\left[\frac{m_\eta}{\sqrt{1+\sigma_\eta^2}}\right]$$

is the dominating term. The second term is a correction. If one replaces m by a natural number m_o such that $m_o \le m < m_o+1$, denoted as $m_o = E[m]$, one can evaluate it as

$$\text{correction} \cong \sqrt{2\pi}\, \frac{c(t)}{(1+\sigma_\eta^2)^{(m_o+2)/2}}\, \varphi\left[\frac{m_\eta}{(1+\sigma_\eta^2)^{1/2}}\right]$$

$$\times \sum_{p=\emptyset}^{E\left[\frac{m_o+1}{2}\right]} \begin{bmatrix} m_o+1 \\ 2p \end{bmatrix} \sigma_\eta^{2p} \left[\frac{m_\eta}{(1+\sigma_\eta^2)^{1/2}}\right]^{m_o+1-2p} (2p-1)\,!!$$

where $E\left[\frac{m_o+1}{2}\right] \in N$ is defined analogously to m_o above. It is further

$$\begin{bmatrix} m_o+1 \\ 2p \end{bmatrix} = \frac{(m_o+1)\,!}{(2p)!\,(m_o+1-2p)\,!}$$

$$\text{and} \quad (2p-1)\,!! = \begin{cases} 1 & \text{if } p=0 \\ (2p-1)(2p-3)\ldots 1 & \text{if } p\ge 1 \end{cases}$$

Because $\sigma_\eta \ll 1$ and $c(t) \ll 1$, the correction term remains small for values of m between 2 and 5 as will also be shown in the later numerical example. Therefore, we finally obtain for the upcrossing rate:

$$\nu_\eta^R(t) = \frac{\omega_R}{1+\sigma_\eta^2}\, \frac{m_\eta}{\sqrt{1+\sigma_\eta^2}}\, \varphi\left[\frac{m_\eta}{\sqrt{1+\sigma_\eta^2}}\right] \qquad (4.1.2)$$

4.2 Correlation between envelope and threshold

The covariance approximately is by using the expansion for $\xi(t)$

$$\mathrm{Cov}(R(t),\eta(t)) = -\frac{1}{\sigma}\frac{K_1K_2}{m-2}\left[1-K_2E\left[\sum_{i=1}^{n} S_i^m\right]\right]^{\frac{1}{m-2}-1} 2^m\sigma^m \mathrm{Cov}\left[R(t),\sum_{i=1}^{n} R_i^m\right]$$

where $R_i = \frac{S_i}{2}$ and $R(t)=R_n$. If a correlation function of the type $\mathrm{corr}(R_i^m,R_n) = k^{|i-n|}$ $k<1$ is assumed then for n such that $k^n \ll 1$,

$$\mathrm{Cov}\left[R(t),\sum_{i=1}^{n} R_i^m\right] \simeq \sigma_{R^m}\,\sigma_R\,\frac{1}{1-k}$$

The variance of the threshold process is:

$$\sigma_\eta^2 = \frac{1}{\sigma^2}\left[\frac{K_1K_2}{m-2}\right]^2\left[1-K_2E\left[\sum_{i=1}^{n} S_i^m\right]\right]^{\frac{2}{m-2}-2}$$

$$\times\, E\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\left[S_i^m-E\left[S_i^m\right]\right]\left[S_j^m-E\left[S_j^m\right]\right]\right]$$

As before we assume $\mathrm{corr}(R_i^m,R_j^m) = \rho^{|i-j|}$. With

$$\sigma_{R^m}^2 = E\left[R^{2m}\right]-E\left[R^m\right]^2 = 2^m\left[\Gamma(1+m)-\Gamma^2(1+\frac{m}{2})\right]$$

where $\Gamma(\ )$ denotes the gamma function we obtain for values of n such that $\rho^n \ll 1$:

$$\sigma_\eta^2 = \frac{1}{\sigma^2}\left[\frac{K_1K_2}{m-2}\right]^2\left[1-K_2\,E\left[\sum_{i=1}^{n} S_i^m\right]\right]^{\frac{2}{m-2}-2} 2^{3m}\sigma^{2m}$$

$$\times\left[\Gamma(1+m)-\Gamma^2(1+\frac{m}{2})\right]\frac{3+\rho}{1-\rho}\,n$$

The correlation between envelope and threshold consequently is:

$$\mathrm{corr}\,(R(t),\eta(t)) = \frac{1}{1-k}\left[\frac{1-\rho}{3+\rho}\right]^{\frac{1}{2}}\frac{1}{\sqrt{n}}$$

for t sufficiently large. It decreases with $1/\sqrt{n}$. After a certain number of cycles, the envelope and the threshold, therefore can be taken as uncorrelated. This confirms the observation that for a regular stress process, the threshold primarily depends on the accumulated damage and only to a small degree on the instantaneous stress range.

4.3 Threshold characteristics as a function of time

For the computation of the upcrossing rates, the mean value and the variance of the threshold and its time derivative are required.

a) Threshold process

Denoting $m=m(t)$ and $\sigma=\sigma(t)$, the mean value is

$$m_\eta(t) = \frac{1}{\sigma}\left[K_1\left[1-K_2\,E\left[\sum_{i=1}^{n} S_i^m\right]\right]^{\frac{1}{m-2}} - m\right]$$

with

$$E\left[\sum_{i=1}^{n} S_i^m\right] = 2^{\frac{3m}{2}}\, n\, \Gamma(1+\tfrac{m}{2})\sigma^m \quad \text{where} \quad n \simeq \frac{\omega_0}{2\pi}\, t .$$

It follows that:

$$m_\eta(t) = \frac{1}{\sigma}\left[K_1\left[1-K_2\, 2^{\frac{3m}{2}}\, \Gamma(1+\tfrac{m}{2})\sigma^m \frac{\omega_0}{2\pi}\, t\right]^{\frac{1}{m-2}} - m\right]$$

The variance has been evaluated in section 4.1. If $\rho^{|i-j|}=\text{corr}(R_i^m,R_j^m)$ is assumed as before one easily determines:

$$\sigma_\eta^2(t) = \frac{1}{\sigma^2}\left[\frac{K_1K_2}{m-2}\right]^2\left[1-K_2 2^{\frac{3m}{2}}\, \Gamma(1+\tfrac{m}{2})\sigma^m \frac{\omega_0}{2\pi}\, t\right]^{\frac{2}{m-2}-2} 2^{3m}\sigma^{2m}$$

$$\times \left[\Gamma(1+m) - \Gamma^2(1+\tfrac{m}{2})\right] \frac{3+\rho}{1-\rho}\, \frac{\omega_0}{2\pi}\, t$$

b) <u>Time derivative of the threshold process</u>

The mean value and the variance of eq.(3.4.2) are:

$$m_{\dot\eta}(t) = -\frac{1}{\sigma}\, \frac{K_1K_2}{m-2}\, 2^{\frac{3m}{2}}\sigma^m\, \Gamma(1+\tfrac{m}{2})\, \frac{\omega_0}{\pi}\left[1-K_2 2^{\frac{3m}{2}}\sigma^m\, \Gamma(1+\tfrac{m}{2})\, \frac{\omega_0}{2\pi}\, t\right]^{\frac{1}{m-2}-1}$$

and

$$\sigma_{\dot\eta}^{2}(t) = \frac{1}{\sigma^2}\left[\frac{K_1K_2}{m-2}\, \frac{\omega_0}{\pi}\left[1-K_2\, 2^{\frac{3m}{2}}\, \Gamma(1+\tfrac{m}{2})\, \frac{\omega_0}{2\pi}\, t\right]^{\frac{1}{m-2}-1}\right]^2$$

$$\times\, 2^{3m}\sigma^{2m}\left[\Gamma(1+m)-\Gamma^2(1+\tfrac{m}{2})\right]$$

5. NUMERICAL EXAMPLE

5.1 <u>Numerical data</u>

We now study the upcrossing rates as given before in the case of a center crack in a large plate [4]. The plate has a crack whose initial size is a_0= 2mm. It is submitted to a narrow-band stationary Gaussian far field stressprocess with mean value m = 50 N/mm^2 and standard deviation σ = 25 N/mm^2. The loading mean period is T_0 = 2[s] . The correlation between two successive maxima is ρ = 0.8 and the regularity factor is α = 0.99. The geometrical parameter is assumed to be constant, i.e. $Y(a)=1$. The material constants are m=3 and $c=10^{-13}$. Crack length is in mm and stress as are in N/mm^2. The fracture toughness is K_{ic} = 2250 $N/mm^{3/2}$.

5.2 <u>Results and discussion</u>

- The rupture law $K=K_{ic}$ introduces a fatigue threshold that is interesting to be considered only if it is below the constant threshold, i.e. the threshold valid for time invariant failure criteria. If one tolerates, under due consideration of this "constant" threshold γ, a failure probability of 10^{-3} in one year (or approximately 1,6 10^7 load cycles for T_0= 2s), in the equation

$$P_f(t) = 10^{-3} = 1-\exp(-t\nu(\gamma)) \simeq t\, \frac{\omega_0}{\sqrt{2\pi}}\, \varphi(\frac{\gamma-m}{\sigma}),$$

one obtains $\gamma \simeq 8{,}8\, \sigma$ for the data given. On the other hand, the fatigue threshold at the beginning of crack propagation is:

$$\xi(0) = \frac{K_{ic}}{\sqrt{\pi a_0}} \simeq 36\, \sigma$$

Consequently, this threshold will not be of any interest as long as $\xi(t) > \gamma$. In figure 1 the evolution of crack length and threshold is demonstrated. It appears that the lifetime T_ξ where $\xi(t)$ is the relevant threshold is relatively short and can almost be neglected in comparison with the total propagation time.

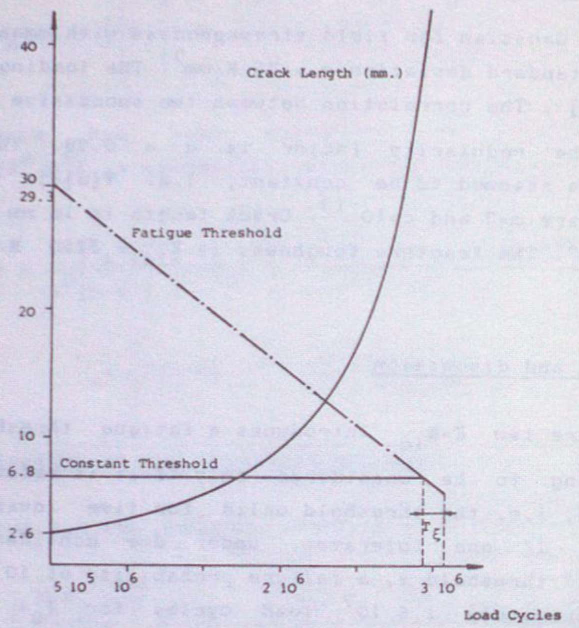

Figure 1: Threshold in the reduced space and evolution of the length of th crack

- The validity of the exapansion can be verified because

$K_2\left[\sum_{i=1}^{n} S_i^m - E\left[\sum_{i=1}^{n} S_i^m\right]\right]$ is of the order of $K_2\sigma\left[\sum_{i=1}^{n} S_i^m\right]$

which is equal to $K_2\sqrt{n}\,\sigma^m\,2^{\frac{3m}{2}}\,\Gamma(1+\frac{m}{2})$ .For example, its value for $n=4\ 10^6$ is $5\ 10^{-4}$. It is, therefore, negligible as compared to 1 as presumed.

- The correction factor in the fatigue formulation can be neglected. It is between 3% (for $\xi(t)=\gamma$) and 5% (in the failure region) of the dominating term. Consequently, eq.(4.1.1) can be simplified to eq.(4.1.2) which is identical to eq.(2.4.2) in this case. Both $\sigma_{\dot\eta}$ and $m_{\dot\eta}$ approach zero. Therefore, we can assume for fatigue applications that $\eta=0$, i.e. replace $\varphi(\ )$ by $(2\pi)^{-1/2}$ and set $\sigma_{\dot\eta}=0$.

- The improvement of eq.(2.2.1) and eq.(2.4.2) by also considering the variance of the threshold usually is of little interest from a numerical point of view as one can see from table 1.

6. SUMMARY AND CONCLUSIONS

Some well known results for the upcrossing rate of narrow-band Gaussian processes above deterministic time variant and instationary Gaussian process thresholds are reviewed and generalized to some extent with particular reference to structural fatigue. It is found that asymptotically, the dependence between the loading process and the process of stress intensity taken as the threshold process vanishes. Furthermore, the threshold process has vanishing variance and may be taken as a deterministic function equal to the mean of the threshold process.

N(x 10^6)	2,89	2,95	3,00	3,05	3,1
Threshold	6,8	6,2	5,7	5,3	4,8
2.1.1	0,548 10^{-10}	0,215 10^{-8}	0,358 10^{-7}	0,477 10^{-6}	0,510 10^{-5}
2.2.1	0,710 10^{-10}	0,268 10^{-8}	0,433 10^{-7}	0,562 10^{-6}	0,585 10^{-5}
2.3.1	0,937 10^{-10}	0,337 10^{-8}	0,518 10^{-7}	0,635 10^{-6}	0,618 10^{-5}
2.4.2	1,20 10^{-10}	0,415 10^{-8}	0,620 10^{-7}	0,738 10^{-6}	0,699 10^{-5}
Relative correction to 2.4.2 (see 2.4)	-3,3 10^{-2}	-2,2 10^{-2}	-1,5 10^{-2}	-0,95 10^{-2}	-0,6 10^{-2}

Table 1: Upcrossing rates computed with different formulae

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An approximation to upcrossing rate integrals

F. Guers and R. Rackwitz

1. Introduction

The outcrossing approach is generally considered to be efficient for the reliability calculation of structural components subjected to a load modelled as a scalar Gaussian process $X(\tau)$ with mean $\mu(\tau)$ variance $\sigma^2(\tau)$ and twice differentiable covariance function $C(\tau_1,\tau_2)$. Assuming a threshold $a(\tau)$, the upcrossing rate $\nu_a(\tau)$ has to be calculated. Then, the following asymptotic relationship for the failure probability can be used [1]:

$$P_f(t) \approx 1 - \exp[- \int_0^t \nu_a(\tau)\, d\tau] \qquad (1)$$

Herein, according to Rice [2], $\nu_a(\tau)$ is given by

$$\nu_a(\tau) = \int_{\dot{a}}^{\infty} (\dot{x} - \dot{a})\, f_{X\dot{X}}(a,\dot{x})\, d\dot{x} \qquad (2)$$

with $\dot{X} = \dot{X}(\tau)$ and $\dot{a} = \dot{a}(\tau)$ the time derivatives of $X = X(\tau)$ and $a = a(\tau)$, respectively. For eq. (1) to hold it is necessary that $X(\tau)$ is sufficiently mixing, i.e. that [1]

$$P(\{X(\tau)\le x\} \cap \{X(\tau+\theta)\le x\}) - P(X(\tau)\le x) \,.\, P(X(\tau+\theta)\le x) \to 0$$

for $\theta \to \infty$ and any τ. Closed form solutions for eq. (2) have been obtained by Cramer/Cryen [3] and others. If, in particular, the process $X(\tau)$ is normalized by

$$U(\tau) = (X(\tau) - \mu(\tau)) / \sigma(\tau) \qquad (3)$$

and the corresponding threshold by

$$\alpha(\tau) = (a(\tau) - \mu(\tau)) / \sigma(\tau) \qquad (4)$$

which makes $U(\tau)$ and $\dot{U}(\tau)$ independent, the following formula is exact [3]:

$$\nu_a(\tau) = \frac{\omega_o(\tau)}{\sqrt{2\pi}} \exp\left[-\frac{\alpha^2(\tau)}{\omega_o(\tau)}\right] \Psi\left(\frac{\dot{\alpha}(\tau)}{\omega_o(\tau)}\right) \tag{5}$$

where

$$\omega_o^2(\tau) = \frac{1}{\sigma^2(\tau)} \left[\left. \frac{\partial^2 C(\tau_1, \tau_2)}{\partial\tau_1 \partial\tau_2} \right|_{\tau_1=\tau_2=\tau} - \dot{\sigma}^2(\tau) \right] \tag{6}$$

and

$$\Psi(x) = \varphi(x) - x\,\Phi(-x) \tag{7}$$

If, however, some parameters $\underline{Q}$ determining $a(\tau)$ and/or $X(\tau)$ are uncertain with distribution function $F_{\underline{Q}}(\underline{q})$, the following formula can be derived for the total failure probability:

$$\begin{aligned} P_f(t) &= \int_{\underline{Q}} P_f(t|\underline{Q})\, dF_{\underline{Q}}(\underline{q}) \\ &\approx 1 - \int_{\underline{Q}} \exp[-\int_0^t \nu(\tau|\underline{q})\, d\tau]\, dF_{\underline{Q}}(\underline{q}) \end{aligned} \tag{8}$$

Then, the time integration but especially the (outer) probability integration can cause serious numerical problems at least for higher dimensions of $\underline{Q}$. In the following, we study several numerical approximations in suitable calculation schemes for eq. (8). In particular, asymptotic approximations for certain integrals proposed by Laplace [4] are applied to eq. (8) which, as will be seen, greatly facilitate the numerical analysis.

2. Approximations for the time integration

Consider an interval [a,b] where h(x) is a smooth continuous function and f(x) an increasing continuous function reaching its largest value at x = b. Assume first that $f'(b) \neq 0$, i.e. f(x) still would increase beyond b.

Then, for the integral

$$F(z) = \int_a^b h(x) \exp[z\, f(x)]\, dx \tag{9}$$

the following asymptotic approximation can be proved [4]:

$$F(z) \underset{z\to\infty}{\sim} \frac{h(b)}{f'(b)} \frac{\exp[z\, f(b)]}{z} \left[1 - \exp[-z\,(b-a)\, f'(b)]\right] \tag{10a}$$

$$\underset{z\to\infty}{\sim} \frac{h(b)}{f'(b)} \frac{\exp[z\, f(b)]}{z} \tag{10b}$$

This formula is obtained by introducing in (9) an expansion of f in the neighbourhood of z = b into a Taylor serie truncated after the second non-vanishing term, and by applying the mean value theorem of integration with respect to h(x).

The role of the large number z is to increase the weight of f in the neighbourhood of b, thus ensuring that exp[z f(x)] is the dominant part in the integrand. Following the ideas of Lindgren [5] and Breitung [6] to use this asymptotic result by introducing a scaling factor, one can use directly the Taylor expansion in the exponential term but retain formula (10a). The validity of this approximation will be demonstrated at a numerical example. Using eq. (5), the time integral (in eq. (1)) can be written as (conditioned on $\underline{Q}$):

$$I(t) = \int_0^t \nu_a(\tau)\, d\tau = \int_0^t \frac{\omega_o}{\sqrt{2\pi}} \Psi\left(\frac{\dot{a}}{\omega_o}\right) \exp\left[-\frac{\alpha^2}{2}\right] d\tau \tag{11}$$

For convenience of notation, reference to τ in the integrand is now being omitted. Eq. (11) is seen to be of the form (9) so that the approximations (10) can be applied for sufficiently high thresholds $\alpha(\tau)$.

The classical approximation arising from (10a) is

$$I_1(t) = \frac{h(t)}{f'(t)} \exp[f(t)]\,[1 - \exp(-t\, f'(t))] \tag{11a}$$

If, further, the second order term in the expansion is added and by using the expansion

$$\exp\left(\tau^2 \frac{f''(t)}{2}\right) \approx 1 + \tau^2 \frac{f''(t)}{2}$$

provided that $f''(t) > 0$, an even better approximation is:

$$I_2(t) = I_1(t) + h(t) \frac{\exp[f(t)]}{f'(t)} \frac{f''(t)}{[f'(t)]^2} \times \left[1 - [\,1 + t\,f'(t) + \tfrac{1}{2}(t\,f'(t))^2]\,\exp[-t\,f'(t)]\right] \tag{11b}$$

In eqs (11) it is

$$h(\tau) = \frac{\omega_0(\tau)}{\sqrt{2\pi}}\, \varphi\left(\frac{\dot{\alpha}(\tau)}{\omega_0(\tau)}\right) \tag{12}$$

and

$$f(t) = -\frac{1}{2}\alpha^2(t) \tag{13a}$$

$$f'(t) = -\alpha(t)\,\dot{\alpha}(t) \tag{13b}$$

$$f''(t) = -\dot{\alpha}^2(t) - \alpha(t)\,\ddot{\alpha}(t) \tag{13c}$$

By applying the mean value theorem of integration to $h(\tau)$, one can replace $h(t_m)$ (where t_m is an element of $[0,t]$) by $h(t)$ assuming that $h(\tau)$ is sufficiently smooth. Formulae (11a) and (11b) are the

main results of this study which are believed to be derived here for the first time.

An other case of interest is when f'' is negative at the largest point of f in $[0,t]$. Assume first that it is $\tau = t$. The following approximation can then be derived in the same manner.

$$I_3(t) = h(t) \exp[f(t)] \left[\frac{2\pi}{-f''(t)}\right]^{1/2} \exp\left[-\frac{f'(t)^2}{2\,f''(t)}\right] \times \left[\Phi\left[(-f''(t))^{1/2}\left[t - \frac{f'(t)}{f''(t)}\right]\right] - \Phi\left[\frac{f'(t)}{(-f''(t))^{1/2}}\right]\right] \tag{14}$$

This formula is especially useful for constant thresholds and for load processes which are instationary either in the variance (earthquakes, for example) or in the mean value (e.g. impact loading).

If f has a maximum at $\tau = t_m \in [0,t]$, integration over $[0,t]$ should be cut into two integrations over $[0,t_m]$ and $[t_m,t]$ and eq. (14) may be applied for $t = t_m$ with straightforward simplifications due to $f'(t_m) = 0$. Note that the maximum point has to be found by some suitable method.

In the following we study in more detail the application of eq. (11a) and (11b) which are relevant when studying structural fatigue.

3. Computation of failure probability

In order to be able to use FORM or SORM [7] for the necessary probability integration in eq. (9), we introduce an auxiliary standard normal variable U_T, i.e. we set:

$$I_3(z; a, b) = h(t_m)\, e^{z f(t_m)} \left(\frac{2\pi}{-z f''(t_m)}\right)^{1/2} \left[\Phi\left(\frac{zb - t_m}{(-f''(t_m))^{-1/2}}\right) - \Phi\left(\frac{za - t_m}{(-f''(t_m))^{-1/2}}\right)\right]$$

$$F_T(t) = P\ (T \le t) = 1 - \exp[-\ I(t)] = P\ (U_T \le u) = \phi(u) \tag{15a}$$

Thus, the random time to (first) failure can be given as

$$T = -\ I^{-1}[\ \mathrm{Ln}\ \phi(-U_T)] \tag{15b}$$

which enables us to write

$$P_f(t) = P\ (g(\underline{X}) \le 0) = P\ (T - t \le 0) \tag{16}$$

as required by FORM or SORM. The inversion of $I(t)$ usually must be carried out numerically. In noting that $\dot{I}(t) = \nu(t)$ we approximately have by iteration

$$T^{k+1} \approx T^k - \frac{I_i(T^k) + \mathrm{Ln}\ \phi(-U_T)}{\nu(T^k)} \tag{17}$$

with $i = 1,2$ denoting one of the approximations in eq. (11) for $I(t)$. It has to be noted that $I(\tau)$ is an increasing function of τ and the following equations, therefore, are equivalent:

$$T - t \le 0$$

$$\leftrightarrow \quad I(T) - I(t) \le 0 \tag{16a}$$

$$\leftrightarrow \quad \mathrm{Ln}\ \phi(-U_T) - I(t) \le 0 \tag{16b}$$

With that formulation the numerical inversion can be avoided and it will be used to check the validity of the inversion scheme (17).

Introducing now the Rosenblatt-transformation $\underline{Q} = \underline{T}_Q(\underline{U}_Q)$, the failure probability can be determined from

$$P_f(t) \approx P\ (\ T(\underline{T}_Q(\underline{U}_Q), U_T) - t \le 0)$$

$$\approx \phi(-\beta) \prod_{i=1}^{n-1} (1 - \beta\, \kappa_i)^{-1/2} \tag{18a}$$

$$\approx \phi(-\beta) \tag{18b}$$

with β the well-known safety index and κ_i the main curvatures in the β-point. As usual, eq. (18a) represents the SORM- and eq. (18b) the FORM-result. This formulation is particularly useful in the analysis of systems (see [8]). It avoids any explicit integration but is only approximate yet likely to be sufficiently accurate for small failure probabilities.

4. Numerical investigations for fatigue applications

The proposed reliability formulation (18) makes use of two numerical approximations (11) and (17) which are now tested at an example. Firstly, the approximations of eq. (11) for $I_i(t)$ ($i = 1,2$) are compared with an "exact" $I(t)$ obtained by numerical integration (see table 1). Then, the accuracy of the numerical inversion of $I(t)$ in eq. (17) is checked by comparing the results according to FORM - SORM applied to the formulations (16a) and (16b) (see table 2). Finally, it is of interest to compare the two approximations (18a) and (18b).

Assume that the threshold function is as in [9]

$$a(t) = a(0) \left[1 - \left(\frac{\gamma}{a(0)}\right)^C K\ E[S^B]\ t \right]^{1/C}$$

where $a(0)$, γ, C, K and B are given parameters and $X(t)$ a stationary, narrow-band Gaussian process with $\mu(t) = 0$, standard deviation $\sigma(t) = a(0)\ /\ N$ and $\omega_0(t) = 2\ \pi$. Further, assume that $\gamma = a(0)$ and $E[S^b] = (2\sqrt{2})^B\ \sigma^B\ \Gamma(1+B/2)$. Then,

$$\alpha(t) = \frac{a(t) - \mu(t)}{\sigma(t)} = N\,[1 - K\,(\frac{2\sqrt{2}\,a(0)}{N})^{B}\,\Gamma(1+B/2)\,t]^{1/C}$$

$$= N\,[1 - \bar{C}\,N^{-B}\,t]^{1/C}$$

where $\bar{C}$ collects all constants not depending on the parameter N. $\alpha'(t)$ and $\alpha''(t)$ are easily obtained by differentiation. Some typical results are given in the following table where the parameters are assumed deterministic with values N = 5, B = C = 12 and $\bar{C}$ = 15000 derived from [9].

From these results and the study of a number of further parameter combinations one concludes that I_1 (t) should only be used when I (t) < 10^{-2} whereas the slightly more complicated formula $I_2(t)$ is suitable for integrals as large as 0.5.

t (x100)	1.	10.	50.	75.	100.	125.	150.
$a(t)/\sigma$	5.00	4.97	4.85	4.75	4.62	4.43	4.07
$I(t)(x10^{-3})$	0.375	3.98	27.1	52.0	95.1	185.2	493.7
$I_1(t)(x10^{-3})$	0.375	3.97	26.1	47.2	77.6	99.5	263.0
$I_2(t)(x10^{-3})$	0.375	3.98	27.2	53.0	98.5	187.2	423.7

Table 1: Approximation formulae for time integration

The table 2 presents the results according to FORM and SORM for N and C being uncertain variables both following log-normal distributions specified by:

N ~ LN (6,6 x0.1) and C ~ LN (15000,15000 x 0.1).

The inversion in eq. (17) has been carried out by requiring a relative precision which is compatible with the precision in the FORM-SORM algorithm. First of all, the β^{I} values are the same for both formulations. If high numerical precision is required, particularly for the case of larger 2nd order corrections, the inversion algorithm (17), or an equivalent algorithm suitable for this task, has to be rather accurate. Then, the curvature corrections are the same for formulations (16a) and (16b). Therefore, the form of eq. (16a) is suitable if the improvement of eq. (18a) is taken into account.

t (x100)	1.	10.	50.	75.	100.	125.	150.
$a^{*}(0)/\sigma$	4.82	4.83	4.75	4.80	4.86	4.92	4.99
$I^{*}(t)(x10^{-2})$	0.09	0.93	13.4	26.5	38.3	50.1	55.6
β^{I}(16a-16b)	3.78	3.17	2.56	2.31	2.11	1.95	1.81
β^{II}(16a-16b)	3.79	3.14	2.31	2.18	1.91	1.81	1.71

Table 2: Reliability calculation for componental fatigue

Finally, the relatively large value of I(t) at the linearisation point in some cases suggest to prefer the approximation (11b) in applications.

5. Summary and conclusion

An analytical approximation for upcrossing integrals is proposed. It is successfully applied to a formulation in fatigue reliability. By avoiding a supplementary numerical integration this approximation seems to be well suited for more complex structural reliability problems to be investigated in further studies.

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