Polar-Coded Non-Coherent Communication[1]

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Outline

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Polar Codes

IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 55, NO. 7, JULY 2009 3051

Channel Polarization: A Method for Constructing Capacity-Achieving Codes for Symmetric Binary-Input Memoryless Channels

Erdal Arıkan, *Senior Member, IEEE*

***Abstract*—A method is proposed, called channel polarization, to construct code sequences that achieve the symmetric capacity $I(W)$ of any given binary-input discrete memoryless channel (B-DMC) W. The symmetric capacity is the highest rate achievable subject to using the input letters of the channel with equal probability. Channel polarization refers to the fact that it is pos-**

A. Preliminaries

We write $W : \mathcal{X} \to \mathcal{Y}$ to denote a generic B-DMC with input alphabet $\mathcal{X}$, output alphabet $\mathcal{Y}$, and transition probabilities $W(y|x)$, $x \in \mathcal{X}$, $y \in \mathcal{Y}$. The input alphabet $\mathcal{X}$ will always be $\{0, 1\}$, the output alphabet and the transition probabilities may

- They are capacity-achieving on binary memoryless symmetric (BMS) channels with low encoding/decoding complexity [Arı09].

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- They are capacity-achieving on binary memoryless symmetric (BMS) channels with low encoding/decoding complexity [Arı09].
- But successive cancellation (SC) decoding performs poorly for small blocks due to imperfect polarization.

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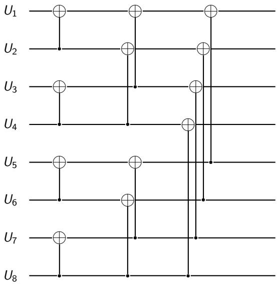
- The technique is lossless in terms of mutual information (required to achieve the capacity).
- The technique is of low complexity (there exists an encoder-decoder pair, realizing the technique with $\mathcal{O}(N \log N)$ complexity, where N is the block length).

Encoding

Let V_1^K denote the random information bits to be encoded:



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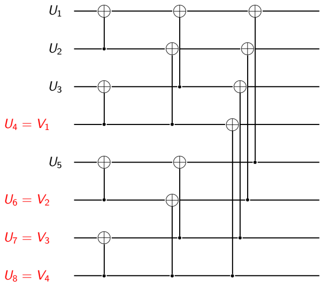

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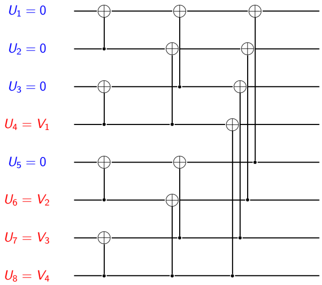

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1. For a given set $\mathcal{A}$, map V_1^K onto $U_{\mathcal{A}}$.
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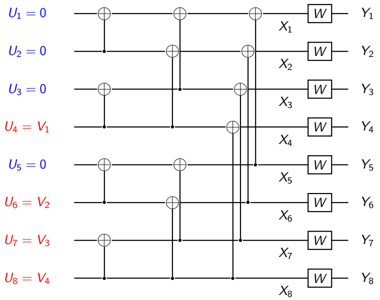
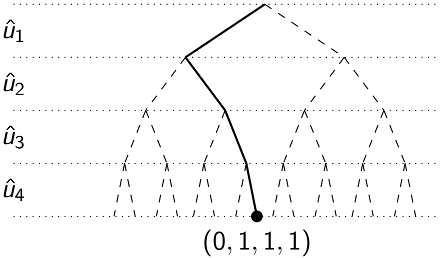
3. Apply polar transform of length$-N$, i.e.,

$$X_1^N = U_1^N \mathbf{G}_2^{\otimes \log_2 N}$$

and transmit X_1^N over the channel after suitable modulation (the figure assumes w.l.o.g. a binary-input channel).



Successive Cancellation Decoding



$$\hat{u}_i = \begin{cases} u_i & \text{if } i \in \mathcal{F} \\ f_i\left(y_1^N, \hat{u}_1^{i-1}\right) & \text{if } i \in \mathcal{A}. \end{cases}$$

$$f_i\left(y_1^N, \hat{u}_1^{i-1}\right) \triangleq \begin{cases} 0 & \text{if } p_{Y^N,U^{i-1}|U_i}(y_1^N, \hat{u}_1^{i-1}|0) \geq p_{Y^N,U^{i-1}|U_i}(y_1^N, \hat{u}_1^{i-1}|1) \\ 1 & \text{otherwise} \end{cases}$$

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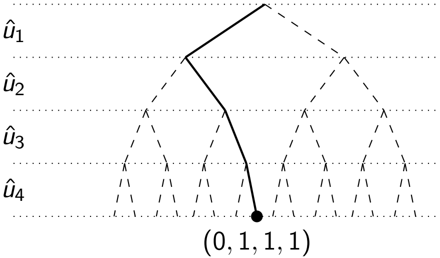

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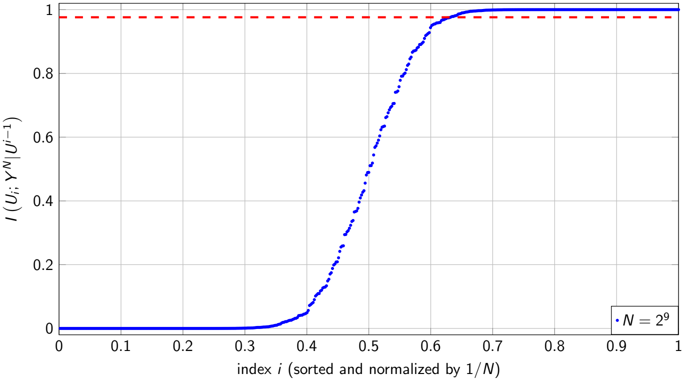
where a frame error occurs if $\hat{u}_i \neq u_i$ for any $i \in \mathcal{A} \longleftrightarrow$ imperfect channel polarization at finite-length regime!

Imperfect Channel Polarization at Finite-Length Regime



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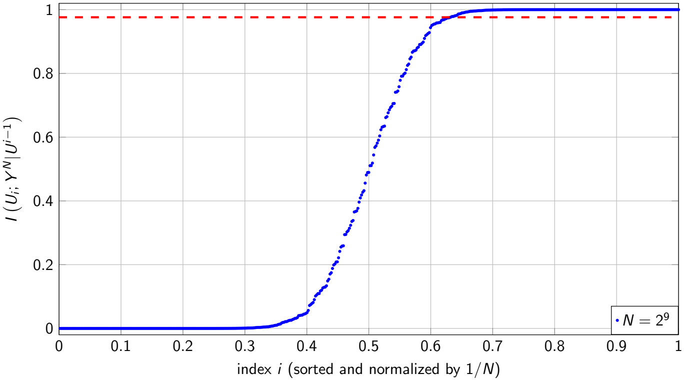

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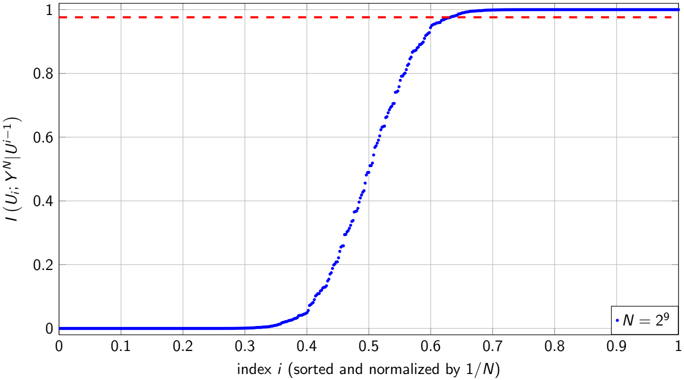

- Choose set $\mathcal{A}$ to contain the most reliable K indices.
- Any error made by SC decoding cannot be corrected $\rightarrow$ use successive cancellation list (SCL) decoding to make use of the frozen bits in reliable positions for error-correction!

Successive List Cancellation Decoding

IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 61, NO. 5, MAY 2015 2213

List Decoding of Polar Codes

Ido Tal, *Member, IEEE* and Alexander Vardy, *Fellow, IEEE*

***Abstract*—We describe a successive-cancellation list decoder for polar codes, which is a generalization of the classic successive-cancellation decoder of Arıkan. In the proposed list decoder, L decoding paths are considered concurrently at each decoding stage, where L is an integer parameter. At the end of the decoding process, the most likely among the L paths is selected as the single codeword at the decoder output. Simulations show that the resulting performance is very close to that of maximum-likelihood decoding, even for moderate values of L. Alternatively, if a genie is allowed to pick the transmitted codeword from the list, the results are comparable with the performance of current state-of-the-art LDPC codes. We show that such a genie can be easily implemented using simple CRC precoding. The specific list-decoding algorithm that achieves this performance doubles the number of decoding paths for each information bit, and then uses a pruning procedure to discard all but the L most likely paths. However, straightforward implementation of this**

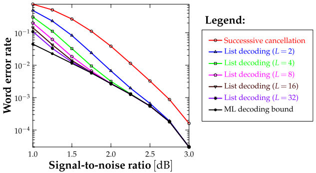

Fig. 1. List-decoding performance for a polar code of length $n = 2048$ and rate $R = 0.5$ on the BPSK-modulated Gaussian channel. The code was constructed using the methods of [15], with optimization for $E_b/N_0 = 2$ dB.

- SC list (SCL) decoding with CRC and large list-size performs very well and approaches maximum-likelihood (ML) decoding performance [TV15].

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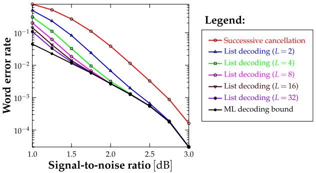

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- SC list (SCL) decoding with CRC and large list-size performs very well and approaches maximum-likelihood (ML) decoding performance [TV15].
- It can also be used to decode other codes (e.g., Reed–Muller codes, PAC codes, etc.).

Successive Cancellation List Decoding

Key idea: Each time a decision is needed on $\hat{u}_i$, both options, i.e., $\hat{u}_i = 0$ and $\hat{u}_i = 1$, are stored. This doubles the number of partial input sequences (paths) at each decoding stage.

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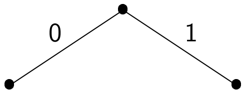

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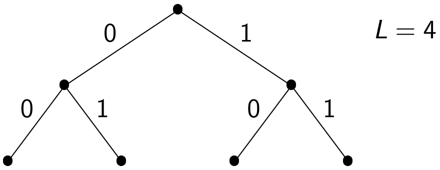

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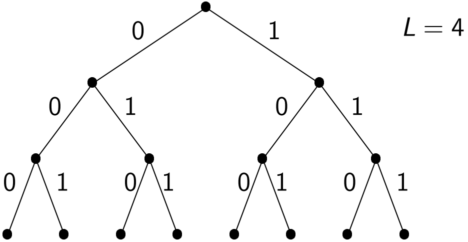

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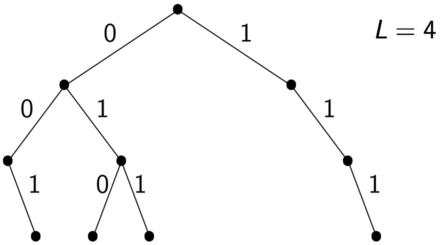

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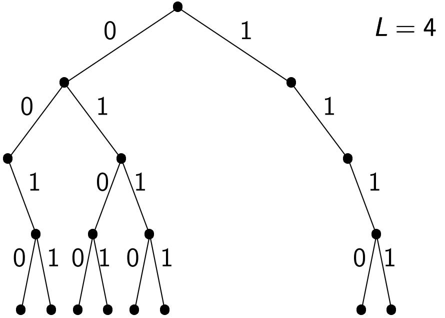

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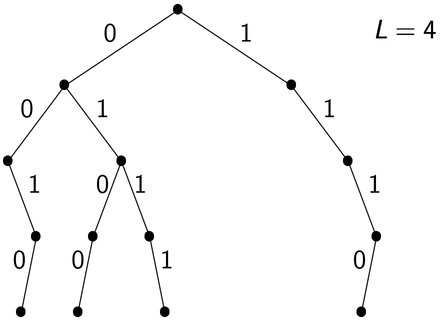

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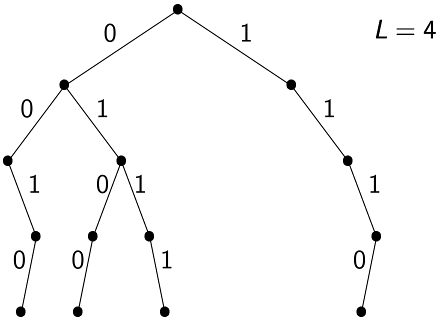

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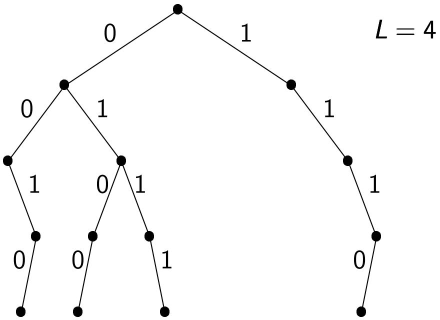

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- Very similar ideas were applied to RM codes (see, e.g., [Sto02, DS06]).

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Based on a joint work with Peihong Yuan and Gerhard Kramer (TUM) [YCK21]


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  - ✓ ... polar codes are usually used with a high-rate outer code [TV15, 5G21] that can resolve CSI ambiguities, e.g., the phase ambiguity due to all-one codeword when using QPSK and Gray labeling [IHG10].

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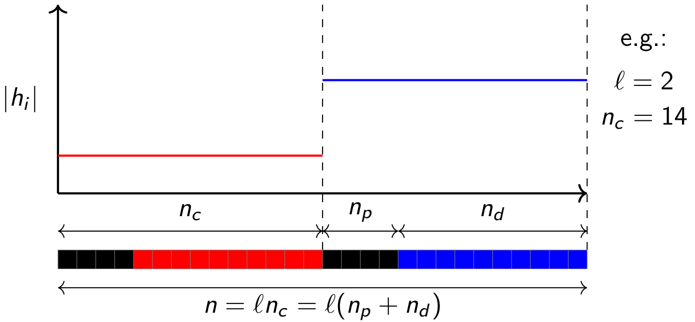
  - ✓ ... polar codes are usually used with a high-rate outer code [TV15, 5G21] that can resolve CSI ambiguities, e.g., the phase ambiguity due to all-one codeword when using QPSK and Gray labeling [IHG10].
  - ✓ ... polar codes concatenated with outer CRC codes are very competitive in short block length regime [CDJ+19], where pilot symbols cost a large overhead [ODS+19].

System Model (PAT)

- The input-output relationship of the channel is given by

$$\boldsymbol{y}_i = h_i \boldsymbol{x}_i + \boldsymbol{n}_i \qquad \text{for} \qquad i = 1, \ldots, \ell$$

where $\boldsymbol{x}_i = \left[\boldsymbol{x}_i^{(p)}, \boldsymbol{x}_i^{(d)}\right] \in \mathcal{X}^{n_c}$, $\boldsymbol{y}_i \in \mathbb{C}^{n_c}$, $H_i \sim P_H$ and $\boldsymbol{N}_i \sim \mathcal{CN}(0, \sigma^2 \boldsymbol{I}_{n_c})$.



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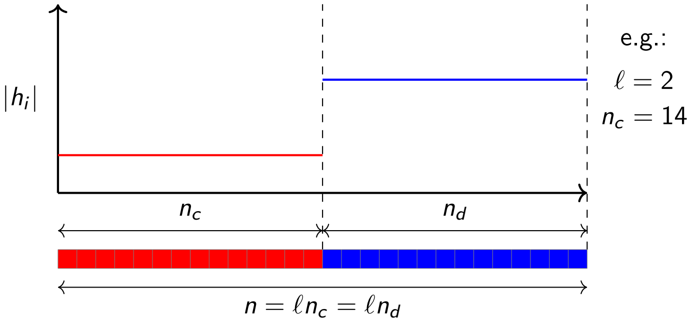
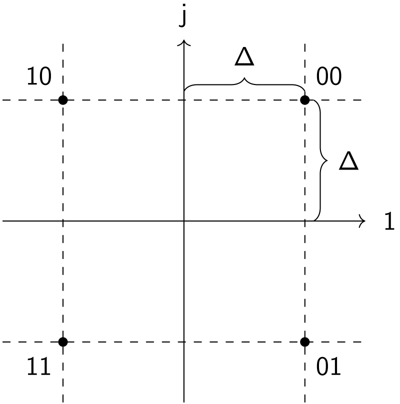


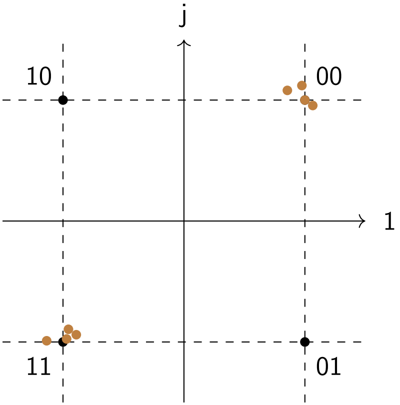
Mini Example



- $\ell = 1$
- Repetition code, $R = 0.5$

$$w_1, w_2, \ldots \mapsto w_1, w_1, w_2, w_2, \ldots$$

Mini Example



$h = 1$

- $\ell = 1$
- Repetition code, $R = 0.5$

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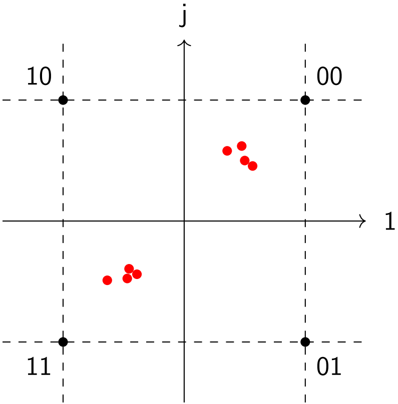

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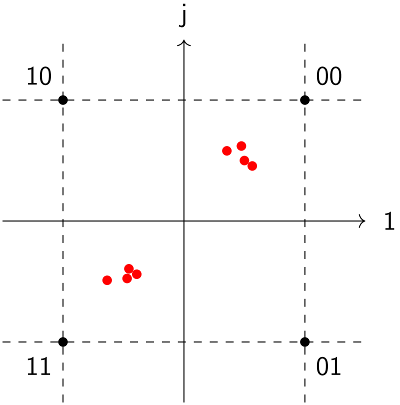

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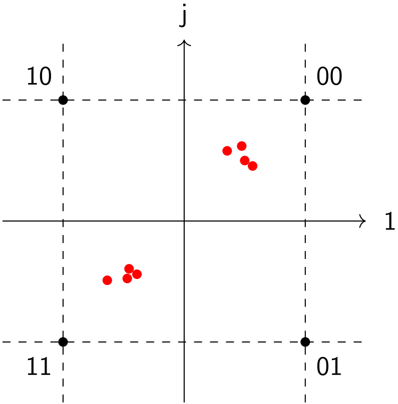

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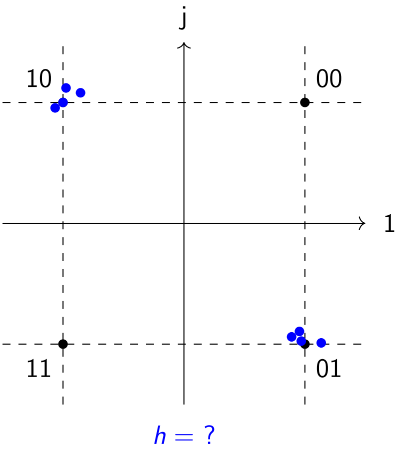

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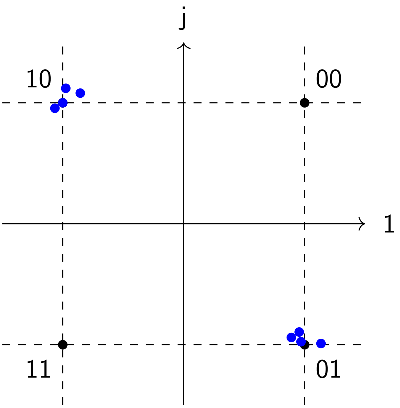

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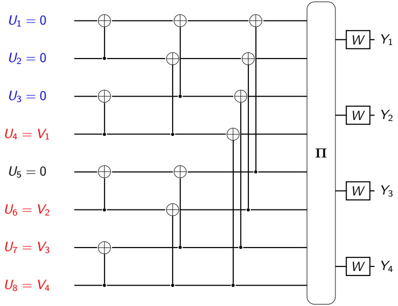

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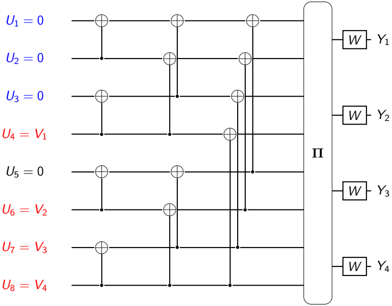
- $\ell = 1$, $\mathcal{A} = \{4, 6, 7, 8\}$ and $\mathcal{F} = \{1, 2, 3, 5\}$
- A channel estimate is obtained as

$$\hat{h} = \arg\max_{h} p_{Y_1^4|U_1^3,H_1}(y_1^4|000, h)$$

where $p_{Y_1^4|U_1^3,H_1}(y_1^4|000, h)$ can be efficiently computed via SCL decoding with $L_e = 1$ and $\beta = 3$ for any h.



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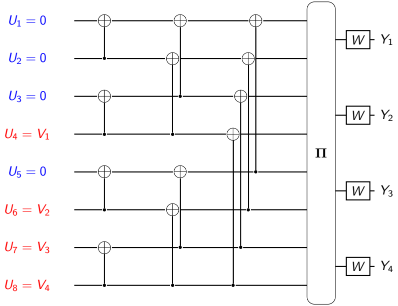
- A more accurate estimate is

$$\begin{aligned}\hat{h} &= \arg\max_{h} p_{Y_1^4|U_1^3,U_5,H_1}(y_1^4|0000, h) \\ &= \arg\max_{h} \left[p_{Y_1^4,U_4|U_1^3,U_5,H_1}(y_1^4, 0|0000, h) \right. \\ &\qquad\qquad \left. + p_{Y_1^4,U_4|U_1^3,U_5,H_1}(y_1^4, 1|0000, h)\right]\end{aligned}$$

where the cost function requires SCL with $L_e = 2$ and $\beta = 5$ for any h.



General Case

- Let $h_i = r_i e^{j\theta_i}$ where $r_i \in [0, \infty)$ and $\theta_i \in [0, 2\pi)$, $i \in [\ell]$.

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1. Estimate the amplitudes $r_i = |h_i|$ as

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2. Use the polar code constraints to estimate the phase as

$$\left\{\hat{\theta}_1, \ldots, \hat{\theta}_\ell\right\} = \underset{\{\theta_1,\ldots,\theta_\ell\}}{\arg\max}\; p_{Y_1^n | U_{\mathcal{F}^{(\beta)}}, H_1^\ell}\left(y_1^n \middle| 0, \hat{h}_1^\ell\right)$$

$$= \underset{\{\theta_1,\ldots,\theta_\ell\}}{\arg\max} \sum_{u_{\mathcal{A}^{(\beta)}}} p_{Y_1^n, U_{\mathcal{A}^{(\beta)}} | U_{\mathcal{F}^{(\beta)}}, H_1^B}\left(y_1^n, u_{\mathcal{A}^{(\beta)}} \middle| 0, \hat{h}_1^\ell\right)$$

Complexity

- The search space grows exponentially in the number of diversity branches ℓ. Although there can be other ways to reduce the complexity, the following observation halves the search space.

Corollary

Polar-coded modulations with the QPSK and Gray labeling over the channel (1) satisfy

$$p_{Y_1^n|U_{\mathcal{F}(\beta)},H_1^\ell}\left(y_1^n\left|0,h_1^\ell\right.\right)=p_{Y_1^n|U_{\mathcal{F}(\beta)},H_1^\ell}\left(y_1^n\left|0,-h_1^\ell\right.\right)$$

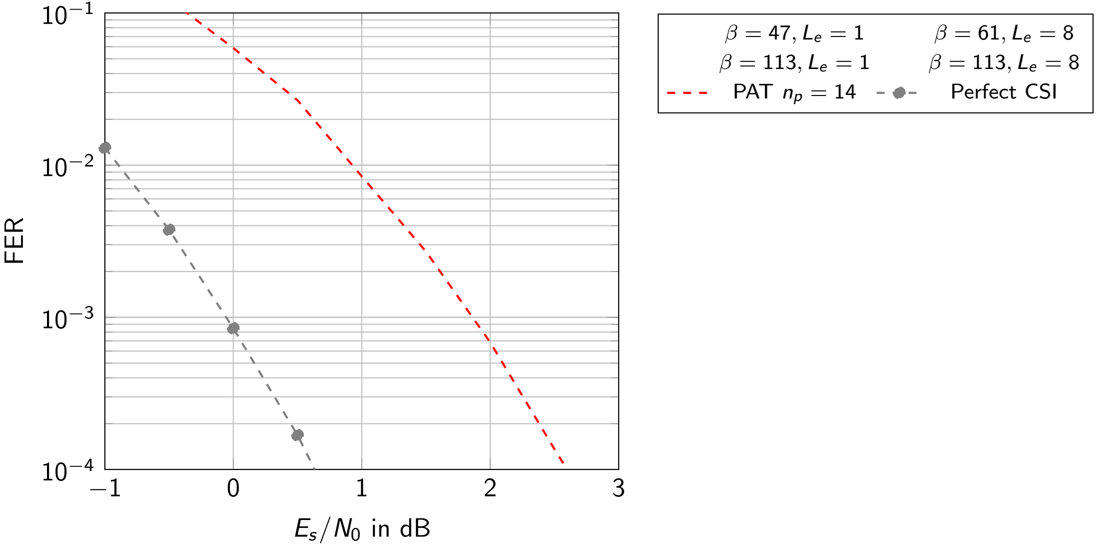
for all y_1^n and h_1^ℓ.

Outline

Example: Single Block ($\ell = 1$, $n_c = 64$)

- $y_i = e^{j\theta} x_i + z_i, \quad i = 1, \ldots, n_c = 64$ where $\theta \sim \mathcal{U}[0, 2\pi)$
- No CSIT/CSIR (including the amplitude)
- $(128, 38)$ 5G polar code with 6 bits CRC, $R = 0.5$ bpcu, Pilot-free
- Random interleaver
- QPSK (Gray)

Example: Single Block ($\ell = 1$, $n_c = 64$)



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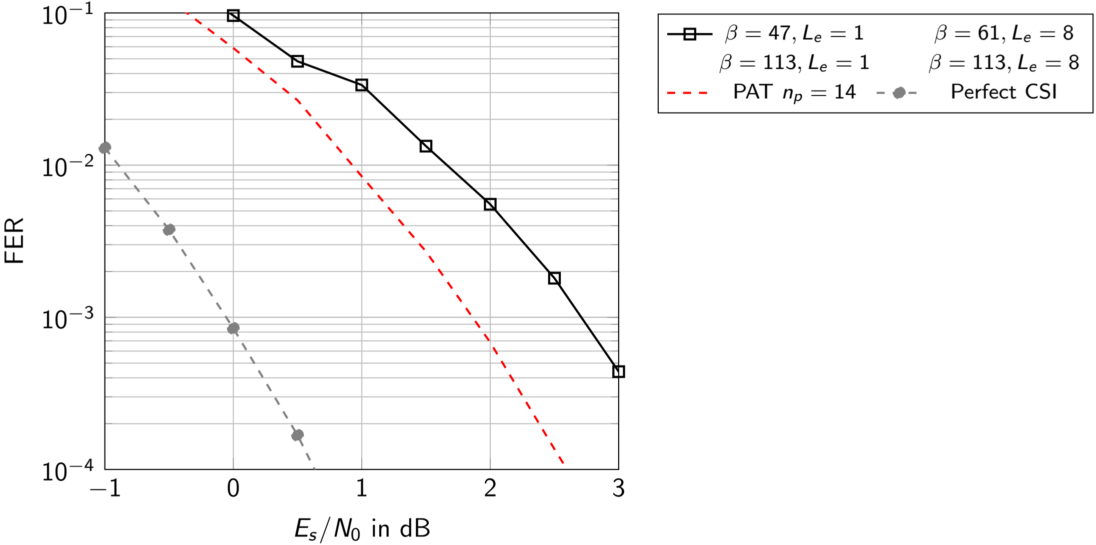

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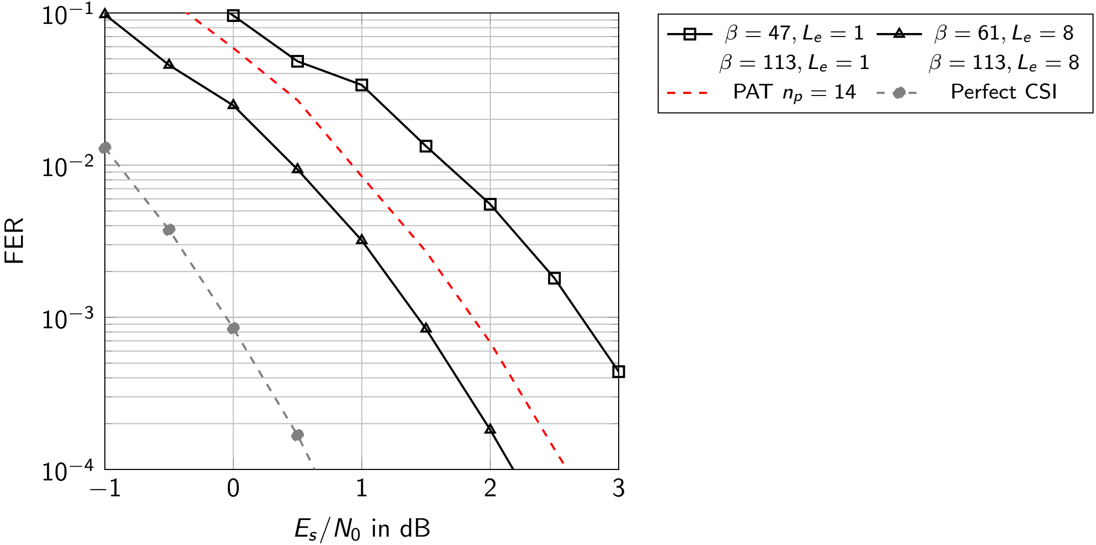

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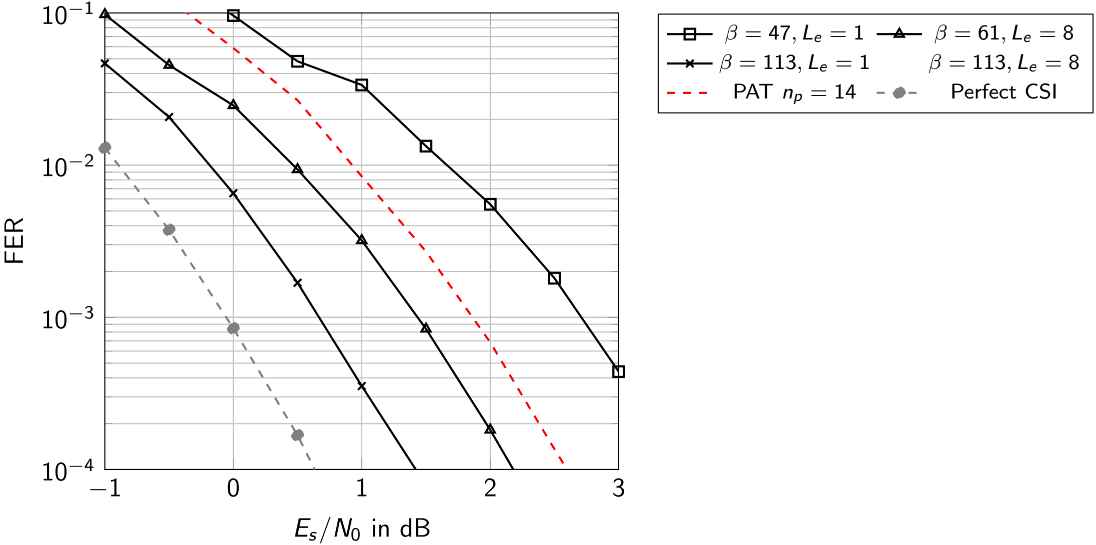

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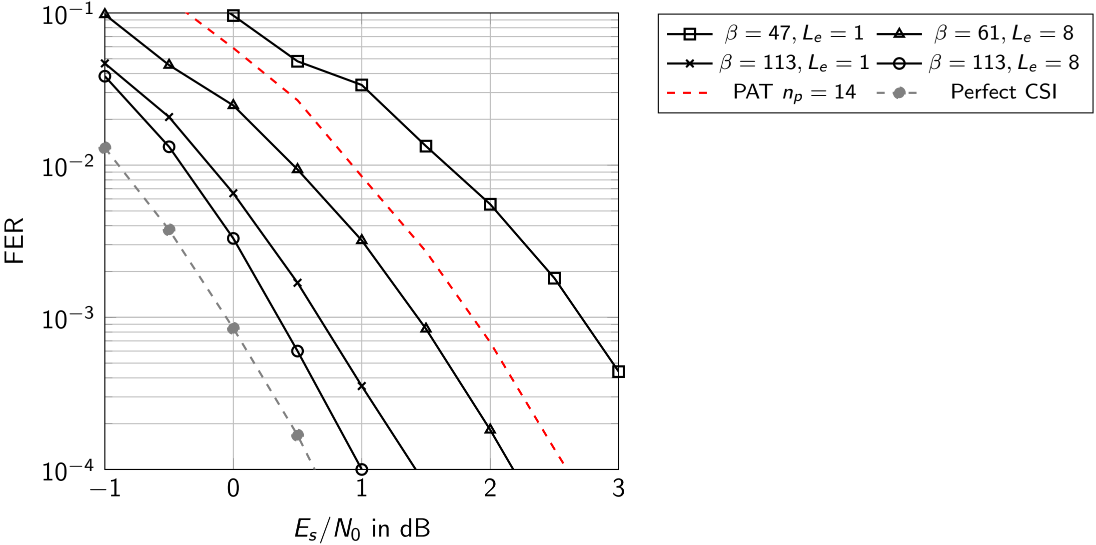

Complexity

Table 1: Number of Visited Nodes per Frame at $E_s/N_0 = 1$ dB

Method	FER	Visited Nodes
PAT $(n_p = 14,\ L = 8)$	8.43×10^{-3}	631
PAT $(n_p = 14,\ L = 32)$	3.16×10^{-3}	2223
PCT $(\beta = 47,\ L_e = 1,\ L = 8)$	3.36×10^{-2}	1383
PCT $(\beta = 61,\ L_e = 8,\ L = 8)$	3.20×10^{-3}	2151
PCT $(\beta = 113,\ L_e = 1,\ L = 8)$	3.50×10^{-4}	2439
PCT $(\beta = 113,\ L_e = 8,\ L = 8)$	1.00×10^{-4}	8807
Perfect CSI $(L = 8)$	2.40×10^{-5}	631

$8 + 8$ coarse-fine search for the optimization

$$\hat{\theta} = \arg\max_{\theta} p_{Y_1^n | U_{\mathcal{F}_\beta} H}\left(y_1^n \middle| 0, \hat{r}e^{j\theta}\right)$$

Example: Rayleigh Block-Fading Channel ($\ell = 2,\ n_c = 32$)

- $\boldsymbol{y}_i = h_i \boldsymbol{x}_i + \boldsymbol{n}_i, \quad i = 1, 2$ where $H_i \sim \mathcal{CN}(0, 1)$ and $\boldsymbol{N}_i \sim \mathcal{CN}(0, \sigma^2 \boldsymbol{I}_{n_c})$.
- No CSIT/CSIR
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(Courtesy of Dr. A. Lancho (Chalmers, MIT))

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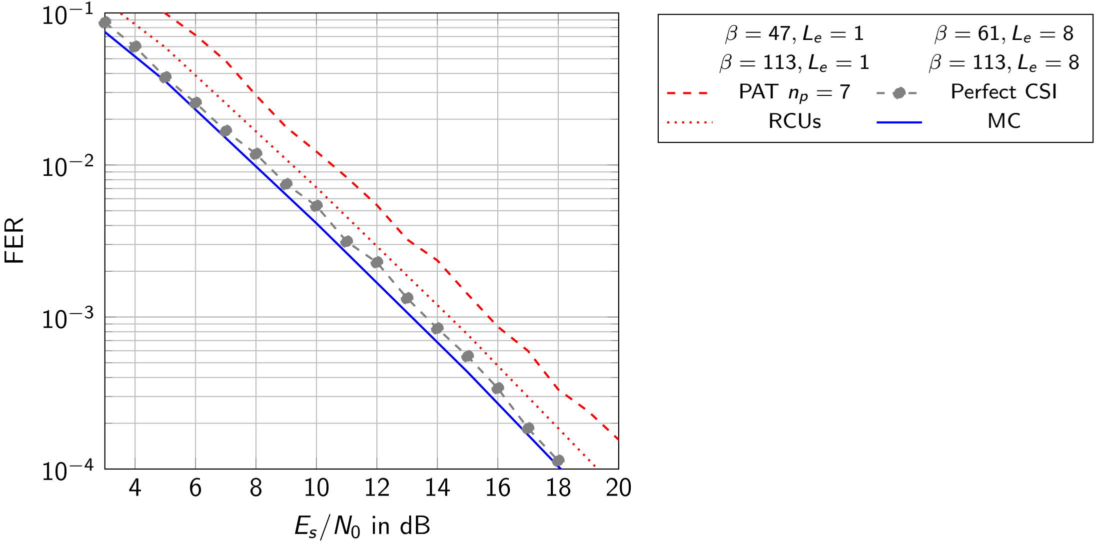

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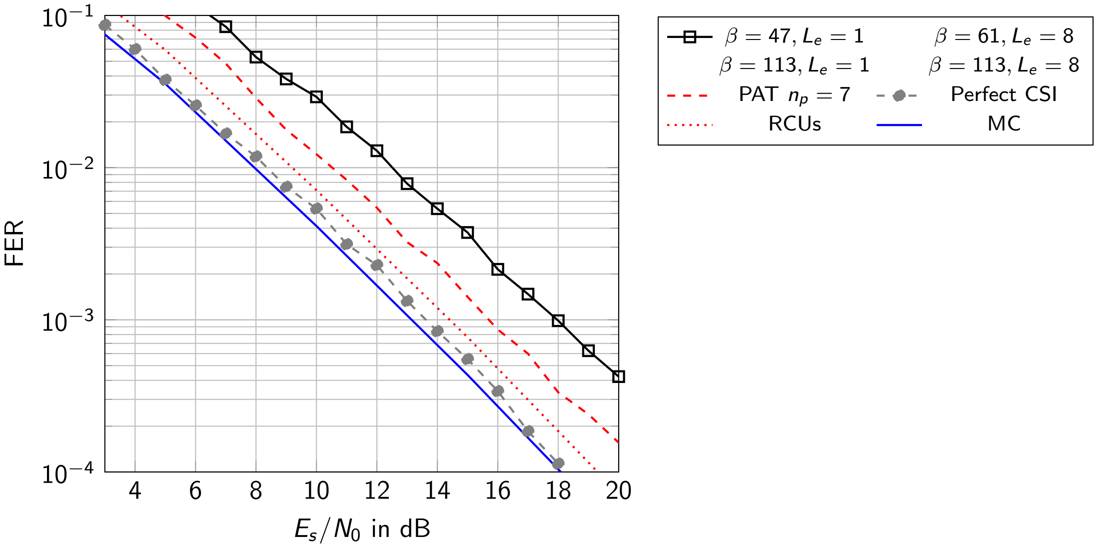

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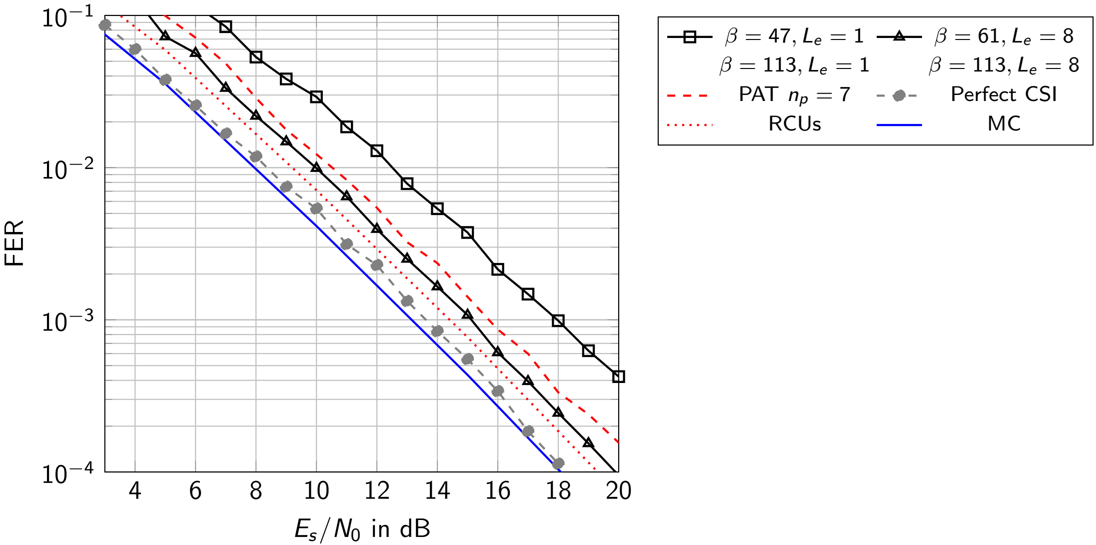

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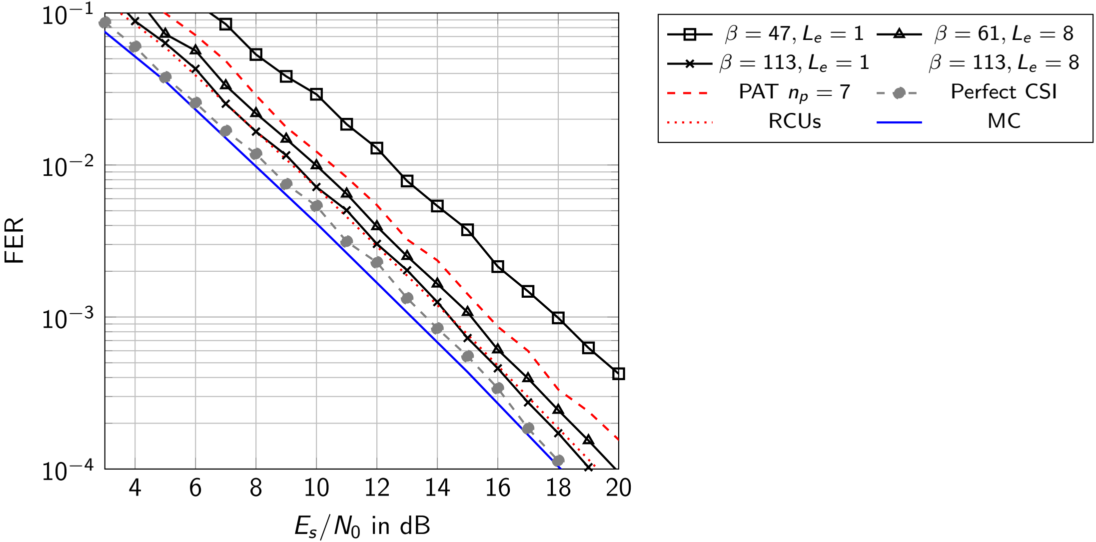

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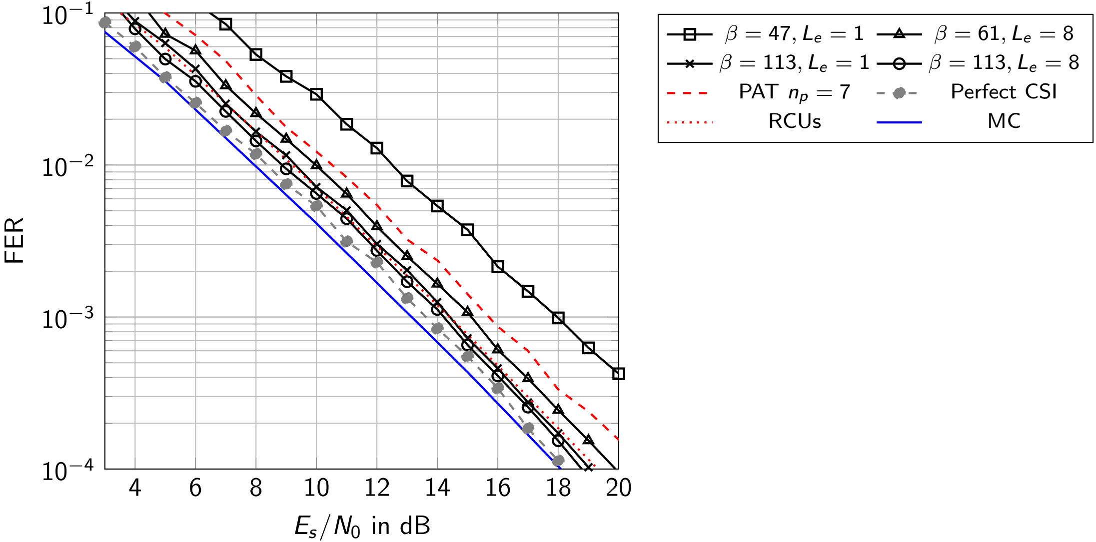

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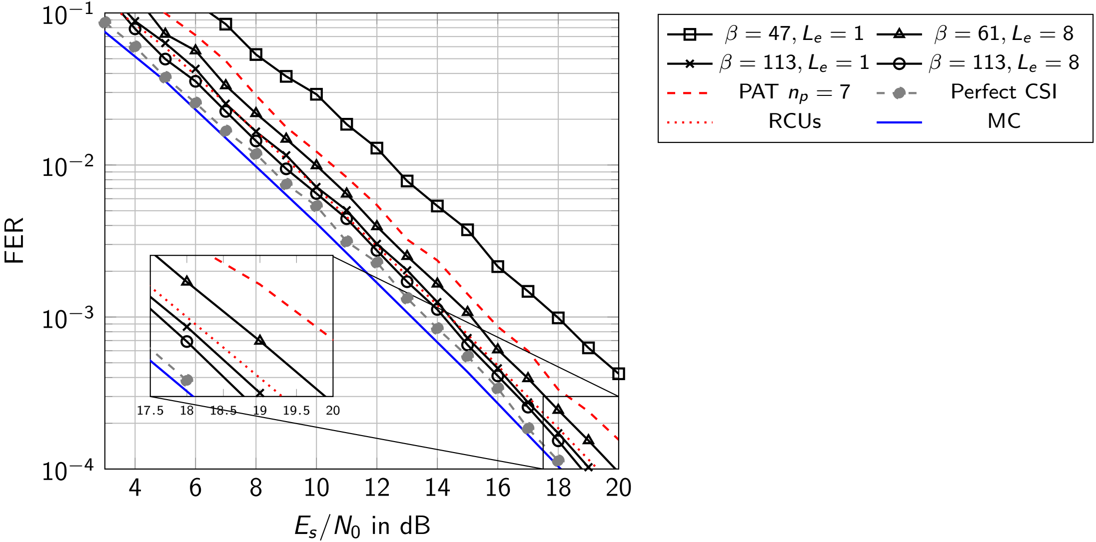

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  - ... freezing reliable bit positions could improve the channel estimation, and this may be reflected in an overall performance gain.

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