Deutsches Geodätisches Forschungsinstitut (DGFI-TUM)
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Development of an empirical thermospheric density model to improve the precise orbit determination of low-Earth-orbiting satellites

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DynamicEarth SPP 1788

TIPOD
Development of High-Precision Thermosphere Models for Improving Precise Orbit Determination of Low-Earth-Orbiting Satellites

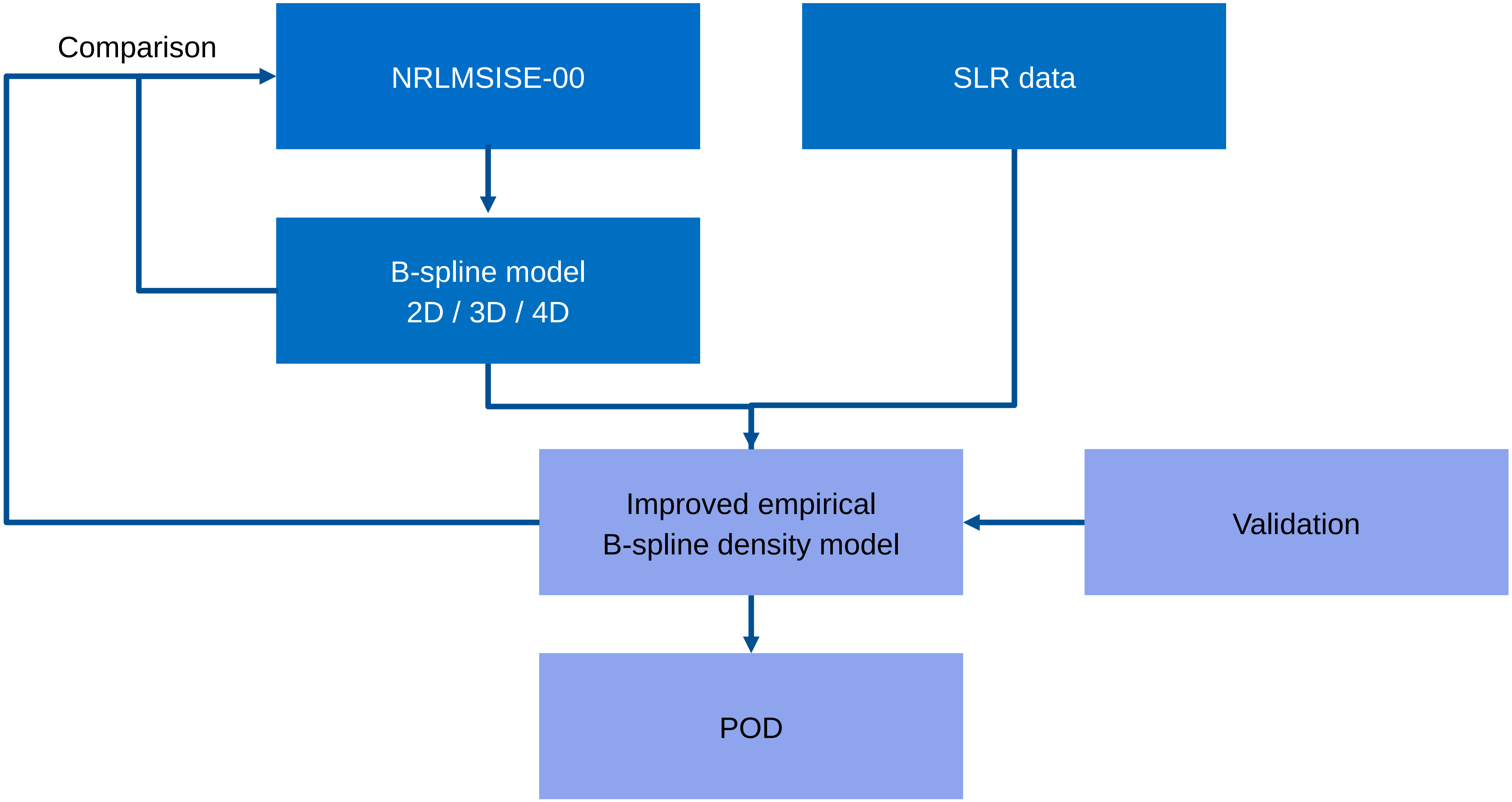
Motivation (1)

- For many geoscientific applications such as satellite altimetry and gravity missions, and precise orbit determination (POD), accurate information of the thermospheric density is of crucial importance.
- The aerodynamic acceleration $\boldsymbol{a}_{\mathrm{aero}}$ is the largest non-gravitational acceleration in the equation of motion:

$$\boldsymbol{a}_{\mathrm{aero}} = -\frac{1}{2}\frac{A_{\mathrm{ref}}}{m}\boldsymbol{c}_{\mathrm{aero}}\,\rho_{\mathrm{M}}\,v_{\mathrm{rel}}^{2}$$

- Low-Earth orbiting (LEO) satellites are sensitive to $\boldsymbol{a}_{\mathrm{aero}}$ so that accelerometers or trackers can be used to derive information about the density ρ_{M}.
- Satellites above 550 km are usually not equipped with accelerometers but a study proved that SLR (satellite laser ranging) measurements are a reliable complement to accelerometer measurements to obtain thermospheric density information (Zeitler et al. 2021).
- Thermosphere models (e.g., NRLMSISE-00) are used in POD to describe the interaction between satellite and atmosphere.
- The framework of the study is the DFG-funded project TIPOD (Development of High-Precision Thermosphere Models for Improving Precise Orbit Determination of Low-Earth-Orbiting Satellites).
- Our goal: …

Motivation (2)


Comparison

NRLMSISE-00

SLR data

B-spline model
2D / 3D / 4D

Improved empirical
B-spline density model

Validation

POD

Thermosphere model NRLMSISE-00 (1)

- Empirical thermosphere model
- Successor of the MSIS-90 model (Picone et al., 2002)

INPUT

Information about
- solar activity (F10.7 cm index)
- geomagnetic activity (AP index)

Epoch
Position
Local apparent solar time

Naval Research

Laboratory Mass

Spectrometer and

Incoherent Scatter

Radar Exosphere

(NRLMSISE-00)

OUTPUT

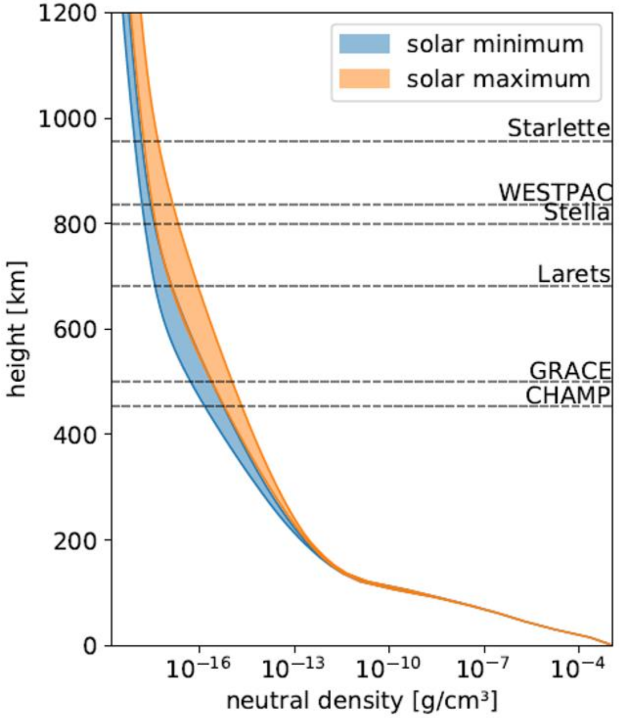
Total mass density

Temperature

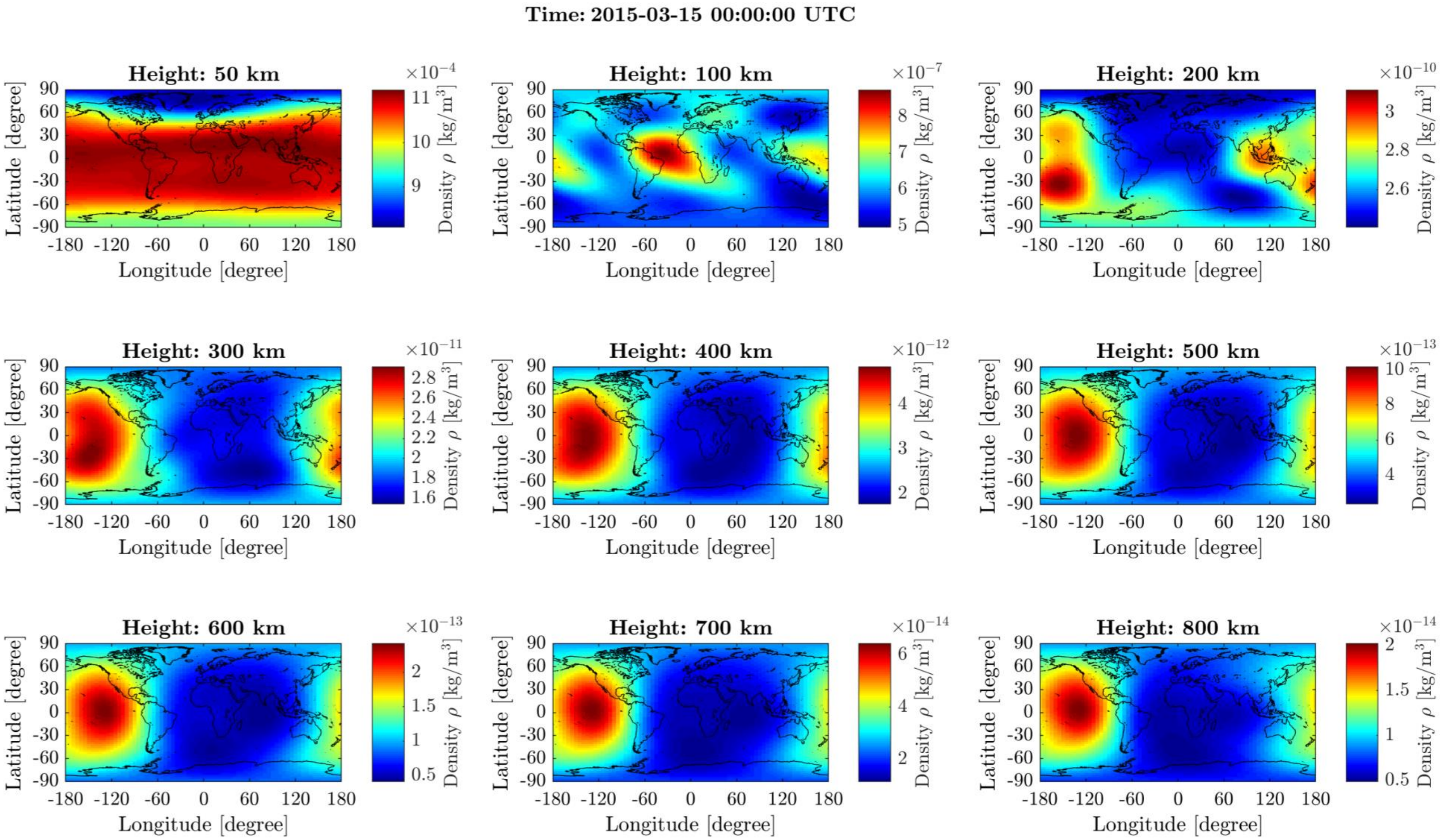
Gas density (H, HE, N, O, N2, O2, AR)

Thermosphere model NRLMSISE-00 (2)

Left: exponential decay of the density with height
Right: Spatial structure of the density by height



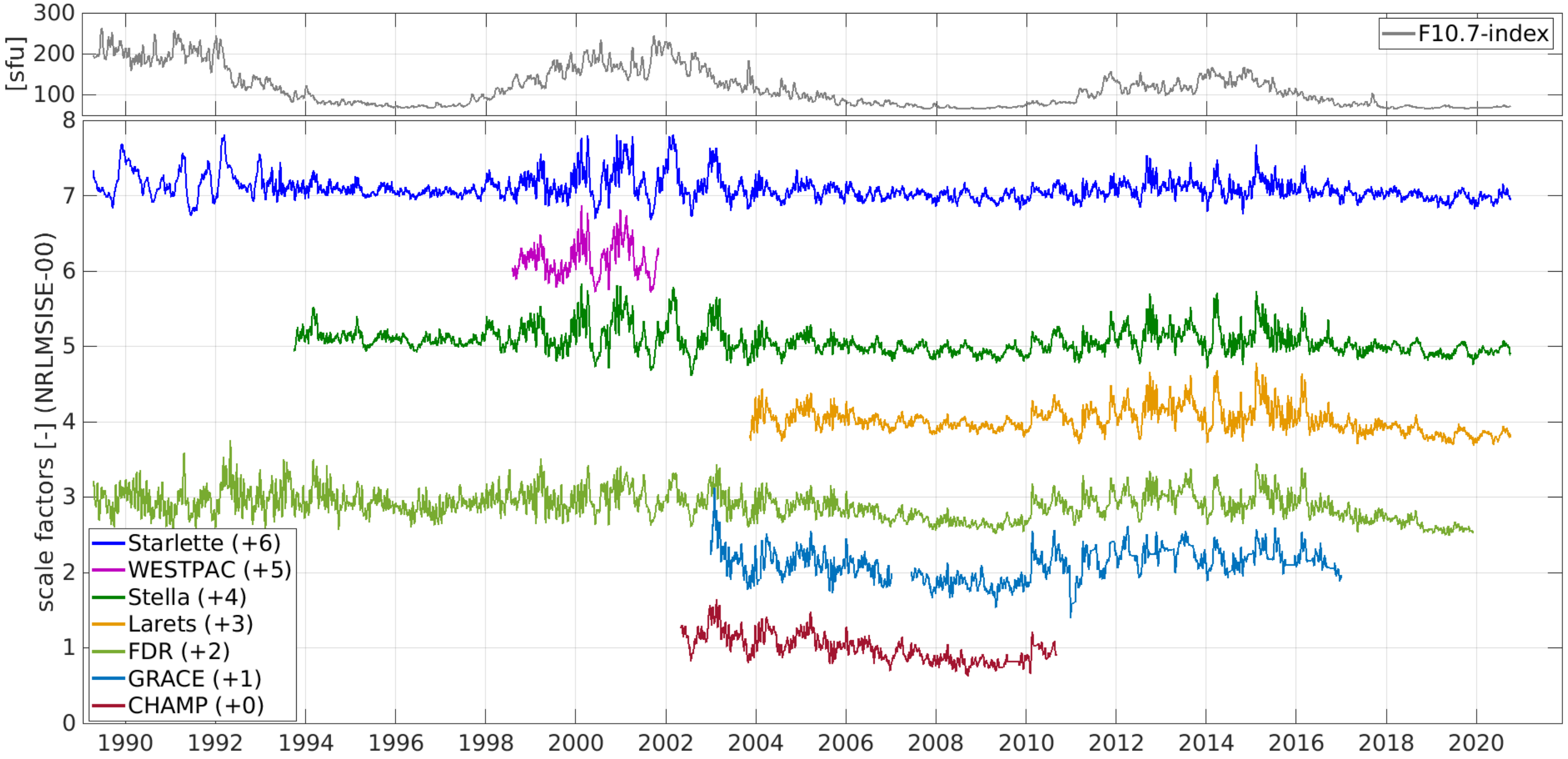
Zeitler et al., 2021



Comparison of accelerometer and SLR data (1)

Research question: Do SLR measurements to LEO satellites provide a suitable complement to accelerometer measurements to obtain thermospheric density information? (Zeitler et al., 2021)

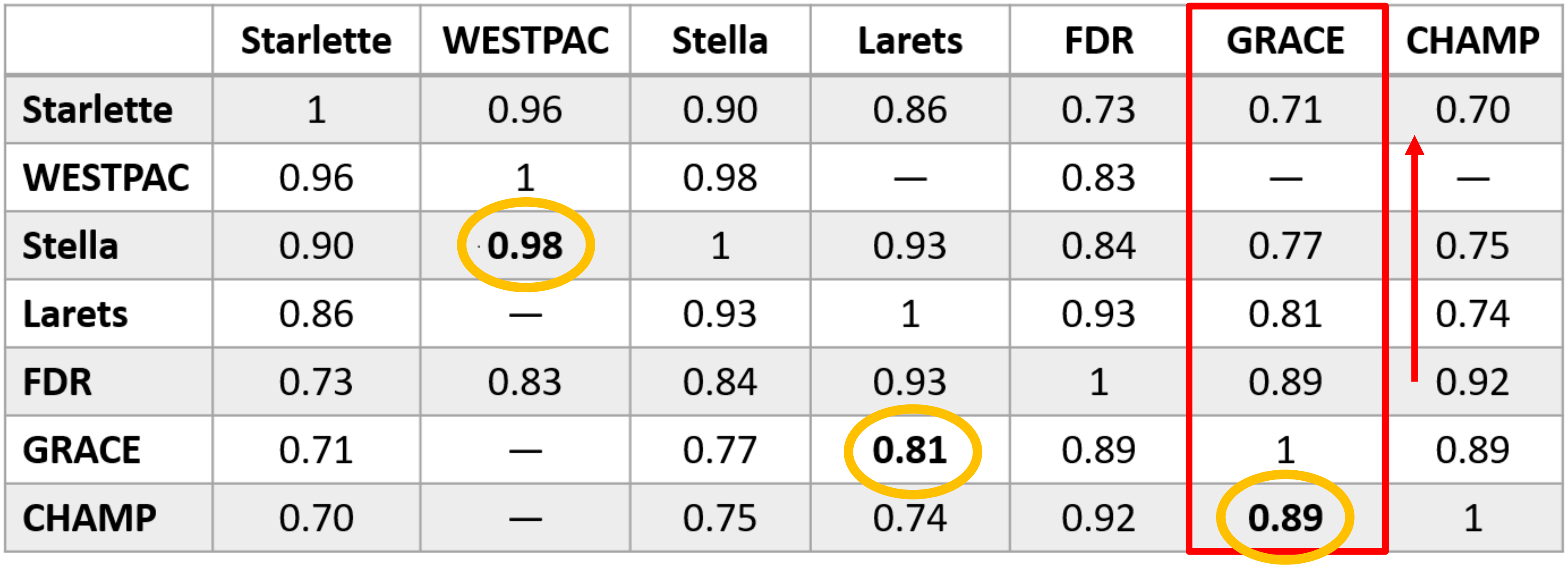
Top panel: solar activity (F10.7 cm index)
Bottom panels: estimated scale factors within the POD using the NRLMSISE-00 model. This indicates to what extent the model density values differ from the measured thermospheric densities derived from SLR and accelerometer measurements.



Zeitler et al., 2021

Comparison of accelerometer and SLR data (2)

The table below shows the correlation coefficients between each two satellite-specific scale factor time series.



	Starlette	WESTPAC	Stella	Larets	FDR	GRACE	CHAMP
Starlette	1	0.96	0.90	0.86	0.73	0.71	0.70
WESTPAC	0.96	1	0.98	—	0.83	—	—
Stella	0.90	**0.98**	1	0.93	0.84	0.77	0.75
Larets	0.86	—	0.93	1	0.93	0.81	0.74
FDR	0.73	0.83	0.84	0.93	1	0.89	0.92
GRACE	0.71	—	0.77	**0.81**	0.89	1	0.89
CHAMP	0.70	—	0.75	0.74	0.92	**0.89**	1

Zeitler et al., 2021

- Estimated scale factor time series derived from SLR and accelerometer data agree very well.
- SLR data is a suitable complement to accelerometer data.

B-spline expansion (1)

- Representation of a spatial signal using mathematical expressions
- Alternative to spherical harmonics

$$\boxed{\rho_M(\boldsymbol{x}_{i_s}, t_s) + e(x_{i_s}, t_s)} = \sum_{k_1=0}^{K_{J_1}-1} \sum_{k_2=0}^{K_{J_2}-1} \boxed{d_{k_1,k_2}^{J_1,J_2}(t_s)} \; \boxed{\phi_{k_1,J_1}(\varphi_{i_s})\, \tilde{\phi}_{k_2,J_2}(\lambda_{i_s})}$$

global 3D function

unknown coefficients

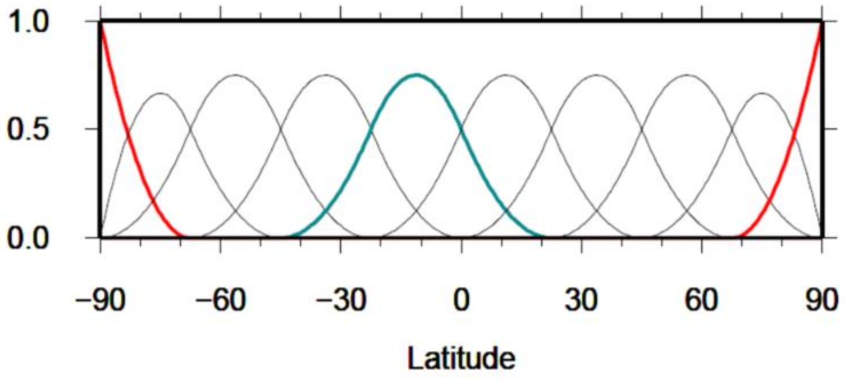
2D basis functions

Polynomial B-splines

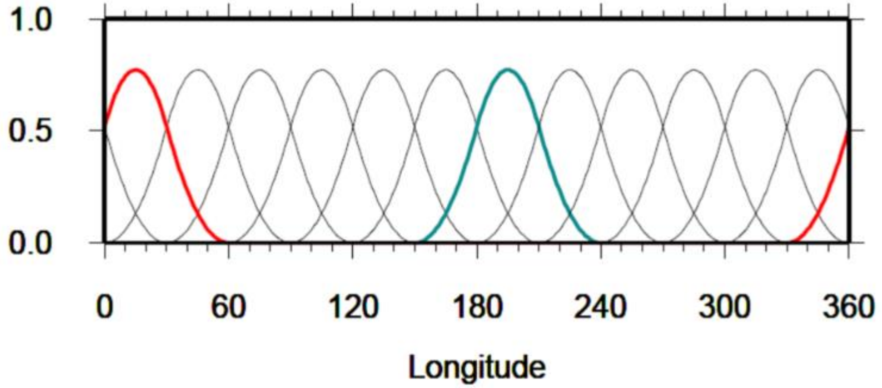
- Representation of latitude-dependent variations
- Number of B-splines: $K_{J_1} = 2^{J_1} + 2$

Trigonometric B-splines

- Representation of longitude-dependent variations
- Number of B-splines: $K_{J_2} = 3 \cdot 2^{J_2}$
- First and last B-spline require “wrapping around” effect



Goss et al., 2020



B-spline expansion (2)

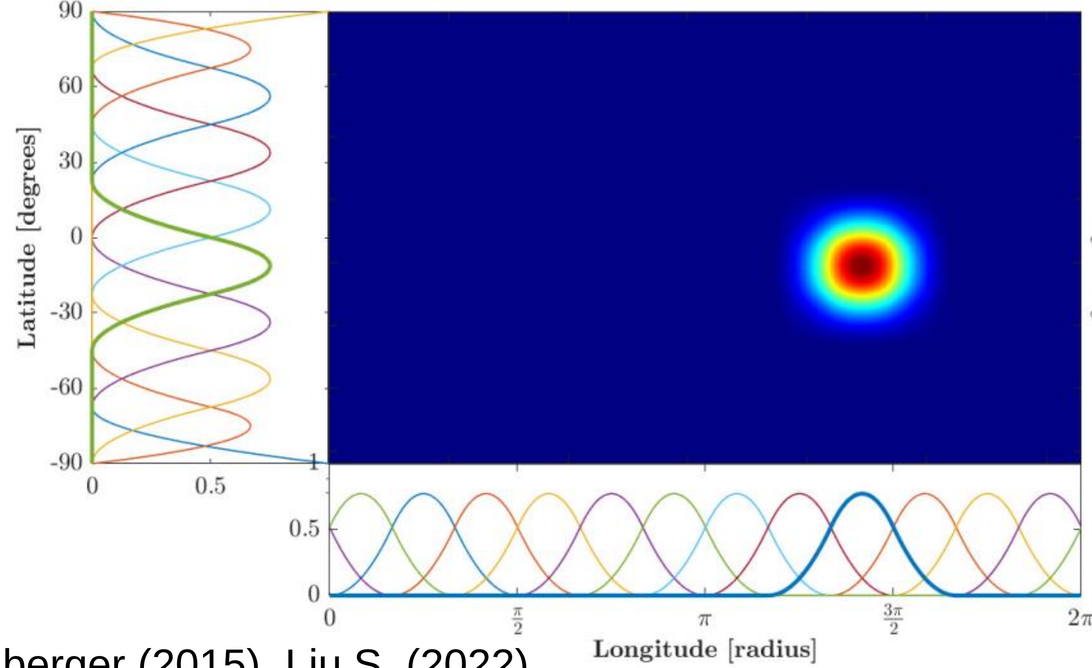
- Representation of a spatial signal using mathematical expressions
- Alternative to spherical harmonics

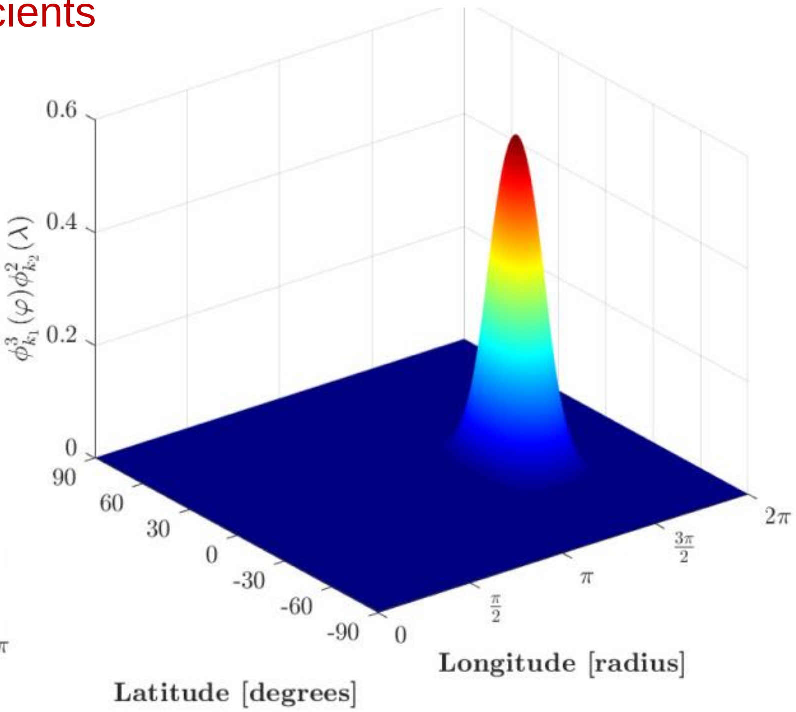
$$\boxed{\rho_M\left(\boldsymbol{x}_{i_s},t_s\right)+e\left(x_{i_s},t_s\right)} = \sum_{k_1=0}^{K_{J_1}-1}\sum_{k_2=0}^{K_{J_2}-1} \boxed{d_{k_1,k_2}^{J_1,J_2}(t_s)}\ \boxed{\phi_{k_1,J_1}\left(\varphi_{i_s}\right)\tilde{\phi}_{k_2,J_2}\left(\lambda_{i_s}\right)}$$

global 3D function

unknown coefficients

2D basis functions

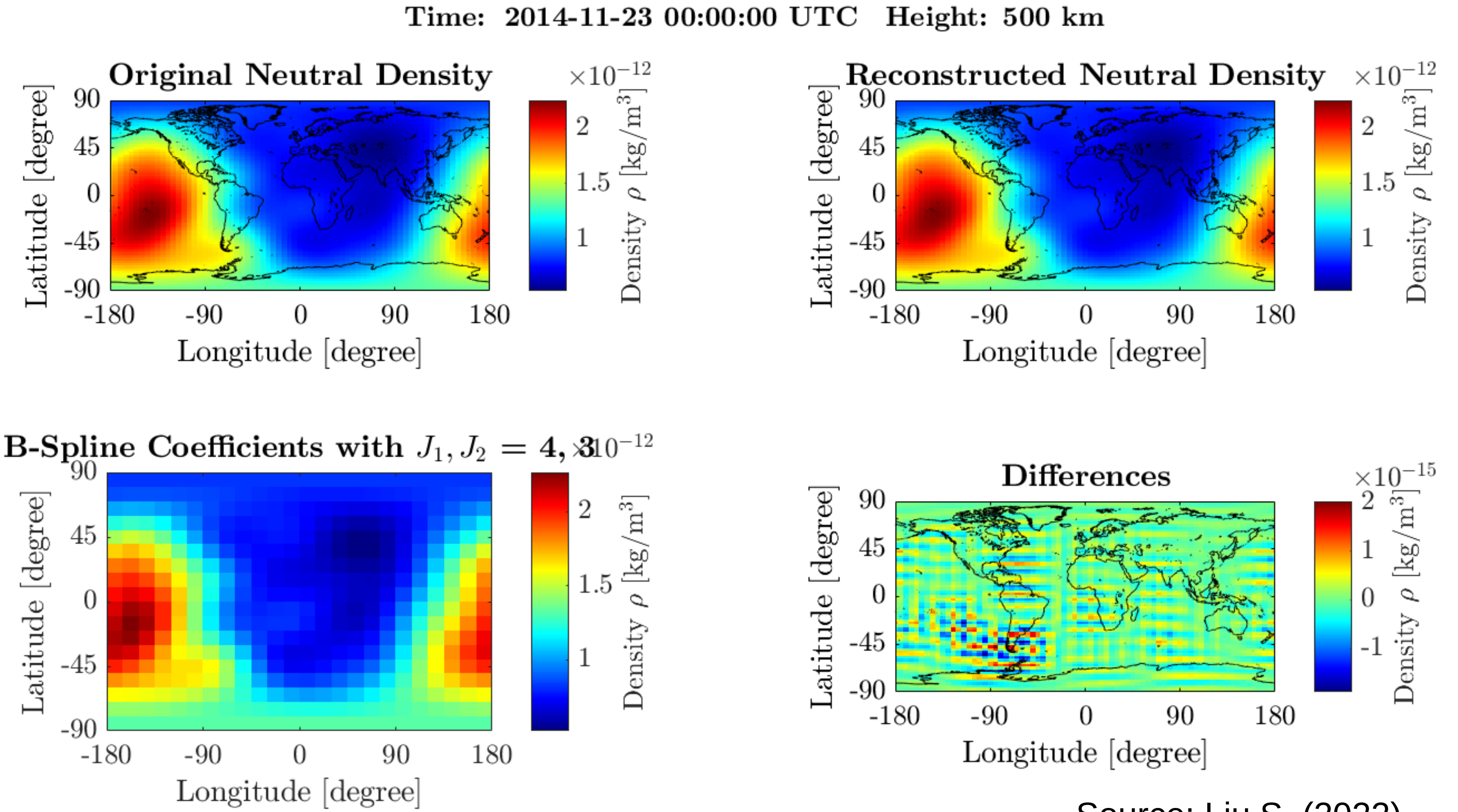




Source: Limberger (2015), Liu S. (2022)

2D representation of thermospheric density

Estimation of B-spline coefficients (latitude and longitude) for a specific epoch and height of the density



Time: 2014-11-23 00:00:00 UTC Height: 500 km

Original Neutral Density

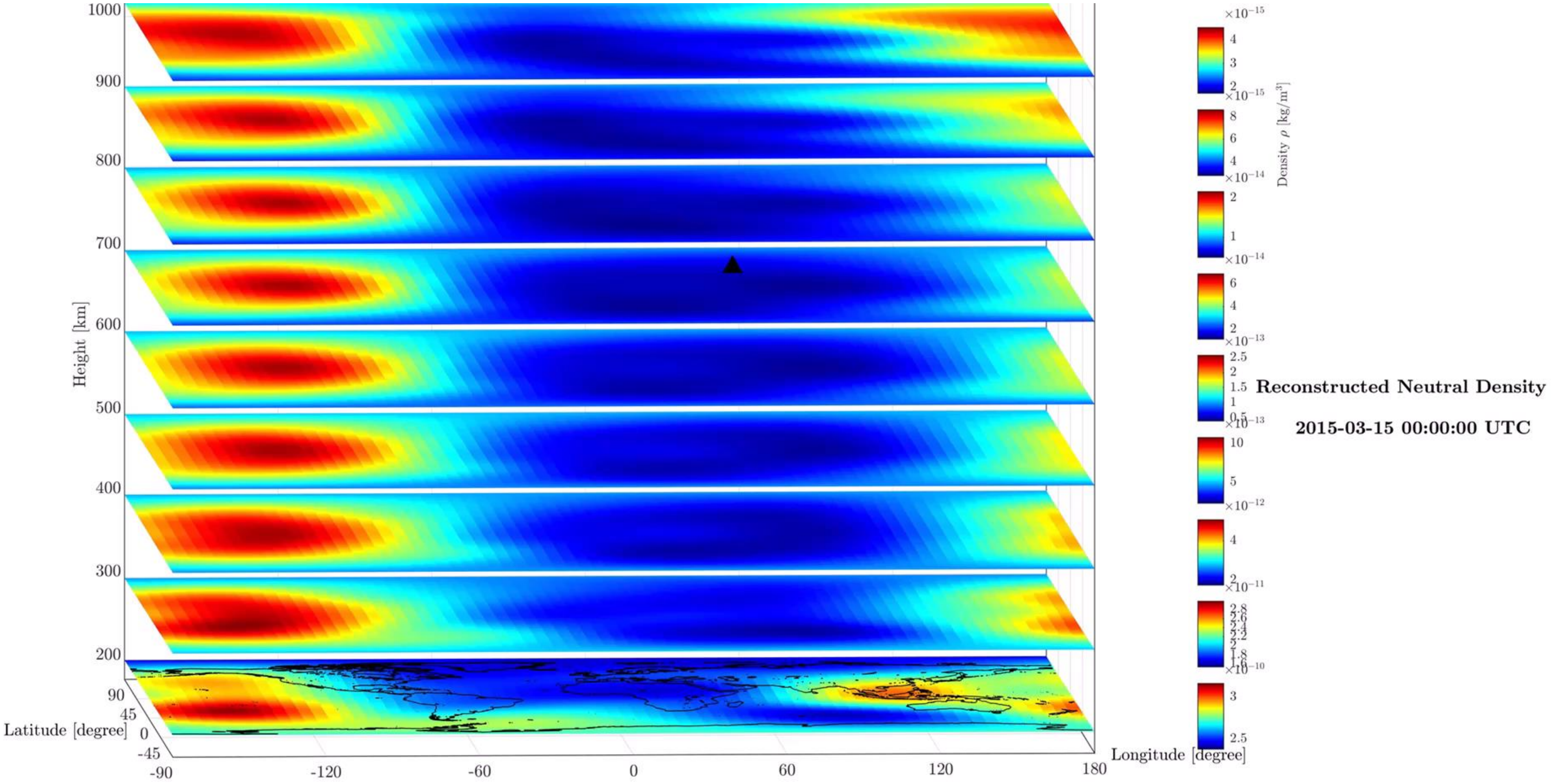
Reconstructed Neutral Density

B-Spline Coefficients with $J_1, J_2 = 4, 3$

Differences

Source: Liu S. (2022)

Reconstructed density and Larets satellite positions



Conclusions and outlook

- It is essential to consider the effects of the aerodynamic acceleration in POD of LEO satellites since it is the largest non-gravitational perturbation acting on the spacecraft.
- A study proved that SLR observations are a suitable complement to accelerometer data to derive density information about 550 km.
- The B-spline expansion is an appropriate approach for the representation of a thermospheric density model.

Next steps:

- Generalization of the approach to the 4D case.
- Inclusion of SLR information in the estimation of B-spline coefficients to obtain improved coefficients.
- Generation of an improved thermospheric density model to be used for the POD of other LEO satellites.

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DFG Deutsche Forschungsgemeinschaft

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