Proportionally Fair Resource Allocation in SD-RAN

Fidan Mehmeti
Chair of Communication Networks
Technical University of Munich, Germany
Email: fidan.mehmeti@tum.de

Wolfgang Kellerer
Chair of Communication Networks
Technical University of Munich, Germany
Email: wolfgang.kellerer@tum.de

***Abstract*—The introduction of Software-Defined Radio Access Networks in 5G, whose main feature is the possibility of decoupling the control plane from the data plane, and associating the former with a controller away from base stations, represents a paradigm shift in the way the network resources are allocated. This property provides an increased flexibility in cellular network operation, yielding significant improvements compared to the pre-5G resource allocation era. However, the full extent to which this amelioration ranges is not yet clear for different metrics of interest and objectives. One such objective is to allocate resources so that proportional fairness is achieved. Therefore, in this paper, we consider analytically the problem of proportionally fair allocation in SD-RAN environments, by deriving the policy which accomplishes that. We do this for two scenarios. In the first, the goal is to provide proportional fairness across all the users in the network, whereas in the second, the objective is to provide proportionally fair allocation in terms of the throughput of all BSs. We evaluate the performance with input parameters from a real trace. Results show that the introduction of SD-RAN increases the value of the objective by up to an order of magnitude compared to the scenario with no SD-RAN.**

***Index Terms*—SD-RAN, 5G, Proportional fairness.**

I. Introduction

In pre-5G networks, both data plane and control plane operations were performed jointly in Base Stations (BSs). This changed in 5G with the emergence of Software Defined Networks (SDNs) [1] and their adaptation in Radio Access Networks (RANs), known as SD-RAN [2], where the control is decoupled from the data plane and transferred to *centralized* entities known as SD-RAN controllers. This represents a paradigm shift in how cellular networks operate in general, and how the assignment of resources is handled in particular.

This way of operation brings a lot of benefits into the mobile network [2]–[4], with *flexibility* being among the principal ones. This increased level of flexibility stems from having a broader view of the entire network, enabled by the centralized SD-RAN approach. In that way, depending on the current distribution of User Equipments (UEs), i.e., users, across BSs, and their channel conditions for which the UEs periodically update their serving BSs [5], and BSs send those information to the SD-RAN controller, the latter can assign resources to BSs according to a given allocation policy. BSs then perform the allocation across UEs within their operational region. Consequently, exploiting the wide network knowledge results in an overall performance improvement as it allows for optimal allocation decisions, depending on the objective. In contrast to SD-RAN, in a classical RAN setup, each BS has its own fixed set of resources, and allocates them to the UEs within its coverage area.

Among the improvements SD-RAN brings to both the cellular operator and UEs is the increased throughput. However, while some open-source SD-RAN prototypes, like FlexRAN [2] and 5G-EmPOWER [3] already exist, it is not yet clear the extent to which the delegation of traditional RAN functions to centralized controllers increases either the overall throughput, or that of individual UEs. Other objectives may be of interest. One such is to *allocate resources effectively*, but at the same time to provide a level of *fairness*. Combining these two objectives is captured best by providing *proportional fairness* [6] in terms of the throughput when allocating the network resources, as the users with better channel conditions experience higher rates, but also the users with worse channel conditions are not penalized. To the best of our knowledge, this problem has not been addressed before in the context of SD-RAN.

Deriving and implementing the proportionally fair allocation policy in cellular networks is challenging mostly due to the dynamic nature of wireless channels, originating from UE mobility and effects inherent to mobile networks, like shadowing [7]. This channel variability drives forward the need to change the amount of allocated resources at the same granularity level at which the channel changes, and to take into account the channel conditions of all UEs when making the allocation decisions.

Some of the important questions that arise related to providing proportional fairness in SD-RAN-led networks are:

- What is the allocation policy that provides proportional fairness in an SD-RAN-enabled cellular network with a given number of BSs, where the number of UEs per BS is known before hand together with their channel conditions at a given time?
- If the goal is to provide proportionally fair resource allocation among BSs, what is the policy that enables the fulfillment of that objective?
- How does an SD-RAN-enabled network perform against a system in which all the BSs have their amount of resources fixed, i.e., against traditional resource allocation?

To answer the aforementioned questions, in this paper we formulate first an optimization problem whose objective is to provide proportional fairness among all UEs in the network, irrespective to which BSs they are associated with, given the constrained resources but having the extra flexibility of adaptive resource allocation to BSs, depending on the number of UEs and their channel conditions. We show that in such a scenario proportional fairness is achieved if all the network resources are split equally among UEs, i.e., BSs with more UEs receive proportionally more resources. On the other hand, if the goal is to provide proportional fairness among BSs, the resources among BSs should be shared equally, while within a BS, the UE with the highest CQI[1] at that slot should receive all the resources. Further, we give the data rates for the users when these proportionally fair policies are used. The results we provide in this work are particularly helpful for the cellular operator as they can provide an exact prediction of the average data rate when sharing resources in a fair way and can also help in network resource planning. The main message of this paper is that the use of SD-RAN can improve the performance. This is especially emphasized when the number of BSs increases. Specifically, our main contributions are:

- We derive the allocation policy which guarantees proportional fairness among all UEs, as well as the policy that guarantees proportional fairness among BSs.
- We evaluate the performance using realistic input data gathered from a measurement campaign [9].
- We show concrete performance improvements when using SD-RAN compared to the traditional RAN approach in terms of proportional fairness.

The remainder of this paper is organized as follows. In Section II, we discuss some related work. The system model and the problem formulation are presented in Section III. The analysis for proportional fairness among all UEs and BSs is presented in Section IV. Section V introduces the benchmark model without SD-RAN. In Section VI, we evaluate the performance and provide some interesting engineering insights. Finally, Section VII concludes the paper.

II. Related work

Since its introduction, the concept of SD-RAN has attracted significant attention in the last years [10], [11]. Among the first works that suggest handing over the control decisions to a centralized controller from BSs are [12] and [13], which also discuss the increased flexibility when using SD-RAN. However, the gains in terms of the increased throughput or the improvements in terms of the effective use of network resources are not discussed neither in [12] nor in [13].

To our best knowledge, different aspects of data rate maximization problem, including fair resource allocation, in SD-RAN environments have not been considered so far. The first prototype implementations of SD-RAN are FlexRAN [2] and 5G-EmPOWER [3]; both constrained to serve only a limited number of UEs with a single server. As opposed to [2] and [3], with our analysis we can predict the performance for the derived optimal policies for any number of BSs, number of UEs, channel conditions, and any amount of resources.

[1]CQI is an information each UE sends to its BS to describe the channel conditions. The value ranges from 1 (very poor channel conditions) to 15 (excellent channel conditions) [8].

In [14], the problem of minimizing the number of assigned resources has been considered in an SD-RAN environment, by taking into account two types of slices, those for delay-sensitive traffic, and those for throughput-critical traffic. The other contribution of [14] is that slice isolation can be maintained. However, there is no discussion on the resource allocation policy that provides proportional fairness.

The general problem of guaranteeing proportional fairness in a wireless network has been considered previously in [15]. However, the problem setup in [15] is not compatible with the SD-RAN environment (i.e., the number of resources is fixed per BS), losing this way the additional flexibility in resource allocation. As will be seen in Section VI, SD-RAN significantly outperforms the no-SD-RAN approach.

On a similar note, the authors in [16] consider the problem of allocating resources where network slices can be spread across multiple BSs. The objective in [16] is to allocate resources so that the overall throughput (across all UEs) is maximized, by guaranteeing a minimum data rate to everyone first. However, the solution in [16] is based on a non-closed form approximation approach, which does not allow to see the explicit dependency of throughput on different input parameters. Also, the proportionally fair resource allocation is not considered in [16]. In contrast, in our work, we solve the problem over the entire network in its most general form for any number of UEs, BSs, and heterogeneous channel statistics while providing a closed-form expression for the throughput obtained via proportionally fair resource allocation.

The most related work in spirit to ours is [8], in which the authors consider the problem of proportional fairness after providing the same minimum data rate to everyone in a single BS, and reallocating afterwards the unused resources to the same group of UEs. While data rates vary from one frame to another, all UEs receive the same rate in a given frame. However, the SD-RAN controller is not used in [8], meaning that the amount of resources belonging to each BS is fixed, loosing the additional flexibility of allocating resources adaptively to BSs, which as will be shown in Section VI of our work increases the objective considerably compared to the no-SD-RAN setup.

III. Performance modeling

First, we introduce the system model, and then define two optimization problems that we solve in this paper.

A. System model

We consider an SD-RAN-led network (Fig. 1) with a single controller responsible for assigning resources (and the allocation decisions further to UEs) to BSs. For every BS there is an SD-RAN agent that communicates with the controller [17], using the Transport Control Protocol (TCP). We denote by $\mathcal{N}$ the set of all BSs. There are in total $n = |\mathcal{N}|$ BSs in the operational area of the controller. Further, we denote by $\mathcal{M}_i$ the set of all UEs within the coverage area of BS i, where $m_i = |\mathcal{M}_i|$ is the number of UEs in BS i. So, the total number of UEs in the network is $\sum_{i=1}^{n} m_i$.

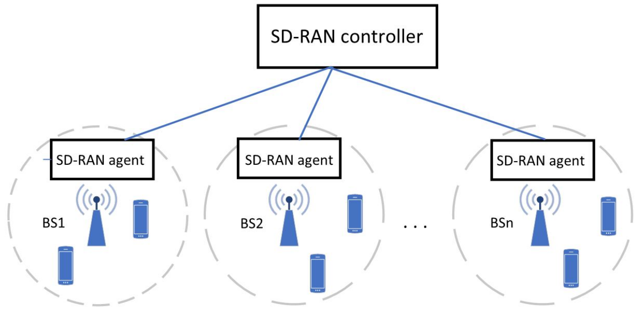

Fig. 1. Illustration of the SD-RAN environment.

5G uses *Physical Resource Blocks (PRBs)* as the unit of allocation on a per-slot basis [18]. Each PRB consists of 12 subcarriers. The slot duration is a function of the subcarrier spacing. Specifically, if the subcarrier spacing is 15 KHz (PRB width of 180 KHz), the slot duration is 1 ms. If the subcarrier spacing is 30 KHz (PRB width of 360 KHz), the corresponding slot duration is 0.5 ms. The slot duration decreases further ($2\times$) when switching to subcarrier spacing of 60 KHz, and another $2\times$ when switching to 120 KHz [5]. Different PRBs are assigned to different UEs within a slot. The assignment varies across slots. Consequently, scheduling needs to be performed across two dimensions, *frequency* and *time*. In total, there are K available PRBs for n BSs.

UEs experience different channel conditions (CQIs) across different PRBs even within the same slot. Because of the UE mobility and time-varying nature of the channels, per-PRB CQI (which is a function of Signal-to-Interference-Plus-Noise-Ratio (SINR)) changes from one slot to another, whose value depending on the Modulation and Coding Scheme (MCS) used sets the per-PRB rate. To maintain analytical tractability, a simplifying assumption is made in this paper. Namely, we assume that the BS splits the transmission power equally among all PRBs it transmits on, and that the channel characteristics for a UE remain static across all PRBs (identical CQI over all PRBs for a given UE), but change randomly (according to some distribution) from one slot to another, and are mutually independent among UEs (i.e., UEs with heterogeneous channel conditions). These assumptions reduce the resource allocation problem to the number of allocated PRBs and not to which PRBs are assigned to a UE.

The previous assumptions imply that in every slot, UE (i,j)[2], where $i \in \mathcal{N}$ and $j \in \mathcal{M}_i$, will have a per-PRB rate (known also as per-block rate), i.e., the rate each assigned PRB brings to a UE, which can be modeled with a discrete random variable, $R_{i,j}$, with values in $\{r_1, r_2, \ldots, r_{15}\}$, such that $r_1 < r_2 < \ldots < r_{15}$, with a Probability Mass Function (PMF) $p_{R_{i,j}}(x)$, which is a function of UE (i,j)'s CQI over time.[3] Note that there are 15 possible values of CQI.

[2]We denote every UE with the ordered pair (i,j), where i stands for the BS, while j denotes the UE receiving service by that BS.

[3]We omit the reference to time throughout this paper in order to simplify the notation.

TABLE I
NOTATION

$\mathcal{N}$	Set of all BSs
$n = \|\mathcal{N}\|$	Number of BSs
$\mathcal{M}_i$	Set of all UEs in BS i
$m_i = \|\mathcal{M}_i\|$	Number of UEs in BS i
K	Total number of PRBs
K_i	Number of PRBs allocated to BS i
$K_{i,j}$	Number of PRBs allocated to UE j in BS i
$R_{i,j}$	Per-PRB rate of UE j in BS i in a slot
$C_{i,j}$	Data rate of UE (i,j) in a slot
$p_{R_{i,j}}(x)$	PMF of per-PRB rate of UE (i,j)

Table I summarizes the notation used throughout this work.

B. Problem formulation

Each UE periodically sends the information about its CQI to its serving BS. Then, every BS collects all the CQI information from the UEs in its area and forwards them to the SD-RAN controller (see Fig. 1). Based on the CQI information from all BSs (and hence all UEs), the controller then, depending on the resource allocation policy used, decides on the number of PRBs to assign to each BS in every slot. Further, from the PRBs it receives, each BS decides how it will allocate those PRBs to the UEs in its coverage area. Therefore, using SD-RAN, the resource allocation process is performed in two levels. First, among BSs, and then each BS allocates the PRBs it received from the controller to the UEs within its area.

Let $K_{i,j}, \forall j \in \mathcal{M}_i$, denote the number of PRBs UE j gets from BS i.[4] If $K_i, \forall i \in \mathcal{N}$, denotes the number of PRBs that BS i receives from the controller in a slot, then it holds $K_i = \sum_{j=1}^{m_i} K_{i,j}$. If $C_{i,j}$ expresses the data rate of UE (i,j) in a slot, then $C_{i,j} = K_{i,j} R_{i,j}$.

In this paper, the goal is to provide proportionally fair resource allocation along two dimensions: (i) across UEs, and (ii) across BSs.

1) Proportional fairness across UEs: First, we strive to provide proportionally fair resource allocation across all UEs in the entire SD-RAN-led network, which is equivalent to maximizing the sum of the logarithms of the data rates of all UEs [6].[5] This results in the following optimization problem:

$$\mathcal{P}_1: \quad \max_{K_{i,j}} \quad \sum_{i=1}^{n} \sum_{j=1}^{m_i} \log(K_{i,j} R_{i,j}) \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \sum_{j=1}^{m_i} K_{i,j} \leq K, \tag{2}$$

$$K_{i,j} \geq 0, \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{M}_i. \tag{3}$$

Constraint (2) expresses the maximum number of PRBs that can be allocated to all UEs, whereas (3) captures the fact that

[4]Each UE can receive resources from one BS only.

[5]An equivalent objective to (1) is to maximize the product of the data rates, i.e., $\prod_{i=1}^{n} \prod_{j=1}^{m_i} K_{i,j} R_{i,j}$. Hence, in Section VI, for some scenarios we show the product of the data rates, and for some others the sum of the logarithms of the data rates. We do this to better visualize the outcomes.

the number of allocated PRBs to UEs cannot be negative. The decision variables are $K_{i,j}$.

2) Proportional fairness across BSs: In the second scenario, the goal is provide proportionally fair allocation of PRBs across BSs, i.e., to maximize the sum of the logarithms of the throughput of all BS. The throughput in BS i in a slot is $\sum_{j=1}^{m_i} C_{i,j} = \sum_{j=1}^{m_i} K_{i,j} R_{i,j}$. The following optimization formulation describes this scenario:

$$\mathcal{P}_2: \quad \max_{K_{i,j}} \quad \sum_{i=1}^{n} \log \left(\sum_{j=1}^{m_i} K_{i,j} R_{i,j} \right) \tag{4}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \sum_{j=1}^{m_i} K_{i,j} \leq K, \tag{5}$$

$$K_{i,j} \geq 0, \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{M}_i. \tag{6}$$

As opposed to $\mathcal{P}_1$, $\mathcal{P}_2$ is not concerned with the fair allocation within users of the BS, but only looks at the BS as a whole. The constraints (5) and (6) are identical to those of $\mathcal{P}_1$. The decision variables are again $K_{i,j}$.

In the next section, we solve optimization problems $\mathcal{P}_1$ and $\mathcal{P}_2$ by obtaining the corresponding optimal policies as well as showing the data rate in a slot of every UE when following the corresponding optimal resource allocation policies.

IV. Performance Optimization

In this section, first we determine the optimal policy and derive the corresponding data rates by solving $\mathcal{P}_1$. Then, we solve $\mathcal{P}_2$.

A. Proportional fairness across all UEs

We proceed with solving $\mathcal{P}_1$. The function in the objective is apparently concave. Namely, the main diagonal elements of its Hessian matrix are equal to $-K_{i,j}^2 < 0$, whereas all the off-diagonal elements are 0, making the Hessian a negative definite matrix, resulting in a concave objective function [19]. Given also that the constraints are linear, there exists a solution to the problem, and the local optimizer is a global optimizer as well. First, we define the Lagrangian of this optimization problem as

$$\begin{aligned} \mathcal{L} &= \sum_{i=1}^{n} \sum_{j=1}^{m_i} \log(K_{i,j} R_{i,j}) - \lambda \left(\sum_{i=1}^{n} \sum_{j=1}^{m_i} K_{i,j} - K \right) \\ &+ \sum_{i=1}^{n} \sum_{j=1}^{m_i} \mu_{i,j} K_{i,j}, \end{aligned} \tag{7}$$

where $\lambda \geq 0$ and $\mu_{i,j} \geq 0$, $\forall j \in \mathcal{M}_i$ are the slack variables. It can be easily shown that $\mathcal{P}_1$ satisfies Slater's condition [6], hence the strong duality holds. Therefore, Karush-Kuhn-Tucker (KKT) conditions [19] can be applied to the dual optimization problem, and the optimal solution would need to satisfy the following system of equations:

$$\frac{\partial \mathcal{L}}{\partial K_{i,j}} = 0, \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{M}_i, \tag{8}$$

$$\lambda \left(\sum_{i=1}^{n} \sum_{j=1}^{m_i} K_{i,j} - K \right) = 0, \tag{9}$$

$$\mu_{i,j} K_{i,j} = 0, \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{M}_i. \tag{10}$$

Substituting Eq.(7) into Eq.(8), we obtain

$$\frac{1}{K_{i,j}} - \lambda + \mu_{i,j} = 0, \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{M}_i, \tag{11}$$

or equivalently,

$$\lambda = \frac{1}{K_{i,j}} + \mu_{i,j}, \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{M}_i. \tag{12}$$

In order to avoid $-\infty$ in the objective (1), the number of assigned resources to any UE has to be strictly positive, $K_{i,j} > 0$. This results in $\lambda > 0$ in Eq.(12). The latter combined with Eq.(9) yields

$$\sum_{i=1}^{n} \sum_{j=1}^{m_i} K_{i,j} = K. \tag{13}$$

Eq.(13) merely states that constraint (2) must be satisfied with strict equality, i.e., there should be no PRBs left unassigned. This is intuitive as we are trying to maximize the sum of logarithms of data rates, i.e., to increase the effectiveness of resource assignments.

The fact that $K_{i,j} > 0$ also leads (from Eq.(10)) to $\mu_{i,j} = 0, \forall i \in \mathcal{N}, \forall j \in \mathcal{M}_i$. Then, Eq.(12) reduces to

$$K_{i,j} = \frac{1}{\lambda}. \tag{14}$$

Substituting Eq.(14) into Eq.(13) and rearranging, we obtain

$$\lambda = \frac{\sum_{i=1}^{n} m_i}{K}, \tag{15}$$

which after being substituted into Eq.(14) leads to:

Result 1. *A proportionally fair resource allocation policy across all UEs in the network with SD-RAN is achieved if the number of assigned PRBs to UE (i, j) follows the policy*

$$K_{i,j} = \frac{K}{\sum_{i=1}^{n} m_i}. \tag{16}$$

There are two important conclusions that can be drawn from Result 1. First, to achieve proportional fairness, the resources have to be split equally among all UEs. Second, which is implied by the first outcome - this allocation policy is *static*. This reduces the computational complexity at the controller side significantly as it does not have to reschedule resources in every slot. In a slot, UE (i, j) experiences the data rate

$$C_{i,j} = K_{i,j} R_{i,j} = \frac{K R_{i,j}}{\sum_{i=1}^{n} m_i}, \tag{17}$$

which implies that UEs with better channel conditions have higher data rates. Hence, *fairness is achieved by having everyone receive the same amount of network resources, and effectiveness is achieved by having UEs with better channel conditions experience higher data rates.*

B. Proportional fairness across BSs

The second objective is to provide proportional fairness in resource allocation across BSs, i.e., solve $\mathcal{P}_2$.

The objective function Eq.(4) is concave. Namely, the main diagonal elements of the corresponding Hessian matrix $\mathbf{A}$ are $-\frac{R_{i,j}^2}{\left(\sum_{j=1}^{m_i} K_{i,j}R_{i,j}\right)^2} < 0$, whereas the off-diagonal elements are $-\frac{R_{i,j}R_{s,t}}{\left(\sum_{j=1}^{m_i} K_{i,j}R_{i,j}\right)^2} < 0$, implying that for any non-zero vector $\mathbf{x}$, it holds that

$$\mathbf{x}^T\mathbf{A}\mathbf{x} < 0,$$

meaning that the Hessian is a negative definite matrix, hence the objective is strictly concave. With constraints (5) and (6) being linear, it turns out that there exists a unique solution to $\mathcal{P}_2$, or said differently, the local maximum is also a global solution.

The Lagrangian function is defined as

$$\begin{aligned}\mathcal{L} &= \sum_{i=1}^{n}\log\left(\sum_{j=1}^{m_i}K_{i,j}R_{i,j}\right) - \lambda\left(\sum_{i=1}^{n}\sum_{j=1}^{m_i}K_{i,j} - K\right) \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{m_i}\mu_{i,j}K_{i,j}, \end{aligned} \tag{18}$$

where $\lambda \geq 0$ and $\mu_{i,j} \geq 0$, $\forall j \in \mathcal{M}_i$, are the slack variables. Similarly to $\mathcal{P}_1$, Slater's condition is valid here as well. KKT conditions can be applied to the dual optimization problem, and as the constraints of $\mathcal{P}_2$ are identical to those of $\mathcal{P}_1$, the solution to $\mathcal{P}_2$ should adhere to the same system of equations as $\mathcal{P}_1$, i.e., Eqs.(8)-(10).

Substituting Eq.(18) into Eq.(8) and rearranging, we obtain

$$\lambda = \frac{R_{i,j}}{\sum_{j=1}^{m_i}K_{i,j}R_{i,j}} + \mu_{i,j},\ \forall i \in \mathcal{N}, \forall j \in \mathcal{M}_i. \tag{19}$$

In order to avoid $-\infty$, every BS has to receive some resources, i.e., $\sum_{j=1}^{m_i}K_{i,j}R_{i,j} > 0$, which together with Eq.(19) implies that $\lambda > 0$. The latter together with Eq.(9) implies full utilization of network resources, the same as when solving $\mathcal{P}_1$, i.e., Eq.(13) applies to this case as well.

Next, let us look at Eq.(19) when it is applied to two UEs that are receiving service from the same BS i. Let us denote these as (i, j_1) and (i, j_2). We have

$$\frac{R_{i,j_1}}{\sum_{j=1}^{m_i}K_{i,j}R_{i,j}} + \mu_{i,j_1} = \frac{R_{i,j_2}}{\sum_{j=1}^{m_i}K_{i,j}R_{i,j}} + \mu_{i,j_2}. \tag{20}$$

Let us assume w.l.o.g. that in the given slot UE (i, j_1) has a higher CQI than UE (i, j_2), i.e., $R_{i,j_1} > R_{i,j_2}$. Then, from Eq.(20), we have

$$\frac{R_{i,j_1} - R_{i,j_2}}{\sum_{j=1}^{m_i}K_{i,j}R_{i,j}} = \mu_{i,j_2} - \mu_{i,j_1} > 0, \tag{21}$$

implying $\mu_{i,j_1} < \mu_{i,j_2}$. This means that $\mu_{i,j_2} > 0$, resulting from the constraint (10) in $K_{i,j_2} = 0$. So, as long as a UE within the area of her BS has a worse channel (lower CQI) than another UE, she will not receive any PRBs at all. The analysis propagates across UEs of all BSs. So, within a BS, only UEs with the highest CQI are eligible to receive PRBs. Then, what is this policy in the second level (within BSs) of resource allocation? It is the well-known *maxCQI policy*, where only the user with the highest CQI will receive all the resources, or if more than one UE have identical (highest) CQI they share the resources equally.

Next, we consider only the UEs with the highest CQI in their BSs. For that purpose, let us pick the corresponding UEs in BS i_1 and i_2, UE (i_1, j_1) and UE (i_2, j_2), i.e., for which we know from the previous discussion that $K_{i_1,j_1} > 0$ and $K_{i_2,j_2} > 0$. For these two UEs, Eq.(19) becomes

$$\frac{R_{i_1,j_1}}{\sum_{j=1}^{m_{i_1}}K_{i_1,j}R_{i_1,j}} + \mu_{i_1,j_1} = \frac{R_{i_2,j_2}}{\sum_{j=1}^{m_{i_2}}K_{i_2,j}R_{i_2,j}} + \mu_{i_2,j_2}. \tag{22}$$

If we denote by $R_{i_1,max}$ and $R_{i_2,max}$ the highest per-PRB rates in BS i_1 and BS i_2, respectively, then as both UEs (i_1, j_1) and (i_2, j_2) are eligible to receive PRBs, it holds that $R_{i_1,j_1} = R_{i_1,max}$ and $R_{i_2,j_2} = R_{i_2,max}$. As the denominators in Eq.(22) represent the total throughput in BSs i_1 and i_2, the following holds:

$$\frac{R_{i_1,j_1}}{K_{i_1}R_{i_1,max}} + \mu_{i_1,j_1} = \frac{R_{i_2,j_2}}{K_{i_2}R_{i_2,max}} + \mu_{i_2,j_2}, \tag{23}$$

where $K_{i_1} = \sum_{j=1}^{m_{i_1}}K_{i_1,j}$ and $K_{i_2} = \sum_{j=1}^{m_{i_2}}K_{i_2,j}$ denote the total number of PRBs allocated to BSs i_1 and i_2, respectively.

From the above discussion, Eq.(23) reduces to

$$\frac{1}{K_{i_1}} + \mu_{i_1,j_1} = \frac{1}{K_{i_2}} + \mu_{i_2,j_2}. \tag{24}$$

Since $K_{i_1,j_1} > 0$ and $K_{i_2,j_2} > 0$, from Eq.(10) it follows that $\mu_{i_1,j_1} = 0$ and $\mu_{i_2,j_2} = 0$. Eq.(24) now reduces to

$$K_{i_1} = K_{i_2},\ \forall i_1, i_2 \in \mathcal{N}. \tag{25}$$

Propagating the last property across all BSs, it results that each BS will receive $\frac{K}{n}$ PRBs in every slot, irrespective of the channel conditions of UEs within the BS. So, for the first level of resource allocation in an SD-RAN-led network, resources are split equally among BSs, i.e., we have the Round-robin policy [20]. Summarizing, we have:

Result 2. *A proportionally fair resource allocation policy among all BSs in the network with SD-RAN is achieved if the combination Round-robin – maxCQI is used. The amount of PRBs UE (i, j) receives in a slot is then*

$$K_{i,j} = \begin{cases} 0, & \text{if } R_{i,j} < R_{i,max} \\ \frac{K}{n|\mathcal{M}_i^{\max}|}, & \text{if } R_{i,j} = R_{i,max}, \end{cases}$$

where $|\mathcal{M}_i^{\max}|$ denotes the number of UEs within BS i that have the highest CQI in the given slot.[6]

As opposed to achieving proportional fairness among UEs, where the allocation is static, when it comes to proportionally fair resource allocation among BSs, the allocation policy is dynamic in the second level of allocation (among UEs within each BS), as it depends on $R_{i,j}$.

[6] We remind the reader that with the maxCQI policy we assume that when more than one UE have the same highest CQI they share resources equally.

The data rate of UE (i, j) from Result 2 is either 0, or $C_{i,j} = \frac{KR_{i,j}}{n|\mathcal{M}_i^{\max}|}$ when she has the highest CQI in the slot within BS i.

V. Benchmark model

In order to assess the performance of the SD-RAN-enabled network in terms of proportional fairness, we need a benchmark model. To that end, the most suitable model is the one in which there is no SD-RAN, but where there are some proportional fairness guarantees. Hence, we choose the benchmark in which the RAN operates in a traditional way, where every BS is pre-assigned its set of PRBs, and the allocation process undergoes proportional fairness within each BS separately. If K is the total number of PRBs in the system, then w.l.o.g. we assume that each BS operates on $\frac{K}{n}$ PRBs, where n as already defined, is the number of BSs.

In the no-SD-RAN setup, the problem formulation for BS i, whose solution guarantees proportionally fair resource allocation to the UEs within its coverage area is

$$\mathcal{P}_0(i): \quad \max_{K_{i,j}} \quad \sum_{j=1}^{m_i} \log\left(K_{i,j} R_{i,j}\right) \tag{26}$$

$$\text{s.t.} \qquad \sum_{j=1}^{m_i} K_{i,j} \leq \frac{K}{n}, \tag{27}$$

$$K_{i,j} \geq 0, \quad \forall j \in \mathcal{M}_i. \tag{28}$$

Essentially, for each BS we would need to solve $\mathcal{P}_0(i)$ separately. The function in the objective is apparently concave. Namely, the main diagonal elements of its Hessian matrix are equal to $-K_{i,j}^{-2} < 0$, whereas all the off-diagonal elements are 0, making the Hessian a negative definite matrix, resulting in a concave objective function [19]. Given also that the constraints are linear, there exists a solution to the problem, and a local optimizer is a global optimizer as well. We define the Lagrangian of this optimization problem as

$$\mathcal{L} = \sum_{j=1}^{m_i} \log(K_{i,j} R_{i,j}) - \lambda \left(\sum_{j=1}^{m_i} K_{i,j} - \frac{K}{n} \right) + \sum_{j=1}^{m_i} \mu_{i,j} K_{i,j}, \tag{29}$$

where $\lambda \geq 0$ and $\mu_{i,j} \geq 0$, $\forall j \in \mathcal{M}_i$. It can be easily shown that $\mathcal{P}_0(i)$ satisfies Slater's condition [6], hence the strong duality holds here as well. Therefore, KKT conditions can be applied to the dual optimization problem, and the optimal solution would need to adhere to the following constraints:

$$\frac{\partial \mathcal{L}}{\partial K_{i,j}} = 0, \quad \forall j \in \mathcal{M}_i, \tag{30}$$

$$\lambda \left(\sum_{j=1}^{m_i} K_{i,j} - \frac{K}{n} \right) = 0, \tag{31}$$

$$\mu_{i,j} K_{i,j} = 0, \quad \forall j \in \mathcal{M}_i. \tag{32}$$

Substituting Eq.(29) into Eq.(30), we obtain

$$\lambda = \frac{1}{K_{i,j}} + \mu_{i,j}, \forall j \in \mathcal{M}_i, \tag{33}$$

implying $\lambda > 0$. Combining this with Eq.(31), the following must hold:

$$\sum_{j=1}^{m_i} K_{i,j} = \frac{K}{n}. \tag{34}$$

Further, $K_{i,j} > 0$ as otherwise would make the objective $-\infty$. This fact combined with Eq.(32) yields $\mu_{i,j} = 0$, $\forall j \in \mathcal{M}_i$. Therefore, from Eq.(33) we obtain $\lambda = \frac{1}{K_{i,j}}$. Substituting the latter into Eq.(34), we get

$$\lambda = \frac{nm_i}{K}, \forall j \in \mathcal{M}_i, \tag{35}$$

resulting in

$$K_{i,j} = \frac{1}{\lambda} = \frac{K}{n} \cdot \frac{1}{m_i}, \tag{36}$$

i.e., within each BS proportional fairness is achieved when all the PRBs of that BS are split equally among UEs of that BS.

Having the benchmark against which we can compare the results obtained with our approaches, we proceed next with assessing the performance under different policies.

VI. Performance Evaluation

In this section, we describe the simulation setup first. Then, we compare the performance of our two approaches, the benchmark, and another policy (same rate to everyone) for different cases. This is followed by results on the impact of channel statistics on the allocation process. Finally, we look at some corner cases.

A. Simulation setup

We have used a 5G trace with data measured in the Republic of Ireland as input parameters. These traces can be found in [21], with a detailed description in [9], and statistical analysis in [22]. Here, the parameter of interest from the trace is CQI with 15 levels, which serves to determine the per-PRB rate of a user in a slot. These measurements were conducted for one UE, but at different days, for different services, and when the user is static and also when moving around. To mimic the dynamic nature of these users, we have picked 8 UEs that were moving around. Based on the frequency of occurrence of a per-PRB rate for every UE, we obtained the corresponding per-PRB rate probabilities (Table II).

The slot duration is 0.5 ms. The subcarrier spacing is 30 KHz, with 12 subcarriers per block, making the PRB width 360 KHz. The total number of PRBs is $K = 273$ [5]. The simulations are conducted in MATLAB R2021b.

In the simulator, every BS in each slot sends the information of CQIs of its UEs to the controller. With the full picture of all CQIs in the network, the controller according to the allocation policy used distributes the resources (PRBs) to BSs together with the information on how to further assign them to UEs in their coverage areas. Depending on the amount of resources assigned, and its per-PRB rate, we determine the data rate each UE experiences in a slot.

Unless stated otherwise, we show results for three cases:

- Case 1: 4 BSs; 2 UEs for BSs 1 and 2, 4 UEs for BSs 3 and 4.

TABLE II
PER-PRB RATES AND THE CORRESPONDING PROBABILITIES FOR EVERY USER FROM THE REPUBLIC OF IRELAND TRACE [9]

R (kbps)	48	73.6	121.8	192.2	282	378	474.2	712	772.2	874.8	1063.8	1249.6	1448.4	1640.6	1778.4
$p_{1,k}$	0	0	0	0	0	0	0.01	0.05	0.11	0.13	0.14	0.18	0.06	0.11	0.21
$p_{2,k}$	0	0	0	0	0	0.01	0.02	0.06	0.13	0.14	0.2	0.21	0.07	0.09	0.07
$p_{3,k}$	0.01	0	0	0	0	0.01	0.01	0.02	0.06	0.13	0.17	0.18	0.08	0.18	0.15
$p_{4,k}$	0	0	0	0	0	0.02	0.03	0.13	0.06	0.2	0.32	0.11	0.01	0.09	0.03
$p_{5,k}$	0	0	0	0	0	0	0.04	0.07	0.13	0.17	0.22	0.2	0.05	0.06	0.06
$p_{6,k}$	0	0	0	0	0.01	0.03	0.11	0.12	0.19	0.15	0.15	0.12	0.05	0.04	0.03
$p_{7,k}$	0	0	0	0	0	0	0.05	0.06	0.15	0.17	0.2	0.2	0.05	0.07	0.05
$p_{8,k}$	0	0	0.01	0.01	0.01	0.03	0.15	0.12	0.18	0.14	0.13	0.11	0.06	0.03	0.02

- Case 2: 5 BSs; 2 UEs for BSs 1 and 2, 4 UEs for BSs 3 and 4, 6 UEs for BS 5.
- Case 3: 8 BSs; 2 UEs for BSs 1 and 2, 4 UEs for BSs 3 and 4, 6 UEs for BSs 5 and 6, 8 UEs for BSs 7 and 8.

Note that in all the cases, a UE is chosen randomly from one of the eight types of Table II. Then, its CQI values across slots are taken from the trace of the corresponding user.

B. Performance comparisons

We start with comparing the performance obtained with our policies against the benchmark. First, we compare our approach for proportional fairness across all UEs in an SD-RAN-enabled network (the solution to $\mathcal{P}_1$), to which we refer as SD-RAN in the figures, with the benchmark and another allocation policy in both of which there is no SD-RAN, i.e., each BS "owns" a fixed set of PRBs. The benchmark is described in Section V, whose results are referred to as $\mathcal{P}_0$. The second comparison policy is "equal-rate" [23] in the BS, i.e., UEs with good channel conditions receive fewer PRBs, whereas UEs with bad channel conditions receive more PRBs.

We show results for the three cases introduced previously. Fig. 2 portrays the results for the product of data rates of all UEs (the equivalent of Eq.(1)) in the network over time with different policies for Case 1. We do this for better visualization purposes. See Footnote 5 for explanations on this. As can be observed, the solution to $\mathcal{P}_1$ always outperforms that of the benchmark $\mathcal{P}_0$, and equal-rate. The difference is very large, up to $2\times$. We are showing results for only 30 slots to better discern the differences. However, as Result 1 is the optimal solution of $\mathcal{P}_1$, it always outperforms the two no-SD-RAN approaches. The equal-rate policy always provides the worst results in terms of proportional fairness, as it penalizes the users with good channel conditions.

Fig. 3 depicts the results for Case 2, whereas Fig. 4 does that for Case 3. Similar to Fig. 2, on the y-axes we show the product of data rates of all UEs. In both scenarios, SD-RAN outperforms the other two approaches in terms of proportional fairness significantly. Note that as the number of BSs and UEs increases, the difference in performance gets larger. For Case 3, our approach (obtained solving $\mathcal{P}_1$) outperforms the other two policies by up to an order of magnitude.

Next, we compare the results in terms of proportional fairness across BSs. Fig. 5 shows the outcomes related to Case 1 for the value of the objective Eq.(4) in the network over time with different policies, whereas Fig. 6 and Fig. 7 depict the results for Case 2 and Case 3, respectively. Our approach now uses the solution to $\mathcal{P}_2$. The other parameters remain unchanged from the previous scenarios. In all three cases, SD-RAN outperforms no-SD-RAN. Equal-rate policy performs worst in this aspect as well. It is worth pointing out that for this set of plots we show on the y-axes the results in logarithmic scale, i.e., as the objective is originally defined in Eq.(4). The reason for this change is that the differences between the results provided by different approaches are much larger now. E.g., in Fig. 7, we see differences (on logarithmic scale) as high as 3 (for Eq.(4)), which means $1000\times$ higher objective with our approach then the benchmark if we show the products of data rates (i.e., the objectives on linear scale).

TABLE III
THE AVERAGE (IN MBPS) AND THE COEFFICIENT OF VARIATION OF PER-PRB RATES FOR USERS OF TABLE II

UE	1	2	3	4	5	6	7	8
$\mathbb{E}[R_i]$	1.25	1.12	1.27	1.02	1.07	0.92	1.06	0.87
$c_{V,R}$	0.31	0.31	0.3	0.32	0.31	0.38	0.31	0.41

The effects shown in the previous results can be observed for other cases too (different input parameters). Common to all these is that SD-RAN is always more proportionally fair.

C. Impact of channel statistics

Next, we look at the impact of channel statistics (expressed through the first and second moments of the per-PRB rate) on the average of the assigned number of PRBs to UEs and their variability. Note that this applies only to proportional fairness among BSs, as in the case of proportional fairness among UEs, all of them receive the same number of PRBs always, i.e., there is no variability neither across users nor in the time dimension. We assume there are 4 BSs, and in the BS of interest there are 8 UEs (those from Table II). Our focus here is to look how much varies the number of assigned PRBs to users with different channel statistics. To quantify the latter, we use the average per-PRB rate $\mathbb{E}[R]$ and the coefficient of variation (c_V), where the latter is defined as the ratio of the standard deviation and the mean of the per-PRB rate. Table III shows those two parameters for users of Table II. As can be seen, all of them have roughly similar channel variability, but there are differences in their first moments of per-PRB rates.

So, how varying is the number of PRBs assigned to users, and even more importantly, what does it depend on? The allocation policies we propose in this work react according

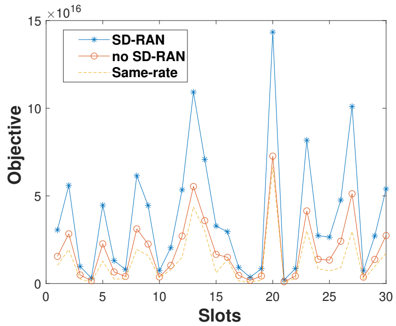

Fig. 2. The evolution of the equivalent objective to Eq.(1) for Case 1.

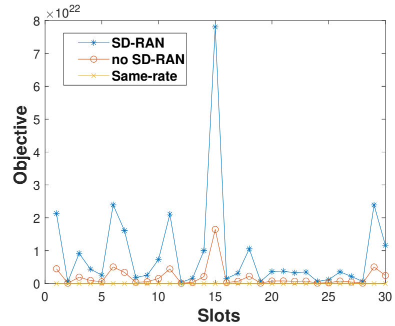

Fig. 3. The evolution of the equivalent objective to Eq.(1) for Case 2.

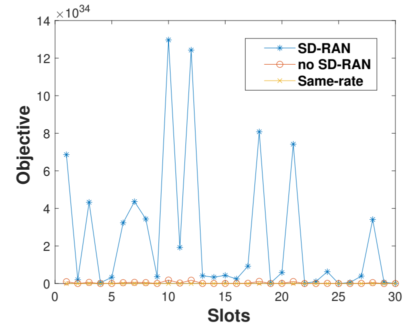

Fig. 4. The evolution of the equivalent objective to Eq.(1) for Case 3.

to channel (CQI) changes at UEs. Fig. 8 depicts the average value of the number of PRBs for these UEs over time. As can be observed from Fig. 8, UE 1 and UE 3 receive the highest number of PRBs over time (between 17 and 18 in every slot on average), whereas UE 8 gets the lowest number of PRBs (on average less than 4 per slot). If these results are compared with the first moment of per-PRB rates (the second row of Table III), we see that they are consistent, i.e., UEs 1 and 3 have the highest average per-PRB rate, while UE 8 the lowest.

How is the situation in terms of the variability of the number of assigned PRBs over time? Fig. 8 also shows the results for the coefficient of variation of the number of assigned PRBs to these users. What can be observed is the fact that UE 6 and UE 8 experience the highest variability in PRB assignments, while UE 1 and UE 3 the lowest. As opposed to the first moment, these outcomes are not fully compliant with the variability of channel conditions of UEs. Namely, from Table III (third row), we can observe that UEs 2, 4, 5, and 7 have as varying channel conditions as UEs 1 and 3, but higher variability in the assigned PRBs over time. So, the first moment of per-PRB rate is far more decisive on the first moment of assigned PRBs than the variability of channel conditions is on the variability of assigned PRBs over time.

The previous findings are explained by Table IV, which shows the probability that a UE will receive any resources in a slot, i.e., the probability that UE will experience the highest (potentially with other UEs) CQI in a given slot. Note that in the BS level the allocation policy is maxCQI. UE 1 and UE 3 have the highest chances to receive any resources, hence they have the highest average PRBs assigned. On the other hand, UE 8 in 93% of the slots will not receive any PRBs, and in only 7% of the slots will have resources allocated, increasing thus its coefficient of variation of the number of assigned PRBs.

D. Policy comparisons: Corner cases

So far, we have compared the allocation policies for various configurations, considering UEs with different CQI distributions. In the following, we demonstrate performance differences between the allocation policies in corner cases in terms of the UE channel conditions. To that end, we consider the following four scenarios (in each scenario there are 2 BSs with 2 UEs each, and 2 other BSs with 4 UEs each, i.e., in total there are 4 BSs and 12 UEs):

- Scenario A: All UEs in all BSs experience similar channel conditions. The CQI of every UE in a slot is drawn uniformly from the entire set.
- Scenario B: All UEs have excellent channel conditions (CQI is drawn uniformly from the range $13-15$).
- Scenario C: All UEs have very bad channel conditions (CQI is drawn uniformly from the range $1-3$).
- Scenario D: UEs in two of the BSs experience excellent channel conditions (their CQI is drawn uniformly from the range $13-15$), whereas UEs in the other two BSs suffer from bad channel conditions (their CQI is drawn uniformly from the range $1-3$).

TABLE IV

THE PROBABILITY THAT THE CQI OF A GIVEN UE IS EQUAL TO THE HIGHEST CQI AMONG ALL UES IN A SLOT

user	1	2	3	4	5	6	7	8
prob.	0.35	0.18	0.34	0.1	0.15	0.09	0.14	0.07

Fig. 9 shows the ratio of the average values of the equivalent objectives to Eq.(1), i.e., the product of data rates, over time in the network with SD-RAN (solving $\mathcal{P}_1$), and no-SD-RAN (the outcome from the solution to $\mathcal{P}_0$) for the four scenarios described above. SD-RAN outperforms no-SD-RAN in all four scenarios, by almost exactly the same factor (near 2). This implies that the average of the product of data rates over time with $\mathcal{P}_1$ is almost twice as high as the average of the product of data rates over time with $\mathcal{P}_0$, obviously implying a more proportionally fair resource allocation.

Fig. 10 depicts the average values of the objective Eq.(4) over time (i.e., on logarithmic scale) for the same setup as previously, but with the results obtained from $\mathcal{P}_2$ and $\mathcal{P}_0$. Again, in all scenarios, the SD-RAN outperforms no-SD-RAN network. The difference between the two policies is the highest in Scenario A, which is around 2 on logarithmic scale, or equivalently, around $100\times$ on linear scale. Obviously, the latter means that the product of the throughput of all BSs is $100\times$ higher with the solution of $\mathcal{P}_2$ than with that of $\mathcal{P}_0$.

VII. CONCLUSION

In this paper, we considered the problem of providing proportional fairness in terms of throughput in an SD-RAN

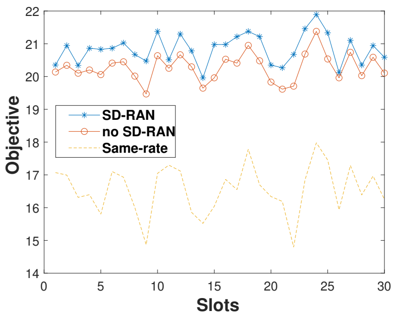

Fig. 5. The evolution of the objective Eq.(4) for Case 1.

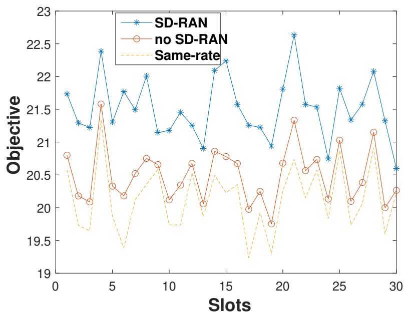

Fig. 6. The evolution of the objective Eq.(4) for Case 2.

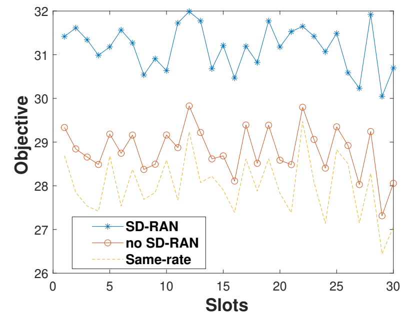

Fig. 7. The evolution of the objective Eq.(4) for Case 3.

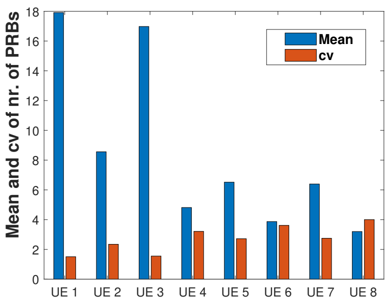

Fig. 8. The average and the coefficient of variation of the number of assigned PRBs to UEs over time.

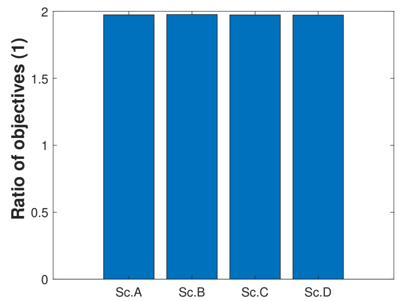

Fig. 9. The ratio of the equivalent objectives to Eq.(1) of $\mathcal{P}_1$ and $\mathcal{P}_0$ for special cases.

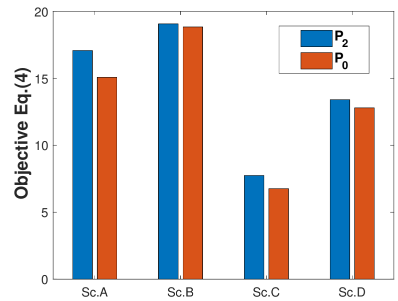

Fig. 10. The value of the objective Eq.(4) for special cases with $\mathcal{P}_2$ and $\mathcal{P}_0$.

environment. We derived the allocation policies that provide proportional fairness. We did this for two cases. In the first, the goal was to provide proportionally fair allocation across all UEs in the network, while in the second case, the objective was to guarantee proportional fairness among BSs. We evaluated the performance on real data sets, and compared it with no-SD-RAN network and another allocation policy, demonstrating the considerable improvements the introduction of SD-RAN brings into play. In the future, we will consider α-fairness.

Acknowledgement

This work was supported by the Federal Ministry of Education and Research of Germany (BMBF) under the project "6G-Life", with project identification number 16KISK002.

References

[1] L. Cui, R. Yu, and Q. Yan, "When big data meets software-defined networking: SDN for big data and big data for SDN," *IEEE network*, vol. 30, no. 1, 2016.

[2] X. Foukas, N. Nikaein, M. M. Kassem, M. K. Marina, and K. Kontovasilis, "FlexRan: A flexible and programmable platform for software-defined radio access networks," in *Proc. of ACM CoNEXT*, 2016.

[3] E. Coronado, S. N. Khan, and R. Riggio, "5G-EmPOWER: A software-defined networking platform for 5G radio access networks," *IEEE Transactions on Network and Service Management*, vol. 16, no. 2, 2019.

[4] A. Papa, R. Durner, L. Goratti, T. Rasheed, and W. Kellerer, "Controlling Next-Generation Software-Defined RANs," *IEEE Communications Magazine*, vol. 58, no. 7, 2020.

[5] ETSI, "5G NR overall description: 3GPP TS 38.300 version 15.3.1 release 15." www.etsi.org, 2018. Technical specification.

[6] R. Srikant, *The Mathematics of Internet Congestion Control*. Birk., 2004.

[7] A. Goldsmith, *Wireless communications*. CUP, 2005.

[8] F. Mehmeti and T. L. Porta, "Reducing the cost of consistency: Performance improvements in next generation cellular networks with optimal resource reallocation," *IEEE Tran. on Mob. Comp.*, vol. 21, no. 7, 2022.

[9] D. Raca, D. Leahy, C. J. Sreenan, and J. J. Quinlan, "Beyond throughput, the next generation: A 5G dataset with channel and context metrics," in *Proc. of ACM MMSys*, 2020.

[10] Z. Zaidi, V. Friderikos, and M. A. Imran, "Future RAN architecture: SD-RAN through a general-purpose processing platform," *IEEE Vehicular Technology Magazine*, vol. 10, no. 1, 2015.

[11] Q. Qin, N. Choi, M. R. Rahman, M. Thottan, and L. Tassiulas, "Network slicing in heterogeneous Software-defined RANs," in *Proc. of IEEE INFOCOM*, 2020.

[12] M. Yang, Y. Li, D. Jin, L. Su, S. Ma, and L. Zeng, "OpenRAN: A software-defined RAN architecture via virtualization," *ACM SIGCOMM Computer Communication Review*, vol. 43, no. 4, 2013.

[13] A. Gudipati, D. Perry, L. E. Li, and S. Katti, "SoftRAN: Software defined radio access network," in *Proc. of ACM SIGCOMM workshop on Hot topics in Software Defined Networking*, 2013.

[14] A. Papa, M. Klugel, L. Goratti, T. Rasheed, and W. Kellerer, "Optimizing dynamic RAN slicing in programmable 5G networks," in *Proc. of IEEE ICC*, 2019.

[15] G. Tychogiorgos, A. Gkelias, and K. K. Leung, "Utility-proportional fairness in wireless networks," in *Proc. of IEEE PIMRC*, 2012.

[16] A. Papa, A. Jano, S. Ayvaşık, O. Ayan, H. M. Gürsu, and W. Kellerer, "User-based Quality of Service aware multi-cell radio access network slicing," *IEEE Tran. on Net. and Ser. Management*, vol. 19, no. 1, 2022.

[17] A. Papa, P. Kutsevol, F. Mehmeti, and W. Kellerer, "Effects of SD-RAN control plane design on user Quality of Service," in *Proc. of IEEE Netsoft*, 2022.

[18] G. Ku and J. M. Walsh, "Resource allocation and link adaptation in LTE and LTE Advanced: A tutorial," *IEEE Communications Surveys & Tutorials*, vol. 17, no. 3, 2015.

[19] S. Boyd and L. Vandenberghe, *Convex optimization*. CUP, 2004.

[20] B. Li, A. Eryilmaz, and R. Srikant, "Emulating round-robin in wireless networks," in *Proc. of ACM MOBIHOC*, 2017.

[21] https://github.com/uccmisl/5Gdataset.

[22] F. Mehmeti and T. L. Porta, "Analyzing a 5G Dataset and Modeling Metrics of Interest," in *Proc. of IEEE MSN*, 2021.

[23] F. Mehmeti and T. F. La Porta, "Minimizing rate variability with effective resource utilization in 5G networks," in *Proc. of ACM MobiWac*, 2021.