

DISCUSSION

A Discussion of “A Note on Universal Inference” by Tse and Davison

Mathias Drton^{1,2} | Hongjian Shi¹ | David Strieder^{1,2}

¹School of Computation, Information and Technology, Technical University of Munich, Garching b. München, Germany

²Munich Center for Machine Learning (MCML), Munich, Germany

Correspondence

Mathias Drton, School of Computation, Information and Technology, Technical University of Munich, Boltzmannstr. 3, 85748 Garching b. München, Germany.
Email: mathias.drton@tum.de

Funding information

European Research Council (ERC), Grant/Award Number: 83818; German Federal Ministry of Education and Research; Bavarian State Ministry for Science and the Arts

We congratulate the authors for their insightful contribution to universal inference, as proposed in Wasserman et al. (2020). The split likelihood ratio test (SLRT) provided by the universal inference framework is valid in finite samples and requires essentially no regularity conditions. However, this amazing universality also comes at a price as the SLRT may be highly conservative.

In their article, Tse and Davison (2022) explore how nuisance parameters impact the conservativeness of the SLRT. They study the loss of power in the basic normal location model, taking up the analysis in Dunn et al. (2022). Tse and Davison also propose a modification to reduce the conservatism of the SLRT. Universal inference is justified by an application of Markov's inequality, and the modification is based on the clever observation that, in the normal location model, the expectation to be considered may be more tightly controlled by exploiting the independence of two terms that emerge in the presence of nuisance parameters. The impact of using this trick for irregular testing problems is illustrated in an example based on a Gaussian mixture model.

An independent recent work of our group studies universal inference from a related but also different angle (Strieder & Drton, 2022). We would like to take the opportunity of this discussion to point out similarities and also differences between this work and the paper of Tse and Davison.

The main results in Strieder and Drton (2022) provide the large-sample theory for the SLRT under both the null hypothesis and local alternatives assuming the classical regularity conditions that lead to Wilks' theorem for the standard likelihood ratio test (see Theorem 3.1 and related Corollaries/Remarks in Strieder & Drton, 2022). The setup allows for general composite null hypotheses. The limiting distributions of the SLRT are then used to illustrate how (loss of) power of the test is impacted by dimensionality of the hypotheses. These limiting distributions coincide with the distributions that are considered in the paper of Tse and Davison (e.g. Equations 8, 13–14).

The two articles differ in the way the (limiting) distribution of the SLRT is used in order to improve power. While Tse and Davison focus on using the independence of the two parts stemming from the parameter of interest and the nuisance parameter, Strieder and Drton concentrate on highlighting the impact of the data splitting. Tse and Davison utilize insights from the regular setting under the perspective of nuisance parameters to motivate the use of a smaller critical value. Such a smaller critical value obviously increases power and continues to control the size in the regular case. However, with the modified critical value, one no longer has a universal guarantee to control the size in general irregular problems. This raises the question: How far should we go in terms of lowering the critical value once we start setting it based on asymptotic distributions derived for regular testing problems? While this would be easy to do, we certainly do not want to go all the way to the relevant quantile of the regular limiting distribution. But then which principles should we invoke to justify a particular lowered critical value when applying it to irregular problems?

As mentioned, the work of Strieder and Drton pursues a different goal and proposes a different modification. Instead of changing the critical value, this modification employs the regular limiting distribution merely to tune the data splitting ratio (at a target power level). Tuning the splitting ratio has a more limited impact on power, but importantly, an SLRT with tuned splitting remains universally valid in finite samples—no matter whether the problem is regular or not.

This is an open access article under the terms of the [Creative Commons Attribution](https://creativecommons.org/licenses/by/4.0/) License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.

© 2023 The Authors. *Stat* published by John Wiley & Sons Ltd.

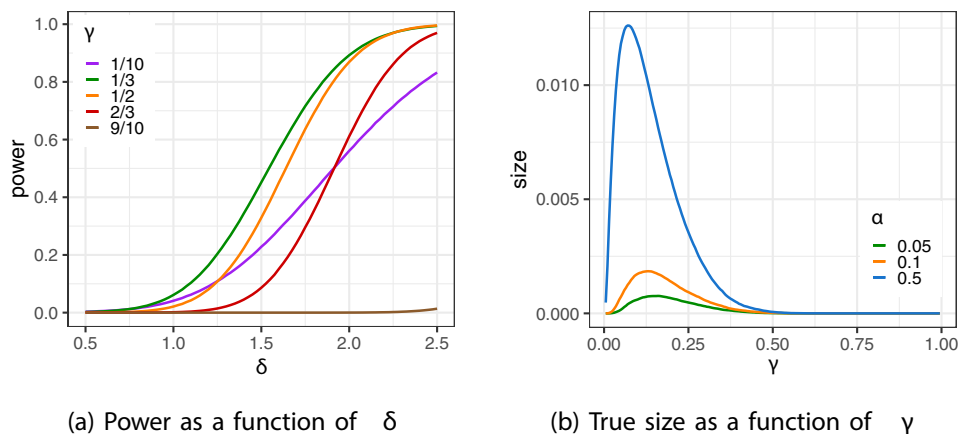


FIGURE 1 Power of the split likelihood ratio test (SLRT) for testing a 45-dimensional hypothesis in 50-dimensional parameter space with departure (δ, \dots, δ) , split ratio γ , and nominal size α .

Tse and Davison also investigate the power of the SLRT across various splitting ratios with the conclusion that a ratio of $\gamma = 2/3$ performs best. We emphasize that this is a feature of the considered low-dimensional setting. Contrasting Figure 2 from the article of Tse and Davison, Figure 1 here displays the power and size of the SLRT when testing a 45-dimensional hypothesis in a 50-dimensional space and shows that in a higher-dimensional setting, a splitting ratio below 0.5 performs best. For a detailed analysis of the optimal choice of the splitting ratio based on the dimensionality of tested null and alternative hypotheses, we refer to Strieder and Drton (2022).

ACKNOWLEDGEMENTS

Open Access funding enabled and organized by Projekt DEAL.

REFERENCES

- Dunn, R., Ramdas, A., Balakrishnan, S., & Wasserman, L. (2022). Gaussian universal likelihood ratio testing. *Biometrika*. <https://doi.org/10.1093/biomet/asac064>
- Strieder, D., & Drton, M. (2022). On the choice of the splitting ratio for the split likelihood ratio test. *Electronic Journal of Statistics*, 16(2), 6631–6650. <https://doi.org/10.1214/22-EJS2099>
- Tse, T., & Davison, A. C. (2022). A note on universal inference. *Stat*, 11(1), e501. <https://doi.org/10.1002/sta4.501>
- Wasserman, L., Ramdas, A., & Balakrishnan, S. (2020). Universal inference. *Proceedings of the National Academy of Sciences*, 117(29), 16880–16890. <https://doi.org/10.1073/pnas.1922664117>

How to cite this article: Drton, M., Shi, H., & Strieder, D. (2023). A Discussion of “A Note on Universal Inference” by Tse and Davison. *Stat*, 12(1), e574. <https://doi.org/10.1002/sta4.572>