POMS PRODUCTION AND OPERATIONS MANAGEMENT

PRODUCTION AND OPERATIONS MANAGEMENT POMS

Vol. 28, No. 3, March 2019, pp. 721–739
ISSN 1059-1478 | EISSN 1937-5956 | 19 | 2803 | 0721

DOI 10.1111/poms.12947
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First-Price Split-Award Auctions in Procurement Markets with Economies of Scale: An Experimental Study

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We experimentally study first-price split-award auction formats as they can be found in procurement markets where suppliers have economies of scale. Our analysis includes sequential and combinatorial auctions, which allow for bids on the package of two shares and single shares. We derive equilibrium predictions as hypotheses for bidder behavior in our laboratory experiments. These equilibrium predictions help explain important patterns in our experimental results. The combinatorial first-price sealed-bid auction yields lower prices than the other mechanisms and is highly efficient independent of the extent of scale economies. With strong economies of scale both combinatorial auction formats let the auctioneer incur significantly lower procurement costs and generate high efficiency compared to the sequential auction. We also find high efficiency of the combinatorial first-price sealed-bid auction in experiments with diseconomies of scale, making these auctions attractive if the buyer has uncertainty about the economies of scale in a market. Our analysis shows that combinatorial split-award auctions can be an attractive alternative for the buyer compared to their sequential counterparts.

Key words: split-award auction; laboratory experiment; procurement; economies of scale; combinatorial auction
History: Received: July 2018; Accepted: August 2018 by Haresh Gurnani, after 4 revisions.

1. Introduction

Procurement managers regularly need to purchase large quantities of certain goods or commodities. This can be raw materials for direct procurement or commodities for maintenance, repair, and operations. Economies of scale in the production are widespread and suppliers in industrial procurement often provide volume discounts when selling larger quantities. If a procurement manager knows that there are substantial economies of scale in a market, he might want to have competition for the entire demand in order to minimize cost. In many cases, however, the procurement manager does not know the economies of scale in the production, and such information is difficult or impossible to get for specific commodities. Even if suppliers are expected to have economies of scale in an industry, there can be capacity constraints leading to diseconomies of scale for some suppliers.

Therefore, buyers are interested in auction formats allowing them to determine the cost-minimal outcome for different environments. We analyze *ex post* split-award auctions and provide evidence that they yield efficient outcomes in situations where the suppliers have economies of scale, but also in environments where this is not the case. In our experimental analysis, we focus on economies of scale as this is by far the most wide-spread environment in procurement markets. We provide evidence that combinatorial first-price sealed-bid auctions yield efficient and low-cost results in environments with different levels of economies of scale, but also with diseconomies of scale.

1.1. Motivation

Companies such as Sun and HP, for example, procure products worth hundreds of millions of dollars using different types of multiple sourcing auctions (Donohue et al. 2017, Elmaghraby 2007). Split-award auctions are regularly used for multi-sourcing in industrial procurement and they auction off shares of the entire demand (aka. lots or line items). For example, a procurement manager might be willing to sign a contract for the entire demand of 100 tons of a raw material, or alternatively two contracts for 50 tons each depending on which allocation yields the lowest total cost. There are two classes of split-award auctions, which allow for different allocations. *Ex ante* split-award auctions always implement an outcome with multiple winners, as no single supplier is allowed to win the 100% share only. Hence, such formats are apt, when the procurement manager commits to multi-sourcing a priori.

Ex post split-award auctions decide endogenously, whether there are multiple winners or one supplier wins the entire demand.[1] Such auctions are either organized as *combinatorial* or *non-combinatorial (typically sequential) auctions*.

The majority of procurement auctions are first-price sealed-bid (FPSB) auctions (Bogaschewsky and Müller 2015), but multi-object first-price auctions are not well understood. The existing game-theoretical literature on combinatorial first-price split-award auctions focuses on designs with two shares only (Anton and Yao 1992, Anton et al. 2010), and so do we. This keeps our analysis tractable and also describes the largest part of the split-award auctions we have found among our industry partners.

One can think of a number of possible first-price split-award auction formats combining first-price sealed-bid and Dutch auctions. We focus on three specific types as they are representative for others and we have evidence for their use in the field. The first format is a *non-combinatorial Dutch-FPSB auction*, in which the two shares are awarded sequentially; the first auction is an ascending Dutch auction, while the remaining share is awarded by a FPSB auction. In contrast to this sequential format, the two combinatorial formats allow for bids on individual objects and on packages. We analyze a simple *combinatorial FPSB auction*, in which bidders simultaneously submit a bid for a share of the demand and the package of both shares. The last auction format is the *combinatorial Dutch-FPSB auction*, which is an extension of the non-combinatorial variant. In the Dutch phase, bidders can not only win a share of the business, but they receive counteroffers for 100% as well. For the case that a bidder only accepts the smaller counteroffer in phase 1, the remaining share is auctioned off by a final FPSB stage. A more detailed description of the formats is provided in section 2.

We analyze the types of auction designs used by one of the largest European electronics and manufacturing multinationals, that is, auctions with a spend of between 250 thousand and 175 million Euros each within one year (April 2015 to March 2016). The total annual spend of this company is in the billions. We concentrated on first-price auctions only, as nearly all of the split-award auctions were first-price auctions (first-price sealed-bid or versions of an ascending (Dutch) auction). Only a small share were descending (English) auctions, and the company never organized a Vickrey–Clarke–Groves mechanism. There were two interesting observations from the empirical analysis which motivated this research.

(i) About every third first-price auction was a split-award auction, most of which included two shares only. About 81% were *ex ante* and 19% are *ex post* split-award auctions.
(ii) Only 5% of the split-award auctions were organized as combinatorial auctions allowing bidders to submit a package bid. The majority was organized as *ex ante* or *ex post* non-combinatorial auction.

Observation (i) shows the importance of split-award auctions on the procurement practices of the electronics multinational. Most of them involved only two predefined shares. The frequent application of *ex ante* split-award auctions arises from the unwillingness of a buyer to allow package bids, when he wants to implement a dual sourcing strategy for sure. The reason can be to keep up competition for future auctions or to have a second source in case the primary supplier defaults. Nevertheless, in about every fifth split-award auction the buyer of the electronics multinational delegated the decision about the sourcing strategy to the market mechanism by applying an *ex post* format.

The most surprising observation (ii), however, is that the majority of these *ex post* split-award auctions did not allow for package bids, but were non-combinatorial auctions. Sequential split-award auctions were employed by the procurement managers, amongst others, in the hope of achieving lower prices in the second stage as competition is more transparent. As in many procurement organizations, only a small proportion of all conducted auctions included package bids.

Unfortunately, not much is known about non-combinatorial and combinatorial first-price auctions with multi-unit demand, which is an important gap in the theoretical literature and an environment with significant importance for procurement practice.

1.2. Contributions
We report on the experimental analysis of bidding behavior in various *ex post* split-award auctions. The combinatorial FPSB auction, as well as the combinatorial and the non-combinatorial Dutch-FPSB auctions are studied for environments with low and strong economies of scale. The resulting efficiency, procurement costs and individual bidding behavior in each auction format are compared to our equilibrium predictions, which explain important patterns of the bidding behavior and the outcomes of the auctions in the laboratory.

With strong economies of scale, we find higher efficiency in the combinatorial FPSB than in the combinatorial Dutch-FPSB auction; although the efficiency of the non-combinatorial format is comparable to the FPSB auction with strong economies of scale, it incurs significantly higher costs than the combinatorial formats.

For weak economies of scale the combinatorial FPSB auction is still highly efficient whereas both Dutch-FPSB auction mechanisms (combinatorial and non-combinatorial) lead to significantly lower efficiency. The costs of the non-combinatorial Dutch-FPSB auction are close to those of its combinatorial counterpart and the FPSB auction has the lowest costs. The combinatorial FPSB auction appears to be more robust against changes in scale economies in terms of procurement costs than the combinatorial Dutch-FPSB auction, and the choice of the auction format matters. Importantly, we find no benefits from the non-combinatorial Dutch-FPSB mechanism in our experimental studies. We also provide experiments with diseconomies of scale where the split-award is efficient. Indeed, the combinatorial FPSB auction is efficient in all of these experiments as well.

The results suggest that the combinatorial FPSB auction is a flexible mechanism compared to alternatives such as single-lot or sequential auctions for multiple lots. Allowing suppliers to bid only on the entire quantity in a single-lot auction will result in low efficiency with diseconomies of scale, while sequential auctions have disadvantages in markets with economies of scale as we show. As a result, the combinatorial FPSB auction provides an attractive alternative in situations where the buyer has uncertainty about the scale economies in a market.

2. The Auctions

We first introduce necessary notation and assumptions largely following Anton and Yao (1992), before we discuss the auction formats. Finally, we provide pointers to the related literature.

2.1. Notation and Assumptions

We analyze *ex post* split-award auctions, in which $n > 2$ *ex ante* symmetric, risk-neutral, and profit-maximizing suppliers can win a contract for 50% or 100% of the business.[2] Bidder i's (with $i \in \{A, B, \ldots\}$ and $n = |\{A, B, \ldots\}|$) costs for 100% of a business θ_i are independently distributed according to $F(\cdot)$ with support $[\underline{\Theta}, \overline{\Theta}]$ $(0 < \underline{\Theta} < \overline{\Theta})$. The density f is both positive and continuous. The cost draw of a bidder i for the 100% share is denoted by Θ_i and is private. The costs for 50% of the business, $C\Theta_i$ are dependent on a constant efficiency parameter $0.5 < C < 1$, which is symmetric and common knowledge amongst all suppliers, but not the buyer. No costs incur in the case of loss of the business. The economies of scale depend on the production technology which is known to suppliers. In contrast, procurement managers need to buy dozens of commodities and there can be significant uncertainty about the scale economies (Anton and Yao 1992). We assume the economies of scale C to be the same across all suppliers of a product, which is a reasonable assumption in production. We focus on markets with economies of scale in which it is always efficient for the buyer to award 100% of the business to a single supplier. It is easy to show that independent of the two draws of the bidders' cost types, the sole source award is always the efficient award if the efficiency parameter C lies above 0.5. We refer to this setting as Sole Source Efficiency (SSE).

We discuss static as well as dynamic formats in this article. In the static mechanisms, each bidder i submits prices for 100% and∖ or 50% of the business, $p^s(\Theta_i) : [\underline{\Theta}, \overline{\Theta}] \longrightarrow \mathbb{R}$ and $p^\sigma(\Theta_i) : [\underline{\Theta}, \overline{\Theta}] \longrightarrow \mathbb{R}$, respectively.

The dynamic mechanisms are modeled as multistage games with observed actions. Considering the history $h^0 = \{\}$, the functions $p^{s1}(\Theta_i, h^0)$ and $p^{\sigma 1}(\Theta_i, h^0)$ denote the level at which a bidder with costs Θ_i approves a price for the respective award in phase 1. In phase 2, it must be differentiated between a bidding strategy $p^{\sigma 2w}(\Theta_w, h^1)$ for the winner of phase 1 with cost type Θ_w and the price(s) $p^{\sigma 2l}(\Theta_l, h^1)$ of the loser(s) with cost type(s) Θ_l with ($l \neq w$ and $\Theta_l, \Theta_w \in [\underline{\Theta}, \overline{\Theta}]$). Both bidding strategies of phase 2 depend on the history $h^1 = \{p^{\sigma 1}(\Theta_w, h^0)\}$. All price functions of phase 1, as well as phase 2 map from $[\underline{\Theta}, \overline{\Theta}] \times \mathbb{R}$ to $\mathbb{R}$.

Bidding functions are assumed to be non-decreasing and continuous in pure bidding strategies; by applying a mixed strategy, the supplier randomizes his bids over an interval $[\underline{b}(\Theta_w), \overline{b}(\Theta_w)]$ (with $\underline{b}(\Theta_w) < \overline{b}(\Theta_w)$ and $\underline{b}(\Theta_w), \overline{b}(\Theta_w) \in [\underline{\Theta}, \overline{\Theta}]$) according to a distribution function H (and density h).

The auctioneer is *ex ante* indifferent between awarding 100% of the business to a single supplier (sole source award) and awarding 50% of the business each to two different suppliers (split award). Hence, determination of the winner in a split award auction must satisfy the auctioneer's indifference condition. Hereafter, the i-th lowest order statistic out of n different cost types is denoted by $\Theta_{i:n}$. All bidders are assumed to be *ex post* individually rational, that is, the equilibrium bids for 50% of the business and 100% of the business must be greater than or equal to the costs of the respective allocation.[3]

Similar to Anton and Yao (1992), we discuss three different equilibrium types: In a *Winner-Takes-All (WTA) equilibrium*, the auction always results in a sole source award. In contrast, when bidders play a *σ equilibrium*, the auction results in a split award. An equilibrium in which both awards can appear with positive probability is called a *hybrid equilibrium*. Most of the equilibria discussed in this article involve pure strategies. If an equilibrium is in mixed-strategies it is noted accordingly.

Bayesian Nash equilibria are considered in the FPSB split-award model, which comprise a set of strategies $S_e^{\text{BNE}} = (p_e^s(\Theta_i), p_e^\sigma(\Theta_i))$ for bidders with cost types Θ_i. We use perfect Bayesian equilibria as equilibrium solution concept for the non-combinatorial and combinatorial Dutch-FPSB auction. The set of strategies $S_e^{\text{PBE}} \in \{S_e^{\text{PBE1}}, S_e^{\text{PBE2}}\}$ and a system of beliefs $\mu = \{\mu^1_{-i}(\Theta_i|h^0), \mu^2_l(\Theta_w|h^1), \mu^2_w(\Theta_l|h^1)\}$, which are probability distributions with $\Theta_i, \Theta_w, \Theta_l \in [\underline{\Theta}, \overline{\Theta}]$ conditional on the history $h^0 = \{\}$ or $h^1 = \{p_e^{\sigma 1}(\Theta_w, h^0)\}$, define the perfect Bayesian equilibrium (S_e^{PBE}, μ). Bidders must be differentiated after phase 1, because there is a winner w of the first unit with cost type Θ_w, whose bid is revealed after phase 1, and one or more loser(s) l with cost types Θ_l ($w \neq l \in \{A, B, \ldots\}$ and $\Theta_w, \Theta_l \in [\underline{\Theta}, \overline{\Theta}]$). Hence, $\mu_l^2(\Theta_w|h^1) = P(\Theta_w \leq \Theta|h^1)$ characterizes the beliefs of bidder(s) l about the cost type of the winner of phase 1, whereas $\mu_w^2(\Theta_l|h^1) = P(\Theta_l \leq \Theta|h^1)$ defines the beliefs of bidder w about the type of a loser of phase 1.

In the Dutch or Dutch-FPSB auction, the set of strategies of bidders with cost types Θ_i in a WTA equilibrium, $S_e^{\text{PBE1}} = (p_e^{s1}(\Theta_i, h^0), p_e^{\sigma 2l}(\Theta_i, h^1))$, is defined by a price function $p_e^{s1}(\Theta_i, h^0)$ for 100% of the business and a (credible) threat $p_e^{\sigma 2l}(\Theta_i, h^1)$ in case of a split deviation of a bidder in phase 1. In the non-combinatorial formats, the strategy set of a WTA or hybrid equilibrium is given by $S_e^{\text{PBE2}} = (p_e^{\sigma 1}(\Theta_i, h^0), p_e^{\sigma 2w}(\Theta_w, h^1), p_e^{\sigma 2l}(\Theta_l, h^1))$.

The model assumptions and the equilibrium types discussed in this section apply for the equilibrium predictions below. We refer to such an environment as *SSE split-award auction model*.

2.2. The Non-Combinatorial Dutch-FPSB Split-Award Auction

In the non-combinatorial Dutch-FPSB auction bidders can only submit offers for shares of the business, not the whole business. We focus on the non-combinatorial Dutch-FPSB split-award auction with two phases as one of the most common formats.

The first phase comprises a Dutch auction for 50% of the business. The price clock starts at a price close to zero or strictly lower than the lowest possible cost type. Then, the split price is raised continuously until a supplier accepts a price. This supplier wins 50% of the business and phase 1 terminates.

A FPSB mechanism is applied to auction off the remaining share of the business amongst all suppliers in the subsequent phase. The winner as well as the loser(s) of phase 1 simultaneously submit offers for the remaining 50% share; the supplier with the lowest quote wins.

Two tie-breaking rules apply for all split-award auctions analyzed in this article: First, the tie is always broken in favor of the split award in the case that the procurement costs of both allocations are equivalent; second, if two or more bids for the same award are equal, a lottery in which each involved supplier has equal chances decides on the winner. Hence, the efficient sole source allocation is awarded when one supplier is the winner of 50% in both phases. Otherwise, the different winners of phase 1 and 2 are awarded 50% of the business each in the inefficient split allocation.

2.3. The FPSB Split-Award Auction

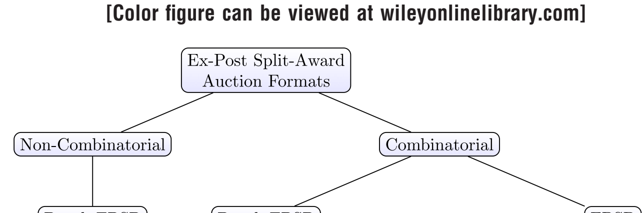
The FPSB split-award auction[4] is the simplest format and identical to the mechanism discussed by Anton and Yao (1992). All bidders simultaneously submit their quotes for 50% and 100% of the business in a concealed manner. Depending on the cost-minimal allocation either the sole source award is won by the supplier with the lowest price for 100% of the business or the split award is awarded to the two most competitive suppliers for 50% of the business.

2.4. The Dutch-FPSB Split-Award Auction

The 50% share as well as the 100% share are up for auction simultaneously in phase 1 of the Dutch-FPSB split-award auction. This is the main difference compared to its non-combinatorial counterpart, in which bidders can only win 50% in phase 1. Beginning from low starting prices, two price clocks, one for the 50% share and one for the 100% share, are raised continuously. In order to incorporate the buyer's indifference condition in the auction, the price for 100% must always be twice as high as the price for 50% share at each point in time during phase 1.

A bidder can win the sole source award either by accepting 100% of the business in phase 1 or by winning 50% sequentially in both phases. Hence, phase 2, in which the remaining share is auctioned off as in the format of section 2.2, only becomes effective when the 50% share is approved in phase 1. In this case, the winner of phase 1 gets at least 50% of the business for sure and can only increase his share in phase 2. Again, the Dutch-FPSB split-award auction comprises a sealed-bid auction in the second phase. Figure 1 gives

Figure 1 Overview of Split-Award Auctions Discussed in this Article [Color figure can be viewed at wileyonlinelibrary.com]



an overview of the different types of *ex post* split-award auctions which are analyzed in this article.

2.5. Related Literature

Closest to the non-combinatorial *ex post* split-award auctions introduced earlier are multi-unit auctions with bidders, who demand more than one unit. Unfortunately, there is not much literature on sequential or simultaneous first-price auctions. Krishna (2010, p. 226) writes "a full treatment of sequential auctions with multi-unit demands is problematic (...) once a particular bidder has won the first unit his behavior and interests are different from those of the other bidders."

Bidding behavior in sequential second-price auctions with bidders having multi-unit demand is discussed in Katzman (1999). They show that the second-price auction has a simple dominant strategy in spite of the asymmetries among bidders. Chakraborty (2006) characterizes equilibria for a first-price (forward) auction, in which bidders simultaneously submit sealed bids for two identical items. Bidders have private and *diminishing* valuations for winning multiple items in a sales auction. There is no analysis of sequential multi-unit procurement auctions with economies of scale as of yet.

There are also only a few papers on equilibrium analyses of combinatorial auctions. In a seminal article, Bernheim and Whinston (1986) provides an equilibrium analysis for a combinatorial first-price sealed-bid format in a complete-information model. Anton and Yao (1989) characterize bidding equilibria of first-price *ex post* split-award auctions with two bidders and two shares in a general efficiency environment under complete information. In this symmetric information setting, the split-award auction is weakly dominated in procurement costs by the WTA auction. However, the former format may raise the incentive for cost-decreasing investments prior to the bidding stage and therefore reduce total procurement costs under those of the WTA auction. Gong et al. (2012) formalize the potential advantages of split-award auctions with the possibility of investment prior to the bidding stage as first reported in Anton and Yao (1989). They derive conditions under which split-awards of generalized second-price auctions dominate sole-sourcing in procurement costs. It should be noted, though, that in their model the buyer determines the final sourcing before the investment stage and therefore it is not endogenously determined by the suppliers (and the underlying cost structure) whether to split the award or not.

Anton and Yao (1992) extend their previous analysis (Anton and Yao 1989) to the usual *incomplete information* environment analyzed in auction theory. There is no investment stage before bidding in their model and they focus on strong *diseconomies of scale* which is modeled via a linear cost function $C\Theta$ for one unit with a cost draw Θ for two units, and an efficiency parameter $C \ll 0.5$ that determines the costs for one unit. Suppliers have cost benefits of winning one unit compared to the package (of both units) and the split-award outcome is always efficient. They establish conditions under which efficient σ equilibria with two winners of *ex post* split-award auctions lead to lower procurement costs than the inefficient WTA equilibrium. The latter corresponds to the unique equilibrium of the WTA auction, where suppliers can only bid on the package of both units. Therefore, whenever the buyer prefers a σ equilibrium to the WTA equilibrium, the *ex post* split-award auction is also preferred to the WTA auction.

Besides Anton and Yao (1992), the closest related theoretical article to our analysis on split-award auctions is by Anton et al. (2010). This article not only discusses the case of a linear cost function $C\Theta$, but also more general nonlinear cost functions $C(\Theta)$. With the linear cost function $C\Theta$ that Anton and Yao (1992) and we use, no bidder can have economies of scale in their model (see their assumption 2). The nonlinear cost functions $C(\Theta)$ can result in environments, in which economies of scale are present for some *but not for all* cost types. An environment where all suppliers have economies of scale (as in our study), or where all suppliers have strong diseconomies of scale as in Anton and Yao (1992) is not covered by the analysis in Anton et al. (2010), even assuming arbitrary cost functions $C(\Theta)$. Their analysis is limited to two suppliers and two units and depends on the possibility of a single supplier to exclude the split unilaterally, similar to the diseconomies of scale-case by Anton and Yao (1992).

In summary, we make a number of contributions to this literature stream on *ex post* split-award auctions:

(1) We focus on a setting with economies of scale ($C > 0.5$) for all suppliers which has not been analyzed in prior literature on split-award auctions. Considering the linear cost function $C\Theta$, the settings of Anton and Yao (1992), Anton et al. (2010) and ours complement each other analyzing different market environments.
(2) We not only analyze the sequential Dutch-FPSB and the combinatorial FPSB split-award auction, but also a combinatorial Dutch-FPSB auction that is being used in practice and show the differences to the earlier formats. So, not only the fact that an auction allows package bids is important, also the type of first-price auction matters.

(3) We analyze the common case with $n > 2$ and do not require a market with two bidders and two units.
(4) We provide experimental results, which none of the two prior papers on split-award auctions does. This is important as our equilibrium predictions explain the experimental results such as their efficiency and cost ranking.

Despite their potential cost advantages in asymmetric information Environments, *ex post* split-award auctions might be preferred by the buyer due to further benefits such as incentivising higher cost-reducing investment. In their recent work, Chaturvedi et al. (2014) discuss benefits of split awards on supply base maintenance as a different aspect. In contrast, we focus on welfare comparisons between different first-price *ex post* split-award auctions as in Anton and Yao (1992) and Anton et al. (2010).

3. Hypotheses

In what follows, we derive equilibrium predictions which lead to hypotheses for our laboratory experiments. Let us introduce some additional notation and the equilibrium solution concepts, before we analyze the different auction formats. While we do not aim for a complete equilibrium analysis, the predictions serve as an benchmark to compare our experimental results against.

3.1. The FPSB Split-Award Auction

First, we discuss equilibrium strategies in the FPSB split-award auction. We show that there are pure WTA equilibria with economies of scale, for which bidders aim to exclude the split award with a high bid and bid competitively on 100% of the business.

PROPOSITION 1. *Consider the SSE split-award auction model including $n > 2$ ex ante symmetric bidders with cost types Θ_i. In the FPSB split-award auction, a WTA equilibrium is given by*

$$p_e^s(\Theta_i) = \Theta_i + \frac{\int_{\Theta_i}^{\overline{\Theta}} (1 - F(t))^{n-1} dt}{(1 - F(\Theta_i))^{n-1}}$$
$$p_e^\sigma(\Theta_i) = p_e^s(\Theta_i).$$

This proof can be found in Appendix A.1. As bidders submit high prices for 50% of the business, there are no profitable deviations which include winning a share of the business. We only present a very easy strategy, in which bidders submit the same bid for 50% and 100% of the business to be sure that the split is excluded. Other bid-to-lose prices are possible, as long as they are high enough to exclude a split award.

The bidding function $p_e^s(\Theta_i)$ assures that there is no sole source deviation for the suppliers.

Furthermore, we can exclude other equilibrium types in a setting with economies of scale.

PROPOSITION 2. *Consider the SSE split-award auction model including $n > 2$ ex ante symmetric bidders with cost types Θ_i. In the FPSB split-award auction, any σ equilibrium can be excluded. Additionally, the WTA equilibrium presented in Proposition 1 is unique within the class of strategies with strictly increasing price functions.*

Please refer to Appendix S1 for the proof of this proposition. By analyzing only strategies with increasing price functions, we show that the WTA equilibrium in Proposition 1 is unique. There are more profitable deviations in all other WTA or hybrid equilibria, such that such strategies can be excluded.

The existence and characteristics of such an equilibrium are completely independent of the specific value of the efficiency parameter C. This leads to the following hypothesis:

HYPOTHESIS 1 (BIDDING STRATEGY IN THE COMBINATORIAL FPSB AUCTION). *Bidders play an efficient WTA equilibrium strategy in the combinatorial FPSB auction with strong and weak economies of scale.*

3.2. The Dutch-FPSB Split-Award Auction

Second, we analyze the bidding behavior in the Dutch-FPSB split-award auction as an alternative combinatorial first-price mechanism. It can be seen as the combinatorial extension of the non-combinatorial format. We first show that pure WTA equilibria, including bundle bids for 100% of the business exist in some but not all settings with economies of scale.

PROPOSITION 3. *Consider the SSE split-award auction model including $n > 2$ bidders with cost types Θ_i. In the Dutch-FPSB split-award auction, there is a WTA equilibrium where the suppliers bid competitively on the package of two units in the first phase, if the economies of scale are strong enough.*

An explicit formal description of the equilibrium strategy and the proof can be found in Appendix A.2. As in Anton and Yao (1992), we characterize conditions on the scale economies (described via C), which allow for pure and efficient equilibria. Note that the combinatorial Dutch-FPSB auction requires stronger assumptions for pure WTA equilibria to exist compared to the combinatorial FPSB auction, where $C > 0.5$ is sufficient. With weak economies of scale some bidders in the combinatorial Dutch-FPSB auction have an incentive to deviate from equilibrium by following a strategy that tries to win 100% sequentially. As the loser of phase 1 can only win at most 50% in phase 2, sequential deviations can be more profitable for some cost types.

The existence of WTA equilibria in combinatorial first-price auctions is due to the fact that bidders can express their strong preferences for 100% by accepting the bundle offer. This is not possible in the non-combinatorial auction formats of section 3.3. In the proof of the equilibrium strategy for the combinatorial Dutch-FPSB auction, we show that sole source deviations for winning 100% in phase 1 as well as split deviations are less profitable than playing the equilibrium strategy.

Similar to the other two auction formats discussed in the previous sections, σ equilibria can be excluded. The same holds trut for hybrid equilibria with strictly increasing price functions.

PROPOSITION 4. *Consider the SSE split-award auction model including $n > 2$ bidders with cost types Θ_i. In the Dutch-FPSB split-award auction, there is no σ equilibrium. Furthermore, it can be shown that, when a WTA equilibrium as in Proposition 4 exists, it is unique considering equilibria with strictly increasing price functions.*

The proof can be found in Appendix S1 as well. Although a complete equilibrium analysis of both auction formats is not in the scope of this article, our analysis indicates that the introduction of bundle bids alone may not guarantee efficiency for all possible settings with economies of scale. This strategic difference between both first-price auction formats underscores the importance of the decision for the right split-award auction format, as not only the choice between non-combinatorial and combinatorial bid language may lead to varying results, but also the choice of the specific auction format. This is different to the single-object setting, in which cost equivalence applies under standard assumptions. This leads to the following hypothesis:

HYPOTHESIS 2 (BIDDING STRATEGY IN THE COMBINATORIAL DUTCH-FPSB AUCTION). *Bidders play an efficient WTA equilibrium strategy for strong economies of scale, but not always with weak economies of scale.*

3.3. The Non-Combinatorial Dutch-FPSB Split-Award Auction

Finally, we analyze bidding behavior in the non-combinatorial Dutch-FPSB split-award auction. In this format, bidders cannot submit combinatorial offers, although they have a preference for winning the bundle with economies of scale. Bidders interested in winning 100% must subsequently win both auctions for 50%. Furthermore, the winner Θ_w of the first phase knows that his additional costs for winning the remaining unit, $(1 - C)\Theta_w$ in phase 2 are lower than those of his opponents with $C > 0.5$. The following hypothesis is based on a lengthy equilibrium prediction that we had to omit from the study for space restrictions. The proof is available in an Appendix S1.

HYPOTHESIS 3 (BIDDING STRATEGY IN THE NON-COMBINATORIAL DUTCH-FPSB AUCTION). *Bidders follow a safe strategy in equilibrium with strong economies of scale, that is, they submit prices as high as their cost for the 50% share in both phases. Hence, the winner of the first unit wins the second share as well in a symmetric equilibrium and the auction outcome is efficient. This is not the case with weak economies of scale. Suppliers with low cost types gamble in both phases of the auction and bid at levels above their costs. Hence, the winner of the first unit might lose the remaining share against a competitor and the outcome becomes inefficient in many cases.*

The bidding behavior in phase 2 of the sequential auction depends on the costs of the bidders as well as the result of phase 1. As the cost type of the winner of phase 1 is revealed in equilibrium, all losers (with cost type Θ_l) have full information about the winner's cost type in phase 2. One can show that only mixed equilibria exist for bidders with *low cost types* Θ_w. The bidders with low cost types randomize their bids, which can lead to inefficiency.

In contrast, a winner of phase 1 with a *high cost type* does not take the risk of losing the remaining 50% share by following a safe strategy. This means, he bids his costs for 50% in equilibrium. In this case, the winner of phase 1 wins phase 2 as well, because the individual rationality assumption applies to all suppliers. A loser of phase 1, Θ_l, has no chance of winning the remaining share due to the scale economies and bid a price equal to his costs $C\Theta_l$ in equilibrium. So, for settings with strong economies of scale, it can be optimal to bid a price as high as the costs for winning the 50% share in phase 2, and this is true even for a winner with the lowest cost type $\underline{\Theta}$. In this case, the sole source award is the only outcome in equilibrium, and the discussed strategies form a WTA equilibrium. As split awards can occur with weak economies of scale, the strategies are part of a hybrid equilibrium.

Let us now compare all three auction formats in terms of efficiency and procurement costs.

3.4. Efficiency

With economies of scale, the result of the auction is only efficient for all possible combinations of cost types in a WTA equilibrium. However, the results of the analysis above show that there are only hybrid equilibria for the non-combinatorial format in some settings.

The FPSB split-award auctions always results in the WTA equilibrium, if $C > 0.5$. In the Dutch-FPSB or non-combinatorial Dutch-FPSB auction the economies of scale need to be stronger than in the FPSB auction to assure the existence of a WTA equilibrium. With strong economies of scale, the winner of phase 1 of the sequential split-award auction format has no incentive to bid higher than his costs for the split award in phase 2 and wins 100% of the business in equilibrium as well. However, inefficient split awards are possible in settings with a lower efficiency parameter C and combinations of cost types, for which the first and second order statistic lie close together. This leads to the following hypothesis.

HYPOTHESIS 4 (EFFICIENCY). *The non-combinatorial Dutch-FPSB auction is as efficient as the combinatorial FPSB and the Dutch-FPSB auction with strong economies of scale. In the combinatorial and non-combinatorial Dutch-FPSB auctions, inefficiencies arise with weak economies of scale.*

3.5. Procurement Costs

Procurement costs are the most important metric to measure success in procurement. Depending on the efficiency parameter C the costs can vary significantly between the different auction formats discussed above. We derived a formula for procurement costs in equilibrium in the non-combinatorial auction, which can be found in Appendix A.3 and which also serves as a baseline for our experiments.

As the bidders play different equilibrium strategies depending on their cost type, the computation of the procurement costs in the non-combinatorial auction and the resulting formula is more elaborate than the derivation of the costs in the FPSB or Dutch-FPSB auction. The procurement costs of the latter are identical to those in the well-known Vickrey-Clarke-Groves mechanism, which follows from the revenue equivalence theorem (Krishna 2010).

HYPOTHESIS 5 (PROCUREMENT COSTS). *The non-combinatorial Dutch-FPSB auction leads to higher procurement costs than the combinatorial FPSB and Dutch-FPSB auction with strong economies of scale. The non-combinatorial Dutch-FPSB auction also leads to higher costs than the combinatorial FPSB with weak economies of scale.*

4. Experimental Evaluation

Before discussing efficiency, procurement costs and the bidder behavior in our experiments in detail, we first describe the experimental design.

4.1. Experimental Design

In multi-period human subjects experiments we tested our hypotheses for a three-bidder environment of one non-combinatorial and two combinatorial first-price split-award auction mechanisms. We examined the non-combinatorial Dutch-FPSB auction and the combinatorial FPSB and Dutch-FPSB formats. Moreover, we analyzed each of the three auctions for two different economies of scale settings. One setting with an efficiency parameter of $C = 0.67$ (strong economies of scale) and the other setting with an efficiency parameter of $C = 0.52$ (weak economies of scale). Thus, our treatment variables correspond to the auction format and the efficiency parameter, which result in the following six treatments (Table 1).

At the beginning of every period in all treatments, the bidders are informed about their own cost draws for the supply of 50% or 100% of a fictitious order. Each bidder's cost parameter Θ is uniformly and independently distributed on the interval [100.00, 140.00]. The efficiency parameter is set to be either $C = 0.52$ or $C = 0.67$. Thus, a bidder's costs for the 100% share, Θ, lie within the range [100.00, 140.00] and his costs for the 50% share, $C\Theta$, lie within [52.00, 72.80] and [67.00, 93.80] for $C = 0.52$ and $C = 0.67$, respectively. Although every bidder knows his own costs only and not those of his competitors, common knowledge of the cost parameter distribution and the efficiency parameter is given. We have also conducted a series of experiments with $\Theta \in [100.00, 200.00]$ and efficiency parameter $C = 0.52$ for the non-combinatorial Dutch-FPSB and combinatorial FPSB auction, in order to check robustness of our results. Due to space restrictions, we focus on the results for the range [100.00, 140.00] and only report an outlook on the results for the higher range in section 5.

Upper and lower bounds are implemented in every auction format. In the FPSB auction, each bidder is allowed to submit one bid of up to 150.00 for the 50% share and one bid of up to 300.00 for the 100% share at the start of every period. Both values can be entered in step sizes of 0.50. Moreover, participants cannot submit bids below their respective costs. In the first phase of the non-combinatorial Dutch-FPSB auction and the combinatorial Dutch-FPSB auction, the price for the 50% share starts at 50.00 and is raised by 0.50

Table 1 Treatments

	Auction format	Efficiency parameter
Non-combinatorial	Dutch-FPSB	0.52
		0.67
Combinatorial	FPSB	0.52
		0.67
	Dutch-FPSB	0.52
		0.67

Table 2 Sample Sizes

			Sample size				
			Group 1	Group 2	Group 3	Group 4	Σ
Non-combinatorial	Dutch-FPSB	0.52	12	12	12	12	48
		0.67	12	12	12	12	48
Combinatorial	FPSB	0.52	12	12	12	12	48
		0.67	12	12	12	12	48
	Dutch-FPSB	0.52	12	12	12	12	48
		0.67	12	9	12	12	45
							285

every half second. In the Dutch-FPSB auction, the price for the 50% share is half the price of the 100% share and is increased according to this rule during the auction. Both prices cannot rise higher than 150.00 and 300.00, respectively. In the second phase each bidder submits a bid of up to 150.00, in step size of 0.5, for the remaining 50% share. Participants cannot accept prices and submit bids below their respective costs.

We conducted two sessions for every treatment and each session consisted of two matching groups in each of which 12 subjects participated. In every matching group the 12 subjects were randomly matched to four first-price split-award auctions of three bidders in each of 20 consecutive periods. No interaction between subjects across matching groups occurred. Each subject participated in one session only. For the Dutch-FPSB auction with efficiency parameter $C = 0.67$, one matching group contained only 9 subjects. In total, 285 subjects participated in the experiments. The sample sizes of the different treatments are summarized in Table 2.

In total 320 auctions took place in every treatment except for the Dutch-FPSB auction with efficiency parameter $C = 0.67$ in which only 300 auctions were conducted.[5] The unit of statistical observation is the matching group average which is used to compute average procurement costs and efficiencies of the six different treatments. In each matching group, four first-price split-award auctions were conducted in each of 20 consecutive periods which resulted in 80 auctions per matching group based on which the average values are calculated. For the Dutch-FPSB auction with efficiency parameter $C = 0.67$ in one matching group only three auctions were repeated for 20 periods which added up to a total of 60 auctions in this matching group. We also discuss individual bidding behavior in which case the unit of analysis is the individual decision.

At the beginning of each session the instructions were read aloud to all subjects. The subjects then had time to go through the instructions on their own and to answer the comprehension questions. The interaction in the experiment was computerized and entirely anonymous. Communication or personal interaction between the subjects was prohibited.

The experiments were conducted at the experimenTUM, the laboratory for experimental economic studies of the Technical University of Munich in 2017. Subjects were undergraduate and graduate students from the Technical University of Munich from a wide range of different study programs. Our experiments were computerized using the experimental software z-Tree (Fischbacher 2007). Besides a show-up fee of 6 EUR (7.09 USD), participants could earn experimental currencies (ECU) during the experiment, which were converted to Euros by a given exchange rate.

The exchange rates have been set based on differences in the equilibrium bids and initial tests such that the average earnings did not vary too much between the different treatments. For the cost parameter range of [100.00, 140.00] the equilibrium bidding strategy of the FPSB auction does not depend on the efficiency parameter and the exchange rate was set to 2.5 ECU/EUR. This was also the rate for the Dutch-FPSB format in case of efficiency parameter $C = 0.67$, as the sole-source outcome results in equilibrium. For $C = 0.52$ we used a more conservative exchange rate of 3.5, as we expected some bidders to deviate sequentially and to earn more than the equilibrium payoff in a FPSB auction. As the non-combinatorial Dutch-FPSB auction involves very different sequential equilibria in pure strategies for $C = 0.67$ and in mixed-strategies for $C = 0.52$, we used exchange rates of 14.00 ECU/EUR and 4.00 ECU/EUR, respectively. Finally, for a different range of [100.00, 200.00] of the cost parameter (and $C = 0.52$ fixed) the equilibrium predictions change significantly which results in adapted exchange rates of 7.5 ECU/EUR and 6.5 ECU/EUR for the FPSB auction and the non-combinatorial Dutch-FPSB format, respectively. These exchange rates helped equalize the subjects' revenues across treatments. The sessions lasted on average two hours and subjects earned 20.10 EUR (23.74 USD) on average in the various treatments. The money was paid anonymously to the participants after the experiment.

4.2. Efficiency and Procurement Costs

We will first discuss our aggregate results on efficiency and procurement costs.

4.2.1. Efficiency

RESULT 1. *The FPSB auction is highly efficient independent of the efficiency parameter C, supporting Hypothesis 4. The Dutch-FPSB auction results in significantly less efficient allocations than the FPSB split-award auction for both efficiency settings. The non-combinatorial auction yields lower efficiency compared to the FPSB auction with weak economies of scale.*

We measure efficiency as the share of the auction outcomes in which the bidder with the lowest cost type wins 100% of the business. First the share per matching group is calculated and based on these values the treatment average is determined. The metric is denoted as efficient allocations and depicted in Table 3. Standard deviations (SD) are written in brackets. For $C = 0.52$ the FPSB auction is highly efficient (87.50%), whereas only 71.25% of all non-combinatorial Dutch-FPSB auctions result in an efficient outcome. The Dutch-FPSB auction is characterized by the same share of efficient allocations as its non-combinatorial counterpart with *p*-value of 0.15.[6] The difference in efficiency between the non-combinatorial format and the FPSB auction (*p*-value of 0.00) is in line with our hypotheses, which predict that bidders with low cost types have an incentive to gamble in phase 2 after having won a 50% share in phase 1. In case at least one of the losers has a cost draw close to the one of the winner, there is a chance that the award will be split. This actually happens in 20.94% of all auctions, which is within the predicted boundary of at most 24.70% split awards on average. The significant difference between the two combinatorial auction formats (*p*-value of 0.00) may be explained by the same reasoning as consecutive strategies for the sole source award can be observed for the Dutch-FPSB auction.

With $C = 0.67$, however, bidders should not gamble in the non-combinatorial Dutch-FPSB format and accept the counteroffers for 50% as soon as they can in both phases. Hence, the winner should always be the lowest cost type in both periods and efficiency levels should be similar to the FPSB auction. This can be supported by the empirical data, because the efficiency of the non-combinatorial auction is significantly higher than in the setting with weak economies of scale (*p*-value of 0.01) and does not differ from the FPSB auction (*p*-value of 0.73). The FPSB and Dutch-FPSB format do not differ statistically in efficient allocations compared to the setting with weak economies of scale (*p*-values of 0.44 and 0.65, respectively). For the FPSB auction, this is in line with our hypotheses, as an increase in the efficiency parameter C should not change the share of efficient outcomes.

In a combinatorial market there is no obvious one-fits-all definition of efficiency and therefore, to validate the robustness of our results, we also analyze a measure for mean allocative efficiency in which sets of split award winners and sole source award winners are denoted by N^{σ}_{winner} and N^{s}_{winner}, respectively. $N^{\sigma}_{optimal}$ comprises the bidder with the lowest cost type per auction. Then, we define the allocative efficiency of a split-award auction with SSE based on the definition of Kwasnica et al. (2005) as

$$\text{Allocative Efficiency} = \frac{\Sigma_{i \in N^{\sigma}_{optimal}} \Theta_i}{\Sigma_{i \in N^{\sigma}_{winner}} C\Theta_i + \Sigma_{i \in N^{s}_{winner}} \Theta_i}.$$

This metric is determined for each matching group and the average then corresponds to allocative efficiency per treatment. As shown in Table 3 with standard deviations (SD) in brackets, allocative efficiency does not significantly change for different scale economies within each auction format (*p*-value of 0.44 for the FPSB format, 0.18 for the Dutch-FPSB auction and *p*-value of 0.22 for the non-combinatorial Dutch-FPSB mechanism). For $C = 0.67$ only the two Dutch-FPSB auction formats do not differ significantly (*p*-value of 0.24) but the FPSB format is significantly more efficient than the combinatorial and non-combinatorial Dutch-FPSB auctions with *p*-values of 0.03 and 0.04, respectively. For $C = 0.52$ the FPSB auction is also most efficient compared to the Dutch-FPSB auction with *p*-value of 0.01 and compared to the non-combinatorial Dutch-FPSB auction with *p*-value of 0.00. Nevertheless, the similarly

Table 3 Efficiency

			Number of auctions	Efficient allocations	Allocative efficiency
Non-combinatorial	Dutch-FPSB	0.52	320	71.25% (SD = 3.95%)	98.51% (SD = 0.23%)
		0.67	320	86.88% (SD = 6.25%)	97.70% (SD = 1.04%)
Combinatorial	FPSB	0.52	320	87.50% (SD = 2.70%)	99.62% (SD = 0.15%)
		0.67	312	85.55% (SD = 3.83%)	99.52% (SD = 0.19%)
	Dutch-FPSB	0.52	320	75.94% (SD = 4.13%)	99.00% (SD = 0.28%)
		0.67	300	77.08% (SD = 2.28%)	98.50% (SD = 0.56%)

high measures for allocative efficiency provide evidence that the inefficient allocations mainly appear when the two lowest cost types are close to each other. This might also explain why, contrary to the predictions, we observe a significantly higher value of efficient allocations for the FPSB auction than for the Dutch-FPSB format (p-value of 0.01). We discuss the bidding behavior for the different treatments in more detail below to better explain the differences.

4.2.2. Procurement Costs

RESULT 2. *The FPSB auction results in lower procurement costs than the combinatorial and non-combinatorial Dutch-FPSB auction. Only for* $C = 0.67$ *the average prices in the FPSB auction are not statistically lower than in the Dutch-FPSB auction. For* $C = 0.52$ *the Dutch-FPSB auction does not differ significantly from its non-combinatorial counterpart whereas it leads to significantly lower costs for* $C = 0.67$. *The results are in line with Hypothesis* 5.

The overall costs per treatment are defined as the mean of the average prices the auctioneer has to pay in each matching group. The procurement costs of all treatments are summarized in Table 4 below, with the standard deviations (SD) given in brackets. For $C = 0.67$ the procurement costs in both combinatorial auction formats do not differ significantly (p-value of 0.07). This is in line with the hypotheses which predict the same equilibrium to be played in both combinatorial auction formats. Although the non-combinatorial auction almost always results in the sole-source award for $C = 0.67$, it is much more expensive for the auctioneer than the combinatorial formats ($p = 0.00$ for both tests) with value of 146.25 and 95% confidence interval of [144.99, 147.51] which includes the predicted costs of 147.40.

The procurement costs of the FPSB auction lie significantly below the predicted value of 120.00 (p-values of 0.00 for the two values of C) with 95% confidence intervals of [114.91, 116.32] and [113.34, 114.76] for $C = 0.67$ and $C = 0.52$, respectively.[7] Bidders appear to exclude the split award not only with high bids on 50% but also with bids lower than predicted on 100%. The latter also holds true for the Dutch-FPSB auction with 95% confidence intervals of [116.70, 118.45] and p-value of 0.01 for $C = 0.67$. Here, participants force the direct winning of 100% of the business instead of a consecutive winning. This reasoning also explains why the FPSB auction is less expensive than the non-combinatorial format for $C = 0.52$.

For $C = 0.52$ both Dutch-FPSB auction formats are statistically more expensive than the FPSB auction (p-value of 0.00 each). Moreover, there is no statistical difference for both ascending auction formats for $C = 0.52$ with p-value of 0.37 which suggests that for weak economies of scale the bidding behavior in the combinatorial format resembles that one in the non-combinatorial mechanism.

With an efficiency parameter of $C = 0.52$ the Dutch-FPSB auction format is statistically as expensive as with a higher efficiency parameter (p-value of 0.49), which is also true for the FPSB auction with a p-value of 0.11. The non-combinatorial mechanism, however, is much less expensive (p-value of 0.00) with a value of 118.62 and 95% confidence interval of [117.80, 119.43] for low efficiency parameter compared to its costs at $C = 0.67$. This average price for low efficiency parameter is even slightly below the lower bound of the predicted interval [119.70, 121.50].

In summary, the non-combinatorial mechanism leads to higher procurement costs than both combinatorial auction formats. Within the latter the FPSB auction stands out with slightly lower procurement costs. It thus might be preferably employed by the procurement manager.

4.3. Bidding Behavior

We next discuss bidding behavior in the six treatments to underline our explanations for the differences in efficiency and procurement costs given in section 4.2. Equilibrium predictions for the combinatorial formats in our experimental setting can be found in Appendix A.4. We estimated multivariate fixed-effects regressions for bids and prices in split and sole-source awards of all treatments. In this section the unit of observation is any bidder's action at any one auction in any one period. In addition, univariate regressions in which the cost draw is the

Table 4 Procurement Costs

			Average procurement costs		
			Overall	Split award	Sole-source award
Non-combinatorial	Dutch-FPSB	0.52	118.62 (SD = 1.00)	117.52 (SD = 1.86)	118.85 (SD = 0.97)
		0.67	146.25 (SD = 3.07)	158.12 (SD = 5.46)	145.68 (SD = 3.00)
Combinatorial	FPSB	0.52	114.05 (SD = 0.63)	114.26 (SD = 3.18)	114.06 (SD = 0.64)
		0.67	115.63 (SD = 1.45)	–	115.63 (SD = 1.45)
	Dutch-FPSB	0.52	117.99 (SD = 0.83)	118 (SD = 2.82)	118.03 (SD = 0.87)
		0.67	117.54 (SD = 0.88)	–	117.54 (SD = 0.88)

single independent variable were implemented. These regressions allow us to interpret all plots of bids and prices on cost draws in this section and provide intuitive insights on the subjects' bidding behavior. Moreover, we conducted fixed-effects logistic regressions to analyze the change in allocation over time. We found no adaptation in bidding behavior and no trend with repeated interactions of the bidders.

4.3.1. The FPSB Split-Award Auction

RESULT 3. *In the FPSB auction, the* 50% *and* 100% *share bids closely follow the respective WTA equilibrium bids for both efficiency settings supporting Hypothesis* 1. *Participants include a small mark-up in their bids for the* 100% *share and the cost structure results in sole-source allocations only.*

The above result is illustrated in Figures 2 and 3 that depict the bids on the 50% and 100% share for the efficiency settings with $C = 0.67$ and $C = 0.52$, respectively. Again, straight lines correspond to the univariate regression of bids against costs and the dotted lines show the corresponding costs.

In our fixed-effects regressions the cost parameter significantly determines the bids for 50% and 100% in both efficiency environments. Moreover, for $C = 0.52$ the higher a subject bids for 100% the higher he has to bid on 50% in order to exclude the split-award with high certainty. With $C = 0.67$ scale economies are so strong that the height of the 50% share bid is entirely independent of the bid for 100%. Nevertheless, as predicted in our hypotheses, the efficiency parameter does not have any impact on the bidding behavior for the 100% share and hence the costs are the same in both settings.

4.3.2. The Non-Combinatorial Dutch-FPSB Split-Award Auction.

Because bidding in the Dutch-FPSB auction exhibits phenomena that we see in the FPSB auction and in the non-combinatorial Dutch-FPSB auction, we first discuss the latter.

RESULT 4. *For* $C = 0.52$ *there is no difference in bidding behavior between split and sole-source allocations which supports the predicted randomization strategy in Hypothesis* 3. *In the setting with higher efficiency parameter* $C = 0.67$ *sole-source winners systematically deviate from the equilibrium strategy in phase* 2 *towards a secure bid of* 67.00 *which guarantees winning.*

A payoff-maximizing winner of the first share will not accept the second share at a price below $C\Theta$ in the symmetric equilibrium of the non-combinatorial Dutch-FPSB auction although he has lower marginal costs of $(1 - C)\Theta$. In equilibrium he can be certain to

Figure 2 Bids of Sole-Source Award Winners for $C = 0.67$ [Color figure can be viewed at wileyonlinelibrary.com]

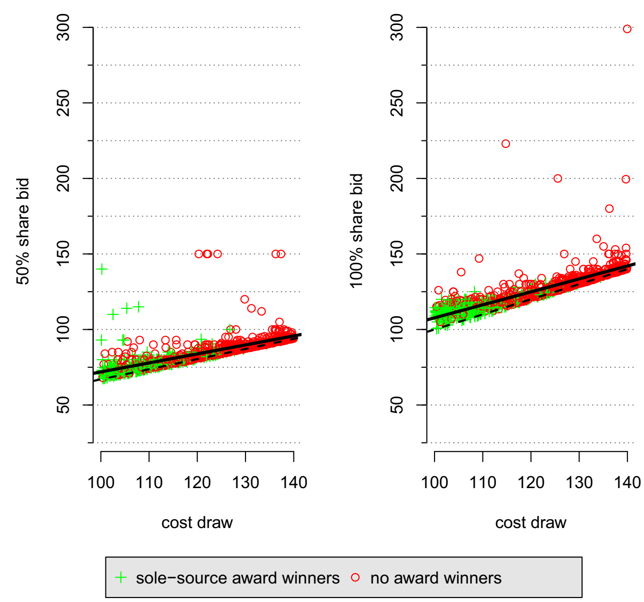
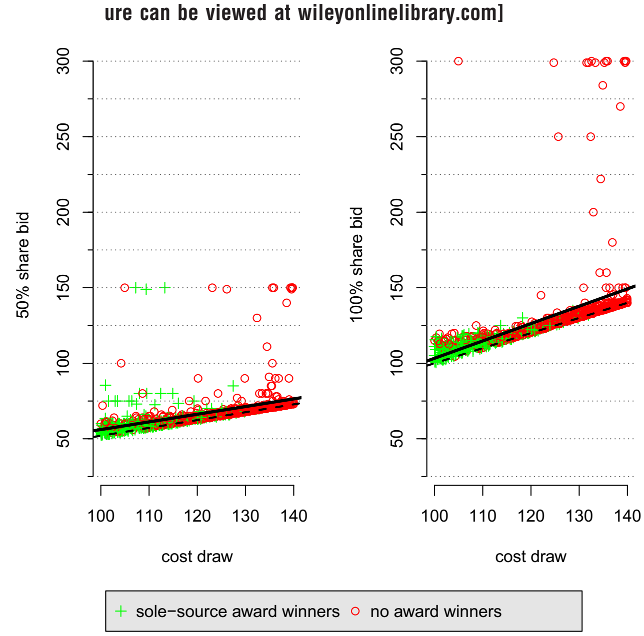


Figure 3 Bids of Sole-Source Award Winners for $C = 0.52$ [Color figure can be viewed at wileyonlinelibrary.com]



have the lowest cost type and his opponents cannot profitably accept the second share at a price below $C\Theta$ as they have higher cost types than Θ. Therefore, the relevant value for the winner of the first share is not his marginal cost for the second share $(1 - C)\Theta$ but his opponents' marginal costs which are greater than $C\Theta$. Moreover, even if the winner of the first share does not infer to have the lowest cost type, no opponent can possibly have 50% share costs below 67.00 and 52.00 in the setting with efficiency parameters of

C = 0.67 and C = 0.52, respectively. Although we predict bidding of costs for the former efficiency setting, the observed average price might reasonably be expected to lie between 67.00 and the cost type of the winner. For the C = 0.52 treatment the equilibrium strategy predicts bids above costs for low types and the lower logical boundary might not serve as a strong reference point anymore.

With large scale economies of C = 0.67, bidders accept the first share at a price close to their costs for one share as predicted in equilibrium. Nevertheless, some bidders gamble and let prices rise slightly above costs in phase 1. This resulted in 18 unpredicted split awards. In the sole-source allocations, the winner of the first share accepts at an average price of 75.70 which exceeds the predicted value of 73.70. The average bid for the second share (69.93) is much lower than predicted and might indicate a tendency towards the logical lower bound of 67.00. Figure 4 shows that, as predicted, prices for the first share follow costs relatively closely. In phase 2, bidding behavior differs amongst subjects. While there are some bidders who follow the equilibrium and submit a price close to $C\Theta$, one can observe some indication of pooling at prices of 67.00 for other bidders. This observation is verified in our fixed-effects regressions in which cost draws do not statistically influence second-share bids of first-share winners in the sole-source awards. This behavior is well illustrated by the separation of the scatter plot for medium to high types in Figure 4.

For C = 0.52 all average split and sole-source bids do not differ statistically. Moreover, average prices for the first and second share in the sole-source allocations are almost identical. This observation might underline the partial randomness in allocation outcomes as predicted by the mixed strategy. Furthermore, as expected, bidders with low cost types tend to gamble in both phases and accept prices, which exceed their costs for the 50% share, while high cost types bid close to their costs. This is nicely depicted in Figure 5. Winners appear to bid closer to their costs of $C\Theta$ for the second share than in the setting with C = 0.67. In particular, the figure shows less indication of pooling for the second share than in the setting with the higher efficiency parameter. Again, this behavior is underlined by our fixed-effects regressions for C = 0.52 in which the second-share bids of first-share-winners vary significantly with cost draws in sole-source allocations. An explanation of this behavior could be that supporting the sole source outcome with a bid close to 52.00 in phase 2 would yield only low or even negative profit in the setting with C = 0.52. This is different to the setting with strong economies of scale and the reason for the higher appearance of split awards with C = 0.52.

Figures 4 and 5 show bids for the first and second share of the sole-source winners for the efficiency settings C = 0.67 and C = 0.52, respectively. The straight lines depict the univariate regression of bids against cost draws, whereas the dotted lines represent costs for the 50% shares.

4.3.3. The Dutch-FPSB Split-Award Auction

RESULT 5. *The bidding behavior of the Dutch-FPSB auction is similar to the FPSB auction with C = 0.67*

Figure 4 Bids of Consecutive Sole-Source-Award Winners for *C* = 0.67 [Color figure can be viewed at wileyonlinelibrary.com]

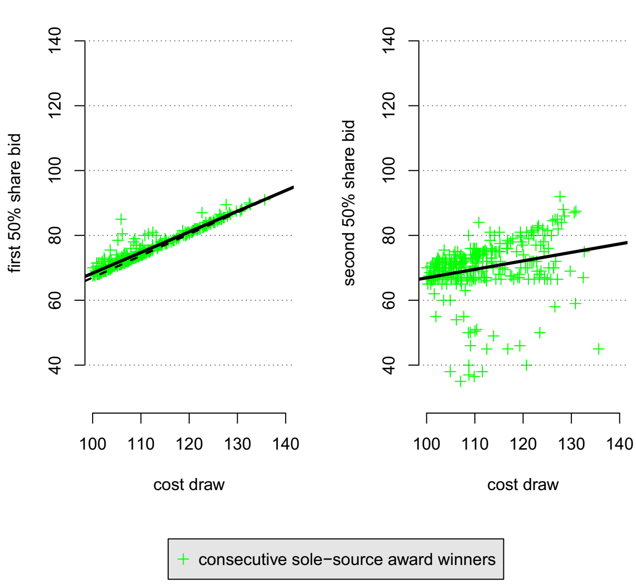
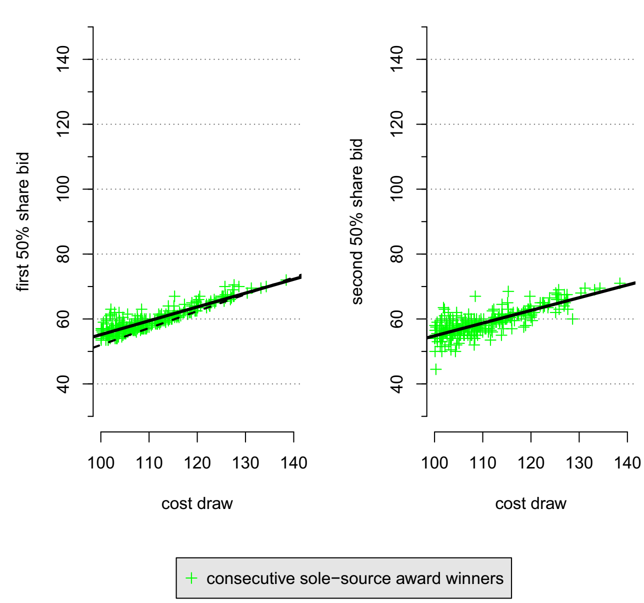


Figure 5 Bids of Consecutive Sole-Source-Award Winners for *C* = 0.52 [Color figure can be viewed at wileyonlinelibrary.com]



supporting Hypothesis 2. *For* $C = 0.52$ *the consecutive sole-source bids follow the pattern of those in the non-combinatorial format.*

We omit the discussion of the ten split awards in the efficiency setting with parameter $C = 0.52$ and treat them as accidental outliers. Moreover, we leave out univariate plots for sole-source allocations of this auction format. For a depiction of the structure of 100% share bids in the direct sole-source awards please consider Figures 2 and 3 of the FPSB auction. Similarly, for an illustration of the form of consecutive sole-source bids we refer to Figure 5 of the non-combinatorial Dutch-FPSB auction for $C = 0.52$.

Note that a possible reason for a bidder to select the consecutive sole-source award instead of the direct alternative is to be able to choose the logical upper bound as explained in section 4.3.2 in the second phase. The winner of the first 50% share might benefit from knowing that he is the lowest cost type as he can then let the price for the second 50% share rise slightly higher than for the first.

With the efficiency parameter $C = 0.52$ the height of average consecutive sole-source bids of the Dutch-FPSB auction is slightly lower than in its non-combinatorial counterpart with a value of 116.97 compared to 118.81. The option of opponents being able to directly select the sole-source award through accepting the 100% share offer poses a credible threat for a bidder aiming at the consecutive sole-source allocation. Therefore, the latter has to accept the first share at a price below half the average 100% share bid in direct sole-source awards. Summarizing, with weak economies of scale bidding behavior in the Dutch-FPSB auction is similar to that of the non-combinatorial Dutch-FPSB and therefore differs from bidding in the FPSB auction. This results in a lower value of efficient allocations and higher procurement costs.

With high enough economies of scale ($C = 0.67$) bidding behavior in the Dutch-FPSB auction corresponds closely to that in the FPSB auction as bidders do not choose the consecutive sole source award. Thus, procurement costs are statistically the same in both auction formats.

4.4. Diseconomies of Scale and Robustness

We focused on procurement markets with economies of scale. One reason for using a combinatorial *ex post* split-award auction rather than a single-lot auction on the entire demand are uncertainties of the buyer about the economies of scale. If there were diseconomies of scale a single-lot auction would lead to high procurement costs, while a combinatorial auction allows for a dual-source solution with lower costs. A complete discussion of procurement markets with diseconomies of scale in theory and in experiments is clearly beyond this study. However, we did run experiments with diseconomies of scale, which indeed always ended up in the efficient solution with three suppliers and no single-source solution (see Appendix A.5). We leave a complete analysis of markets with diseconomies of scale for future research.

We also conducted another series of experiments for the FPSB and the non-combinatorial Dutch-FPSB auction with weak economies of scale and a larger range of cost types ($\Theta \in [100, 200]$) in order to check the robustness of our results with economies of scale. Again, the procurement costs of the FPSB format proved to be lower than those of the non-combinatorial Dutch-FPSB auction with values of 136.22 and 142.04, respectively. The efficiency of the FPSB auction remained on an equally high level as in the experiments with the smaller cost range (85.63%). The efficiency of the non-combinatorial format improved (82.50%) but remained significantly below that of the FPSB auction format.

5. Summary and Managerial Insights

There is a wide variety of multi-object first-price auctions available to buyers and making a choice is difficult. Buying the entire demand as a single-lot auction will lead to low efficiency and high costs, if there are diseconomies of scale. In contrast, the wide-spread sequential auctions lead to inefficiency in case of economies of scale as we show. If an auctioneer was sure that there are economies of scale, he would use a single lot auction. Procurement managers often lack this information. Our study shows that a combinatorial FPSB split-award auction still yields highly efficient and low-cost outcomes if the suppliers indeed have economies of scale. We also conducted experiments with diseconomies of scale which all led to efficient and low-cost outcomes.

Table 5 provides a summary of our results. The combinatorial FPSB auction achieves high efficiency and low costs with weak and strong economies of

Table 5 Efficiency of Equilibrium Outcomes and Laboratory Experiments

	Equilibrium predictions		Experiment	
Auction format	Weak ES	Strong ES	Weak ES	Strong ES
Non-combinatorial Dutch-FPSB	ME	FE	ME	HE
Combinatorial FPSB	FE	FE	HE	HE
Combinatorial Dutch-FPSB	NP	FE	ME	ME

Note: ES (economies of scale), FE (fully efficient, = 100%), HE (highly efficient, ≥ 85%), ME (medium-efficient, ≥ 70%), NP (no prediction) scale. For the combinatorial Dutch-FPSB auction we get significantly lower efficiency in both experimental settings, although the equilibrium predictions suggest full efficiency for the setting with strong economies of scale. Non-combinatorial Dutch-FPSB auctions are efficient only with strong economies of scale, but lead to higher procurement costs compared to the combinatorial formats with weak and strong economies of scale. With weak economies of scale, the non-combinatorial mechanism leads to significant inefficiencies.

Overall, our results provide evidence that combinatorial first-price sealed-bid auctions are robust mechanisms that achieve high efficiency and low costs in a wide variety of procurement markets.

Acknowledgments

The financial support from the Deutsche Forschungsgemeinschaft (DFG) (BI 1057/1-8) is gratefully acknowledged. We thank the participants of the 2017 ZEW Workshop on Market Design, especially Joszef Sakovics, Nicolas Fugger, and Vitali Gretschko, for helpful comments.

Appendix A. Proofs

A.1. Proof of Proposition 1

Consider the SSE split-award auction model including $n > 2$ *ex ante* symmetric bidders with cost types Θ_i. In the FPSB split-award auction, a WTA equilibrium S_e^{BNE} is given by

$$p_e^s(\Theta_i) = \Theta_i + \frac{\int_{\Theta_i}^{\overline{\Theta}} (1 - F(t))^{n-1} dt}{(1 - F(\Theta_i))^{n-1}}$$
$$p_e^\sigma(\Theta_i) = p_e^s(\Theta_i).$$

Proof. In equilibrium, there is no bidder, who benefits from a deviation. In the FPSB auction one distinguishes sole source, split and hybrid deviations. Deviations for the sole source award are excluded by the equilibrium strategy of $p_e^s(\Theta_i)$ which maximizes the expected payoff to win 100% of the business. The high bid for the split award, $p_e^\sigma(\Theta_i)$, assures that the probability to win the split award with a deviating price $\hat{p}^\sigma(\hat{\Theta}) > \hat{\Theta}C$ is zero. Hence, any split or hybrid deviation can be excluded.

A.2. Proof of Proposition 3

Consider the SSE split-award auction model including $n > 2$ bidders with cost types Θ_i. In the Dutch-FPSB split-award auction, there is a WTA equilibrium given by

$$p_e^{s1}(\Theta_i, h^0) = \Theta_i + \frac{\int_{\Theta_i}^{\overline{\Theta}} (1 - F(t))^{n-1} dt}{(1 - F(\Theta_i))^{n-1}}$$
$$p_e^{\sigma 2l}(\Theta_i, h^1) = C\Theta_i$$

and

$$\mu_{-i}^1(\Theta_i | h^0) = F(\Theta)$$
$$\mu_l^2(\Theta_w | h^1) = F(\Theta)$$
$$\mu_w^2(\Theta_l | h^1) = \begin{cases} 0 & \text{if } \Theta < \Theta_w \\ \frac{F(\Theta) - F(\Theta_w)}{(1 - F(\Theta_w))} & \text{if } \Theta \geq \Theta_w \end{cases}$$

if either

$$C \geq \frac{\overline{\Theta}}{2\underline{\Theta}} \tag{A1}$$

or

$$\int_{\Theta_i}^{\overline{\Theta}} (1 - F(t))^{n-1} dt - E[\hat{\Pi}^s(x^*)] > 0 \tag{A2}$$

applies to all possible types $\Theta_i < \frac{\overline{\Theta}}{2C}$ with $E[\hat{\Pi}^s(x^*)]$as defined below.

Proof. We distinguish between split and sole source deviations.

Split deviations. A split deviation $\hat{p}^{\sigma 1}(\hat{\Theta}, h^0)$ is only possible in phase 1, as there is no second phase in equilibrium. The price, for which the split deviation is accepted, must yield a higher payoff than accepting the sole source award, that is,

$$2\hat{p}^{\sigma 1}(\hat{\Theta}, h^0) - \hat{\Theta} < \hat{p}^{\sigma 1}(\hat{\Theta}, h^0) - \hat{\Theta}C$$
$$\hat{p}^{\sigma 1}(\hat{\Theta}, h^0) < \hat{\Theta}(1 - C)$$

However, a bidder would make a loss by accepting such prices, as $C > 0.5$ applies. Hence, no split deviations are possible.

Sole source deviations. The structure of the WTA equilibrium assures that there is no profitable sole source deviation for 100% of the business in phase 1. However, deviations which try to win 100% sequentially by accepting the price for 50% in phase 1 at $\hat{p}^{\sigma 1}(\hat{\Theta}, h^0)$ as well as in phase 2 at $\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1)$, must also be excluded.

Case 1: $C > \frac{\overline{\Theta}}{2\underline{\Theta}}$

When the efficiency parameter C is high enough, it is not possible for individual rational bidders to follow a sequential deviation, as even the lowest cost type can first accept 50% of the business at a price of $\hat{p}^{\sigma 1}(\underline{\Theta}, h^0) = \underline{\Theta}C$. However, when all other bidders follow the equilibrium strategy, the auction terminates at a price lower than this price level. Even the highest cost type accepts the 100 % share for a price of $\overline{\Theta}$, which is higher than $2\hat{p}^{\sigma 1}(\underline{\Theta}, h^0)$.

Case 2: $C \leq \frac{\underline{\Theta}}{2\overline{\Theta}}$

First, note that only cost types $\hat{\Theta} < \frac{\overline{\Theta}}{2C}$ are possible candidates for a sequential deviation as we assume that bidders cannot accept offers which are lower than their costs for that share. We start by determining a potential deviating strategy in the second phase. The $n-1$ losers know that one bidder did not stick to the equilibrium strategy. Otherwise, phase 2 does not appear in this WTA equilibrium.

Furthermore, they do not know the cost type of the deviating bidder, but the deviating bidder knows that all other bidders have a cost type higher than $\Theta_d(x)$ with

$$\hat{p}^{\sigma 1}(x, h^0) = 0.5 p_e^{s1}(\Theta_d(x), h^0)$$
$$\Theta_d(x) = (p_e^{s1})^{-1}(\hat{p}^{\sigma 1}(x, h^0)).$$

We assume that the losing bidders may credibly threat to accept the second share for a price which equals their costs for 50%, that is, $C\Theta_l$. Threats below are not credible, as bidders are individual rational.

There are two different types of strategies in phase 2: A bidder can either play a strategy $\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1) = \Theta_d(x)C$, which is winning with probability 1, or a strategy $\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1) > \Theta_d(x)C$. However, note that the winner of phase 1 has the additional information that the other bidders have a cost type which is not only higher than his cost type but also higher than $\Theta_d(x)$. Hence, he has to maximize the expected payoff of

$$\begin{aligned} E[\hat{\Pi}_e^{\sigma 2}(\hat{\Theta}, h^1)] &= (\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1) - (1-C)\hat{\Theta}) P(\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1) \\ &\quad < \Theta_{1:n-1} C | \Theta_{1:n-1} > \Theta_d(x)) \\ &= (\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1) - (1-C)\hat{\Theta}) P\Big(\frac{\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1)}{C} \\ &\quad < \Theta_{1:n-1} | \Theta_{1:n-1} > \Theta_d(x)\Big) \\ &= (\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1) - (1-C)\hat{\Theta}) \\ &\quad \frac{P(\Theta_{1:n-1} > \frac{\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1)}{C}, \Theta_{1:n-1} > \Theta_d(x))}{P(\Theta_{1:n-1} > \Theta_d(x))} \\ &= (\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1) - (1-C)\hat{\Theta}) \\ &\quad \frac{(1 - F(\frac{\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1)}{C}))^{n-1}}{(1 - F(\Theta_d(x)))^{n-1}}, \end{aligned}$$

because $\frac{\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1)}{C} \in [\Theta_d(x), \overline{\Theta}]$. Assume $\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1)$ maximizes the expected payoff for cost types $\hat{\Theta} < \Theta_{\kappa 1}$ and bidders with $\hat{\Theta} \in [\Theta_{\kappa 1}, \frac{\overline{\Theta}}{2C}]$ play a safe strategy $\Theta_d(x)C$ in phase 2.

The deviating strategy in phase 1 has to consider the different strategies in phase 2. For high cost types $\hat{\Theta} > \Theta_{\kappa 2}$, a deviation with $\hat{p}^{\sigma 1}(\hat{\Theta}, h^0) \geq \hat{\Theta}C$ could be optimal. Hence, these bidders maximize

$$E[\hat{\Pi}_1^s(x)] = (\hat{p}^{\sigma 1}(x, h^0) + \Theta_d(x)C - \hat{\Theta})(1 - F(\Theta_d(x)))^{n-1}.$$

In contrast, deviating bidders with cost types $\Theta_{\kappa 1} \leq \hat{\Theta} < \Theta_{\kappa 2}$ play $\hat{p}^{\sigma 1}(x, h^0) > xC$ such that

$$E[\hat{\Pi}_2^s(x)] = (\hat{p}^{\sigma 1}(x, h^0) + \Theta_d(x)C - \hat{\Theta})(1 - F(\Theta_d(x)))^{n-1}$$

is optimal and bidders with low cost types $\hat{\Theta} < \Theta_{\kappa 1}$ optimize

$$\begin{aligned} E[\hat{\Pi}_3^s(x)] =& (\hat{p}^{\sigma 1}(x, h^0) - \hat{\Theta}C + \\ &+ (\hat{p}^{\sigma 2w}(\hat{\Theta}, h^1) - \hat{\Theta}(1-C)) \\ &\frac{(1 - F(\frac{\hat{p}^{\sigma 2w}(x, h^1)}{C}))^{n-1}}{(1 - F(\Theta_d(x)))^{n-1}})(1 - F(\Theta_d(x)))^{n-1}. \end{aligned}$$

Define by $x_1^*(\hat{\Theta})$, $x_2^*(\hat{\Theta})$ and $x_3^*(\hat{\Theta})$ the optimal sequential deviations for the different classes of deviating bidders, that is, the values maximizing $E[\hat{\Pi}_1^s(x)]$, $[\hat{\Pi}_2^s(x)]$ and $E[\hat{\Pi}_3^s(x)]$, respectively. Additionally, let

$$E[\hat{\Pi}^s(x^*)] = \begin{cases} E[\hat{\Pi}_1^s(x_1^*)] & \text{if } \hat{\Theta} < \Theta_{\kappa 1} \\ E[\hat{\Pi}_2^s(x_2^*)] & \text{if } \Theta_{\kappa 1} \leq \hat{\Theta} < \Theta_{\kappa 2} \\ E[\hat{\Pi}_3^s(x_3^*)] & \text{if } \Theta_{\kappa 2} \leq \hat{\Theta} < \frac{\overline{\Theta}}{2C} \end{cases}$$

be the expected payoff of these optimal deviations. In equilibrium

$$\int_{\hat{\Theta}}^{\overline{\Theta}} (1 - F(t))^{n-1} dt - E[\hat{\Pi}^s(x^*)] > 0$$

must apply for all possible types $\hat{\Theta} < \frac{\overline{\Theta}}{2C}$, which is considered by condition (A2). This is for example fulfilled for a setting with $n = 3$, $C = \frac{2}{3}$, $\Theta \sim [100, 140]$.

A.3. Procurement Costs

The procurement costs of the non-combinatorial auction formats are stated in the following corollary, which follows from our equilibrium predictions.

Corollary 1. *Consider the SSE split-award auction model including $n > 2$ bidders with cost types Θ_i. In the non-combinatorial Dutch-FPSB split-award auction, the expected procurement costs for the buyer are*

$$\begin{aligned} E[p_b^{hybrid}] = \int_{\underline{\Theta}}^{\Theta_{\kappa 1}} &\{p_{e1}^{\sigma 1}(x, h^0) + + \int_x^{(\bar{b})^{-1}(x)} [p_e^{\sigma 2l}(y, p_e^{\sigma 1}(x, h^0)) \\ &(1 - H(p_e^{\sigma 2l}(y, p_e^{\sigma 1}(x, h^0)))) + H(p_e^{\sigma 2l}(y, p_e^{\sigma 1}(x, h^0))) \\ &+ \int_{\underline{b}(x)}^{p_e^{\sigma 2l}(y, p_e^{\sigma 1}(x, h^0))} z\, h(z) dz] f_2^n(y | \Theta_{1:n} = x) dy \\ &+ \int_{\bar{b}^{-1}(x)}^{\overline{\Theta}} \int_{\underline{b}(x)}^{\bar{b}(x)} z\, h(z) dz\, f_2^n(y | \Theta_{1:n} = x) dy\} f_1^n(x) dx + \\ &+ \int_{\Theta_{\kappa 1}}^{\Theta_{\kappa 2}} (p_{e2}^{\sigma 1}(x, h^0) + xc) f_1^n(x) dx + \int_{\Theta_{\kappa 2}}^{\overline{\Theta}} 2Cx f_1^n(x) dx \end{aligned}$$

with

$$f_k^n(y) = \frac{n!}{(k-1)!(n-k)!}[F(y)]^{k-1}[1-F(y)]^{n-k}f(y)$$

$$f_2^n(y|\Theta_{1:n-1}=x) = \frac{(n-1)(1-F(y))^{n-2}f(y)}{(1-F(x))^{n-1}}$$

A.4. Predictions for the Experiments

In this section, we discuss the equilibrium predictions for the parameters in our experiments.

A.4.1. The FPSB Split-Award Auction. Bidders in the FPSB split-award auction with cost type Θ_i are expected to submit prices $p_e^s(\Theta_i) = \frac{2}{3}(\Theta_i + 70.00)$ for 100% of the business. The prices for 50% should be high enough to exclude the split award independent of the quote of the other bidders, that is, $p_e^\sigma(\Theta_i) > p_e^s(\Theta_i) - C\underline{\Theta}$. As the efficiency parameter does not influence the equilibrium bids for the sole source award, we do not expect any difference in bidding behavior for the settings with $C = 0.52$ and $C = 0.67$. The auction should always result in the efficient sole source allocation for the lowest cost type and the expected procurement costs are $E[p_b^{WTA}(\cdot,\cdot)] = E[\Theta_{2:n}] = 120.00$.

A.4.2. The Dutch-FPSB Split-Award Auction. The experimental setting with $C = 0.67$ was chosen such that condition 2 is true. Hence, we predict that a bidder with costs Θ_i accepts the counteroffer for 100% of the business at a price of $p_e^{s1}(\Theta_i, h^0) = \frac{2.00}{3.00}(\Theta + 70.00)$. The prediction for the efficiency and expected procurement costs are the same as for the FPSB split-award auction. We do not have predictions for the Dutch-FPSB format in the setting with $C = 0.52$.

A.4.3. The Non-Combinatorial Dutch-FPSB Split-Award Auction. The experimental setting with an efficiency parameter $C = 0.67$ was chosen such that bidders are expected to play an equilibrium strategy of $p_{e3}^{\sigma1}(\Theta_i, h^0) = C\Theta_i$, $p_e^{\sigma2l}(\Theta_l, h^1) = C\Theta_l$ and $p_e^{\sigma2w}(\Theta_w, h^1) = C\Theta_w$. This means that they should accept the first share as soon as they get offered a price equal to their costs for 50% and follow the same logic in phase 2. As the lowest cost type wins both shares sequentially with probability 1, we expect full efficiency and procurement costs of $E[p_b^{WTA}(\cdot,\cdot)] = 2CE[\Theta_{1:n}] = 147.40$.

In the setting with efficiency parameter $C = 0.52$, there is an equilibrium in mixed strategies, for which we differentiate between three ranges of cost types. High cost types $\Theta_i \in [130.00, 140.00]$ play a safe strategy $p_{e3}^{\sigma1}(\Theta_i, h^0) = C\Theta_i$, $p_e^{\sigma2l}(\Theta_l, h^1) = C\Theta_l$ and $p_e^{\sigma2w}(\Theta_w, h^1) = C\Theta_w$ similar to the setting with $C = 0.67$, whereas medium-size cost types $\Theta_i \in [121.33, 130.00)$ gamble in phase 1 and submit a safe bid in phase 2, that is $p_{e2}^{\sigma1}(\Theta_i, h^1) > C\Theta_i$, $p_e^{\sigma2l}(\Theta_l, h^1) = C\Theta_l$ and $p_e^{\sigma2w}(\Theta_w, h^1) = C\Theta_w$. Low cost types with $\Theta_i \in [100, 121.33)$ are expected to gamble in phase 1 as well as in phase 2 (in case of winning), that is, $p_{e1}^{\sigma1}(\Theta_i, h^0) > C\Theta_i$, $p_e^{\sigma2l}(\Theta_l, h^1) = C\Theta_l$ and $p_e^{\sigma2w}(\Theta_w, h^1) \geq \underline{b}(\Theta_w) > C\Theta_w$. In phase 2, the winner of phase 1 is expected to randomize his bid according to a distribution H over the interval $[\underline{b}(\Theta_w), \overline{b}(\Theta_w)]$ with

$$\overline{b}(\Theta_w) = \frac{(75.83 + \Theta_w)}{3.13}$$

$$\underline{b}(\Theta_w) = \frac{(211390.00 + \Theta_w(5226.67 + (-106.83 + 0.42\Theta_w)\Theta_w))}{(-140.00 + \Theta_w)^2}.$$

As the full analytical solution considering the distribution function H is hard to compute, we only provide boundaries for the predicted efficiency and procurement costs. An inefficient split award only occurs in the event that the winner of phase 1 loses the remaining share against one of the opponents. As this is never the case, when $p_e^{\sigma2w}(\Theta_w, h^1) = \underline{b}(\Theta_w)$, the upper bound for the expected efficiency in our setting is 1. The lower bound of the efficiency can be determined by calculating the probability that the second highest order statistic is lower than $\frac{\overline{b}(\Theta_w)}{C}$, that is, that the most competitive loser of phase 1 can underbid the highest possible bid of the winner of phase 1:

$$E[efficiency] = 1 - \int_{100.00}^{121.33} \int_{x}^{\frac{(75.83+x)}{3.13\cdot 0.52}} f_{1,2}^n(x,y)\,dy\,dx = 75.30\%$$

Hence, the non-combinatorial Dutch-FPSB auction should result at most in about 1 out of 4 times in the inefficient split award.

We use a similar approach in order to predict the procurement costs. When the lowest order statistic has a cost type higher than 121.33 the expected costs can be calculated by solving the respective integrals in Corollary 1. When the most competitive supplier has a cost type lower than 121.33 and accepts the counteroffer for $p_{e1}^{\sigma1}(\Theta_i, h^0)$ in phase 1, the highest possible costs in phase 2 for the buyer would be $\overline{b}(\Theta_w)$. Hence, we determine the upper boundary of the expected procurement costs by replacing the expected costs in phase 2 by $\overline{b}(\Theta_w)$:

$$E[p_b^{hybrid}(\cdot,\cdot)] = \int_{100.00}^{121.33} (p_{e1}^{\sigma1}(x,h^0) + \overline{b}(x))f_1^n(x)dx + \int_{121.33}^{130.00} (p_{e2}^{\sigma1}(x,h^0) + xC)f_1^n(x)dx + \int_{130.00}^{140.00} 2Cxf_1^n(x)dx = 121.52$$

Table A1 Efficiency

	Efficiency				
	Total auctions	Omitted auctions	Split awards	Efficient allocations	Allocative efficiency
FPSB	180	10	100%	71.82% (SD = 0.04)	98.80% (SD = 0.01)

Similarly, the lower boundary for the procurement costs can be determined by substituting the expected price in phase 2 by $\underline{b}(\Theta_w)$:

$$E[p_b^{hybrid}(\cdot,\cdot)] = \int_{100.00}^{121.33} (p_{e1}^{\sigma 1}(x,h^0)) + \underline{b}(x))f_1^n(x)dx + \int_{121.33}^{130.00} (p_{e2}^{\sigma 1}(x,h^0) + xC)f_1^n(x)dx + + \int_{130.00}^{140.00} 2.00Cxf_1^n(x)dx = 119.72$$

Hence, we predict that the procurement costs lie in the interval [119.72, 121.52].

A.5. Experiments with Diseconomies of Scale

We conducted additional experiments with three bidders and diseconomies of scale. To exemplify our analysis, we first summarize the experimental design and then the results for the combinatorial FPSB auction. Every period in all treatments starts with an information stage for the bidders, in which they are informed about their own costs for supplying 50% or 100% share of a fictitious order. The information about the cost draws is private and the participants do not know about the costs of their opponents. However, it is common knowledge that the cost parameter Θ is uniformly and independently distributed on [100.00, 140.00] and that the efficiency parameter remains constant at $C = 0.3$ in every period. Hence, the costs of a bidder for the 100% share, Θ, range from 100.00 to 140.00 and his costs for the 50% share, $C\Theta$, from 30.00 to 42.00. We followed the exact procedures of our main experiments on economies of scale with the only exception that only 15 consecutive auctions were conducted in each matching group. At the end of the session, each subject was anonymously paid his cumulative earnings from all periods including a show-up fee of 6 EUR (6.56 USD). On average subjects earned 20.85 EUR (22.78 USD) and participated between one and a half to two hours in the experiments.

The results can be found in Table A1 showing that all auctions ended up in a split-award.

In summary the combinatorial FPSB auction is as efficient (efficient allocations) and leads to the same procurement costs than the combinatorial Dutch-FPSB format in a setting with diseconomies of scale with p-values of 0.73 and 0.20, respectively. As the former auction is cheaper and also more efficient than the latter for economies of scale, the combinatorial FPSB auction appears to have advantages over other multi-unit auction formats independent of the efficiency environment as long as at least three suppliers participate in the procurement auction.

Notes

[1]This is different to unit-demans (Krishna 2010) or *ex ante* split-award auctions, where each bidder is allowed to win at most one lot (Bichler et al. 2015).

[2]We apply notation in line with Anton and Yao (1992). The terms auctioneer and buyer as well as bidder and supplier are used interchangeably.

[3]Sales managers are typically not allowed to make a loss (go below cost). A reason can be found in principal-agent problems within the firm. It is the sales representative, not the owner of the firm, who bids. Often sales representatives are incentivized by the volume of sales, which can lead to aggressive bidding of sales representatives. A lower bound on the bids that a sales representative must not underbid shall avoid losses for the firm that can accumulate in the large number of auctions.

[4]For the sake of simplicity, the addition combinatorial is sometimes dropped when we talk about the combinatorial auction formats. Non-combinatorial auctions are written out fully in order to be distinguishable.

[5]One session of the FPSB auction with efficiency parameter $C = 0.67$ was conducted in trial experiments in which we did not prohibit the submission of bids below costs. In eight auctions losses were made by at least one bidder and for comparability we deleted these observations. Thus, although 320 auctions were carried out in total we can only include 312 in our analysis.

[6]A Welch t-test is used for all significance tests between two samples in this section. The Welch *t*-test is based on the matching group average values.

[7]Student *t*-tests are used for all single sample significance tests in this section.

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Supporting Information

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Appendix S1: Proofs.