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Supersymmetric SO(10) unification and flavor-changing weak decays

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Abstract

We study flavor-changing decays of hadrons and leptons in supersymmetric grand unified theories with universal soft-breaking terms at the Planck scale. Specifically, we study an $SO(10)$ model with a flavor structure motivated by the observed large atmospheric mixing angle.

In such models, the large top Yukawa coupling leads to a predictive pattern of flavor violation among the sfermion mass matrices both in the slepton and squark sectors. The steps taken are the following.

- We perform the first study of this model utilizing a full renormalization-group analysis to relate Planck-scale parameters to weak-scale parameters. This allows us to impose constraints from direct searches and vacuum stability. We provide a prescription to compute the relevant weak-scale parameters from a few weak-scale inputs.
- We compute and discuss the effective Lagrangians for $B - \bar{B}$ mixing, for $B_d \to \phi K_S$ for $\tau \to \mu\gamma$ decay.
- A detailed numerical study of $B_s - \bar{B}_s$-mixing and $BR(\tau \to \mu\gamma)$ allows us to assess the best way to search for signals of, and possibly falsify, this model.
- As a by-product of our computation, we compute the full one-loop renormalization-group equations in matrix form of the most general renormalizable SUSY-$SO(10)$ couplings of the matter fields.

We find that in $B_s - \bar{B}_s$ mixing, a significant (about a factor of four), but not an order-of-magnitude enhancement of the standard model prediction is possible. For $\tau \to \mu\gamma$ on the other hand, we find large signals close or even above the experimental upper bound, so that in fact non-observation of this decay already exludes part of the otherwise allowed parameter space.

Contents

Chapter 1

Introduction

The standard model of strong and electroweak interactions is one of the great success stories of physics. Starting from a situation in the 1960s when all one had were phenomenological theories of limited predictivity describing the hundreds of baryons and mesons, typically nonrenormalizable and satisfying various approximate symmetries and conservation laws, a number of groundbreaking developments culminated in a simple and predictive description of essentially all known phenomena of particle physics.

The first major development was the description of electromagnetic and weak interactions by an $SU(2) \times U(1)$ gauge theory that is spontaneously broken to electromagnetism by the Higgs mechanism [1, 2, 3], and which was soon shown to be renormalizable [4]. Later it was realized that strong interactions could also be described by a gauge theory [5, 6, 7, 8], unbroken, asymptotically free and based on the group $SU(3)$. At the end there stood a fully consistent theory of quarks and leptons, with exact gauge symmetry (albeit partly hidden) and renormalizable, and with a relatively small number of parameters: after the inclusion of the charm, bottom, and top quarks, but before the inclusion of neutrino masses, there are just nineteen parameters, which already includes the masses of all fundamental particles.

Soon after it was speculated from the fermion content of the standard model that the nonsimple gauge group of the standard model may be the result of another instance of spontaneous symmetry breaking, and that there may be an underlying simple gauge group, first taken to be $SU(5)$, which is the smallest possible choice [9, 10]. This involved a unification of different standard-model multiplets into irreducible representations of $SU(5)$.

Furthermore, within the standard model the extrapolated running gauge couplings approximately meet at a very high grand-unification scale of about 10^{15} GeV, which was seen as an additional hint at unification [10]. An even higher degree of unification was possible with the gauge group $SO(10)$ [11, 12], where all fermions of a standard-model generation are unified within one irreducible representation, at the cost of having to introduce a right-handed neutrino. Being a gauge singlet below the GUT scale, nothing keeps it from acquiring a large Majorana mass. In interplay with Yukawa couplings to the lepton and Higgs doublets, this could give rise to the seesaw mechanism [13, 14, 15], predicting tiny Majorana masses for the left-handed neutrinos. Together with proton decay, this is one of the generic predictions of $SO(10)$ grand unified theories.

Nevertheless, the standard model, even when embedded within a grand unified theory, is unsatisfying on theoretical grounds in a number of regards, the most important of them being the hierarchy problem: the smallness of the electroweak scale—setting the mass scale for the W and Z bosons—with respect to a fundamental (Planck or grand unification) scale. This hierarchy is unstable under radiative corrections, which are driving the electroweak scale toward the highest mass scale in the theory.

An attractive solution, and indeed, within weakly coupled four-dimensional quantum field theory, the unique one, was found in (softly broken) supersymmetry. Making the theory supersymmetric, which involves introducing a bosonic (fermionic) partner for each standard-model fermion (boson), can protect the electroweak scale, or equivalently the Higgs mass, from quadratically divergent corrections. Supersymmetry became even more attractive when more precise determinations of the weak mixing angle from LEP data showed that unification does not take place in the standard model without the introduction of new degrees of freedom [16]. On the other hand, the situation is much more promising in the MSSM [17, 18], which is consistent with unification and at the same time raises the grand unification scale to about

$$M_{\mathrm{GUT}} \approx 10^{16}\mathrm{GeV},$$

which might help with explaining the stability of the proton.

These nice theoretical arguments have, of course, to be confronted with the experimental fact that not a single supersymmetric particle has been observed, and that at this time, there is no conclusive indirect evidence pointing towards supersymmetry (or at least against an SM valid up to the GUT scale). Both directions are continually pursued, with the current potential for direct searches given by the energy reach and performance of the Fermilab Tevatron, while a rich potential source of indirect evidence comes, among other experiments, from the B-factories Babar and Belle. This includes information on $B-\bar{B}$ mixings and decays as well as on lepton decays such as those of the τ.

A particularly interesting and important point in this respect is that supersymmetric theories have, in their soft breaking terms, additional sources of flavor violation beyond the CKM matrix of the standard model and that in grand-unified schemes where the SUSY breaking is transmitted close to the Planck scale in a flavor-universal form, the pattern as well as the size of flavor violation can be related to the Yukawa couplings and mixings (such as the CKM matrix) present in the low-energy theory. The nonuniversality arises from the GUT renormalization of the soft-breaking terms. The possibility of this effect was discovered long ago [19], but it was found to be small. Later, when the top quark was found to be heavy, the authors of [20, 21, 22] reconsidered the effect and found it to be potentially large in decay modes such as $\mu \to e\gamma$ and $\tau \to \mu\gamma$, with weaker signals in hadronic observables such as B_d-mixing, ϵ_K, and ϵ'. However, they assumed the CKM matrix to be the only source of flavor (and CP) violation and consequently found some processes, such as the flavor-changing τ decay, to occur at a rather small rate.

The discovery of atmospheric neutrino oscillations [23] and a corresponding large mixing angle has sparked renewed interest in supersymmetric seesaw models with and without $SO(10)$ unification. Reference [24] and a number of other authors have since then investigated the situation quantitatively within supersymmetric $SU(5)$ with right-handed neutrinos. Notably, there have been no such studies within a true $SO(10)$ model. In 2002, a simple $SO(10)$ model incorporating the seesaw mechanism and the near-degenerate observed neutrino mass spectrum was proposed by Chang, Masiero, and Murayama [25]. The alignment of the right-handed (and consequently left-handed) neutrino Majorana mass matrix with the up-type Yukawa couplings in that model automatically implies that the large atmospheric mixing angle manifests itself in a large mixing not only among sleptons, but also among down-type squarks.

In this dissertation, we will study the pattern of flavor violation of the CMM model quantitatively, doing a full renormalization-group analysis and computing several flavor-changing weak decays. In a numerical study, we use these results to study and compare the mass difference ΔM_{B_s} and the branching ratio of $\tau \to \mu\gamma$. Unlike the work of Barbieri et al. in the context of $SO(10)$ with the CKM matrix as only source of flavor violation, we find potentially large rates for the latter decay.

The remainder of this thesis is organized as follows. In chapter 2, we review the standard model and its minimal supersymmetric version and define our notation. We study in detail the new sources of flavor violation and two often considered patterns of the sfermion mass matrices. Chapter 3 is devoted to supersymmetric grand unification. After a brief review of the field, we study the modifications that arise in the scalar mass matrices at the qualitative level. In chapter 4, we introduce the CMM model in detail and derive the Feynman rules needed to obtain the effective Lagrangian for weak decays. Subsequently, in chapter 5, we collect the renormalization group equations and their solutions for the relevant parameters of the model. In the course of this, we derive renormalization-group equations for the most general superpotential and soft-breaking terms involving the $SO(10)$ matter fields. We also discuss some issues related to the IR fixed-point structure of $SO(10)$. We conclude the chapter with a compact recipe giving the pattern of the breaking terms at the weak scale in terms of a few inputs, also defined at the weak scale. The relevant effective Lagrangians for $B_s - \bar{B}_s$ mixing, $B_d \to \phi K_S$ decay, and $\tau \to \mu\gamma$ are computed in chapter 6 and compared to existing results in the literature. It is then argued qualitatively that effects in

$\Delta F = 1$ B_d decay are small in the CMM model and that a quantitative prediction, particularly of the CP asymmetry, is beset by large hadronic uncertainties. Chapter 7 is concerned with a detailed numerical study of ΔM_s and $\tau \to \mu\gamma$. We find a possible enhancement of the former of up to 300 % with respect to the standard model expectation. For the latter, the predicted rate either comes close to or even exceeds the experimental upper bound in much of the otherwise allowed parameter space. We conclude and give an outlook in chapter 8.

Chapter 2

The standard model and its supersymmetric extension

In this chapter we give a brief overview of the standard model, introducing our notation and focusing on flavor violation. We review the basics of softly broken supersymmetry together with the minimal supersymmetric standard model, again focusing on the additional sources of flavor violation. For other issues related to gauge theory, the standard model, and supersymmetry, we refer to the rich text book and review literature on these subjects where more detailed treatments can be found.

2.1 Aspects and problems of the standard model

2.1.1 Field content and Lagrangian

The standard model is a quantum gauge theory of fermions augmented by a fundamental (Higgs) scalar. The gauge group is $SU(3)_C \times SU(2)_W \times U(1)_Y$, and the fermions can be described (for example) by left-handed Dirac spinors. Then the irreducible representations under which the fermions and the Higgs scalar transform are listed together with their quantum numbers in table 2.1. The gauge singlet ν_i^c acts as a right-handed neutrino. It is not present in the "traditional" standard model but has been included as it will appear later in this thesis.

Denoting the gauge fields of the $SU(3)$, $SU(2)$, and $U(1)$ factors by G, W, and B, and introducing the gauge covariant derivative

$$D_\mu = \partial_\mu + ig_1 Y B_\mu + ig_2 T_W^a W_\mu^a + ig_3 T_C^a G_\mu^a, \qquad (2.1)$$

the Lagrangian of the standard model takes the form

$$\mathcal{L}_{SM} \;\; = \;\; \bar{q}_i i\not{D} q_i + \bar{u}_i^c i\not{D} u_i^c + \bar{e}_i^c i\not{D} e_i^c + \bar{d}_i^c i\not{D} d_i^c + \bar{l}_i i\not{D} l_i$$

Table 2.1: Standard model field (irrep) content. $i = 1, 2, 3$ denotes generations.

Field	$(R_{SU(3)}, R_{SU(2)}, Y)$
q_i	(3, 2, 1/6)
u_i^c	($\bar{3}$, 1, -2/3)
e_i^c	(1, 1, 1)
d_i^c	($\bar{3}$, 1, 1/3)
l^i	(1, 2, -1/2)
ν_i^c	(1, 1, 0)
h	(1, 2, -1/2)

$$
\begin{aligned}
&-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} \\
&-\left\{Y^U_{ij}q_i u^c_j h^* + hY^D_{ij}q_i d^c_j + hY^E_{ij}l_i e^c_j + \text{h.c.}\right\} \\
&-\frac{\lambda}{4}\left(h^*h\right)^2 - \mu_0^2 h^* h. \qquad (2.2)
\end{aligned}
$$

Here h^* is the adjoint of the Higgs field, which has the opposite hypercharge of h. Dirac and gauge group representation indices have been suppressed, as have charge-conjugation matrices and ϵ-tensors.

In the standard model, one has $\mu_0^2 < 0$, which leads to a vacuum expectation value for h and spontaneous symmetry breakdown of the $SU(2)_W \times U(1)_Y$ factor to electromagnetic $U(1)_Q$. By the Higgs mechanism, three gauge bosons, termed W^+, W^-, and Z^0, acquire masses, and so do the fermions through the Yukawa terms in (2.2).

In order to describe neutrino masses, we omit a Yukawa term involving the singlet neutrinos ν_i^c (which would give neutrinos Dirac masses) but instead allow for a nonrenormalizable term

$$
\mathcal{L}_M = -\frac{1}{2}\frac{X^M_{ij}}{M_R}\left(l_i h^*\right)\left(l_j h^*\right) + \text{h.c.} \qquad (2.3)
$$

Such a term can arise via the seesaw mechanism [13, 14, 15] if flavor singlet neutrinos exist and have very large Majorana masses as well as Yukawa couplings (at a high scale) to h. Integrating out the ν_i^c then leads to the term $\mathcal{L}_M$. This is the case in the model of Chang, Masiero, and Murayama [25] studied later in this dissertation. After electroweak symmetry breaking, $\mathcal{L}_M$ leads to a Majorana mass matrix for the left-handed neutrinos.

2.1.2 Flavor violation

What is important for flavor physics is that the only generation-nonuniversal couplings in the Lagrangian are the Yukawa matrices Y^U, Y^D, Y^E, and the matrix X^M. They are proportional to the fermion mass matrices M^u, M^d, M^e, and M^ν and can be made diagonal and real via unitary field redefinitions. In a first step, we diagonalize Y^U and X^M,

$$\begin{aligned}
q_i &\to U^*_{Lik} q_k = (q_u, q_c, q_t), &(2.4)\\
u^c_j &\to U_{Rjk} u^c_k = (u^c, c^c, t^c), &(2.5)\\
l_i &\to V_{ik} l_k = (l_e, l_\mu, l_\tau), &(2.6)\\
Y^U &\to U^\dagger_L Y^U U_R = \mathrm{diag}(y_u, y_c, y_t), &(2.7)\\
X^M &\to V^T X^M V = \mathrm{diag}(x_1, x_2, x_3). &(2.8)
\end{aligned}$$

In the new basis, the remaining structures Y^D and Y^E have the form

$$\begin{aligned}
Y^D &= K^* \hat{Y}^D U_D, &(2.9)\\
Y^E &= U^\dagger \hat{Y}^E U_E, &(2.10)\\
\hat{Y}^D &= \mathrm{diag}(y_d, y_s, y_b), &(2.11)\\
\hat{Y}^E &= \mathrm{diag}(y_e, y_\mu, y_\tau). &(2.12)
\end{aligned}$$

The unitary matrices U_D and U_E can be eliminated via redefinitions of the fields d^c_i and e^c_i without affecting the remainder of the Lagrangian. They are therefore unphysical. We remark already here that this is different in the minimal supersymmetric standard model.

The unitary matrices K and U are physical. (However, some of their complex phases can be eliminated, leaving three physical angles and one phase in K and three angles and three phases in U.) Eliminating K and U from the Yukawa terms by field redefinitions of the $T_3 = -1/2$ components of the q_i destroys the manifest $SU(2)$ invariance, and the matrices reappear in the couplings of the $W^\pm$ bosons. K is known as the Cabibbo-Kobayashi-Maskawa matrix [26] and U as the Maki-Nakagawa-Sakata matrix [27, 28], and they are the only physical sources of flavor and CP violation in the standard model with massive left-handed neutrinos.

2.1.3 Hierarchy problem

The standard model has worked extremely well and survived all experimental tests so far. One may ask up to what energy scale the standard model remains valid, so that only after that scale new degrees of freedom or more fundamental changes have to be introduced. There are several indications that this fundamental scale may be very large compared to the electroweak scale. First, if one considers the running of the three gauge couplings, they meet approximately at a scale of about 10^{15} GeV. This could find its interpretation in a grand unification of gauge couplings within a simple gauge group, sponaneously broken at this grand unification (GUT) scale. Second, if neutrino masses are described by a nonrenormalizable term as in (2.3), the mass differences inferred from oscillation experiments suggest that M_R appearing in that equation should be of a similar order of magnitude, if the elements of X^M are not extremely small. Furthermore, the Planck scale of roughly 10^{19} GeV, where gravitation is expected to become strong, lies beyond that scale, although not too far so. This suggests that there may be no need to give up four-dimensional flat-space field theory at a low-energy scale.

Under these conditions, the smallness of the electroweak scale, or equivalently of the parameter μ_0^2, is technically unnatural [29, 30]. That is, it receives quadratically divergent radiative corrections which cannot be eliminated by any internal symmetry of the Lagrangian. This is related to the fact that there exists no internal symmetry which forbids mass terms for any of the scalars in a theory, unlike fermion masses which can be protected from quadratic divergences by approximate chiral symmetries that—were they exact—would prohibit the mass term.

One could attempt to "tune" the bare μ_0^2 such that the desired hierarchy results. However, the presence of any heavy particles, such as those associated with the scale M_R, will also contribute to the quadratic renormalization of μ_0^2 so one would have to do an extreme fine-tuning of the whole Lagrangian, if that is possible at all. Otherwise, any hierarchy of scalar masses (or vacuum expectation values) will be at most of order $1/\alpha$, where α is the relevant coupling constant at a high-mass scale [30].

It is clear then that the addition of new particles between the weak and GUT scales will aggravate instead of ameliorate the problem. There are two main ways out of this problem beyond ignoring it. Either the four-dimensional description breaks down not far beyond the weak scale, such as in models of large extra dimensions [31, 32], or the theory or one has to make the theory supersymmetric. (There are other possibilities that involve strongly coupled fields, such as technicolor theories.)

2.2 The supersymmetric solution to the hierarchy problem

$N = 1$ supersymmetry [33, 34, 35] makes the hierarchy of the standard model technically natural by relating bosonic and fermionic degrees of freedom. For instance, supersymmetry implies equal masses for bosons and fermions, which in turn can protect small scalar masses if the associated fermion mass is protected by a chiral symmetry. In fact, supersymmetry eliminates all quadratic divergences from the theory, leaving only logarithmic wave function renormalization. Supersymmetry can be broken softly, i.e. without destroying the aforementioned desirable properties, in a sufficiently general way to build phenomenologically acceptable models. The selection of topics in supersymmetry here is guided by what will be needed to define the model of Chang, Masiero, and Murayama and discuss it together with relevant properties of the minimal supersymmetric standard model, which is its low-energy effective theory. We will use the elegant superfield formalism [36, 37] which is suitable both for unbroken and softly broken supersymmetry and allows for simple elimination of unphysical parameters from the Lagrangian. For an exposition of the supersymmetry algebra, the superspace differential operators $\mathcal{D}$ and $\bar{\mathcal{D}}$, and the construction of irreducible representations and invariant Lagrangians in terms of superfields, we refer to [38, 39]. We use the standard two-component notation as in [38] for this section, utilizing, however, the standard $(+,-,-,-)$ metric signature. General reviews of phenomenological SUSY models, in particular the minimal supersymmetric standard model, are found, for example, in [40, 41, 42].

2.2.1 Superfields

Superfields are fields defined on superspace, which is parameterized by a set $(x_\mu, \theta, \bar{\theta})$ of the usual four space-time coordinates and two anticommuting (Grassmann) coordinates which are Weyl spinors. The two relevant irreducible superfield representations for us are the chiral and the vector superfield. An (ordinary, left) chiral superfield is a scalar function $\Phi(\theta, y)$ of θ and the "holomorphic" coordinate

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta} \tag{2.13}$$

and has the Taylor expansion

$$\Phi(\theta, y) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y). \tag{2.14}$$

Its adjoint is a function of $\bar{\theta}$ and

$$\bar{y}^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta} \tag{2.15}$$

and is called a right chiral superfield. The component fields ψ and ϕ describe a chiral fermion and a complex scalar, called a sfermion. One can show that any product of left (right) chiral superfields again is left (right) chiral and that the F component is invariant under supersymmetry transformations if integrated over all spacetime. Thus the coefficient of $\theta\theta$ in the superpotential

$$\mathcal{W}(\{\Phi_i\}) = \sum_n c^{(n)}_{i_1\cdots i_n}\Phi_{i_1}(\theta, y)\cdots\Phi_{i_n}(\theta, y) \tag{2.16}$$

can be used in invariant actions.

A vector (gauge) superfield, on the other hand, is self-adjoint and has the expansion (in Wess-Zumino supergauge) of

$$V(\theta, \bar{\theta}, x) = \theta\sigma^\mu\bar{\theta}v_\mu(x) + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x) \tag{2.17}$$

In this case, the integral of the D component is invariant. The component $v_\mu(x)$ will act as a gauge field, and the neutral fermion λ is known as gaugino. A composite term that transforms like a vector superfield is given by

$$\Phi^* \exp\left(2g_A V_A^a T_A^a\right) \Phi \tag{2.18}$$

where we have generalized to an arbitrary semisimple gauge group with gauge couplings g_A and generators T_A^a. Furthermore it is useful to define the chiral spinor fields

$$2g_A T_A^a W_A^a = \frac{1}{4}\bar{\mathcal{D}}\bar{\mathcal{D}} \exp\left(-2g_A T_A^a V_A^a\right) \mathcal{D} \exp\left(2g_A T_A^a V_A^a\right) \tag{2.19}$$

where now

$$W_A^a(\theta, y) = \lambda_A^a(y) + \left(D_A^a(y) - \frac{i}{2}\sigma^\mu \bar{\sigma}^\nu v^a_{A\mu\nu}\right)\theta + i\theta\theta\sigma^\mu \partial_\mu \bar{\lambda}_A^a(y). \tag{2.20}$$

The most general gauge and supersymmetry invariant action is then given via

$$S = \int \mathrm{d}^4x \left(\mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, \Phi_i^* \exp\left(2g_A T_A^a V_A^a\right) \Phi_i + \left\{\mathrm{d}^2\theta \left[\mathcal{W}(\{\Phi_i\}) + \frac{1}{4} W_A^a W_A^a\right] + \text{h.c.}\right\}\right). \tag{2.21}$$

We remark that the D and F component fields are auxiliary since they do not have a kinetic term, so they can be eliminated in favor of polynomials in the scalar fields ϕ by the equations of motion. Before doing this, in order to interpret the terms in (2.21), we spell them out in terms of component fields. First,

$$\begin{aligned}
&\int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \Phi_i^* \exp\left(2g_A T_A^a V_A^a\right) \Phi_i = \\
&\quad \sum_i |D_\mu \phi_i|^2 + i\psi_i \sigma^\mu D_\mu \overline{\psi}_i - g\sqrt{2}\left(\phi_i^* T_A^a \lambda_A^a \psi_i + \bar{\lambda}_A^a \overline{\psi}_i T_A^a \phi_i\right) \\
&\quad + F_i^* F_i + g_A D_A^a d_A^a
\end{aligned} \tag{2.22}$$

where

$$d_A^a = \phi_i^* T_A^a \phi_i \tag{2.23}$$

Thus this term describes gauge kinetic terms for the scalars and fermions of the chiral multiplets. Furthermore, it contains a coupling between a fermion, a sfermion and a gaugino. There is also a contribution to the scalar potential.

The superpotential term gives Yukawa couplings between fermions and sfermions as well as another contribution to the scalar potential:

$$\int \mathrm{d}^2\theta \mathcal{W}(\{\Phi_i\})\Big|_{\theta\theta} = -Y_{ij}(\{\phi_i\})\psi_i\psi_j + F_i f_i \tag{2.24}$$

where

$$\begin{aligned}
Y_{ij}(\{\phi_i\}) &= \frac{\partial^2 \mathcal{W}}{\partial \Phi_i \partial \Phi_j}(\{\phi_i\}), &(2.25)\\
f_i &= \frac{\partial \mathcal{W}}{\partial \Phi_i}(\{\phi_i\}). &(2.26)
\end{aligned}$$

Finally,

$$\int \mathrm{d}^2\theta\, \frac{1}{4} W_A^a W_A^a + \text{h.c.} = \\ -\frac{1}{4} v_{A\mu\nu}^a v_A^{a\mu\nu} + \left(i\lambda_A^a \sigma^\mu \partial_\mu \bar{\lambda}_A^a - g_A f_A^{abc} \lambda_A^a \sigma^\mu v_{A\mu}^b \bar{\lambda}_A^c \right) + \frac{1}{2} D_A^a D_A^a \tag{2.27}$$

provides kinetic and self-coupling terms for gauge fields and gauginos.

It is now easy to read off the equations of motion for the F and D fields,

$$F_i = -f_i^*, \tag{2.28}$$
$$D_A^a = -d_A^a, \tag{2.29}$$

and the scalar potential thus is the sum of the two terms

$$V_F(\{\phi_i\}) = f_i^* f_i, \tag{2.30}$$
$$V_D(\{\phi_i\}) = \frac{1}{2} d_A^a d_A^a. \tag{2.31}$$

It is clear from eq. (2.25) and the form of V_F that the theory is renormalizable if and only if the degree of the superpotential is at most three. One can also derive that the complex scalars and the chiral fermions come in supermultiplets of degenerate mass.

2.2.2 Nonrenormalization theorem

The crucial property of the supersymmetric action (2.21) is that the superpotential $\mathcal{W}$ is not renormalized [43]. More precisely, the only divergences of the effective action have the form of $\theta\theta\bar{\theta}\bar{\theta}$ terms and only a logarithmic wave function renormalization for the Φ and V superfields is required. Thus the coefficients in $\mathcal{W}$ get only logarithmically renormalized, too. The same is true of the vacuum expectation values in spontaneously broken gauge theories. Thus a large hierarchy, while not explained, becomes stable under radiative corrections.

2.2.3 Soft breaking

The fact that no scalar particles with the mass and electric charge of e.g. the electron have been observed implies that supersymmetry must be broken. While spontaneous supersymmetry breaking is possible by giving a vacuum expectation value to an F or D component of a superfield, it generally is difficult to do this in a phenomenologically viable way because the mass formulas [44] applicable to such cases require at least some supersymmetric particles to be lighter than their standard model partners. (Another concern is that the vaccuum energy no longer vanishes (as it does in exact SUSY), leading to a large cosmological constant.)

For this reason, it is customary to break supersymmetry explicitly. Doing this, however, is restricted by requiring that the breaking be soft, i.e., that it should not spoil the nonrenormalization theorem. The allowed soft breaking terms have been analyzed for the renormalizable case by Girardello and Grisaru [45]. They can all be expressed in a manifestly supersymmetric notation by introducing external vector and chiral spurion superfields U and η whose D and F components (respectively) are given a nonzero value (thus breaking supersymmetry). The most general renormalizable gauge-invariant soft-breaking action can be written [45, 46]

$$S_{\text{soft}} = -\int \mathrm{d}^4x \left(\mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, U \Phi_i^* m_{ij}^2 \Phi_j + \left\{ \mathrm{d}^2\theta \left[\eta \tilde{\mathcal{W}}(\{\Phi_i\}) + \frac{1}{2} m_A \eta W_A^a W_A^a \right] + \text{h.c.} \right\} \right) \tag{2.32}$$

where

$$\eta = \theta\theta, \qquad U = \theta\theta\bar{\theta}\bar{\theta}. \tag{2.33}$$

$\tilde{\mathcal{W}}$ is again a polynomial of at most degree three, now containing coefficients of mass dimensions one to three. It simply gives a contribution to the scalar potential of the form

$$V_{\tilde{\mathcal{W}}} = \tilde{\mathcal{W}}(\{\phi_i\}). \tag{2.34}$$

The hermitian matrix m_{ij}^2 contributes to the scalar mass matrix and is restricted by gauge invariance, while the m_A, which may be complex, give mass to the gauginos.

2.2.4 Nonrenormalizable case

The nonrenormalization theorem has not long ago been generalized by Weinberg [47] to the case of a very large class of nonrenormalizable theories; in particular, the superpotential may have arbitrary degree. This is important to us as the specific model to be studied later contains nonrenormalizable terms. Unfortunately, the last reference does not reanalyze the structure of the allowed soft terms. We will assume, as is customary in the literature, that at least the terms of (2.32) do not reintroduce quadratic divergences.

2.2.5 Eliminating parameters in softly broken supersymmetry

We now come to a technical application of the superfield formalism, the question of counting physical parameters or equivalently of what parameters can be eliminated by field redefinitions. One can do this in component field language, but one has to be careful about what redefinitions keep the manifest softly broken symmetry. For example, different rephasings for fermions and sfermions will make the fermion-sfermion-gaugino couplings complex. Rephasing the gauginos can compensate for this and restore the manifest soft breaking, but in turn changes the phase of the mass parameters, and so on.

In superfield notation, there are just two types of transformation that do not spoil the general form of (2.21), (2.32). First, there are unitary transformations among the chiral superfields Φ,

$$\Phi_i \rightarrow U_{ik}\Phi_k. \tag{2.35}$$

Second, one can rephase the Grassmann coordinates $\theta, \bar{\theta}$, which can be considered a special type of R transformation:

$$\begin{aligned} \theta' &\equiv e^{i\alpha}\theta, &(2.36)\\ \Phi(\theta, y) &= \Phi(e^{-i\alpha}\theta', y), &(2.37)\\ V(\theta, \bar{\theta}, x) &= V(e^{-i\alpha}\theta', e^{i\alpha}\bar{\theta}', x), &(2.38)\\ W(\theta, y) &= e^{i\alpha}W(e^{-i\alpha}\theta', y) &(2.39)\end{aligned}$$

(Note that the field strength superfield W automatically has R character 1/2. Also note that $\mathrm{d}\theta' = e^{-i\alpha}\mathrm{d}\theta$.) Going from θ to θ', the dynamic component fields transform as

$$\begin{aligned} \lambda_A^a &\rightarrow e^{-i\alpha}\lambda_A^a, &(2.40)\\ \psi_i &\rightarrow e^{-i\alpha}\psi_i, &(2.41)\\ v_\mu &\rightarrow v_\mu, &(2.42)\\ \phi_i &\rightarrow \phi_i. &(2.43)\end{aligned}$$

Thus the rephasing is equivalent to a set of field redefinitions. One has

$$\begin{aligned} \int \mathrm{d}^2\theta\, \Phi_1 \cdots \Phi_n &= e^{-2i\alpha}\int \mathrm{d}^2\theta'\, \Phi_1 \cdots \Phi_n, &(2.44)\\ \int \mathrm{d}^2\theta\, (\theta\theta)\Phi_1 \cdots \Phi_n &= \int \mathrm{d}^2\theta' (\theta'\theta')\Phi_1 \cdots \Phi_n, &(2.45)\\ \int \mathrm{d}^2\theta(\theta\theta)W_A^a W_A^a &= e^{-2i\alpha}\int \mathrm{d}^2\theta' (\theta'\theta')W_A^a W_A^a, &(2.46)\end{aligned}$$

with the remaining terms in the action invariant. A general R transformation can be composed of the two transformations discussed so far. Both types of transformation keep the form of (2.21) and (2.32).

As an application, we consider the following situation which arises in the MSSM and the CMM model. Let

$$\mathcal{W} = y\Phi_1\Phi_2\Phi_3, \qquad \tilde{\mathcal{W}} = A\Phi_1\Phi_2\Phi_3. \tag{2.47}$$

Furthermore consider any one of the gaugino masses, m. For this case, one finds the physical combination

$$\arg y - \arg A + \arg m = \text{invariant} \tag{2.48}$$

where two of the three phases can be freely adjusted. Similarly, one can show that the phase differences between the different gaugino masses are physical.

Table 2.2: MSSM superfield content. $i = 1, 2, 3$ denotes generations.

superfield	component fields
Q_i	$(q_i\ ,\ \tilde{q}_i)$
U_i^c	$(u_i^c,\ \tilde{u}_i^c)$
E_i^c	$(e_i^c,\ \tilde{e}_i^c)$
D_i^c	$(d_i^c,\ \tilde{d}_i^c)$
L_i	$(l_i,\ \tilde{l}_i)$
H_u	$(\tilde{h}_u,\ h_u)$
H_d	$(\tilde{h}_d,\ h_d)$
V_1	$(B_\mu,\ \tilde{g}_1)$
V_2	$(W_\mu,\ \tilde{g}_2)$
V_3	$(G_\mu,\ \tilde{g}_3)$

2.3 The MSSM

The minimal supersymmetric standard model is found by promoting the standard model chiral fermions and scalars to chiral supermultiplets as well as the gauge bosons to vector multiplets and constructing the corresponding supersymmetric gauge theory. The notation for all superfields and component fields is given in table 2.2. Superpartners are denoted throughout with a tilde. The superpotential is determined by the standard model Yukawa potential; however, the holomorphy condition implies that one has to use two separate Higgs multiplets H_u, H_d in place of h^* and h. This is also necessary to make the theory anomaly free, as there are now additional fermions, the higgsinos, in the theory. (Gauginos do not contribute to anomalies.) One has

$$\mathcal{W} = Y_{ij}^U Q_i U_j^c H_u + H_d Y_{ij}^D Q_i D_j^c + H_d Y_{ij}^E L_i E_j^c + \mu H_u H_d. \tag{2.49}$$

This is also the most general superpotential invariant under R parity (a Z_2 subgroup of an R symmetry where H_u, H_d have R character zero and all other chiral superfields have R character 1/2). We note that the R parity of a particle can be written [48]

$$R = (-)^{3B+L+2S} \tag{2.50}$$

where B, L, and S are the baryon number, lepton number, and spin of the particle. Under R parity, all standard model particles are even and all their SUSY partners odd, which has the desirable consequence that the lightest supersymmetric particle is stable. R parity conservation also ensures that baryon and lepton number are preserved by the superpotential.

2.3.1 Soft breaking terms and particle masses

The polynomial $\tilde{\mathcal{W}}$ of eq. (2.32) is restricted by gauge invariance and R parity to the form

$$\tilde{\mathcal{W}} = A^U_{ij} Q_i U^c_j H_u + H_d A^D_{ij} Q_i D^c_j + H_d A^E_{ij} L_i E^c_j + m^2_{12} H_u H_d. \tag{2.51}$$

m^2_{12} is often parameterized as $B\mu$. The trilinear couplings are called A-terms. Beyond this, there are mass terms for the three gauginos and explicit scalar masses,

$$\begin{aligned}\mathcal{L}_m \;=\;& -\frac{1}{2} m_{\tilde{g}_1} \tilde{g}_1 \tilde{g}_1 - \frac{1}{2} m_{\tilde{g}_2} \tilde{g}_2 \tilde{g}_2 - \frac{1}{2} m_{\tilde{g}_3} \tilde{g}_3 \tilde{g}_3 \\ & - m^2_{\tilde{q}\,ij} \tilde{q}^*_i \tilde{q}_j - m^2_{\tilde{u}\,ij} \tilde{u}^{c*}_i \tilde{u}^c_j - m^2_{\tilde{e}\,ij} \tilde{e}^{c*}_i \tilde{e}^c_j - m^2_{\tilde{d}\,ij} \tilde{d}^{c*}_i \tilde{d}^c_j - m^2_{\tilde{l}\,ij} \tilde{l}^*_i \tilde{l}_j \\ & - m^2_{h_u} |h_u|^2 - m^2_{h_d} |h_d|^2, \end{aligned} \tag{2.52}$$

where i, j are generation indices. Now there is no a priori reason for the scalar mass matrices to have a specific form (other than merely hermitian). In particular, diagonalizing the superpotential couplings in generation space as in (2.4)–(2.12) in general does not lead to diagonal sfermion masses. Moreover, the matrices U_D and U_E of equations (2.9), (2.10) now are physical, as their absorption into redefinitions of D^c_i and E^c_i affects the form of $m^2_{\tilde{d}}$ and $m^2_{\tilde{e}}$. In general, the diagonalization of the sfermion mass matrices therefore introduces flavor-changing fermion-sfermion-gaugino couplings beyond those involving charginos. This is a potential source of large flavor and CP violation [49]. The same is true of the A-terms.

2.3.2 Electroweak symmetry breaking

Without the soft terms, there would be no breakdown of $SU(2) \times U(1)$ to electromagnetism. After introducing the parameters $m^2_{h_u}$, $m^2_{h_d}$, and $B\mu$, breaking is possible and both h_u and h_d can obtain vacuum expectation values v_u, v_d, giving mass to the fermions and sfermions. An important parameter is the ratio

$$\tan\beta = \frac{v_u}{v_d}. \tag{2.53}$$

For large $\tan\beta$, the bottom and tau Yukawa couplings become large.

The scalar potential need not, however, have its minimum at this point. In that case, charge and/or color would be broken. Demanding that the physical vacuum be the true one (i.e., stable) leads to strong constraints on the A-terms. See [50] and the review [51] as well as references therein. It is common and useful to define rescaled A-terms ($X = U, D, E$)

$$a^X_i \equiv \frac{A^X_{ii}}{Y^X_{ii}}. \tag{2.54}$$

Then at tree level, one has the conditions

$$
\begin{aligned}
|a_1^U| &< \sqrt{3(m_{\tilde{q}_1}^2 + m_{\tilde{u}_1}^2 + m_{H_u}^2)}, && (2.55)\\
|a_1^D| &< \sqrt{3(m_{\tilde{q}_1}^2 + m_{\tilde{d}_1}^2 + m_{H_d}^2)}. && (2.56)
\end{aligned}
$$

These bounds are imposed at a low energy (electroweak) scale; there is no corresponding condition e.g. at the GUT scale where the tree potential is a bad approximation to the effective potential. There are similar bounds for the flavor-offdiagonal entries in the A matrices [50]. The A-term phases are generally constrained from CP-violating observables [49], at least for the first two generations.

The structure of the sfermion and bosino mass matrices is affected by electroweak symmetry breakdown. There are both D-term corrections to the gauge invariant sfermion masses in (2.52) and $SU(2) \times U(1)$-breaking terms mixing squarks of different chiralities (e.g. $\tilde{d}_i$ and $\tilde{d}_j^c$, where $\tilde{d}_i$ is the $T_3 = -1/2$ component of the doublet $\tilde{q}_i$). They depend on the A-terms and the combinations μY^X; for instance,

$$(M_{\tilde{d}}^2)_{LR} \propto -\mu^* M^d \tan\beta + A^D. \tag{2.57}$$

This left-right mixing is never large for the first two generations due to the smallness of the Yukawa couplings and the A-terms; barring large $\tan\beta$, it is only relevant for the stops at all. Since in this dissertation no scenarios of large $\tan\beta$ will be considered and stop mixing will not be important either, we will here ignore left-right mixing among the sfermions altogether.

The $U(1)$ and $SU(2)$ gauginos mix with the higgsinos into four Majorana neutralinos and two Dirac charginos. Their respective mass matrices read (we are using the basis of [52]; the notation there is different but can be related to the one used here as $\mu \leftrightarrow \mu, M_1 \leftrightarrow m_{\tilde{g}_1}, M_2 \leftrightarrow m_{\tilde{g}_2}$)

$$
M_{\chi^0} =
\begin{pmatrix}
m_{\tilde{g}_1} & 0 & -M_Z\cos\beta\sin\theta_W & M_Z\sin\beta\sin\theta_W \\
0 & m_{\tilde{g}_2} & M_Z\cos\beta\cos\theta_W & -M_Z\sin\beta\cos\theta_W \\
-M_Z\cos\beta\sin\theta_W & M_Z\cos\beta\cos\theta_W & 0 & -\mu \\
M_Z\sin\beta\sin\theta_W & -M_Z\sin\beta\cos\theta_W & -\mu & 0
\end{pmatrix} \tag{2.58}
$$

and

$$M_{\chi^+} = \begin{pmatrix} m_{\tilde{g}_2} & \sqrt{2}M_W\sin\beta \\ \sqrt{2}M_W\cos\beta & \mu \end{pmatrix} \tag{2.59}$$

It is clear that even for small $\tan\beta$, mixing between gauginos and higgsinos can in general not be neglected, except when μ or the gaugino masses become large.

We now discuss two specific kinds of flavor structure.

2.3.3 Minimal flavor violation

One way to reduce the number of parameters is to simply impose that the flavor and CP structure of the new couplings is the same as in the standard model. That is, the standard model field redefinitions (2.4)–(2.12) already diagonalize the A-terms and the squark mass matrices. Note that this implies that $m^2_{\tilde{q}}$ is proportional to the unit matrix. Then the only flavor- and CP-violating couplings are the couplings of charginos to quarks and squarks and of charged Higgs bosons to quarks. Furthermore, these couplings contain the appropriate CKM matrix element as a factor. This "scenario" is known as minimal flavor violation [53, 54]. This is not related to any specific breaking mechanism but is a limit which is predictive and testable phenomenologically. As has been mentioned above, it can be justified at least for the first two generations by the constraints coming from K physics.

The contributions to the effective Lagrangians describing weak decays have a very specific form within minimal flavor violation. In particular, there are, to a good approximation, no new flavor and CP structures and no new operators introduced in the Lagrangian, such that the physics is fully described by modified Inami-Lim-type vertex functions [54]. This leads to specific signatures in flavor- and CP-violating observables; in particular, it still allows for an extraction of the unitarity triangle withouth knowledge of SUSY particle masses [55].

The MFV contributions persist in a situation with more general flavor violation; they therefore represent the "minimial" impact of supersymmetry on flavor physics.

2.3.4 Universal boundary conditions

Many different mechanisms of supersymmetry breaking have been considered. One particularly popular approach is minimal supergravity. Here the soft-breaking terms are assumed to arise at the Planck scale from gravitational interactions and have a universal form. That is, there is a universal scalar soft (mass)^2, and the A-terms are related to the Yukawa couplings by a universal parameter a_0. Thus at the Planck scale,

$$(m^2_{\tilde{f}})_{ij} = \delta_{ij} m_0^2, \qquad m^2_{H_u} = m^2_{H_d} = m_0^2, \qquad A^r_{ij} = a_0 Y^r_{ij}, \tag{2.60}$$

with $\tilde{f} = \tilde{q}, \tilde{d}^c, \tilde{u}^c, \tilde{l}, \tilde{e}^c$ and $r = U, D, E$.

However, this universality is not preserved under renormalization group evolution. Evolving the MSSM renormalization group equations [56, 57] down to the weak scale, the nonuniversality of the Yukawa couplings is transferred to the Higgs sector and the sfermion mass matrices. The former may have the consequence that the electroweak symmetry is broken radiatively, in the sense that the tree potential at the Planck scale has its minimum at the origin, while at the electroweak scale the minimum is at nonvanishing values of the neutral Higgs fields. This was first discussed in [58]. With the measured large top quark mass, radiative symmetry breaking is guaranteed.

Among the sfermions, for small $\tan\beta$ the effect is only large for the matrices $m^2_{\tilde{q}}$ and $m^2_{\tilde{u}}$, since all Yukawa couplings except that of the top are very small. Under this condition, in a basis where the matrix Y^U is diagonal, they remain diagonal under evolution and have the schematic form

$$m^2_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}_1} & 0 & 0 \\ 0 & m^2_{\tilde{f}_2} & 0 \\ 0 & 0 & m^2_{\tilde{f}_3} - \Delta_{\tilde{f}} \end{pmatrix}. \tag{2.61}$$

at the weak scale. The nonvanishing splittings $\Delta_{\tilde{u}}, \Delta_{\tilde{q}}$ are calculable by solving the renormalization group equations.

As the sdown and slepton masses keep their universal form, the matrices U_D and U_E are again rendered unphysical. Just as in the case of minimal flavor violation, the righthanded sdowns and the slepton doublets therefore do not have chirality-preserving FCNC couplings. (As their left-right-mixings masses according to (2.57) are also small (for small $\tan\beta$), chirality-flipping FCNC couplings will be small; their effects may however be enhanced in certain processes such as $b \to s\gamma$ by a factor $m_{\tilde{g}3}/m_b$ and can be constrained from phenomenology.)

Above the GUT scale the situation may change. First, there can be additional large Yukawa couplings that are not simultaneously diagonal with Y^U, destroying the simple form of (2.61). Second and more model-independently, unification of matter multiplets generally implies that above the GUT scale, the large top Yukawa coupling affects the evolution of the other sfermion mass matrices as well. This will be discussed in chapter 3.

Chapter 3

Supersymmetric grand unification

In this chapter we review grand unification in the context of supersymmetry. With respect to flavor physics, there are two main differences to the MSSM. The first is qualitative: unification of what are separate gauge multiplets in the standard model into larger representations implies correlations between flavor structures, and the large top Yukawa coupling now affects additional sfermion matrices. Furthermore, within $SO(10)$ grand unification there is the prediction of a right-handed neutrino, which by means of the seesaw mechanism can induce small Majorana masses for the standard model left-handed neutrinos. The other difference is quantitative, due to the larger group theoretical factors the renormalization group running of the soft terms becomes much faster.

3.1 $SO(10)$ unification of gauge and matter fields

We have said before that the MSSM improves the prediction for gauge coupling unification, while at the same time raising the unification scale to

$$M_{\rm GUT} \approx 10^{16}\text{GeV}. \tag{3.1}$$

Let us now turn to the unification of the irreducible representations of the standard model into one irreducible **16** of $SO(10)$ per generation and the consequences for flavor physics.

Within the original $SU(5)$ model [9], the fermions of one standard model generation are unified into one five-dimensional and one ten-dimensional irreducible representation of the gauge group, and they completely fill these representations. Thus no additional unobserved fermions are predicted. Furthermore, this representation content happens to make the GUT anomaly free, just as is the case with the standard model. As a consequence of this unification, certain Yukawa couplings become related at M_{GUT}, for instance $y_\tau = y_b$ in the absence of high-dimensional Higgs representations.

Making the absence of additional fermions a requirement, $SU(5)$ becomes the unique GUT [9]. However, there is one other simple gauge group that almost fulfils the requirement, this is $SO(10)$ [11, 12]. This group has a sixteen-dimensional complex representation, and the additional degree of freedom is a standard model gauge singlet, i.e., a right-handed neutrino (cf. table 2.1). Moreover, $SU(5)$ is a subgroup of $SO(10)$ and the ν^c field is a singlet under $SU(5)$, as well. We denote the singlet neutrino superfields of the three generations by N_i.

3.1.1 Lagrangian

The sixteen-dimensional representation of $SO(10)$ is a spinor representation. This has, as is the case with Dirac spinors, the consequence that it can only enter the Lagrangian in the form of bilinears. Decomposing

$$\mathbf{16} \times \mathbf{16} = \mathbf{10} + \mathbf{120} + \mathbf{126} \tag{3.2}$$

and considering that a renormalizable superpotential has degree three (or less), this implies that there are only three choices of Higgs representation that can have couplings to the matter fields in a renormalizable context. Schematically, one can write

$$\mathcal{W} = \frac{1}{2} Y^{10}_{ij} \mathbf{16}_i \mathbf{16}_j\, \mathbf{10} + \frac{1}{2} Y^{120}_{ij} \mathbf{16}_i \mathbf{16}_j\, \mathbf{120} + \frac{1}{2} Y^{126}_{ij} \mathbf{16}_i \mathbf{16}_j\, \overline{\mathbf{126}}. \tag{3.3}$$

Here we have suppressed group structure and omitted couplings not involving the spinors. The necessary group theory is collected in appendix A.

Each of the three Higgs representations contains MSSM doublets with the quantum numbers of H_u and H_d. The light doublets will, in general, be some superposition of them. The resulting fermion mass matrices have been given in [59, 60].

One important point is that the group-theoretical structure of the three terms in (3.3) forces the matrices Y^{10} and Y^{126} to be symmetric and the matrix Y^{120} to be antisymmetric. This means that if one wants to have a nonsymmetric Yukawa matrix for one of the light doublets, the corresponding $SO(10)$ Higgs field must transform reducibly, that is, it must be a linear combination of two or more irreducibly transforming fields or it must be replaced by a nonrenormalizable operator, as is done in the Chang-Masiero-Murayama model.

The choice of one of the large representations has the side effect that the $SO(10)$ gauge coupling is no longer asymptotically free, although it does not become infinite before the Planck scale unless several large representations are introduced. The A-terms have a structure analogous to $\mathcal{W}$, and there is now just one three-by-three soft mass matrix for the three generations of sfermions.

3.1.2 Seesaw mechanism

The N_i, being singlets, can acquire a Majorana mass once $SO(10)$ is broken:

$$\mathcal{W}_R = \frac{1}{2} M_{Rij} N_i N_j. \tag{3.4}$$

A typical mass will be a bit below the GUT scale unless couplings are fine tuned. In general, there will also be a Yukawa term; below M_{GUT} this has the form

$$\mathcal{W}_\nu = Y^\nu_{ij} L_i N_j H_u. \tag{3.5}$$

Integrating out the N fields then gives a nonrenormalizable operator that, after breaking of GUT and SUSY to the standard model, will lead to the term $\mathcal{L}_M$ of eq. (2.3). This is the seesaw mechanism [13, 14, 15]. Small neutrino Majorana masses are therefore a generic prediction of $SO(10)$ grand unification. The seesaw mechanism can be relevant in the $SU(5)$ case as well, but in this case there is no a priori reason to have right-handed neutrinos.

3.2 Universality breaking in SUSY GUTs

We now reexamine the issue of sfermion mass matrices and their flavor structure first studied in sec. 2.3.1.

3.2.1 Radiative effects

Assuming that the SUSY breaking is again connected with Planck-scale physics and has universal form at that scale, GUT radiative effects on the Higgs and sfermion sectors were first considered in [61, 62, 63].

The situation is reminiscent of the MSSM case. This is true particularly when there is only one large (top) Yukawa coupling in one of the matrices Y^{10}, Y^{120}, and Y^{126}. In this case all soft mass matrices remain simultaneously diagonalizable with relevant matrix. At the scale of $SO(10)$ breaking, the unique $SO(10)$ sfermion (mass)^2 matrix splits into the five of the MSSM (six if including the righthanded sneutrinos). (This may happen in several steps if $SO(10)$ is not broken directly to the SM.) From there to the weak scale, the masses renormalize differently; in the case of small $\tan\beta$, only the matrices $m^2_{\tilde{q}}$ and $m^2_{\tilde{u}}$ receive additional nonuniversal corrections. All sfermion mass matrices have the form of eq. (2.61) in a basis diagonalizing Y^U.

3.2.2 D-terms from gauge group rank reduction

When, in breaking a softly broken supersymmetric gauge theory, the rank of the gauge group is reduced, additional nonuniversal contributions to the scalar masses arise [64]. This can only happen when the soft terms are already nonuniversal at the breaking scale. Both conditions are satisfied in $SO(10)$ with minimal supergravity boundary conditions. The rank is 5, one greater than that of the standard model group, and it contains a $U(1)$ factor that commutes with the $SU(5)$ containing the standard model. This $U(1)$ is commonly labeled X. The various $SU(5)$ (or standard model) representations arising after $SO(10)$ breakdown have different $U(1)_X$ quantum numbers. Each multiplet gets a (mass)^2 shift proportional to its X charge. For the CMM model, this will be taken into account in chapter 5. All shifts are generation universal for each MSSM multiplet.

The general sparticle spectrum including radiative and breaking effects has been studied by a number of authors [65, 66, 67, 68]. (The last reference considers very large $\tan\beta$.)

3.2.3 Flavor and CP phenomenology

Because of the similar form of the sfermion mass matrices under the conditions spelled out above, also the flavor phenomenology resembles the MSSM. However, there are now possibly large mass splittings also among the righthanded sdowns $\tilde{d}_i^c$ and charged sleptons $\tilde{e}_i^c$ (as well as the lefthanded sleptons $\tilde{l}_i$), and as was mentioned at the end of section 2.3.4, this now gives physical relevance to the matrices U_E and U_D related to the diagonalization of down and electron masses (eqs. (2.9) and (2.10)). Diagonalizing the fermion masses leads to flavor-changing couplings of gluinos, charginos and neutralinos. This in turn implies contributions to flavor- and CP-violating observables. The effect was considered in [19] and claimed to be small. After the top quark was found to be heavy, it was reconsidered [20, 21, 22] and, after a quantitative analysis of the radiative effects, found to be potentially important in decay modes such as $\mu \to e\gamma$ and $\tau \to \mu\gamma$. Generally, the effects on hadronic observables —B_d-mixing, ϵ_K, and ϵ'—were found to be less important.

Nevertheless, the authors consider $SU(5)$ without right-handed neutrinos and $SO(10)$ with only symmetric Yukawa matrices, and this implies that the effects are "unnecessarily" small (governed by CKM elements) or absent.

Once neutrino masses are included, the MNS matrix U with its large mixing angles and unconstrained phases enters. The situation has been investigated within $SU(5)$ unification in various places, for instance in [24]. For the case of $SO(10)$, a flavor structure was proposed in [25] which relates the matrix U_D to U, however their analysis of the phenomenology is restricted to qualitative considerations. In this dissertation, we explore their model quantitatively, calculating both the radiative effects and the necessary low-energy effective interactions.

3.2.4 A word on generic GUT problems

GUT theories make several generic predictions, most importantly the unification of at least the bottom and tau Yukawa couplings, the decay of the proton and, in the case of $SO(10)$, the nonvanishing of neutrino masses.

Of the three, the last has been confirmed by experiment and the second can be considered open but giving constraints on model parameters such as $\tan\beta$ (see e.g. [69]). The first is becoming more and more constraining due to the increasing lower bound on the proton lifetime. In fact, nonsupersymmetric $SU(5)$ is considered ruled out, while the situation for the supersymmetric variants is becoming difficult. The main contributions here come from color triplet Higgs exchange and suppressing them is related to the so-called doublet-splitting problem. For a recent analysis, see [70]. While the problem has to be resolved for any SUSY GUT theory, it appears to us to have little relation to the specific flavor structure of the theory. For this reason, we will ignore the issue in this thesis and leave it to be resolved elsewhere.

Chapter 4

The Chang-Masiero-Murayama model

The SUSY $SO(10)$ theory introduced in this chapter has a very simple flavor structure. The only large, renormalizable Yukawa coupling is that responsible for the top quark mass, and the right-handed neutrino Majorana masses are simultaneously diagonal with the up-type-quark Yukawa couplings at a high-energy scale. There is only one additional flavor structure that is not diagonal in this basis—it is responsible for the down-type-quark and charged-lepton masses. In particular, both the CKM and the MNS mixing matrices are encoded within it. Radiative corrections from the range of validity of the GUT theory, and from the (effective) MSSM below the GUT scale, evolve the universal soft parameters at the Planck scale into nonuniversal soft masses at the weak scale, which are, however, aligned (simultaneously diagonal) with the up-type-quark and neutrino mass matrices. Therefore, mixing angles appear between the right-handed down-type quarks and the corresponding squarks and also between the left-handed leptons (charged ones as well as neutrinos) and their sfermionic partners. Furthermore, these angles are correlated with the MNS mixing angles. The known large atmospheric neutrino mixing then implies a large mixing among sfermions.

4.1 Field content and Lagrangian

An interesting supersymmetric $SO(10)$ model has recently been proposed by Chang, Masiero, and Murayama [25]. At the Planck scale, the quarks and lepton fields of each standard model generation are unified together with a right-handed neutrino into an irreducible, 16-dimensional (spinor) representation of the gauge group $SO(10)$. In addition, the model contains several Higgs multiplets: a $\mathbf{10}_H$, a $\overline{\mathbf{16}}_H$ or $\overline{\mathbf{126}}_H$, as well as a further nonrenormalizable term $\mathbf{H}'$ (for definiteness, $\mathbf{45}_H \times \mathbf{10}'_H$) transforming reducibly. All of them have superpotential couplings to the matter fields. Using matrix notation to describe the generation structure of the couplings, the superpotential of the model reads

$$W_{10} = \frac{1}{2}\mathbf{16}^T Y^U \mathbf{16}\,\mathbf{10}_H + \frac{1}{M_{\rm Pl}}\frac{1}{2}\mathbf{16}^T \tilde{Y}^D \mathbf{16}\,\mathbf{H}' + \frac{1}{M_{\rm Pl}}\frac{1}{2}\mathbf{16}^T Y^M \mathbf{16}\,\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H, \quad (4.1)$$

with $Y^U, \tilde{Y}^D$, and Y^M three-by-three matrices, and $\mathbf{16}$ a vector, in generation space. In the last term, the two factors of $\overline{\mathbf{16}}_H$ could be replaced by a $\overline{\mathbf{126}}_H$, giving instead a renormalizable term

$$\frac{1}{2}\mathbf{16}^T Y^M \mathbf{16}\,\overline{\mathbf{126}}_H \quad (4.2)$$

In either case, there must be a Higgs in the conjugate representation to avoid D-term SUSY breaking at the GUT scale. We will refer to the two variants of the model as **NR** (nonrenormalizable) and **R** (renormalizable), according to the Y^M term. One difference between both cases is that in the **NR** case, the gauge coupling is asymptotically free while in the **R** case it is not.

The soft SUSY-breaking terms involving the fields with large renormalizable couplings to the matter fields in the CMM model take the form

$$\begin{aligned}\mathcal{L}_s^{10} &= -\tilde{\mathbf{16}}^\dagger m_{16}^2 \tilde{\mathbf{16}} - m_{10}^2 \tilde{\mathbf{10}}_H^* \tilde{\mathbf{10}}_H - \frac{1}{2}\tilde{\mathbf{16}}^T A^U \tilde{\mathbf{16}}\,\tilde{\mathbf{10}}_H \\ &\quad - m_{126}^2 \tilde{\overline{\mathbf{126}}}_H^* \tilde{\overline{\mathbf{126}}}_H - \frac{1}{2}\tilde{\mathbf{16}}^T A^M \tilde{\mathbf{16}}\,\tilde{\overline{\mathbf{126}}}_H \qquad (4.3)\end{aligned}$$

where the tildes indicate the scalar components of the superfields defined before. The terms on the second line are absent in the nonrenormalizable case. The breaking terms are assumed to be universal at the Planck scale:

$$\begin{aligned} m_{16}^2 &= m_0^2 \mathbf{1}, & (4.4)\\ m_{10_H}^2 &= m_{45}^2 = m_{10'}^2 = m_{16_H}^2 = m_{126}^2 = m_0^2, & (4.5)\\ A^U &= a_0 Y^U, & (4.6)\\ A^M &= a_0 Y^M. & (4.7)\end{aligned}$$

The matrix Y^M, after breaking to $SU(5)$, is responsible for Majorana masses for the right-handed neutrinos once the $\overline{\mathbf{16}}_H$ acquires a vacuum expectation value breaking $SO(10)$ to $SU(5)$.

Y^U and $\tilde{Y}^D$ give mass to up- and down-type quarks, respectively, at the weak scale. The symmetry factors are such that these, as well as the neutrino masses, have their conventional normalization.

Because of the reducible transformation of $\mathbf{H}'$, the matrix $\tilde{Y}^D$ can have arbitrary flavor symmetry. On the other hand, Y^U and Y^M are complex symmetric on group theoretical grounds (see appendix A) and can be diagonalized by a unitary transformation in generation space.

The crucial assumption on the flavor structure in the Chang-Masiero-Murayama model is that Y^U and Y^M can simultaneously be made diagonal (and hierarchical) due to appropriate flavor symmetries. The only large Yukawa couplings contributing to the renormalization of the theory are y_t and possibly the third-generation eigenvalue y_m in Y^M .

4.2 Symmetry breaking chain

The symmetry breaking down to the MSSM is assumed to take place via $SU(5)$ as an intermediate symmetry and in such a fashion that Y^U and $\tilde{Y}^D$ give Dirac masses to the up-type-quark and neutrino supermultiplets and to the down-type-quark and charged lepton fields, respectively. $SO(10)$ is broken by vevs for the $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ (or the $\mathbf{126}_H$ and $\overline{\mathbf{126}}_H$) to $SU(5)$ at the scale

$$M_{10} \approx 10^{17}\text{GeV}, \tag{4.8}$$

which gives reasonable values for low-energy parameters if the largest eigenvalues in ${Y^U}^\dagger Y^U$, $\tilde{Y}^{D\dagger}\tilde{Y}^D$, and ${Y^M}^\dagger Y^M$ are of order one and $\tan\beta$ is not too large. (This is for case **NR**, in the renormalizable model the largest eigenvalue in Y^M must be smaller.) This step also breaks the $U(1)_X$ factor. The $SO(10)$ multiplets decompose as

$$\begin{aligned}
\mathbf{10}_H &= (*, \mathbf{5}_H) = (*, (\mathbf{3}_H, H_u)), && (4.9)\\
\mathbf{10}'_H &= (\overline{\mathbf{5}}_{H'}, *) = ((\overline{\mathbf{3}}_H, H_d), *), && (4.10)\\
\mathbf{16}_i &= (\Psi_i, \Phi_i, N_i) = ((Q_i, U_i^c, E_i^c), (D_i^c, L_i), N_i). && (4.11)
\end{aligned}$$

Here the asterisks denote fields assumed to acquire masses of order M_{10}. The further decomposition into MSSM multiplets has already been indicated. Ψ, Φ, and N transform as $\mathbf{10}$, $\overline{\mathbf{5}}$, and $\mathbf{1}$ under $SU(5)$, respectively. $\overline{\mathbf{3}}_H$ and $\mathbf{3}_H$ are color-triplet Higgses which must be made heavy through the details of the symmetry breaking and nonrenormalizable couplings; this is the usual doublet-triplet splitting problem besetting all four-dimensional GUT theories with a simple gauge group that was mentioned in section 3.2.4.

The effective $SU(5)$ superpotential, for the **NR** variant and up to nonrenormalizable terms, is given by

$$W_5 = \frac{1}{2}\Psi^T Y^U \Psi\, \mathbf{5}_H + \Psi^T Y^D \Phi\, \overline{\mathbf{5}}_{H'} + \Phi^T Y^\nu N\, \mathbf{5}_H + \frac{1}{2}\frac{1}{M_{\text{Pl}}} N^T Y^M N\, \mathbf{1}_H^2. \tag{4.12}$$

Y^D is related to $\tilde{Y}^D$ via

$$Y^D = \frac{v_{45}}{M_{\text{Pl}}}\tilde{Y}^D, \tag{4.13}$$

where v_{45} is the vacuum expectation value (also $SO(10)$-breaking) of the $\mathbf{45}_H$. $\mathbf{1}_H$ is the $SU(5)$ singlet in the $\overline{\mathbf{16}}_H$; for the **R** scenario a single power of the singlet in $\overline{\mathbf{126}}_H$ appears instead and the suppressing factor of $1/M_{\text{Pl}}$ is absent.

The $SU(5)$ singlets N_i can be identified with right-handed neutrino superfields, which obtain a Majorana mass somewhat below M_{10} from the vev of the Higgs singlet. We only discuss the **NR** case, where a mass term

$$\frac{1}{2}N^T M_N N, \qquad M_N = \frac{v_{16}^2}{M_{\rm Pl}} Y^M \tag{4.14}$$

appears. At the scale M_{10}, the following relation holds (up to threhold effects and corrections from nonrenormalizable terms):

$$Y^U = Y^\nu. \tag{4.15}$$

It becomes invalid below that scale due to the different renormalization group running of the Yukawa couplings. Likewise, the soft masses of the scalars in Ψ and Φ are equal at M_{10} for each generations. At a scale close to their Majorana masses, the right-handed neutrinos should be integrated out, giving a dimension-five operator leading, after $SU(5)$ and electroweak symmetry breakdown, to a seesaw Majorana mass matrix for the left-handed neutrinos.

The effective $SU(5)$ theory is valid down to the scale

$$M_{\rm GUT} \approx 10^{16}\text{GeV}, \tag{4.16}$$

where it is broken further to the MSSM. Again neglecting effects from threshold corrections and possible additional nonrenormalizable operators, the MSSM superpotential relevant below the GUT scale becomes

$$W_{SM} = Q^T Y^U U^c H_u + L^T Y^\nu \nu^c H_u + Q^T Y^D D^c H_d + E^c Y^{E^T} L H_d + \frac{1}{2} N^T M_N N. \tag{4.17}$$

We do not speculate on the origin of the μ term in eq. (2.49) and treat it as a free parameter. At $M_{\rm GUT}$ the $SU(5)$ relation

$$Y^D = Y^{E^T} \tag{4.18}$$

holds; like (4.15) it is valid up to corrections from threshold effects and non-renormalizable operators and becomes invalid as one scales down from the GUT breaking scale, and one also has

$$m^2_{\tilde{d}ij} = m^2_{\tilde{l}\,ji}, \tag{4.19}$$

$$m^2_{\tilde{u}ij} = m^2_{\tilde{q}ij} \ = \ m^2_{\tilde{e}ij}. \tag{4.20}$$

The further evolution to the weak scale is the usual MSSM one discussed qualitatively in chapter 2 and quantitatively in chapter 5.

4.3 Radiative corrections and flavor violation

Before we study the weak-scale flavor violation of the CMM model, we consider the phases of the important parameters above the GUT scale that enter the renormalization group equations. These are the soft mass m_0^2, the universal $A-$parameter a_0, the gaugino mass $m_{\tilde{g}}$ and the large Yukawa couplings y_t and, if applicable, y_m. Of these, all but m_0^2 can a priori be complex. Performing an R transformation as in section 2.2.5, the gaugino mass can be made real, at the cost of changing the phase of a_0. y_t and y_m can then be made real by respective redefinitions of $\mathbf{10_H}$ and $\overline{\mathbf{126}}_H$, leaving a_0 as the only complex parameter. This phase must be physical, as was shown in section 2.2.5. At the symmetry breaking scales, the parameters are continuous at leading order, so that no new phases are introduced. In particular, the large Yukawa y_{ν_3} is real, and there will be no complex gaugino masses within the MSSM.

We now specialize to a basis for the $SO(10)$ spinors $\mathbf{16}$ where Y^U and Y^M are diagonal. We will call this the (U) basis. Then, defining the $SU(5)$ and SM fields to be components of these basis spinors according to eq. (4.11), also the matrix Y^ν and the left-handed neutrino Majorana mass operator,

$$Y^\nu M_N^{-1} Y^\nu, \tag{4.21}$$

have a diagonal flavor structure.

The *only* nondiagonal structures now are the matrices Y^D and Y^E, i.e., all flavor mixing is contained in these matrices, parameterizable as in eqs. (2.9) and (2.10).

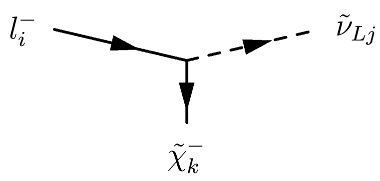
Radiative corrections due to the large Yukawa coupling in Y^U (and possibly in Y^M) above the GUT scale affect the soft masses of the third generation sfermions. As we have argued in sections 3.2 and 2.3.4, the soft masses become nonuniversal but stay diagonal in the (U) basis, which therefore is the universal mass eigenbasis for all sfermions, if, as required above, Y^D is assumed small. We are interested in the right-handed sdowns and the doublet sleptons, for which one has

$$m_{\tilde{d}}^{2,(U)} = \begin{pmatrix} m_{\tilde{d}_R}^2 & 0 & 0 \\ 0 & m_{\tilde{d}_R}^2 & 0 \\ 0 & 0 & m_{\tilde{d}_R}^2 - \Delta_{\tilde{d}} \end{pmatrix}, \tag{4.22}$$

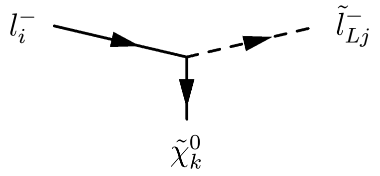
$$m_{\tilde{l}}^{2,(U)} = \begin{pmatrix} m_{\tilde{l}_1}^2 & 0 & 0 \\ 0 & m_{\tilde{l}_1}^2 & 0 \\ 0 & 0 & m_{\tilde{l}_1}^2 - \Delta_{\tilde{l}} \end{pmatrix}. \tag{4.23}$$

The splittings $\Delta_{\tilde{d}}$ and $\Delta_{\tilde{l}}$ denote the difference in radiative corrections between the third and first generations from the Planck scale to the weak scale.

Below the electroweak symmetry breaking scale, however, the preferred bases for down-type quarks and charged leptons are (respectively) those in which Y^D



$$-i\left(\frac{e}{\sin\theta_W}Z_+^{1i}P_L + y_{l_i^-}Z_-^{2i^*}P_R\right)U_{ij} \quad (4.27)$$



$$i\left(\left[\frac{e}{\sqrt{2}\cos\theta_W}Z_N^{1k} + \frac{e}{\sqrt{2}\sin\theta_W}Z_N^{2k}\right]P_L\right.$$

$$\left.+y_{l_i^-}Z_N^{3k^*}P_R\right)U_{ij} \quad (4.28)$$

Figure 4.1: The flavor-changing charged-lepton-sneutrino-chargino and charged-lepton-charged-slepton-neutralino couplings. (All sfermions left chiral.)

and Y^E, and with them the fermion mass matrices, are diagonal. According to (2.9) and (2.10), and considering that the neutrino masses are also diagonal in the (U) basis, the two matrices Y^D and Y^E are diagonalized by the biunitary transformations

$$Y^E = U^\dagger \hat{Y}^E U_E, \quad (4.24)$$

$$Y^D = K^* \hat{Y}^D U_D, \quad (4.25)$$

where K and U are the CKM and MNS matrices. Thus the right-handed down-type squark and doublet slepton mass eigenstates are related to the interaction partners of the right-handed down-type quarks and left-handed charged leptons by

$$\begin{pmatrix} \tilde{d}_R^* \\ \tilde{s}_R^* \\ \tilde{b}_R^* \end{pmatrix} = U_D \begin{pmatrix} \tilde{d}_{R_1}^* \\ \tilde{d}_{R_2}^* \\ \tilde{d}_{R_3}^* \end{pmatrix} \equiv U_D \begin{pmatrix} \tilde{d}_1^{c,(U)} \\ \tilde{d}_2^{c,(U)} \\ \tilde{d}_3^{c,(U)} \end{pmatrix}, \qquad \begin{pmatrix} \tilde{l}_e \\ \tilde{l}_\mu \\ \tilde{l}_\tau \end{pmatrix} = U^* \begin{pmatrix} \tilde{l}_1 \\ \tilde{l}_2 \\ \tilde{l}_3 \end{pmatrix} \equiv U^* \begin{pmatrix} \tilde{l}_1^{(U)} \\ \tilde{l}_2^{(U)} \\ \tilde{l}_3^{(U)} \end{pmatrix}. \quad (4.26)$$

Consequently, there are flavor-changing chargino and neutralino couplings. For the lepton sector, they are governed by MNS elements, and the Feynman rules are shown in fig. 4.1. The definition of the chargino and neutralino mixing matrices Z_+, Z_-, and Z_N is that of ref. [52]. These flavor-violating vertices suggest that there may be large contributions to the decays $\tau \to \mu\gamma$ and $\mu \to e\gamma$. Due to the unitarity of the MNS matrix, these amplitudes are, respectively, proportional to the products of matrix elements $U_{\mu 3}^* U_{\tau 3}$ and $U_{e3}^* U_{\mu 3}$. We emphasize that there is generally a large effect in the former decay but not necessarily in the latter because there is only an upper bound on the element U_{e3}.

This type of flavor-changing vertex also arises in the MSSM with righthanded neutrinos and in supersymmetric $SU(5)$ grand unification with gauge singlets; consequently it has been studied by a number of authors after experimental evidence for neutrino masses and mixings became available [71, 24, 72, 73, 74, 75].

However, the effects are generally smaller than in $SO(10)$ and also depend on the structure of the Majorana mass matrix for the singlets.

The new effect in the CMM model is that there is another vertex predicted to have large flavor- and possibly CP violation. If (4.18) would hold exactly, the matrices U and U_D would be related by

$$U_D = PU^* \tag{4.29}$$

where P is a diagonal matrix of phase factors; these serve to fix U in its six-parameter standard parameterization. However, the bad GUT Yukawa relations for the first two generations throw this assumption into question. Due to the approximate third-generation bottom-tau unification, however, one still expects the third rows of U_D and U^* to be roughly proportional, i.e., the physical bottom and tau to be members of GUT multiplet. Then we can define a unitary matrix M satisfying $U_D = MU^*$, having the form

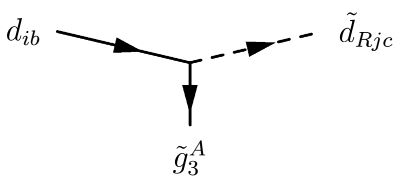
$$M = \begin{pmatrix} M_{11} & M_{12} & 0 \\ M_{21} & M_{22} & 0 \\ 0 & 0 & e^{i\phi} \end{pmatrix}, \tag{4.30}$$

where ϕ is an unknown phase. Bounds from reactor neutrino oszillations [76, 77, 78, 79] imply that $|U_{e3}| < 0.17$ at 90% C.L. while atmospheric neutrino observations suggest [79] that $|U_{\mu 3}| \approx |U_{\tau 3}| \approx 1/\sqrt{2}$. Thus, appoximately,

$$|U_{D13}| = \frac{1}{\sqrt{2}}|M_{12}|, \qquad |U_{D23}| = \frac{1}{\sqrt{2}}|M_{22}|, \qquad |U_{D33}| = \frac{1}{\sqrt{2}}. \tag{4.31}$$

The matrix elements U_{Dij} enter the down-quark-down-squark-gluino vertex for a right-handed down quark mass eigenstate d_{Ri} and a right-handed down squark mass eigenstate $\tilde{d}_{Rj}$ (fig. 4.2). These vertices give potentially large contributions to hadronic $\Delta F = 1$ and $\Delta F = 2$ processes.

A nonvanishing M_{12} will lead to potentially huge contributions to the $B_d - \bar{B}_d$ mixing and even to ΔM_K and ϵ_K. This does not seem likely given the good agreement between their standard model predictions as exhibited in CKM fits. Then either the SUSY particles are very heavy, so that they decouple and their flavor-changing effects are suppressed, or M_{12} must be small (and M_{22} close to one in magnitude). Since decoupling the sparticles is against our wish to solve the hierarchy problem, in the remainder of this dissertation we take the matrix M to be diagonal. This, in turn, means that equation (4.29) remains valid.



$i\sqrt{2}g_3 T^A_{cb} U_{Dij} P_R$

Figure 4.2: The flavor-changing quark-squark-gluino coupling.

Chapter 5

Renormalization of the CMM model

In this chapter we discuss the renormalization group equations for the CMM model and their solutions for the parameters needed in this dissertation. After reviewing the situation in the MSSM and deriving the evolution of the relevant parameters, we derive the renormalization-group equations of the most general renormalizable $SO(10)$-invariant superpotential for the matter fields and of the soft SUSY-breaking terms, which to our knowledge has not been done before in the literature. The equations are then solved analytically for the CMM model and the results compared to the literature. In parallel, we review the $SU(5)$ evolution. After the treatment of additional sources of nonuniversality due to the reduction of the gauge group rank and the GUT threshold, a recipe for obtaining weak-scale parameters from a number of weak-scale inputs concludes the chapter.

5.1 MSSM renormalization

5.1.1 SUSY threshold corrections at the electroweak scale

Our input parameters are physical particle masses and gauge couplings, with the latter defined in the $\overline{MS}$ scheme with five active quark flavors. We convert $\tilde{\alpha}_s(M_Z)$ to $\tilde{\alpha}_3^{\overline{DR}}(M_Z) \equiv (g_3^{\overline{DR}}(M_Z))^2/(4\pi)^2$ and $m_t^{\rm pole}$ to $\tilde{y}_t^{\overline{DR}}(M_Z)$, both corresponding to the $\overline{DR}$ scheme suitable for softly broken supersymmetry. By a tilde over any coupling constant, we denote here and in the following a factor of $1/(4\pi)$ to simplify many expressions. We take the full MSSM particle spectrum to be present (i.e. dynamical) above the conversion scale M_Z. The conversion is achieved via the formulas [80, 69]

$$\tilde{\alpha}_3(M_Z) = \frac{\tilde{\alpha}_s(M_Z)}{1 - \Delta\alpha_s}, \tag{5.1}$$

$$\Delta\alpha_s = 2\tilde{\alpha}_s(M_Z)\left[\frac{1}{2} - \frac{2}{3}\log\left(\frac{m_t^{\rm pole}}{M_Z}\right) - 2\log\left(\frac{m_{\tilde{g}}}{M_Z}\right) - \frac{1}{6}\sum_{\tilde{q}}\log\left(\frac{m_{\tilde{q}}}{M_Z}\right)\right], \quad (5.2)$$

$$\tilde{y}_t^{\overline{DR}}(M_Z) = \frac{1}{4\pi v\sin\beta}\left(\frac{m_t^{\rm pole}}{1+\frac{\Delta m_t}{m_t}}\right), \quad (5.3)$$

$$\begin{aligned}\frac{\Delta m_t}{m_t} &= \tilde{\alpha}_3(M_Z)\left[4\ln\left(\frac{M_Z^2}{m_t^{\rm pole 2}}\right) + \frac{20}{3} - \frac{4}{3}\Big(B_1(0,m_{\tilde{g}},m_{\tilde{t}_1}) + B_1(0,m_{\tilde{g}},m_{\tilde{t}_2})\right.\\ &\left.\quad - \sin(2\theta_t)\frac{m_{\tilde{g}}}{m_t^{\rm pole}}(B_0(0,m_{\tilde{g}},m_{\tilde{t}_1}) - B_0(0,m_{\tilde{g}},m_{\tilde{t}_2}))\Big)\right] \quad (5.4)\end{aligned}$$

Eq. (5.4) is valid up to (small) electroweak corrections. The functions B_0 and B_1 are defined in [80]. We neglect weak-scale threshold corrections to g_1, g_2. That is,

$$\tilde{\alpha}_2(M_Z) = \frac{\alpha_e}{s_W^2}, \quad (5.5)$$

$$\tilde{\alpha}_1(M_Z) = \frac{5}{3}\frac{\alpha_e}{c_W^2}. \quad (5.6)$$

The soft term inputs are also taken at M_Z in the $\overline{DR}$ scheme. In the following, all masses and couplings are running $\overline{DR}$ parameters, even when the labels "$\overline{DR}$" are omitted, unless stated otherwise.

5.1.2 Renormalization-group equations

The renormalization group equations for the MSSM including arbitrary soft-breaking parameters are known up to two loops and are collected for the MSSM for example in [81].

For small $\tan\beta$, the top Yukawa coupling becomes close to its infrared quasi-fixed point [82, 83, 84], and consequently the ratio y_t/g can approach or exceed its fixed point values in SU(5) and SO(10) discussed in section 5.2. This is important because it may lead to a qualitative change of the evolution above $M_{\rm GUT}$. Therefore, we use two-loop equations for the dimensionless coulings g_1, g_2, g_3, y_t. This is also important for reasons of scheme and scale independence, because we also take threshold corrections into account and these are next-to-leading order effects.

In view of the large uncertainties in supersymmetric parameters, we will work with the one-loop equations for the soft parameters that afford simple and numerically unproblematic analytical solutions.

For convenience, we define the variable $t = \ln \mu$. Then the dimensionless couplings satisfy [81]

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\alpha}_1 &= 2\tilde{\alpha}_1^2 \left(\frac{33}{5} + \left(\frac{199}{25}\tilde{\alpha}_1 + \frac{27}{5}\tilde{\alpha}_2 + \frac{88}{5}\tilde{\alpha}_3 \right) - \frac{26}{3}\tilde{y}_t^2 \right), && (5.7)\\
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\alpha}_2 &= 2\tilde{\alpha}_2^2 \left(1 + \left(\frac{9}{5}\tilde{\alpha}_1 + 25\tilde{\alpha}_2 + 24\tilde{\alpha}_3 \right) - 6\tilde{y}_t^2 \right), && (5.8)\\
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\alpha}_3 &= 2\tilde{\alpha}_3^2 \left(-3 + \left(\frac{11}{5}\tilde{\alpha}_1 + 9\tilde{\alpha}_2 + 14\tilde{\alpha}_3 \right) - 4\tilde{y}_t^2 \right), && (5.9)\\
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{y}_t &= \tilde{y}_t \Bigg(-\frac{16}{3}\tilde{\alpha}_3 - 3\tilde{\alpha}_2 - \frac{13}{15}\tilde{\alpha}_1 + 6\tilde{y}_t^2 \\
&\quad + \left(-\frac{16}{9}\tilde{\alpha}_3^2 + 8\tilde{\alpha}_3\tilde{\alpha}_2 + \frac{136}{45}\tilde{\alpha}_3\tilde{\alpha}_1 + \frac{15}{2}\tilde{\alpha}_2^2 + \tilde{\alpha}_2\tilde{\alpha}_1 + \frac{2743}{450}\tilde{\alpha}_1^2 \right) \\
&\quad + \tilde{y}_t^2 \left(16\tilde{\alpha}_3 + 6\tilde{\alpha}_2 + \frac{6}{5}\tilde{\alpha}_1 \right) - 22(\tilde{y}_t^2)^2 \Bigg). && (5.10)
\end{aligned}
$$

For the dimensionful parameters, we first note that

$$m_{\tilde{g}_i} \propto \alpha_i \tag{5.11}$$

and that gaugino masses unify up to NLO threshold corrections at M_{GUT}.

Let us consider the A-term matrices. Here we only study A^D, because the vacuum stability bound (2.56) can be used to constrain the Planck-scale universal parameter a_0, as we will see. This is not possible for A^U, which has a more involved RG evolution.

We neglect all Yukawa couplings except y_t and work in a basis where Y^U is diagonal. Due to the fact that there is only wave-function renormalization in supersymmetric theories, the renormalization group equations for the matrices Y^D and A^D have the schematic form (we do not need them in more detail)

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}Y^D, A^D &= \text{flavor-universal terms} \times Y^D + \text{flavor-universal terms} \times A^D \\
&\quad + y_t^2 \begin{pmatrix} 0 & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix}. && (5.12)
\end{aligned}
$$

This is true as long as the Higgs multiplets appearing in the Y^D term have no large Yukawa coupling that might contribute to their own wave-function renormalization. This condition is satisfied both in the MSSM and in the CMM model above the GUT scale (with universal soft terms at M_{Pl}). As a consequence, the evolution of the upper-left 2×2 submatrices is universal. This in turn implies that, if $A^D \propto Y^D$ at some scale, the same will be true at any scale up to a term of the form of the matrix in eq. (5.12). Now considering the form (2.9) of Y^D in this basis together with (4.29) and the fact that $U_{e3} \approx 0$, the matrix A^D in the basis where Y^D is diagonal will have the form

$$A^D\Big|_D = \text{diagonal} + f(y_t^2)\begin{pmatrix} 0 & * & * \\ 0 & * & * \\ * & * & * \end{pmatrix}. \tag{5.13}$$

But this means that to find the (11) element in the basis where Y^D is diagonal the whole y_t-dependent term can be dropped. (This works as well in the basis where Y^U is diagonal, but that basis is less useful for applying the vacuum stability bound.) Defining now a_1^D as in (2.54), the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}a_1^D = +2(D_3^D\tilde{\alpha}_3^2 + D_2^D\tilde{\alpha}_2^2 + D_1^D\tilde{\alpha}_1^2)\frac{\bar{m}_{\tilde{g}}}{\bar{\tilde{\alpha}}} \tag{5.14}$$

holds, where

$$D_3^D = 16/3, \qquad D_2^D = 3, \qquad D_1^D = 7/15, \tag{5.15}$$

and $\bar{m}_{\tilde{g}}/\bar{\tilde{\alpha}}$ is the RG-invariant and universal ratio of gaugino mass and coupling constant. It can be evaluated from any pair of corresponding gaugino mass and coupling constant renormalized at an arbitrary scale. For definiteness, we will evaluate it as $m_{\tilde{g}3}(M_Z)/\tilde{\alpha}_3(M_Z)$, which is useful because in the numerical analysis $m_{\tilde{g}3}(M_Z)$ will be chosen one of the input parameters.

A relation similar to (5.14) holds above M_{GUT}, as will be seen shortly. Of course the simple form of (5.14) depends crucially on neglecting U_{e3}; if this could not be done, it would be difficult to find a simple correlation between a_0 and weak-scale parameters.

The evolution of a_1^D is purely inhomogeneous and can be integrated, leading to a mere shift. Because of gaugino mass unification, this shift is proportional to any one of the gaugino masses and independent of other SUSY parameters. In particular, the relation between a_1^D at the weak and GUT scales is calculable in terms of weak-scale inputs.

Now let us turn to the soft masses. Again neglecting small Yukawas, the soft-mass evolution is very simple for the right-handed down-type squarks and the sleptons of all three generations as well as for the up-type and doublet (left-chiral) squarks of the first two generations, which is all we need in this thesis. For the (mass)^2 eigenvalues, one has

$$\frac{\mathrm{d}}{\mathrm{d}t}m^2_{\tilde{u}_{R_j}} = \left(-\frac{32}{3}\tilde{\alpha}_3^3 - \frac{32}{15}\tilde{\alpha}_1^3\right)\frac{m^2_{\tilde{g}3}(M_Z)}{\tilde{\alpha}_3^2(M_Z)} - \frac{4}{5}\tilde{\alpha}_1 S, \tag{5.16}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}m^2_{\tilde{d}_{R_i}} = \left(-\frac{32}{3}\tilde{\alpha}_3^3 - \frac{8}{15}\tilde{\alpha}_1^3\right)\frac{m^2_{\tilde{g}3}(M_Z)}{\tilde{\alpha}_3^2(M_Z)} + \frac{2}{5}\tilde{\alpha}_1 S, \tag{5.17}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}m^2_{\tilde{q}_j} = \left(-\frac{32}{3}\tilde{\alpha}_3^3 - 6\tilde{\alpha}_2^3 - \frac{2}{15}\tilde{\alpha}_1^3\right)\frac{m^2_{\tilde{g}3}(M_Z)}{\tilde{\alpha}_3^2(M_Z)} + \frac{1}{5}\tilde{\alpha}_1 S, \tag{5.18}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}m^2_{\tilde{e}_i} = \left(-\frac{24}{5}\tilde{\alpha}_1^3\right)\frac{m^2_{\tilde{g}3}(M_Z)}{\tilde{\alpha}_3^2(M_Z)} + \frac{6}{5}\tilde{\alpha}_1 S, \tag{5.19}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} m^2_{\tilde{l}_i} = \left(-6\tilde{\alpha}_2^3 - \frac{6}{5}\tilde{\alpha}_1^3\right) \frac{m^2_{\tilde{g}_3}(M_Z)}{\tilde{\alpha}_3^2(M_Z)} - \frac{3}{5}\tilde{\alpha}_1 S, \tag{5.20}$$

where $i = 1, 2, 3$, $j = 1, 2$. The quantity S is defined in eq. (4.27) of [81]. For us, it is only important that it is equal to $m^2_{h_u} - m^2_{h_d}$ at the GUT scale and satisfies

$$\frac{\mathrm{d}}{\mathrm{d}t} S = \frac{66}{5}\tilde{\alpha}_1 S. \tag{5.21}$$

Comparing with (5.7), $S/\tilde{\alpha}_1 = \text{inv.}$ follows. Thus in (5.16)–(5.20) one can replace

$$\tilde{\alpha}_1 S \longrightarrow \tilde{\alpha}_1^2 \frac{S(M_{\mathrm{GUT}})}{\tilde{\alpha}(M_{\mathrm{GUT}})}. \tag{5.22}$$

Again the equations (5.16)– (5.20) can be integrated and lead to shifts linear in $m^2_{\tilde{g}_3}(M_Z)$ and $S(M_{\mathrm{GUT}})$.

5.1.3 MSSM evolution

Integrating the right-hand sides of (5.14) and (5.16)–(5.20) gives the relations

$$m^2_{\tilde{u}_{R_1}}(M_Z) = m^2_{\tilde{u}_{R_1}}(M_{\mathrm{GUT}}) + (\delta_3 + w_u^2\delta_1) m^2_{\tilde{g}_3}(M_Z) - w_u \xi S(M_{\mathrm{GUT}}), \tag{5.23}$$

$$m^2_{\tilde{q}_1}(M_Z) = m^2_{\tilde{q}_1}(M_{\mathrm{GUT}}) + (\delta_3 + \delta_2 + w_q^2\delta_1) m^2_{\tilde{g}_3}(M_Z) - w_q \xi S(M_{\mathrm{GUT}}), \tag{5.24}$$

$$m^2_{\tilde{d}_{R_i}}(M_Z) = m^2_{\tilde{d}_{R_i}}(M_{\mathrm{GUT}}) + (\delta_3 + w_d^2\delta_1) m^2_{\tilde{g}_3}(M_Z) - w_d \xi S(M_{\mathrm{GUT}}), \tag{5.25}$$

$$m^2_{\tilde{l}_i}(M_Z) = m^2_{\tilde{l}_i}(M_{\mathrm{GUT}}) + (\delta_2 + w_l^2\delta_1) m^2_{\tilde{g}_3}(M_Z) - w_l \xi S(M_{\mathrm{GUT}}), \tag{5.26}$$

$$m^2_{\tilde{e}_{R_i}}(M_W) = m^2_{\tilde{e}_{R_i}}(M_{\mathrm{GUT}}) + w_e^2\delta_1 m^2_{\tilde{g}_3}(M_Z) - w_e \xi S(M_{\mathrm{GUT}}), \tag{5.27}$$

$$a_1^D(M_Z) = a_1^D(M_{\mathrm{GUT}}) - (\delta_3' + \delta_2' + \frac{1}{2}(w_d^2 + w_q^2 + w_{h_d}^2)\delta_1') m_{\tilde{g}_3}(M_Z), \tag{5.28}$$

for $i = 1, 2, 3$. w_f for $f = u, q, d, l, e, h_d$ denotes the hypercharges of the standard model fields given in table 2.1. Furthermore, at the GUT scale,

$$m^2_{\tilde{u}_{R_1}} = m^2_{\tilde{q}_1} = m^2_{\tilde{e}_{R_i}} = m^2_{\psi_1}, \qquad m^2_{\tilde{d}_{R_i}} = m^2_{\tilde{l}_i} = m^2_{\phi_1}. \tag{5.29}$$

The other symbols have the following meaning:

$$\delta_3 = \frac{32}{3} \frac{1}{\tilde{\alpha}_3^2(M_Z)} \int_{t_Z}^{t_{\mathrm{GUT}}} \tilde{\alpha}_3^3(t)\mathrm{d}t = 0.78, \tag{5.30}$$

$$\delta_2 = 6 \frac{1}{\tilde{\alpha}_3^2(M_Z)} \int_{t_Z}^{t_{\mathrm{GUT}}} \tilde{\alpha}_2^3(t)\mathrm{d}t = 0.059, \tag{5.31}$$

$$\delta_1 = \frac{24}{5} \frac{1}{\tilde{\alpha}_3^2(M_Z)} \int_{t_Z}^{t_{\mathrm{GUT}}} \tilde{\alpha}_1^3(t)\mathrm{d}t = 0.011, \tag{5.32}$$

$$\xi = \frac{1}{\tilde{\alpha}_1(M_{\mathrm{GUT}})} \int_{t_Z}^{t_{\mathrm{GUT}}} \tilde{\alpha}_1^2(t)\mathrm{d}t = 0.027, \tag{5.33}$$

Table 5.1: Sample set of weak-scale parameters

m_t^{pole}	174 GeV
M_Z	91.2 GeV
M_{GUT}	10^{16} GeV
M_{Pl}	10^{19} GeV
$\alpha_s(M_Z)$	0.121
$\alpha_2(M_Z)$	0.034
$\alpha_1(M_Z)$	0.017
$\tan\beta$	3
$m_{\tilde{q}}$	300 GeV
$m_{\tilde{t}_1}$	200 GeV
$m_{\tilde{t}_2}$	300 GeV
$\theta_{\tilde{t}}$	$\pi/6$
$m_{\tilde{g}}$	400 GeV

$$\delta_3' = \frac{32}{3}\frac{1}{\tilde{\alpha}_3(M_Z)}\int_{t_Z}^{t_{\text{GUT}}}\tilde{\alpha}_3^2(t)\mathrm{d}t = 1.16, \tag{5.34}$$

$$\delta_2' = 6\frac{1}{\tilde{\alpha}_2(M_Z)}\int_{t_Z}^{t_{\text{GUT}}}\tilde{\alpha}_3^2(t)\mathrm{d}t = 0.186, \tag{5.35}$$

$$\delta_1' = \frac{1}{\tilde{\alpha}_3(M_Z)}\int_{t_Z}^{t_{\text{GUT}}}\tilde{\alpha}_1^2(t)\mathrm{d}t = 0.075. \tag{5.36}$$

Here we have defined $t_Z = \ln(M_Z)$, $t_{\text{GUT}} = \ln(M_{\text{GUT}})$. To get an impression of the relative importance of the terms, we show typical numerical values that correspond to the parameters of table 5.1.

We have not discussed the evolution of the soft parameters that are affected by the top Yukawa coupling, as we will not need them later. Their evolution is more involved but could be treated along the lines of the "bottom-up approach" [85], after correcting for nonuniversal soft terms at M_{GUT}.

The Higgs masses in particular depend on superpotential couplings above M_{GUT} that cannot be inferred from low-energy parameters, and we consider them free parameters. They are not needed in our analysis either; we assume that their values are such that no additional phenomenological constraints are violated.

5.2 GUT renormalization

5.2.1 Renormalization-group equations

Unlike in the MSSM case, the renormalization-group equations for a general SUSY $SO(10)$ theory are not known to the literature. There is one work [21] giving results for minimal SUSY $SO(10)$, which would apply to the CMM model if only the large Yukawa coupling to the $\overline{\mathbf{16}}_H$ is kept, as we do, but upon checking the computation we found disagreement. Because of this and possible general interest in the needed techniques, which are hard to find in the literature, we describe the calculation in some detail. Necessary group theory is collected in appendix A along with the full one-loop RGEs in matrix form.

For the case of $SU(5)$, [24] give sufficiently general results. We checked them, too, since this is possible with little effort, and find agreement.

5.2.1.1 Gauge coupling

It is well known that the evolution of the gauge coupling constant at one loop only depends on the representations of the gauge group present in the theory, according to the formula

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\alpha} = -2\left(3\,C_2(G) - \sum_R T(R)\right)\tilde{\alpha}^2 \equiv 2\beta_0\tilde{\alpha}^2 \tag{5.37}$$

with R labeling the irreducible representations of the non-gauge superfields appearing in the theory. $C_2(R)$ and $T(R)$ denote the quadratic Casimir invariant and the Dynkin index of a representation. Specializing to the CMM model, one obtains for the $SU(5)$ and $SO(10)$ theories:

$$\beta_0^{SU(5)} = -3, \tag{5.38}$$
$$\beta_0^{SO(10),NR} = -4, \tag{5.39}$$
$$\beta_0^{SO(10),R} = 53. \tag{5.40}$$

The superscripts “NR” and “R” pertain to the variants of the model with non-renormalizable and renormalizable neutrino Majorana masses, respectively.

5.2.1.2 $SO(10)$ case

The renormalization of the $SO(10)$ superpotential and soft-breaking terms requires first to find the group-theoretic structure of the same, which has always been suppressed so far. This is somewhat more complicated than in the Standard model due to the use of spinor representations for the matter fields. In turn, this implies similarities to the Lorentz structures appearing in relativistic theories of fermions. As explained in appendix A, irreducible $SO(10)$ spinors are analogous to chiral fermions, and their bilinears can (respectively) be coupled to 10-, 120- and 126-dimensional tensor representations via antisymmetrized products of 1, 3, or 5 $SO(10)$ gamma matrices and a “charge conjugation matrix”.

Following the notation introduced in said appendix, let us take m, n, r, s, t to be fundamental $SO(10)$ indices and enforce the constraint $m < n < \cdots$ in sums. Furthermore, in the case of the 126-dimensional Higgs, we only sum over one half-set of possible five-indices. Let ψ, h^R denote the scalar components of the corresponding $SO(10)$-spinor and $SO(10)$-tensor superfields Ψ, H^R. We identify by $\tilde{g}$ the gaugino component of the gauge superfield and allow several generations of spinors, distinguished by indices i and j. Then the most general $SO(10)$-symmetric renormalizable superpotential along with the corresponding soft terms reads

$$\begin{aligned}\mathcal{W} &= \frac{1}{2}Y^{10}_{ij}\ \Psi_i^T\left(C\gamma_m P_S^-\right)\Psi_j\ H^{10}_m + \frac{1}{2}Y^{120}_{ij}\ \Psi_i^T\left(C\gamma_m\gamma_n\gamma_r P_S^-\right)\Psi_j\ H^{120}_{mnr}\\ &\quad+\frac{1}{2}Y^{126}_{ij}\ \Psi_i^T\left(C\gamma_m\gamma_n\gamma_r\gamma_s\gamma_t P_S^-\right)\Psi_j\ H^{126}_{mnrst},\end{aligned} \tag{5.41}$$

$$\begin{aligned}\mathcal{L}_s &= -\Big\{\frac{1}{2}A^{10}_{ij}\psi_i^T\left(C\gamma_m P_S^-\right)\psi_j\ h^{10}_m + \frac{1}{2}A^{120}_{ij}\ \psi_i^T\left(C\gamma_m\gamma_n\gamma_r P_S^-\right)\psi_j\ h^{120}_{mnr}\\ &\quad+\frac{1}{2}A^{126}_{ij}\ \psi_i^T\left(C\gamma_m\gamma_n\gamma_r\gamma_s\gamma_t P_S^-\right)\psi_j\ h^{126}_{mnrst} + \psi_i^\dagger(m_\psi^2)_{ij}\psi_j - \frac{1}{2}m_{\tilde{g}}\tilde{g}\tilde{g}\Big\} + \text{h.c.}\\ &\quad-m_{10}^2 h^{10\dagger}h^{10} - m_{120}^2 h^{120\dagger}h^{120} - m_{126}^2 h^{126\dagger}h^{126}.\end{aligned} \tag{5.42}$$

In the bilinear terms, we have omitted the summed-over $SO(10)$ indices. Applying the general expressions of [46, 81] leads to two types of loop expressions, which are evaluated in the appendix,

$$\operatorname{tr}(C\Gamma_{i_1\cdots i_k}P_S^-(C\Gamma_{j_1\cdots j_l}P_S^-)^\dagger) = 16\,\delta_{i_1\cdots i_k,j_1\cdots j_l}\delta_{kl} \equiv L\,\delta_{i_1\cdots i_k,j_1\cdots j_l}\delta_{kl} \tag{5.43}$$

$$C\Gamma_{i_1\cdots i_k}P_S^-(C\Gamma_{i_1\cdots i_k}P_S^-)^\dagger = \left[\frac{1}{2}\right]\binom{10}{k}P_S^- = \dim([k])P_S^- \tag{5.44}$$

The factor 1/2 in the second equation applies to the case of the $\overline{\mathbf{126}}$. $[k]$ denotes an irreducible rank-k tensor representation ($k = 1, 3$, or 5). Compact formulas using matrix notation for the flavor structures now subsume the renormalization group equations:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\tilde{Y}^R &= \sum_S \dim(S)\left(\tilde{Y}^S\tilde{Y}^{S\dagger}\tilde{Y}^R + \tilde{Y}^R\tilde{Y}^{S\dagger}\tilde{Y}^S\right) + \frac{1}{2}L\operatorname{tr}(\tilde{Y}^R\tilde{Y}^{R\dagger})\tilde{Y}^R\\ &\quad-2\tilde{\alpha}\left(2C_2(\Psi)+C_2(R)\right)\tilde{Y}^R\end{aligned} \tag{5.45}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\tilde{A}^R &= \sum_S \dim(S)\left((\tilde{Y}^S\tilde{Y}^{S\dagger}\tilde{A}^R + \tilde{A}^R\tilde{Y}^{S\dagger}\tilde{Y}^S) + 2(\tilde{A}^S\tilde{Y}^{S\dagger}\tilde{Y}^R + \tilde{Y}^R\tilde{Y}^{S\dagger}\tilde{A}^S)\right)\\ &\quad+L\left(\frac{1}{2}\operatorname{tr}(\tilde{Y}^R\tilde{Y}^{R\dagger})\tilde{A}^R + \operatorname{tr}(\tilde{A}^R\tilde{Y}^{R\dagger})\tilde{Y}^R\right)\\ &\quad+2\tilde{\alpha}\left(2C_2(\Psi)+C_2(R)\right)(2m_{\tilde{g}}\tilde{Y}^R - \tilde{A}^R)\end{aligned} \tag{5.46}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}m_\psi^2 &= \sum_S \dim(S)\Big(\tilde{Y}^{S\dagger}\tilde{Y}^S m_\psi^2 + m_\psi^2\tilde{Y}^{S\dagger}\tilde{Y}^S + 2\tilde{Y}^{S\dagger}m_\psi^{2\,T}\tilde{Y}^S\\ &\qquad\qquad+2\tilde{Y}^{S\dagger}\tilde{Y}^S m_S^2 + 2\tilde{A}^{S\dagger}\tilde{A}^S\Big)\\ &\quad-8\alpha C_2(\Psi)|m_{\tilde{g}}|^2\mathbf{1}\end{aligned} \tag{5.47}$$

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}m_R^2 &= L_R\left(\mathrm{tr}(\tilde{Y}^R\tilde{Y}^{R\dagger})m_R^2 + 2\mathrm{tr}(\tilde{Y}^{R\dagger}m_\psi^{2\,T}\tilde{Y}^R) + \mathrm{tr}(\tilde{A}^R\tilde{A}^{R\dagger})\right) \\
&\quad -8\tilde{\alpha}C_2(R)|m_{\tilde{g}}|^2 \\
&\quad + \text{contributions from additional superfields} \qquad (5.48)
\end{aligned}
$$

As indicated in (5.48), the Higgs soft masses receive contributions from superpotential couplings to fields with no couplings to the spinors. These are highly model dependent; for the case of the CMM model discussed here, which at least contains one **45** and a **126** in addition to the fields appearing above, we assume that they are not large enough to significantly affect the evolution.

For the CMM model we need to also consider the evolution of the nonrenormalizable couplings Y^D and the corresponding A-term A^D. The first can be found by applying the one-loop wavefunction renormalization to the term

$$
Y_{ij}^D \mathbf{16}_i \mathbf{16}_j \mathbf{45_H} \mathbf{10'_H}. \qquad (5.49)
$$

Only the renormalization of the spinors is affected by large Yukawa couplings, therefore the y_t-dependent term renormalization group equation for Y^D has the form of the matrix in (5.12). Following the reasoning below that equation, we only need to calculate the y_t-independent contribution. It is straightforward to do this (for example by generalizing equations (2.7)–(2.9) in ref. [81]). One obtains

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{Y}^D &= -2\,\tilde{\alpha}\,(2\,C_2(16) + C_2(10) + C_2(45))\,\tilde{Y}^D + \mathcal{O}(y_t^2) \\
&= -\frac{95}{2}\tilde{\alpha}\tilde{Y}^D + \mathcal{O}(y_t^2). \qquad (5.50)
\end{aligned}
$$

The matrix A^D satisifies

$$
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{A}^D = -\frac{95}{2}\tilde{\alpha}\left(\tilde{A}^D - 2m_{\tilde{g}}\tilde{Y}^D\right) + \mathcal{O}(y_t^2). \qquad (5.51)
$$

This brings us into a position where we can give the complete set of equations for the relevant parameters in the range of $SO(10)$ applicability. For convenience, let us here and in the $SU(5)$ case to follow denote by A_t, A_m, A_{ν_3} the large eigenvalues of A^U, A^M, A^ν, as well as define y_d and A_d to be the (11) elements of the matrices Y^D and A^D in the basis where those matrices are diagonal. Neglecting higher orders of the small Yukawa couplings, but for now still allowing for a $\overline{\mathbf{126}}$, we obtain

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{y}_t &= (28\,\tilde{y}_t^2 + 252\,\tilde{y}_m^2 - \frac{63}{2}\,\tilde{\alpha})\tilde{y}_t, \qquad (5.52) \\
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{y}_m &= (260\,\tilde{y}_m^2 + 20\,\tilde{y}_t^2 - \frac{95}{2}\,\tilde{\alpha})\tilde{y}_m \qquad (5.53) \\
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{y}_d &= -\frac{95}{2}\,\tilde{\alpha}\,\tilde{y}_d, \qquad (5.54)
\end{aligned}
$$

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{A}_t &= (84\,\tilde{y}_t^2 + 252\,\tilde{y}_m^2 - \frac{63}{2}\,\tilde{\alpha})\tilde{A}_t + 504\,\tilde{y}_t\tilde{y}_m\tilde{A}_m + 63\,\tilde{\alpha}\tilde{y}_t m_{\tilde{g}} && (5.55)\\
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{A}_m &= (20\,\tilde{y}_t^2 + 780\,\tilde{y}_m^2 - \frac{95}{2}\,\tilde{\alpha})\tilde{A}_m + 40\,\tilde{y}_m\tilde{y}_t\tilde{A}_t + 95\,\tilde{\alpha}\tilde{y}_m m_{\tilde{g}} && (5.56)\\
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{A}_d &= -\frac{95}{2}\,\tilde{\alpha}\tilde{A}_d + 95\,\tilde{\alpha}\,\tilde{y}_d\,m_{\tilde{g}} && (5.57)\\
\frac{\mathrm{d}}{\mathrm{d}t}m_{16_3}^2 &= (40\,\tilde{y}_t^2 + 504\,\tilde{y}_m^2)m_{16_3}^2 + 20\,\tilde{y}_t^2 m_{10}^2 + 252\,\tilde{y}_m^2 m_{126}^2 \\
&\quad +20\,|\tilde{A}_t|^2 + 252\,|\tilde{A}_m|^2 - 45\tilde{\alpha}m_{\tilde{g}}^2 && (5.58)\\
\frac{\mathrm{d}}{\mathrm{d}t}m_{16_1}^2 &= -45\tilde{\alpha}m_{\tilde{g}}^2 && (5.59)\\
\frac{\mathrm{d}}{\mathrm{d}t}m_{10}^2 &= 16\,\tilde{y}_t^2 m_{10}^2 + 32\,\tilde{y}_t^2 m_{16_3}^2 + 16\,|\tilde{A}_t|^2 - 36\,\tilde{\alpha}m_{\tilde{g}}^2 && (5.60)\\
\frac{\mathrm{d}}{\mathrm{d}t}m_{126}^2 &= 16\,\tilde{y}_m^2 m_{126}^2 + 32\,\tilde{y}_m^2 m_{16_3}^2 + 16\,|\tilde{A}_m|^2 - 50\,\tilde{\alpha}m_{\tilde{g}}^2, && (5.61)
\end{aligned}$$

5.2.1.3 $SU(5)$ case

Again we start by writing the group-theoretical structure of the superpotential. Using a single lower greek letter for the fundamental representation and an upper greek letter for its conjugate, the fields transform have the tensor structures

$$\Psi_{\alpha_1\alpha_2}, \qquad \Phi^{\alpha}, \qquad \overline{\mathbf{5}}^{\alpha}_{H'}, \qquad \mathbf{5}_{H\alpha} \tag{5.62}$$

where Ψ is antisymmetric in its indices. With the help of the totally antisymmetric tensor $\epsilon^{\alpha\beta\gamma\delta\lambda}$ and requiring $\alpha_1 < \alpha_2$ for Ψ in sums, the superpotential reads (setting the small Y^D to zero and dropping the Majorana mass of the singlets):

$$W_5 = \frac{1}{2}\epsilon^{\alpha_1\alpha_2\beta_1\beta_2\gamma}\Psi^T_{\alpha_1\alpha_2}Y^U\Psi_{\beta_1\beta_2}\,\mathbf{5}_{H\gamma} + \Phi^{\alpha T}Y^{\nu}N\,\mathbf{5}_{H\alpha} \tag{5.63}$$

Introducing the notation

$$\begin{aligned}
I &= (\iota_1\iota_2) \qquad (\iota_1 < \iota_2), && (5.64)\\
E^{IK\lambda} &= \epsilon^{\iota_1\iota_2\kappa_1\kappa_2\lambda} \qquad (\iota_1 < \iota_2, \kappa_1 < \kappa_2), && (5.65)
\end{aligned}$$

one has

$$W_5 = \frac{1}{2}E^{AB\gamma}\Psi^T_A Y^U\Psi_B\,\mathbf{5}_{H\gamma} + \Phi^{\alpha T}Y^{\nu}N\,\mathbf{5}_{H\alpha} \tag{5.66}$$

and can compute

$$\begin{aligned}
E^{IK\lambda}E^{IK\mu} &= 6\delta^{\lambda\mu}, && (5.67)\\
E^{IK\lambda}E^{MK\lambda} &= 3\delta^{IM}. && (5.68)
\end{aligned}$$

This is sufficient to find the renormalization-group equations for the couplings in W_5 evaluating the general expressions of [46, 81]. The group-theoretic structure of the soft-breaking terms likewise is straightforward to find, and (5.67, 5.68) are sufficient to find their beta functions. Altogether, one obtains the matrix equations listed in appendix B.2. They agree with the results of [24] when one neglects higher orders of Y^D. For the CMM model, keeping only y_t, y_ν,

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{y}_t &= (9\,\tilde{y}_t^2 + \tilde{y}_{\nu_3}^2 - \frac{96}{5}\,\tilde{\alpha})\tilde{y}_t && (5.69)\\
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{y}_d &= -\frac{84}{5}\tilde{\alpha}\,\tilde{y}_d, && (5.70)\\
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{y}_{\nu_3} &= (7\,\tilde{y}_{\nu_3}^2 + 3\,\tilde{y}_t^2 - \frac{48}{5}\,\tilde{\alpha})\tilde{y}_{\nu_3}, && (5.71)\\
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{A}_t &= (27\,\tilde{y}_t^2 + \tilde{y}_{\nu_3}^2 - \frac{96}{5}\,\tilde{\alpha})\tilde{A}_t + 2\tilde{y}_t\tilde{y}_{\nu_3}\tilde{A}_{\nu_3} + \frac{192}{5}\tilde{\alpha}\tilde{y}_t\,m_{\tilde{g}}, && (5.72)\\
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{A}_d &= \frac{84}{5}\,\tilde{\alpha}(2\,m_{\tilde{g}}\,\tilde{y}_d - \tilde{A}_d), && (5.73)\\
\frac{\mathrm{d}}{\mathrm{d}t}\tilde{A}_{\nu_3} &= (21\,\tilde{y}_{\nu_3}^2 + 3\,\tilde{y}_t^2 - \frac{48}{5}\tilde{\alpha})\tilde{A}_{\nu_3} + 6\tilde{y}_{\nu_3}\tilde{y}_t\tilde{A}_t + \frac{96}{5}\,\tilde{\alpha}\,\tilde{y}_{\nu_3}m_{\tilde{g}}, && (5.74)\\
\frac{\mathrm{d}}{\mathrm{d}t}m_{\psi_3}^2 &= 6\,\tilde{y}_t^2(2m_{\psi_3}^2 + m_u^2) + 6\,|\tilde{A}_t|^2 - \frac{144}{5}\tilde{\alpha}m_{\tilde{g}}^2 && (5.75)\\
\frac{\mathrm{d}}{\mathrm{d}t}m_{\psi_1}^2 &= -\frac{144}{5}\tilde{\alpha}m_{\tilde{g}}^2 && (5.76)\\
\frac{\mathrm{d}}{\mathrm{d}t}m_{\phi_3}^2 &= 2\,\tilde{y}_{\nu_3}^2(m_{\phi_3}^2 + m_{N_3}^2 + m_u^2) + 2\,|\tilde{A}_{\nu_3}|^2 - \frac{96}{5}\tilde{\alpha}m_{\tilde{g}}^2 && (5.77)\\
\frac{\mathrm{d}}{\mathrm{d}t}m_{\phi_1}^2 &= -\frac{96}{5}\tilde{\alpha}m_{\tilde{g}}^2 && (5.78)\\
\frac{\mathrm{d}}{\mathrm{d}t}m_{N_3}^2 &= 10\,\tilde{y}_{\nu_3}^2(m_{\phi_3}^2 + m_{N_3}^2 + m_u^2) + 10\,|\tilde{A}_{\nu_3}|^2 && (5.79)\\
\frac{\mathrm{d}}{\mathrm{d}t}m_{N_1}^2 &= 0 && (5.80)\\
\frac{\mathrm{d}}{\mathrm{d}t}m_u^2 &= 6\,\tilde{y}_t^2(2m_{\psi_3}^2 + m_u^2) + 6\,|\tilde{A}_t|^2 + 2\,\tilde{y}_{\nu_3}^2(m_{\phi_3}^2 + m_{N_3}^2 + m_u^2)\\
&\quad +2\,|\tilde{A}_{\nu_3}|^2 - \frac{96}{5}\,\tilde{\alpha}m_{\tilde{g}}^2 && (5.81)
\end{aligned}$$

5.2.2 Solution for y_t, fixed point of y_t/g, and related constraint

5.2.2.1 $SO(10)$ case

We start considering $SO(10)$ with a slight digression. If the renormalizable Y^M term is used, the evolution of the two Yukawa couplings y_t and y_m is coupled, which leads to an interesting RG flow. Fig. 5.1 depicts the evolution from M_{Pl} to M_{10} of the pair $(y_t/g, y_m/g)$. There is a "critical line" which separates two regions, and if the starting value is in one of the two regions, the couplings stay there forever. The only "true" IR fixed point has $y_t = 0$, while there is a saddle point at $y_m = 0, y_t \neq 0$. However, experimental data suggests [79] that the left-

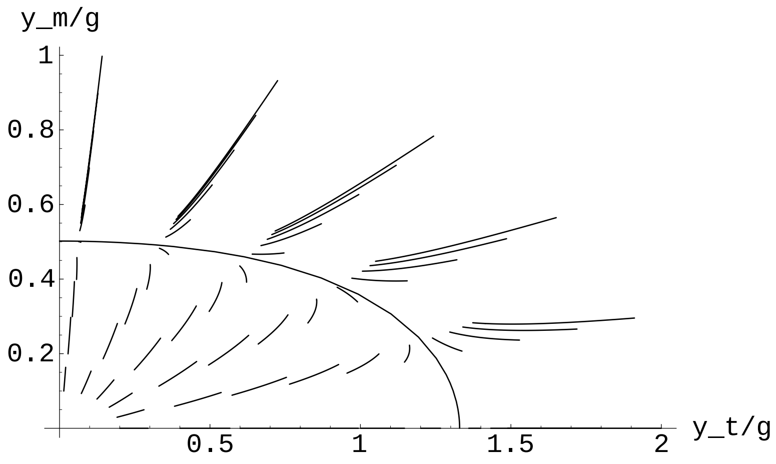


Figure 5.1: $SO(10)$ Yukawa evolution. Trajectory pieces correspond to evolution from M_{Pl} to M_{10}.

handed neutrino Majorana masses are of order 10^{-2} eV. For a vev of the $\overline{\mathbf{126}}$ of $\mathcal{O}(M_{10})$, $y_m = \mathcal{O}(10^{-2})$ and can be neglected. The saddle point therefore effectively acts as IR fixed point for the evolution of y_t.

For the further study of the GUT RGEs, it is useful to define the variable

$$X_t \equiv \frac{\alpha}{y_t^2}. \tag{5.82}$$

From equations (5.39), (5.40), and (5.52), one sees that X_t satisfies

$$\frac{\mathrm{d}}{\mathrm{d}t} X_t = 2(\beta_0 + d_t)\tilde{\alpha}(X_t - X_t^c) \tag{5.83}$$

(with one of the two β_0 values) and has a fixed point at

$$X_t = X_t^c \equiv \frac{c_t}{d_t + \beta_0}. \tag{5.84}$$

In (5.83) and (5.84),

$$c_t = 28, \qquad d_t = \frac{63}{2}. \tag{5.85}$$

Equation (5.83) has the analytical solution

$$X_t = X_t^c + (X_t(0) - X_t^c)\left(\frac{\alpha}{\alpha(0)}\right)^{1+d_t/\beta_0}. \tag{5.86}$$

Here $X_t(0), \tilde{\alpha}(0)$ correspond to some arbitrary initial scale.

5.2.2.2 $SU(5)$ case

At the scale M_{10}, the single Yukawa coupling y_t splits into two, corresponding to Dirac masses for the top and the third-generation neutrino, which are denoted by y_t and y_{ν_3}. Their evolution is coupled. However, since they start at a common value, one can to a good approximation set $y_{\nu_3} \to y_t$ in eq. (5.69) and $y_t \to y_{\nu_3}$ in eq. (5.71). We have checked explicitly that the error is less than one percent for reasonable values of the Yukawa couplings. Then X_t and X_{ν_3} (defined analogously to (5.82)) satisfy (5.83),(5.84), (5.86), mutatis mutandis, with

$$c_t = 10, \qquad d_t = \frac{96}{5}, \qquad c_{\nu_3} = 10, \qquad d_{\nu_3} = \frac{48}{5}. \tag{5.87}$$

5.2.2.3 Qualitative behavior of y_t

If y_t/g is above (or below) the fixed point value at one scale, it will be so at any other scale (within the same GUT). As can be seen from fig. 5.1, if y_t/g is large, the precice information about its starting value at $M_{\rm Pl}$ is quickly lost. Conversely, y_t/g above the fixed point at M_{10} means that y_t can become very large and even infinite before the Planck scale. In that case, the one-loop RGEs cannot be trusted. This situation typically corresponds to very small $\tan\beta \lesssim 2$ and leads to a very large splitting between soft masses. On the other hand, if y_t/g lies below its fixed point at M_{10}, it will remain there all the way to the Planck scale. Then the radiative effects are bounded and, for given soft parameters at the Planck scale (cleanly related to parameters at the weak scale, as we will show in the next subsection), upper bounds on the splittings are given by taking y_t/g at its $SO(10)$ fixed point. The numerical analysis in this dissertation focuses on this situation. It is doubtful if one can make quantitative predictions in perturbation theory if y_t/g becomes large. See also [21], whose authors mention this fact but still allow nonperturbatively large y_t (and use the one-loop evolution) in their investigation. Smaller y_t corresponds to larger $\tan\beta$, and considering the y_t-dependence would seem particularly interesting in connection with processes that depend directly on $\tan\beta$ instead of only indirectly through the sparticle mass splittings; however, in this dissertation we do not study $\tan\beta$-dominated effects.

5.2.3 Soft-term evolution

The renormalization of the soft terms is crucial for computing the down squark mass splitting entering the phenomenological analysis.

The $SU(5)$ and $SO(10)$ evolution we need to know is more involved than in the MSSM because we need to take into account the effects of the large Yukawa couplings y_t and y_{ν_3}. We need to consider the matrices A^U (and A^ν in $SU(5)$) and the nonrenormalizable term A^D. A^U enters the soft mass evolution; however, here we only need the large third-generation eigenvalue A_t. Knowing the evolution of A^D is necessary to find a_0 from $a_1^D = a_d/y_d$ at the GUT scale. However, this ratio satisfies an equation as simple as in the MSSM. It is possible to find analytical solutions for all relevant A-terms in both $SO(10)$ and $SU(5)$ and for the soft masses in $SO(10)$. Only the relatively small corrections to the soft masses in $SU(5)$ have to be solved for numerically.

We explain here in some detail the analytical solutions. They solve the corresponding equations exactly and differ from analytical formulas given in the literature by only higher-order terms, which means the latter also correctly resum the leading logarithms.

5.2.3.1 A-terms

We define $a_t = A_t/y_t$. In $SO(10)$, it obeys

$$\frac{\mathrm{d}}{\mathrm{d}t}a_t = \frac{2C_t\,\tilde{\alpha}}{X_t}a_t + 2d_t\tilde{\alpha}^2\frac{m_{\tilde{g}}(0)}{\tilde{\alpha}(0)} \tag{5.88}$$

with (cf. section 5.2.2)

$$d_t = \frac{63}{2}, \qquad C_t = c_t = 28. \tag{5.89}$$

and "0" denoting an initial scale. Changing the variable results in

$$\frac{\mathrm{d}a_t}{\mathrm{d}\tilde{\alpha}} = \frac{c_t}{\beta_0}\frac{1}{\alpha X_t}a_t + \frac{d_t}{\beta_0}\frac{m_{\tilde{g}}(0)}{\tilde{\alpha}(0)}. \tag{5.90}$$

Defining

$$\gamma_t = 1 + \frac{d_t}{\beta_0}, \qquad u = \left(\frac{\alpha(t)}{\alpha(0)}\right)^{\gamma_t}, \tag{5.91}$$

$$I_{10} = u\frac{X_t(0)}{X_t}, \qquad \tilde{a}_{10} = \frac{d_t}{\beta_0 + d_t}\frac{1}{X_t(0)}j\left(\frac{1}{\gamma_t} - 2; u\right), \tag{5.92}$$

this has the solution

$$a_t(t) = I_{10}\left(a_t(0) + \tilde{a}_{10}m_{\tilde{g}}(0)\right) \tag{5.93}$$

where I_{10} solves the homogeneous equation and the function j is defined in appendix C.1.

Between M_{10} and M_{GUT}, we set $y_{\nu_3} \to y_t$ and vice versa on the right-hand sides of eqs. (5.72), (5.74), as we did in the Yukawa evolution. The $SU(5)$ solutions for a_{ν_3} and a_t are given by (5.91)–(5.93) with the replacements $I_{10} \to I_5^{\nu_3,t}$, $X_t \to X_t, X_{\nu_3}$, and $d_t \to d_t, d_\nu$ where now

$$d_t = \frac{96}{5}, \qquad d_\nu = \frac{48}{5}. \tag{5.94}$$

The equation for a_1^D in $SO(10)$ and $SU(5)$ is

$$\frac{\mathrm{d}}{\mathrm{d}t} a_1^D = 2\, c_d\, \tilde{\alpha}\, m_{\tilde{g}} \tag{5.95}$$

with $c_d = 95/2$ in $SO(10)$ and $c_d = 84/5$ in $SU(5)$. As with eq. (5.14), it can be integrated to give a simple shift proportional to $m_{\tilde{g}}$. Then the relation between the GUT-scale A-term $a_1^D(M_{\mathrm{GUT}})$ and the universal A-term at the Planck scale, a_0, is

$$a_0 = a_1^D(M_{\mathrm{GUT}}) - \left(\delta_5' + \frac{\tilde{\alpha}(M_{10})}{\tilde{\alpha}(M_{\mathrm{GUT}})} \right) m_{\tilde{g}}(M_{\mathrm{GUT}}). \tag{5.96}$$

In analogy with the MSSM, we have defined

$$\begin{aligned} \delta_5' &= \frac{168}{5} \frac{1}{\tilde{\alpha}(M_{\mathrm{GUT}})} \int_{t_{\mathrm{GUT}}}^{t_{10}} \tilde{\alpha}^2(t) \mathrm{d}t, \\ \delta_{10}' &= 95 \frac{1}{\tilde{\alpha}(M_{10})} \int_{t_{10}}^{t_{\mathrm{Pl}}} \tilde{\alpha}^2(t) \mathrm{d}t. \end{aligned} \tag{5.97}$$

5.2.3.2 Soft mass parameters

The coupled $SO(10)$ evolution (5.60), (5.58) of m_{10}^2 and $m_{16_3}^2$ can also be solved analytically. Defining

$$\mathbf{m} = \begin{pmatrix} m_{10}^2 \\ m_{16_3}^2 \end{pmatrix} \tag{5.98}$$

and

$$b = 4 \qquad c = 9, \tag{5.99}$$

a change of basis

$$\hat{\mathbf{m}} = B^{-1}\mathbf{m}, \qquad B^{-1} = \begin{pmatrix} 4 & 2 \\ 5 & -1 \end{pmatrix}, \tag{5.100}$$

decouples the equations to

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{m}_1 = \frac{2 c_t \tilde{\alpha}}{X_t} \hat{m}_1 + \left(b \frac{\tilde{\alpha}}{X_t} |a_t|^2 - c \tilde{\alpha}^3 \frac{m_{\tilde{g}}(M_{\mathrm{GUT}})}{\tilde{\alpha}(M_{\mathrm{GUT}})} \right), \tag{5.101}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{m}_2 = 0. \tag{5.102}$$

The equation for $\hat{m}_1$ is of the same linear inhomogeneous form as (5.88) and can be solved analogously, with all integrals having analytical solutions. All expressions are collected in appendix C.1.

In $SU(5)$, there does not seem to be an analytical solution to eqs. (5.75)–(5.81). The solutions given in [21] for the A-terms and soft masses solve the renormalization-group equations and agree with our analytical solutions where present, both up to higher-order terms. For the numerical analysis in this dissertation, the coupled soft mass evolution in $SU(5)$ is solved numerically. Both because the group theoretical factors are smaller than in $SO(10)$ and because the evolution between M_{10} and $M_{\rm GUT}$ is a short one, the radiative effects are small compared to those from the $SO(10)$ evolution.

For the first two generations, the soft-mass evolution is again very simple in both $SO(10)$ and $SU(5)$, allowing to write

$$\begin{aligned} m^2_{\psi_1}(M_{\rm GUT}) &= m^2_{\psi_1}(M_{10}) + 3\delta_5 m^2_{\tilde g}(M_{\rm GUT}) \\ m^2_{\phi_1}(M_{\rm GUT}) &= m^2_{\phi_1}(M_{10}) + 2\delta_5 m^2_{\tilde g}(M_{\rm GUT}) \\ m^2_{\mathbf{16}_1}(M_{10}) &= m_0^2 + \delta_{10}\frac{\alpha^2(M_{10})}{\alpha^2(M_{\rm GUT})} m^2_{\tilde g}(M_{\rm GUT}) \end{aligned} \tag{5.103}$$

where

$$\begin{aligned} \delta_5 &= \frac{48}{5}\frac{1}{\tilde\alpha^2(M_{\rm GUT})}\int_{t_{\rm GUT}}^{t_{10}} \tilde\alpha^3(t){\rm d}t, \\ \delta_{10} &= 45\,\frac{1}{\tilde\alpha^2(M_{10})}\int_{t_{10}}^{t_{\rm Pl}} \tilde\alpha^3(t){\rm d}t, \end{aligned} \tag{5.104}$$

5.3 Additional sources of nonuniversality

5.3.1 D-terms from gauge group rank reduction

As has been mentioned in section 3.2.2, the breaking from $SO(10)$ to $SU(5)$ can be thought of as taking place in two steps, via $SU(5)\times U(1)_X$ to $SU(5)$. Upon breakdown of the $U(1)$ factor, the gauge group rank is reduced by one, and D-term contributions to the soft masses appear [64], which are proportional to the $U(1)$ charge of a multiplet and to mass splittings between $SO(10)$ multiplets already present at that scale.

Consequently, splittings between fields with identical $SU(5)\times U(1)$ quantum numbers (but in different generations), like $m^2_{\phi_3} - m^2_{\phi_1}$, are not modified. On the other hand, additional splittings between the soft masses of up squarks, down squarks, and Higgses will appear. Their precise values depend on the soft masses and $U(1)$ charges of all scalars present in the theory. Their relative sizes however, being proportional to their $U(1)$ charges, can simply be parameterized [67] by a dimensionful quantity D of order $M^2_{\rm SUSY}$. For the CMM model,

$$\Delta^D m^2_N = 5D, \qquad \Delta^D m^2_\psi = D, \qquad \Delta^D m^2_\phi = -3D, \qquad \Delta^D m^2_{H_u} = -2D. \tag{5.105}$$

5.3.1.1 Independence of $SU(5)$ evolution and $U(1)$ breakdown

Substituting the shifts of (5.105) into (5.75)–(5.81) leaves these equations invariant. That means the exact scale of $U(1)_X$ breakdown is immaterial except for the universal $U(1)_X$ gauge contributions which have been neglected in writing the equations but should be small. Furthermore, in the CMM model as introduced in chapter 4 the $U(1)$ factor is directly broken at M_{10} (although this might by different if the $\mathbf{45}_H$ receives the larger vev). In light of this, we perform the D-term shifts at the scale M_{10}. Then at that scale, the soft mass $m^2_{\mathbf{16}_1}$ is found without ambiguity from the D-terms as

$$m^2_{\mathbf{16}_1}(M_{10}) = \frac{1}{4}\left(3m^2_{\psi_1}(M_{10}) + m^2_{\phi_1}(M_{10})\right) \tag{5.106}$$

which combined with (5.103) gives

$$\begin{aligned} m^2_{\mathbf{16}_1}(M_{10}) &= \frac{1}{4}\left(3m^2_{\tilde{u}_{R_1}}(M_{\mathrm{GUT}}) + m^2_{\tilde{d}_{R_1}}(M_{\mathrm{GUT}})\right) - \frac{11}{4}\delta_5 m^2_{\tilde{g}}(M_{\mathrm{GUT}}), & (5.107)\\ m_0^2 &= \frac{1}{4}\left(3m^2_{\tilde{u}_{R_1}}(M_{\mathrm{GUT}}) + m^2_{\tilde{d}_{R_1}}(M_{\mathrm{GUT}})\right) \\ &\quad - \left(\frac{11}{4}\delta_5 + \frac{\alpha^2(M_{10})}{\alpha^2(M_{\mathrm{GUT}})}\delta_{10}\right) m^2_{\tilde{g}}(M_{\mathrm{GUT}}) & (5.108) \end{aligned}$$

5.3.2 GUT threshold

Neglecting higher-order effects, the three gauge couplings should meet at the GUT scale if grand unification occurs. NLO threshold corrections change this. Unfortunately, these depend on the exact particle spectrum and Lagrangian of the $SU(5)$ GUT theory and are therefore not calculable. However, we adopt the point of view that exact unification is too restrictive and allow for a slight non-universality. This can be done in several ways. Our strategy is to require $g_5(M_{\mathrm{GUT}}) = g_1(M_{\mathrm{GUT}}) = g_2(M_{\mathrm{GUT}})$ while allowing g_3 to differ. This seems justified by the following observation: g_1 and g_2 at the weak scale are subject to threshold corrections [69] which we also neglect. Furthermore, their mixing into g_3 under renormalization is an NLO effect and their mixing into y_t is also smaller because of both smaller coefficients and the relative smallness of $\tilde{\alpha}_1, \tilde{\alpha}_2$ with respect to $\tilde{\alpha}_3$ over most of the range of evolution. Conversely, g_3 is larger and much more strongly mixing with y_t so a small change in g_3 at M_{GUT} can have a more significant effect.

5.4 Weak-scale mass splitting from weak-scale inputs

The relations between parameters at different scales derived so far in this chapter enable one to translate the constraints imposed by the CMM structure into a recipe for finding the set of weak-scale parameters of interest from a minimal set of input parameters.

The procedure consists of the following steps.

1. First the inputs $\alpha_s(M_Z)$, $\alpha_e(M_Z)$, and $\sin^2\theta_W$, the top mass, and $\tan\beta$ determine the values of the $\overline{DR}$ couplings $\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, y_t$ at the scale M_Z by means of eqs. (5.1)–(5.6). There is dependence on the details of the squark mass spectrum, however it is a next-to-leading effect. Consequently, the exact choice of squark masses is not too important and one can choose a universal squark mass $m_{\tilde{q}}$ in this step. The gluino mass $m_{\tilde{g}_3}$ also enters.

   The resulting couplings are scaled up to $M_{\rm GUT}$, M_{10}, and $M_{\rm Pl}$ via numerical integration of the system (5.7)– (5.10), the one-loop-running unified gauge coupling, and the analytical expression (5.86).

2. Next, the Planck-scale universal value a_0 is found from the weak-scale value of a_1^D via equations (5.28) and (5.95). The universal soft mass m_0^2 is obtained from $m^2_{\tilde{d}_{R_1}}$ and $m^2_{\tilde{u}_{R_1}}$ at the weak scale through equations (5.23), (5.25), and (5.108). In both steps, the gluino mass enters as a parameter.

3. Lastly, the mass splittings between the light and heavy right-handed down-type squarks and between the light and heavy left-handed sleptons at the weak scale are found by computing the GUT splittings according to sec. 5.2.3.2. The left-handed slepton masses can be related to the right-handed sdown masses by means of (5.29),(5.26). (The parameter D can also be found from $m_{\tilde{u}_{R_1}}$ and $m_{\tilde{d}_{R_1}}$, and the remaining sfermion masses that are not affected by y_t can be computed.)

The inputs and “outputs“ needed at each step are summarized in table 5.2.

Table 5.2: Input parameters and knowledge gained from them

Step	input	output	equations
1.	$\alpha_s(M_Z)$, $\alpha_e(M_Z)$, $\sin^2\theta_W$, $m_{\tilde{q}}$, $m_{\tilde{g}_3}$, $\tan\beta$	$\tilde{\alpha}_3(\mu)$, $\tilde{\alpha}_2(\mu)$, $\tilde{\alpha}_1(\mu)$, $y_t(\mu)$	5.1–5.6, 5.7–5.10, 5.86
2.	$m_{\tilde{g}_3}$, a_1^D, $m_{\tilde{d}_{R_1}}$, $m_{\tilde{u}_{R_1}}$	a_0, m_0^2	5.28, 5.95, 5.23, 5.25, 5.108
3.	—	$m_{\tilde{d}_{R_3}}$, $m_{\tilde{l}_3}$	5.103; section 5.2.3.2

Chapter 6

Effective Lagrangian and weak-scale observables

The present chapter is devoted to the study of the effective Lagrangians that describe flavor-changing weak decays. We calculate the one-loop matching corrections for the hadronic $\Delta B = 2$ and $\Delta B = 1$ Lagrangian as well as the leptonic operator Q_7' mediating the decay $\tau \to \mu\gamma$. They form the basis of our study of $B_s - \bar{B}_s$ mixing and $\tau \to \mu\gamma$ in chapter 7. The $\Delta B = 1$ Lagrangian could be applied to the CP asymmetry in $B_d \to \phi K_S$, however this involves additional steps that we have no room to discuss in this thesis. We will, however, argue that we do not expect any significant change in the prediction from its standard value from the contributions we compute.

6.1 $\Delta F = 2$ processes

The $\Delta F = 2$ hadronic effective Lagrangian at a low-energy scale μ takes the form

$$-\mathcal{L} = \frac{G_F^2 M_W^2}{(4\pi)^2} \lambda_t^2 \left(C_L(\mu) O_L(\mu) + C_R(\mu) O_R(\mu) \right) + \text{h.c.}, \tag{6.1}$$

where

$$\frac{G_F^2 M_W^2}{(4\pi)^2} = \frac{\alpha_2^2}{8 M_W^2}, \tag{6.2}$$

$$\lambda_t = V_{ti}^* V_{tj}, \tag{6.3}$$

$$O_L = \bar{q}_{iL} \gamma_\mu q_{jL}\, \bar{q}_{iL} \gamma^\mu q_{jL}, \tag{6.4}$$

$$O_R = \bar{q}_{iR} \gamma_\mu q_{jR}\, \bar{q}_{iR} \gamma^\mu q_{jR}. \tag{6.5}$$

The flavor labels i, j take the pairs of values $(i, j) = (s, b)$, (d, b), or (d, s) dependently if the B_s, B_d, or K^0 system is considered.

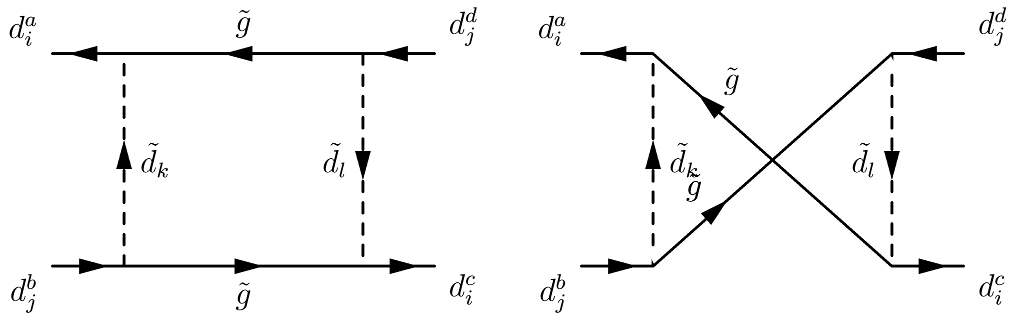


Figure 6.1: Gluino-squark box diagrams contributing to the $\Delta F = 2$ and $\Delta F = 1$ Lagrangian (with suitable external quark flavors).

Several remarks are in order. First, our matching computation is done in the NDR-$\overline{MS}$ scheme, where also the standard-model renormalization-group evolution is conventionally performed. To be consistent with chapter 5, this means the strong coupling constant $\alpha_3^{\overline{DR}}$ has to be converted back to α_s, while the sparticle masses, treated at the leading order in view on their unknown experimental values, are unchanged, as are α_1 and α_2. In eq. (6.2) and below, we therefore refer to NDR-$\overline{MS}$ quantities, and $\alpha_s = \alpha_s^{(5)}(M_Z)$, unless otherwise stated.

In the standard model, only the operator O_L has a non-vanishing Wilson coefficient, which (factoring out the CKM factors as in (6.1)) is real and positive and originates solely from box diagrams. The same is true separately for the charged-Higgs and chargino contributions of supersymmetry with minimal flavor violation, see e.g. [54]. Altogether one has

$$\begin{aligned} C_L &= 4\eta S(x_t) + 4\eta^{(H^+)} S^{(H^+)}(x_t, x_{HW}) \\ &\quad + 4\eta^{(\chi^+)} S^{(\chi^+)}(x_{\tilde{u}W}, x_{\tilde{t}_i W}, x_{\chi_i^+ W}) \end{aligned} \tag{6.6}$$

with $x_{ab} = m_a^2/m_b^2$. See the reference [54] for bounds on C_L. The coefficients η are discussed below.

The effects peculiar to the CMM model are encoded in the Wilson coefficient C_R of the parity-reflected operator O_R. The dominant contribution is expected to come from the squark-gluino box diagrams of fig. 6.1. Because there is flavor mixing only among right-handed squarks and the gluinos couple right-handed squarks only to their right-handed fermionic partners, no other chirality structures arise. Evaluating the diagrams, one obtains

$$C_R = \frac{\Lambda_3^2}{\lambda_t^2} \frac{(4\pi)^2 \alpha_s^2}{4 G_F^2 M_W^2 m_{\tilde{g}}^2} \eta^{(\tilde{g})} S^{(\tilde{g})}(x, y) = 2 \frac{\Lambda_3^2}{\lambda_t^2} \frac{\alpha_s^2}{\alpha_2^2} \frac{M_W^2}{m_{\tilde{g}}^2} \eta^{(\tilde{g})} S^{(\tilde{g})}(x, y), \tag{6.7}$$

$$\Lambda_3 = U_{D\,i3}^* U_{Dj3}, \tag{6.8}$$

where $x = m_{\tilde{d}_{R_1}}^2/m_{\tilde{g}}^2, y = m_{\tilde{d}_{R_3}}^2/m_{\tilde{g}}^2$, and where we now write $m_{\tilde{g}}$ for the gluino mass.

The loop functions are given in appendix C.2. Note that in general at least for one of the two neutral B meson systems, the Wilson coefficient C_R gets a large enhancement factor due to the large atmospheric neutrino mixing angle. Following the reasoning of section 4.3, we take the $(1,3)$ element of U_D to be small. We then restrict our study to $B_s - \bar{B}_s$-mixing, where

$$|\Lambda_3| = |U_{\mu 3}||U_{\tau 3}| \approx \frac{1}{2}, \tag{6.9}$$

with an unknown phase. The CKM element V_{ts} is known from semileptonic decays arising at the tree level, combined with the unitarity of the CKM matrix. It has small phase.

In principle there also are neutralino and mixed gluino-neutralino contributions; they should be small due to the smaller couplings.

The coefficients η, $\eta^{(H^+)}$, and $\eta^{(\chi^+)}$ account for QCD corrections including the renormalization-group evolution from the matching scale μ_M to the hadronic scale where the mixing amplitude is evaluated. One chooses $\mu_b \approx m_b$ for the B system. The scale μ_M is chosen of the order of the heavy particles that are integrated out. The factors also serve to make the amplitude stable against variation of the matching scale and change of the renormalization scheme [86]. However, our η factors are not to be identified with η_2, η_B, and so on, as the dependence on the low-energy scale is included in our factors, following [87]. The relevant expressions for the standard-model coefficient η at NLO are given in [87].)

For the gluino contribution to $B - \bar{B}$ mixing we use the LO evolution, which is consistent with the LO matching calculaction, and set $\mu_M = M_Z$. Then

$$\eta^{(\tilde{g})} = \left(\frac{\alpha_s(M_Z)}{\alpha_s(\mu_b)}\right)^{\frac{6}{23}} \tag{6.10}$$

While it might be more rigorous to choose $\mu_M = \mathcal{O}(m_{\tilde{g}})$, we do not expect large logarithmic corrections to be present that would invalidate our choice.

6.1.1 Mass differences

The fact that the only new operator is the parity reflection of the standard-model operator O_L is fortunate: Because QCD is parity symmetric, the hadronic matrix elements of both are equal. This means that in amplitudes, the two Wilson coefficients C_L and C_R can be summed into an "effective" coefficent $C_L^{\text{eff}} = C_L + C_R$, and the standard-model expressions for the mixing amplitudes in terms of C_L and hadronic parameters can be taken over. Unfortunately, however, the unknown relative phase between C_L and C_R introduces an uncertainty into the amplitude.

The neutral B-meson mass difference ($B = B_d, B_s$) is given by [87]

$$\Delta M_B = \frac{G_F^2}{24\pi^2} M_W^2 M_B f_B^2 B_1^{VLL}(\mu) |C_L^{\text{eff}}(\mu)|. \tag{6.11}$$

Here f_B is the B-meson decay constant and $B_1^{VLL}(\mu)$ is related to the matrix element of the operator O_L renormalized at the scale μ via

$$\langle O_L(\mu)\rangle = \frac{1}{3} M_B f_B^2 B_1^{VLL}(\mu). \tag{6.12}$$

It is related to the RG- and scheme-invariant bag parameter $\hat{B}_B$ through the expression

$$\hat{B}_B = B_1^{VLL}(\mu)\left[\alpha_s^{(5)}(\mu)\right]^{-6/23}\left[1 + \frac{\alpha_s^{(5)}(\mu)}{4\pi} J_5\right] \tag{6.13}$$

with

$$J_5 = \frac{\gamma^{(0)}\beta_1}{2\beta_0^2} - \frac{\gamma^{(1)}}{2\beta_0} \tag{6.14}$$

a (scheme-dependent) NLO coefficient. This is an NLO relation, and strictly speaking it should be combined with an NLO computation of the Wilson coefficient $C_L^{\text{eff}}(\mu)$. However, as we have said above, the scheme and scale dependence of the new-physics contribution is relatively small, and by the same token one can combine (6.13) with the mixed LO-NLO expression we have for C_L^{eff}. Lattice-QCD investigations [88] give a value of

$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 276(38)\,\text{MeV}. \tag{6.15}$$

Note that, unlike in the case of B_d mixing, this is not affected by potential large extrapolation errors such as chiral logs [89], receiving much attention over the last year and a half. After squaring, the relative error is less than 30 percent. Combined with the rather precise knowledge of V_{ts}, which is independent of loop-level new-physics effects, a quite precise prediction of the mass difference for the B_s system can be made both in the standard model and in the CMM model. Experimentally, there is only a lower bound at 95% CL of [90]

$$\Delta M_{B_s} > 14.4\,\text{ps}^{-1}. \tag{6.16}$$

For purposes of the numerical analysis in chapter 7, however, we prefer to consider C_R normalized with respect to the SM value for C_L, which eliminates the uncertainties from the hadronic matrix element as well as from the unknown phase of C_R and from the SUSY-MFV contributions (that depend on stop and charged-Higgs masses).

6.1.2 CP violation

We have mentioned above that the phase of the CMM contribution C_R in the B_s system is unconstrained. This has to be contrasted with the standard-model contribution, which is almost real. As a result, the prediction for the CP asymmetry in decays such as $B_s \to \psi\phi$ is affected and in fact, for large C_R, any value for the asymmetry is possible.

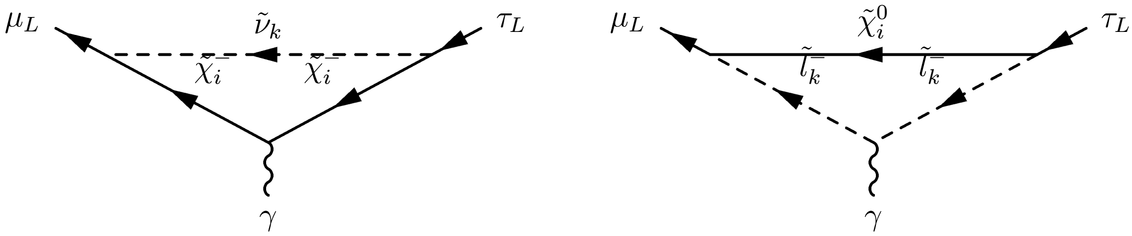


Figure 6.2: Chargino and neutralino penguins contributing to the effective Lagrangian for $\tau \to \mu\gamma$.

6.2 $\Delta F = 1$ processes

6.2.1 $\tau \to \mu\gamma$

Within the standard model, this decay is strongly GIM-suppressed because the neutrinos are almost degenerate in mass. As a consequence, the predicted decay rate is many orders of magnitude below the experimental upper bounds coming from Belle [91] and Babar [92], with the Belle 90% CL bound of $BR(\tau \to \mu\gamma) < 6 \times 10^{-7}$ being the stronger one. Within the CMM model, the splitting between the second- and third-generation slepton masses together with the large atmospheric MNS angle leads to a potentially large decay rate. Neglecting left-right mixing among sleptons, the effective Lagrangian contributing to the decay $\tau \to \mu\gamma$ computed from the diagrams of fig. 6.2 reads

$$-\mathcal{L} = C_7' Q_7' + \text{h.c.} \tag{6.17}$$

where

$$Q_7' = m_\tau \bar{\mu}_L \sigma^{\mu\nu} F_{\mu\nu} \tau_R, \tag{6.18}$$

$$C_7' = \frac{e^3}{(4\pi)^2 \sin^2\theta_W} \sum_J U_{\tau J} U_{\mu J}^* \left(A_+ + A_0\right) \tag{6.19}$$

Here A_+ and A_0 denote contributions from chargino and neutralino loops, which have the form

$$A_+ = -Z_+^{1i*} Z_+^{1i} \frac{H_1(x_{Ji})}{m_{\chi_i^+}^2} + \frac{Z_+^{1i*} Z_-^{2i*}}{\sqrt{2}\cos\beta} \frac{H_2(x_{Ji})}{m_{\chi_i^+} M_W} \tag{6.20}$$

$$\begin{aligned} A_0 &= \frac{1}{2\cos^2\theta_W} \left| Z_N^{1i} \sin\theta_W + Z_N^{2i} \cos\theta_W \right|^2 \frac{H_3(y_{Ji})}{m_{\chi_i^0}^2} \\ &\quad - \frac{1}{2\cos^2\theta_W \cos\beta} \left(Z_N^{1i*} \sin\theta_W + Z_N^{2i*} \cos\theta_W \right) Z_N^{3i*} \frac{H_4(y_{Ji})}{m_{\chi_i^0} M_W} \end{aligned} \tag{6.21}$$

with

$$x_{Ji} = \frac{m_{\tilde{\nu}_J}^2}{m_{\chi_i^+}^2}, \qquad y_{Ji} = \frac{m_{\tilde{l}_J}^2}{m_{\chi_i^0}^2} \tag{6.22}$$

$\tan\beta$ is, as usual, the ratio of the vacuum expectation values v_u and v_d giving mass to up- and down-type particles, respectively. The matrices Z_+, Z_- and Z_N arise from the diagonalization of the chargino and neutralino mass matrices given in (2.59), (2.58). The notation follows [52]. Our results appear to disagree with those of [24], which still has to be resolved.

Neglecting gaugino-higgsino mixing, Z_+ and Z_- would be unit matrices, and so would be Z_N except for the (3,4) sub-block, which describes maximal mixing (i.e. all entries have magnitude $1/\sqrt{2}$). Then the second terms in each of the expressions for A_+ and A_0, which involve the higgsino components of the charginos and neutralinos and their couplings to the tau, would vanish.

Because this technically corresponds to the limit $M_W \to 0$ and M_W appears in the denominators of those terms, however, this might be a bad approximation even though only a rough estimate is needed to test if the experimental bound is satisfied for given parameter values; certainly it is not the leading term in an expansion in M_W or $g_{1,2}$.

A better way is to expand the masses and Z matrices to first order in the electroweak VEVs in the terms containing factors of $1/(\cos\beta M_W)$. One has

$$m_{\chi_1^+} = m_{\tilde{g}_2}, \qquad m_{\chi_2^+} = |\mu|, \tag{6.23}$$

$$m_{\chi_1^0} = m_{\tilde{g}_1}, \qquad m_{\chi_2^0} = m_{\tilde{g}_2}, \qquad m_{\chi_3^0} = m_{\chi_4^0} = |\mu|, \tag{6.24}$$

$$\begin{aligned}
Z_+^{11*} Z_-^{21*} \frac{1}{\sqrt{2}\cos\beta m_{\chi_1^+} M_W} &= -\frac{m_{\tilde{g}_2} + \mu\tan\beta}{m_{\tilde{g}_2}(\mu^2 - m_{\tilde{g}_2}^2)}, && (6.25)\\
Z_+^{12*} Z_-^{22*} \frac{1}{\sqrt{2}\cos\beta m_{\chi_2^+} M_W} &= \frac{\mu + m_{\tilde{g}_2}\tan\beta}{\mu(\mu^2 - m_{\tilde{g}_2}^2)}, && (6.26)\\
Z_N^{11*} Z_N^{31*} \frac{1}{2\cos\beta m_{\chi_1^0} M_W} &= \frac{\sin\theta_W}{2\cos\theta_W} \frac{m_{\tilde{g}_1} + \mu\tan\beta}{m_{\tilde{g}_1}(\mu^2 - m_{\tilde{g}_1}^2)}, && (6.27)\\
\frac{1}{2\cos\beta M_W}\left(\frac{Z_N^{13*} Z_N^{33*}}{m_{\chi_3^0}} + \frac{Z_N^{14*} Z_N^{34*}}{m_{\chi_4^0}}\right) &= -\frac{\sin\theta_W}{2\cos\theta_W} \frac{\mu + m_{\tilde{g}_1}\tan\beta}{\mu(\mu^2 - m_{\tilde{g}_1}^2)}, && (6.28)\\
Z_N^{22*} Z_N^{32*} \frac{1}{2\cos\beta m_{\chi_2^0} M_W} &= -\frac{1}{2}\frac{m_{\tilde{g}_2} + \mu\tan\beta}{m_{\tilde{g}_2}(\mu^2 - m_{\tilde{g}_2}^2)}, && (6.29)\\
\frac{1}{2\cos\beta M_W}\left(\frac{Z_N^{23*} Z_N^{33*}}{m_{\chi_3^0}} + \frac{Z_N^{24*} Z_N^{34*}}{m_{\chi_4^0}}\right) &= \frac{1}{2}\frac{\mu + m_{\tilde{g}_2}\tan\beta}{\mu(\mu^2 - m_{\tilde{g}_2}^2)}. && (6.30)
\end{aligned}$$

These expansions become invalid if $\tan\beta$ is large, in which case the neglect of left-right mixing can no longer be justified either. It is evident that the higgsino-like contributions cannot be neglected and may even dominate the amplitudes. For our numerical study, we therefore keep the complete amplitudes and do not perform an expansion.

The inclusive decay rate is given by

$$\Gamma(\tau \to \mu\gamma) = \frac{m_\tau^5}{16\,\pi}|C_7'|^2. \tag{6.31}$$

From this, one computes the branching ratio according to

$$BR(\tau \to \mu\gamma) = \tau_\tau \Gamma(\tau \to \mu\gamma). \tag{6.32}$$

This branching ratio will also be studied numerically in chapter 8.

6.2.2 $B_d \to \phi K_S$

A prototype decay mode probing new physics is the exclusive decay $B_d \to \phi K_S$, which is triggered by the quark level decay $b \to s\bar{s}s$. In particular, the mixing-induced CP asymmetry $a_{CP}(B_d \to \phi K_S)$ in this decay has been proposed as a theoretically clean test of the Standard Model [93]: the CKM mechanism of CP violation predicts the mixing-induced CP asymmetries in $B_d \to \phi K_S$ and $B_d \to \psi K_S$ to coincide within a few percent, and new physics in the loop-induced $b \to s\bar{s}s$ decay amplitude can easily alter this prediction. Recently branching ratios and CP asymmetries of B_d decays into light pseudoscalar and vector mesons have been analyzed in the framework of QCD factorization [94]. On the experimental side both Belle and Babar had consistently found a significant deviation of $a_{CP}(B \to \phi K_S)$ from its SM value, the combined measurement being at variance at the $2.8\,\sigma$ level [95, 96]. In summer 2003, the situation changed. The new Babar result, which includes a reanalysis of the old data published in [95, 96], is now consistent with the SM, while the new Belle result exhibits an even larger discrepancy with the SM [97].

The CMM model involves new sources of $b_R - s_R$ transitions stemming from the squark mass matrix. It is well known that $B_s - \bar{B}_s$ mixing is far more sensitive to this effect than $\Delta B = 1$ processes such as the decay amplitude of $b \to s\bar{s}s$. For a recent study in generic SUSY models, see [98]. We have computed the gluino-gluon-penguin and gluino-box contributions to the $\Delta B = 1$ Wilson coefficients in the CMM model. The effective Lagrangian receives contributions from the penguin and box diagrams of figs. 6.3 and 6.1 It has the form

$$-\mathcal{L} = \frac{G_F}{\sqrt{2}} \left(\sum_{q=u,d,s,c,b} \sum_{i=3}^{6} C_i' Q_i^{q\prime} + C_8' Q_8' \right) + \text{h.c.}, \tag{6.33}$$

where the operators are spelled out

$$\begin{aligned} Q_3^{q\prime} &= \bar{s}_{R\alpha}\gamma_\mu b_{R\alpha}\; \bar{q}_{R_\beta}\gamma^\mu q_{R\beta}, & (6.34)\\ Q_4^{q\prime} &= \bar{s}_{R\alpha}\gamma_\mu b_{R\beta}\; \bar{q}_{R_\beta}\gamma^\mu q_{R\alpha}, & (6.35)\\ Q_5^{q\prime} &= \bar{s}_{R\alpha}\gamma_\mu b_{R\alpha}\; \bar{q}_{L_\beta}\gamma^\mu q_{L\beta}, & (6.36) \end{aligned}$$

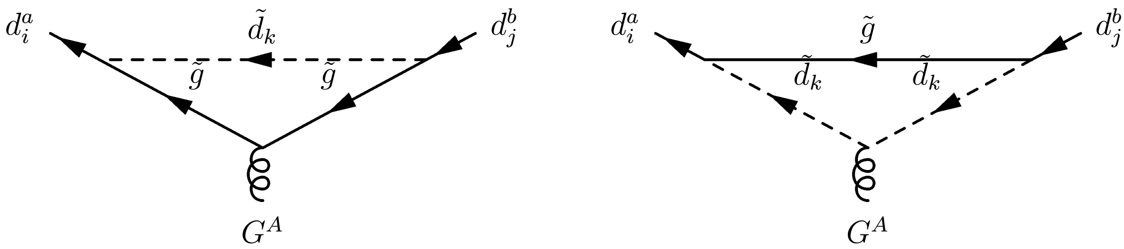


Figure 6.3: Gluino-gluon penguins contributing to the hadronic $\Delta F = 1$ Lagrangian (with suitable external quark flavors).

$$Q_6^{q\prime} = \bar{s}_{R\alpha}\gamma_\mu b_{R\beta}\, \bar{q}_{L\beta}\gamma^\mu q_{L\alpha}, \tag{6.37}$$

$$Q_8' = -\frac{1}{16\pi^2} m_b \bar{s}_R \sigma^{\mu\nu} G^a_{\mu\nu} T^a b_L, \tag{6.38}$$

and the Wilson coefficients are given by

$$C_i^{q\prime} = C_i^{q,\text{box}} + C_i^{q,\text{peng}} \qquad (i = 3 \dots 6), \tag{6.39}$$

$$C_3^{q,\text{peng}} = C_5^{q,\text{peng}} = -\frac{\sqrt{2}}{G_F}\frac{1}{2\,N_c} P\, G_4(x_b, x_s), \tag{6.40}$$

$$C_4^{q,\text{peng}} = C_6^{q,\text{peng}} = \frac{\sqrt{2}}{G_F}\frac{1}{2} P\, G_4(x_b, x_s), \tag{6.41}$$

$$C_3^{q,\text{box}} = \frac{\sqrt{2}}{G_F} P \left(\frac{1}{36} G_1(x_{\tilde{q}_R}) - \frac{5}{9} F_1(x_{\tilde{q}_R})\right) \quad (q = u, d, c), \tag{6.42}$$

$$C_3^{q,\text{box}} = \frac{\sqrt{2}}{G_F} P \left(\frac{1}{36} G_1^{b,s} - \frac{5}{9} F_1^{b,s}\right) \quad (q = b, s), \tag{6.43}$$

$$C_4^{q,\text{box}} = \frac{\sqrt{2}}{G_F} P \left(\frac{7}{12} G_1(x_{\tilde{q}_R}) + \frac{1}{3} F_1(x_{\tilde{q}_R})\right) \quad (q = u, d, c), \tag{6.44}$$

$$C_4^{q,\text{box}} = \frac{\sqrt{2}}{G_F} P \left(\frac{7}{12} G_1^{b,s} + \frac{1}{3} F_1^{b,s}\right) \quad (q = b, s), \tag{6.45}$$

$$C_5^{q,\text{box}} = \frac{\sqrt{2}}{G_F} P \left(\frac{1}{18} F_1(x_{\tilde{q}_L}) - \frac{5}{18} G_1(x_{\tilde{q}_L})\right), \tag{6.46}$$

$$C_6^{q,\text{box}} = \frac{\sqrt{2}}{G_F} P \left(\frac{7}{6} F_1(x_{\tilde{q}_L}) + \frac{1}{6} G_1(x_{\tilde{q}_L})\right), \tag{6.47}$$

$$C_8' = \frac{g_3^2}{4m_{\tilde{g}}^2} \sin(2\,\theta_R) e^{i\delta_R} G_8(x_b, x_s). \tag{6.48}$$

Here we have defined

$$(U_D)_{s3} = \sin\theta\, e^{i\delta}, \qquad (U_D)_{b3} = \cos\theta \tag{6.49}$$

(where $\theta \approx \pi/4$ and we have pulled out an unphysical phase),

$$P = \frac{\alpha_s^2}{2m_{\tilde{g}}^2} \sin(2\theta) e^{i\delta}, \tag{6.50}$$

$$
\begin{aligned}
G_8(x_b, x_s) &= (-C_2(8)C(x_b) + C_2(3)D(x_b) - (x_b \to x_s)), & (6.51)\\
G_4(x_b, x_s) &= (C_2(8)A(x_b) + C_2(3)B(x_b) - (x_b \to x_s)), & (6.52)\\
F_1(z) &= F(x_b, z) - F(x_s, z), & (6.53)\\
G_1(z) &= G(x_b, z) - G(x_s, z), & (6.54)\\
F_1^b &= \cos^2\theta F_1(x_b) + \sin^2\theta F_1(x_s), & (6.55)\\
F_1^s &= \sin^2\theta F_1(x_b) + \cos^2\theta G_1(x_s), & (6.56)\\
G_1^b &= \cos^2\theta G_1(x_b) + \sin^2\theta F_1(x_s), & (6.57)\\
G_1^s &= \sin^2\theta G_1(x_b) + \cos^2\theta G_1(x_s), & (6.58)
\end{aligned}
$$

$$
x_b = \frac{m^2_{\tilde{q}_{R3}}}{m^2_{\tilde{g}}}, \qquad x_s = \frac{m^2_{\tilde{q}_{R1}}}{m^2_{\tilde{g}}}, \qquad x_{\tilde{q}_{R,L}} = \frac{m^2_{\tilde{q}_{R,L}}}{m^2_{\tilde{g}}}, \tag{6.59}
$$

with $C_2(8) = 3, C_2(3) = 4/3$. The loop functions A, B, C, D are defined in [99], while F and G are defined in [100]. All are listed in appendix C.2.

For the penguin contributions, our results agree with the earlier computation of [99]. We have compared the box contributions to eq. (5.1) of ref. [101], with which we agree except for the sign of the G_1 terms in our expressions for $C_4^{q,\mathrm{box}}$. It should be noted that most authors treat the hadronic effective Lagrangian in the mass-insertion approximation, which is inadequate if large sfermionic mixing angles are present. For more general results in the MIA, see the recent paper [98], which differs in the form of the LR-mixing contributions but agrees for the RR-mixing with the older study in [49].

Qualitatively, it is clear that in our model effects are small and we can reproduce the Babar result but not the Belle result. A better quantitative statement is difficult to make: once the SM and new physics contributions (which involve different operators) interfere, hadronic uncertainties do not drop out anymore from $a_{CP}(B_d \to \phi K_S)$. Unfortunately, the global analysis in [94] has shown that QCD factorization fails badly in cases like $B_d \to \phi K_S$, where a $\bar{q}$ and a q field of the weak decay operator create a vector meson. This feature makes it difficult to derive quantitative bounds on $a_{CP}(B \to \phi K_S)$ in our model.

Chapter 7

Numerical study of the phenomenology

Given the results of chapter 5, as summarized in section 5.4, we are now in a position to study the sparticle spectrum and, subsequently, the Wilson coefficients and observables of 6. In this chapter, after giving some details concerning the numerical implementation of the formulas and procedures described in chapter 5, we first discuss the mass differences between the sfermions of different generations as functions of weak-scale input parameters, taking into account constraints from direct searches and from the condition of correct electroweak symmetry breaking. A subsequent section is devoted to the implications for B phenomenology and leptonic flavor violation.

7.1 Fixed-point constraint and implementation

In section 5.2.2 we mentioned that upper bounds on the sparticle mass splittings are given by taking the ration y_t/g at its fixed-point value within $SO(10)$, unless the top Yukawa coupling is very large. We also argued that in the latter case, the predictivity of the theory suffers and it may also become strongly coupled below the Planck scale. For these reasons, we study the case where y_t/g is at its GUT fixed point, while varying the other parameters subject to this constraint.

This simply implies a relation between $y_t^{\overline{DR}}(M_Z)$ and $\alpha_{1,2,3}(M_Z)$, which can be used to find $y_t^{\overline{DR}}(M_Z)$ from the three other parameters. In fact it turns out that $y_t^{\overline{DR}}(M_Z)$ is a rather smooth function of $\alpha_3^{\overline{DR}}(M_Z)$ with very little dependence on the remaining parameters. Consequently, we only need to compute $\alpha_3^{\overline{DR}}(M_Z)$ according to (5.1), (5.2) from the input parameters.

The remainder of this section gives some details on the numerical procedure and is rather technical. Unless interested in reproducing the results, the reader mainly interested in the physical results will lose little by skipping to the next section.

The formulas relevant for obtaining the right-handed sbottom mass and other sfermion masses, and the gauge, Yukawa, and universal soft parameters in the intermediate steps, were all programmed in Mathematica.

The FP constraint can be solved either iteratively, which is very slow due to the numerical solution of the MSSM gauge and Yukawa evolution, or by interpolation. Here we show that to a good approximation, $y_t(M_Z)$ is a linear function of $\alpha_3(M_Z)$, almost independent from the other parameters.

With exact unification of gauge couplings, there would be only one independent gauge coupling, which could e.g. be chosen to be g_1. Fig. 7.1 shows,

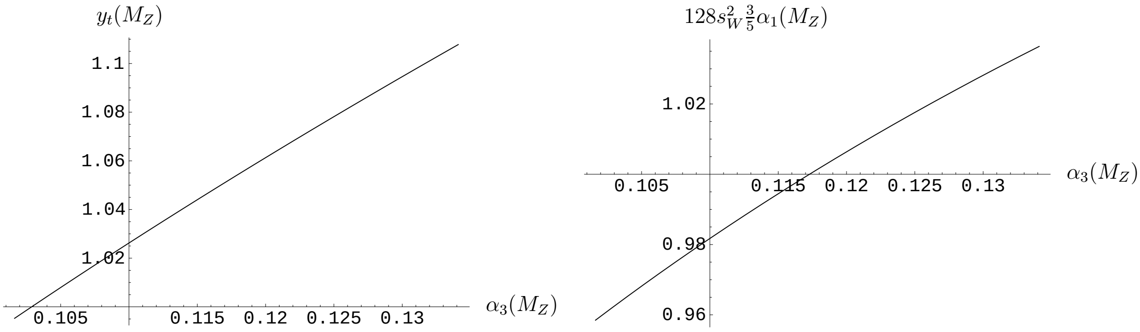


Figure 7.1: Correlations from exact gauge coupling unification

unfortunately, a correlation between $\sin^2\theta_W$ with $\alpha_3(M_Z)$ for this case that is quite strong. (We plot on the vertical axis the ratio of $\alpha_1(M_Z)$, computed from $\alpha_3(M_Z)$ and the requirement of unification, and its value as computed from the electromagnetic coupling and the measured value of $\sin^2\theta_W$, which we fix at 0.23.) While it might seem then that the correct value of $\alpha_3(M_Z)$ could be found from the measured value of $\sin^2\theta_W$, the latter is affected by threshold corrections at both the weak and GUT scales that we do not take into account. In accordance with what was said in section 5.3.2, we therefore allow for $\alpha_3(M_{\rm GUT})$ to differ from $\alpha_5(M_{\rm GUT}) = \alpha_{12}(M_{\rm GUT})$, by which we denote the assumed unified value of α_1 and α_2. This unification is also subject to corrections, but we ignore these consistently with the neglected weak-scale threshold corrections. The modified

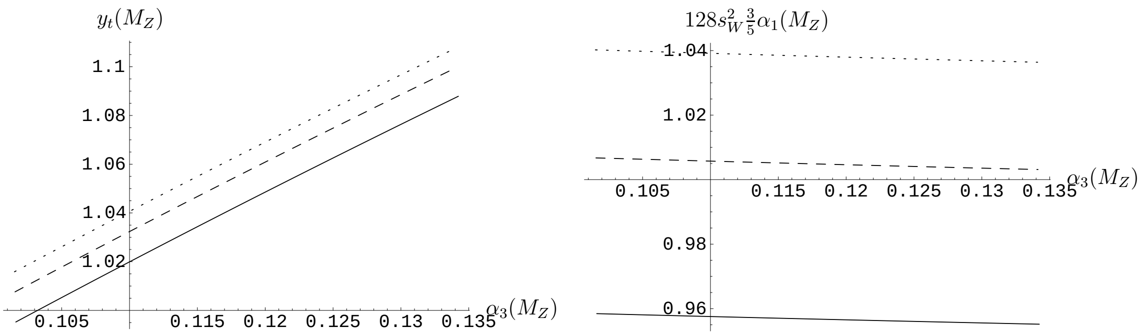


Figure 7.2: Correlations with partial coupling unification, as defined in text

correlations for this "partial coupling unification" case are shown in fig. 7.2, for three different values of $\alpha_{12}(M_{\rm GUT})$. The correlation between $\alpha_3(M_Z)$ and $\sin^2\theta_W$ is now much weaker. This means that we have found a consistent way of varying $\alpha_3(M_Z)$ (or, equivalently, $\alpha_s(M_Z)$ and the supersymmetric particle masses) while keeping $\sin^2\theta_W$ phenomenologically acceptable and, more importantly, unchanged. (Even though the latter may receive large corrections at the weak scale, these have little to do with the strong coupling constant.) One could relax the unification condition further, allowing $\alpha_1(M_{\rm GUT})$ and $\alpha_2(M_{\rm GUT})$ to differ. In view of the relatively small impact of the precise values of these couplings on the evolution, we do not pursue this further.

Let us now consider the leading-order relation $\alpha_5(M_{\rm GUT}) = \alpha_{12}(M_{\rm GUT})$. The value of $\alpha_5(M_{\rm GUT})$ now depends sensitively on $\sin^2\theta_W$, which, like the relation itself, is subject to corrections that we either do not know or neglect. One can consider the alternative (in partial unification) continuity relation $\alpha_5(M_{\rm GUT}) = \alpha_3(M_{\rm GUT})$. We will refer to the two conditions as C12 and C3, respectively. The differences between both approaches give an indication of the uncertainties due to GUT- and weak-scale threshold corrections. Both are compared in fig. 7.3. An uncertainty of $y_t(M_Z)$ is present but is not large. For given top and SUSY

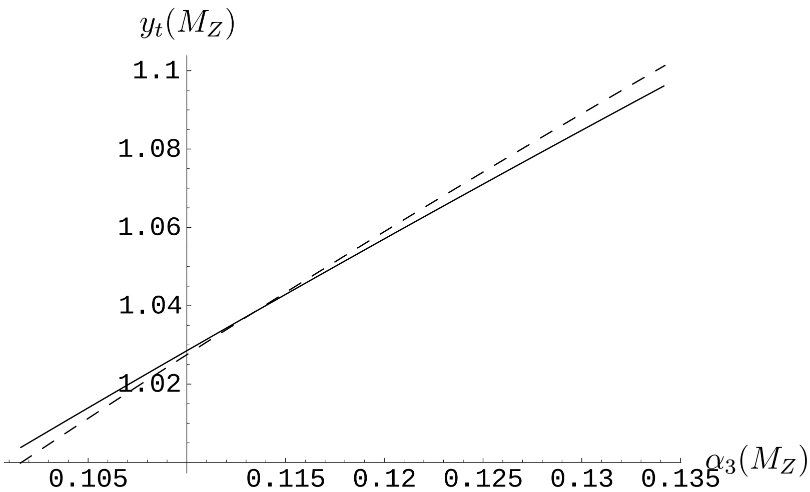


Figure 7.3: $y_t(M_Z)$ vs. $\alpha_3(M_Z)$ for two different GUT continuity conditions

particle masses, this translates into a value of $\tan\beta$ with an uncertainty in the 1-2 percent range, which is quite precise. Furthermore, the dependence of y_t on $\alpha_3(M_Z)$ is evidently well approximated by a linear function.

Next, one can investigate the impact of different values of $\alpha_{12}(M_{\rm GUT})$ and the two GUT-scale continuity conditions on the evolution of α_3. The left plot in fig. 7.4 compares four combinations; the differences are minimal. (Note that only a very small range of $\alpha_3(M_Z)$ is shown, to make the deviations visible at all.)

To complete the picture, the impact of the same set of choices on the evolution of α_1, α_2, more precisely on the unified value $\alpha_{12}(M_{\rm GUT})$, is investigated on the

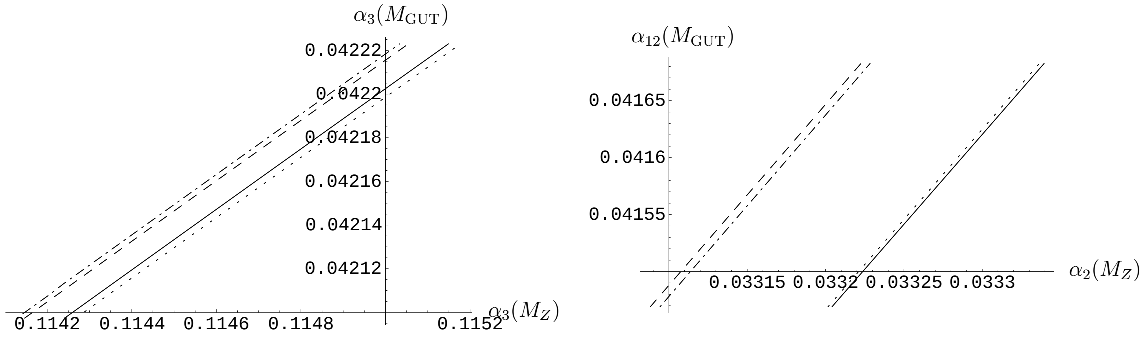


Figure 7.4: Impact of different strategies on the evolution of α_3, α_1, α_2.

right of fig. 7.4. The variation due to $\alpha_3(M_Z)$ is below one percent, while the choice of GUT continuity condition has tiny impact. We conclude that $\alpha_3(M_Z) = \alpha_3(M_Z)(y_t(M_Z))$ is well approximated by linear interpolation.

7.2 Constraints on the input parameters

The supersymmetric particle masses needed to compute the radiative effects by way of the procedure of section 5.4 are bounded from below from direct SUSY searches. Additional bounds on the A-terms come from the requirement that the vacuum exhibit the observed pattern of electroweak symmetry breaking.

For parameters not explicitly specified, we use the values of table 5.1. Note that we do not need any input on m_t and $\tan\beta$, because y_t is directly fixed by the gauge couplings and the fixed-point constraint.

7.2.1 Direct searches

The experimental lower limits from Tevatron and LEP data are summarized in table 7.1. Although strictly speaking the soft mass parameters of the first two generations are universal in the CMM model only for vanishing GUT-scale D-term and up to small corrections due to the different SU(2)×U(1) gauge quantum numbers, we use a universal squark mass $m_{\tilde{q}}$ in computing $\alpha_3^{\overline{DR}}(M_Z)$ and neglect the D-term shifts between members of the five- and ten-dimensional $SU(5)$ multiplets. Adjusting presumably would allow one to relax somewhat the limits on the parameter space we are going to find, as well as blur our predictions for the flavor-violating observables.

7.2.2 Vacuum stability and correct symmetry breaking.

The A-terms at the weak scale can be constrained by requiring that the minimum of the scalar potential is the physical one (and equivalently the latter is stable). The relevant tree-level bounds have been given in eqs. (2.55),(2.56). The

Table 7.1: Input parameter ranges. Source: ref. [79]

$\tan\beta$	FP constraint
$\lvert a_1^D\rvert$	as allowed by eq. (2.56)
$\arg(a_0)$	free, see text
$m_{\tilde{d}_{R_1}}$	> 250 GeV
$m_{\tilde{u}_{R_1}}$	> 250 GeV
$m_{\tilde{g}_3}$	> 195 GeV
$m_{\tilde{d}_{R_3}}$	> 91 GeV
$m_{\tilde{l}_3}$	> 76 GeV
$m_{\tilde{e}_{R_3}}$	> 76 GeV

parameters $m^2_{h_u}$ and $m^2_{h_d}$ are affected by large Yukawa couplings and unknown couplings among GUT Higgs multiplets. In light of these uncertainties, we will use the condition

$$|a_1^D| < 2.5\, m^2_{\tilde{q}}. \tag{7.1}$$

We will see that other constraints are always stronger than this bound.

7.3 Third-generation soft masses

Varying the input parameters in their allowed ranges, we will now investigate the impact on the soft masses $m_{\tilde{d}_{R_3}}$ and $m_{\tilde{l}_3}$ entering the formulas for the observables of chapter 6. We also consider $m_{\tilde{e}_{R_3}}$ because its lower experimental bound gives an additional constraint.

7.3.1 Correlation of $m_0, a_0, m_{\tilde{g}}(M_{\rm Pl})$ with weak-scale inputs

The procedure of sec. 5.4 allows to find the universal Planck-scale parameters m_0 and a_0 (as well as the Planck-scale gaugino mass) from the weak-scale inputs a_1^D, $m_{\tilde{q}}$, $m_{\tilde{g}_3}$, and the parameter S defined at the GUT scale. Figs. 7.5 and 7.6 show the image in the $(\bar{m}_0, a_0)$ plane of the phenomenologically allowed area in the $(m_{\tilde{q}}, a_1^D)$ plane (cut off at $m_{\tilde{q}} = 1500$ GeV) for two different values of the gluino mass. $\bar{m}_0$ is defined as

$$\bar{m}_0 = \mathrm{sgn}(m_0^2)\sqrt{|m_0^2|}. \tag{7.2}$$

We observe that the relation between weak-scale and Planck-scale parameters is rather smooth and conlude that choosing weak-scale inputs not only is desirable on practical phenomenological grounds but also a good way of parameterizing the radiative effects. The "focussing" effect on the left of the plots arises because we constrain the ratio $a_1^D/m_{\tilde{q}}$ instead of a_1^D itself, as the vacuum stability bound

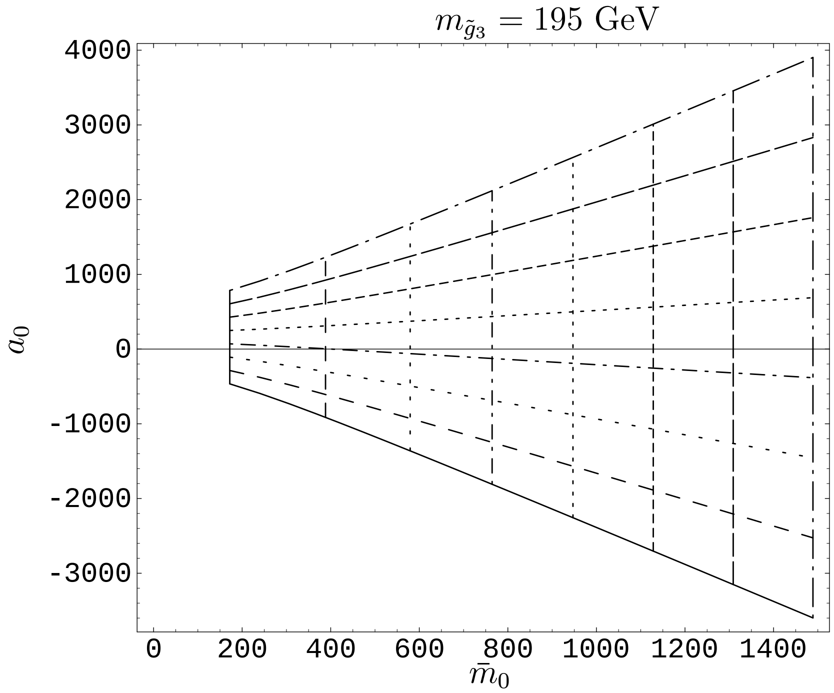


Figure 7.5: Image of an $(m_{\tilde{q}}, a_1^D/m_{\tilde{q}})$ grid in the $(\bar{m}_0, a_0)$ plane, $m_{\tilde{g}_3} = 195$ GeV, $m_{\tilde{g}}(M_{\rm Pl}) = 63 - 67$ GeV. For discussion of the grid, see text.

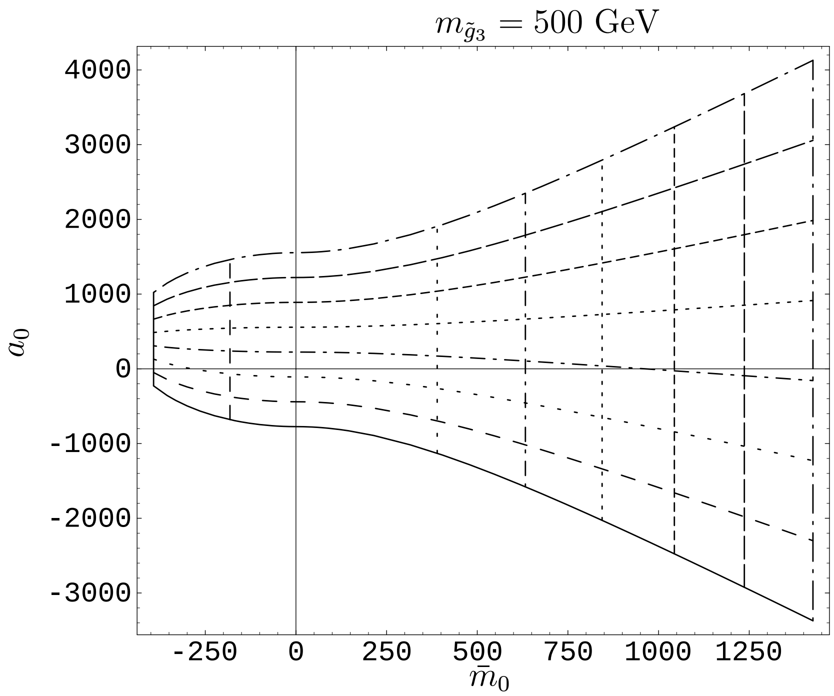


Figure 7.6: $(m_{\tilde{q}}, a_1^D/m_{\tilde{q}})$ grid in $(\bar{m}_0, a_0)$ plane, $m_{\tilde{g}_3} = 500$ GeV, $m_{\tilde{g}}(M_{\rm Pl}) = 168 - 177$ GeV

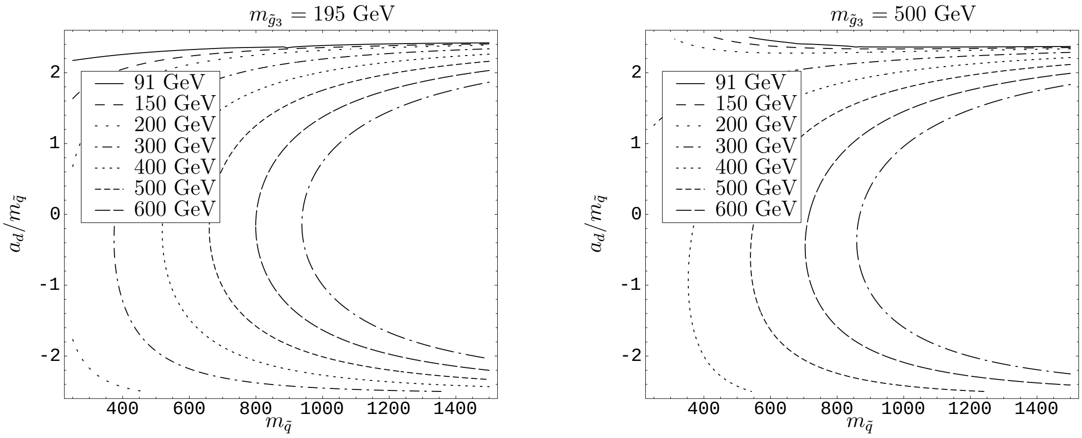


Figure 7.7: $m_{\tilde{d}_{R_3}}$ in the $(m_{\tilde{q}}, a_1^D)$ plane. The left plot has $m_{\tilde{g}_3} = 195$ GeV, the right plot corresponds to $m_{\tilde{g}_3} = 500$ GeV.

mandates. For this reason, the range of allowed a_0 also becomes larger for larger m_0^2.

Also, no "conspiracy" of Planck-scale parameters is needed to obtain phenomenologically allowed low-energy parameters, particularly not for light gluino mass. If the gluino is heavier, for small/negative m_0^2 large positive a_0 is favored. The phase of a_0 is equal to that of a_1^D, but it is not directly constrained from (2.56). This phase does not enter the renormalization-group evolution of the soft masses; however, it does have an impact on the magnitude of a_0 for given $|a_1^D|$ at M_Z. This is because a_0 and a_1^D are related by a linear shift proportional to $m_{\tilde{g}_3}$ due to the RG evolution, which can combine destructively or constructively with $|a_1^D|$. It is clear from this argument that it is sufficient to consider real a_1^D of both signs to obtain the full allowed range for the radiative effects.

7.3.2 $m_{\tilde{d}_{R_3}}$, $m_{\tilde{l}_3}$, and $m_{\tilde{e}_{R_3}}$

We now discuss the relevant third-generation soft masses. Figs. 7.7–7.9 show contour plots of $m_{\tilde{d}_{R_3}}$, $m_{\tilde{l}_3}$, and $m_{\tilde{e}_{R_3}}$ in the $(m_{\tilde{q}}, a_1^D)$ plane, again for two values of the gluino mass. It is reassuring that for positive a_1^D, the experimental lower bounds, particularly that on $m_{\tilde{e}_{R_3}}$, give stronger constraints than the somewhat arbitrary vacuum-stability bound $|a_1^D| < 2.5\, m_{\tilde{q}}$. In other words, the parameter values allowed from direct searches correspond to a stable physical vacuum.

Therefore, while smallest third-generation masses (corresponding to large splittings and therefore to possibly large weak decay and mixing amplitudes) do appear for large A-terms, there will be no dependence of the weak phenomenology on the details of that bound.

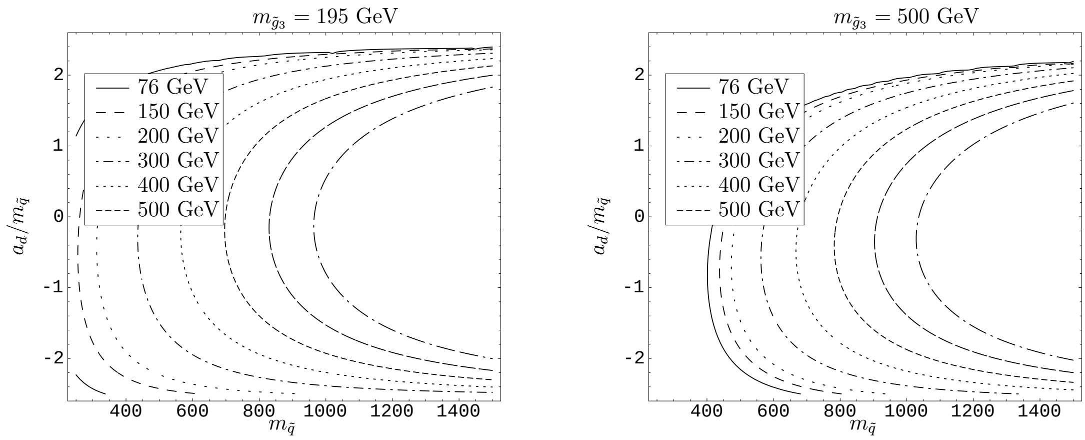


Figure 7.8: $m_{\tilde{l}_3}$ in the $(m_{\tilde{q}}, a_1^D)$ plane.

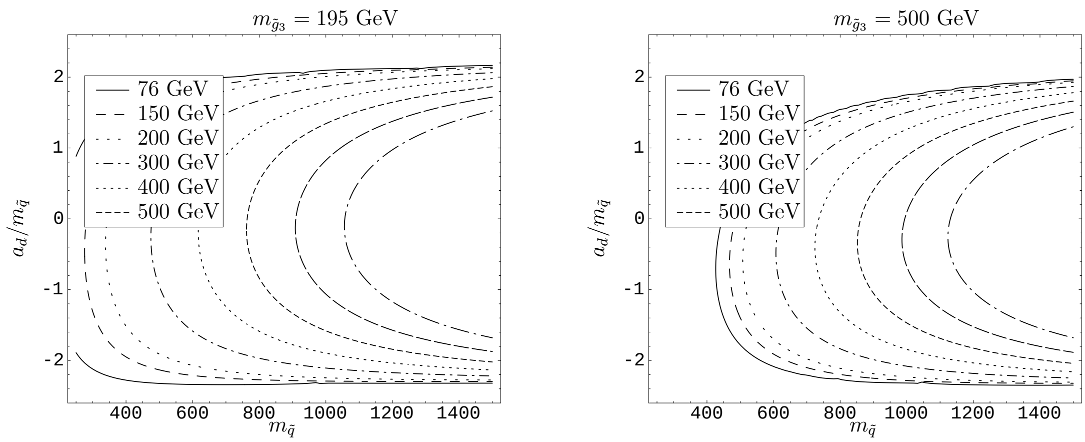


Figure 7.9: $m_{\tilde{e}_{R_3}}$ in the $(m_{\tilde{q}}, a_1^D)$ plane.

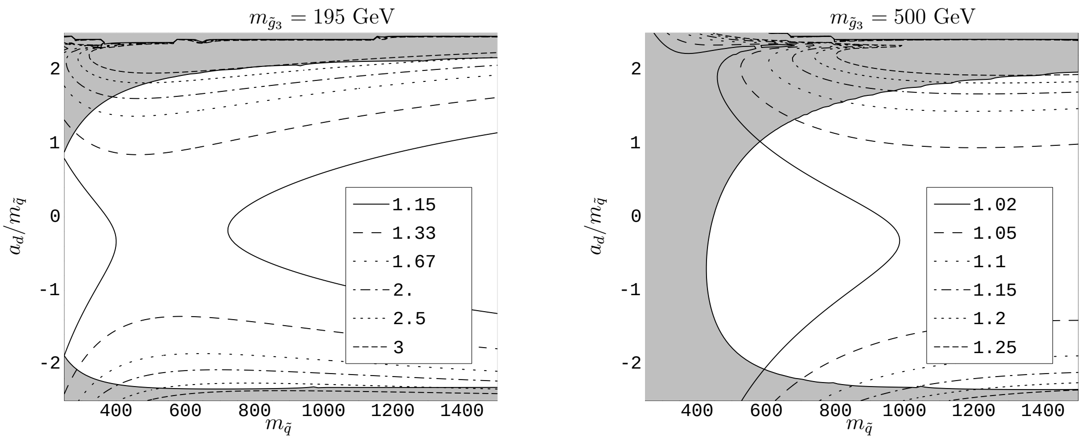


Figure 7.10: $\Delta M_{B_s}^{(\max)}/\Delta M_{B_s}^{(SM)}$ including impact of the $m_{\tilde{\tau}_R}$ constraint, for $m_{\tilde{g}_3} =$ 195 and 500 GeV. The shaded area is excluded. (Cf. fig. 7.9.)

7.4 $\Delta F = 1$ and $\Delta F = 2$ FCNC processes

We now turn to the investigation of the flavor-violating observables from chapter 6.

7.4.1 B_s–$\bar{B}_s$ mixing

The ratio of the mass difference in the B_s system, normalized to its standard model value, is shown in the contour plots of fig. 7.10. In doing this, we add the magnitudes of the standard and SUSY contributions linearly to obtain the maximal effect. It is important to notice that the phase of the CMM contribution to B_s-mixing is unconstrained, and so it is also possible to have destructive interference between the standard and SUSY contributions. The shaded area is the one excluded by direct searches for the $\tilde{\tau}$ particle. We find that the largest effects arise for smallest gluino mass. Discarding the region excluded by the $\tilde{e}_{R_3}$ mass, an enhancement of the mass splitting of a factor of 3 with respect to the standard model is possible. Larger splitting would be found for larger a_1^D, which is ruled out, showing the strong constraining impact of already the current status of SUSY particle searches. For a larger gluino mass of 500 GeV, the effect is unobservably small throughout the parameter space, due to the parametric suppression of the mixing amplitude by $m_{\tilde{g}_3}^2$.

7.4.2 $\tau \to \mu\gamma$ decay

The radiative flavor-changing τ decay, completely negligible in the Standard model even with Neutrino masses, receives much larger contributions in the CMM model due to the mass nonuniverality between left-handed sleptons of different generations. However, there is additional dependence on parameters from the Higgs sector, notably $\tan\beta$ and the μ parameter, both of which enter the chargino and neutralino mass matrices. $\tan\beta$ also appears in the chargino coupling to charged leptons and sneutrinos. However, the fixed-point constraint imposed by us makes $\tan\beta$ a dependent parameter. One has to keep in mind, however, the lower bounds on chargino and neutralino masses from direct searches. They translate to constraints on μ, but also on $m_{\tilde{q}}$, that are strongest for low gluino mass due to the gaugino mass unification. (Recall that $m_{\tilde{q}}$ enters the formula (5.2) determining $\alpha_3(M_Z)$.).

Let us consider the impact in the $(m_{\tilde{q}}, \mu)$ plane. This is shown in fig. 7.11. The shaded areas are excluded due to an unphysically low lightest neutralino or chargino mass. We see that for $m_{\tilde{g}_3}$ at its lowest experimentally allowed value, only a small range of μ between about -75 GeV and -200 GeV is allowed. (The lower limit depends on $m_{\tilde{q}}$. The constraint gets relaxed very quickly with increasing gluino mass.) The dependence on $m_{\tilde{q}}$ is relatively mild; however, very light squark masses < 350 GeV are excluded altogether. For heavier gluino mass, there is just an excluded interval for μ ranging from about -55 GeV to 150 GeV, practically independent of $m_{\tilde{q}}$.

At this point we must mention that the sign of μ can constrained from the decay $b \to s\gamma$, see for instance [102, 103]. In our case, where the top squark A-term A_t is close to its IR fixed point of about $-2m_{\tilde{g}_3}$, positive μ would be favored. That, in turn, would seem to exclude the light gluino mass of $m_{\tilde{g}_3}$ altogether, where we found above that only $\mu < 0$ is allowed. We caution, however, that $\mu > 0$ becomes allowed already for $m_{\tilde{g}_3} \approx 210$ GeV, or in any case for rather light gluino. Furthermore, refs. [102, 103] work under the assumption of a (very) heavy gluino, so their conclusion does not necessarily apply to the case at issue. Finally, we opine that the status of theoretical errors in $b \to s\gamma$ is not fully clear at this time.

Figs. 7.12, 7.13 and 7.14 show $BR(\tau \to \mu\gamma)$ for six representative cases. In the figures, the shaded area has a branching ratio above the 90% upper limit from Belle [91]. The impact of the $\tilde{\tau}$ search is also shown, in the form of the continuous black lines. Within much of the parameter space allowed by the direct searches, the branching ratio is still below the experimental upper limit; however, for very light third-generation sparticle the limit would generally be exceeded. In that respect, the situation is similar to the B_s-mixing.

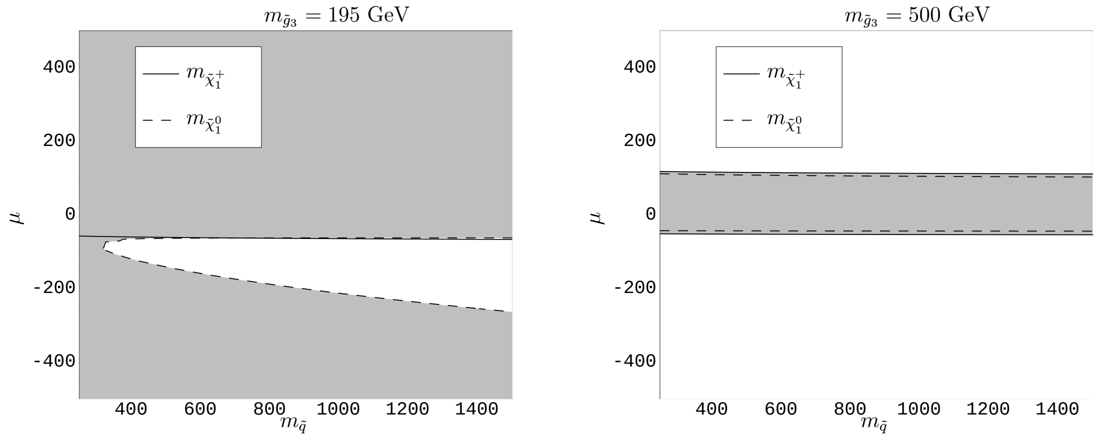


Figure 7.11: Impact of the experimental lower bounds on $m_{\chi_1^0}$ and $m_{\chi_1^+}$ in the $(m_{\tilde{q}}, \mu)$ plane. Left plot corresponds to $m_{\tilde{g}_3} = 195$ GeV, right plot to $m_{\tilde{g}_3} = 500$ GeV. The shaded areas are excluded. The dashed and continuous contours correspond to the experimental lower limits on $m_{\chi_1^0}$ and $m_{\chi_1^+}$.

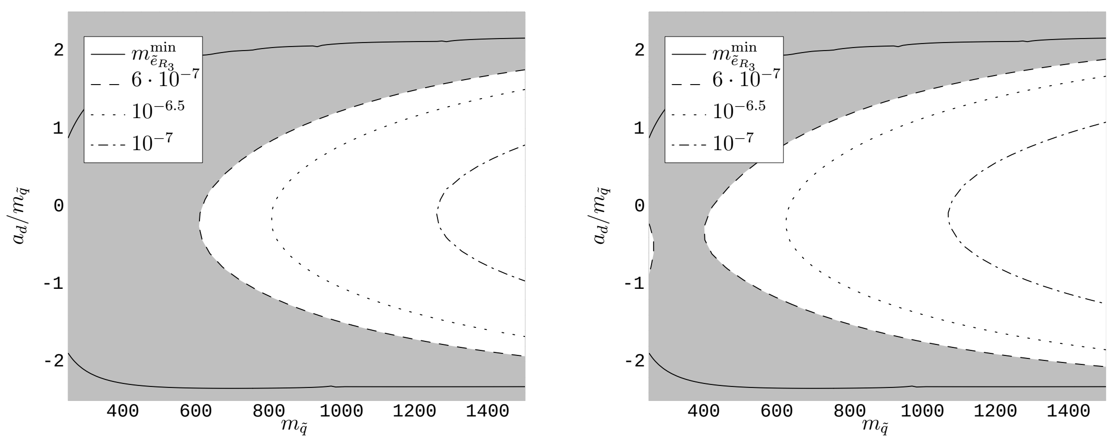


Figure 7.12: $BR(\tau \to \mu\gamma)$ in the $(m_{\tilde{q}}, a_{d_1})$ plane for $m_{\tilde{g}_3} = 195$ GeV and $\mu = -75$ GeV (left) and $\mu = -150$ GeV (right).

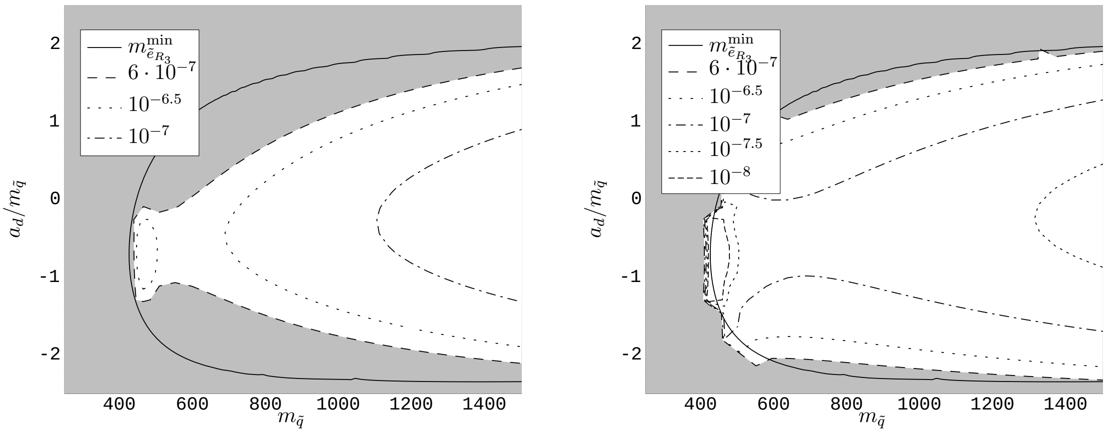


Figure 7.13: $BR(\tau \to \mu\gamma)$ in the $(m_{\tilde{q}}, a_{d_1})$ plane for $m_{\tilde{g}_3} = 500$ GeV and $\mu = -55$ GeV (left) and $\mu = -250$ GeV (right).

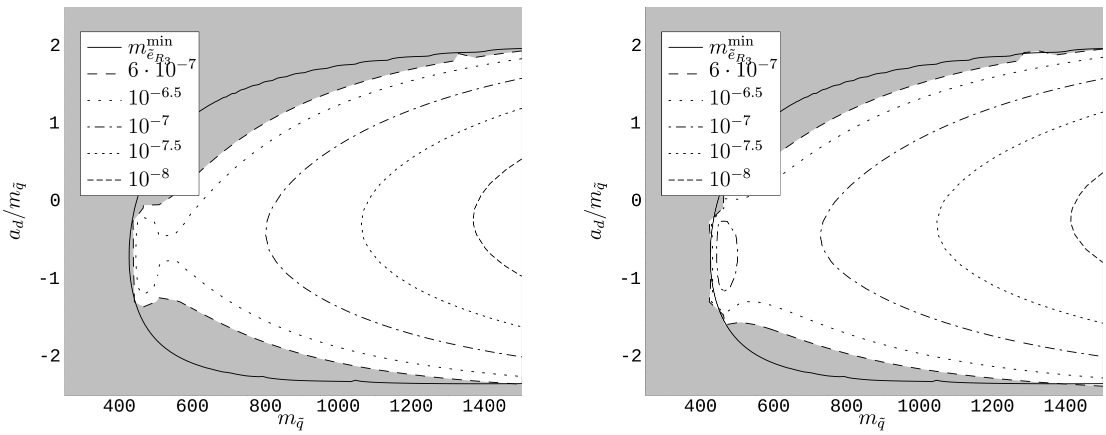


Figure 7.14: $BR(\tau \to \mu\gamma)$ in the $(m_{\tilde{q}}, a_{d_1})$ plane for $m_{\tilde{g}_3} = 500$ GeV and $\mu = 175$ GeV (left) and $\mu = 300$ GeV (right).

Chapter 8

Conclusions and outlook

In this dissertation, we have studied hadronic and leptonic flavor violation in supersymmetric unified theories. In particular, we have studied the impact of the supersymmetric $SO(10)$-grand-unified model of Chang, Masiero, and Murayama, which is motivated by the observed large atmospheric mixing angle together with the approximate observed degeneracy of neutrino masses.

Our treatment is the first in this neutrino-mixing-inspired $SO(10)$-unified model that relates the Planck-scale and weak-scale parameters via the renormalization group and takes into account the corresponding correlations and experimental constraints.

We have computed the renormalization-group running of the relevant parameters and provided a compact recipe to find their weak-scale values from a small set of weak-scale input parameters, in the spirit of the "bottom-up approach" to minimal supergravity [85]. As a by-product, we have computed the full one-loop renormalization-group equations in matrix form of the most general superpotential for the matter sector of softly broken SUSY $SO(10)$.

Furthermore we have computed the one-loop effective Lagrangian within the CMM model for B_s mixing, for the hadronic $\Delta F = 1$ decay $B_d \to \phi K_S$, and for the leptonic flavor-violating decay $\tau \to \mu\gamma$. We find that the B_s-mixing amplitude can get large contributions from gluino exchange due to the large mixing angle in the right-handed down-squark mass matrix. For the case of $\tau \to \mu\gamma$, our result differs in its analytical form from the computation by Hisano et al. [24], which needs to be investigated. In the case of the decay $B_d \to \phi K_S$, our penguin contributions to the Wilson coefficients agree with [99]. The box contributions have been computed by the authors of [101]. We agree with them except for two signs. We have argued that due to the chirality structure of flavor violation in the CMM model, no large contributions to $B_d \to \phi K_S$ are to be expected. A quantitative analysis of this decay and of its CP asymmetry, which has been getting much interest due to the unclear experimental status, which shows a possible discrepancy with the standard model prediction and more recently between the Belle and Babar experiments, involves a number of hadronic matrix elements and phases which at present are poorly known.

In a numerical study, we therefore focused on $B_s - \bar{B}_s$ mixing as well as $\tau \to \mu\gamma$. We first studied the correlation between the universal mass parameters of the first two generations and the A-term of the down squark and the Planck-scale values m_0^2 and a_0 and found a smooth relation, justifying the use of the former in parameterizing the radiative effects of the CMM model. We considered the constraining impact of lower bounds on sparticle masses from direct searches and studied the flavor-violating hadronic and leptonic observables, illustrated by suitable contour plots. In the B_s system, the standard model prediction for the mass difference ΔM_{B_s} can be increased by a factor of up to four. For the τ decay, we find a large effect over much of the parameter space, which, particularly in the case of a light gluino, already rules out a significant portion of the otherwise allowed parameter space. In conclusion, both findings suggest that this model is most easily confirmed or excluded from the flavor-violating mode $\tau \to \mu\gamma$.

In both the hadronic and leptonic $\Delta F = 1$ modes, it would be worthwile to study the additional contributions for larger $\tan\beta$, when left-right mixing among the sfermions is no longer negligible. In the standard model, the $\Delta B = 1$ operator Q_8 is of order (or has a Wilson coefficient of order) m_b. Due to the left-right mixing in the gluino mass term, there generally are corrections to the standard-model coefficient C_8 and to the coefficient C_8' of the parity-reflected operator Q_8' in eq. (6.38) that are enhanced by $m_{\tilde{g}}/m_b$. However, a consistent study in the case of B decays must also include contributions from Higgs penguins, beyond the gluon penguins and boxes computed in this dissertation.

In the B_s mixing, there are no penguin contributions at one loop, and the standard-model operator is not suppressed by m_b/M_W. Correspondingly, additional effects for moderate $\tan\beta$ are not expected to be large: while additional chirality structures may arise, there is no parametric enhancement and also, unlike in the K^0 system, no chiral enhancement of the hadronic matrix element.

Appendix A

Crash review of SO(10)

A.1 Lie Algebra of SO(10)

SO(N) is the group of orthogonal N-row matrices of determinant one. We are mainly interested in the case of even $N = 2n$, making use of the symbol n when talking about results special to this case. Sometimes we put $N = 10$, $n = 5$ to obtain concrete SO(10) results.

The tangent space of SO(N) at the identity is the set of all traceless symmetric matrices. Thus an obvious basis of the Lie algebra is given by

$$\left(\tilde{T}_{ij}\right)_{mn} = i\left(\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}\right) \tag{A.1}$$

and the basis vectors satisfy

$$[\tilde{T}_{ij}, \tilde{T}_{kl}] = i\left(\delta_{jk}\tilde{T}_{il} + \delta_{il}\tilde{T}_{jk} - \delta_{jl}\tilde{T}_{ik} - \delta_{ik}\tilde{T}_{jl}\right) \tag{A.2}$$

Here i, j, k, l, m, n run from 1 to N. This is the fundamental, N-dimensional representation. It is real, as is evident from the structure constants. The $\tilde{T}_{ij}$ are antisymmetric and also $\tilde{T}_{ji} = -\tilde{T}_{ij}$, so there are only $N(N-1)/2 = 45$ linearly independent generators.

To have a continous gauge coupling when breaking to SU(5), one should rescale the generators by a factor $1/\sqrt{2}$:

$$T_{ij} = \frac{1}{\sqrt{2}}\tilde{T}_{ij} \tag{A.3}$$

This affects the Casimir invariants as conventionally defined in the particle physics literature, and in this note.

A.2 Some tensor representations

From the tensor products of the fundamental representation, some useful irreducible representations can be found.

A.2.1 Adjoint representation

This is given by the antisymmetric tensor s_{mn}. Dimension $N(N-1)/2 = 45$. It is of course also found by the action of the Lie algebra on itself.

A.2.2 120-dimensional representation

The three-index antisymmetric tensor a_{mnp} contains $\binom{N}{3} = 120$ independent entries. It forms the basis for an irreducible representation. We fix by convention $m < n < p$, and likewise for higher antisymmetric tensors. This eliminates overcounting factors if we sum over repeated indices.

A.2.3 252-dimensional reducible representation. Levi-Civita tensor, duality transform, $252 = 126 + \overline{126}$

$a_{m_1 \cdots m_n}$ contains $\binom{N}{n} = 252$ independent entries. However, it is not irreducible: There exists a unique N-dimensional completely antisymmetric tensor $\epsilon_{m_1 \cdots m_N}$ with $\epsilon_{1 \cdots N} = 1$. It is invariant under SO(N). If $a_{m_1 \cdots m_k}$ is antisymmetric, so is $\epsilon_{n_1 \cdots n_{N-k} m_1 \cdots m_k} a_{m_1 \cdots m_k}$. This defines a duality transformation from the rank-k antisymmetric tensors to the rank-$(N-k)$ ones. In the case of $N = 2n$, the space of rank-n tensors is invariant. Let us define ϵ_{LM} to be a shorthand for n-indices $L = (l_1, \ldots, l_n)$ and $M = (m_1, \ldots, m_n)$. As before, we restrict to $l_1 < l_2 < \ldots$ and so on. (Note also that for n odd, $\epsilon_{LM} = -\epsilon_{ML}$.) Then

$$\tilde{a}_L \equiv i^n \epsilon_{LM} a_M \tag{A.4}$$

is the dual of a and, furthermore, $\tilde{\tilde{a}} = a$. Consequently,

$$\begin{aligned} a^\pm &\equiv \frac{1}{2}(a \pm \tilde{a}) && \text{(A.5)} \\ a_L^\pm &= \frac{1}{2}(\delta_{LM} \pm i^n \epsilon_{LM})\, a_M && \text{(A.6)} \\ &\equiv (P_T^\pm)_{LM} a_M && \text{(A.7)} \end{aligned}$$

are eigentensors of the duality transform with eigenvalues ± 1. They are invariant, 126-dimensional subspaces of the 252-dimensional representation. The tensor projectors $P_T^\pm$ satisfy (with a a rank-five antisymmetric tensor):

$$\begin{aligned} \left(P_T^\pm\right)^2 &= P_T^\pm && \text{(A.8)} \\ P_T^\pm P_T^\mp &= 0 && \text{(A.9)} \\ P_T^+ + P_T^- &= 1 && \text{(A.10)} \\ P_T^\pm a &= a P_T^\mp && \text{(A.11)} \end{aligned}$$

A.3 Spinor representations

The procedure follows that for the construction of Dirac spinors. This similarity stems from the fact that the Lorentz group is SO(3,1), a close relative of SO(4). For details, particularly on the construction of gamma matrices, see [104].

A.3.1 Clifford algebra and spinor generators

One can explicitly construct N matrices γ_m with 2^n rows, satisfying

$$\{\gamma_m, \gamma_n\} = 2\delta_{mn}\mathbf{1} \tag{A.12}$$

(This holds also for odd N, with $N = 2n + 1$.) Then the matrices

$$\tilde{\sigma}_{ij} \equiv \frac{i}{4}[\gamma_i, \gamma_j] \tag{A.13}$$

represent the Lie algebra, since they satisfy (A.2). Consequently, they generate a 2^n-dimensional (spinor) representation. As with the generators of the fundamental representation, a rescaling

$$\sigma_{ij} = \frac{1}{\sqrt{2}}\tilde{\sigma}_{ij} \tag{A.14}$$

is needed to have continuous coupling at the SO(10) and SU(5) breaking scales. We also have

$$[\overset{(\sim)}{\sigma}_{ij}, \gamma_m] = -(\overset{(\sim)}{T}_{ij})_{mn}\gamma_n \tag{A.15}$$

which allows to construct bilinears transforming like tensors of the fundamental representation.

A.3.2 γ_5, chirality projectors, Chisholm identity

If $N = 2n$, the matrix

$$\gamma_5 \equiv (-i)^n \gamma_1 \cdots \gamma_N \tag{A.16}$$

anticommutes with all γ's and commutes with the σ's. Furthermore we have $\gamma_5^2 = \mathbf{1}$. Thus the spinor projectors

$$P_S^{\pm} \equiv \frac{1}{2}(1 \pm \gamma_5) \tag{A.17}$$

satisfy eqs. (A.8,A.9,A.10) and project onto invariant subspaces. That is, the 2^n-dimensional spinor representation decomposes into two 2^{n-1}-dimensional "chiral" irreps. (For odd $N = 2n+1$, there is no such extra matrix and there is only one, 2^n-dimensional, spinor irrep.)

A.3.2.1 Chisholm identity

Multiplying both sides of (A.16) from the left with $\gamma_{m_1}\cdots\gamma_{m_k}$ where all γ's are different, we can permute the product $\gamma_1\cdots\gamma_N$ on the right so that the first k matrices appear as the sub-product $\gamma_{m_k}\cdots\gamma_{m_1}$. We obtain

$$\gamma_{m_1}\cdots\gamma_{m_k}\gamma_5 = (-i)^n(-)^{\frac{(k-1)k}{2}}\epsilon_{m_1\cdots m_k m_{k+1}\cdots m_N}\gamma_{m_{k+1}}\cdots\gamma_{m_N}. \tag{A.18}$$

(No sum over repeated indices.) The indices $m_{k+1}\dots m_N$ label the remaining γ's in some arbitrary order, and the antisymmetric tensor element gives the sign coming from permuting the γ's. Multiplying both sides by this ϵ element (and relabeling some indices as well as putting $k\to N-k$), one obtains

$$\gamma_{m_1}\cdots\gamma_{m_k} = (-i)^n(-)^{\frac{(k-1)k}{2}}\epsilon_{m_1\cdots m_N}\gamma_{m_{k+1}}\cdots\gamma_{m_N}\gamma_5 \tag{A.19}$$

This formula can be cast into one with summed indices if a symmetry factor $1/(N-k)!$ is introduced on the r.h.s. Note that it is only valid if all indices on the left are different, which makes the expression on the l.h.s. completely antisymmetric in its indices.

A.3.2.2 Connection between tensor and spinor projectors

Let $\Gamma_{m_1\cdots m_k}\equiv\gamma_{m_1}\cdots\gamma_{m_k}$. For strings of n gamma matrices, with $L=(l_1,\cdots,l_n)$, (A.19) gives

$$\Gamma_L = (-i)^{n^2}\epsilon_{LM}\Gamma_M\gamma_5. \tag{A.20}$$

Because we require $m_1<m_2<\cdots$, there is only one M with non-vanishing ϵ_{LM} for a given L, so we can take the sum over M. Then

$$\Gamma_L P_S^{\pm} = \Gamma_L\frac{1}{2}(1\pm\gamma_5) = \frac{1}{2}(\Gamma_L \pm (-i)^{n^2}\epsilon_{LM}\Gamma_M) \tag{A.21}$$

A.3.3 Charge conjugation matrix, irreducible bilinears

The application of SO(10) to particle physics involves putting the fermions into irreducible spinor representations both under SO(10) and the Lorentz group:

$$\psi^{--}_{a\alpha} = P^-_{ab}\Pi^-_{\alpha\beta}\psi_{b\beta} \tag{A.22}$$

where latin and greek letters pertain to SO(10) and the Lorentz group, respectively. To avoid gauge invariant mass terms and obtain the correct $V-A$ structure at low energies, only one type of irrep should be present (by convention, we give it negative chiralities under SO(10) and SO(3,1)).

The possible bilinears entering the Yukawa terms then have the SO(10) structure $\psi^{--}_a M_{ab}\psi^{--}_b$ with a suitable matrix M. (There may also be several different incarnations (copies) of this irrep, in which case there is also a flavor structure.)

A possible basis is the set of matrices $C\gamma_{m_1}\cdots\gamma_{m_k}$ where C is an arbitrary non-singular matrix and the indices m_i can be chosen to be all different (making the product antisymmetric in the indices). It is possible [104] to find unitary C such that

$$C\gamma_m C^{-1} = (-)^n \gamma_m^T, \tag{A.23}$$

which has the consequence

$$C\gamma_5 C^{-1} = (-)^n \gamma_5^T \tag{A.24}$$

and motivates the name "charge-conjugation matrix". Then using Fermi symmetry one can show that all bilinears containing even numbers of γ's vanish identically. Furthermore, by the Chisholm identity and the fact that our spinors are chiral, it suffices to consider the cases $k = 1, 3, 5$. One finds that for $k = 3$ the flavor structure is antisymmetric while for $k = 1, 5$ it is symmetric (for $N = 2n = 10$). The spinor generators obey

$$C\sigma_{mn} C^{-1} = -\sigma_{mn}^T \tag{A.25}$$

(for any n). Combining this with (A.15), we find that the bilinears transform like antisymmetric tensors of ranks 1, 3, and 5 under SO(10). Invariant Yukawa terms can now be formed by contracting the bilinears with suitable scalars transforming like antisymmetric tensors. In the case of the rank-5 tensor, (A.21) means

$$\Gamma_L P_S^- h_L = \Gamma_X (P_T^+)_{XL} h_L = \Gamma_X h_X^+, \tag{A.26}$$

so only the self-dual part of h couples. However, if h_L was canonically normalized, then h^+ is not, because the "transformation" P_T^+ defining it is not unitary. Furthermore every component appears twice ($h_L = ih_M$ for $\epsilon_{LM} = +1$), which would require a suitable symmetry factor for the coupling term.

One way to fix this is to restrict the summation over L to one half-set of the possible 256 indices and define $h_L^+ = 1/\sqrt{2}(h_L + i\epsilon_{LM} h_M)$. This is what we will do here.

Now all Yukawa couplings get the same symmetry factor 1/2 because of the (anti)symmetry of the bilinears. Thus with i, j denoting families, the most general Yukawa potential involving the matter fields is

$$\begin{aligned}\mathcal{L} \;=\;& \frac{1}{2} Y_{ij}^{10}\, \psi_i^T \,(C\gamma_m P_S^-)\, \psi_j\, h_m^{10} + \frac{1}{2} Y_{ij}^{120}\, \psi_i^T\, (C\Gamma_{[mnr]} P_S^-)\, \psi_j\, h_{[mnr]}^{120} \\ &+ \frac{1}{2} Y_{ij}^{126}\, \psi_i^T\, (C\Gamma_{[mnrst]} P_S^-)\, \psi_j\, h_{[mnrst]}^{126}. \end{aligned} \tag{A.27}$$

Remark: In a supersymmetric theory, the superpotential has the same form with all fields replaced by the superfields they belong to. The Fermi symmetry is changed to Bose symmetry, but the Dirac charge conjugation matrix is also absent. Both changes compensate each other such that the allowed bilinears are again those transforming as tensors of rank 1, 3, or 5.

(The gauge coupling terms contain the antisymmetric bilinear $\psi^{--\dagger}\tilde{\sigma}_{mn}\psi^{--}$, the kinetic term $\psi^{--\dagger}\psi^{--}$).

A.4 Loop calculations

SO(10) loop calculations involve group-theoretical factors resembling in part the color factors in QCD calculations and, because SO(10) spinors are involved, the traces over gamma matrices in Dirac algebra.

A.4.1 Casimir invariants

The Casimir invariants

$$\begin{aligned}\mathrm{tr}_R(T^a(R)T^b(R)) &= T(R)\delta^{ab} && \text{(A.28)}\\ (T^a(R)T^a(R))_{mn} &= C_2(R)\delta_{mn} && \text{(A.29)}\end{aligned}$$

can be found for the spinor and fundamental representations by explicit computation (we fix $i<j, k<l$):

$$\begin{aligned}\mathrm{tr}(\sigma_{ij}\sigma_{kl}P_S^-) &= 2^{n-4}\delta_{ij,kl} && \text{(A.30)}\\ \sigma_{ij}\sigma_{ij}P_S^- &= \frac{N(N-1)}{16}P_S^- && \text{(A.31)}\\ \mathrm{tr}(T_{ij}T_{kl}) &= \delta_{ij,kl} && \text{(A.32)}\\ (T_{ij}T_{ij})_{mn} &= \frac{1}{2}(N-1)\delta_{mn} && \text{(A.33)}\end{aligned}$$

and consequently

$$T(N)=1, \qquad C_2(N)=\frac{1}{2}(N-1)=\frac{9}{2}, \tag{A.34}$$

$$T(2^{n-1})=2^{n-4}=2, \qquad C_2(2^{n-1})=\frac{N(N-1)}{16}=\frac{45}{8} \tag{A.35}$$

For the higher representations, we follow the procedure given in [105] and find

$$\binom{2n}{3}=120 \;:\; T=(n-1)(2n-3)=28, \tag{A.36}$$

$$C_2=\frac{3}{2}(2n-3)=\frac{21}{2} \tag{A.37}$$

$$\frac{1}{2}\binom{2n}{n}=126 \;:\; T=\frac{1}{2}\binom{2n-2}{n-1}=35, \tag{A.38}$$

$$C_2=\frac{1}{2}n^2=\frac{25}{2} \tag{A.39}$$

Remark: C, C_2 as defined above obviously depend on the choice of basis (including normalization) of the Lie algebra. A true Casimir operator, constant on each irrep and invariant under change of basis, can be defined making use of the Killing form. See e.g. [105].

A.4.2 Loop factors and traces

The Feynman rules corresponding to the Yukawa couplings when combined with a closed loop will lead to expressions like for example $\mathrm{tr}(C\Gamma_L P_S^-)^\dagger C\Gamma_M P_S^-$ which we need to evaluate. To do this, we need the hermiticity properties of C, Γ, P_S^-, and trace rules.

A.4.2.1 Hermiticity properties

From $\gamma_m^\dagger = \gamma_m$ we have

$$\Gamma_L^\dagger = (\gamma_{l_1} \cdots \gamma_{l_k})^\dagger = \gamma_{l_k} \cdots \gamma_{l_1} = (-)^{(k-1)k/2}\Gamma_L \tag{A.40}$$

The sign is positive for simple indices and five-indices, negative for a three-index. Furthermore, γ_5 is hermitian and so are the projectors. The charge conjugation matrix is unitary.

A.4.2.2 Trace rules

This is completely analogous to the Lorentz case (including proofs):

$$\begin{aligned}
\mathrm{tr}(\gamma_{m_1} \cdots \gamma_{m_{2k+1}}) &= 0 && \text{(A.41)}\\
\mathrm{tr}(\gamma_{m_1} \cdots \gamma_{m_{2k+1}}\gamma_5) &= 0 \qquad (k<n) && \text{(A.42)}\\
\mathrm{tr}(\gamma_{m_1} \cdots \gamma_{m_{2k}}\gamma_5) &= 0 \qquad (k<n) && \text{(A.43)}\\
\mathrm{tr}(\gamma_{m_1} \cdots \gamma_{m_N}\gamma_5) &= -i^n \epsilon_{m_1\cdots m_N} && \text{(A.44)}\\
\mathrm{tr}(\gamma_{m_1} \cdots \gamma_{m_{2k}}) &= 2^n s_{m_1\cdots m_{2k}} && \text{(A.45)}
\end{aligned}$$

In the last line, the tensors s satisfy the recursion relation

$$\begin{aligned}
s_{m_1\cdots m_{2k}} &= \delta_{m_1m_2} s_{m_3\cdots m_{2k}} - \delta_{m_1m_3} s_{m_2m_4\cdots m_{2k}} + \cdots - \cdots, && \text{(A.46)}\\
s_{m_1m_2} &= \delta_{m_1m_2}. && \text{(A.47)}
\end{aligned}$$

A.4.2.3 Collection of loop factors

The two kinds of object that appear in one-loop calculations are

$$\begin{aligned}
&\mathrm{tr}(C\Gamma_{i_1\cdots i_k} P_S^- (C\Gamma_{j_1\cdots j_l} P_S^-)^\dagger), && \text{(A.48)}\\
&C\Gamma_{i_1\cdots i_k} P_S^- (C\Gamma_{i_1\cdots i_k} P_S^-)^\dagger, && \text{(A.49)}
\end{aligned}$$

the second of which involves an implicit sum over $i_1, \ldots, i_k$. $k, l = 1, \ldots, 5$, and we restrict to $i_1 < \cdots < i_k$ etc.

We obtain

$$\begin{aligned}
\mathrm{tr}(C\Gamma_{i_1\cdots i_k} P_S^- (C\Gamma_{j_1\cdots j_l} P_S^-)^\dagger) &= 2^{n-1}\delta_{i_1\cdots i_k, j_1\cdots j_l}\delta_{kl} && \text{(A.50)}\\
C\Gamma_{i_1\cdots i_k} P_S^- (C\Gamma_{i_1\cdots i_k} P_S^-)^\dagger &= \left[\frac{1}{2}\right] \binom{2n}{k} P_S^- && \text{(A.51)}
\end{aligned}$$

In the case of an n-index in (A.51), if the summation is only over half the allowed indices, one has to insert the indicated factor of 1/2. Then for all k the factor on the right-hand side is simply the dimension of the rank-k antisymmetric tensor irreducible representation.

Appendix B

Renormalization group equations

In this appendix we collect the matrix-valued one-loop renormalization group equations for the most general renormalizable SO(10) softly broken SUSY GUT, neglecting only unknown Higgs self couplings. The superpotential and soft-breaking terms are defined in equations (5.41) and (5.42).

We also list the RGEs for the minimal SU(5) model augmented with right-handed neutrinos appearing as an intermediate effective theory in the CMM model, with superpotential given in (4.12). In this case, however, we neglect the Yukawa matrix Y^D. Equations including the effect of Y^D are given in [24]. They agree with our equations if Y^D is neglected.

B.1 SO(10)

$$\begin{aligned}
\dot{Y}^{10} &= 20\, Y^{10} Y^{10\dagger} Y^{10} + 8\,\mathrm{tr}(Y^{10} Y^{10\dagger}) Y^{10} - \frac{63}{2}\, g^2 Y^{10} \\
&\quad +120\,(Y^{120} Y^{120\dagger} Y^{10} + Y^{10} Y^{120\dagger} Y^{120}) \\
&\quad +126\,(Y^{126} Y^{126\dagger} Y^{10} + Y^{10} Y^{126\dagger} Y^{126}) && \text{(B.1)} \\
\dot{Y}^{120} &= 240 Y^{120} Y^{120\dagger} Y^{120} + 8\,\mathrm{tr}(Y^{120} Y^{120\dagger}) Y^{120} - \frac{87}{2}\, g^2 Y^{120} \\
&\quad +10\,(Y^{10} Y^{10\dagger} Y^{120} + Y^{120} Y^{10\dagger} Y^{10}) \\
&\quad +126\,(Y^{126} Y^{126\dagger} Y^{120} + Y^{120} Y^{126\dagger} Y^{126}) && \text{(B.2)} \\
\dot{Y}^{126} &= 252\, Y^{126} Y^{126\dagger} Y^{126} + 8\,\mathrm{tr}(Y^{126} Y^{126\dagger}) Y^{126} - \frac{95}{2}\, g^2 Y^{126} \\
&\quad +10\,(Y^{10} Y^{10\dagger} Y^{126} + Y^{126} Y^{10\dagger} Y^{10}) \\
&\quad +120\,(Y^{120} Y^{120\dagger} Y^{126} + Y^{126} Y^{120\dagger} Y^{120}) && \text{(B.3)} \\
\dot{A}^{10} &= 30\,(Y^{10} Y^{10\dagger} A^{10} + A^{10} Y^{10\dagger} Y^{10}) + 8\,\mathrm{tr}(Y^{10} Y^{10\dagger}) A^{10} \\
&\quad +16\,\mathrm{tr}(A^{10} Y^{10\dagger}) Y^{10} + \frac{63}{2}\, g^2 (2\, m_{\tilde{g}} Y^{10} - A^{10})
\end{aligned}$$

$$
\begin{aligned}
& +120\,(Y^{120}Y^{120\dagger}A^{10} + A^{10}Y^{120\dagger}Y^{120} \\
& \qquad +2(A^{120}Y^{120\dagger}Y^{10} + Y^{10}Y^{120\dagger}A^{120})) \\
& +126\,(Y^{126}Y^{126\dagger}A^{10} + A^{10}Y^{126\dagger}Y^{126} \\
& \qquad +2(A^{126}Y^{126\dagger}Y^{10} + Y^{10}Y^{126\dagger}A^{126})) \qquad \text{(B.4)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{A}^{120} &= 360\,(Y^{120}Y^{120\dagger}A^{120} + A^{120}Y^{120\dagger}Y^{120}) + 8\,\mathrm{tr}(Y^{120}Y^{120\dagger})A^{120} \\
& +16\,\mathrm{tr}(A^{120}Y^{120\dagger})Y^{120} + \frac{87}{2}\,g^2(2\,m_{\tilde{g}}Y^{120} - A^{120}) \\
& +10\,(Y^{10}Y^{10\dagger}A^{120} + A^{120}Y^{10\dagger}Y^{10} \\
& \qquad +2(A^{10}Y^{10\dagger}Y^{120} + Y^{120}Y^{10\dagger}A^{10})) \\
& +126\,(Y^{126}Y^{126\dagger}A^{120} + A^{120}Y^{126\dagger}Y^{126} \\
& \qquad +2(A^{126}Y^{126\dagger}Y^{120} + Y^{120}Y^{126\dagger}A^{126})) \qquad \text{(B.5)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{A}^{126} &= 378\,(Y^{126}Y^{126\dagger}A^{126} + A^{126}Y^{126\dagger}Y^{126}) + 8\,\mathrm{tr}(Y^{126}Y^{126\dagger})A^{126} \\
& +16\,\mathrm{tr}(A^{126}Y^{126\dagger})Y^{126} + \frac{95}{2}\,g^2(2\,m_{\tilde{g}}Y^{126} - A^{126}) \\
& +10\,(Y^{10}Y^{10\dagger}A^{126} + A^{126}Y^{10\dagger}Y^{10} \\
& \qquad +2(A^{10}Y^{10\dagger}Y^{126} + Y^{126}Y^{10\dagger}A^{10})) \\
& +120\,(Y^{120}Y^{120\dagger}A^{126} + A^{126}Y^{120\dagger}Y^{120} \\
& \qquad +2(A^{120}Y^{120\dagger}Y^{126} + Y^{126}Y^{120\dagger}A^{120})) \qquad \text{(B.6)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{m}^2_{\Psi} &= 10(Y^{10}Y^{10\dagger}m^2_{\Psi} + m^2_{\Psi}Y^{10\dagger}Y^{10} + 2Y^{10\dagger}m^{2\,T}_{\Psi}Y^{10} \\
& \qquad\quad +2Y^{10\dagger}Y^{10}m^2_{10} + 2A^{10\dagger}A^{10}) \\
& +120(Y^{120}Y^{120\dagger}m^2_{\Psi} + m^2_{\Psi}Y^{120\dagger}Y^{120} + 2Y^{120\dagger}m^{2\,T}_{\Psi}Y^{120} \\
& \qquad\quad +2Y^{120\dagger}Y^{120}m^2_{120} + 2A^{120\dagger}A^{120}) \\
& +126(Y^{126}Y^{126\dagger}m^2_{\Psi} + m^2_{\Psi}Y^{126\dagger}Y^{126} + 2Y^{126\dagger}m^{2\,T}_{\Psi}Y^{126} \\
& \qquad\quad +2Y^{126\dagger}Y^{126}m^2_{126} + 2A^{126\dagger}A^{126}) \\
& -45g^2|m_{\tilde{g}}|^2\mathbf{1} \qquad \text{(B.7)}
\end{aligned}
$$

$$
\begin{aligned}
\dot{m}^2_{10} &= 16\mathrm{tr}(Y^{10}Y^{10\dagger})m^2_{10} + 32\mathrm{tr}(Y^{10\dagger}m^{2\,T}_{\Psi}Y^{10}) \\
& +16\mathrm{tr}(A^{10}A^{10\dagger}) - 36g^2|m_{\tilde{g}}|^2 \qquad \text{(B.8)} \\
\dot{m}^2_{120} &= 16\mathrm{tr}(Y^{120}Y^{120\dagger})m^2_{120} + 32\mathrm{tr}(Y^{120\dagger}m^{2\,T}_{\Psi}Y^{120}) \\
& +16\mathrm{tr}(A^{120}A^{120\dagger}) - 84g^2|m_{\tilde{g}}|^2 \qquad \text{(B.9)} \\
\dot{m}^2_{126} &= 16\mathrm{tr}(Y^{126}Y^{126\dagger})m^2_{126} + 32\mathrm{tr}(Y^{126\dagger}m^{2\,T}_{\Psi}Y^{126}) \\
& +16\mathrm{tr}(A^{126}A^{126\dagger}) - 100g^2|m_{\tilde{g}}|^2 \qquad \text{(B.10)}
\end{aligned}
$$

B.2 SU(5)

$$\dot{Y}^U = 6\,Y^U Y^{U\dagger} Y^U + 3\,\mathrm{tr}(Y^U Y^{U\dagger}) Y^U + \mathrm{tr}(Y^\nu Y^{\nu\dagger}) Y^U - \frac{96}{5}\,g^2 Y^U \tag{B.11}$$

$$\dot{Y}^\nu = 6\,Y^\nu Y^{\nu\dagger} Y^\nu + \mathrm{tr}(Y^\nu Y^{\nu\dagger}) Y^\nu + 3\,\mathrm{tr}(Y^U Y^{U\dagger}) Y^\nu - \frac{48}{5}\,g^2 Y^\nu \tag{B.12}$$

$$\begin{aligned}\dot{A}^U =\ & 9\,\Big(Y^U Y^{U\dagger} A^U + A^U Y^{U\dagger} Y^U\Big) + \Big(3\,\mathrm{tr}(Y^U Y^{U\dagger}) + \mathrm{tr}(Y^\nu Y^{\nu\dagger})\Big)\,A^U \\ & + \Big(6\,\mathrm{tr}(Y^{U\dagger} A^U) + 2\,\mathrm{tr}(Y^{\nu\dagger} A^\nu)\Big)\,Y^U + \frac{96}{5}\,g^2(2\,m_{\tilde{g}} Y^U - A^U)\end{aligned} \tag{B.13}$$

$$\begin{aligned}\dot{A}^\nu =\ & 11\,Y^\nu Y^{\nu\dagger} A^\nu + 7\,A^\nu Y^{\nu\dagger} Y^\nu + \Big(\mathrm{tr}(Y^\nu Y^{\nu\dagger}) + 3\,\mathrm{tr}(Y^U Y^{U\dagger})\Big)\,A^\nu \\ & + \Big(2\,\mathrm{tr}(Y^{\nu\dagger} A^\nu) + 6\,\mathrm{tr}(Y^{U\dagger} A^U)\Big) + \frac{48}{5}\,g^2(2 m_{\tilde{g}} Y^\nu - A^\nu)\end{aligned} \tag{B.14}$$

$$\begin{aligned}\dot{m}^2_\psi =\ & 3\,Y^{U\dagger} Y^U m^2_\psi + 3\,m^2_\psi Y^{U\dagger} Y^U + 6\,Y^{U\dagger}(m^2_\psi)^T Y^U + 6\,Y^{U\dagger} Y^U m^2_u \\ & +6\,A^{U\dagger} A^U - \frac{144}{5}\,g^2 |m_{\tilde{g}}|^2 \mathbf{1}\end{aligned} \tag{B.15}$$

$$\begin{aligned}\dot{m}^2_\phi =\ & Y^{\nu *} Y^{\nu T} m^2_\phi + m^2_\phi Y^{\nu *} Y^{\nu T} + 2\,(Y^\nu m^2_N Y^{\nu\dagger})^T + 2\,Y^{\nu *} Y^{\nu T} m^2_u \\ & +2\,A^{\nu *} A^{\nu T} - \frac{96}{5}\,g^2 |m_{\tilde{g}}|^2 \mathbf{1}\end{aligned} \tag{B.16}$$

$$\begin{aligned}\dot{m}^2_N =\ & 5\,Y^{\nu\dagger} Y^\nu m^2_N + 5\,m^2_N Y^{\nu\dagger} Y^\nu + 10\,Y^{\nu\dagger} m^2_\phi Y^\nu + 10\,Y^{\nu\dagger} Y^\nu m^2_u \\ & +10\,A^{\nu\dagger} A^\nu\end{aligned} \tag{B.17}$$

$$\begin{aligned}\dot{m}^2_u =\ & \Big(6\,\mathrm{tr}(Y^U Y^{U\dagger}) + 2\,\mathrm{tr}(Y^\nu Y^{\nu\dagger})\Big)\,m^2_u + 12\,\mathrm{tr}(Y^U m^2_\psi Y^{U\dagger}) \\ & +2\,\mathrm{tr}(Y^{\nu\dagger}(m^2_\phi)^T Y^\nu) + 2\,\mathrm{tr}(Y^\nu m^2_N Y^{\nu\dagger}) + 6\,\mathrm{tr}(A^{U\dagger} A^U) + 2\mathrm{tr}(A^{\nu\dagger} A^\nu) \\ & -\frac{96}{5}\,g^2 |m_{\tilde{g}}|^2\end{aligned} \tag{B.18}$$

Appendix C

List of functions

Here we collect the various analytic functions appearing in chapters 5 and 6.

C.1 Renormalization group solutions

We list here the functions entering the analytic solutions of the GUT soft-term evolution described in sec. 5.2.3. We denote $X \equiv X(u)$, $X_0 = X(0)$, $\Delta_0 = X_0 - X_c$.

$$
\begin{aligned}
j(p,u) &= u^{p+1}\left(\frac{X(u)}{p+2} + \frac{X_c}{(p+1)(p+2)}\right), && \text{(C.1)}\\
\hat{m} &= I(u)\Big(\frac{b}{2\beta_0\gamma} X \left(|a_0|^2\, i_1(u) + 2\,\mathrm{Re}(a_0) m_{\tilde{g}}\, i_2(u) + m_{\tilde{g}}^2 i_3(u)\right) \\
&\quad -\frac{c}{2\beta_0\gamma X}\, m_{\tilde{g}}^2\, j\left(\frac{2}{\gamma} - 2, u\right)\Big), && \text{(C.2)}\\
I(u) &= u\frac{X_0}{X}, && \text{(C.3)}\\
i_1(u) &= \frac{1}{\Delta_0 X_0} - \frac{1}{\Delta_0(\Delta_0 u + X_c)}, && \text{(C.4)}\\
i_2(u) &= \frac{d}{\gamma\beta_0} l_1\left(\frac{1}{\gamma} - 2, u\right), && \text{(C.5)}\\
i_3(u) &= \left(\frac{d}{\gamma\beta_0 X_0}\right)^2 l_2\left(\frac{1}{\gamma} - 2, u\right), && \text{(C.6)}\\
l_1(p,u) &= \frac{1}{X_0 - X_c}\frac{1}{p+1}(u^{p+1} - 1) - \frac{1}{X} j(p,u), && \text{(C.7)}\\
l_2(p,u) &= \frac{X_c}{\Delta_0(p+1)^2}(u^{2p+2} - 1) + \frac{2}{(p+2)(2p+3)}(u^{2p+3} - 1) \\
&\quad -\frac{2J(p)}{\Delta_0(p+1)}(u^{p+1} - 1) - \frac{1}{\Delta_0 X(u)} j(p,u)^2, && \text{(C.8)}
\end{aligned}
$$

$$J(p) = \frac{X_c}{p+1} + \frac{\Delta_0}{p+2} \tag{C.9}$$

C.2 Loop functions

This section lists the loop functions entering the effective Lagrangians for B_s mixing, $\tau \to \mu\gamma$, and $B \to \phi K_S$.

B_s mixing

$$F(x,y) = -\frac{1}{x-y}\left[\frac{x}{(x-1)^2}\log x - \frac{1}{x-1} - (x \leftrightarrow y)\right] \tag{C.10}$$

$$G(x,y) = \frac{1}{x-y}\left[\frac{x^2}{(x-1)^2}\log x - \frac{1}{x-1} - (x \leftrightarrow y)\right] \tag{C.11}$$

$$S(x,y) = \frac{11}{18}G(x,y) - \frac{2}{9}F(x,y) \tag{C.12}$$

$$S^{(g)}(x,y) = S(x,x) - 2S(x,y) + S(y,y) \tag{C.13}$$

$B \to \phi K_S$

$$A(x) = \frac{3 - 3x + \ln x + 2x\ln x}{6(x-1)^2} \tag{C.14}$$

$$B(x) = \frac{-11 + 18x - 9x^2 + 2x^3 - 6\ln x}{18(x-1)^4} \tag{C.15}$$

$$C(x) = \frac{1 - x^2 + 2x\log x}{4(x-1)^3} \tag{C.16}$$

$$D(x) = \frac{-2 - 3x + 6x^2 - x^3 - 6x\ln x}{6(x-1)^4} \tag{C.17}$$

$\tau \to \mu\gamma$

$$H_1(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2\ln x}{12(x-1)^4} \tag{C.18}$$

$$H_2(x) = \frac{-1 + 4x - 3x^2 + 2x^2\ln x}{2(x-1)^3} \tag{C.19}$$

$$H_3(x) = \frac{1}{2}D(x) \tag{C.20}$$

$$H_4(x) = -2C(x) \tag{C.21}$$

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