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Acceptability of Civil Engineering Decisions Involving Human Consequences

Albrecht Lentz

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1. apl. Prof. Dr.-Ing., Dr.-Ing. habil. R. Rackwitz
2. Univ.-Prof. Dr.-Ing., Dr.-Ing. E.h. P. Schießl
3. Prof. Dr. M. H. Faber,
   Eidgenössische Technische Hochschule Zürich / Schweiz

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Abstract

In most cases, heightened structural safety leads to higher costs and therefore to a reduction of average disposable income. At the same time, average life expectancy rises because of the ensuing lower failure rates. From income and life expectancy, it is possible to derive socio-economic utility functions such as the life quality index. A safety-relevant decision is deemed acceptable if the utility function value rises or remains at least equal. The thesis extends this approach in order to cover not only mortality effects, but equally effects upon morbidity. Furthermore, the effect of delays (latency) is investigated. A generalised consequence model facilitates the determination of mortality and life expectancy from failure rates and toxical emission rates. Realistic case studies illustrate the application of the proposed methods.

Zusammenfassung

Höhere Zuverlässigkeit von Bauwerken führt meist zu höheren Kosten und somit zu einer Verminderung des durchschnittlichen verfügbaren Einkommens. Gleichzeitig steigt die durchschnittliche Lebenserwartung aufgrund der niedrigeren Versagensraten. Aus Einkommen und Lebenserwartung lassen sich volkswirtschaftliche Nutzenfunktionen wie etwa der Lebensqualitätsindex bilden. Eine sicherheitsrelevante Entscheidung gilt als akzeptabel, wenn der Funktionswert steigt oder zumindest gleich bleibt. Die Arbeit erweitert diesen Ansatz, um neben den Auswirkungen auf die Mortalität auch jene auf die Morbidität zu berücksichtigen. Ebenso wird die Wirkung von Verzögerungen (Latenz) und Verzinsung aufgearbeitet. Ein umfassendes Konsequenzenmodell erleichtert die Bestimmung von Mortalität und Lebenserwartung aus Versagensraten oder toxischen Emissionsraten. Konkrete Fallbeispiele verdeutlichen die Anwendung der Methoden.

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Another person I would like to thank in particular is Prof. Dr. Michael H. Faber, whom I owe a lot. He was the one who introduced me to engineering risk analysis, when I wrote my diploma thesis as his student at ETH Zürich in 2001 and who established the contact with my to-be doctoral thesis supervisor. He also acted as one of the co-examiners of this thesis. Apart from all that I have always regarded him as a mentor who has been providing me with feed-back and encouragement ever since my graduation.

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Albrecht Lentz

Contact: albrecht.lentz <at> gmx.net

Contents

Chapter 1

Introduction

1.1 The Problem

Whenever an engineering facility fails, the owner—public or private—faces a loss of assets. Apart from this most obvious consequence, there are usually a number of follow-up events such as loss of business. From a societal point of view, the most serious consequences of failure events are those concerning human life and limb. In this respect, civil engineering facilities can be considered as particularly sensitive, both because of their omnipresence in everyday life and because of their sheer size and the ensuing hazard potential.

Risk is commonly defined as the product of failure probability and failure consequences. For potential loss of assets, risk assessment is comparatively straightforward: The failure probability follows from applying reliability theory to the physical relations describing the structure, while the consequences are more or less equal to the original construction cost. Estimating indirect consequences in terms of loss of business proves already more delicate a task. However, the most challenging problem probably lies in predicting the *physical consequences for human beings*, i.e. injury, disease and death. The psychological and physiological causes of human behaviour and vulnerability are mostly hard to model, as they are usually subject to a number of complex underlying phenomena.

At the same time, potential loss of life or health evokes the question of *societal risk acceptability.* With these existential values at stake, the question is apparently more delicate to answer than for merely material consequences. Human life does not carry a price tag, but is generally agreed to be of infinite value. Still, resources are limited and investments into a specific risk mitigation measure will be lost for other essential needs of society. This brings up the question of the societal acceptability of spending resources on a risk measure, i.e. of *societal affordability.* The two concepts, societal risk acceptability and societal affordability, add up to the broader concept of the *acceptability of risk-relevant decisions.*

In brief, human consequences of failure events in the civil engineering domain are not only more complex to predict, but also more difficult to valuate, as compared to other consequences. Besides, other areas of the engineering domain, especially those involving the release of toxic substances, are facing the same type of challenge. In this respect, the present issue is of considerable societal relevance.

1.2 Existing Approaches

All acceptability criteria for safety-relevant decisions appear to have one common principle: The outcome of a project is compared to the status quo. If the outcome is more desirable than the present situation (or at least equal), it is deemed acceptable. Otherwise, the project should not be realised. In this sense, all safety criteria can be called relative. This understanding of acceptability can be extended to existing facilities from an earlier era: They are deemed acceptable, if their safety level corresponds to that of state-of-the-art projects. This is usually the case for built structures, but less often for processes involving toxic emissions.

The difference between the different types of criteria lies in the yardstick that is used in order to compare a project to the status quo. In this respect, three major types of safety criteria can be identified:

Conventional Criteria for Human Safety

At present, the most commonly applied criteria valuate projects more or less exclusively by their effect upon safety levels (expressed as victims per year), while attributing little or no importance to the socio-economic costs of a safety measure. Because of this focus on the safety aspect of a decision, 'tolerable risk' is more common as an expression than 'acceptable decision'. 'Acceptable risk' equally exists in this framework, but this class includes only minor hazards that do not raise any special concern in case of a fatality. In some cases, tolerable risks are defined by the ALARP condition (as low as reasonably practicable). Although the socio-economic overall effect is partly taken into account by this condition, there is no objective definition of what is 'reasonable', i.e. societally affordable.

Conventional safety criteria are calibrated by analysing the safety levels of previous projects. These levels are obtained either as fatality rates or as occurrence rates of hazardous events (failure rates, emission rates). The first approach is preferable, because it accounts for the effects upon human safety in a more explicit way.

Two important examples of this type of criterion are constituted by the Eurocode [25] and the prescriptions of the British Health & Safety Executive (HSE), as summarised in [11]. The former applies to civil engineering, while the latter aims at more general applications. These and some other regulations are briefly presented in Appendix A.

Cost-benefit Analysis as a Criterion for Human Safety

In principle, cost-benefit analysis (CBA) is a method that aims at assessing the profitability of a project and has little to do with human safety issues. A project is deemed acceptable, if it makes the owner (private or public) wealthier, or at least not poorer. Formally, this criterion can be written as $Z \geq 0$, where Z denotes profit. It is obtained by subtracting investment costs C and damage costs D from benefit (revenues) B:

$$Z(p) = B(p) - C(p) - D(p) \tag{1.1}$$

In this relationship, p is a safety parameter which heightens costs $C(p)$ on one hand, but reduces the probability of undesirable events resulting in economic damage $D(p)$ on the other hand. Furthermore, it has an effect upon $B(p)$, because higher safety reduces the likelihood of down-times during which no revenues can be generated. It was Rosenblueth & Mendoza [110] who introduced this concept to the civil engineering domain in 1971. Appendix B provides a brief overview over its mathematical implications.

Damage can occur in two ways, as a loss of investment goods or as external damage. In the first case, $D(p)$ can be as high as the original investment costs $C(p)$. This corresponds to the complete loss of a facility requiring total reconstruction. External damages, on the other hand, are independent of investment costs $C(p)$ and can exceed them by far in certain cases (e.g. chemical and nuclear industries). They include loss of off-site property as well as loss of life. However, these damages will only be considered in the cost-benefit analysis, if the owner can be held liable regardless of default (strict liability), as it is commonly the case in the energy and transportation sectors (see e.g. German Liability Act [2]).

Compensation costs for fatal victims can amount to considerable sums, depending on the jurisdiction of the respective country. The loss of expected future earnings can serve as a rough lower estimate, but values can also be significantly higher.For the owner of a facility, averting these compensation costs (or reducing the insurance premium) is an economic incentive for providing a minimum level of safety. In consequence, this type of criterion is rather ineffective in the case of highly profitable projects (compare Chapter 4).

Besides, utility-theoretic and empirical findings indicate that people are willing to afford safety payments that are well above the potentially gained lifetime earnings (see Chapter 2).

Utility-based Criteria for Human Safety

Instead of valuating projects exclusively by means of fatality rates or by means of money, it is possible to use a joint indicator that unites both aspects. The foundations for such an approach are laid by socio-economic utility theory, which regards personal utility as the basic measure in decision analysis. Utility is a measure of its own, but can equally be transformed into other units, such as life expectancy or income. However, money is only seen as a means of conversion when balancing different essential goods such as longevity, health, education and time for leisure and recovery against each other. In this context, safety is regarded primarily as a source of longevity.

Utility theory has a long tradition in economics and is used to explain some basic phenomena, such as consumer behaviour and the shape of demand functions (see e.g. [111]). From the 1970ies onwards, Usher [136] and other economists [4, 22, 109, 118] adopted these foundations in order to describe trade-offs between wealth and lifetime. This involved the derivation of people's *willingness to pay* (WTP) as a criterion for safety-related decisions, with a focus on health-policy questions. In 1994, Lind [67] introduced this type of criterion to engineering problems. In his reasonings, he incorporated other utility measures such as the U.N. Human Development Index (HDI) and developed a measure called the *life quality index*. This utility measure was further developed by Nathwani, Lind and Pandey [84, 92] and Rackwitz [101, 103, 104, 105].

As already foreshadowed in the problem outline, the present thesis focusses on the utility-based approach in order to assess the societal acceptability of safety-related decisions. For this reason, the derivation of the WTP criterion and its underlying principles are dealt with at length in Chapter 2 and some of the following chapters.

Investigating the interplay of utility-based criteria with cost-benefit analysis is an issue that has been brought up repeatedly by Rackwitz, e.g. [101, 103, 104, 105], but equally by Streicher [126] and Kübler [63].

1.3 Aim of the Thesis

The first objective of the present thesis is to derive and validate willingness to pay (WTP) as a utility-based criterion, which assesses the societal acceptability of safety-related decisions. To this purpose, the state of the art needs to be reviewed, before some open questions can be addressed. These include the extension of the WTP criterion from potential loss of life to general health effects (injury and disease), as well as its correct application to different types of projects. Projects with potential failure events require a different approach than those that pose a hazard by permanently emitting some noxious matter.

The primary effect of a safety-related decision consists in a change of the hazard level. It is expressed either as occurrence rate (acute failure) or as emission rate (continuous release of toxicants). Both phenomena potentially entail human consequences in the form of death, injury or disease. Determining occurrence rates is the subject of reliability theory, which has established a consistent and generally agreed methodology to that purpose. The same can be said about the methods describing industrial emission rates of all sorts. However, it appears that no generalised approach has emerged yet in order to assess the human consequences, especially in the case of acute failure events. For continuous toxic impacts, the case is less drastic. In order to provide a meaningful result, the WTP criterion relies upon the acuracy of the human consequence model. Hence, it appears little consistent to establish a methodology for acceptable decisions, while lacking a systematic basis for human consequence modelling.

The present thesis is an attempt to address both objectives in a joint effort.

1.4 Structure of the Thesis

With respect to societal risk acceptability of safety-related decisions, loss of life can be seen as a special case of health-compromising effects in general. Most studies actually limit themselves to this special case and derive the WTP criterion on this basis. **Chapter 2** provides an introduction to the state of the art, apart from a few further going refinements. In the next step, **Chapter 3** extends the realm of acceptability modelling to the more general case of disability (i.e. injury and disease).

Chapter 4 outlines the differences between projects involving acute failure and projects leading to continuous release of noxious matter and explains how to apply the WTP criterion to these differing cases. This is also the point, at which the corresponding human consequence models described in the following chapters are brought into the centre of interest. In this respect, Chapter 4 can be seen as the interface connecting the other parts of the thesis (see Figure 1.1).

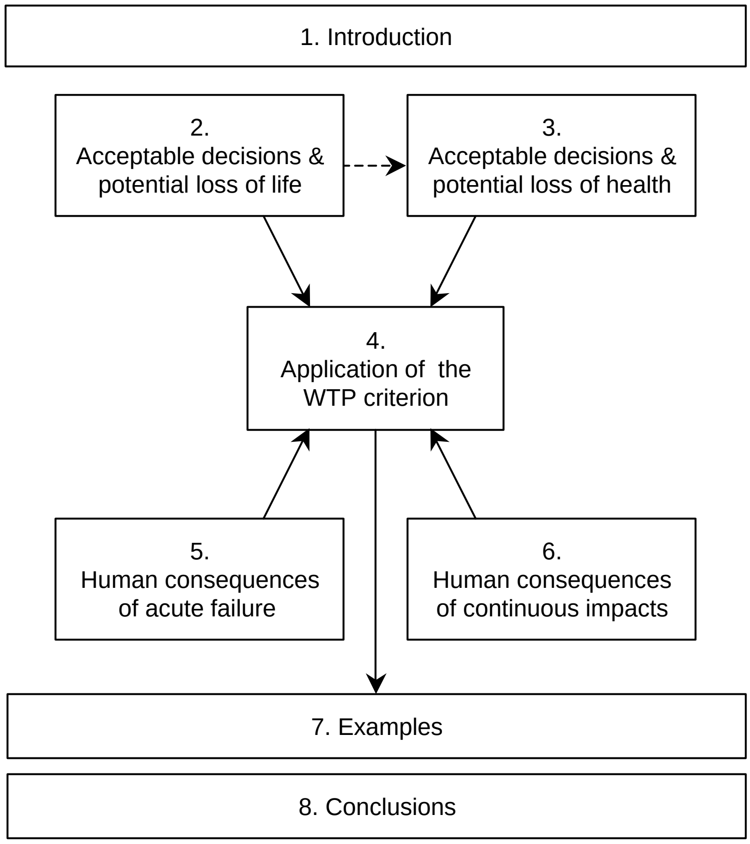

Figure 1.1: Structure of the thesis

Chapter 5 develops a generalised human consequence model for the case of acute failure. The analysis is based on three of the most important failure types in civil engineering, namely building collapse due to earthquakes, dam failure and tunnel fire. Nevertheless, the resulting methodology aims at acute failure events as a whole. The human consequence model for continuous toxic emissions is dealt with less extensively in **Chapter 6**; here, the existing methodological knowledge is considerably more extensive and touches aspects, which are sciences of their own, including meteorology and toxicology. Unlike the methodological ambitions of Chapter 5, the aim is simply to make the existing knowledge accessible for acceptability assessments by providing a compact introduction.

Chapter 7 presents some illustrative examples, followed by the conclusions in **Chapter 8**.

Chapter 2

Societal Acceptability of Decisions Involving Potential Loss of Life

The present chapter aims to show how socio-economic utility theory can be used to assess safety-relevant decisions. Loss of life is the most drastic human consequence a decision may have. At the same time, it is the most basic case from a methodological point of view, as opposed to injury or disease. In fact, the prevailing number of previous publications limits itself to this case. The following pages are meant to review the existing knowledge.

2.1 Preliminary Considerations

2.1.1 Ethical Concepts

Making right and good decisions is the subject of ethics, one of the main issues of philosophy. There are a number of ethical theorems and principles, among which Kant's categorical imperative is maybe the most famous one. In the present context, two further concepts deserve closer attention because of their omnipresence in the public discourse: One is part of constitutional law in most Western nations and concerns the *intangibility of human life*. The other one, *utilitarianism*, provides the philosophical basis for utility-theory and the use of quantitative risk analysis in decision-making. The different concepts can lead to contradictory conclusions in some cases.

Categorical Imperative

The categorical imperative forms the essence of Immanuel Kant's (1724–1804) moral philosophy. It is the most prominent example of *deontological* philosophy, a school postulating the existence of a priori moral obligations. Kant devised several formulations of the categorical imperative [60]. Among these, the following two formulations are the most commonly known:

1. 'Act only according to that maxim whereby you can at the same time will that it become a universal law.'

2. 'Act so as to use humanity, whether in your own person or in others, always as an end, and never merely as a means.'

Intangibility of Human Life

In the Judeo-Christian tradition, which has determined much of the development of modern Western value concepts, man is understood as the counterpart of God and human life is seen as holy. The modern concept of intangibility of life [1, 133] is essentially a secularised version of the holiness concept (see e.g. Lenzen [65]). Each of these concepts amounts to the principle of *infinite value of human life*, which bears two major implications (e.g. [115]):

- The value of any human life is higher than that of a non-human life or that of an unenlivened object.
- When each life is of infinite value, each fraction of life must equally be of infinite value. In consequence, the value of each human life is equal, regardless of age, health or remaining life expectancy [53, 65].

Intangibility of human life is a fundamental concept that cannot be further derived by rational means, e.g. by some kind of optimisation. If such a derivation were possible, the result would depend on the specific assumptions and every change in these assumptions would lead to a different conclusion. As a result, intangibility would become negotiable and thus relative. However, absoluteness is a central property of the intangibility concept, a necessity that became evident during the great moral breakdowns of the past century[1].

Utilitarianism

In the 18$^{\text{th}}$ century, the English philosopher and reformer Jeremy Bentham established an ethical theory known as utilitarianism. His *principle of happiness* interprets pain and pleasure as the only absolutes in the world, leading to what is known as the central ethical demand of utilitarianism, 'the greatest happiness for the greatest number'. John Stuart Mill, who lived in England during the 19$^{\text{th}}$ century is the second philosopher commonly associated with utilitarianism. He considered cultural and spiritual happiness as more important than the material aspects of happiness. This view differs from the contemporary understanding of utilitarianism serving as the philosophical basis for economic utility theory.

In order to prevent situations, in which society as a whole obtains a net benefit on the expenses of a few of its members, the principle has to be subjected to a major constraint. Some argue that this constraint is already implied in Bentham's original reasoning [62]. In [85], it is referred to as the *Kaldor-Hicks Compensation Principle*, demanding that

> 'A policy is to be judged socially beneficial if the gainers receive enough benefits that they can compensate the losers fully and still have some net gain left over.'

[1]This reasoning roughly corresponds to the theory of *natural law*, limiting the realm of *legal positivism* in contemporary constitutional law.

Utilitarianism belongs to the group of *consequentialist* theories. Consequentialism is opposed to deontology and judges actions only by their final outcome. The existence of a-priori moral principles is denied. In its most radical form, utilitarianism advocates some thoughts that are directly opposed to the equality reasoning derived from infinite value of life: Higher remaining life expectancy means higher remaining lifetime utility, such that the payment for saving a young person's life would be higher than in the case of an older person. Similar conclusions apply for wealthy and healthy persons, for health and wealth both contribute to utility per time unit. In his 'Practical Ethics', Singer [119] takes a similar direction, when he puts the value of a life into a direct relationship with a person's degree of rationality, consciousness and ability to suffer, much to the disadvantage of newborn children and mentally handicapped persons.

2.1.2 Discussion

From an engineer's point of view, it is hardly possible to participate in the discourse, a small part of which is sketched above. Yet, engineers as well as economists implicitly take the side of utilitarianism, simply by relying on quantitative analysis of risk and utility as the basis for their decisions. Such a proceeding implies using numbers and ultimately prices with respect to human life, which can conflict with the infinite value of human life following from the constitutional principle of intangibility. Therefore, engineers and generally decision makers cannot circumvent these fundamental issues altogether, at least not if potential fatalities and their prevention are involved.

With respect to life-saving measures, it may be helpful to distinguish between rescue measures and prevention. *Rescue* is required, when an individual is in an acute state of emergency. Judging by the commonly observable way the public and its representatives react in case of accidents, severe illness and catastrophes, it appears that society is willing to undertake almost anything to save the life of a threatened individual. The question of affordability seems to be only of secondary interest. This behaviour possibly reflects the conception of human life being of infinite value. In the opposed case of *preventive* measures against future fatalities, there is strong evidence that people take affordability into account to a much higher degree. People's actual choices reveal their implied *willingness to pay* (WTP) for life-saving measures, e.g. when purchasing safety-enhancing products [84]. Other indicators include workers' readiness to accept higher wages as a compensation for higher work risks (see Section 2.5.3).

As an interpretation of these observations, one could deduce that making trade-offs with respect to human life is admissible as long as the fatal outcome is only an uncertain possibility in the future. The fatal event might not occur at all or some other fatal event might happen beforehand. In fact, society's members are facing a multitude of different threats. Even if the complete national income were spent on risk prevention, some risks would always be remaining, while no resources would be left for other essential needs such as food, housing, education etc. On the other hand, it seems equally pointless not to invest any money at all into risk prevention. A number of measures, such as obligatory safety belts, can save many lives at a reasonable expense. Following this line of thought, there must be a limit to affordability somewhere between these extremes, i.e. the WTP. This corresponds to the viewpoint, according to which 'the core of all risk management is a problem of allocation of a scarce commodity (the public's money) to serve the public good in the best way' [68].

2.2 Expressing Utility

By *willingness to pay* (WTP) the socio-economic literature understands the amount of money the average member of society is ready to sacrifice in order to prevent a fatality [4, 84, 118]. The life at stake can be one's own or that of an anonymous fellow citizen. If the individual is willing to trade money against safety at this rate, it implies that the additional survival probability provides the same amount of personal utility as using the money for consumption instead. In order to determine the WTP, utility needs to be described and quantified beforehand.

The idea of measuring utility in absolute values appears impracticable, beginning with the difficulty of finding an appropriate unit of measurement. In fact, utility theory does not deal with absolute measures, but with comparing the relative utility of two options ('ordinal utility', see e.g. [111]). The strength of the utility concept consists in making monetary and non-monetary goods comparable by depicting them on one scale. If the value of something like leisure time is to be compared to that of something like extra income, the following question arises: What is the use—or: *utility*—of the one good, when compared to the other? The fact that people actually do make choices of this sort everyday provides the basis for the analytical and empirical derivation of utility functions.

Many goods contribute to life quality and therefore to utility in an essential way. Among these are personal wellbeing, longevity, self-realisation, wealth, intact family relations and many more. On an *individual level*, people take all of these factors into account for choices concerning their own lives. When deciding about utility-relevant interventions on a *societal level*, it is very hard if not impossible to quantify the effect some of these factors, such as the mentioned integrity of family ties. For this reason, the more tangible factors among these have to serve as a substitute for the remaining factors, as far as utility-based decision-making is concerned [84].

In the following, two existing approaches for determining lifetime utility are presented and discussed.

2.2.1 The Classical Socio-economic Approach

Lifetime Utility

The basic idea in economic utility theory is that a person's enjoyment of life originates from a permanent stream of consumption [136]. Let $c(t)$ denote consumption per time unit, so that $u(c(t))$ is utility from consumption, also per time unit. Then, the (remaining) lifetime utility of a person aged a who is going to live until a_D follows as

$$U(a, a_D) = \int_a^{a_D} u(c(t))dt \tag{2.1}$$

Due to different psychological and economic reasons, people value future utility less than present utility. This phenomenon is described by discounting future utility at a rate $\gamma(t)$:

$$
\begin{aligned}
U(a, a_D) &= \int_a^{a_D} u(c(t)) \exp\left[-\int_a^t \gamma(\tau^*) d\tau\right] dt \\
&= \int_a^{a_D} u(c(t)) \exp[-\gamma(t-a)] dt
\end{aligned}
\tag{2.2}
$$

with[2] $\tau^* = \tau - a$. The second line in (2.2) holds for constant $\gamma(t) = \gamma$. In a societal context, one cannot simply use capital market rates for discounting future utility. The precise reasons deserve closer attention and are discussed in Section 2.3, which is entirely dedicated to this question. Equation (2.2) uses continuous (or *exponential*) discounting in the form $\exp[-\int_0^t \gamma(\tau) d\tau]$ instead of yearly discounting, i.e. $\prod_{\tau=0}^{t}(1 - \gamma'(\tau))$. For mathematical derivations, this form is much more convenient. Continuous and yearly discount rates can be easily converted by $\gamma(t) = \ln(1 + \gamma'(t))$. For small discount rates not higher than 5%, the relation can be simplified as $\gamma(t) \approx \gamma'(t)$.

Life Expectancy

Apparently, being alive is a prerequisite for being able to spend money and enjoy consumption. Yet, the remaining life span of an individual cannot be predicted in the form $a_D - a$, but has to be estimated by calculating the remaining *life expectancy* from statistical data on mortality, so-called life tables. The following lines are meant to provide a brief introduction into life-table calculations (compare e.g. [38, 42]):

Let $F_D(a)$ be the probability of *not* having survived up to age a and $f_D(a)$ be the corresponding probability density function. Then, age-dependent mortality is defined as the probability of dying at age a, under the condition of having reached age a in the first place:

$$
\mu(a) = \frac{f_D(a)}{1 - F_D(a)}
\tag{2.3}
$$

Thus, $\mu(a)$ is easily determined: It only takes to find out how many people have reached age a at the beginning of a given year t and how many of these persons did not survive until the end of year t. Mortality follows from dividing the second number by the first one. Performing this simple calculation for all age classes yields the so-called period life table for period (calendar year) χ. These tables differ from country to country and are publicly available from different sources, e.g. [69, 144]. Figure 2.1 (left part) displays $\mu(a)$ from a French table. Usually, separate period life tables are published for men and women. In the present context, it is preferred to use joint life tables for both sexes, which is more consistent with the goal of establishing a generalised societal safety criterion.

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[2]Note that in (2.2), t and τ are both relative to the birth year of the individual aged a. Therefore, $\gamma(\tau)$ is not the discount rate τ years from *now*, but that of the year in which the individual aged a today will be τ years old. The discount rate, however, does not depend on the age of one specific person. Instead, it is a socio-economic parameter which applies to all persons living in a given year in the same way, regardless of their respective age. The decision point (i.e. *now*) is usually indicated as year zero, so that *now* means $t^* = 0$.

It is possible to and admissible to use $\gamma(\tau)$ instead of $\gamma(\tau^*)$ in simple expressions as the one above. However, it will produce a considerable amount of unclarity in equations like (2.8) and even more so, when people of different birth years enter into one single expression as in (2.11).

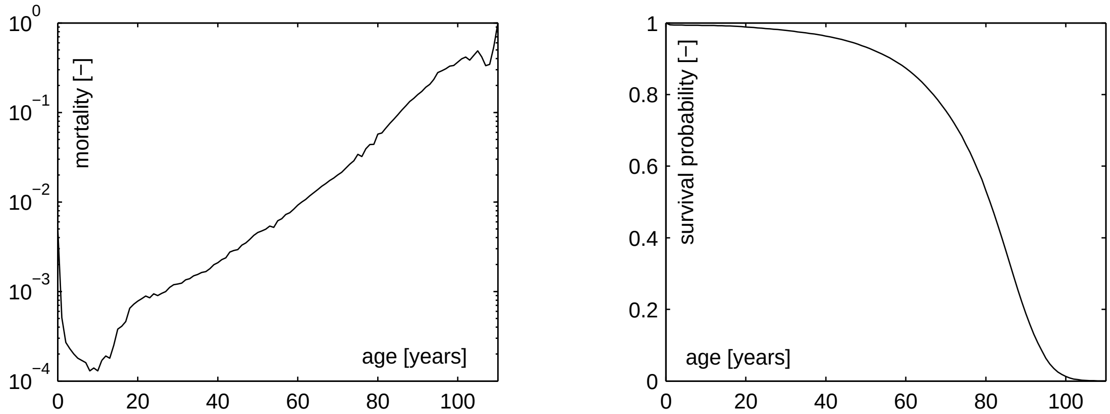

Figure 2.1: Mortality $\mu(a)$ and survival probability $S(a)$ for France (period life table [144])

The number $S(a)$ expresses the number of persons, who have survived up to age a:

$$S(a) = S(0)(1 - F_D(a)) \tag{2.4}$$

When setting $S(0) = 1$, then $S(a)$ is a person's probability of survival up to (at least) age a. It can be calculated from $\mu(a)$ by resolving (2.3)[3] for $S(a)$:

$$S(a) = \exp\left[-\int_0^a \mu(t)dt\right] \tag{2.5}$$

Figure 2.1 (right part) displays the typical shape of this function. Integrating the survival probability from 0 to the maximum attainable age a_u, which is frequently assumed to be 110 years, yields life expectancy at birth:

$$e(0) = e_0 = \int_0^{a_u} S(t)dt = \int_0^{a_u} \exp\left[-\int_0^t \mu(\tau)d\tau\right] dt \tag{2.6}$$

This result becomes a little more intuitive, when imagining an age cohort of 100 000 persons born in the same year, i.e. $S(0) = 100\,000$. Every year, the number of survivors $S(a)$ becomes a little smaller until reaches zero after a_u years. Adding the $S(a)$ numbers of all years (ages) from 0 to a_u and dividing by a_u apparently yields the average duration of life among the members of the age cohort, i.e. their life expectancy. In industrialised societies, life expectancy at birth usually amounts to approximately 79 years (typically +3 years for women, −3 years for men). Country-specific values are listed at the end of the chapter in Table 2.5.

For a person aged a, the remaining life expectancy follows as

[3] Because of

$$\mu(a) = \frac{f_D(a)}{1 - F_D(a)} = \frac{-\frac{d}{dt}(1 - F_D(a))}{1 - F_D(a)} = -\frac{d}{dt}\ln[1 - F_D(a)] = -\frac{d}{dt}\ln[S(a)]$$

$$\begin{aligned} e(a) &= \int_a^{a_u} S(t|a)dt \\ &= \frac{1}{S(a)} \int_a^{a_u} S(t)dt = \frac{\int_a^{a_u} \exp\left[-\int_0^t \mu(\tau)d\tau\right] dt}{\exp\left[-\int_0^a \mu(\tau)d\tau\right]} \\ &= \int_a^{a_u} \exp\left[-\int_a^t \mu(\tau)d\tau\right] dt \end{aligned} \tag{2.7}$$

$S(t|a)$ is the probability of surviving until t under the condition of having survived up to a.

Using $\mu(a)$ values from a period life table leads to a moderate underestimation of $e(a)$ in the present situation of permanently decreasing mortality rates. It implies that someone aged 30 today will be subject to the same mortality rate in 20 years as it can be observed with 50-year old persons today. This error can be corrected by using predictive *cohort life tables*, attributing a different mortality function $\mu(a)$ to each cohort, i.e. to each group of persons sharing one birth year ϑ.

Essentially, these tables are obtained by selecting an age class, e.g. $a = 50$ years and analysing how the mortality rate of 50-year old persons has been changing over the years. This is done by comparing historical period life tables from different calendar years (periods) χ. In a second step, the observed trend is extrapolated into the future, leading to a mortality rate $\mu_\chi(50)$ depending upon the period χ. This proceeding is repeated for all other age groups a in order to predict the age- and period-dependent mortality rate $\mu_\chi(a)$. Now, it is possible to recompose these data in order to establish a cohort life tables for different birth years θ. For persons born in 2006, the cohort life table has the form $\mu_{2006}(0) = \dots$, $\mu_{2007}(1) = \dots$, $\mu_{2008}(2) = \dots$ etc.

For the youngest generations presently alive, the resulting life expectancies are about 7% higher than for using a period life table for the present year [104] (see Table 2.5 at the end of the chapter).

Expected Lifetime Utility

With this input, it is now possible to calculate the *expected* remaining lifetime utility of someone aged a [4, 22, 109, 118]:

$$\begin{aligned} L(a) = E[U(a)] &= \int_a^{a_u} S(t|a)u(c(t)) \exp\left[-\int_a^t \gamma(\tau^*)d\tau\right] dt \\ &= \int_a^{a_u} \exp\left[-\int_a^t \mu(\tau)d\tau\right] u(c(t)) \exp\left[-\int_a^t \gamma(\tau^*)d\tau\right] dt \\ &= \int_a^{a_u} u(c(t)) \exp\left[-\int_a^t \mu(\tau) + \gamma(\tau^*)d\tau\right] dt \\ &= \int_a^{a_u} u(c(t))S_d(t|a)dt \end{aligned} \tag{2.8}$$

Because of the mathematical analogy between mortality and discounting, it is possible to include the discounting effect in the survival term, which leads to $S_d(t|a)$, something that could be called *discounted probability of survival.* For the special case $c(a) = c$ discussed further below, (2.8) further simplifies:

$$L(a) = u(c) \int_a^{a_u} S_d(t|a)dt = u(c)e_d(a) \tag{2.9}$$

Variable $e_d(a)$ is called *discounted life expectancy* at age a. On one hand, it can be seen as a merely mathematical construction, making use of the formal analogy between discounting and mortality. On the other hand, people actually do value future utility less than present utility and being alive.

Society consists of people of all age classes. When lifetime utility is meant to serve as a yardstick for societal decisions, it is necessary to regard the remaining lifetime utility of the average member of society. If population growth n remains stable during a_u years or longer, people are distributed over different age classes with

$$h(a, n) = \frac{\exp[-na]S(a)}{\int_0^{a_u} \exp[-na]S(a)da} \tag{2.10}$$

In reality, population growth rates are changing, but (2.10) can serve as a helpful approximation, nevertheless. Note, that (2.10) uses $S(a)$ and not $S_d(a)$, which is necessary in order to figure who is actually alive *now.* Averaging $e_d(a)$ over all age classes yields the *age-averaged discounted life expectancy* of a society:

$$\begin{aligned} \bar{e}_d = E_A[e_d(a)] &= \int_0^{a_u} \int_a^{a_u} \exp\left[-\int_a^t \mu(\tau) + \gamma(\tau^*)d\tau\right] dt\, h(a, n)da \\ &= \int_0^{a_u} e_d(a)\, h(a, n)da \end{aligned} \tag{2.11}$$

Without discounting, age-averaged life expectancy amounts roughly to $\bar{e} \approx e_0/2 \approx 40$ years for industrialised countries. Applying a typical average discount rate between 3 and 4% leads to values of $\bar{e}_d \approx 20$ years.

Now, societal lifetime utility can be written as

$$L = u(c)\bar{e}_d \tag{2.12}$$

Utility of Consumption

Up to here, the derivation in (2.1) to (2.9) has shown how to express life time utility $L(a)$ on the basis of utility of consumption per time unit $u(c(a))$, while the relation between utility u and consumption $c(a)$ remained unclear, just as the dependency of c upon age a.

Formally speaking, a utility function is a function $u(c)$ that can be differentiated twice and is defined for $c > 0$ [75]. The first derivative has to fulfill $\frac{du}{dc} > 0$, which is called the condition of *non-satiation*. It says that an additional unit of consumption or wealth is always appreciated at least a little bit, even by a very affluent individual. For the second derivative, $\frac{d^2u}{dc^2} < 0$ has to apply. This condition contains the phenomenon of *diminishing marginal utility*: If an individual has only five Euros to spend, one extra Euro has much more worth to him than to someone who has 50 € for consumption. The same condition is also referred to as *risk aversion*, which is due to a slightly different mental image: Consider an individual that owns 30 € and has the chance of playing a game which will bring him 20 € in case he or she wins, but cost equally 20 € in case of losing. The respective chances are both 50%, so that the expected gain equals zero. Still, the individual realises that the loss of utility caused by losing 20 € and owning only 10 € is greater than the gain in utility derived from additional 20 €. The reason for this is the same as in the first case: People buy essential things first and the less essential only after the basic needs are fulfilled. Therefore, the 11^{th} up to the 30^{th} Euro are dedicated to much more useful purchases than the 31^{st} up to the 50^{th} Euro disposable in case of winning. As a logical consequence, the individual will behave risk-aversely and abstain from playing the game.

In the literature [4, 118, 136], a special class of utility functions is preferred, namely such with constant proportional risk aversion (CPRA). When a utility function is proportional, the relative loss or gain of utility depends on the proportion of assets lost. For the owner of 100 €, winning 10 € has the same utility as an extra 50 € has for the owner of 500 €. Constant proportional risk aversion means that the elasticity[4] of $u'(c)$, i.e. of the marginal utility of consumption, needs to be constant:

$$\frac{du'(c)}{dc}\frac{c}{u'(c)} = \frac{d^2u(c)}{dc^2}\frac{c\,dc}{du(c)} = \varepsilon = \text{const} \tag{2.13}$$

If zero consumption is assumed to yield zero utility, i.e. $u(0) = 0$, this leads to a formulation of the type

$$u(c) = c^{1-\varepsilon} = c^q \qquad \text{with } \; 0 < q < 1 \tag{2.14}$$

which fulfils (2.13) when inserting.

The second question concerns the form of the consumption function. An individual may choose a certain consumption path $c(a)$ describing the level of consumption at different ages. There is, however, an optimal consumption path $c^*(a)$ leading to maximum lifetime utility. It can be found by applying calculus of variation to (2.8). In this optimisation, $c(a)$ is subject to a few constraints:

$$\begin{aligned} \frac{dk(a)}{dt} &= \gamma' k(a) + g(a) - c(a) + i(a) \\ k(0) &= k(a_D) = 0 \\ c(a) &\geq 0 \end{aligned} \tag{2.15}$$

[4]The elasticity of a function y with respect to x is defined as $\varepsilon = \frac{dy(x)}{dx}\frac{x}{y(x)}$ [111]. The case $\varepsilon = \text{const}$ is called *iso-elasticity*. It implies that a relative change in x will always cause the same relative change in $y(x)$, regardless of the absolute value of x.

Here, $k(a)$ is wealth, $g(a)$ is income and $i(a)$ denotes the receipts and payments from a fair insurance: At the beginning and the end of an average life, people don't earn any money. Nevertheless, they have to consume in order to stay alive. If legacies and bequests are excluded from the considerations for reasons of transparency, this means that people first have to borrow money until they enter working life ($i(a) > 0$). Then, they pay the loan back and simultaneously start investing into a life insurance ($i(a) < 0$). After retirement (again, $i(a) > 0$), life insurance frees people from the risk of exhausting their financial resources in case they outlive their initial life expectancy at retiring age. This model is somewhat hypothetical, but pictures reality well, nevertheless. To a large extent, the interrelations between the generations of a family assume the role of an insurance. For these assumptions, Shepard & Zeckhauser [118] show a constant consumption rate $c^*(a) = c$ to be optimal. This leads to the convenient simplification in (2.9) above.

Inserting (2.14) in (2.12) allows to rewrite societal lifetime utility as

$$L = c^q \bar{e}_d \tag{2.16}$$

The value of the exponent is usually given as $q = 0.2$ in the literature [4, 22, 70, 118]. However, there is no explanation or derivation with respect to this value. Instead, it seems that different authors cite each another in order to justify their choice. The life quality method described in Section 2.2.2 which proposes an alternative quality ranking approach may provide a rational derivation of the exponent q.

2.2.2 The Life Quality Approach

General Considerations

At the beginning of the 1990s, Lind was comparing different social indicators in order to derive a criterion for acceptable risks and affordable risk mitigation in the engineering domain [67]. Soon afterwords, this work resulted in a comprehensive publication by Natwhani, Lind & Pandey with the programmatic title *Affordable Safety by Choice: The Life Quality Method* [84]. The thought behind the life quality index or LQI is similar to the reasoning on lifetime utility above. Again, wealth (the prerequisite of consumption) and life expectancy serve as the two representative quantities out of a number of factors describing life quality. However, the emphasis on the representational character of the two parameters is much stronger. In the words of Preston, Keyfitz & Schoen, as quoted in [84],

> 'The circumstances under which men die are closely related to the conditions under which they live. The extent of violence, poverty, passivity, and ignorance in a population is reflected in the statistics of its causes and ages of death. Vigorous attempts to delay death are so universal that accurate mortality statistics provide a reliable touchstone of a population's level of social organization and technological sophistication.'

This reasoning confirms the significance of life expectancy as a life quality measure (In [84], this is also referred to as the *life measure principle*). At the same time it underscores the representative role of money: Money can be understood as 'frozen lifetime', in a sense that a certain part of lifetime has to be spent in order to generate a given amount of income[5]. This concept is central to the derivation of the LQI.

A fraction of our lifetime which we will call w is dedicated to earning an income g. For the beginning, g is sufficiently well described as a number closely related to the GDP (gross domestic product) per capita. A detailed discussion on the actual difference will follow after the basic derivation. If p is productivity per work time unit, then a person's total expected lifetime earnings follow as pwe_0, with life expectancy at birth e_0 given in (2.6). A person's yearly income amounts to $g = pw$. This income can be consumed and enjoyed during leisure time $l = (1 - w)e_0$. Since 1870, when industrialisation was at the point of penetrating the economic lives of the broad population in the Western world, real GDP (per capita) rose from some 2000 PPP US$[6] to 20 000 PPP US$ in the year 2000 [71]. This corresponds to an average yearly per capita growth of about 1.9% per year (industrialised countries). Total growth rates were even higher because of the simultaneous increase in population. At the same time, the average yearly time spent at work dropped from approximately 2900 to 1600 hours [71]. The change in w, i.e. with respect to total lifetime, is even more accentuated, due to the absolute and relative extension of educational and retirement phases in the wake of life expectancies rising from around 45 to 80 years.

The explanation for an increasing g parallelly to a decrease in w is to be sought in a productivity growth rate exceeding that of economical growth by 0.3 to 0.5%, depending on the country. Instead of becoming richer at the pace of productivity, people chose to use this productivity surplus in order to extend their leisure periods. At one time, this confirms the concept of time 'as the ultimate source of utility' [149] as well as the idea that leisure time and wealth are indeed two states of aggregation of one single matter, which are balanced against each other in the way that suits individuals or society best (work-leisure optimisation).

The Original Derivation

Similarly to the lifetime utility approach, Nathwani, Pandey & Lind use a product approach, combining an income function $f(g)$ with a leisure time function $h(l)$ with $l = (1 - w)e_0$, so that

$$L_w = f(g)h(l) \tag{2.17}$$

The two functions $f(g)$ and $h(l)$ are assumed to be monotonically increasing, differentiable and mutually independent. g and e_0 (and therefore l) are highly correlated among different countries (Figure 2.2). Kübler [63] demonstrates that a similar correlation can be observed when analysing the historical development of some industrialised countries. However, the correlation is very weak with respect to the income classes within one country, as shown in Table 2.1. Differences in life expectancy at birth are in the order of 5% between the poorest and the richest quantiles, whereas income differs by a factor of 7 to 10.

[5]In [68], Lind goes as far as to convert everything—money, death risks etc.—into life years, which serve as the means of comparison in an acceptability and affordability criterion (*time principle of acceptable life risk*). This is an alternative way of formulating the LQI concept as presented in this section.

[6]International US$ adjusted for *purchase power parity* according to the World Bank [145]

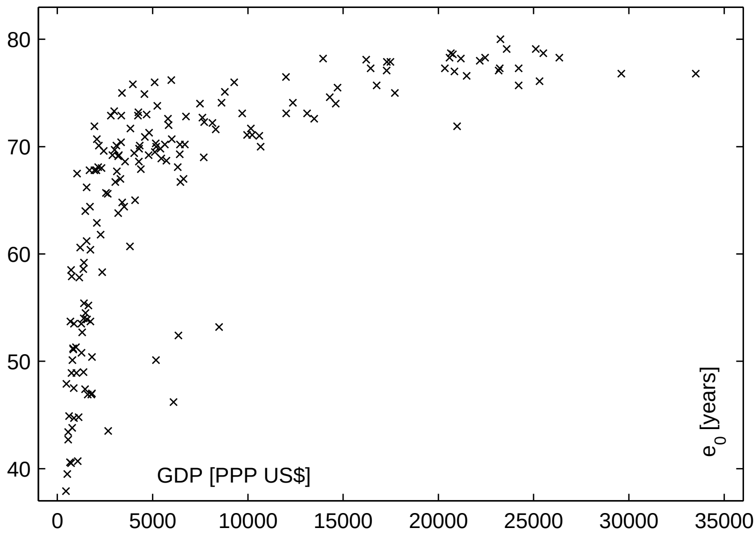

Figure 2.2: Life expectancy at birth versus GDP per capita in 174 countries [132]

When regarding society as a whole, one single risk-relevant project or decision has only a small impact upon average income and upon life expectancy at birth. Therefore, these impacts can be treated as infinitesimal increments. The resulting absolute change in L_w then is

$$\begin{aligned} dL_w &= \frac{\partial L_w}{\partial g} dg + \frac{\partial L_w}{\partial l} dl \\ &= \frac{df(g)}{dg} h(l)\, dg + f(g) \frac{dh(l)}{dl} dl \end{aligned} \tag{2.18}$$

and the relative change follows as

$$\frac{dL_w}{L_w} = \frac{g}{f(g)} \frac{df(g)}{dg} \frac{dg}{g} + \frac{l}{h(l)} \frac{dh(l)}{dl} \frac{dl}{l} \tag{2.19}$$

For $c_1 = \frac{g}{f(g)} \frac{df(g)}{dg}$ and $c_2 = \frac{l}{h(l)} \frac{dh(l)}{dl}$, (2.19) can be rewritten as

$$\frac{dL_w}{L_w} = c_1 \frac{dg}{g} + c_2 \frac{dl}{l} \tag{2.20}$$

The two factors c_1 and c_2 are the elasticities[7] of $df(g)$ and $dh(l)$, respectively. They describe the relative impact a change in g or l has upon L_w. When l rises by 1% because of some risk-relevant project ($dl/l = 0.01$), this means that L_w is going to increase by c_2%. It can be reasonably assumed that the relative impact of the two factors is independent of their absolute values, so that $c_1/c_2 = \text{const}$. Nathwani et al. [84] refer to this property as *indifference*. Under this assumption, both elasticities need to have constant values (iso-elasticity), so that

[7]Compare footnote 4 (p. 15).

Table 2.1: Life expectancy at birth versus income

Germany [107]: relative income position	e_0 (men)	e_0 (women)	**Canada** [91]: relative income position	e_0
1st quartile	77	82	1st quintile	74.0
2nd quartile	82	85	2nd quintile	76.9
3rd quartile	81	84	3rd quintile	77.5
4th quartile	83	86	4th quintile	78.1
			5th quintile	78.5

$c_1 = \frac{g}{f(g)} \frac{df(g)}{dg} = \text{const}$ and $c_2 = \frac{l}{h(l)} \frac{dh(l)}{dl} = \text{const}$. These two differential equations can be solved as $f(g) = g^r$ and $h(l) = l^s$, leading to

$$L_w = g^r l^s = (pw)^r ((1-w)e_0)^s \qquad \text{with } 0 < r, s < 1 \tag{2.21}$$

In order to determine r and s, it is necessary to recall the concept of *work-leisure optimisation* mentioned in the beginning. A person can increase his or her leisure time $(1-w)e_0$ in two ways: One is to prolong life expectancy e_0 by reducing some fatal risk. The other one consists in working less, i.e. reducing w. As stated by Nathwani et al. [84],

> 'People's choices reveal their preferences. Presumably, people on the average work just enough so that the marginal value of the wealth produced, or income earned, is equal to the marginal value of the time they lose when at work.'

Apart from the statement that people do optimise w with respect to L_w, this hypothesis implies that they already *are* in a state of optimality $w = w^*$. Both statements will have to be verified later on in the text. As a mathematical consequence of having attained optimality,

$$\frac{dL_w}{dw} = 0 \tag{2.22}$$

Inserting (2.21) in (2.22) leads to

$$r = s\frac{w^*}{1-w^*} \tag{2.23}$$

Without any loss of generality, we choose $r + s = 1$, leading to $r = w^*$ and $s = 1 - w^*$, so that finally

$$\begin{aligned} L_w &= (pw)^{w^*} ((1-w)e_0)^{1-w^*} \\ &= g^{w^*} e_0^{1-w^*} (1-w)^{1-w^*} \approx g^{w^*} e_0^{1-w^*} \end{aligned} \tag{2.24}$$

In industrialised countries, the term $(1-w)^{1-w^*}$ varies very little between countries and does not change significantly with time. Therefore, it can be treated as a constant. Given that L_w

serves as an *ordinal* measure of utility (establishing a ranking between options instead of an absolute measure), the term can be dropped in (2.24). Because of its ordinal character, taking the $(1 - w^*)^{\text{th}}$ root of L_w is equally permissible and leads to a further simplification of the life quality index:

$$L_q = g^q e_0 \qquad \text{with} \;\; q = \frac{w^*}{1 - w^*} \tag{2.25}$$

This result resembles that in (2.16) to a high degree, although e_0 stands in the place of $\bar{e}_d$ and income g takes that of consumption c. Again, a power function with a constant exponent serves to describe the monetary aspect of the indicator. However, q is no longer subject to qualified guessing, but is derived from the (optimal) fraction of time in life spent at work as $w^*/(1 - w^*)$. Yet, a number of questions are left open, with respect to the validity of this derivation as well as to the appropriate determination of w and g. These are addressed on the following pages.

The final question, whether lifetime utility and LQI can be combined into one synthesis is subsequently discussed in Section 2.2.3.

The Production of the GDP

The result in (2.25) holds under the assumption $g = pw$ made in the beginning. This view is in agreement with the definition of labour productivity, a key indicator that is empirically determined as $p = g/w$ for an economy. However, it reflects the way in which the GDP and thus g is generated only in part. To clarify this, it is helpful to briefly recall the way in which the GDP is determined:

The GDP or gross domestic product has become the standard measure for the output and income of a nation, as used e.g. by the United Nations [131]. It differs only slightly from the gross national income (GNI), in as far it limits itself to measuring production within a country's borders, while disregarding cross-boarder income transfers. GNI includes these transfers and thereby accounts for the economical activities of a country's nationals, regardless of their country of residence or work. In this sense, GNI is the more significant indicator when it comes to explaining income, while GDP is more easily determined in statistical terms and also has some advantages when evaluating short-term production changes.

Being an indicator for income and output at one time, the GDP can be interpreted in two ways, as a *flow of costs* and as a *flow of (final) products* (see e.g. [111]). Business has to pay for means of production, i.e. labour, land and capital goods. Private households receive money in the form of salaries (cost of workforce) and business profits (as interests and dividends). This is the flow-of-cost aspect, also termed *earnings approach*.

Under the flow-of-product aspect (*product approach*), private households pay money in order to purchase products from enterprises. Purchases between enterprises are not considered; they serve to create the final product, which is ultimately bought by a private or public household. For purchases of public interest, private households delegate a part of their income to the state (taxation). Further, net exports (exports minus imports) have to be added. The rest of the income is used for capital formation (saving), e.g. when a private household buys stocks

Table 2.2: National Accounting (after [125, 111])

product approach	earnings approach
private consumption	wages
+ public consumption	+ interests & profits
+ gross investment	+ indirect taxes
+ net exports	− trans-boarder income transfers
	+ depreciation
= GDP	= GDP
+ trans-boarder income transfers	+ trans-boarder income transfers
= GNI	= GNI
− depreciation	− depreciation
= NNI	= NNI

of an enterprise and the enterprise uses the revenues for buying capital goods (investments into production equipment, buildings etc[8]). As both views are aspects of one single cycle, they result in the same GDP value.

In order to describe how means of production are turned into the actual product, it is necessary to regard the flow of costs. In 1927, Cobb and Douglas established a function based on the observation that the relative contributions to economical output Y by labour and capital had maintained a constant relation over a long period of time (e.g. [72]). From a different point of view—since total output equals national income—this implies that the yearly total income of workers and that of capital owners grew at almost exactly the same rate. The *Cobb-Douglas production function* has the form

$$Y = AK^{\alpha}\Lambda^{\beta} \qquad \text{with} \quad \begin{cases} \alpha + \beta = 1 \\ 0 < \alpha, \beta < 1 \end{cases} \tag{2.26}$$

where K denotes the share of capital in the production process and Λ that of labour. A is a factor describing the productivity of the available technology. For $\beta = \alpha - 1$, the marginal products of capital and labour, respectively, are

$$\frac{\partial Y}{\partial \Lambda} = (1-\alpha)AK^{\alpha}\Lambda^{-\alpha} = \frac{(1-\alpha)Y}{\Lambda} \tag{2.27}$$

and

$$\frac{\partial Y}{\partial K} = \alpha AK^{\alpha-1}\Lambda^{1-\alpha} = \frac{\alpha Y}{K} \tag{2.28}$$

If K and Λ are increased by the same factor, then Y will rise by that very factor, too. This effect is called constant returns to scale. Raising only one of the two, say Λ, produces the following effect: The marginal product of labour $\partial Y/\partial \Lambda$ drops, while that of capital rises.

[8] In this sense, the amount of savings (theoretically) equals the amount of investments, but they correspond to two different actions. Saving is what a private household does when it puts money into an enterprise (directly or indirectly, via a bank). Investment is what the enterprise does with the assets it receives from a private household.

This is an incentive to raise K. In the end, this mechanism brings K/Λ back to its previous level so that the relation keeps constant on the long run. When production factors always earn their marginal products, total labour income equals $\Lambda(\partial Y/\partial\Lambda) = (1-\alpha)Y$ and total capital income equals αY. In many industrialised countries, $\beta = 1-\alpha$ has remained more or less constant until today, with values close to $\beta = 0.7$ [72, 104].

Most recently, Pandey [90] remarked the relevance of this production theory for the LQI concept and substituted $\Lambda = wN_{pop}$[9] in (2.26), so that $Y = AK^\alpha (wN_{pop})^\beta$, where N_{pop} is the total population[10] of a country. The output (and income) per person then is

$$g = \frac{Y}{N_{pop}} = A\left(\frac{K}{N_{pop}}\right)^\alpha w^\beta \tag{2.29}$$

Substituting for g in (2.21) and following through the same procedure as in (2.22) to (2.25) leads to

$$r = \frac{w^*}{\beta - \beta w^* + w^*} \tag{2.30}$$

Inserting r in (2.24) produces an inconveniently complicated expression. However, one may take the $(1-r)^{\text{th}}$ root with the same justification as for taking the $(1-w^*)^{\text{th}}$ root in the original derivation, so that the exponent in (2.25) is finally rewritten as

$$q = \frac{1}{\beta}\frac{w^*}{1-w^*} \qquad \text{with} \quad \beta \approx 0.7 \tag{2.31}$$

The Relation between g and GDP

When people balance total leisure time against wealth, it can be assumed that they have no interest in compromising the bases of their wealth. This has some consequences for the definition of g, i.e. the amount of income per capita disposable for risk mitigation measures with respect to the GDP. For the following considerations, it is necessary to shift from the flow-of-costs view towards the flow of products.

Every year, a large share of the production side of the GDP is made up by purchases of new investment goods (capital formation), such as buildings, machinery and other productive equipment. Together, they are referred to as yearly gross investments. At the same time, existing investment goods lose some or all of their value, due to physical deterioration or obsolescence. In economical terms, this effect is described as depreciation. Replacing these losses is an utter economical necessity. Otherwise, the economic output begins to shrink. In this sense, the share of depreciation in the GDP is definitely off limits for trade-offs between l and g. In consequence, GDP minus depreciation, the so-called net domestic product (NDP)

[9] Note that w is not only the fraction of life time spent at (paid) work but also the yearly workhours, because of $w \cdot 1$ year $= w$ (compare p. 17).

[10] When N_{pop} is used instead of the working population N_w, this is due to the definition of w as fraction of *total lifetime* spent at work. Besides, it is in agreement with the definition of GDP per capita, which equally refers to N_{pop} and not to N_w.

Table 2.3: Economic Indicators for Germany, 2003 [125]

(per capita values)	absolute [€]	absolute [PPP US$[a]]	relative to GDP
GDP	25,800	26,400	100.0%
GNI	25,600	26,200	99.2%
NNI	21,770	22,280	84.4%
gross investment	4,510	4,620	17.5%
depreciation	3,860	3,950	15.0%
net investment	650	670	2.5%

[a]Conversion into PPP US$ according to [145]

can be interpreted as an *upper limit* for g. However, it is more common and also more appropriate to use the net national income (NNI), i.e. GNI minus depreciation. Because NNI (just as GNI) accounts for cross-boarder income transfers, it indicates the actual income of a nation (i.e. the money disposable for decisions) more accurately. In any case, NNI and NDP are more significant as economic indicators than GNI or GDP. When GNI and—nowadays especially—GDP are commonly used as standard indicators, it is due to the difficulties in obtaining quick and reliable data on depreciation [111].

The next question is whether to include net investments in the trade-off. The volume of net investments is obtained as the difference between gross investments and depreciation. While the share of gross investments flowing into the replacement of existing goods keeps the economy from shrinking, net investments lay the foundation for actual growth. In our present economic system, steady growth is seen as a requirement for stability. From this point of view it would appear reasonable not to just exclude depreciation and assume g = NNI, but to further exclude net investments.

However, a number of payments apart from investments in the direct sense contribute to a stable or growing level of economic output. In this respect, it may become hard to draw a precise boarder. Education and public infrastructure are very obvious investments on behalf of a country's economy. Then there are those public institutions which do not directly contribute to economic production but provide the framework which economy needs in order to function. These include legislation, jurisdiction, police and military, equally requiring a certain share of the national income. Furthermore, a country's economy also relies on 'soft' parameters, such as the general culture leading to a minimum of social coherence. If people are unable to put a minimum of trust into one another, this will hamper or prevent many deals and compromise the economical climate as a whole. Building and maintaining this coherence can also come at a cost (public and private social and cultural initiatives, NGOs, religious communities and so forth). In this sense, it appears hard to determine, where the realm of non-tradeable investments in the broader sense actually ends.

Apart from such considerations on the foundations of economic stability, people will nor swap money for life expectancy or leisure time, if this decision pushes them beneath the subsistence level. Consequently, this minimum amount would also have to be excluded from g. Ultimately, it appears that the major part of the NNI is required in order to cover the basic needs of the economy, those of society as a basis for a functioning economy and those of the individual subject, requiring food, shelter, clothing, health-care etc.

As the NNI fraction that is 'taken away' and used for risk reduction measures becomes larger, the basic needs that are no longer covered by the remaining part become more and more essential to the population and the economy. With every further step, abandoning the next need becomes more painful. This effect is reflected by the law of diminishing marginal utility, leading to a utility function as the one in (2.14). Given the analogy between the resulting lifetime utility in (2.16) and the LQI formulation in (2.25), it appears that this effect is already included in the LQI concept. In this respect, accounting for it by means of choosing $g \ll \text{NNI}$ is probably redundant. Because of the sharply rising marginal utility of *not* trading yet another income unit for risk mitigation measures, it is most unlikely that these additional measures will occupy more than a few percentage points of the GDP or NNI.

Besides, one of the main assumptions in deriving a utility-based willingness-to-pay criterion as in Section 2.5 is actually the infinitesimal smallness of any change that g may undergo due to a risk-related decision. This assumption has mathematical reasons, but is equally and explicitly founded on the thought that any larger change would modify the macroeconomic set-up. Such a dependency, however, would be mutually exclusive with a derivation that treats macroeconomy as an exogenous factor with the aim of yielding a simple and clear criterion.

Therefore, it is proposed to assume

$$g = \text{NNI} \tag{2.32}$$

or, alternatively,

$$\begin{aligned} g &= \text{NNI} - \text{net investments} \\ &\approx \text{GDP} - \text{gross investments} \end{aligned} \tag{2.33}$$

In the second line of (2.33), GNP was replaced by GDP which is a reasonable approximation (compare Table 2.3; the situation is almost the same in other Western economies). In fact, reliable data for GDP and gross investments are easier to find than for GNI and NNI, which speaks in favour of (2.33). Generally, the difference between the two alternatives (2.32) and (2.33) is not very accentuated. In years of poor conjecture, net investments are especially low, such that both results become about equal. Table 2.3 contains some values in the case of Germany. Generally, typical values for g range at 75% to 85% of the yearly GDP in the industrialised world. At the end of the chapter, Table 2.5 contains GDP and g values for a number of countries.

Apart from the above rationale, the choice of $g \leq \text{NNI}$ offers the convenience of not producing any inconsistencies with the definition of c in Section 2.2.1: Under the constraints in (2.15), no money is inherited at the beginning of life and none is left at the end. This assumption is a strong simplification of economical reality and has been chosen in [118] only in order to facilitate a (comparatively) simple mathematical derivation. However, it implies that all savings (and not just a part) are expected to be consumed in the end. Savings equal investments[11]. Considering that only net investments can theoretically be reconverted into consumption while depreciation cannot, the derivation basically implies $E[c(a)] \leq E[g(a)] \leq \text{NNI}$, apart from some discounting effects.

[11] See footnote 8 (p. 21).

Calibration of w

The LQI derivation assumes that people balance between total lifetime spent at (paid) work we_0 and leisure time $l = (1 - w)e_0$ in an optimal way. As with g and GDP earlier, this raises the question as to which proportion of life is actually disposable for such an optimisation. First, there is the time needed for sleep. As argued by Ditlevsen [26], 'sleep time in general is necessary time for all persons in order both to do work and to enjoy life'. In a medical sense too, sleep is a basic requirement of life with a required daily amount of seven to eight hours for most people and as little as four for others. One possible approach is to exclude sleeping time from the optimisation and reduce the daily time available for work and leisure to 16 hours as in [26].

Nathwani et al. [84] as well as Rackwitz [103], on the other hand, define w with respect to a 24-hour day which is the theoretical upper limit. There is little doubt that people with children or those with excessively long working hours, e.g. well-paid executives, are ready to sacrifice a part of their daily amount of sleep, while those belonging to neither group take advantage of the possibility of sleeping longer. In this respect, daily sleeping time appears negotiable and therefore equally subject to the optimisation process. Undoubtedly, a certain absolute minimum amount of sleep is always required. Nevertheless, these considerations point in a similar direction as the argumentation applied to the relation between g and GDP above: Instead of defining a fixed minimum or average for sleep time, it is proposed to rely upon the effect of decreasing marginal utilities of extra work and extra leisure, as expressed in (2.24), where $L_w = g^{w^*} l^{1-w^*}$. In order to maximise the value of L_w, the optimal work time fraction $w = w^*$ has to stay well off the upper and lower boundary values, i.e. 0 and 1. The intrinsic logic of this relationship depicting human psychology[12] excludes the possibility of 'permanent lack of sleep by choice'. Hence, *it is proposed to define w relative to a 24-hour day* as in [84, 103], both because of the theoretical support and because the practical convenience of not having to decide which fraction of sleeping time is essential and which one is disposable (apart from the difficulty of statistically measuring either of them).

Then, there is the question whether the entire lifespan from birth to death is subject to the work-leisure trade-off. Again, the mentioned publications are strongly supportive of using this theoretical maximum [84, 103]. Nowadays, child labour (<14 years) is banned in industrialised countries, but it used to be common in the agricultural society and during the early phases of industrialisation. In some poor societies child labour still exists. At old age, it is principally possible to engage in paid work, usually self-employed, provided a person's health status permits such a decision. People who feel the financial necessity of working beyond the normal retirement age will mostly do so, as can be observed in countries with very low retirement pensions (e.g. Russia). In other countries, where pension insurance is not compulsory to the Western European extent, there are always those without sufficient funds for retiring at an acceptable standard of living (e.g. USA).

Lind [67] proposes to correct e_0 for background morbidity, i.e. the prevalence of disability (injury, disease). A critically disabled person is unable to earn money or to fully enjoy consumption, so that the corresponding period of time is not disposable for the work-leisure trade-off. On the other hand, Lind, as well as Nathwani et al. [84] and most other publications [4, 54, 109, 118, 136] valuate human consequences only with respect to their fatal impacts

[12] Its validity is discussed in the next subsection, 'Verification of the Work-Leisure Optimisation Principle'.

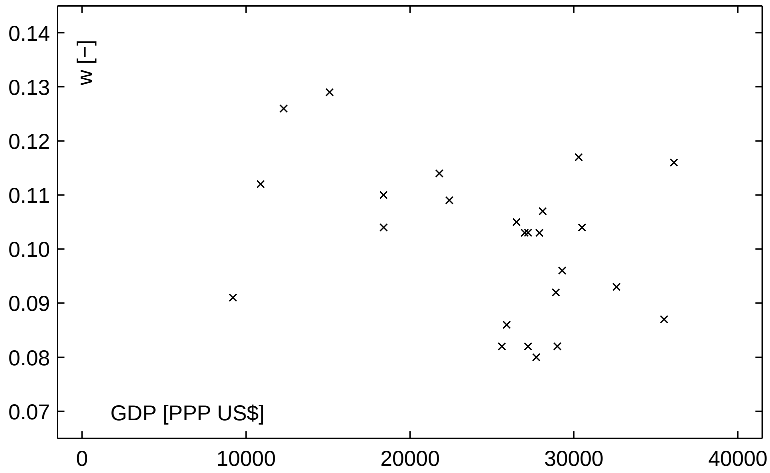

Figure 2.3: GDP per capita versus w in 25 OECD member countries [88]

(mortality), whereas non-fatal impacts are neglected. If background morbidity is taken as a reference, it appears consequential to regard the morbidity consequences of a decision too as proposed in Chapter 3.

In order to determine the average value of w in a society, two approaches can be followed. The first regards the total life span of some average individual [84]:

$$w_{\mathrm{LWT}} = \frac{\text{life working time}}{e_0} \cdot \frac{\text{ywh per employee}}{24 \cdot 365} \tag{2.34}$$

where ywh stands for '(average) yearly working hours'. In (2.34), w includes one hour per day for commuting and is estimated as being equal to 1/8. The second approach was proposed by Rackwitz [104] and describes the momentary state as

$$w_{\mathrm{LF}} = \frac{\text{participating labour force}}{\text{total population}} \cdot \frac{\text{ywh per employee}}{24 \cdot 365} \cdot \frac{9}{8} \tag{2.35}$$

Here, the factor 9/8 corresponds to Nathwani et al.'s assumption that one hour of commuting is necessary per eight hours of labour. Strictly speaking, $w_{\mathrm{LWT}} = w_{\mathrm{LF}}$ is only fulfilled under very special conditions, e.g. if life working time and life expectancy at birth have remained constant for a long period $\geq a_u$, together with a population growth of $n = 0$. Otherwise, w_{LF} represents reality more correctly. Nowadays, with $n \approx 0$ in many Western countries, w_{LWT} is a convenient approximation, as used by Nathwani et al. [84].

Country-specific data can be found in detailed labour statistics [34, 88]. However, as noted in [35] and elsewhere, the respective numbers are hardly ever comparable between different countries. Survey techniques differ, and so do other issues, such as the question whether to account for the self-employed, the unemployed, for overtime as well as part-time work and for multiple jobs. This is reflected by Figure 2.3. Nevertheless, the scatter plot shows a trend towards shorter working hours in richer countries, which is consistent with the work-leisure

Table 2.4: Working hours of Western European couples (added) vs. financial situation [9]

financial situation	**actual situation**	**preferred**
comfortable	66 hours	61 hours
adequate	59 hours	61 hours
difficult	53 hours	64 hours
all couples	*62 hours*	*61 hours*

optimisation principle. The w-values have been moderately adjusted for part time work and can also be found in Table 2.5.

Verification of the Work-Leisure Optimisation Principle

Apart from the life measure principle, the work-leisure optimisation principle can be seen as the central concept of the LQI derivation. Its mathematical equivalent in (2.22), $dL/dw = 0$, has two implications that need to be verified:

- People optimise w with respect to their lifetime utility (or life quality).
- On average the actual value w equals the optimal value w^*, i.e. society has already attained a state of optimality.

In order to prove the validity of these two assumptions, Rackwitz [104] proposes to take advantage of an empirical investigation conducted by Bielenski et al. [9]. They conducted a survey in 16 Western European countries, asking couples about their preferences. Their findings are resumed in Table 2.4 in a compressed way: It appears that those couples with a sufficient income are satisfied with their work-leisure balance, whereas those who are doing very well would prefer less working time. Those with a subjectively unsatisfactory income would be happy to exchange some of their leisure time for the possibility of earning more money. These findings confirm the assumption that people do optimise and strongly speak in favour of the hypothesis that the average member of society is close to his or her optimum.

2.2.3 Discussion and Synthesis

The LQI approach can be seen under two aspects, as an approach of its own or as a means to provide a derivation for the exponent q in the classical socio-economic lifetime utility approach. Most recently, Pandey & Nathwani [92] adopted the second view. Judging by its origins [67], on the other hand, it is an independent approach rooting in the UN Human Development Index rather than socio-economic utility theory. Regardless of the point of view, both approaches—classical socio-economic and LQI—deal with the same problem under almost the same premises. In this respect, the similarity between the two results can hardly be called accidental. Rather, one would suggest that there is one single underlying law that necessarily produces two very similar formulations. Therefore, the question arising is maybe not, whether to prefer one or the other approach, but instead, whether it is possible to find sound arguments in order to bridge the remaining minor differences and establish one final joint formulation.

It may be helpful to recall the two formulations. The socio-economic approach yields expected lifetime utility as $L = c^q \bar{e}_d$ in (2.16). The life quality index has the form $L_q = g^q e_0$ according to (2.25).

The first question is whether the solution $q = \frac{1}{\beta}\frac{w}{1-w}$ from (2.31) is valid only within the LQI framework or also with respect to the lifetime utility approach. If both formulations are indeed two realisations of one single principle, it does not only seem natural that they resemble each other formally, but it can also be deduced that the respective items represent the same sub-aspect of the common underlying principle[13]. Following this line of thought, it appears sensible to regard the LQI derivation for q as a general solution. Besides, the numerical implications do not speak against such a conclusion either. For an average $w = 0.105$ and $\beta = 0.7$, there is $q = 0.18$, which is in good agreement with the original (underived) estimate of $q = 0.2$ from the socio-economic literature.

The next issue concerns the difference between c and g. As previously discussed (p. 24), c is determined under the somewhat virtual assumption that people do not inherit or leave any bequests. Using up all savings until the (expected) end of life is equivalent with reconverting all (net) investments made with that particular individual's savings into money and consuming it. Under these assumptions, c is in about equal to g. Considering that g does not depend on any assumptions such as 'no bequests' and that it can be derived from basic economic data, it is probably preferable to c.

Then, there is the question, whether to use life expectancy at birth e_0 or age-averaged life expectancy $\bar{e} = \int_0^{a_u} e(a)h(a,n)da$. The classical socio-economic approach uses life expectancy in order to quantify the amount of utility that is enjoyed by the average citizen. For this purpose, $\bar{e}$ is the appropriate choice. The LQI also understands life expectancy as a multiplier for utility, but first of all as an indicator for the state of society. Yet, $\bar{e}$ can serve this second purpose just as well. Therefore, $\bar{e}$ is undoubtedly the preferable choice.

Finally, the issue of discounting needs to be decided on. From a utility-theoretic point of view, valuing future gains and losses less than present ones is a fact of economics and psychology. In this respect, the classical socio-economic approach is clearly in favour of discounting. Judging by the considerations behind the LQI approach, the impression is somewhat indifferent. It appears difficult to find strong arguments either in favour or in disfavour of discounting, as long as the LQI is seen primarily as an indicator. If it is understood as a utility function, the same argumentation as for the classical the socio-economic approach applies and discounting becomes as a necessity. In brief, there are strong arguments advocating discounting, whereas convincing counter-arguments are hard to find. Therefore, it appears reasonable to discount age-averaged life expectancy as in (2.11), i.e. to replace $\bar{e}$ by $\bar{e}_d$.

In the light of all these considerations, the final formulation for lifetime utility is proposed as

$$L = g^q \bar{e}_d \qquad \text{with} \quad q = \frac{1}{\beta}\frac{w^*}{1-w^*} \tag{2.36}$$

[13] It may be hard to precisely identify this principle, but that is beside the point of the argument. In any case, the presumed principle is probably closely related to the principle of decreasing marginal utility.

2.3 Societal Discounting of Utility

In the previous section, discounting was referred to as a fundamental fact of economy and psychology. Accordingly, the final lifetime utility formulation in (2.36) was chosen such that it accounts for this phenomenon. This section deals with the theoretic background of discounting and discusses the question of the appropriate height of a societal discount rate.

In a private business context, the case is comparatively clear: No investor is willing to lend money without demanding interest or to invest into a project with a yield on capital below the market interest rate. The market rate is almost equal to the interest someone can obtain without taking any risks, typically by purchasing fixed-interest government bonds. As a long-term average, Nordhaus [86] proposes a rate of $\gamma \approx 5\%$. For public projects, on the other hand, such a high interest rate can be problematic, especially when long-term investments are concerned: Investments into traffic infrastructure or other public facilities are meant to satisfy the needs of present and future generations. However, a high discount rate gives very little weight to costs and benefits arising in the medium and distant future. For $\gamma = 5\%$, a cost or benefit will be valued at only 30% of its present weight 25 years from now. 50 years from now, the weight will have dropped to as little as 9% of its present value. Considering that civil engineering structures are frequently designed for service periods well above 50 years, it appears problematic to underweight the interests of future generations in such an extreme way.

Motives of Discounting

According to Ramsey's classical approach [3, 106, 123], the market rate under perfect market conditions is written as

$$\gamma = \rho + \varepsilon\zeta \tag{2.37}$$

Here, ρ denotes the pure time preference rate and $\varepsilon\zeta$ is the growth time preference rate. All rates are understood per capita and net of inflation.

Growth time preference follows from the principle of decreasing marginal utility described on p. 14: People become richer every year at the pace of the economic growth rate ζ, so that the same real amount of money will provide less additional utility next year compared with the present year (e.g. [97]). In consequence, a shrinking GDP leads to negative growth discounting. Comparing GDP per capita from 1870 with that of 1992 [71] yields $\zeta \approx 1.9\%$ for industrialised countries (Western Europe, USA and Canada, Japan, Australia and New Zealand), $\zeta \approx 1.8\%$ for Southern Europe, $\zeta \approx 1.4\%$ for Eastern Europe and as little as $\zeta \approx 0.9\%$ for Africa. The (real) economic growth rate of a country as a whole is obtained by adding the population growth rate, i.e. $\zeta + n$. For constant proportional risk aversion (CPRA, p. 14), the effect of increased wealth upon marginal utility is expressed by a constant elasticity value ε. It is frequently assumed as $\varepsilon = 1$, which is the exact solution for a logarithmic utility function and a possible approximation otherwise[14] (e.g. [7]).

[14] According to (2.14) , $q = 1-\varepsilon$. For $q \approx 0.2$, it appears more consequential to assume $\varepsilon \approx 0.8$. Unfortunately, looking into the literature seems to provide no explanation regarding the contradictory common estimates of

Pure time preference is a psychological phenomenon broadly described in the economic literature (e.g. [97]). It is called 'pure', because people value certain needs less than others for the sole reason that they occur in the future and not now. This behaviour might partly be due to the uncertainty about one's future (as expressed by the mortality rate). However, the main reason lies within human impatience and economic myopia. When $\gamma = \rho + \varepsilon\zeta$ is commonly estimated as 5% [86], ρ contributes $\approx 3\%$, given the ε and ζ value proposed above (However, Arrow [3] assumes $\gamma \approx 3\%$, implying $\rho \approx 1$).

The Dispute on Sustainable Public Discounting

In recent years, sustainability has become a key concept in the discourse on public policy making. In the words of the UN Commission on Environment and Development ('Brundtland Commission' [130]), a development is regarded as sustainable, if it 'meets the needs of the present without compromising the ability of future generations to meet their own needs'. Due to this definition, sustainability can be understood as an ethical principle aiming to establish intergenerational equity.

In abstract terms, sustainability is a state in which resources are consumed at a rate that can be infinitely maintained. *Environmental sustainability* regards the depletion of renewable and non-renewable natural resources, whereas *economic sustainability* deals with the production capacity of a country. Apparently, the availability of natural resources influences the production capacity, so that both types of sustainability are closely linked to each other. From the viewpoint of *weak sustainability*, environmental factors are primarily understood as contributors to personal utility (raw materials for the production of consumption goods, natural environments for recreational activities). According to this approach, environmental goods can be substituted by other goods. Thus, non-renewable resources can be depleted, as long as society invests into education or other goods that will provide a maintained or growing level of utility in the future. This concept is less comprehensive than that of *strong sustainability*, which regards environmental integrity as an asset that ought to be preserved for its own sake, even if no socio-economic utility can be derived. Strong sustainability in the strictest sense is impracticable, whereas weak sustainability provides little or no protection for rare species and ecosystems. There is an ongoing debate on the appropriate compromise between these two extremes ('How strong is strong enough?'). The current state in the debate and a more in-depth introduction to the mentioned sustainability concepts can be found e.g. in the GREENSENSE report [33].

In the context of utility-based decision making, it is unavoidable to limit the scope to weak sustainability. Considering strong sustainability concepts appears to be incompatible with the utility definition from the previous section and cannot be integrated into that methodology. However, this means in no way that environmental preservation without an economic benefit is regarded as a principally unnecessary issue.

If sustainability is about intergenerational equity, then the choice of the discount rate will play a central role. High discount rates lead to an undervaluation of costs and benefits arising

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$q \approx 0.2$ and $\varepsilon \approx 1$, although economists such as Arrow [3] or Bayer & Cansier [7] leave little doubt that the same phenomenon is involved in both cases. Maybe, the contradiction could be alleviated by accepting that $0.8 \approx 1$ is a defendable approximation, whereas $0.2 \approx 0$ is not.

in the future. In consequence, decisions with a high damage potential in the far future (e.g. nuclear waste disposals) may appear acceptable, whereas those with high benefits for future generations (e.g. restricted release of greenhouse gases) may appear unnecessary and ineffective. Zero discounting, on the other hand, neglects the fact that people become richer from year to year at the pace of economical growth. Therefore, a number of economists [13, 106, 114] propose to discount by growth time preference $\varepsilon\zeta$ alone, while setting pure time preference ρ to zero (i.e. $\gamma = \varepsilon\zeta$). Rabl [99] even proposes a rate of $0 \leq \gamma \leq \varepsilon\zeta$.

Arrow [3], on the other hand, argues that $\rho = 0$ puts 'an incredible and unacceptable strain on the present generation'. Quite convincingly, he argues in favour of a morality understanding, which does not treat everyone but *everyone else* (i.e. everyone except oneself) equally. However, the resulting discount rate γ is only little higher than in the previous case, because Arrow proposes a pure time preference rate as low as $\rho = 1\%$.

Generation-adjusted Discounting

The previous considerations indicate that there is a conflict between the preferences of living persons ($\rho > 0$) and the demands of sustainability and intergenerational equity. As a possible solution, several authors [6, 7, 43, 99] suggest using the full market rate of $\rho + \varepsilon\zeta$, if the interests of living generations are concerned (*intra*generational discounting) and discounting by $\varepsilon\zeta$ alone, if unborn generations are concerned (*inter*generational discounting). Pure time preference, i.e. the impatient wish for instant gratification ($\rho > 0$) is an attribute of living individuals. Unborn individuals are unable to show any preferences whatsoever. In their case, society should act as a trustee.

In consequence, Bayer & Cansier [7] propose a model, in which *inter*generational discounting is used from *now* on (decision point) until the birth of an individual while *intra*generational discounting is applied to the time between birth and the cost or benefit effect of our present decision. If we consider an individual who will be born 20 years from now and a cost effect C_{50} that will occur $t^* = 50$ years from now, its present value is obtained as $\mathrm{PV} = C_{50} \cdot \exp[-20\,\varepsilon\zeta] \cdot \exp[-30(\rho+\varepsilon\zeta)]$. However, public decisions usually affect society as a whole, i.e. people of all age classes. For the determination of the PV, equal weight has to be attributed to every living person. Bayer & Cansier operate with a simplified population model, in which generations are 25 or 40 years apart with no births occurring in the meantime. Principally, this model can be replaced by a real-world life table model as in (2.3) to (2.11) without any loss of generality. This leads to the following formulation [105]:

$$\mathrm{PV}(t^*) = \begin{cases} \int_0^{t^*} C_{t^*} \exp[-a(\rho+\varepsilon\zeta)] \exp[(-t^*+a)\varepsilon\zeta] h(a,n)\,da \\ \quad + \int_{t^*}^{a_u} C_{t^*} \exp[-t^*(\rho+\varepsilon\zeta)] h(a,n)\,da & \text{for } t^* \leq a_u \\ \int_0^{a_u} C_{t^*} \exp[-a(\rho+\varepsilon\zeta)] \exp[(-t^*+a)\varepsilon\zeta] h(a,n)\,da & \text{for } t^* > a_u \end{cases} \tag{2.38}$$

After a_u years, all presently living individuals are assumed to have died. This explains the more compact formulation for $t^* > a_u$. Under the assumption of stable population growth, the age distribution $h(a, n, t^*) = h(a, n) = \text{const}$ can be obtained from (2.10).

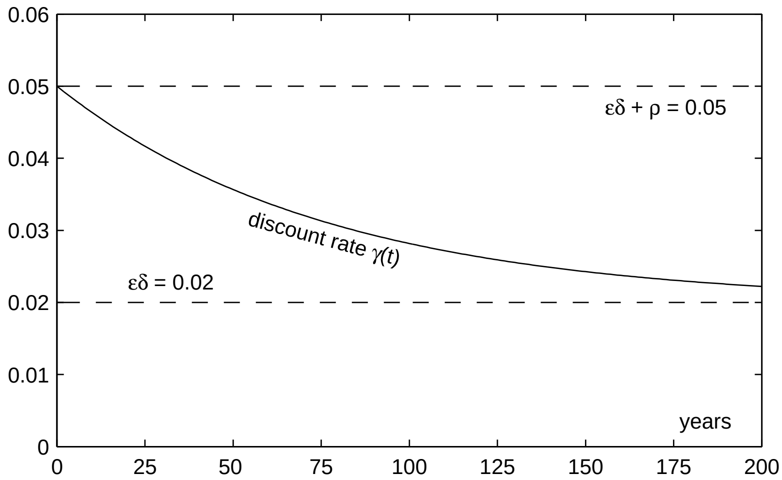

Figure 2.4: Generation-adjusted discount rate for $\varepsilon\delta = 0.02$ and $\rho = 0.03$

The discounting model in (2.38) appears quite complicated for ordinary applications. However, it can be used in order to determine an equivalent time-dependent discount rate:

$$\gamma_{eq}(t^*) = \frac{\ln[C_{t^*}/\mathrm{PV}(t^*)]}{t^*} \tag{2.39}$$

The result can be approximated as

$$\gamma_{eq}(t^*) \approx \varepsilon\zeta + \rho \exp[-bt^*] \tag{2.40}$$

For industrialised countries, $b \approx 0.013$ is a suitable approximation [104]. As shown in Figure 2.4, $\gamma(t^*)$ drops quickly during the first years and approaches the level of $\varepsilon\zeta$ asymptotically for later years.

Independently, Faber [36] presented the concept of a virtual *joint decision maker*, who unites the viewpoints of several successive decision makers, each adhering to a different generation. Each of the successive decision makers follows from the viewpoint of his own generation by discounting at full market rate $\rho + \varepsilon\delta$. Costs related to long-term maintenance or depletion of non-renewable resources can be transferred to the viewpoint of other generational decision makers by discounting at economic growth rate δ. In [37], Faber & Nishijima demonstrate that this approach is equivalent to the one in [7], leading to the same result as in (2.38).

Discounting Utility Instead of Real Prices

Bayer & Cansier [7] argue that utility must not be growth-discounted, because the effect of diminishing marginal utility is already covered by the utility function. According to them, it can only be applied to consumption or income values (c and g, dimension = [money]), but not to utility functions such as $u(c) = c^q$ (dimension = [money^q]). Following this argumentation, growth-discounting would then correspond to calculating the utility of a utility. Shepard &

Zeckhauser [118], on the other hand, think that utility ought to be discounted at the real market rate (here, too, estimated at 5%), which apparently includes the growth time preference rate.

2.4 Expressing Risk

The Mathematical Definition

Risks to human life are commonly expressed by their effect upon mortality μ. In agreement with (2.3), raw mortality can be obtained by dividing the total number of deaths per year by the population size N_{pop}. The life-threatening potential of a decision can be described by the expected number of fatalities per year N_D involved or by the *change* dN_D in expected fatalities respectively. If any member of society might potentially find himself among that number, the corresponding change in mortality follows as:

$$d\mu = \frac{dN_D}{N_{pop}} \tag{2.41}$$

In the case of life-saving decisions, dN_D takes a negative value (and so does $d\mu$). According to (2.7), a change in mortality leads to a change in life expectancy.

In civil engineering, most fatalities are not caused by some steady immission, e.g. air pollution, but typically by some failure event F with a yearly probability of occurrence r (failure rate). Modifying r leads to $dN_D = dr\, N_{D|F}$. The corresponding change in mortality is obtained as

$$d\mu = \frac{dr\, N_{D|F}}{N_{pop}} \tag{2.42}$$

The Role of Risk Perception

Unfortunately, human mind does not perceive risk in such a linear way. According to psychometric observations, people reject certain adverse events more strongly than other ones, depending on the degree of involuntariness and the number of people simultaneously affected [58, 102, 120]. When this phenomenon is commonly referred to as 'risk aversion', this terminology is very misleading [8], because it has nothing to do with risk aversion in the utility-theoretic sense as introduced in Section 2.2.1. The term *disaster aversion*[15] seems to be more to the point.

A typical example is the public attitude towards air versus road traffic accidents. Following the definition in (2.42), travelling on an airliner bears about a tenth of the risk of going by car, when relating to passenger kilometers. With respect to passenger hours, both risks are just about equal [98]. However, being a passenger on an airliner means having practically no control over the situation (involuntariness) and involves a greater number of people simultaneously

[15]Bedford proposed this term during his oral presentation of [8] in Gdańsk, Poland, June 2005.

being killed in case of a crash. In consequence, people dread airplane crashes much more than fatal car accidents.

Considering disaster aversion typically leads to a slight modification of (2.42), in a sense that the conditional number of lives lost is raised to the power of some coefficient α or multiplied with some factor dependent on $N_{D|F}$ (see [58] for overall view). Unlike (2.42), the result of a risk definition like $dr(N_{D|F})^{\alpha}/N_{pop}$ is not a mortality change, but has some other, odd dimension. In consequence, it is also impossible to convert such a result into a gain or loss of life expectancy. Such a risk definition is principally admissible, but completely incompatible with the concept of lifetime utility, which regards life expectancy as a key parameter (life measure principle).

Upon closer inquiry, the incompatibility of utility theory and a disaster-averse modelling originates from a different view upon death. From a utility-theoretic point of view, dying essentially means losing future lifetime (and thereby future utility). Accordingly, dying young is worse than dying old, whereas the cause of death is irrelevant. Considering disaster-aversion means attributing negative utility—or *disutility*—to the physical process of dying. The degree of disutility depends on the degree to which a certain death cause is dreaded. This approach is in disagreement with the concept of lifetime utility, where negative utility does not exist in an absolute sense, but only relatively as a loss of positive utility.

Then, which of the two approaches should be preferred? It appears that both views have their merits. However, the present framework does not primarily deal with the psychologically adequate description of risk. Instead, it aims at deriving an acceptability criterion for risk-related decisions that is consistent with more general principles such as the intangibility of human life (see Section 2.1). Utility theory permits meeting this goal, whereas risk-averse modelling does not provide a consistent alternative in this respect. Therefore, the risk definition in (2.41) and (2.42) is deemed adequate for the present context.

2.5 Willingness to Pay as an Acceptability Criterion

Decisions leading to a higher chance of survival at no cost are generally welcome and even more so, when they are associated with a positive income effect. Inversely, a heightened risk of losing one's life is never acceptable, as long as it does not involve any advantages on other levels, such as income, or even leads to monetary disadvantages. Most cases however, belong to neither of these two extremes. Mitigating fatal risks usually comes at a cost and undergoing a higher risk is usually rewarded by a higher income. The readiness to accept such an exchange is called *willingness to pay* (WTP) in the first case and *willingness to accept* a payment (WTA) in the second.

2.5.1 Deriving WTP from Lifetime Utility

As discussed in Section 2.1.2, either type of willingness—WTP and WTA—shows that the marginal utility of the accepted entity is equal or higher than that of the exchanged entity. In consequence, an individual's total lifetime utility L remains equal or rises. Formally, this acceptability criterion can be written as [84]

$$dL \geq 0 \tag{2.43}$$

The incremental change of $L = L(g, \bar{e}_d)$ is obtained by total differentiation:

$$dL = \frac{\partial L}{\partial g} dg + \frac{\partial L}{\partial \bar{e}_d} d\bar{e}_d \tag{2.44}$$

The exact value of the WTP (or the WTA), i.e. the boundary of the acceptable domain follows from $dL = 0$:

$$\text{WTP} = -dg = \frac{\frac{\partial L}{\partial \bar{e}_d}}{\frac{\partial L}{\partial g}} d\bar{e}_d \tag{2.45}$$

Inserting (2.36) in (2.44) leads to

$$\text{WTP} = -dg = \frac{g}{q} \frac{d\bar{e}_d}{\bar{e}_d} \tag{2.46}$$

Note that a payment corresponds to a loss in disposable income, so that dg is negative in that case and $-dg$ has a positive value. Because of $E[X]/E[Y] \neq E[X/Y]$[16], the result is written more correctly as

$$\text{WTP} = -dg = \frac{g}{q} E_A\left[\frac{de_d(a)}{e_d(a)}\right] \tag{2.47}$$

According to (2.43), the acceptable domain is finally delimited by

$$-dg \leq \frac{g}{q} E_A\left[\frac{de_d(a)}{e_d(a)}\right] \qquad \text{or} \qquad \frac{dg}{g} + \frac{1}{q} E_A\left[\frac{de_d(a)}{e_d(a)}\right] \geq 0 \tag{2.48}$$

This boundary condition is graphically displayed in Figure 2.5. In the original version of Nathwani et al. [84], the LQI formulation in (2.25) is used, leading to

$$-dg \leq \frac{g}{q} \frac{de_0}{e_0} \qquad \text{or} \qquad \frac{dg}{g} + \frac{1}{q} \frac{de_0}{e_0} \geq 0 \tag{2.49}$$

When a decision leads to a rise in (age-averaged and discounted) life expectancy, the right hand side of (2.48), i.e. $\frac{g}{q} E_A[\frac{de_d(a)}{e_d(a)}]$, delimits the *maximum* amount an average individual is assumed to be willing to pay for this specific gain of lifetime. Because the derivation is based on people's general preferences (as described by the work-leisure optimisation principle), it is a helpful tool for making decisions with public relevance. The criterion helps to distinguish

[16] i.e. $\dfrac{d\bar{e}_d}{\bar{e}_d} = \dfrac{E_A[de_d(a)]}{E_A[e_d(a)]} \neq E_A\left[\dfrac{de_d(a)}{e_d(a)}\right]$

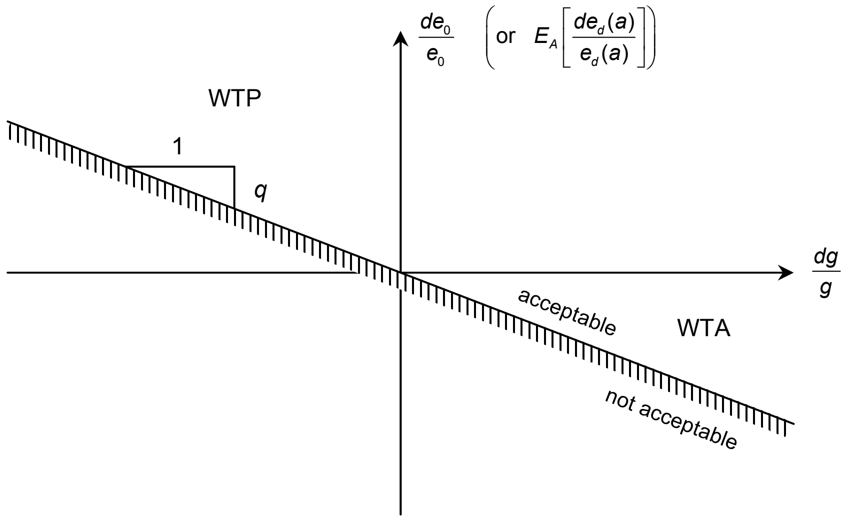

Figure 2.5: Graphic representation of the WTP criterion, after [84]

appropriate risk mitigation measures from those that are wasting public resources. In this respect, it is consequential to speak of an *affordability criterion*. For the inverse case of a heightened fatal risk (and lowered life expectancy), the criterion expresses the *minimum* gain that needs to arise from a decision in order to appear acceptable (or rather *tolerable*).

These implications of (2.48) and (2.49) are equally illustrated by Figure 2.5. Decisions located in the 1st quadrant are generally always acceptable, while those in the 3rd quadrant are principally unacceptable. In the 2nd and 4th quadrants, the WTP criterion distinguishes between acceptable and unacceptable decisions. Hereby, the quadrant 2 corresponds to the WTP case (less income for more safety) and quadrant 4 to the WTA case (less safety for more income).

It should be noted that the utility-based WTP criterion is a relative criterion and can only serve to determine, whether a potential change is preferable to the status quo. If there are several options, the criterion helps to determine the optimal one. According to Nathwani et al. [84], 'The best option among several options is the one from which any change will reduce the LQI [*here:* lifetime utility]'.

2.5.2 Quantification of WTP

The potentially fatal consequences of a decision are usually expressed as a change in the mortality function $d\mu(a)$ as discussed in Section 2.4. In the WTP criterion, they enter as a change in discounted life expectancy $de_d(a)$. The most obvious way of obtaining $de_d(a)$ from $d\mu(a)$ consists in inserting in (2.11) and calculating the difference as

$$de_d(a) = e_d(a, d\mu(a)) - e_d(a) \tag{2.50}$$

leading to

$$
\begin{aligned}
E_A\left[\frac{de_d(a)}{e_d(a)}\right] &= \int_0^{a_u} \frac{de_d(a)}{e_d(a)} h(a,n)\,da \\
&= \int_0^{a_u} \frac{e_d(a, d\mu(t)) - e_d(a)}{e_d(a)} h(a,n)\,da \\
&= \int_0^{a_u} \frac{e_d(a, d\mu(t))}{e_d(a)} h(a,n)\,da - 1
\end{aligned}
\tag{2.51}
$$

because of $\int_0^{a_u} h(a,n)\,da = 1$. For complex mortality functions, this is probably the preferable way of proceeding. However, a lot of time can be saved by using approximative relationships for some less complicated standard cases:

When regarding changes in mortality $d\mu(a)$ resulting from some specific decision, the general all-cause level of mortality $\mu(a)$ of a population is commonly referred to as *background mortality*. One standard case consists in a mortality change that affects every member of society regardless of his or her age and age-specific background mortality in the same way, so that

$$
\mu_\Delta(a) = \mu(a) + \Delta \tag{2.52}
$$

and $d\mu(a) = \Delta = \text{const}$. This scheme is commonly referred to as *absolute* [142] and corresponds to typical cases of accidents and technical failure as those described in Chapter 5. When a building collapses, everybody inside faces more or less the same risk of dying, regardless of sex or age[17]. Besides, it is assumed that the occupants cover different age-classes in a representative way, i.e. according to $h(a,n)$. This second assumption may not always be true but assuming it is required by the equality principle (see Section 5.2.3 for discussion). With respect to the WTP criterion (2.48), Rackwitz [101] proposes to linearise the relation between $d\mu(a) = \Delta$ and $d\bar{e}_d$ by using the first term of a McLaurin series (as before, $\tau^* = \tau - a$[18]):

$$
\begin{aligned}
E_A\left[\frac{de_d(a)}{e_d(a)}\right] &\approx \int_0^{a_u} \frac{-\frac{d}{d\Delta} e_d(a,\Delta)\big|_{\Delta=0} \cdot \Delta}{e_d(a)} h(a,n)da \\
&= \int_0^{a_u} \frac{-\frac{d}{d\Delta} \int_a^{a_u} \exp\left[-\int_a^t \mu(\tau) + \Delta + \gamma(\tau^*)d\tau\right] dt\Big|_{\Delta=0} \cdot \Delta}{\int_a^{a_u} \exp\left[-\int_a^t \mu(\tau) + \gamma(\tau^*)d\tau\right] dt} h(a,n)da \\
&= -\int_0^{a_u} \frac{\int_a^{a_u} (t-a) \exp\left[-\int_a^t \mu(\tau) + \gamma(\tau^*)d\tau\right] dt}{\int_a^{a_u} \exp\left[-\int_a^t \mu(\tau) + \gamma(\tau^*)d\tau\right] dt} h(a,n)da \cdot \Delta \\
&= -J_\Delta\,\Delta
\end{aligned}
\tag{2.53}
$$

Such a linearisation is only permissible for small (infinitesimal) changes in g and $\bar{e}_d$. Besides, large changes are generally beyond the scope of the entire lifetime utility derivation, because

[17] Rackwitz [104] remarks that fatalities (or averted fatalities) are distributed according to the general age distribution, so that $n_D(a) = N_D h(a,n)$ in case of an accident. Because the general population is distributed as $n_{pop}(a) = N_{pop} h(a,n)$, this leads to $d\mu(a) = \frac{n_D(a)}{n_{pop}(a)} = \text{const}$.

[18] Compare footnote 2 (p. 11).

their impact upon the stability of economy as a whole is not included in the model lifetime utility model.

The *proportional* or *relative* risk model is another important standard case:

$$\mu_\delta(a) = \mu(a)(1+\delta) \tag{2.54}$$

Here, the impact of δ is proportional to a person's background mortality. It implies that those persons who are more susceptible to death causes of any type—typically because of a weakened physical state—are also more likely to succumb to the phenomenon behind δ. Such a relationship is typical for diseases of all sort, as they are caused by infections or the case of toxic exposure described in Chapter 6. Essentially, (2.54) is the simplest version of the various relationships introduced there. Except for the first few months after birth, $\mu(a)$ is an exponentially rising function of a (compare Figure 2.1). Therefore, this mortality scheme affects elder people more strongly than young ones. Linearisation follows the same principle as in (2.53):

$$\begin{aligned}
E_A\left[\frac{de_d(a)}{e_d(a)}\right] &\approx \int_0^{a_u} \frac{-\frac{d}{d\delta} e_d(a,\delta)\big|_{\delta=0}\cdot\delta}{e_d(a)} h(a,n)da \\
&= \int_0^{a_u} \frac{-\frac{d}{d\delta}\int_a^{a_u}\exp\left[-\int_a^t \mu(\tau)(1+\delta)+\gamma(\tau^*)d\tau\right]dt\Big|_{\delta=0}\cdot\delta}{\int_a^{a_u}\exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right]dt} h(a,n)da \\
&= -\int_0^{a_u} \frac{\int_a^{a_u}\exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right]\cdot\int_a^t \mu(\tau)d\tau\, dt}{\int_a^{a_u}\exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right]dt} h(a,n)da\cdot\delta \\
&= -j_\delta\,\delta = -\bar{\mu}\, J_\delta\, \delta
\end{aligned} \tag{2.55}$$

where $\bar{\mu} = \int_0^{a_u}\mu(a)h(a,n)da$ denotes the so-called crude mortality, i.e. the average mortality rate in a society. The two formulations in (2.53) and (2.55) appear a little complex, but in fact they help to greatly simplify the application of the WTP criterion: For each country, J_Δ and J_δ need to be calculated only once and can subsequently be taken from a list like the one in Table 2.5. The criterion in (2.48) then reduces to

$$-dg \le -\frac{g}{q} J_z z \tag{2.56}$$

where z is a general symbol for a fatal impact like Δ or δ. Similar derivations can be found for other, more complicated relations such as toxicological dose-response relations (see Sections 4.3 and 6.4). Note that both sides of (2.56) have negative signs[19].

For a correct calculation of J_z, it is theoretically necessary to establish cohort life tables for each age-group presently alive (compare Section 2.2.1). As an alternative, one may use the

[19] As mentioned before, an investment into safety corresponds to a loss of income, so that $-dg$ has a positive value. Furthermore, a decrease in mortality (negative z) leads to an increase in life expectancy (positive $d\bar{e}_d$). Thus, both sides of (2.56) have positive values if safety is increased.

Table 2.5: Societal and socio-economic indicators for different countries

country	**GDP**[a]	$\boldsymbol{g}^b$	$\boldsymbol{\zeta}^c$	$\boldsymbol{n}^d$	$\boldsymbol{e_0}^e$	$\boldsymbol{e_0}^f$	$\bar{\boldsymbol{\mu}}^g$	$\boldsymbol{w}^h$	$\boldsymbol{q}^i$	$\boldsymbol{J_\Delta}^j$	$\boldsymbol{J_\delta}^j$	$\boldsymbol{G_\Delta}^j$
Austria	28.900	22.800	1.8	0.24	77.9	84.8	0.89	9.2	14	14.4	16.6	$2.35 \cdot 10^6$
Belgium	27.700	22.600	1.5	0.16	77.3	85.2	1.01	8.0	12	14.7	15.9	$2.77 \cdot 10^6$
Canada	30.300	24.200	2.0	0.99	78.1	84.3	0.73	11.7	19	15.2	18.2	$1.95 \cdot 10^6$
Czech Rep.	15.100	11.100	1.4	−0.07	74.0	76.9	1.08	12.9	21	14.3	18.8	$0.76 \cdot 10^6$
Denmark	29.300	23.600	1.8	0.30	76.6	82.2	1.09	9.6	15	13.9	14.9	$2.19 \cdot 10^6$
Finland	26.500	21.400	2.1	0.16	77.4	82.5	0.98	10.5	17	13.9	16.2	$1.75 \cdot 10^6$
France	27.200	21.900	1.9	0.37	77.8	85.8	0.90	8.2	13	14.5	15.9	$2.44 \cdot 10^6$
Germany	25.900	21.500	1.9	0.27	77.8	87.0	1.04	8.6	13	14.4	14.0	$2.38 \cdot 10^6$
Italy	25.600	20.700	2.0	0.07	78.6	81.6	1.07	8.2	13	13.9	14.7	$2.21 \cdot 10^6$
Japan	27.000	20.400	2.7	0.17	80.1	93.1	0.83	10.3	16	13.4	14.1	$1.71 \cdot 10^6$
Netherlands	29.000	23.400	1.5	0.55	77.7	84.0	0.87	8.2	13	15.1	18.8	$2.72 \cdot 10^6$
New Zealand	21.800	16.700	1.2	1.14	78.7	83.7	0.76	11.4	19	16.9	16.3	$1.49 \cdot 10^6$
Norway	35.500	28.900	2.1	0.49	78.3	82.0	0.98	8.7	14	14.3	14.9	$2.95 \cdot 10^6$
Spain	22.400	16.100	1.8	0.10	78.8	84.8	0.91	10.9	18	14.3	14.8	$1.30 \cdot 10^6$
Switzerland	30.500	24.000	1.9	0.27	79.6	85.1	0.88	10.4	17	14.5	16.5	$2.05 \cdot 10^6$
UK	27.900	23.400	1.3	0.23	77.4	78.8	1.07	10.3	16	15.2	17.2	$2.22 \cdot 10^6$
USA	36.100	30.000	1.8	0.90	75.1	85.7	0.87	11.6	19	15.3	15.5	$2.42 \cdot 10^6$

[a] in PPP US$[87]
[b] in PPP US$, calculated according to (2.33) from GDP and gross investment rate [17]
[c] in % [71]
[d] in % [17]
[e] From period life [144] table for $\chi = 2000$.
[f] From cohort life table for $\vartheta = 2000$, following from a simple extrapolation of past developments into the future (analysis by Rackwitz [104]).
[g] in %
[h] in % [88]
[i] After (2.31).
[j] For $\varepsilon = 1 - q$ and a country-independent pure time preference rate $\rho = 3\%$.

period table for the present year ($\chi = 2006$) as an upper bound for age-dependent mortality $\mu(a)$, whereas the cohort life table for the youngest generation presently alive (i.e. those with birth year $\vartheta = 2006$) can serve as a lower bound. This approach holds if mortality rates continue to decrease strictly monotonically with each period as in the past. Table 2.6 illustrates the range of possible values for data from 2000. In the case of the additive mortality scheme, the period table leads to smaller and thus more conservative estimates of J_Δ. However, the differences are rather small and fall within the range of statistical uncertainties. For the proportional effect, J_δ is significantly lower, if the cohort table is used. It is proposed to use the lower and thus more conservative estimate for J_Δ and J_δ, respectively. This rationale is followed in the overall data view of Table 2.5.

In the case of the additive risk model, there is another interesting interpretation. Because the values of g, q and J_Δ are country-specific values independent of the respective application, it is possible to simplify further:

Table 2.6: The effect of different model choices upon J_z (Germany/Japan/USA)

all values for Ger. / *Jap.* / USA	for life expectancy at birth e_0			for discounted LE at birth $e_{0,d}$			for age-averaged life expectancy $\bar{e}$			for age-averaged discounted LE $\bar{e}_d$		
J_Δ for...												
period $\chi = 2000$	40.0	*41.0*	39.2	21.7	*19.8*	22.0	23.0	*22.6*	23.8	14.4	*13.4*	15.3
cohort $\vartheta = 2000$	44.4	*46.9*	44.2	22.9	*21.0*	23.6	24.6	*25.1*	26.7	15.2	*14.3*	16.5
J_δ for...												
period $\chi = 2000$	15.7	*16.1*	19.9	3.7	*3.3*	6.3	33.8	*34.7*	34.9	17.9	*18.7*	19.4
cohort $\vartheta = 2000$	11.8	*10.5*	16.2	2.9	*1.7*	4.7	24.6	*26.0*	28.5	14.0	*14.1*	15.5

$$-dg \leq -G_\Delta \Delta \qquad \text{with } G_\Delta = \frac{g}{q} J_\Delta \tag{2.57}$$

According to the mortality definition in (2.41), one may substitute for $\Delta = d\mu = N_D/N_{pop}$, where dN_D are the additional and $-dN_D$ the averted fatalities per year. Considering that $-dg$ corresponds to the yearly safety costs per person and year, one may write

$$\frac{\text{safety costs}}{\cancel{N_{pop} \cdot \text{year}}} \leq G_\Delta \frac{\text{averted fatalities}}{\cancel{N_{pop} \cdot \text{year}}} \tag{2.58}$$

From this result it can be seen that G_Δ is actually the absolute *WTP for averting one fatality*. In the socio-economic literature, G_Δ is commonly referred to as *value of a statistical life* or VSL [55, 118]. The term 'VSL' appears to be somewhat unluckily chosen, given the reasoning on the infinite value of human life in Section 2.1. However, it has become the standard denomination for G_Δ. The definition in (2.58) is in accordance with the frequently encountered formulation VSL = WTP/dm (as e.g. in [118]). Country-specific values for G_Δ are given in Table 2.5. Typical values for Western economies are in the order of 1.5 to 2.5 million PPP US$.

Another standard measure from the literature (e.g. [33, 73]) is the so-called *value of a life year lost*, abbreviated as VLYL or VOLY. It is obtained by dividing by the age-averaged discounted life expectancy:

$$\text{VLYL} = \frac{G_\Delta}{\bar{e}_d} \tag{2.59}$$

2.5.3 WTP from Empirically-based Investigations

The WTP per averted fatality or VSL equally serves as a useful standard measure, when it comes to comparing the previous results to those from other studies. Apart from the classical socio-economic and life quality approaches described above, three more approaches can be identified [77, 137]. All of them use empirical evidence for calibration and in part also instead of a theoretical derivation:

The *contingent valuation method* (CVM) is a very direct and purely empirical approach based on interviews with random individuals. It was conceived in order to determine the monetary value of non-market goods, basically by asking interviewees what amount they would be willing to pay e.g. in order to save a rare species from extinction or, as in the present case, in order to save the life of an anonymous fellow citizen. The challenge lies in formulating the question skillfully that interviewees are able to imagine something that is intuitively meaningful. Apart from a large number of individual interviews ($>$2000 in [54]), this requires the—sometimes repeated—adaption of the question.

Wage-risk studies use a lifetime utility model of the same type as that in (2.8), see e.g. [139]. The parameters in the model are determined by statistically analysing workers' employment data. It is assumed that workers are ready to take more risky jobs against higher payments and that this behaviour is reflected by actual wage numbers. Apart from risk and wage, other factors are equally considered in order to compare workers of a similar category (education, experience, sex, region etc.) with the purpose of avoiding statistical errors. This analysis of workers' choices does not only reveal their implied VSL; it is equally possible to derive other parameters, such as the implied discount rate γ [137, 139].

Product-risk studies function in a similar way as wage-risk studies. In this case, the willingness to purchase safety-protective products (bicycle helmets etc.) is taken as the basis for determining the implied VSL. This approach holds on the premise that consumers are able to figure the amount of additional safety correctly. As with wage-risk studies, it appears hard to tell, whether such an assumption is justified.

There is a wide range of VSL results in the socio-economic literature. Meta-analyses, i.e. studies surveying dozens of previous analyses, serve as a valuable digest [23, 77, 78, 137]. Viscusi & Aldy [137] compare over 60 different studies and obtain average VSL values of 6 to 8 million US$ (all values in year 2000 US$). The values in Miller [77] range between 2.5 and 3.5 million US$ for most industrialised countries. Mrozek & Taylor [78], finally, indicate VSL numbers of 1.5 to 2.5 million US$, which is more or less comparable to the results derived in Section 2.5.2. It is not clear, why the other studies deviate from these values so significantly. The discussion in the following section tries to find some possible interpretations.

Similarly to the utility-based derivation, empirical evidence equally indicates a clear correlation between income g and the VSL. Miller [77] observes an elasticity with respect to income[20] of 0.85 to 1.00, which is in good agreement with the previous derivations. Equation (2.57) is based on a linear relationship between the two indicators, which basically implies an elasticity equal to one. Yet, Viscusi & Aldy [137] find values between 0.15 and 1.00, with an average of approximately 0.5. A study by Costa & Kahn [20] points in the opposite direction: Their results of $\varepsilon_{\mathrm{VSL}} \approx 1.5$ to 1.7 is based on the developments in the United States between 1940 and 1980. This high elasticity outcome derived from the historical evolvement of VSL and income is possibly related to a phenomenon briefly described on p. 17: While productivity has been growing by 2.3% yearly on long-term average, only 1.8% of this growth are realised in the GDP. There too, people gave higher priority to non-consumption goods such as leisure

[20]Following the definition of elasticity (see footnote 4, p. 15) the elasticity of the VSL with respect to income is obtained as

$$\varepsilon_{\mathrm{VSL}} = \frac{d\mathrm{VSL}}{dg} \frac{g}{\mathrm{VSL}}$$

time than to maximising their consumption. However, this allegation would require further attention in order to be substantiated.

The implicit discount rates in [137] scatter significantly between 1% and more than 15%.

2.5.4 Discussion

The comparison between the utility-derived VSL values in Section 2.5.2 and the empirical results in Section 2.5.3 reveals significant differences. The latter results are significantly higher than the former ones, but within the same order of magnitude. For the following discussion it is helpful to recall that the utility-derived approach has its empirical foundation too, consisting of the empirically observable work-leisure trade-off. Although it is difficult to locate the precise reason for the inconsistencies, it is possible to find some potential explanations:

The reason might simply consist in people's inability to judge risks correctly. Even in the cases where they are aware of the actual numbers (as with CVM surveys) the possibility of being killed remains somewhat abstract for most individuals. In part, it can be assumed that the low probabilities involved contribute to the difficulty of obtaining an intuitive access. It can be argued that empirical evidence from the work-leisure trade-off reflects peoples' actual will in a less distorted way: First, every individual will face the consequences of his work-leisure decisions with almost absolute certainty. Second, the choice can usually be reconsidered and adapted according to personal experience. All this is not possible with fatal low-probability events.

However, the divergence between the two approaches might equally have another cause: According to the utility-derived approach, death is essentially an event that reduces personal utility for the rest of their possible lifespan. However, people fear fatal events not only because of death (in the sense of not being alive), but equally because the process of dying is a dreadful event in most people's awareness. The perceived disutility tempts people to pay more for preventive measures than predicted by utility theory (compare the discussion on risk perception in Section 2.4). The utility-derived approach determines people's utility function from their behaviour in a situation with little emotional content (work-leisure trade-offs) and transfers it to highly emotional decisions involving the prospect of death. The empirical approach includes this emotional bias from the beginning.

It appears that the conflict between the two approaches cannot be solved on an economic or decision-theoretic level. In fact, it is a political and philosophical question whether public decisions on safety should be based on people's rational behaviour in general situations or on their emotionally biased behaviour regarding safety-related issues.

Chapter 3

Societal Acceptability of Decisions Involving Health Consequences

3.1 Introduction

Loss of life is the most drastic health consequence of a decision, but not necessarily the only one. Most life-threatening events do equally involve non-fatal consequences, i.e. injury and disease. The present chapter seeks to adapt the WTP criterion and the underlying utility-based rationale in order to cover this type of outcome.

The previous chapter described lifetime utility as a product of two items: One is utility of consumption, a monotonous function of a person's available income, the second one is the time, during which consumption can be enjoyed (lifetime in good health). At the instant of death both values are reduced to zero: Once dead a person stops earning an income and ceases to have time in which to spend his or her income. In the case of disease or injury—summarised by the term *disability*—equally both terms are affected. On one hand, income is reduced due to the temporary or permanent inability to work and due to the arising medical treatment costs. On the other hand, disability reduces lifetime spent in good health during which consumption can actually be enjoyed. Disability may additionally reduce leisure time because of the time required for therapy.

Sections 3.2 and **3.3** investigate the impact of morbidity changes upon lifetime in good health and upon income, respectively. In **Section 3.4**, these considerations are integrated into an adapted version of the WTP criterion and compared to empirical findings from the literature.

3.2 The Effect of Disability upon Lifetime in Good Health

With fatal consequences as in Chapter 2, matters were comparatively simple: Life and death are mutually exclusive states with no intermediate steps in between. Moreover, death is an irreversible state. This explains the very simple and distinct definition of fatal risk expressed by the mortality equation (2.41). Although morbidity ν is defined in the same way, i.e. as the

Table 3.1: Severity weights of different types of disabilities

source	**type of disability b**	**severity weight s_b**
Murray/ Lopez [83]	vitiligo (white spots) on face, weight-for-height < 2 std. dev.	0–0.020
	watery diarrhoea (five episodes per day), severe sore throat	0.021–0.120
	radius fracture in stiff cast, rheumatoid arthritis	0.121–0.240
	below-knee amputation, deafness	0.241–0.360
	mental retardation (IQ 55-70)	0.361–0.500
	unipolar major depression, blindness, paraplegia	0.501–0.700
	active psychosis, dementia, severe migraine, quadriplegia	0.701–1.000
Hofstetter/ Hammitt [49]	chronic respiratory symptoms / children	0.14
	chronic bronchitis / adults	0.14
	respiratory hospital admission	0.44
	neurocognitive development deficits (from lead)	0.06
	severe annoyance (from noise)	0.09
	sleep disturbance (from noise)	0.08
	ischaemic heart disease hospital admission	0.44
Perenboom et al. [94]	major problems with activities of daily life[a]	0.11
	inability to perform activities of daily life	0.65

[a] ability to carry a 5 kg object 10 metres, walk 400 metres in one go, dress and undress oneself, get in and out of bed, move from one room to another etc.

number of cases divided by the population size, this value alone is only of limited help and needs to be combined with information on both severity and duration of a disabling state.

Prevalence P_b can be understood as the probability of being in a state of disability of type b on a given day in the future[1] or as the product of occurrence rate (morbidity) ν_b and average duration D_b of disability b:

$$P_b(a) = D_b \nu_b(a) \tag{3.1}$$

Another way of expressing prevalence is to account for restricted activity days RAD_b and symptom days SD_b in the general population. A RAD is defined as a day on which an individual is forced to alter his or her normal activity (days off work and/or in bed) [32]. If indicated in a different dimension, these numbers need to be divided by the number of years and the population size:

$$P_b(a) = \frac{RAD_b}{\text{years} \cdot N_{pop}} \quad \text{and} \quad P_b(a) = \frac{SD_b}{\text{years} \cdot N_{pop}} \tag{3.2}$$

The common approach in morbidity valuation is to adjust periods spent in poor health by the relative loss of ability to enjoy one's life (referred to as quality or disability adjustment). A state of coma or agony reduces this ability practically to zero whereas catching a cold only subtracts a few percentage points. Medical science and health economics have defined

[1] In this sense it appears reasonable to use the probability symbol P equally for prevalence

several indicators for morbidity impacts upon life quality. In 1971, Sullivan [128] introduced an adjusted life-expectancy by subtracting the expected lifetime spent in a state of disability. Depending on whether periods of disability are subtracted from life expectancy completely or only in proportion to their severity, the concept was later referred to as *disability-free life expectancy* (DFLE) or *disability-adjusted life expectancy* (DALE), respectively (see e.g. [94]). The latter, more specific measure is written as[2]

$$e_{DA}(a) = \int_a^{a_u} S(t|a)[1 - \sum_b s_b P_b(t)]dt \tag{3.3}$$

This formulation differs from (2.7) only in as far as survival probability $S(t|a)$ is corrected for severity weight s_b and age-dependent prevalence $P_b(a)$ of all existing disabilities b. Since s expresses the relative loss of life enjoyment capability, it can adopt values between 0 and 1. Table 3.1 lists up severity weights for different types of disabilities. Attributing such weights to different types of disabilities is a delicate task, given that a relative degree of objectivity can only be reached by assessing the subjectively felt impacts of as many patients as possible by means of psychometric measures. In this light, the discrepancies between the numbers originating from different studies in Table 3.1 are not surprising.

For the calculation of the current $e_{DA}(a)$, it is more convenient to group all existing types of disabilities into severity classes ι of similar severity weight. The sum expression in (3.3) then changes to $\sum_\iota s_\iota P_\iota(a)$. The values for severity class prevalence in Table 3.2 represent the situation in established market economies. For these countries, the age-averaged value of $e_{DA}(a)$ is generally very close to 10% lower than the corresponding value for $e(0)$. Numbers for other parts of the world can be taken directly from [83].

Discounting and age-averaging can be introduced in analogy to (2.11):

[2]For persons of age a, the average observed disability-induced loss of lifetime enjoyment capability is $\sum_b s_b P_b(a)$. If we want to make a prediction about an a years old person's loss of lifetime enjoyment capability, say, five years from now, we need to consider that the person can only be affected in case he or she is still alive at age $a+5$. Therefore, we need to correct the result by the probability of survival up to $a+5$ (given survival up to $S(a)$):

$$\frac{S(a+5)}{S(a)} \sum_b s_b P_b(a)$$

If this step is repeated for each potentially remaining year of life, we obtain the expected loss of life expectancy due to disability adjustment:

$$\frac{1}{S(a)} \int_a^{a_u} S(t) \sum_b s_b P_b(t)dt$$

When subtracting this expression from life expectancy we see that both terms can be joined in one single integral:

$$\begin{aligned} e_{DA}(a) &= \frac{1}{S(a)} \int_a^{a_u} S(t)dt - \frac{1}{S(a)} \int_a^{a_u} S(t) \sum_b s_b P_b(t)dt \\ &= \frac{1}{S(a)} \int_a^{a_u} S(t)[1 - \sum_b s_b P_b(t)]dt \\ &= \int_a^{a_u} S(t|a)[1 - \sum_b s_b P_b(t)]dt \end{aligned}$$

Table 3.2: Prevalence of severity classes in established market economies [83]

severity class ι	defined by severity weight s_ι	prevalence $P_\iota(t) \times 10^{-3}$ in age group 0–4	5–14	15–44	45–59	≥ 60
I	0.000–0.020	62.7	60.4	99.2	179.8	339.5
II	0.021–0.120	71.3	59.0	89.4	176.7	384.3
III	0.121–0.240	21.3	17.6	53.9	73.6	157.0
IV	0.241–0.360	8.7	7.9	27.9	37.8	81.8
V	0.361–0.500	4.8	4.5	14.7	19.7	52.4
VI	0.501–0.700	1.9	1.9	30.1	32.6	55.9
VII	0.701–1.000	1.1	1.2	6.4	11.9	46.8

$$\begin{aligned} \bar{e}_{DA,d} &= \int_0^{a_u} e_{DA,d}(a) h(a,n) da \\ &= \int_0^{a_u} \int_a^{a_u} S_d(t|a)[1 - \sum_b s_b P_b(t)] \, dt \, h(a,n) da \end{aligned} \tag{3.4}$$

Again, the results of (3.4) differ from $\bar{e}_d$ by around 10%.

The prevalence information from Table 3.2 is absolutely sufficient in order to calculate the current value of $e_{DA,d}(a)$. However, they cannot be used for predicting the correct *change* in DALE $de_{DA,d}(a)$, as it is required when assessing the acceptability of a decision. For this purpose, it is necessary to know which decision typically leads to which disability type and severity. In Chapter 6, Tables 6.3 and 6.4 contain dose-response coefficients in order to express the effect of a change in toxic exposure as changes in ν_b and P_b, respectively. Numbers on average disability durations D_b are indicated in Table 6.2, whereas severity weights can be taken from Table 3.1 or similar listings in the literature. With this information a change in DALE due to a change in $P_b(a)$ can be determined as

$$de_{DA,d}(a, dP_b) = -\int_a^{a_u} S_d(t|a) \, s_b dP_b(t) \, dt \tag{3.5}$$

Apart from disability-adjusted life expectancy, other measures have been introduced. Although the DALE concept is certainly the most adequate approach in the present context, they should be briefly introduced in order to avoid confusion due to the similarity of the respective names and definitions. As mentioned before, disability-free life expectancy (DFLE) is almost identical to DALE but includes only periods spent in complete health. The World Health Organization [146] also uses the term 'healthy life expectancy' instead of DALE. Quality-adjusted life years (QALY) are a purely descriptive number which multiplies severity weights with life years instead of multiplying them with survival probabilities as in (3.3). Another frequently encountered concept are damage-adjusted life years (DALY), which describe the loss of quality-adjusted life time due to both morbidity *and* mortality as consequence of a disease.

3.3 The Effect of Disability upon Available Income

Apart from its impact upon lifetime in good health, disability can have an effect upon the working capability (i.e. capability of earning money) and cause therapy costs. Combined values considering both effects can provide a quick estimate. Calculating a 'disability-adjusted income' g_{DA} in analogy to disability-adjusted life expectancy $e_{DA}(a)$ appears to make little sense, since statistically measured values for g are already net of all such effects. Only *changes* in disease-specific morbidity or prevalence can affect the current income g. Under the assumption of constant consumption levels throughout lifetime (as in Chapter 2) and age-independent mortality changes $d\nu_b$, Friedrich & Bickel propose a simple estimate for the change in g due to damage-adjustment:

$$dg_{DA}(d\nu_b) = -K_b d\nu_b \tag{3.6}$$

In this relation, K_b is the cost associated with one case of disability type b. This number already includes information on average duration, treatment costs per time unit and income loss per time unit. For Finland, the estimated overall cost for cases leading to an emergency room visit (ERV) amounts to 160 €, while it is 1430 € on average when entailing a hospital admission (HA). These estimates are taken from [39] and refer to disabilities caused by air pollution.

For a more accurate model, loss of working capacity and therapy costs need to be modelled separately. The two terms for worktime adjustment and therapy adjustment add up as

$$dg_{DA} = dg_{WA} + dg_{TA} \tag{3.7}$$

These two components are regarded more closely on the following pages.

3.3.1 Loss of Working Capacity

Loss of Work Time

Disability can cause a temporary or permanent loss of work time and thus income. As a first step, it is necessary to differentiate whether a person is economically active or not. Persons who are not, but have an income from savings or rent insurance do not lose any income due to not being able to work. Unpaid housework and child education should probably be considered as paid work regarding the costs that would arise if somebody else had to do the work in the place of the disabled person. As long as we are regarding an anonymous, 'average' representative of society, the question of whether there is social insurance covering disability-induced work losses appears to be secondary. Theoretically, the expected value of income is the same regardless of whether the loss of income is redistributed to all other members of society or not.

The average income of the working age, which begins at age a^- and lasts until a^+ can be determined from average income g as

$$g_w = \frac{g}{\int_{a^-}^{a^+} h(a,n)da} \tag{3.8}$$

or, vice versa, $g = g_w \int_{a^-}^{a^+} h(a,n)da$, in agreement with the life table mathematics in Section 2.2.1. Typical upper and lower ages are 20 and 65 years, respectively. Alternatively, g_w can be obtained e.g. from [39], which names 55 ECU (1997) as average daily productivity cost in the EU[3]. Yet, it appears more consistent with the above reasoning on unpaid house and family work to calculate g_w after (3.8). However, it does not account for unemployment.

When regarding the effect of a change in disability prevalence $dP_b(a)$, one might think of a case where some medium severe disability forces a person to work part-time. If this case is excluded and only complete losses of working ability—be it for a day, a month or permanently—are considered, it is not the severity weight s_b in itself which matters, but whether it exceeds the critical severity weight for working incapability s_{cr}. Then the product $s_b P_b(a)$ from (3.3) can be replaced by $P_b(a)$ and the condition $s_b \geq s_{cr}$. After a change in prevalence $dP_b(a)$, average income needs to be adjusted for loss of work time:

$$g(dP_b) = g_w \int_{a^-}^{a^+} [1 - dP_b(a)]\, h(a,n)da \qquad \text{for } s_b \geq s_{cr} \tag{3.9}$$

The change in income due to worktime adjustment follows as

$$\begin{aligned} dg_{WA}(dP_b) &= g(dP_b) - g \\ &= -g_w \int_{a^-}^{a^+} dP_b(a)\, h(a,n)da \\ &= -g \frac{\int_{a^-}^{a^+} dP_b(a)\, h(a,n)da}{\int_{a^-}^{a^+} h(a,n)da} \qquad \text{for } s_b \geq s_{cr} \end{aligned} \tag{3.10}$$

Loss of Productivity

The yearly income of an individual can be expressed as to the product of total work time w and productivity p (see Section 2.2.2). Hence, productivity is defined as the salary per time unit. If an injury or disease does not render professional activity impossible, it can reduce the working capability during a limited period or for good. Not every disability has consequences in every job. Physical disabilities concern persons engaged in physical labour much more severely than desk workers whose working capability may remain unaffected depending on the case. In order to quantify the degree of lost productivity, it is possible to use invalidity scales similar to the disability severity weight from Table 3.1.

If the only effect of a disability consists in restricting a person's daily or weekly work hours at constant hourly payment, the consequences can be listed as 'loss of work time' in the way described above. However, limited working capability frequently means that a person has to

[3]Some country-specific values (in ECU/day): Germany: 68, Spain: 46, Italy: 54, Netherlands: 61, UK: 53

Table 3.3: Therapy costs from [39]

country	emergency room visit: K_{ERV} [€][a]	hospital day: κ_{HD} [€/day]
France	24	340
Germany	20	280
Italy	20	–
Netherlands	–	340
Spain	100	–
United Kingdom	80	300

[a]Local currency values dating from 1994–1997 were converted into $€_{1999}$ values

look for a less demanding job instead of continuing with the previous job at a lower level of activity. Clearly this effect is very difficult to quantify. Nevertheless, Muir & Zegarac [80] undertook the effort of estimating the effect of IQ losses upon total societal income. Lifelong IQ deficits can occur due to exposure with neurotoxic substances during fetal and breastfeeding phases. As a result, corresponding income losses range between 1.5 and 3% per lost IQ point. GREENSENSE [33] names an estimate for the effect of lead concentrations in the air upon IQ levels (see Chapter 6 on toxic consequences).

3.3.2 Therapy Costs

Therapy costs and the corresponding loss in average income can be written as

$$\begin{aligned} dg_{TA}(dP_b) &= -\int_0^{a_u} \kappa_b(a)\, dP_b(a) h(a,n) da \\ &\approx -\kappa_b \int_0^{a_u} dP_b(a) h(a,n) da \\ &= -\kappa_b\, dP_b \end{aligned} \tag{3.11}$$

Here, κ_b denotes therapy cost per time unit. Elderly people fall sick more frequently and for longer periods. Both effects are reflected by prevalence numbers. The daily costs arising in case of a treatment however, do not appear to depend on age in such an obvious way. This is reflected by the approximation in the second line of (3.11). Alternatively, one may calculate $dg_{TA}(dP_b)$ using total treatment costs per case K_b:

$$\begin{aligned} dg_{TA}(d\nu_b) &= -K_b\, d\nu_b \\ &= -\kappa_b\, D_b\, d\nu_b \\ &= -\kappa_b\, dP_b \end{aligned} \tag{3.12}$$

Table 3.3 indicates some values on κ_b and K_b. Costs per day for hospitalised patients κ_{HD} are consistent over the countries displayed, whereas two separate definitions seem to exist for the costs of an emergency room visit K_{ERV}: Germany, France and Italy show very similar values, yet those for Spain and the U.K. are about four times higher. Specific values for $d\nu_b$, dP_b and D_b are presented in Chapter 6 on toxic consequences. When applying (3.12), note that D_b and κ_b are usually expressed in terms of days, whereas P_b is an expected duration per year.

For chronic disabilities, average daily therapy costs can be assumed to be significantly lower than the values in Table 3.3. If visiting a medical practice is required every two weeks, then $\kappa_b = K_{ERV}/14$.

3.4 WTP and Non-fatal Consequences

3.4.1 Adaption of the WTP Criterion

Vulnerability—i.e. susceptibility for changes in morbidity—depends on an individual's general health which, again depends upon his or her age. This applies to the consequences of toxic exposure in particular (proportional risk model), whereas acute failure events affect different age groups more evenly (absolute risk model, compare Section 2.5.2).

Sections 3.2 and 3.3 discusses how a change in disability prevalence $dP_b(a)$ leads to a change in disability-adjusted (discounted) life expectancy $de_{DA,d}(a)$ and disability-adjusted income $dg_{DA} = dg_{WA} + dg_{TA}$. Both effects can be included in the WTP criterion (2.48):

$$\frac{dg + dg_{DA}}{g} + \frac{1}{q}\, E_A\left[\frac{de_{DA,d}(a)}{e_{DA,d}(a)}\right] \geq 0 \tag{3.13}$$

As outlined on p. 47, the statistical value for g is already net of all disability effects, whereas $e_d(a)$ needs to be replaced by the adjusted value $e_{DA,d}(a)$. Note that dg is the cost for *preventing* disability whereas dg_{DA} is the follow-up cost of *actually occurred* cases (therapy cost and loss of worktime). More prevention leads to less actual occurrences and less prevention to more occurrences. For a given decision the respective values of dg and dg_{DA} always have opposite algebraic signs.

In many cases the available data on morbidity consequences are quite rudimentary and do not provide age-dependent information. Therefore, it may become necessary to model prevalence changes as being age-independent[4] (absolute risk model), i.e. $dP_b(a) = dP_b$. This is close to reality for cases of acute failure but only a rudimentary approximation for toxic exposure. For an age-independent change in prevalence the relative change in disability-adjusted life expectancy simplifies to

$$\begin{aligned}\frac{de_{DA,d}(a, dP_b)}{e_{DA,d}(a)} &= \frac{-s_b\, dP_b \int_a^{a_u} S_d(t|a)dt}{e_{DA,d}(0)} \\ &= -s_b\, dP_b \frac{e_d(a)}{e_{DA,d}(a)}\end{aligned} \tag{3.14}$$

[4]Compare (6.7) as well as Tables 6.3 and 6.4.

so that

$$E_A\left[\frac{de_{DA,d}(a, dP_b)}{e_{DA,d}(a)}\right] = s_b\, dP_b\; E_A\left[\frac{e_d(a)}{e_{DA,d}(a)}\right] \tag{3.15}$$

Furthermore, age-independent changes in prevalence lead to a simplification of (3.10), describing the worktime effect:

$$dg_{WA}(dP_b) = -g\, dP_b \qquad\qquad \text{for } s_b \geq s_{cr} \tag{3.16}$$

The income effect of therapy costs dg_{TA} remains the same as in (3.11).

Subsequently, the WTP criterion for age-independent prevalence changes can be re-written by inserting these results in (3.13):

$$\frac{dg - (\kappa_b + g)dP_b}{g} - \frac{s_b\, dP_b}{q}\, E_A\left[\frac{e_d(a)}{e_{DA,d}(a)}\right] \geq 0 \qquad \text{for } s_b \geq s_{cr} \tag{3.17}$$

For most industrialised countries, $E_A\left[\frac{e_d(a)}{e_{DA,d}(a)}\right] \approx \frac{1}{0.88}$ is a sufficiently precise approximation ($\pm 1\%$). Therefore, it is proposed to simplify further:

$$\frac{dg - (\kappa_b + g)dP_b}{g} - \frac{s_b\, dP_b}{0.88\, q} \geq 0 \qquad\qquad \text{for } s_b \geq s_{cr} \tag{3.18}$$

Resolving for dg yields

$$-dg \leq -dP_b\left[\frac{g\, s_b}{0.88\, q} + \kappa_b + g\right] \qquad\qquad \text{for } s_b \geq s_{cr} \tag{3.19}$$

For $s_b < s_{cr}$, there is no worktime adjustment:

$$-dg \leq -dP_b\left[\frac{g\, s_b}{0.88\, q} + \kappa_b\right] \qquad\qquad \text{for } s_b < s_{cr} \tag{3.20}$$

As previously with (2.56), an investment into health-saving measures leads to negative values for dg as well as for dP_b so that both sides of the criterion adopt positive values.

In order to illustrate the order of magnitude, it is proposed to consider the case of chronic bronchitis. If it were possible to reduce its prevalence by one fifth, i.e. $dP_{\mathrm{CB}} = -0.01$, the willingness to pay for the corresponding measures would amount to 200 PPP US$ or 180 € per person and year[5] in Germany. Here, loss of lifetime utility contributes 97% and therapy costs contribute 3%. Eliminating the cause for one fifth of the emergency respiratory hospital admissions in the UK would correspond to a WTP of 11 PPP US$ per person and year[6].

[5] Prevalence estimated from U.S. data as $P_{\mathrm{CB}} \approx 0.05$ [50], $s_{\mathrm{CB}} = 0.14$ (from Table 3.1), $\kappa_{\mathrm{CB}} \approx 500$ €/year (estimated as $K_{ERV}/14d$ from Table 3.3).

[6] Incidence $\nu_{\mathrm{ERHA}} = 0.0077$ [51], duration $D_{\mathrm{ERHA}} = 14$ days (from Table 6.2), $s_{\mathrm{ERHA}} = 0.44$ (from Table 3.1), $\kappa_{HD} = 300$ €/day (from Table 3.3)

Loss of lifetime utility contributes 30%, therapy costs 60% and loss of income 10%. Loss of income can contribute up to 50% of the WTP in case of severe working accidents leading to permanent working inability ($s_b > s_{cr}$).

* * *

Some authors [137, 138] establish a WTP per averted case of disability in analogy to the WTP per averted fatality or VSL ('value of a statistical life'). It appears that such a number can only be derived for *irreversible*, i.e. permanent states of disability. For age-independent changes in morbidity $d\nu(a) = d\nu$, the WTP criterion can then be approximated as

$$\begin{aligned} -dg &\geq -\frac{g}{q} J_\Delta \, s_b \, d\nu \\ &= -G_\Delta \, s_b \, d\nu \end{aligned} \tag{3.21}$$

In (3.21), $s_b G_\Delta$ is the WTP per averted case of disability. G_Δ is the VSL and is given in Table 2.5. The result is simple and intuitive but the derivation is lengthy and has been moved to the back of the chapter (Section 3.4.5). If the disability entails working inability ($s_b > s_{cr}$), then the WTP may rise by as much as 50%, i.e. $\sim 1.5 s_b G_\Delta$, see (3.28).

3.4.2 WTP Criterion for Combined Consequences

Many hazards do not have an exclusive impact either upon morbidity or upon mortality but affect the two of them at the same time. Formally speaking both parameters depend upon one common third parameter. However, these two dependencies upon one hazard parameter (failure rate, toxic concentration etc.) cannot be separated in many cases. As an example, one might consider the consequences of particulate matter emissions (the standard illustration example, Section 7.3). Apparently, developing lung cancer is causally linked with dying of lung cancer. Every victim of lung cancer has to live through a considerable period of illness and most people sick with it will eventually succumb to its effect. On the other hand, bronchitis does not appear to be linked with dying of lung cancer although particulate matter can be the common cause for both events. As a consequence of this reasoning it is possible to distinguish between the following levels of modelling:

1. Mortality and morbidity effects are treated as if they were completely independent.
2. Although mortality and morbidity effects are treated as if they were completely independent, the morbidity assessment uses a life expectancy $e_d(a, d\mu(a))$ that takes account of the reduced mortality (instead of simply using $e_d(a)$).
3. Mortality is treated as a function of morbidity if a dependency exists.

Because the WTP criterion is only valid for *small* changes in mortality, level 2 does not lead to a noticeably more precise result than level 1. Evaluations according to level 3 are very complex and are only indicated if the dependency between morbidity and mortality is very pronounced.

For the simplest case (level 1) the combined criterion is obtained by adding (2.56) and (3.19):

$$-dg \leq -\frac{g}{q} J_z z - dP_b \left[\frac{g\, s_b}{0.88\, q} + \kappa_b + g \right] \tag{3.22}$$

Mortality does not necessarily outweigh morbidity with respect to the WTP, see e.g. Example 7.3. Morbidity can play an equally significant or even dominant role

- if a decision influences morbidity much more strongly than mortality *or*
- if it leads to long-term or even permanent disability and an elevated severity of $s_b \geq \sim 0.25$.

Both conditions are frequently fulfilled in the case of toxic exposure. With acute failure on the other hand, it appears that most severe injuries heal within weeks and months, while those victims that do not recover are outnumbered by the fatal victims (compare Chapter 5).

3.4.3 WTP from Empirically-based Investigations

Viscusi, Magat & Huber [138] asked a population sample about their preferences in risk-risk and risk-money trade-offs for the specific case of chronic bronchitis. Depending on the method, the WTP per averted case varied between 0.5 and 2.3 million PPP US$. Risk-risk trade-offs imply that a case of chronic bronchitis leads to 32% of the WTP for averting a fatality. Viscusi generally obtains very high estimations for the WTP per case as observed with respect to his VSL values in Section 2.5.3.

In their meta study, Viscusi & Aldy [137] name numbers for the 'statistical value of an injury' that are significantly lower in the order of 20000 to 70000 PPP US$. These numbers are based on about 30 U.S. studies and eight studies from other countries.

3.4.4 Discussion

The values in [138]—especially the finding that averting a case of chronic bronchitis is worth 32% of the VSL—confirm the result in (3.21). The results from the meta analysis [137] on the other hand, do not match these numbers at all. The WTP criterion for prevalence changes in (3.19) is defined in a different way that does not permit indicating a WTP *per averted case*. Therefore, it is hard to compare these numbers to the results in the literature.

3.4.5 *Some Remarks on Irreversible States of Disability*

Some impacts can lead to irreversible states of disability. Viscusi and varying co-authors [137, 138] actually limit their empirical investigations to this type of consequences, e.g. injuries from work accidents. In this case, this disability lasts until death and can occur only once.

The following derivation is a proposal of how to apply lifetable mathematics to the specific assumptions in [137, 138].

Let b denote some specific work accident. Then, the probability of not being disabled at age a is $\exp\big[-\int_0^a \nu(t)dt\big]$. Under the condition of survival until t, the expected disability state of a person will by that time be

$$\overline{I}(t) = s_b \cdot \left(1 - \exp\left[-\int_0^t \nu_b(\tau)d\tau\right]\right) \tag{3.23}$$

t years from now. The case $\overline{I} = 0$ corresponds to a state of complete health. The expected health state (under the condition of survival until t) is

$$\begin{aligned} I(a,t) &= 1 - \overline{I}(t) \\ &= 1 - \left[s_b \cdot \left(1 - \exp\left[-\int_0^t \nu_b(\tau)d\tau\right]\right)\right] \end{aligned} \tag{3.24}$$

Following this line of thought, discounted damage-adjusted life expectancy would then be obtained as

$$\begin{aligned} e_{DA,d}(a) &= \int_a^{a_u} S_d(t|a)I(t)dt \\ &= \int_a^{a_u} \exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right] \cdot \left\{1 - \left[s_b \cdot \left(1 - \exp\left[-\int_0^t \nu_b(\tau)d\tau\right]\right)\right]\right\} dt \end{aligned} \tag{3.25}$$

For the case of age-independent changes in disability occurrence $d\nu(a) = d\nu = \Gamma$ assumed in [137, 138], it is possible to proceed in analogy with (2.53):

$$\begin{aligned} & E_A\left[\frac{de_{DA,d}(a)}{e_{DA,d}(a)}\right] \\ \approx & \int_0^{a_u} \frac{-\frac{d}{d\Gamma} e_{DA,d}(a,\Gamma)\big|_{\Gamma=0} \cdot \Gamma}{e_{DA,d}(a)} h(a,n)da \\ = & \int_0^{a_u} \frac{\left\{\begin{array}{l} -\frac{d}{d\Gamma}\int_a^{a_u} \exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right] \\ \cdot\left[1 - \left[s_b\cdot\left(1-\exp\left[-\int_0^a \nu_b(\tau)\,d\tau - \int_a^t \nu_b(\tau)+\Gamma\,d\tau\right]\right)\right]\right] dt|_{\Gamma=0}\cdot\Gamma \end{array}\right\}}{\int_a^{a_u} \exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right] \cdot \left\{1 - \left[s_b \cdot \left(1 - \exp\left[-\int_0^t \nu_b(\tau)d\tau\right]\right)\right]\right\} dt} \cdot h(a,n)\,da \end{aligned}$$

$$
\begin{aligned}
&= -\int_0^{a_u} \frac{\int_a^{a_u} (t-a) \exp\left[-\int_0^a \nu_b(\tau) d\tau - \int_a^t \mu(\tau) + \nu_b(\tau) + \gamma(\tau^*) d\tau\right] \, dt}{\int_a^{a_u} \exp\left[-\int_a^t \mu(\tau) + \gamma(\tau^*) d\tau\right] \cdot \left\{1 - \left[s_b \cdot \left(1 - \exp\left[-\int_0^t \nu_b(\tau) \, d\tau\right]\right)\right]\right\} dt} h(a,n) da \cdot s_b \Gamma \\
&= -J_\Gamma \, s_b \Gamma \qquad (3.26) \\
&\approx -\int_0^{a_u} \frac{\int_a^{a_u} (t-a) \exp\left[-\int_a^t \mu(\tau) + \gamma(\tau^*) d\tau\right] \, dt}{\int_a^{a_u} \exp\left[-\int_a^t \mu(\tau) + \gamma(\tau^*) d\tau\right] \, dt} h(a,n) da \cdot s_b \Gamma \\
&= -J_\Delta \, s_b \Gamma
\end{aligned}
$$

The approximation in the last two lines is only valid for accident types b with a low background occurrence rate $\nu_b(a) \ll \mu(a)$. The approximated result equals that in (2.53). In other cases, $J_\Gamma \approx J_\Delta$ can serve as a very rough approximation. Values for J_Δ are given in Table 2.5. The WTP criterion could thus be written as

$$
\begin{aligned}
-dg &\geq -\frac{g}{q} J_\Gamma \, s_b \Gamma \\
&\approx -\frac{g}{q} J_\Delta \, s_b \Gamma = -G_\Delta \, s_b \Gamma \qquad (3.27)
\end{aligned}
$$

In analogy with (2.58), $s_b G_\Delta$ is the WTP in order to avert one case of disability.

If b leads to working incapacity ($s_b \geq s_{cr}$ fulfilled) , it is possible to derive the change in income dg_{WA} in a similar derivation ultimately leading to $dg_{WA} \approx g J_\Delta \Gamma$. The criterion can then be extended to

$$
-dg \geq -g \, J_\Delta \, \Gamma \left[\frac{s_b}{q} + 1\right] \qquad \text{for } s_b \geq s_{cr} \qquad (3.28)
$$

Chapter 4

Application of the WTP Criterion

4.1 Overview

The previous two chapters have introduced willingness to pay as a criterion for acceptable decisions involving human safety. Theoretically, the WTP criterion in (2.48) could be applied to various problems right away. However, many typical applications from civil engineering and related fields (especially environmental engineering) involve a number of characteristic side-constraints that need to be taken into account in order to reach a viable decision. These constraints involve financial and biological phenomena as well as the interaction with other criteria.

Two basic application cases can be identified. The first basic case shall be referred to as *acute failure*. It denotes those cases in which a facility fails at an unpredictable moment causing loss of life instantaneously. The other basic case involves *chronic impacts* and shall be named accordingly. It refers to those situations in which permanent immission of toxic or radio-active substances lead to sickness and death in the long run (Note that chemical accidents belong to the acute failure type, however). The two cases differ in a number of aspects that are relevant for the application of the WTP criterion; Table 4.1 provides a brief overview.

The two basic application cases are dealt with successively in **Sections 4.2 and 4.3**. In practical applications, decisions makers are frequently forced to comply with other acceptability criteria for legal or economic reasons. **Section 4.4** discusses how the WTP criterion interacts with the other criteria introduced in Section 1.2.

Table 4.1: Comparison of the two basic application cases

aspect	**acute failure**	**chronic impacts**
type of impact:	failure event	permanent immission
type of human consequences:	mainly loss of life	disease and loss of life
occurrence of impact:	at unpredictable moment	during well-known period
occurrence of human consequences:	simultaneous with impact	delayed (latency)
quantification of impact:	failure rate	toxic concentration
quantification of human consequences:	see Chapter 5	see Chapter 6

4.2 Acceptable Decisions Regarding Acute Failure

In the present context, failure is understood in a broader sense: Primarily, it refers to those cases where a structure fails as a whole (e.g. building collapse, dam failure), but it also comprises those cases in which a system fails although structural failure is not necessarily involved. As an important example from the civil engineering domain, one might consider fires in tunnels: As a system, a tunnel is meant to provide safe passage beneath some obstacle (mountain, river, settled area). In case of an uncontrollable fire, the system fails to fulfil its purpose and puts human life at risk.

4.2.1 Failure Rates and the WTP criterion

Changes in the failure rate $r(t)$ of a facility are directly linked to changes in mortality $\mu(a)$. For acute failure, it can be reasonably assumed that all age classes are affected in a similar way, so that a mortality change can be modelled as age-independent[1], i.e. $d\mu(a) = d\mu$. It can be obtained as

$$d\mu = k\, dr(t) \tag{4.1}$$

where k denotes the probability of death in case of failure, given that the individual at risk is permanently present on site. In reality, only N_{PAR} persons out of a population of N_{pop} can be expected to be present. An individual's probability of presence is therefore equal to N_{PAR}/N_{pop}. Furthermore, most individuals attempt to escape in case of danger. They are likely to succeed at a probability P_Q. Including these conditions leads to

$$d\mu = \frac{N_{PAR}}{N_{pop}}(1 - P_Q)\, k\, dr(t) \tag{4.2}$$

Chapter 5 deals exclusively with the systematic quantification of N_{PAR}, P_Q and k for some typical types of failure from the civil engineering domain. These include structural collapse, dam failure and tunnel fire. Morbidity consequences (i.e. non-fatal injuries) can be modelled in a similar way. However, they are usually outweighed by fatal consequences in the case of acute failure (compare Section 5.1, but also Section 3.4.2).

There is an ongoing need for functioning infrastructure. Therefore, facilities are usually rebuilt in case of failure. On a mathematical level, this situation is described by renewal theory [21]. Failure rate $r(t)$ is equal to the so-called renewal density, which is obtained as

$$r(t) = \sum_{n=1}^{\infty} f_n(t) \tag{4.3}$$

where $f_n(t)$ is the density of $F_n(t)$, the probability of at least n failures (renewals) until t. For deteriorating structures, $r(t)$ rises monotonically until the facility fails and/or needs to

[1] Compare footnote 17 (p. 37).

be replaced. At this point, $r(t)$ adopts its initial value and starts to rise again. It is however, possible to form an average value. For an infinitely long period,

$$\lim_{t\to\infty} r(t) = \frac{1}{E[T]} \tag{4.4}$$

where T is the (randomly distributed) timespan between two successive renewals. For $t < \infty$, it serves as upper bound [5] and reasonable approximation [104].

For Poissonian events, such as earthquakes or floods, the failure rate can be modelled as

$$r = \lambda P_F \tag{4.5}$$

If the loading event occurs, which happens at rate λ, failure will take place only if load S exceeds resistance R, i.e. $P_F = P\{R - S < 0\}$. Both parameters, R and S, are randomly distributed.

Substituting for $d\mu = \Delta$ in (2.57) leads to an adapted version of the WTP criterion:

$$-dg(p) \leq -G_\Delta \frac{N_{PAR}}{N_{pop}} (1 - P_Q)\, k\, dr(p) \tag{4.6}$$

Here, p is some technical parameter which influences both safety (via r) and costs. It is assumed that $dg(p)$ and $dr(p)$ are differentiable with respect to dp.

As before, $-dg$ is the per capita income change due to additional safety measures. If the absolute yearly cost of a measure amounts to C_y, then $-dg = \frac{dC_y}{N_{pop}}$. Multiplying (4.6) with N_{pop} yields

$$\begin{aligned} dC_y(p) &\leq -G_\Delta\, N_{PAR}\, (1 - P_Q)\, k\, dr(p) \\ &= -G_\Delta\, N_{D|F}\, dr(p) \end{aligned} \tag{4.7}$$

In this way, it can be seen that the criterion is independent from N_{pop} and therefore from the size of a city or country, which could be very big or very small. Only the change in safety costs $dC_y(p)$ and the change in the yearly expected number of fatalities $N_{D|F}\, dr(p)$ determine the outcome. Note that $N_{D|F} = N_{PAR}\, (1 - P_Q)\, k$ equals the expected number of fatalities given failure.

In order to decide whether increasing p by yet another marginal increment is acceptable, Rackwitz [103] introduced a derivation after dp:

$$\frac{dC_y(p)}{dp} \leq -G_\Delta\, N_{D|F}\, \frac{dr(p)}{dp} \tag{4.8}$$

If there is more than just one such parameter, p is replaced by a vector of parameters $\mathbf{p} = \{p_1, p_2, \ldots, p_n\}^{\mathrm{T}}$. In this case, (4.8) the criterion is solved as an optimisation task:

$$\text{minimise:}\;\; W(\mathbf{p},t) = -dC_y(\mathbf{p},t) - G_\Delta\, N_{D|F}\, dr(\mathbf{p},t) \tag{4.9}$$

In some cases, a facility is not renewed but replaced by a completely different facility. As an example, one might consider a long mountain road I that is replaced by a short road with a tunnel II. In this case, it is not correct to calculate $dr = r^{\mathrm{II}}(p) - r^{\mathrm{I}}$, because failure entails different consequences. Instead, (4.7) has to be re-written as

$$dC_y(p) \leq -G_\Delta \cdot \left[N^{\mathrm{II}}_{D|F}\, r^{\mathrm{II}}(p) - N^{\mathrm{I}}_{D|F}\, r^{\mathrm{I}} \right] \tag{4.10}$$

4.2.2 The Effect of Different Service Lives

The WTP criterion is based on one implicit but important assumption: Prior to a safety decision, mortality has been $\mu(a)$ for a very long time and from the decision point onwards it is going to equal $\mu_\Delta(a) = \mu(a) + \Delta$ with $\Delta < 0$ for an equally very long time (longer then a_u). In Figure 4.1, this is indicated as case A). In order to maintain this change in mortality, it is necessary to renew the safety investment periodically, after t_s years. Otherwise, mortality will rise back to its original rate, as shown in case B) in the figure. The derivations in Chapter 2 are only valid for case A), although it is theoretically possible to adapt them to case B), as shown by Johannesson et al. [54]. Some safety investments need to be repeated every few days (e.g. spraying de-icing salt on winter roads), while others can last for decades or even more than 100 years (e.g. safety-relevant civil engineering structures). In the end, they too, need to be renewed or replaced. The only difference consists in the planned service life or re-investment period t_s.

Safety Investments without Financing Costs

The total cost of constructing a safety-relevant facility is $C(p)$, while $dC(p)$ corresponds to the additional costs for safety enhancement. It can be converted into yearly values as $dC_y(p) = dC(p)/t_s$, at least if financing costs are not considered so that money can be borrowed for free. Let this assumption hold for the next few steps. It appears that the WTP criterion as given in (4.7) corresponds to the special case of yearly re-investments or $t_s = 1$, leading to $dC_y(p) = dC(p)$ and an absolute number of fatalities $t_s r N_{D|F} = r N_{D|F}$ per re-investment. For an arbitrary $t_s > 0$, (4.7) can be re-written as

$$dC(p) = t_s\, dC_y(p) \leq -t_s\, G_\Delta\, N_{D|F}\, dr(p) = \mathrm{WTP} \tag{4.11}$$

According to utility theory, society's willingness to pay depends solely on its economical resources (expressed by factor G_Δ) and the expected number of averted fatalities. If an investment prevents accidents during a very short time $t_s \to 0$, then the probability of failure $t_s r(p)$ equally tends towards zero and so does the number of averted fatalities (i.e. $t_s N_{D|F} dr(p) \to 0$). The willingness to pay during t_s should then be very small too. It appears that the re-formulated criterion in (4.11) complies with this reasoning.

It can be helpful to convert all investments into yearly investments, i.e. to divide (4.11) by t_s:

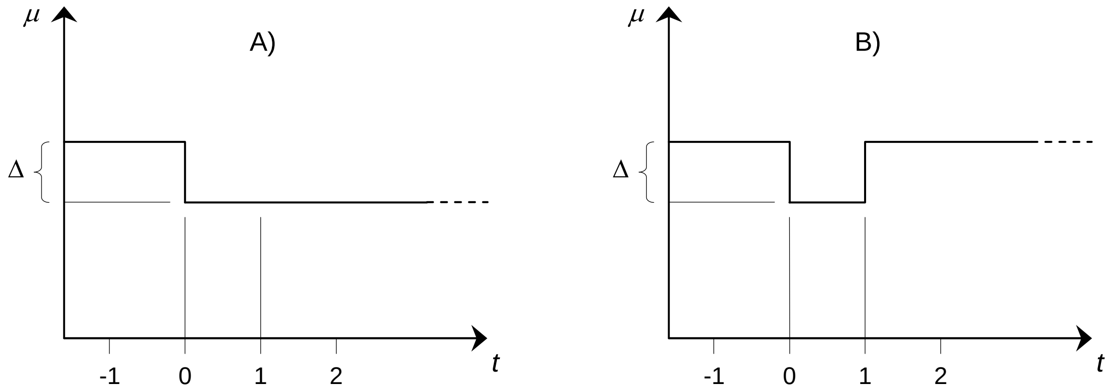

Figure 4.1: The effect of continuing (A) and singular (B) safety investments upon raw mortality

$$dC_y(p) = \frac{dC(p)}{t_s} \leq -G_\Delta \, N_{D|F} \, dr(p) \tag{4.12}$$

The same adaptation applies to the marginal WTP criterion in (4.8).

Safety Investments Involving Financing Costs

Of course, (4.12) applies not only to small service lives $t_s \leq 1$, but equally to very long periods $t_s \gg 1$, which are characteristic for built structures. However, this usually involves considerable sums of money which need to raised during the construction phase. In many cases, enterprises or governments do not have the required sum $dC(p)$ at hand and have to borrow from the capital market, either by issuing stocks and bonds or by taking out loans. After the construction has been finished, the owner of the facility has to return the borrowed sum by yearly instalments plus interest[2]. In consequence, financing costs have to be added to the construction costs, which means that the WTP criterion becomes more difficult to fulfil. For a discount rate γ, paying back a loan in the height of $dC(p)$ over t_s years by constant instalments requires yearly payments of[3]

$$dC_y(p) = dC(p) \frac{\exp[\gamma] - 1}{1 - \exp[-\gamma t_s]} \qquad \text{for } \gamma > 0 \tag{4.13}$$

Of these yearly costs, only $dC_{m,y} = dC/t_s$ corresponds to safety measures and reduces N_D, while the remaining $dC_{f,y} = dC_y - dC/t_s$ are pure financing costs. The share of dC_y available for safety is illustrated in Table 4.2 as a function of t_s and γ.

[2]If the enterprise or government does in fact possess the required assets, the situation is nevertheless the same as with taking out a loan: Instead of investing the assets into a facility, the owner could equally 'put them to work' on the capital market, in order to earn interest revenues at the market rate (so-called shadow cost of investing one's own resources into a project).

[3]The relationship is obtained by adapting equation (1.84) in Bronstein et al. [12] for continuous instead of yearly discounting.

Table 4.2: Fraction of payments contributing to actual safety measures $dC_{m,y}/dC_y$

discount rate γ:	**1%**	**3%**	**5%**	**7%**
$t_s = 10$ yrs:	0.95	0.86	0.79	0.72
$t_s = 25$ yrs:	0.88	0.70	0.57	0.47
$t_s = 50$ yrs:	0.79	0.52	0.37	0.28
$t_s = 100$ yrs:	0.63	0.32	0.20	0.14

For long service lives (more than 25 years), the financing period t_f is usually not identical with t_s, because loans of such durations are not available on the capital market. In these cases, it is proposed to insert t_f instead of t_s. If the facility is financed with proprietary capital, this discrepancy does not occur, i.e. $t_f = t_s$ even for very long service lives.

Average income does not remain the same during t_s, but is expected to grow parallelly with economic growth at a rate ζ. Therefore, income during t_s totals

$$g_{\text{tot}} = \int_0^{t_s} g \exp[\zeta t] dt = g \frac{\exp[\zeta t_s] - 1}{\zeta} \tag{4.14}$$

The results in (4.13) and (4.14) can be directly inserted in (4.12), leading to

$$dC(p) \frac{\exp[\gamma] - 1}{1 - \exp[-\gamma t_s]} \leq -\frac{\exp[\zeta t_s] - 1}{\zeta} G_\Delta \, N_{D|F} \, dr(p) \tag{4.15}$$

or, equivalently

$$dC(p) \leq -\frac{1 - \exp[-\gamma t_s]}{\exp[\gamma] - 1} \frac{\exp[\zeta t_s] - 1}{\zeta} G_\Delta \, N_{D|F} \, dr(p) \tag{4.16}$$

Rackwitz [104] remarks that the two newly introduced fraction terms almost cancel, if the difference between ζ and γ is small (which is generally the case, compare Chapter 2). Again, the adaptation applies analogously to the marginal WTP criterion in (4.8).

4.3 Acceptable Decisions Regarding Continuous Toxic Emissions

Continuous immissions of toxic agents can originate from different emittents, such as production processes, traffic, energy production or natural processes. The ongoing decomposition of materials used for buildings and products is another source. Decisions involving such processes change the local concentrations x_q of an agent q. The concentration is expressed in relation to the exposure medium (air, water, food). It depends on the distance from the source as well as on the transport mechanism, as described in Chapter 6.

A change in exposure medium concentration dx_q leads to a change in mortality $d\mu(a, dx_q)$ and to a change in morbidity. The latter is expressed as a change in prevalence $dP_b(dx_q)$ of disability b. The corresponding relationships are equally described in Chapter 6.

4.3.1 Latency Periods and the WTP Criterion

When applying the WTP criterion to continuous toxic impacts, the main challenge consists in the fact that human consequences usually follow a concentration change with a certain delay. This *latency period* shall be denoted by T_μ and T_ν for the cases of mortality and morbidity, respectively.

Changes in Mortality

The mortality effect of chronic toxic exposure is usually proportional to a person's background mortality. This type of risk model was introduced in (2.54). It assumes that a change in mortality leads to a modified mortality rate $\mu_\delta(a) = \mu(a)(1+\delta)$. In the simplest case, δ is obtained as the product of the change in exposure medium concentrations and a so-called slope factor, i.e. $\delta = \eta\, dx_q$.

When latency is introduced, the most obvious and robust procedure is to determine life expectancy for

$$d\mu(t, dx_q) = \begin{cases} 0 & \dots\ t < a + T_\mu \\ \delta = \eta\, dx_q & \dots\ t \geq a + T_\mu \end{cases} \tag{4.17}$$

and to calculate the corresponding change in life expectancy as $de_d(a) = e_d(a, dx_q, T_\mu) - e_d(a)$ as in (2.50). The result can be inserted in (2.48), leading to the criterion

$$-dg \leq \frac{g}{q}\, E_A\left[\frac{e_d(a, dx_q, T_\mu) - e_d(a)}{e_d(a)}\right] \tag{4.18}$$

Alternatively, the procedure can be accelerated by linearising the relation between $d\mu(a)$ and $de_d(a)$ as in Section 2.5.2. With respect to a latency period T_μ, the linearisation coefficient J_δ from (2.55) is re-written as

$$\begin{aligned}
E_A\left[\frac{de_d(a)}{e_d(a)}\right] &\approx \int_0^{a_u} \frac{de_d(a,\delta,T_\mu)}{e_d(a)}\, h(a,n)da \\
&= \int_0^{a_u - T_\mu} \frac{de_d(a,\delta,T_\mu)}{e_d(a)}\, h(a,n)da + \int_{a_u - T_\mu}^{a_u} \frac{\cancel{de_d(a,\delta,T_\mu)}}{e_d(a)}\, h(a,n)da \\
&= \int_0^{a_u - T_\mu} \frac{-\frac{d}{d\delta}\int_a^{a_u} S_d(t,\delta,T_\mu|a)dt\Big|_{\delta=0}\cdot\delta}{\int_a^{a_u} S_d(t|a)dt}\, h(a,n)da \\
&= \int_0^{a_u - T_\mu} \frac{-\frac{d}{d\delta}\left\{\int_a^{a+T_\mu} \cancel{S_d(t,\delta,T_\mu|a)}dt + \int_{a+T_\mu}^{a_u} S_d(t,\delta,T_\mu|a)dt\right\}\Big|_{\delta=0}\cdot\delta}{\int_a^{a_u} S_d(t|a)dt}\, h(a,n)da
\end{aligned}$$

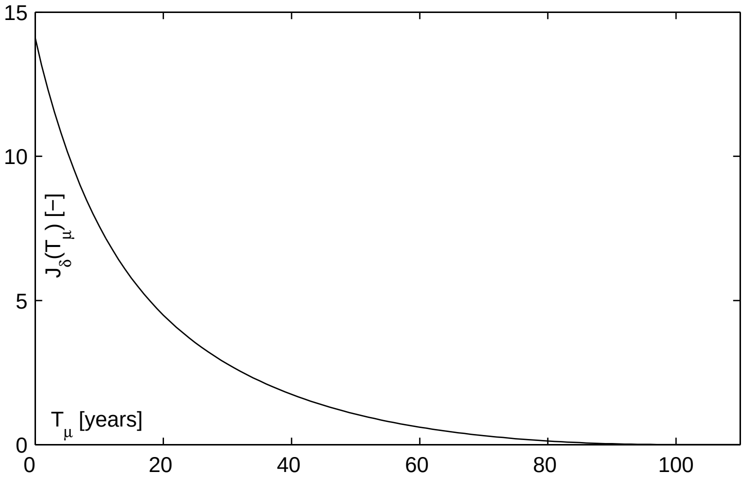

Figure 4.2: The dependency of linearisation coefficient C_δ upon latency period T_μ (for German demographic data)

$$
\begin{aligned}
&= \int_0^{a_u - T_\mu} \frac{-\frac{d}{d\delta}\int_{a+T_\mu}^{a_u} \exp\left[-\int_a^{a+T_\mu}\mu(\tau)+\gamma(\tau^*)d\tau - \int_{a+T_\mu}^{t}\mu(\tau)(1+\delta)+\gamma(\tau^*)d\tau\right] dt\Big|_{\delta=0} \cdot \delta}{\int_a^{a_u}\exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right] dt} \\
&\quad \cdot h(a,n)da \\
&= -\int_0^{a_u - T_\mu} \frac{\int_{a+T_\mu}^{a_u}\exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right]\cdot \int_{a+T_\mu}^{t}\mu(\tau)d\tau\, dt}{\int_a^{a_u}\exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right] dt}\, h(a,n)da\cdot\delta \qquad (4.19) \\
&= -J_\delta(T_\mu)\,\delta \\
&= -J_\delta(T_\mu)\,\eta\, dx_q = J_x(T_\mu)\, dx_q
\end{aligned}
$$

The result is graphically illustrated in Figure 4.2. However, the present risk model represents only the simplest possible case. Section 6.4 introduces some more specific relationships that are frequently encountered in the epidemiological literature. They too can be linearised analogically, which is demonstrated in the same section.

Changes in Morbidity

If a morbidity effect is delayed by a latency period T_ν, this can be accounted for comparatively easily by altering (3.14):

$$
\begin{aligned}
\frac{de_{DA,d}(a, dP_b)}{e_{DA,d}(a)} &= \frac{-s_b\, dP_b \frac{1}{S(a)}\int_{a+T_\nu}^{a_u} S(t)\exp[-\int_a^t \gamma(\tau^*)d\tau]dt}{e_{DA,d}(a)} \\
&= -s_b\, dP_b \frac{e(a+T_\nu)}{e_{DA,d}(a)} \qquad (4.20)
\end{aligned}
$$

Inserting this formulation in (3.19) yields an adapted WTP criterion taking latency into account:

$$-dg \leq -dP_b(dx_q) \cdot \left[\frac{g\, s_b}{q}\, E_A\!\left[\frac{e_d(a+T_\nu)}{e_{DA,d}(a)}\right] + \kappa_b + g\right] \tag{4.21}$$

For the simplest possible risk model, the change in prevalence is obtained as $dP_b(dx_q) = \eta'\, dx_q$ (see Section 6.4).

Combined Changes

As outlined in Section 3.4.2, mortality and morbidity effects can simply be added in most practical situations. This leads to the following criterion:

$$-dg \leq -\frac{g}{q} J_x(T_\mu)\, dx_q \;-\; dP_b(dx_q) \cdot \left[\frac{g\, s_b}{q}\, E_A\!\left[\frac{e_d(a+T_\nu)}{e_{DA,d}(a)}\right] + \kappa_b + g\right] \tag{4.22}$$

4.3.2 Project Costs

The considerations on project costs from Section 4.2 apply to decisions regarding chronic emissions in practically the same way. As before, yearly additional costs of life-saving dC_y are obtained from $C_y = -dg\, N_{pop}$. For mortality changes, the criterion can be re-written as

$$\begin{aligned} dC_y &\leq \frac{g}{q}\, E_A\!\left[\frac{de_d(a, dx_q, T_\mu)}{e_d(a)}\right] N_{pop} \\ &= -\frac{g}{q} J_x(T_\mu) x_q\, N_{pop} \end{aligned} \tag{4.23}$$

for the case of mortality effects. Note that $d\bar{e}_d(dx_q) \cdot N_{pop}$[4] is the absolute number of life years gained, if the life expectancy of all members of society are added up. If the toxic exposure concerns only a part of the population N_{pop}, it is more correct to insert the number of people at risk N_{PAR} instead.

Local concentrations can equally depend on some parameter p controlling the emissions. As with the acute case, it is possible to establish a marginal WTP criterion:

$$\frac{dC_y(p)}{dp} \leq -\frac{g}{q} J_x(T_\mu) \frac{dx_q(p)}{dp}\, N_{pop} \tag{4.24}$$

Different service lives and financing costs can be considered as in (4.16), by multiplying with $\frac{1-\exp[-\gamma t_s]}{\exp[\gamma]-1} \frac{\exp[\zeta t_s]-1}{\zeta}$.

[4] $= E_A[de_d(a, dx_q)] \cdot N_{pop}$

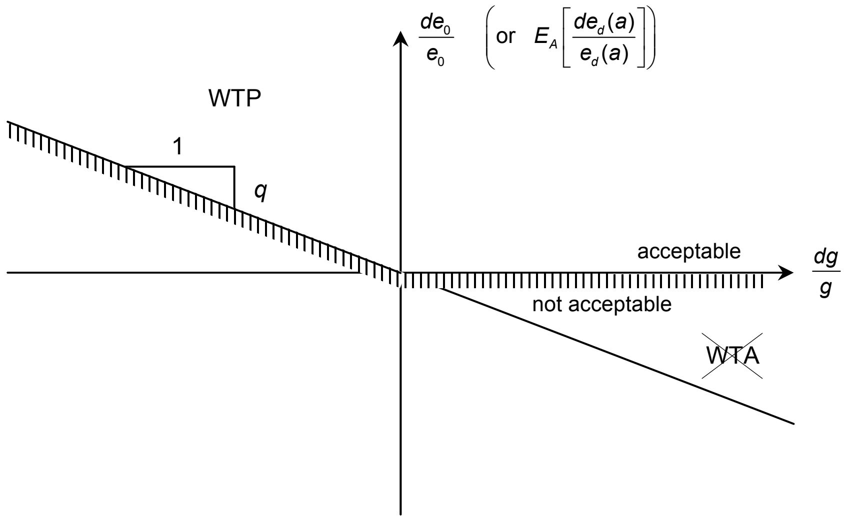

Figure 4.3: Interaction of the WTP criterion with conventional (not utility-based) criteria

4.4 Interaction of WTP with Other Criteria

In the introduction, Section 1.2 distinguished three basic types of safety-related decision criteria. These included conventional safety criteria (without reference to economy or utility), cost-benefit analysis (CBA) and utility-based criteria such as the WTP criterion. In many situations, a decision-maker does not have the freedom of choice to use societal willingness-to-pay as the sole criterion for his or her decisions. Quite frequently, conventional and CBA criteria need to be respected for legal or economic reasons. This leads to an interaction between WTP and other criteria.

4.4.1 Interaction of WTP and Conventional Criteria

The use of specific conventional criteria is often prescribed by standards and prescriptions like the ones described in Appendix A. These criteria valuate a decision mainly by its impact upon fatality or disability rates, while treating other aspects of human life in a secondary, qualitative manner (if at all). In most cases, they are calibrated against mortality and morbidity consequences of state-of-the-art technology (compare Section 1.2). If expressed in terms of life expectancies (disability-adjusted or not), these criteria can essentially be written as

$$d\bar{e} \geq 0 \tag{4.25}$$

In this relation, it does not influence the result, if average life expectancy $d\bar{e}$ is replaced by its discounted counterpart $d\bar{e}_d$.

The interaction of (4.25) and the WTP criterion is demonstrated by Figure 4.3, which adapts Figure 2.5 to the interaction case. Essentially, the non-utility-based contribution cuts off the willingness-to-accept area (WTA), i.e. the case in which people give away some of their safety in order to obtain extra income.

4.4.2 Interaction of WTP and CBA Criteria

General Considerations

Cost-benefit analysis (CBA) is a tool by which owners can assess the profitability of a project. In economic terms, a project is deemed acceptable if the benefits exceed or at least match the initial investment costs in the long run. Applying CBA only has an influence upon safety levels if the project involves potential hazards leading to at least one of the following outcomes:

- Loss of investment goods $C(p)$: If the facility is lost due to failure, it needs to be rebuilt. This causes costs in the height of the original project costs.
- External damage H: Compensation payments are due, if the owner can be held liable for lost property of third parties (H_X) or for fatal victims (H_D). If the operation of a facility is subject to strict liability, the owner has to pay compensation even if the failure has been caused neither by negligence nor intention. This is the typical situation in the transport and energy sectors (see e.g. [2]).

If a civil engineering facility threatens to fail, both consequences are usually involved. If the safety level is too low, this will lead to excessive damage costs in the long run. If it is chosen too high, increased investment costs will impair the profitability of the project. This relation is described in mathematical terms in Appendix B.

Neither of the two conditions applies to chronic emissions of toxic substances: First, the released substance doesn't destroy the the emitting facility in most cases. Second, most toxic substances lead to diseases which also occur without any exposure, yet, at a lower rate. Although epidemiology can clearly prove the effect of ambient exposure with a certain substance and changes in morbidity and mortality, it is usually impossible to attribute an individual case to a given influence, because the observed disease might be due to some other influence as well. Latency makes a clear attribution even more difficult. If causality cannot be proved in the individual case, it becomes almost impossible to hold the emittent liable by legal means.

It is important to realise that 'costs' do not refer to the same quantity in the WTP criterion as in cost-benefit analysis. $C_{\text{society}}(p)$ are the costs that society pays for a project. In the WTP criterion, additional costs for safety $dC_{\text{society}}(p)$ are understood as a negative change in average societal income $-dg = dC_y(p) = dC_{\text{society}}(p)\frac{\exp[\gamma]-1}{1-\exp[-\gamma t_s]}$. In the case of a cost-benefit investigation, (B.9) expresses the effect of a project upon the owner's income:

$$Z(p) = B(p) - C(p) - (C(p) + H)h^*(\gamma, p) \tag{4.26}$$

If the facility is privately owned, revenues correspond to the payments effected by society's members in exchange for using the facility[5] (e.g. privatised motorways), i.e. $B(p) \equiv C_{\text{society}}(p)$. This includes additional safety costs $dC_{\text{society}}(p)$. If profit $Z(p)$ and failure intensity $h^*(\gamma, p)$

[5] Following the introduction of production functions in Section 2.2.2 (p. 21f), it could be argued that society benefits from private revenues as well, in as far as they are used to pay employees' wages, which has an effect on average societal income.

are small, $C_{\text{society}}(p)$ is only little higher than the owner's original investment $C_{\text{invest}}(p)$ included in (4.26). If the facility is publicly owned, profits are equal to (negative) societal costs, i.e. $Z(p) \equiv -C_{\text{society}}(p)$.

The interaction between the WTP criterion and the CBA criterion becomes evident, if the two possible conflicting cases are regarded:

CBA criterion fulfilled, WTP criterion violated: In this case a private owner profits at the expense of the population, whose life quality (or lifetime utility) is reduced. Such projects need to be dejected. If such a facility is publicly owned, its realisation must be regarded as self-destructive behaviour from a societal point of view.

WTP criterion fulfilled, CBA criterion violated: If the facility is privately owned, this can be interpreted as the case, where consumers are willing to pay for safety ($C_{\text{society}}(p) \leq \text{WTP}$), but the owner is not receiving enough payments $B(p) = C_{\text{society}}(p)$ in order to cover his costs ($Z(p) < 0$). As a consequence, the owner is going to put up the prices for users of his facility until he reaches a profitable region. If this cannot be done without violating the WTP criterion, the project will be abandoned. If the facility is publicly owned, the CBA criterion may be violated, as long as the WTP criterion is fulfilled: Apparently, the societal utility is higher than the economic costs.

In conclusion, it can be said that the WTP criterion needs to be fulfilled in any case. If the facility at question is privately owned, the CBA criterion (profitability) needs to be met as well. Otherwise, it is not going to be realised. A private owner cannot be forced to operate at deficit cost in favour of society.

Estimation of Compensation Costs for Human Consequences

In case of a fatality, the relatives of the victims need to be compensated. The rate of the compensation is decided in a court case or in direct negotiations between the owner of the facility and the relatives. As a lower bound, one can adopt the loss of future earnings. If the victim was a years old at the time of death, this corresponds to

$$H_D(a) = g\, e(a) \tag{4.27}$$

Income is expected to grow parallelly with economic growth at rate ζ. However, future income needs to be discounted in order to determine its net present value which, again, requires economic growth rate ζ. The two effects cancel and therefore do not appear in (4.27). Alternatively, one might use full market rate $\gamma = \rho + \varepsilon\zeta$. In this case, only part of the discount rate cancels. Life table mathematics and notations follow those in Section 2.2. In the case of *potential* victims, the respective ages are unknown. Therefore, it is necessary to average over all age classes:

$$\begin{aligned} \bar{H}_D &= g \int_0^{a_u} S(a) h(a,n)\, da \\ &= g\bar{e} \approx g \frac{e_0}{2} \end{aligned} \tag{4.28}$$

The approximation $\bar{e} \approx e_0/2$ corresponds to the case of slowly growing or stagnating populations. For higher population growth rates, as observed in developing countries, the population structure resembles a triangle, so that $\bar{e} \approx 2e_0/3$. In developed countries, $\bar{H}_D \approx g\frac{e_0}{2}$ leads to values of 0.9–1.2 million PPP US$. When discounting at full market rate, $\bar{H}_D$ reduces to less than half of the value. As mentioned above, this number can only save as a lower estimate for compensation costs. In the case of the WTC Twin Tower collapse, recompensation amounted to as much as 2.9 million US$ per fatality (see Kübler [63]).

Alternatively, Rackwitz [104] proposes to integrate the formulation in (2.49), leading to

$$H_D(e_0) = g\left[1 - \left(1 + \frac{de_0}{e_0}\right)^{-\frac{1}{q}}\right] de_0 \tag{4.29}$$

for people aged 0 (newborn babies). In (4.29), de_0 is the expected number of lost life years. Averaging over all ages leads to

$$\bar{H}_D = \int_0^{a_u} H(e(a))h(a,n)\, da \approx H_D(\frac{e_0}{2}) \tag{4.30}$$

Chapter 5

Human Consequences of Acute Failure of Civil Engineering Facilities

5.1 General Framework

For buildings or other technical facilities, the probability of failure is well described by means of reliability theory. However, the question of societal acceptability addressed in the previous chapters requires translating technical risks into human risks. In other words: How big is the probability of people being killed (or injured), given failure of a building, and how many dead and injured are to be expected? A considerable number of authors have addressed this question in the past. Yet, in almost all cases they each limited themselves to one specific event type while applying a multitude of diverging approaches. The present chapter aims at developing a generally applicable methodology for the quantitative estimation of loss of life due to acute failure events. The focus of this chapter will rest on three types of failure from the civil engineering domain which are at the same time among the most frequently observed and the most deadly ones: Building collapse due to earthquakes, dam failure and fire in tunnels.

One of the main arguments for developing a general methodology are the parallels existing even between the most different fields of application. Even considering conditional probability of death is an event-type specific phenomenon subject to entirely different laws in each case, there are two phenomena of universal validity: In each case the same considerations will apply with respect to the expected number of people at risk, i.e. those persons who are in the reach of the event at the time the first signs of the imminent danger can be perceived. The human reaction of running away when facing an impending danger is equally universal, even though it may be forced into different pathways depending on the situation. These observations lead to a simple relationship that will be the basis for all further considerations:

$$\underset{\text{number of lives lost given failure}}{N_{D|F}} = \underset{\text{number of people at risk}}{N_{PAR}} \times \underset{1-\text{probability of escape}}{(1-P_Q)} \times \underset{\text{conditional probability of death}}{k} \tag{5.1}$$

The yearly expected number of fatalities follows from the failure rate r of a facility as

$$N_D = r\, N_{D|F} \tag{5.2}$$

Apart from fatalities, acute failure equally causes non-fatal injuries. As previously discussed in Chapter 3 (p. 53), disabilities either have to outnumber fatalities by several orders of magnitude or they need to be severe and permanent in order to have a significant effect upon lifetime utility. With acute failure, a large share of severe injuries can be healed and those that cannot are usually outweighed by the fatalities. In the case of dam failure, persons who almost drowned do not carry any long-term consequences unless they have been hit by debris. The same applies to tunnel fires and suffocation. In the case of earthquake-induced collapse, fatalities outnumber hospitalisation by a factor of three and injuries requiring major surgery by more than one order of magnitude [18].

Sections 5.2 to 5.4 deal with the three basic quantities N_{PAR}, P_Q and k successively. In doing so, each quantity is split up into as many sub-quantities as possible, which seems essential for two reasons: On one hand, a single-parameter approach appears too limited, when describing a phenomenon subject to several independent influences and will necessarily cause substantial scatter. One the other hand, many of the real-life phenomena those sub-quantities apply to more than just one single event type. This again addresses the issue of taking advantage of parallels.

Section 5.5 is dedicated especially to those cases in which probability of escape and conditional probability of death are mutually conditional, thereby necessitating an iterative procedure. This is the case with all events which do not fulfil the condition of suddenness as defined in section 5.3.

Section 5.6 adds a brief remark on the assumed independence between the expected number of fatalities given failure $N_{D|F}$ and failure rate r, followed by a discussion in **Section 5.7**.

5.2 Presence of People at Risk

5.2.1 Basic Methodological Aspects

Before approaching the details of presence modelling, it should be made clear whose presence is at issue. The so-called *population at risk* includes all individuals who are already present at the location of an event before the first signs of its outbreak becoming noticeable. Therefore, all those who will in the further evolvement of events succeed in escaping, are principally members of the population at risk. N_{PAR} is the number of persons expected to be present on the occasion of an event.

This number is a sub-quantity of the reference population *pop* ($N_{PAR} \leq N_{pop}$). In the case e.g. of a residential building all inhabitants taken together constitute N_{pop}, but only those who are *actually there* at the moment of the failure event are counted as part of N_{PAR} (especially during daytime most residents can be expected to be away).

Except for events of very limited reach, it is recommendable to subdivide the population at risk into subpopulations PAR_i. The sub-area in which PAR_i are located shall be denoted as i. In this manner the locally varying intensity of an event is accounted for more accurately: In case of flooding, the water depth and velocity will not be equal at all points; some sub-areas will remain entirely unaffected. Similar considerations apply to earthquakes with respect to different buildings — here, one would typically consider each building as a separate sub-area. At most, each complex of identically designed buildings would make up one sub-area.

To abstain from this refinement means to risk overestimating $N_{D|F}$, the expected number of lives lost in case of an unfortunate event. Inversely, it leads to an underestimation of the conditional probability of death k, when analysing historical disasters.

In the literature, three types of presence models can be observed, the third being a combination of the first two. The two 'pure' models view the endangered persons in a way that could be called object-based in the first and distribution-based in the second case.

The *object-based approach* (as e.g. in [95]) uses directly collected data of the number of people present at a given location at a given time. This way of proceeding is very accurate but requires a significant data collection effort. It is especially recommendable for those objects, which strongly differ from other objects, be it by their size or by their singularity. For such structures it is problematic to adopt existing data from other objects. Moreover, the significance of the respective structure frequently justifies an increased effort. Airports and hospitals are typical representatives of this category together with traffic infrastructure buildings such as tunnels and bridges. In many cases it is precisely these buildings which are characterised by a high level of organisational dispositions with the effect that the relevant data may have already been collected previously for some other purpose. In the case of road infrastructure, this would apply to data recorded by traffic planners. Data from turnpikes, turnstiles and ticket sales can equally be useful.

The idea behind the *distribution-based approach* is to split up the persons N_{pop} living in some greater area between several sub-areas and the buildings located therein. This way of proceeding takes advantage of general statistical knowledge on the behaviour of populations as a function of spatial, temporal and demographical categories. The approach is suitable for events covering a large area such as floodings or earthquakes (compare [108, 124]). For large scale events, moderate efforts can yield realistic estimations that it is highly impracticable to map the presence of the inhabitants directly when hundreds or thousands of buildings are concerned. To add to it, such data is required with respect to the course of a day, week and year. Instead, the idea is to obtain the necessary data by questioning a population sample. This can be combined with data from registry offices or population censuses. Generally, many statistical results can be transferred from one region to another within one country, which helps to facilitate work further.

In practical applications, elements of the two approaches are often combined. Even when the vehicle frequency is unknown for some traffic infrastructure, the number of vehicles has to be multiplied with the average number of passengers (depending on the vehicle type). Typically, one would not collect this information directly, for it can be taken from general traffic statistics and then be applied to the given case. An interesting approach—although seeming a bit complicated at first sight—is brought up in the HAZUS manual [134] (see Appendix C.1): Within a population, several groups are identified (residential population, working population etc.). Here, the issue is the principal affiliation to a certain group but not the realisation of this affiliation at a given moment. The sum of all these groups amounts to more than 100%, since most members of society belong to more than just one group:

Most of us spend some time at home commuting or working part of the day. Others attend school or visit an area as tourists. The list may be continued arbitrarily. At this point, HAZUS defines another number expressing the probability with which an individual will realise a given affiliation at a given moment. In most cases, this involves being present at a specific location (as an example, the probability that an office worker will be present at his office will be fairly high at 3 p.m. on a work day). Finally, it is essential for the consequences of a failure—in this case structural collapse—whether a person staying at a given location is indoors or outdoors. A third number is introduced to account for the corresponding probability (see Appendix C.1 for detailed explanation). As an illustrative example, one might imagine a person who is barbecuing in the garden in the evening. Obviously, he or she is dedicated to 'residential occupation' (technically speaking) and is present at the corresponding location (his or her home) though outside and not inside.

From here it is only a small step towards an actual synthesis of object- and distribution-based approaches: In order to model human presence for a particular object (instead of aiming at a group of objects associated with a specific occupation), the first step is to find out, how many persons are associated with an object. In case of an ordinary residential or office building, this does not cause any difficulties in particular. It is precisely this type of object for which the expenses associated with the object-based approach are not worthwhile. At the same time a purely distribution-based proceeding is adequate for an accumulation of buildings but not for a particular object. Subsequently, the number of associated persons is multiplied by the two probabilities from HAZUS, i.e. realisation of affiliation and being inside and not just around the building. With respect to the two previously introduced approaches, one might refer to this one as *conditionally distributed approach*.

5.2.2 Temporal Categories

Presence with Respect to Season

The effect of season upon the number of people present at a location has a number of different aspects depending on local conditions. The largest seasonal fluctuations are observed in the type of touristic regions specialised to either winter or summer tourism. Apart from this annual large scale events such as festivals etc. have to be considered [108]. Students are only present at school or college during lecture periods; nonresident students leave town not just the school building. Apart from the temporal aspect, it can be stated that this group of persons congregates at places which differ from other areas not just by the demography of the occupants but also with respect to the building types.

Depending on the seasonally varying climatic conditions (cold/warm or rain season vs. dry season), a changing fraction of the population stays outdoors during the day. Murakami [81] partly attributes the large number of fatalities in the earthquake of Spitak, Armenia (1988) to the prevailing winterly temperatures. Only few people were actually outdoors at the time of the event.

All of the mentioned factors appear to be influential, although not each of them will necessarily be significant under given circumstances. This needs to be judged separately in the individual case.

Presence with Respect to Weekday

Apparently, workdays differ from work-free days (weekend, public holidays), when it comes to the spatial distribution of the population. In the first case, the working population is located at their workplaces and students are located at school which is obviously not the case on work-free days. What is less obvious is the answer to where they can be found instead. The next caption, *Presence with Respect to Daytime*, shows how a day can be divided into a small number of blocks. The question arises, whether it is admissible to transfer the time blocks describing a workday situation onto work-free days. Presumably, people will use their few hours of evening leisure time during the week in a different way than on weekends when the entire daytime awake is available for leisure activities. People go for a trip or do some sport, which however also depends on the season (see above). Their are also differences with respect to the part of the day *not* spent awake, i.e. sleeping: Most people sleep late on weekends. Furthermore, saturdays differ from sundays, at least in those countries where shops are closed on sundays. In any case, one should take care to include only those factors which have a significant impact instead of considering every aspect having some empirically provable, but minor effect.

Hazus [134] entirely ignores the elementary difference between work and work-free days. In this model, people work seven days a week. Reiter [108] assumes that there is a difference between days, but only in as far as students and commuters are present only during workdays. Strictly speaking, residents in this model are meant to work or attend school on weekends too, which is certainly unintended by the author. Hartford et al. [46] accounts for the fact that leisure includes the whole day except for sleep on weekends. Yet, it is not evident whether he attributes a different quality to weekday leisure time than to leisure at the weekend.

The question of how to describe the respective daily time blocks—including their dependency on the weekday under the above mentioned influences—is subject of the following caption. The relative weight of the different blocks for a whole year can be read from an ordinary calender.

Presence with Respect to Time of Day

When modelling the distribution of a population over different places and buildings over the course of a day, it is recommendable to subdivide the day into a small number of time blocks. This helps to limit complexity while maintaining a sufficient proximity to reality. Most of the cited literature sticks to this approach at least in those cases where presence modelling is under close consideration. Reiter [108] defines three categories: Bedtime, work time and leisure time at home. Hartford et al. [46] apply the same scheme. The Hazus manual [134] described in Appendix C.1 equally uses three categories yet, replacing leisure time at home by commuting time. This difference in perception is presumably influenced by the reality of

American suburban conditions where the role of commuting is far more important than, e.g., for a small town in Finland (the subject of Reiter's investigation).

Apparently, both Reiter and Hartford attach some importance to distinguishing leisure time at home from time asleep, although both occupations take place at home. Even if no previous warning is issued in case of an approaching flood wave—the subject of both of these studies—the fast moving water can only be perceived optically and acoustically shortly before its arrival, thus facilitating escape at the last moment. It has great influence, whether a person is awake or asleep (compare Section 5.3). Actual numbers on the percentage of people present at a certain place during a given time block can be taken from the earthquake literature (see Section 5.2.4 below).

5.2.3 Demographic Categories

Affiliation to a certain population group strongly depends on the age of the respective individual. Depending on a society's degree of traditionality, a person's sex also play a more or less important role. When regarding members of the working or studying population, the affiliation to their respective population group has an influence on their presence being bound to office or school buildings at given times of day. Due to their simultaneous absence from residential buildings, the population structure changes in these locations too. There are other places, such as homes for the aged where the age distribution deviates from that of the general population. In retrospective analyses, one will usually consider this effect, e.g. [81]. Although the number of fatalities and the size of the population are usually known from past events, there is frequently a lack of information on the distribution of the population over different buildings and danger zones.

In order to give a sensible estimation of a population's spatial distribution, it is helpful to know the size and typical whereabouts of different age groups [134, 81]. However, once a population has been allocated to different places and objects—by making use of data on age and other characteristics—one should treat these persons as *uniform* and without property. This may appear odd at first, but is actually nothing but a mathematical consequence of the commonly accepted principle of *equality* of all humans as discussed in Section 2.1. Including the actual age-distribution for appointing an appropriate safety-related decision according to the WTP criterion (2.48), would lead to an ethically doubtful result: The safety of a home for the aged would then appear to deserve less resources than the safety of buildings with less aged occupants. The same considerations apply to age-dependent effects with respect to P_Q (probability of successful escape) and k (conditional probability of death) in Sections 5.3 and 5.4.

5.2.4 Quantification of the Individual Categories

The following compilation of quantitative information from the literature is not intended as a recommendation but rather as a simple juxtaposition. Additional information concerning the context in which the quoted data were gathered is meant to help the reader with making his own mind and getting an idea of the typical orders of magnitude. The respective comments may also give a hint whether the respective data can be transferred to the specific problem of

Table 5.1: Duration of the daily time blocks

source	application	working	leisure at home	sleeping
Hartford [46]	Canada (general)	8 a.m. – 5 p.m.	5 p.m. – 10 p.m.	10 p.m. – 8 a.m.
Reiter [108]	Finland (small town)	7 a.m. – 5 p.m.	5 p.m. – 10 p.m.	10 p.m. – 7 a.m.
Yeo/Cornell [147]	USA (offices only)	9 a.m. – 5 p.m.	5 p.m. – 1 a.m.	1 a.m. – 9 a.m.

Table 5.2: Distribution of the population during work time

source	application	residential buildings	other buildings	outdoors/ commuting
Spence et al. [124]	New Zealand/City	22%	58%	20%
Coburn et al. [19]	Agrarian Society	all buildings: 20%		80%

Table 5.3: Distribution of the population during bedtime

source	application	residential buildings	other buildings	outdoors/ commuting
Spence et al. [124]	New Zealand/City	96%	3%	1%
Murakami [82]	Japan/City, 05:45 a.m.	96.6%	–	–
Coburn et al. [19]	Agrarian Society	all buildings: 94%		6%

the applying engineer and—if this should be the case—in which direction additional investigations and estimations could go. The compilation also shows how little useful data is actually available for this seemingly self-evident theme. Still, in combination with the above-described methodology, it should be possible to obtain reasonable results.

Three authors give practical numbers for the duration of the respective daily time blocks (Table 5.1). At weekends and public holidays the work-time block has to be added to the leisure time block. The numbers given by Yeo & Cornell [147] differ from the rest because they do not express the behaviour of the general population, but aim at modelling the presence in office buildings exclusively. In terms of the previous pages, this corresponds to object-based rather than to distribution-based thinking. 17.00 to 01.00 hours is the time where a certain part of the staff works overtime whereas, practically nobody can be expected to work after 01.00 hours.

In the case of the *object-based* approach, basic data have to be collected anew for every case which is not the case for other numbers. Numbers on average household size or passengers per vehicle (relevant e.g. for tunnels) can be transferred to similar objects at least within one country. Although such data are generally easy to obtain, Tables 5.7 and 5.8 give some general idea. Passenger numbers for cars and minivans in Table 5.8 appear to be conservatively estimated.

Table 5.4: Conditional distribution of the population during evening rush hour

source	application	condition: part of ...	residential buildings	other buildings	outdoors/ commuting
		nighttime residential pop.	35%	10%	5%
		daytime residential pop.	–	–	–
		commuters	–	–	100%
Hazus [134]	USA	commercial sector	0%	49%	1%
		industrial sector	0%	45%	5%
		age ≤ 16	–	–	–
		college students	0%	10%	0%

Table 5.5: Conditional distribution of the population during work time

source	application	condition: part of ...	residential buildings	other bldg.	outdoors/ commuting
		age 0 – 6	95%	0%	5%
Murakami [81]	Armenia/	age 7 – 16	0%	95%	5%
	Winter[a]	age 16 – 60(m)/55(f)	$\frac{1}{4} \cdot 95\%$	$\frac{3}{4} \cdot 95\%$	5%
		age 60+(m) / 55+(f)	95%	0%	5%
Hartford [46]	Canada	residential pop.	20%	–	–
Coburn	cities	working/studying	–	85%	–
et al. [19]		residential pop.	47%	–	–
		nighttime residential pop.	–	–	–
		daytime residential pop.	52.5%	16%	31.5%
		commuters	–	–	–
Hazus[b] [134]	USA	commercial sector	0%	97%	3%
		industrial sector	0%	72%	18%
		age ≤ 16	0%	72%	18%
		college students	0%	80%	20%

[a] Only a small fraction of population outdoors due to winterly temperatures

[b] Horizontal sums do not necessarily amount to 100%, because each person belongs to several population groups at a time (see Appendix C.1)

Table 5.6: Conditional distribution of the population during bedtime

source	application	condition: part of ...	residential buildings	other buildings	outdoors/ commuting
Coburn et al. [19]	cities	working/studying	–	12%	–
		residential pop.	78%	–	–
HAZUS [134]	USA	nighttime residential pop.	99%	0%[a]	0.1%
		daytime residential pop.	–	–	–
		commuters	–	–	–
		commercial sector	0%	2%	0.002%
		industrial sector	0%	10%	0.01%
		age ≤ 16	–	–	–
		college students	–	–	–

[a]Note, that it is 0%, not 0.9%: The 'missing' 0.9% is to be found in the industrial sector!

Table 5.7: Average household size in numbers of persons

source	application	persons
Murakami [81]	Armenia	4.2
Murakami [82]	Japan/city	2.6
Coburn et al. [19]	Europe/city	2.5
	Iran, Eastern Turkey / countryside	8

Table 5.8: Average number of passengers in road vehicles

source	application	type of vehicle	persons
Persson [95]	Sweden	car	3
		minivan	5
		bus	20
		heavy goods vehicle	1

5.3 Probability of Successful Escape

5.3.1 Methodology

Most authors dealing with the quantification of loss of life embrace the phenomena of warning and escaping in some way, either explicitly or implicitly. Their efforts to represent these effects differ considerably, which is little surprising given the multitude of equally differing applications. In part, the various approaches even appear to be mutually exclusive or even in clear contradiction to one another. This section seeks to demonstrate the possibility of establishing a model which is sufficiently general and comprehensive in order to let all previous approaches appear as special cases of the latter, while not drifting into vagueness at the same time. It should be mentioned already at this point that applying such a generalised approach to a specific problem usually means neglecting one or the other sub-item, which basically means getting closer again to one of the already existing case-specific approaches.

In the context of a comprehensive methodology, the term ‘warning’ needs to be understood in a most general way. It can refer to a warning (or pre-warning) in the conventional sense: Professionals or casual observers give notice of an impending disaster to the population which, otherwise would have been unsuspectingly threatened. In this case, one can speak of a *direct* form of warning. At the same time we propose to equally refer to the *indirect* self-announcement of a danger as a form of warning. As an example, one may consider a flood wave thundering down a valley, which can be heard and seen a few minutes prior to arrival [74] or the beginning of an earthquake which usually antecedes building collapse by several seconds [41].

Whether a person is successful in escaping or not depends not only on the properties of the escape path, but also on the type, the time, the perception and first of all, the actual existence of a warning. The existence of a warning and that of its perception by the endangered population are expressed by probabilities. If warning is defined as above in the most general way, it can be said that escape Q principally requires Warning W as a precondition:

$$P_Q = P(Q) = P(W) \cdot P(Q|W) \tag{5.3}$$

with

$$\begin{aligned} P(W) &= P(W_0) \cdot P(W_{PRC}|W_0) \cdot P(W_{DC}|W_0 \cap W_{PRC}) \\ &= P(W_0) \cdot P(W_{prc}) \cdot P(W_{dc}) \end{aligned} \tag{5.4}$$

Here, W_0 is the fact of a warning, while W_{prc} stands for perception of a warning and W_{dc} for the decision to flee in the face of a perceived warning. The latter decision is not self-understood. Often, people are not aware of the consequences of a certain hazard (typically in the case of floodings; compare Graham’s model [45] in Appendix C.2). In other cases the attempt to escape appears pointless, e.g. for persons who are staying at a high building floor during an earthquake or who trust in the resistance of the building [41]. As for the perception itself, there are several reasons which can make the latter impossible for parts of the population: sleep, preoccupation with some activity, absence of a TV set or radio, deafness, dementia etc.

In case the condition of warning is fulfilled and a person decides to flee, he or she can only succeed, if the time span remaining after the warning event T_W (‘warning time’ in [45]) is greater than the required time span T_Q[1]:

$$P(Q|W) = P\{T_W - T_Q > 0\} \tag{5.5}$$

When modelling the different time spans as deterministic, $P(Q|W)$ can only take the values 0 and 1. For randomly distributed time spans, (5.5) emerges as a classical limit state problem whose solution can adopt any value lying in between. A few publications on the escape problem more or less follow this line of thought but do not make the reasoning explicit in the present form.

[1] T_Q includes reaction time, which, again, is the sum of perception time and decision time.)

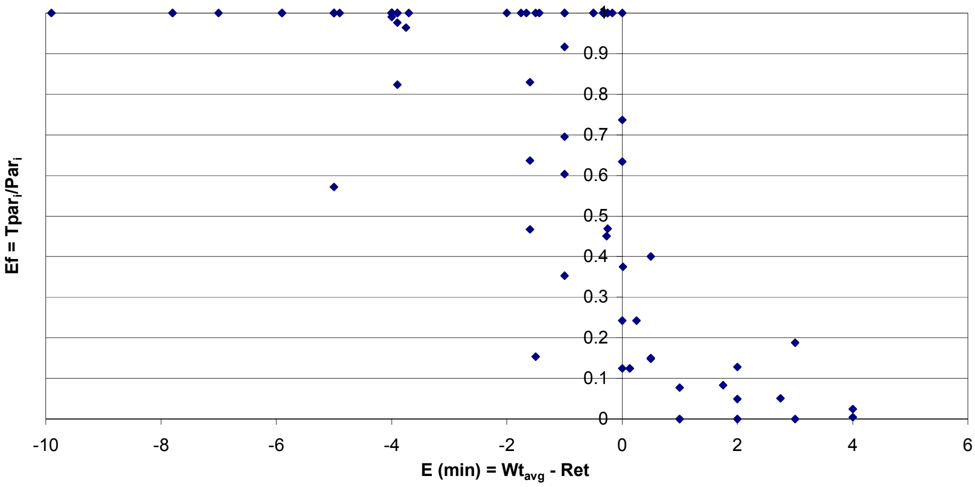

Figure 5.1: Observed rate of unsuccessful escape $(1 - P_Q)$ as a function of excess evacuation time E [minutes], from McClelland & Bowles [74]

Georgescu [41] thoroughly investigates the anatomy of the escape process in the course of an earthquake or fire and is frequently referred to in the earthquake literature. The reaction time (here: 'indecision phase') depends on the type of room in which a person is located. A person will first have to turn off the oven or cooker before leaving the kitchen as an example. The escape path within the flat and in the stairway are treated as separate units both depending on various factors (lighting etc.). In this way, the required escape time including reaction time can be calculated (deterministically). The presented information on T_W is rather vague. According to the nature of an earthquake, the interval in which available time spans can range is indicated to be quite extensive. Georgescu does not seem to want to make up the balance between required and available time in the sense of (5.5) (which however, is precisely what Coburn & Spence [18] are doing, using his data). Regarding the uncertainties with respect to the time span between the onset of an earthquake and the collapse of a building, a probabilistic calculation would suit the earthquake case best.

For McClelland & Bowles [74], the aim is not to develop a model but to analyse the probability of death in the course of dam breaks ('An important statement: This is not a model'). They calculate the excess evacuation time $E = T_W - T_Q$ and plot the rate of successful escapes against it (Figure 5.1 shows a section with $-10\,\text{min} < E < +6\,\text{min}$; each point corresponds to a flooding event with numbers of people affected ranging between a few dozen and several thousand respectively). This leads to a scatter plot roughly following an S-shape. It is important not to misinterpret E as the safety margin, i.e. as the difference between the two random variables T_W and T_Q, which would have to be randomly distributed itself. In order to obtain the latter the second moments of the two stochastic quantities would have to be considered.

Implicitly, McClelland & Bowles acknowledge the random nature of the available escape time by differentiating between individual and average warning time; however, the excess time E is the difference between two expected values, i.e. *average* warning time and *representative* evacuation time. This also explains why successful escape is possible for a negative excess time in Figure 5.1. Generally, escape is never successful for $E < -5\,\text{min}$ while always being successful for $E > +5\,\text{min}$ (data for these extreme cases do exist but are cut off in Figure 5.1). Yet, with the help of McClelland & Bowles's original data material it ought to be possible to model T_W and T_Q as random distributions. From there on one would then be able to calculate the (conditional) probability of escape $P(Q|W)$ for future events. The rates of successful escape from the historical events could serve to validate the model.

The initial discussion of concepts (see p. 80) showed that the phenomenon of warning actually consists of two separate elements: (pre)warning in the conventional sense and self-announcement of an impending disaster. According to this separation, the (unconditional) probability of successful escape equally resolves into two components:

$$\begin{aligned} P(Q) &= P(Q') + P(Q'') \cdot P(1 - P(Q')) \\ &= P(W_0') \cdot P(W_{prc}') \cdot P(W_{dc}') \cdot P(Q|W') \\ &\quad + P(W_{prc}'') \cdot P(W_{dc}'') \cdot P(Q|W'') \cdot P(1 - P(Q')) \end{aligned} \tag{5.6}$$

Here, one apostrophe denotes the first, two apostrophes the second type of warning. The expression $P(W_0'')$ is omitted in the second line of (5.6), because self-announcement of a disaster is a certain event. This of course, does not exclude the possibility of not being perceived (i.e. $P(W_{prc}'') = 0$) or the possibility that the disaster already develops its full effect at the very moment of self-announcement (i.e. $T_W'' = 0$ and therefore $P(Q|W'') = 0$). Depending on the event one or the other type of warning dominates. For earthquake-caused building collapse, the first mode is practically inexistent whereas, it outweighs the second mode for dam-break events, just as for different types of natural disasters not treated here (tsunamis, volcano eruptions etc.). Depending on the situation, one or several components of (5.6) adopt the values 0 or 1 out of principal considerations so that the equation simplifies in many cases (compare Table 5.9).

The model expressed in (5.5) and (5.6) is apt to cover all applications requiring a quantitative assessment of warning and escape. However, this only holds true under two conditions not mentioned up to here: The whole surface of the affected area needs to be struck *at once* and the disaster needs to reach its maximum intensity right away. 'At once' means that the disaster, *after* its arrival, spreads faster than a person can move. Then the time available for escape is exactly the time *until* the arrival of the disaster in the affected area whereas it is impossible after this moment to race against the disaster which is following just behind. If it were possible (i.e. if one or both assumptions are being violated), the disaster might catch the person at some later stage, given that he or she does not succeed in quitting the affected area altogether in time. If the two assumptions are justified this will be referred to as the criterion of *suddenness* being fulfilled. The criterion of suddenness exclusively relates to the way in which an event evolves exclusively but does not make any assertion about the kind or existence of an eventual warning.

The two assumptions are justified for flash floods in more or less narrow valleys (people escape to the hill side prior to the arrival of the water or not; the current does not become more violent as time passes), just as for earthquakes (the whole area is struck almost at once; the process of building collapsing lasts very short). In contrast, flood events, whose danger originates from quickly rising water levels rather than from a wall of water progressing at high velocities count as non-sudden, a view that is implicitly followed e.g. by Hartford et

Table 5.9: Overview for the application of the escape methodology (§ = chapter/section)

proba-	**—structural failure—**		**—flood after dam failure—**		**—fire—**	
bility	**earthquake**	**spontaneous**	**sudden**	**non-sudden**	**explosive**	**non-expl.**
$P(Q')$	0	0	§ 5.3.2	§ 5.5	0	0
$P(Q'')$	§ 5.3.2	0	§ 5.3.2	§ 5.5	0 / § 5.3.2	§ 5.5

al. [46]. The situation in tunnel fires is comparable (e.g. [95]): The relevant question is, which temperature or toxic gas concentration occurs at which point in time at which point and along the tunnel axis. Depending on the situation, a fleeing person succeeds or does not succeed in moving from one position to the next before conditions become lethal at the first of the two positions (and so forth).

Two assertions can be made about the methodological approach to tunnel fires and the flood type treated by Hartford et al.: The quantitative analysis proceeds in an iterative manner for successive time intervals. And: Each iteration step requires not only determining the probability of successful escape, but also that of the conditional probability of death k. In order to determine the latter, information from the following section is required (Section 5.4, *Conditional Probability of Death*). For this reason, a separate section is dedicated to the iterative approach (Section 5.5, *The Iterative Approach*).

Table 5.9 provides an overview over the numeric values of $P(Q')$ and $P(Q'')$ and respectively on the question, which section provides the necessary information for their determination.

5.3.2 Modelling and Quantification of the Model Parameters

The following pages deal with each of the parameters needed for calculating (5.5) and (5.6):

Available Escape Time T_W

The available escape time is strongly case-dependent, such that it is difficult to name absolute values.

This holds true especially for T'_W, the time available between an indirect warning (i.e. warning in the 'conventional sense', see above) and the irruption of the disaster in the affected area. From all the applications treated here, this escape mode only concerns flood events after dam breaks[2] (compare Table 5.9). Once a warning is issued by means of broadcasting, alarm sirens, loudspeakers and similar channels, it can be assumed that all persons are reached roughly at the same time (regardless of their respective perception and reaction). However, the flood wave arrives at the different sub-areas at different times, even when *suddenness* in the above-defined sense is fulfilled. The flood wave sets off at time $t = 0$ and reaches sub-area i containing the group of persons at risk PAR_i at time t_i. Thereby, it covers a distance x_i.

[2]As an exception, one might consider the automatic warning system that registers earthquakes off the Japanese coast and uses the 20 seconds prior to the arrival of the shock waves on the mainland for shutting down nuclear plants and high-speed trains. For most earthquake areas worldwide, however, the distance between populated areas and epicentre is not sufficiently great for such measures.

Warning is issued after a period T'_{ini} following dam failure. Therefore, the regarded case leads to

$$T'_W = \int_0^{x_i} \frac{dx}{v(x)} - T'_{ini} \tag{5.7}$$

with $v(x)$ denoting the velocity of the flood wave. This function needs to be determined by the laws of hydraulics[3] for the specific case. If the warning is issued before the dam failure, T'_{ini} takes a negative value. In Table C.3, Graham [45] indicates actual numbers for T'_W and T'_{ini}, respective of whether he relates the values to the moment of failure or to the moment when the flood wave arrives in the area at risk. His approach consists in allocating the data to different categories[4] instead of approximating the underlying relationship by a mathematical expression.

In order to determine T''_W for the case of flood waves, (5.7) is changed into

$$T''_W = \int\limits_{x_i - \Delta x_{sens}}^{x_i} \frac{dx}{v(x)} \tag{5.8}$$

Herein, Δx_{sens} is the distance at which a direct sensory perception (sonic or visual) of the approaching flood wave becomes possible. Obviously there is no delay in such a self-announcement, so that T''_{ini} equals zero and does not appear in (5.8). Given that Δx_{sens} is a more or less constant parameter and that v can be expected to range within certain limits, it appears admissible to name a global interval between perceptability and arrival of the flood wave of T''_W = 1 to 4 minutes , as done by McClelland [74]. For a first estimation, if more specific data are not available, it is proposed to assume T''_W as being uniformly distributed within the interval named by McClelland. It should be noted that a similar first estimation is not possible for T'_W, due to the case dependency of x_i (cancelling in (5.8)).

When regarding the various parameters from a stochastic point of view, one should be aware of the difference between x_i and the other parameters: The uncertainty of x_i depends on the spatial extent of the considered sub-area i and further in the spatial distribution of the persons at risk PAR_i within the sub-area. The larger we choose i, the larger the variance of x_i will be.

In the case of earthquake-induced building collapse and fires, only T''_W comes into effect (compare Table 5.9). Georgescu gives some—rough, but at least quantitative—indications for both applications: According to [41], the initial phase of an earthquake during which oscillations are noticeable but not yet destructive (0.001 to 0.02 g) lasts *2 to 18 seconds*, whereas the main phase spans *10 to 50 seconds* (for magnitudes between 5.5 and 8 on the Richter scale). Coburn & Spence [18] describe the phase during which the underground is

[3] The Manning-Strickler Equation found in any hydraulics handbook is the standard formula for such applications. The roughness coefficient can be taken from Karvonen et al. [61]: They indicate specific values for currents flowing through forests and groups of house, which they obtained on an experimental basis. The velocity can be remarkably high: In the course of the devastating Banqioa dam failure causing tens of thousands of deaths (China, 1975), the flood wave progressed at almost 50 km/h (approx. 15 m/s) [66].

[4] In the case of Table C.2, these are failure cause, daytime, presence of spectators at the dam site and some special conditions.

Table 5.10: Duration of earthquakes

source	phase	magnitude	duration [s]
Georgescu [41]	initial phase	not specified	2–18
	main phase	not specified	10–50
Coburn/Spence [18]	overall	5	10
		6	25
		7	60
		8	120

‘strongly’ shaking as a function of magnitude (see Table 5.10); their numbers also have the purpose of facilitating an estimation of the available time and therefore at least in part include the initial phase after Georgescu’s definition.

In any case, it can be said that an eventual collapse does occurs at the end of the initial phase at the earliest and closely after the end of the main phase at the latest[5]. The information on T''_W gained in this way is admittedly very vague. For any prediction that is meant to be only a bit more precise, site-specific earthquake statistics and object-specific investigations of the dynamics of the structure are indispensable.

In the case of earthquake-induced building fires, Georgescu names *200 seconds* as the earliest possible time until a flash-over. This value is probably realistic for other fire causes too. For non-explosive fire events however, this way of modelling would be overly simplistic and requires the iterative approach discussed in Section 5.5 (just as for non-sudden flood events). Explosive fire events usually lead to $P_Q = 0$, unless people are able to interpret eventual signs of a due explosion, e.g. after a traffic accident in which a vehicle with hazardous freight catches fire.

Required Escape Time T_Q

Calculating the time it takes for a person to escape resembles the travelling time calculation for a flood wave. In analogy to 5.7 and 5.8, there is

$$\begin{aligned} T_Q &= \int_0^{x_i} \frac{dy}{u(y)} + T_{rea} \\ &\approx \sum_{c=1}^{n} \frac{y_c}{u_c} + T_{rea} \quad \text{with} \quad \sum_{c=1}^{n} y_c = y_j \end{aligned} \tag{5.9}$$

Here, j denotes the single individual, u_c the speed at which a person moves along sub-distance c and y_j the whole distance, which has to be covered to reach a safe place. In the case of an earthquake, y_j is not necessarily the distance to the building exit, at least for those houses, in

[5]Although—regarding Table 5.10—this leads to larger available time spans for stronger earthquakes (given that a building does not collapse already at the beginning), the arising contradiction is only seeming as such. Stronger and longer-lasting earthquakes also destroy those buildings that would have resisted a weaker and shorter event for which the question of successful or unsuccessful escape would have remained meaningless.

Table 5.11: Reaction time

source	application	condition	T_{rea} [s]
Persson [95]	tunnel fires[a]	position 0–150 m behind accident site	210
		position 150–250 m behind accident site	240
		position 0–150 m behind accident site	270
SFPE [121]	fire, generally[b]	sleeping + burnt smell	100 (45 to 205)
		sleeping + burning sound (45 dBa)	< 30
		sleeping + flames (fire)	30
		sleeping + alarm (55 dBa)	13.6
		sleeping + alarm (70 dBa)	9.5
		sleeping + alarm (85 dBa)	7.4
		awake/public place + alarm bell	13
		awake/public place + coded signal	30 to 50
Georgescu [41]	earthquake	staying in living room	5
		staying in kitchen/sleeping room	10
		staying in bath/toilet	15

[a]Although perception time T_{prc} increases with the distance from the accident location, decision time T_{dc} decreases at the same time, because people have more time for reflecting on eventualities *and* can watch people from earlier sections escaping, which encourages them to follow suit. Therefore, the dependency of T_{rea} upon position remains small.

[b]For sleeping persons, perception time T_{prc} dominates the value of T_{rea}, whereas the effect of decision time T_{dc} is small. In public places, such as shopping centers or underground station, the respective weights are distributed exactly the other way.

which staircases can be considered as safe havens [41]. T_{rea} is the reaction or, more concisely, *pre-movement* time; it includes the times required for the perception of the warning (*prc*) and the decision to flee (*dc*):

$$T_{rea} = T_{prc} + T_{dc} \tag{5.10}$$

Furthermore, it can comprise the time of all other actions an individual needs to undertake before beginning the escape [41, 46, 74] (e.g. turning off the cooker etc.). Apparently, the values as given in Table 5.11 are only relevant, if an individual perceives the warning and decides to flee, respectively. Both events are quantified in terms of probabilities later on in the text. When technical alarm devices are involved as with the SFPE values in Table 5.11, it is not only the type of device (bell, siren, automatic loudspeaker voice) that matters, but also the training of the involved persons (see [10, 121]). In the case of an immediately approaching flood wave, which is not included in the table, McClelland's [74] findings on the overall required escape time T_Q can help. While, at daytime, T_Q is equal to 1 to 2 minutes, the nighttime value ranges between 2 and 4 minutes. The difference of approximately 90 seconds can be attributed to reaction time. These numbers are based on a typical small-valley situation where the flood wave is no wider than about 350 m.

The values for y_j need to be determined for the specific case just as those for x_i in 5.7. When combining the escape paths y_j of all individuals PAR_i present in sub-area i into one single quantity y_i, the latter one is randomly distributed due to the spatial distribution of

Table 5.12: Base speeds from [121]

sub-distance type	**riser [mm]**	**tread [mm]**	u_0 **[m/s]**
aisle	–	–	1.40
stair	190	255	1.00
	270	280	1.08
	165	305	1.16
	165	330	1.23

the individuals (the same argumentation was applied above to the stochastic nature of x_i). In the case of building collapse, one house or a group of identically designed houses appears to be a reasonable size for a sub-area. For flood events after dam failure, larger sub-areas are more appropriate.

The *SFPE Engineering Guide on Human Behavior in Fire* [121] features a useful model for the human escaping speed u:

$$u = \begin{cases} u_0 - a \cdot u_0 \cdot \rho & \text{for} \quad \rho > 0.55 \\ 0.85 \cdot u_0 & \text{for} \quad \rho \leq 0.55 \end{cases} \tag{5.11}$$

In this formulation, u_0 is a (calculatory) base velocity [m/s], ρ is the density of the person flow [pers/m^2] and a a constant of $0.266\,\mathrm{m}^2$/pers. The consideration of ρ allows to determine the flowing speed and quantity of persons at narrow points such as doors or aisles. Since the escape speed depends on whether a person is indoors or outdoors, in an aisle or in a staircase etc., subdivision of y_j into sub-distances y_c has to follow these qualitative features. Table 5.12 reproduces some values for u_0 from [121] (which also contains values applicable to persons with different types of handicaps). Some values for u from other sources are listed in Table 5.13. Georgescu [41] introduces additional correction factors to be multiplied with T_Q or one of its sub-steps (Table 5.14). When using these factors, it should be considered that factor r_3 expresses the effect of high person flow densities which is also covered by (5.11). Therefore, these two models should not be combined in one calculation without care.

Apart from the values in Tables 5.12 and 5.13, it can be reasonably assumed that a healthy adult can easily maintain an escape speed of 2 to 3 m/s during several minutes given that there are no major obstacles, as it is the case in open terrain. However, this assumption holds only if the considered individuals are not occupied with helping others—aged, handicapped or injured persons and children—with their escape.

The Actual Probability of a Warning $P(W_0)$

The actual probability of a warning expresses the likelihood that there is actually going to be a warning. As outlined in Section 5.1, this number is only a matter of discussion when *indirect* warning W_0' is concerned; in the case of direct warning, i.e. self-announcement of the impending disaster, warning is a certain event ($P(W_0'') = 1$). Of course, self-announcement may arrive so late that available escape time T_W'' reduces to zero but this is a separate issue.

Of all the disaster types discussed here, it only makes sense to talk of indirect warning for

Table 5.13: Escaping speed

source	application	condition	u [m/s]
Persson [95]	tunnel fires	–	0.70
PIARC[a], cited after [95]	–	–	0.5 to 1.5
		${C_s}^b < 0.2$	1.2
Jin cited after [56]	fire	$0.2 \leq C_s < 0.5$	$1.2 - 3 \cdot C_s$
		$C_s \geq 0.5$	0.3
		aisle/adults	0.3 to 0.5
		aisle/children and old	0.3
Georgescu [41]	earthquake:	stair/teenagers	0.3
	$0 < a < 2\,\mathrm{m/s^2}$	stair/adults	0.2
		stair/children and old	0.1
Melinek/Booth cited after [41]	–	aisle	1.3
		stair	0.5

[a]PIARC: World Road Association based in La Defense, France
[b]Extinction coefficient [m^{-1}]: Optical term, expresses the fraction of light lost per distance unit in a semi-transparent medium due to absorption and scattering

Table 5.14: Correction factors proposed by Georgescu [41]

factor	to be multiplied with	condition	value
r_1	time in staircase	no/one/several landings per storey	1.0 / 1.1 / 1.2
r_2	whole time in building	day / night + illumination / night + no illum.	1.0 / 1.1 / 1.3
r_3	whole time in building	1–4/5–8/> 8 apartments per staircase	1.0 / 1.1 / 1.2

the case of dam and dyke failures, apart from a few special cases (compare footnote 2, p. 83). Unfortunately, the literature does not provide any recommendations on the estimation of P'_W. Graham [45] implicitly deals with the matter, when he introduces the category 'no warning' alongside the 'warning' category in Table C.2. However, it is hard to convert this—nevertheless quantitative—information into explicit values.

In order to determine P'_W, there is obviously no way around an individual investigation of the specific case. This involves information on permanent or temporary presence of staff and nearby living resident population as well as on the type of warning (automatic, human or combined) and the means by which the warning is conveyed to the population at risk (radio transmission, broadcasting, telephone, sirens etc.).

The Probability of Perceiving a Given Warning $P(W_{prc})$

In order for a warning to be perceived, the involved visual or acoustic signs have to act with sufficient intensity upon the persons concerned. Especially with persons asleep, the perception of an issued warning in not self-understood. The *SFPE Guide on Human Behavior in Fire* [121] contains some tangible indications which can in part be used in the context of other event types specifically, where the efficiency of technical alarm devices is addressed.

Table 5.15: Probability of awaking due to sensory stimuli

source	warning type	condition	$P(W_{prc})$
Murakami [82]	direct	earthquake of hazardous intensity	1.00
DLR [40]	direct	noise <33 dBA	0
SFPE [121]	direct	smoke odour / male subject	0.29 (!)
		smoke odour / female subject	0.80
		crackling sound (42–48 dBA)	0.90
		shuffling sound (42–48 dBA)	0.83
		light (flames, 1–5 lux)	0.49
SFPE [121]	indirect	acoustic alarm (55–60 dBA) / child (6–17 yrs)	0.15 (!)
		acoustic alarm (55–60 dBA) / adult	0.80–1.00
		acoustic alarm (70–85 dBA) / adult	1.00
		strobe flashes: deaf/hearing subject	0.86 / 0.82
		incand. light flashes: deaf/hearing subject	0.92 / 0.59

Unfortunately, there was no information available as far as the perception probability of persons awake is concerned.

The probability of interpreting an already perceived warning as such is included in the probability of deciding to flee $P(W_{dc})$ in the following thematic item.

A few details stick out in Table 5.15: Male individuals asleep react weakly to smoke odour, while children asleep are almost deaf to any sort of noise. The deaf persons who have to rely more strongly on visual stimulations than other persons react much more strongly to incandescant light while stroboscope light is apparently so unfamiliar to everybody that deafness does not make a difference here.

The Probability of Deciding to Escape $P(W_{dc})$

There are several reasons why a person would *not* decide to flee after having perceived a warning (given free choice and absence of physical hindrances): In some cases, a person does not understand the seriousness of a warning and underestimates the threat whereas, other persons are unable to cope psychologically and therefore behave in a plainly irrational manner. Finally, there is the case in which a person perfectly understands the degree of threat but deliberately chooses not to react. This last case is mentioned by Georgescu [41] with respect to earthquake-prone areas where people have sufficient experience of past events and deem their homes to be safe. Numbers for $P(W_{dc})$ are said to range between 25 and 60% for the USA and Japan, whereas Georgescu's numbers indicate a drop from 60 to 10% for the case of Romania due to constructional progress in the second half of the twentieth century.

Graham [45] regards the correct appreciation of a hazard by the population as one of the main factors with respect to flood consequences. However, quantitative information on $P(W_{dc})$ is only given implicitly once again (in Tables C.2 and C.3). McClelland & Bowles [74] document cases of irrational behaviour in floods as well as reactions based on a lack of knowledge. People paralysed by shock, denying escape when rising flood waters are still shallow are an example for the first case, others staying in their cars because of not knowing the dangers of such behaviour correspond to the second one. Lack of correct appreciation can also account in case of the Vajont dam break in 1963 (1269 fatalities) equally described in [74], where 158 already evacuated persons chose to slip past police patrols back into their village.

Several studies on human behaviour in fires [48, 95, 121] show that the decision to flee is largely a group-dynamic phenomenon: Many people refuse to undertake anything despite having heard an alarm go off. Boer [10] undertook a field study with even more striking results: After a 'burning' HGV stopped in a tunnel and blocked the passage for following cars, it repeatedly took several minutes (!) before anybody would react and leave their cars. As soon as one person started escaping or instructions were issued via loudspeakers, others followed suit. Therefore a major role is played, as if there are trained personnel in public places and are able to trigger the escape process.

5.4 Conditional Probability of Death

5.4.1 Methodology

If a person is present on the occasion of a failure event and fails to escape from the danger zone in time, his or her probability of death shall be denoted by the factor $k = P(D|\overline{Q})$. However, death D is not only conditional on failure F and on the non-occurrence of successful escape $\overline{Q}$, but also happens to be a function of the extent or severity A of the event:

$$k = k(A|F) \tag{5.12}$$

Failure F is equivalent to the case $A \geq A_{cr}$, where A_{cr} denotes the critical and A the actual extent of an event. Inserting in (4.5) leads to

$$r = \lambda\, P_F = \lambda\, P\{A > A_{cr}\} \tag{5.13}$$

In fact, the common dependency of P_F and k upon A requires closer attention. Some thoughts on this issue are presented in Section 5.6.

There are different ways of characterising the extent of an event. Three hierarchical concepts are used in earthquake engineering which are of equal use in other fields of application:

- Magnitude M describes the global extent of an event and is usually chracterised by the total amount of energy released (earthquake, fire). It is conceivable to apply this measure equally to flood events. In this case it is left open whether M should be applied to the energy or the amount of water released and whether either of these numbers should be taken per time unit or as an overall sum.

- Intensity I quantifies the locally acting impacts and therefore differs between different sub-areas in a large-scale event. For earthquakes, I can be e.g. the local acceleration spectrum and for a flood it would typically be the combined value of water depth and water velocity whereas, for fires it would comprise both local temperatures and toxic gas concentrations.

- The degree of damage D describes the damages a technical object suffers due to an impact of intensity I.

Formally, the above reasoning can be summarised as

$$\begin{cases} A & = \{M, I, D\} \\ I & = f(M) \\ D & = f(I) = f(I(M)) \end{cases} \tag{5.14}$$

In some cases it is appropriate to dissolve k into a 'net k-value' k_0 and further factors z_i:

$$k(A) = k_0(A) \cdot z_1(A) \cdot z_2(A) \cdot \ldots \tag{5.15}$$

This allows separating the actually lethal mechanism from other effects. As an example one may consider the partial collapse factor in Section 5.4.2. These sub-parameters of k can equally depend on the event's extent A.

Regardless of whether k is subdivided in the sense of (5.15) or not, there are different possibilities of dealing with dependencies, be it on A or on other influences. Here, the question is not about 'right' or 'wrong', but about which approach suits a given application best:

- One possibility is to represent a dependency *discretely by means of a table*. This approach is empirical as a matter of principle and has the advantage of not tempting to draw false conclusions when data and knowledge are sparse. In contrast, statistical inter- and extrapolation formulae may prove problematic especially, if the phenomena leading to the described dependency are unknown. This argumentation is described in more detail in the context of Graham's approach (Appendix C.2). Table C.2 shows that it is possible to display even multiple dependencies in a discrete way by introducing several categories to a table. In this way, the precise effect of each single influence remains unknown as it affects the overall value only in an implicit way.
- The other possibility is to formulate the dependency *as a mathematical function* of one or several influences. The function itself can be either continuous or discrete. This approach works both for physically and empirically derived relationships. In the second case, the restrictions form the preceding paragraph hold.

Some reports treat the product of probability of no successful escape $(1 - P_Q)$ and conditional probability of death k as one single quantity. This number can be referred to as *lethality* (after [81]) or as *unconditional probability of death* P_D (unconditional on $\overline{Q}$, only conditional on F). Although merging these two separate phenomena into one is contradictory to the general methodology expressed in (5.1), the approach cannot be completely ignored. A few of the reports cited in the following sections containing valuable mortality data express them as P_D instead of using $k = P(D|\overline{Q})$.

In contrast to the two previously discussed quantities N_{PAR} and P_Q, it is not sensible to draw comparisons between different event types in the case of k. The physical and physiological processes leading to fatalities are fundamentally different for each hazard. Therefore, the following sections are structured in accordance with the event types under consideration: Building collapse due to earthquakes, flooding due to dam failure and tunnel fire.

5.4.2 Building Collapse due to Earthquakes

Comparatively few studies have been performed on occupants' probability of losing their lives in building collapse. The most notable efforts were undertaken by Coburn & Spence [18, 19, 124] and by Murakami [81, 82]. The methodology of Coburn & Spence is the one that reaches out further and is therefore the main foundation for the considerations in the present section. As one of their major results, they identify the partial collapse factor z_{pc}[6] as a key phenomenon that can clearly be separated from the net k_0-value. Furthermore, the methodology differentiates between the probability of immediate death k_0' and the probability of dying later on under the rubble from not being rescued in time, k_0''. In that case, the typical death causes are (untreated) injuries, hypothermia and dehydration. Partial collapse factor and the two death modes lead to the expression

$$\begin{aligned} k &= z_{pc} k_0 \\ &= z_{pc}(k_0' + k_0''(1 - k_0')) \end{aligned} \tag{5.16}$$

with

$$z_{pc} = z_{pc}(I) = \frac{\Delta V(I)}{V_0} \tag{5.17}$$

ΔV denotes a building's absolute loss in volume that depends on the earthquake intensity I and some other factors. A loss of volume will only occur if the damage exceeds the critical level of damage D_{cr}[7]. V_0 is the original volume. Table 5.16 showes some values for z_{pc} from the literature. For buildings surpassing the maximum five floors regarded in the table, z_{pc} values tend towards 1, as the amount of released energy becomes greater with increasing building height.

Immediate and follow-up death probabilities k_0' and k_0'' depend upon a vast number of factors. Data do exist for the effect of the following of influences:

- Construction type: Concerns both construction material and and design; strong dependency on local building traditions
- Number of floors
- Sequence of events in the course of failure

[6] If multiplied with the complementary probability of successful escape, it corresponds to the *probability of entrapment* (denoted by factor M3 in [18, 19], see Appendix C.3):

$$P_{trap} = (1 - P_Q) \cdot z_{pc}$$

[7] D_{cr} is defined e.g. by the 'Cambridge Definition' in [19]: 'More than one wall collapsed or more than half of the roof dislodged or failure of structural members to allow fall of roof or slab'.

Table 5.16: Partial collapse factor

source	construction type	condition	z_{pc} [a]
Spence et al. [124]	timber (1 floor)	–	*0.03*
Coburn et al. [19]	masonry (≤ 3 floors)	I_{MSK} = VII	*0.05*
		I_{MSK} = VIII	*0.40*
		I_{MSK} = IX	*0.80*
		I_{MSK} = X	*1.00*
	RC frame (3–5 floors)	bottom-up collapse	0.75
		top-down collapse	0.50
		pounding of 2 blds.	0.30
		overturning	0.75
Murakami [81]	RC (multi-storey, Ex-USSR)	$I_{MSK} \geq$ VIII	*0.50*
	stone masonry (one-storey, Ex-USSR)	$I_{MSK} \geq$ VIII	*1.00*

[a] Values in italics were originally given as P_{trap} values in [19, 81] and have been converted by the author according to Footnote 6. Thereby, the assumption from [19] was followed, according to which all persons on the ground floor, half of the persons on the first floor and none of the persons on the second floor and higher are able to escape during an earthquake.

Table 5.17: Distribution of the population during work time

source	type of building	region	parameter	value
Coburn et al. [19]	masonry	worldwide	k'_0	0.20
	reinforced concrete			0.40
Spence et al. [124]	masonry (2–3 floors)	New Zealand	k_0	0.175
	RC frame (2–3 floors)			0.21
	RC shear wall (2–3 fl.)			0.10
	steel frame (2–3 fl.)			0.16
	timber (1 floor)			0.006
Murakami [81]	RC slab (multi-storey)	Armenia	k	0.48
	RC frame (multi-storey)		k	0.73
	stone masonry (1 floor)		k	0.64
	masonry (1 floor)		P_D	0.20
Murakami [82]	one-family house (timber)	Japan	P_D	0.036
	tenement (timber)			0.058
	apartment (not timber)			0.0058
Durkin & Murakami [27]	RC (8 floors)	Mexico	P_D	0.47
	RC (5 floors)	El Salvador	P_D	0.21
HAZUS [134]	light steel and timber frames	USA	P_D	0.05
	other types			0.10

Table 5.18: Post-collapse mortality (from Coburn et al. [19])

condition	k_0'' **for masonry**	k_0'' **for RC**
community incapacitated by high entrapment (and fatality) rate	0.95	1.00
community capable of organising rescue activities	0.60	0.90
community + emergency squads after 12 hours	0.50	0.80
community + emergency squads + SAR experts after 36 hours	0.45	0.70

Table 5.17 displays some numbers for $k_0' \in k_0 \in k \in P_D$. The values show a strong dependency upon the factors listed above. As a general tendency, it can be said that persons in lower and more lightly built houses are less likely to succumb to a collapse than those in higher and more heavily built houses. Japanese buildings are an exception which seems to be due to the heavy roof constructions topping the traditionally lightly built houses. Generally, it should be noted that building traditions and standards differ from country to country. On the other hand, data can be transferred between countries with a common cultural background, so that values for New Zealand houses can be assumed to apply equally to other Commonwealth countries such as the U.K. or Australia. At first, it would seem that collapses cause fewer fatalities in Japan or the U.S. This is a statistical effect and has to do with the use of unconditional probabilities P_D instead conditional probabilities k. In addition, the tendency to build light and mostly single-storey houses in Japan and the U.S. can explain some of the difference. When using the approximative relation on escape probabilities in [19][8] in order to convert k values into P_D values, New Zealand numbers appear to be in good accordance with Japanese and U.S. data.

As Durkin & Murakami [27] outline, total collapse (i.e. $z_{pc} = 1$) does not lead to the death of every person that has failed to escape, not even for the case of the collapsed eight-storey house they analyse in their example.

As for post-collapse mortality k_0'', Table 5.18 displays some values. Similarly to the escape probability in (5.5), it is basically determined by the balance between actual and critical times until rescue. The critical time period is mainly influenced by the degree and type of injury as well as by the climatic conditions. Hot weather accelerates the dehydration process in the human body, whereas cold weather causes hypothermia. The actual time until being rescued depends on the availability measures. According to [19], it is the unharmed and untrapped part of the population which carries out the predominant number of successful rescues. This assertion is confirmed by what happened in the Kobe earthquake of 1995 [82]: Out of 14.000 persons rescued alive, 12.500 cases were due to the untrapped population, 1200 due to the fire brigade and several dozens due to civil defense units. Consequently, a high rate of entrapment has the strong negative impact upon k_0'' indicated in Table 5.18. If more than 50% of the population are entrapped, this incapacitates the remaining population not only due to lack of manpower but also due to psychological and social reasons.

[8] See footnote in Table 5.16.

5.4.3 Flooding after Dam Failure

In case of dam or dyke failure the retained water floods the area located downstream with all possible consequences for persons present. Generally, one needs to distinguish between persons at risk indoors and outdoors.

The magnitude of a flooding event describes its overall extent (severity) and can be expressed by the total amount of water or by the amount of water per time unit. This view applies to both the hydrological event triggering dam or dyke failure (precipitation, storm surge etc.) and to the hydrological consequences of the failure itself. However, when assessing the actual impact upon persons and structures, it is the local extent (severity), i.e. intensity which counts. It is characterised by the *hydraulic* conditions which vary from location to location within the flooded area. Here, the product of water velocity v and water depth d—also referred to as 'product number' vd—is a frequently used measure.

Persons Located Outdoors

When a person does not succeed in escaping (Chapter 5.3), it is the same as saying that he or she has been outrun by the advancing flood. However, being caught does not automatically mean losing firm contact with the ground. If ground contact is maintained, the individual concerned can still wade into safety or wait until water levels have fallen again (in case of a flash flood). When z_{su} is the probability of losing firm ground contact and being suspended in the water, the net value k_0 expresses the probability of death conditional on having lost firm ground contact:

$$k = z_{su}(I) \cdot k_0(I) \tag{5.18}$$

Two research groups have performed experiments on the stability of humans in water currents, a U.S. team around Abt in 1989 (cited after [61, 66]) and a Finnish team around Karvonen [61] in 1999. Generally, the U.S. values are more favorable than the Finnish ones. Karvonen et al. interprets the difference mainly as a consequence of the clothing the test subjects were wearing. In the U.S. tests persons were dressed in everyday clothes, whereas the Finnish test persons were wearing protective clothing, which means that a greater surface is exposed to the current, while air enclosures lead to a higher buoyancy at the same time. Both research groups propose a deterministic human stability model. Since the Finnish experiment was performed more recently than the American, Karvonen et al. based their model on both experiments. However, the scattering observed in both tests speaks in favour of a probabilistic model, as the one that Lind & Hartford [66] established based on the data of Abt et al.:

$$z_{su} = P\{vd \geq R_0\sqrt{m_j}\} \tag{5.19}$$

Herein, v denotes the water speed [m/s], d is the water depth [m] and m_j the body mass [kg] of the individual j. The general factor R_0 [$\mathrm{kg}^{-1/2}\,\mathrm{m}^2\,\mathrm{s}^{-1}$] is normally distributed with $\mathsf{N}(0.10, 0.018)$. These values are possibly on the optimistic side, because unsteady flow conditions and floating debris have to be expected under real life conditions [61]. According to (5.19), it is possible to maintain a firm ground contact for water speeds below approximately

0.5 m/s, even if water depth exceeds body height. Therefore, a slight modification shall be proposed in this place:

$$z_{su} = P\left\{(vd \geq R_0\sqrt{m_j}) \cup (d \geq l_j - 0.15)\right\} \tag{5.20}$$

The body height of individual j is given as l_j [m]. This reasoning fits well with the observation made by McClelland & Bowles [74], which is fatalities are rare as long as people wading in still water manage to keep their heads above the water surface. For several individuals, m and l can be modelled as random distributions. Given that the individuals are unlikely to occupy the same location (as well as for hydraulic model uncertainties), v and d equally have to be seen as randomly distributed.

As for the determination of the net probability of death k_0, there unfortunately doesn't exist any quantitative information. The qualitative information available can be hard to transform into actual numbers because some factors can have the opposite effect depending on the circumstances. Floating debris is a main threat in fast flowing currents whereas, it can be the only rescue possibility in extensive and deep inundations. A value of $k_0 = 1$ is a very conservative assumption which is justified only under rapid flowing conditions ($v > 2$ m/s). For water speeds below 1 m/s it should be significantly lower than 1. However, this reasoning has to be questioned if the shore is far away and no floating items are within reach in order to serve as a raft. If a strong current carries a person away and moves him or her into deeper waters, the person's initially low probability of survival rises significantly [74] ('If people know how to swim, velocity is the killer and depth is the accomplice [i.e. the accomplice of the swimmer]'). According to the same source the death rate k is ≈ 1 for car passengers whose vehicle immerges into 1.20 m or more of water depth.

In the case of *slowly rising* floods, which do not fulfill the suddenness criterion from Section 5.3.1, the probability of successful escape P_Q and the conditional probability of death k need to be determined *iteratively*, as described in Section 5.5.

Persons Located Indoors

For persons located indoors, determining the probability of death in case of a flooding proves more complex. Generally speaking, there are three different ways of getting killed for this group of individuals:

- The *building collapses*, before occupants have found time to escape. In this case, the dangers of being slain by the collapsing structure are the same as with earthquake-induces collapse (Section 5.4.2). To add to it, people who are trapped alive by the rubble will drown ($k_0 = 1 \;\Rightarrow\; k = z_{pc}$ according to (5.16)). Informations on critical water speeds for building collapse is provided in [140].
- *Water rises higher than the highest floor* or the roof top before occupants have found time to escape. In this case, people will inevitably drown. If a person does not escape from a lower floor in time, even partial flooding will cause death ($k = 1$).
- A person escapes from the building in time, i.e. before any of the above mentioned scenarios have evolved, but *drowns outside the building* because of the prevailing hydraulic

conditions. As for determinating the probability of death in this case, it corresponds to that of a person outdoors according to (5.20).

In the last case, it is important to correct the probability of death by the probability of having succeeded in fleeing from within the house.

5.4.4 Tunnel Fire

In tunnel fires, it is important to differentiate between explosive and non-explosive events. It is hardly surprising that explosive events are categorised as sudden in the sense of Section 5.3.1, while non-explosive ones belong to the category of non-sudden events requiring iterative calculation. The term 'fire' shall be understood in the widest possible sense in order to include accidents with explosive and non-chemical phenomena such as BLEVE (see below). The following thoughts on non-explosive fire events, i.e. 'fire' in a conventional sense, are equally valid for tunnel and building fires.

Explosive Events

Apart from accidents with blasting agents, there are two types of explosion which can occur in transportation of liquid and gaseous goods: Vapour cloud explosions (VCE) and boiling liquid expanding vapour explosion (BLEVE). BLEVE is a physical phenomenon that does not necessarily require or involve a chemical reaction such as fire. If it sets free chemical energy, it is referred to as *hot*, else as *cold* BLEVE.

The released energy is indicated in kilograms of TNT equivalent. As a first step [95], the charge density CD [kg/m^3] is calculated from tunnel section A [m^2], distance x_j [m] between explosion and individual j and the TNT equivalent m_{TNT}:

$$CD = \frac{m_{TNT}}{A \cdot x_j} \tag{5.21}$$

From there, Persson [95] estimates local peak overpressure p_s [Pa] as

$$p_s = 10000 \cdot (CD^{0.852} + 16.63) \tag{5.22}$$

According to Merx [76], the impulse is calculated as

$$i_s = \frac{p_s \cdot T_p}{2} \tag{5.23}$$

Here, T_p is the phase duration quantified by Merx as 0.05 s in his numeric example. This allows drawing a conclusion about the typical order of magnitude of this parameter. Persson [95] proposes using a constant value of 0.04 s as an approximation, but skips the factor $\frac{1}{2}$ for some unnamed reason.

Figure 5.2 indicates the conditional probability of survival, i.e. $(1 - k)$, for three different death causes. These include lung damage, skull-base fracture and impact of the whole body.

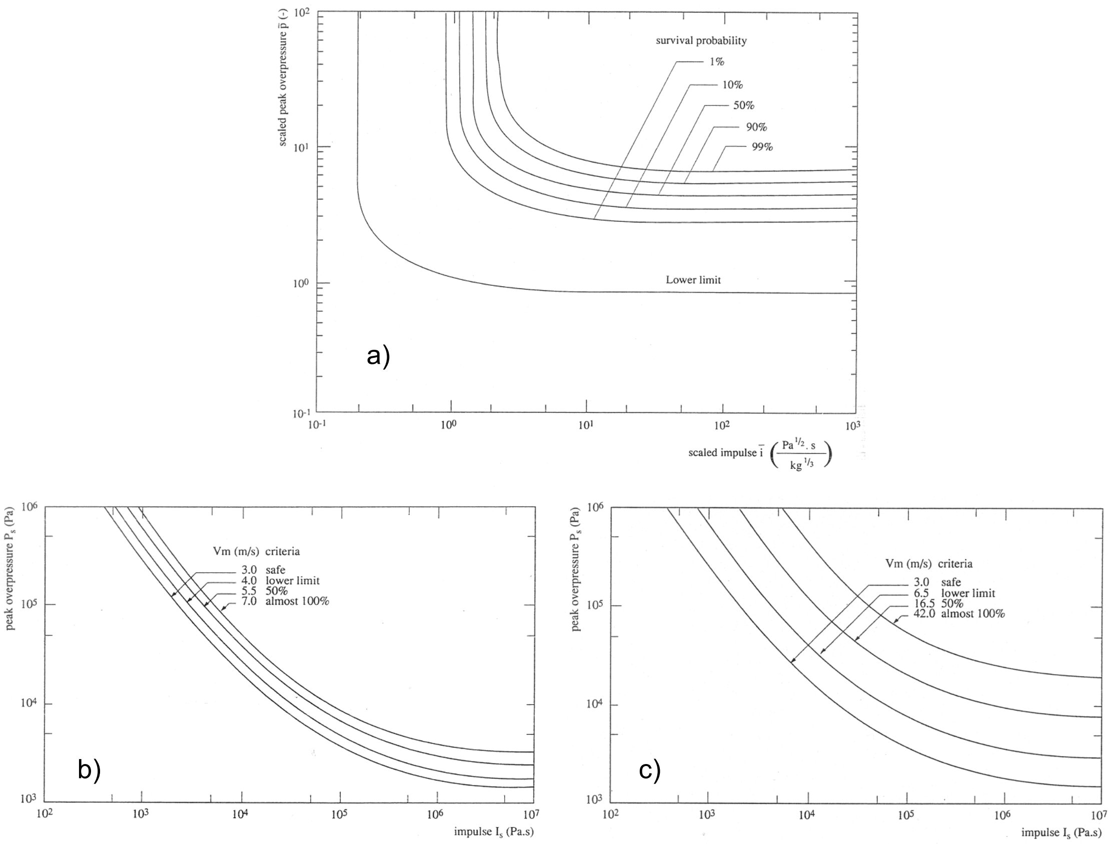

Figure 5.2: Probability of survival $(1-k)$ as function of p_s and i_s: a) for lung damage; b) for skull-base fracture; c) for impact of the whole body (from [76])

Note that in the first case, p_s and i_s are replaced by scaled values, i.e. $p_{se} = p_s/p_0$ and $i_{se} = i/(p_0^{1/2} \cdot m_j^{1/3})$. Herein, m_j is the body mass of individual j.

Non-Explosive Events

When a fire does not involve any explosive events, it is the released substances and the prevailing temperatures, which pose the main threat to human life. Departing from the origin of the fire, gas concentrations and temperatures develop over time and space. Depending on their initial position and the speed at which the life-threatening zone progresses, occupants have a chance of running away. Persons who are already within the danger zone may also succeed in escaping, because lethal effects require a minimum exposure time in order to unfold. The measure for these effects is called the *fractional effective dose* (FED). It is defined as such that an effect will occur if the *actual* dose FED reaches the *critical* dose, which equals 1 per definition (i.e. $FED_{cr} = 1$). Depending on the effect, a difference is made between the fractional incapacitating dose (FID) and the fractional lethal dose (FLD). As a conservative assumption, one can use $FLD = FID$, implying that anybody who has been incapacitated is bound to die [95]. Unless rescue forces are available quickly, this assumption is very realistic.

For the determination of *FID*, several European and North American sources [95, 113, 122, 148] use Purser's model, which shall be presented in the version of Persson [95], apart from some minor adaptions:

The fractional incapacitating dose is made up by the partial effects of CO, CO_2 and lack of O_2. Among these effects, carbon monoxide (CO) is the actual poison compromising the oxygen transport capacity of red blood cells. Carbon dioxide (CO_2) amplifies this effect by provoking hyperventilation, if concentrations are sufficiently high. Besides, the formation of CO and CO_2 is oxygen consuming, which further exacerbates the situation for human beings:

$$FID_{gas} = \int_0^{t_{\max}} fid_{CO}(t) \cdot Z_{CO_2}(t)dt + \int_0^{t_{\max}} fid_{O_2}(t)dt \tag{5.24}$$

Here, FID_q and fid_q denote the fractional incapacitating dose of impact q and its first derivative after time respectively. Unconsciousness occurs when $FID_q \geq 1$. The parameter Z_{CO_2} denotes the amplificatory effect of CO_2 upon the toxic effects of CO. The partial effects are determined as follows:

$$FID_{CO} = \int_0^{t_{\max}} \underbrace{\frac{B}{100D} x_{CO}(t)^{1.036}}_{fid_{CO}(t)} dt \tag{5.25}$$

$$Z_{CO_2}(t) = \frac{1}{7.1}\exp[0.1903 x_{CO_2}(t) + 2.0004] \tag{5.26}$$

$$FID_{O_2} = \int_0^{t_{\max}} \underbrace{\frac{1}{\exp[8.13 - 0.54(20.9 - x_{O_2}(t))]}}_{fid_{O_2}(t)} dt \tag{5.27}$$

Herein, x_q denotes the air concentration of gas q [ppm]. In order to calculate the respective concentrations as functions of time, space and characteristics of the specific fire event, there are different one- and three-dimensional models. Some of them are based on simple approximative relations, while others involve extensive numerical simulations [95, 113, 121, 122]. The factor B represents the physical activity of the subject ($B = 8.30 \cdot 10^{-4}$ for light activity), while D denotes the critical fraction of carboxyhemoglobin compounds (COHb) relative to the original number of hemoglobin molecules in the blood. COHb originates from a reaction between CO and hemoglobin and inhibits the oxygen transportation ability of the latter. On average, unconsciousness and thus incapacitation occurs at 30% COHb, but the individual limit value varies with age and physical constitution. Death typically follows at 50% COHb. At $t_{\max}$, the exposure ends ideally, because the subject has left the danger zone in less fortunate cases, because he or she has succumbed to the toxic impact.

Apart from its amplificatory effect upon the toxicity of carbon monoxide, carbon dioxide exerts a toxic effect of its own, but only in air concentrations of 10% upwards where it leads to unconsciousness:

$$FID_{CO_2} = \int_0^{t_{\max}} \frac{dt}{\exp[6.16 - 0.519 \cdot x_{CO}(t)]} \tag{5.28}$$

Independently of any gas concentration, heat alone causes unconsciousness if the absorbed amount becomes to large (temperature $H(t)$ in [C]):

$$FID_{heat} = \int_0^{t_{\max}} \frac{dt}{\exp[5.18 - 0.0273 \cdot H(t)]} \tag{5.29}$$

If either (5.24), (5.28) or (5.29) fulfill ≥ 1, the respective individual loses consciousness and subsequently dies under the assumptions made above (no rescue team at hand). Hence, the conditional probability of death in case of a (non-explosive) fire is written as

$$\begin{aligned} k &= P\{(FID_{gas} \geq 1) \cup (FID_{CO_2} \geq 1) \cup (FID_{heat} \geq 1)\} \\ &= P\{\max_q [FID_q] \geq 1\} \end{aligned} \tag{5.30}$$

The cited literature only shows how to calculate the expected values of the respective quantities, which would lead to 0 and 1 as the only possible outcomes for k. In the case of B and D in (5.25), the variation among individuals is acknowledged but not quantified. Furthermore, calculating local concentrations and temperatures x_q involves various uncertainties. The first step in order to account for this randomness might consist in replacing the number 1, serving as critical threshold in (5.30), by a normal distribution with $\mu = 1$ and a coefficient of variation between 10 and 20%.

Temperatures and gas concentrations are not the only things that are developing over space and time in case of a fire. People themselves change their location as a function of time when they attempt to escape. Therefore, the interplay of probability of escape P_Q and conditional probability of escape k is rather complex. Because P_Q and k are mutually dependent, $N_{D|F}$ needs to be determined iteratively as described in the following section.

5.5 The Iterative Approach

5.5.1 Methodology

Some failure consequences—or more generally, disasters—spread so slowly that it is possible to outrun the hazard, even if no early warning has been issued (available warning time $T_W = 0$). In Section 5.3.1, these cases were termed *non-sudden.* The escape turns into a series of small time steps. In each of these steps, the question of successful escape and/or survival arises again. In the opposed case of a sudden event (implicitly regarded as the standard case), the situation is more simple: Either someone has left the entire area that is ultimately going to be affected *before* the disaster has reached him or her or the escape will automatically be unsuccessful: With a quickly progressing danger zone it is impossible to keep running ahead of the flood wave. Earthquake waves reach all parts of an area (e.g. a city) virtually at the same instant. Whether someone succeeds in escaping is decided *before* the disaster has arrived. The previous sections have shown how to deal with the predicition of P_Q and k in such cases.

In the case of non-sudden events, it is impossible to treat the possibility of successful escape P_Q and the conditional probability of death k as two independent events the way it was proposed in Sections 5.3 and 5.4. Instead, they are conditional upon one another with all the mathematical consequences such an interdependency can bring about. As a solution, the course events is subdivided into small time intervals. If chosen though infinitesimally small, each interval fulfils the condition of suddenness. For practical purposes, it is common to use some small finite interval instead (e.g. [95]). The expected number of fatalities on the condition of failure is then obtained by forming the union of respective probabilities in each of these intervals. Therefore, the iterative approach leads to a re-formulation of (5.1), so that

$$N_{D|F} = N_{PAR}\, P_D \tag{5.31}$$

where P_D (previously introduced as *lethality*) contains the inseparable contributions of escape probability and conditional probability of death and is obtained as

$$\begin{aligned} P_D &= \bigcup_{s=1}^{s_{\max}} [(1 - P_{Q,s}) \cdot k_s] \\ &= (1 - P_{Q,1}) \cdot k_1 + (1 - P_{Q,2}) \cdot k_2 \cdot P_{Q,1} + \ldots + (1 - P_{Q,s}) \cdot k_s \cdot \prod_{\alpha=1}^{s-1} [P_{Q,\alpha}] + \ldots \\ &= (1 - P_{Q,1}) \cdot k_1 + \sum_{s=2}^{s_{\max}} \left[(1 - P_{Q,s}) \cdot k_s \cdot \prod_{\alpha=1}^{s-1} [P_{Q,\alpha}] \right] \end{aligned} \tag{5.32}$$

If people at risk PAR are widely spread over an area, it is reasonable to create sub-groups PAR_i and apply (5.32) separately to each of them. The last iteration interval $s_{\max}$ is chosen as such that the event has passed its peak level or that the event is completely over at $t(s_{\max})$. For the iteration one should not use overly long intervals; several seconds up to a few minutes appear reasonable, as e.g. in [95], where $\Delta t \approx 30\,\text{s}$.

In this chapter, we have been considering two types of events requiring an iterative approach: Slowly rising floodwaters due to failed dams or dykes and (non-explosive) fires. In the literature, several authors use iterative procedures, such as Hartford [46] in the flood case and Persson [95] in the fire case.

5.5.2 Slow Flooding after Dam Failure

Persons Located Outdoors

For people located outdoors at the onset of a flooding, lethality follows from (5.18) and (5.32) as

$$\begin{aligned} P_D &= \bigcup_{s=1}^{s_{\max}} [(1 - P_{Q,s}) \cdot z_{su,s} \cdot k_{0,s}] \\ &= (1 - P_{Q,1}) \cdot z_{su,1} \cdot k_{0,1} + \sum_{s=2}^{s_{\max}} \left[(1 - P_{Q,s}) \cdot z_{su,s} \cdot k_{0,s} \cdot \prod_{\alpha=1}^{s-1} [1 - (1 - P_{Q,\alpha}) \cdot z_{su,\alpha}] \right] \end{aligned} \tag{5.33}$$

As an alternative to such a complex model it is possible to use frequentist approaches from the literature, such as the so-called *standard method* [140], which regards the typical Dutch application of dyke failure in a plain:

$$P_D \begin{cases} = 0 & \ldots \text{ for } \{d < 3 \cup v < 0.3\} \\ = \min\left[\max\left[8.5 \cdot e^{0.6 \cdot d - 6} - 0.15,\ 0\right],\ 1\right] \\ \quad \cdot \min\left[\max\left[8.5 \cdot e^{1.2 \cdot d' - 4.3} - 0.15,\ 0\right],\ 1\right] \\ = 1 & \ldots \text{ for } \{d > 6.25 \cap v > 2\} \end{cases} \tag{5.34}$$

Here, d [m] denotes water depth and $d' = dd/dt$ [m/h] is the speed at which the water level rises. The (horizontal) speed of the current is indicated as v [m/s].

Persons Located Indoors (Without Building Collapse)

Presumably, a slowly rising flood does not only fulfil the criterion of non-suddenness, but is also associated with comparatively slow (horizontal) water currents. In this respect building collapses due to dynamic water pressure are rather unlikely, although single-sided static water preasure can destroy a house if water is kept from flowing inside. If collapse is excluded, two possible causes of death remain: A person may drown within the building or else drown outside after a successful escape from within the building. Whether a person can drown inside depends on the depth of the water in comparison with the builing height. Altogether, this type of combined analysis is very case-specific so that it appears sensible to refer to the general model in (5.32) instead of proposing some detailed model.

Fire (Non-explosive)

It is virtually impossible to seperate the probability of successful escape P_Q and the conditional probability of death k in the case of non-explosive fire events. This is due to the accumulative nature of the fractional incapacitating dose (FID): If someone manages to leave the burning facility, the condition $FID < 1$ must be fulfilled (which automatically implies $FLD < 1$). If an individual absorbes an excessive dose of any of of the harmful impacts (gases or heat), he or she is incapacited, i.e. unable to flee and inevtiably faces death (unless rescue forces are available in order to evacuate those who have lost consciousness). Therefore, one central question needs to be answered: What is attained first—the critical incapacitating dose or the safe end of the escape path? In mathematical terms, this leads to the following expression, based on (5.30) and (5.32):

$$\begin{aligned} P_D &= \bigcup_{s=1}^{s_{\max}} P\left\{\max_q\left(\sum_{\alpha=1}^{s} FID_{q,\alpha}\right) \geq 1\right\} \\ &= P\left\{\max_q\left(FID_{q,1}\right) \geq 1\right\} \\ &\quad + \sum_{s=2}^{s_{\max}}\left[P\left\{\max_q\left(\sum_{\alpha=1}^{s} FID_{q,\alpha}\right) \geq 1\right\} \cdot \prod_{\alpha=1}^{s-1}\left[1 - P\left\{\max_q\left(\sum_{\beta=1}^{\alpha} FID_{q,\beta}\right) \geq 1\right\}\right]\right] \end{aligned} \quad (5.35)$$

The escape aspect does not show up in the formulation, but is implied in the FID criterion: While escaping, people change their location at each time interval s and so do zones of equal gas concentrations or temperatures (isotherms). Escaping speeds are given in Section 5.3. For the time-dependend evolution of a fire and the corresponding gas concentrations and temperatures x_q, it is common to use specialised simulation software as proposed in [95, 113, 121, 122]. In tunnels, the situation can be reduced from a three-dimensional to a one-dimensional problem, as shown e.g. in [113].

5.6 Additional Remarks

At the beginning of the chapter, it was postulated that probability of failure P_F and conditional number of lives lost $N_{D|F}$ are two independent phenomena. The basic relationship in (5.1) and the following methodological considerations are written under this assumption, which is deemed to produce sufficient accuracy in most practical cases. The following reasoning seeks to widen the scope of the methodology by demonstrating how to deal with problems in which the independence assumption does not hold.

The methodological considerations in Section 5.4.1 showed that the conditional probability of death k and its condition—i.e. failure—both depend upon the extent A of some triggering event. The probability of successful escape P_Q frequently depends upon A as well. The triggering event is usually exogenous to the facility and is expressed in the form of magnitudes or intensities. Inserting (5.13) in (5.2) leads to

$$N_D(A) = \lambda\, P\{A > A_{cr}\} \cdot N_{PAR} \cdot (1 - P_Q(A)) \cdot k(A) \quad (5.36)$$

and further

$$N_D = \int_0^{A_{\max}} \lambda\, P\{A > A_{cr}\} \cdot N_{PAR} \cdot (1 - P_Q(A)) \cdot k(A) \cdot f_\Xi(A)\, dA \quad (5.37)$$

where $\Xi = \{A_1, A_2, \ldots\}$ denotes the set of all possible extent levels and $f_\Xi(A)$ is the corresponding probability density function.

5.7 Discussion

The present chapter seeks to develop a methodology for the quantitative assessment of loss of life in case of failure. A methodology intends to provide a practicable general procedure for a given issue. In this respect, the above pages might serve as a useful tool.

It was less easy to provide numbers for all of the introduced quantities and sub-quantities. In those cases where precise information is not available, the respective order of magnitude has to serve as a substitute. There are two main reasons for these unsatisfactory cases: On one hand, the literature on many of the respective sub-quantities does not provide sufficiently detailed numerical information. Here, additional surveys and experiments could principally help. On the other hand, many parameters do not permit any general numerical statement at all, since some quantities depend heavily upon the specific situation (consider the probability of drowning given loss of firm ground contact in a flood, Section 5.4.3). In such cases, only a case-specific analysis dealing with the local conditions can help.

The analysis limits itself to three of the most important failure types in civil engineering. Nevertheless, it is principally possible to adapt the proposed methodology to other cases as well. The number of people at risk N_{PAR} and the probability of successful escape P_Q are universal quantities, as stated in the very beginning. Only the conditional probability of death requires a separate model for each event type.

Chapter 6

Human Consequences of Continuous Toxic Impacts

6.1 General Framework

Both consequences of acute failure and consequences of long-term impacts are conditional events and subject to uncertainty. Therefore, their occurrence is described by conditional probabilities. In the case of acute failure (Chapter 5), the event of loss of life is conditioned by the event of technical failure. The occurrence of this initial event is uncertain in itself and quantified as a probability. With impacts originating from long-term toxic exposure, the situation is different. Apart from chemical accidents (which are not in the realm of civil engineering), emissions occur permanently over a known period of time. As far as their occurrence is concerned, we are dealing with a certain event. There are of course uncertainties with respect to magnitude and transport process, but basically there is one degree of uncertainty less as compared to the acute case.

Yet, at the same time a degree of uncertainty is added with respect to the conditional event, i.e. the health consequences of long-term toxic exposure. With acute failure, it was possible to state that casualties occur on the spot[1] or not at all. This however, does not hold true in the present case. Health consequences will follow exposure with a certain delay (latency) and equally, health-saving effects will lag behind an eventual reduction of exposure levels. This requires some extra modelling.

Furthermore, the previous chapter assumed that non-fatal casualties are insignificant in comparison to fatal ones (p. 72). In fact non-fatal consequences of acute failure events are mostly made up by injuries; in the long run, most, except some permanently disabling injuries can be expected to heal. This assumption again is not realistic in the case of long-term toxic exposure. Here chronic sickness during years and decades, restraining the enjoyment of life, is a typical outcome. Death is practically always anteceded by comparatively long periods of sickness.

There are several sources of chronic intoxication inside and from buildings. Building materials

[1] As previously stated (Chapter 5): 'Immediate' consequences may well include follow-ups within the first weeks (due to diseases etc.)—a short period in comparison to a lifespan.

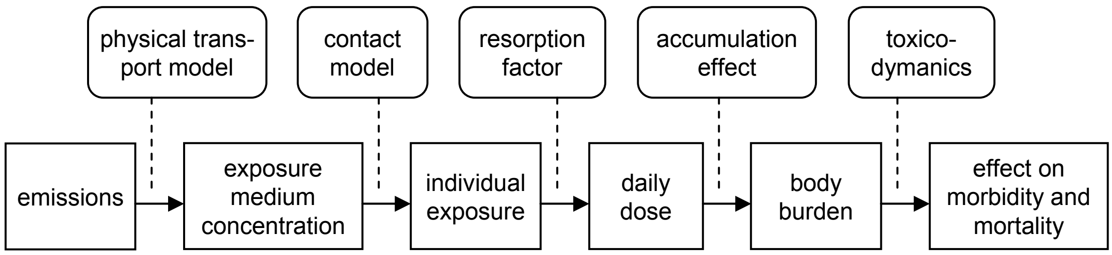

Figure 6.1: Causal chain

may contain noxious substances harmfull to humans, but also to animals and plants within a building or structure and in its surroundings (e.g. heavy metals). Then, parts of the building equipment including furniture etc., may release toxic substances (e.g. components of flame retardants). Finally, a building can also have the effect of concentrating a toxic from an external source. Here, natural radon seeping in from the underground can serve as a classical example.

The chain linking toxic emissions to health effects in Figure 6.1 can be separated into a few main parts. The first part departs from the emission source and deals with the *fate* of a substance which determines the exposure medium concentrations (i.e. local exposure levels) occurring at some remote place. This part is referred to as transport model or *exposure model* and is dealt with in **Section 6.2**. The second part ties in with the exposure level, determining the corresponding individual exposure and from there on the daily dose and body burden (**Section 6.3**). The modelling of the final consequences does not necessarily depart from the body burden (i.e. concentration on tissue level), but can also start from any other point in the causal chain. The corresponding model which can be referred to as *exposure-response model*, is the subject of **Section 6.4**.

6.2 Exposure Modelling

The present section deals with determining exposure medium concentrations of toxic substances. Exposure media include all entities with which a person has direct physical contact and which can serve as carriers for toxic substances. Apart from air, water and soil, this applies especially to food.

As long as the task is merely to link exposure to dose (and dose to response), no exposure modelling in a causal sense is needed. Rather, simple exposure measurements will do. However, in the given context, consequence modelling is not an end to itself, but stands in the larger context of acceptability modeling, which puts health-saving measures and their respective costs into relation. If measures are taken in order to reduce exposure levels, this usually means changing something about the source of emission, typically by making an additional investment (air filters etc.). Natural emissions form an exception: Here, exposure has to be reduced by local protective measures (e.g. air-tight foundations to keep off natural radon seeping in from the underground).

The difficulty in relating emissions to exposure lies in the fact that transport and transformation processes are involved. In many cases, a toxic substance is emitted at one specific location, such as a chemical plant and then it spreads over a whole region and further until it reaches an exposed person at some other location. This type of physical modelling involving transport and transformation processes in air, water and soil is rather demanding and a research field of its own. Therefore, the present section limits itself to providing a brief introduction with references for further reading.

6.2.1 Exposure next to Emissions Sources

This comparatively simple case concerns those situations in which the exposed person is close or very close to the emittent, as it is the case along a road with heavy traffic or inside a room, where toxic substances are released. Exposure here is very direct and does not take any detours through the ecosystem and the food chain. Therefore, exposure is basically determined as a function of release rates and the distribution processes outdoors and indoors.

6.2.2 Transportation through the Ecosystem

Once a substance is released into one ecological compartment (i.e. air, water or soil), it soon starts to propagate geographically, to migrate to the other compartments and to change its nature due to chemical reactions. All these transportation and transformation processes together are referred to as ecological *fate*. Within the ecological matrix, migration between the compartments is not a one-way phenomenon, so that multiple feedback processes can be observed. Humans are subsequently exposed to the emitted substance either directly, through contact with one of the ecological compartments, or indirectly, via the food chain. In the indirect case, plants or animals are subject to direct exposure prior to consumption.

Toxic substances—as well as any other substance—can enter the human body in three ways: ingestion, inhalation and dermal contact. This last step however is already closely intertwined with the determination and application of the effect model (dose-response relationship) and is therefore treated in Section 6.4.

In accordance with Greensense [33], transport of toxic substances is treated by two separate models. The first describes transport through the air, the second treats water and soil as parts of an integrated system. The Greensense framework assumes that substances can move from the air compartment into the soil and water compartment, but not backwards, a simplification, which is shown to be admissible for many substances in [33]. Both models also include some aspects on the question of chemical transformation of substances.

Air Transport Modelling

A European Commission research project on the externalities of energy (ExternE) [30, 31, 32] follows and quantifies the whole chain of events from stack emissions of power and heating plants to their ecological effects and costs. The report proposes a two-tier approach for modelling the transport of noxious matter in the air.

The first tier deals with transportation and exposure in the vicinity of the emittent, i.e. at distances of up to 50 km from the stack. A comparatively simple Gaussian plume model describes local concentrations depending on emission rate, stack height and wind speed. According to the model, the plume widens because of turbulent diffusion and vertical mixing. The approach implies two assumptions: One is the absence of vertical shear winds disturbing the horizontal plume structure much more strongly than the two phenomena described by the Gaussian model. The other one consists in assuming that no chemical reactions occur, which is reasonable for distances from 10 to 50 km.

For distances above 50 km, the second tier becomes relevant. It assumes that the pollutants have now been vertically mixed throughout the height of the considered atmosphere layer. The corresponding approach, named Windrose Trajectory Model (WTM, see [31] for references), describes the movement of an air parcel with constant height at a representative wind speed by means of a trajectory model. The exposure at an individual receptor point is estimated by accounting for the arrival of 24 trajectories, each weighted by the frequency of wind in the corresponding 15° sector. This second tier considers the occurrence of chemical reactions. In the ExternE case, this concerns the atmospheric reactions of NO_x, SO_2 and NH_3 leading to dry and wet deposition. The air pollution and toxic substance chapters of Greensense [33], display Europe-wide values both for emissions and local exposure levels and for different substances (NO_x, SO_2, particulate matter, Cd, Pb, Ni, Cr) in separate 50×50 km grids. Additionally, the report validates the reliability of the WTM by comparing measured to calculated values.

For general applications, including those related to civil engineering, the first tier of the approach may prove little help in some cases, considering the fact that many emissions are not bound to high-rising smokestacks. The second tier however, can be seen as generally applicable to all substances with significant airborne transport.

Multimedial Transport Modelling for Soil and Water

As described in the introductory part, the reasoning in the Greensense approach followed here is based on the assumption that substances may migrate from air into all other compartments, but not backwards. Therefore, the air transport model is only an input to the reduced multimedial model, which considers the remaining two compartments, i.e. water and soil, as well as the concentrations in the food chain. This however, still brings along a complex set of cross-related migration paths between these residual domains. Therefore, the Greensense approach takes a further simplifying step and limits itself to describing how an airborne substance migrates into soil, ground water and surface water and from there on into the human food chain (or directly from the air into the food chain). Herein, transformation processes are included in the form of degradation and decay. When emissions cannot be modelled as being constant over a very long period of time—which can be in the order of decades and centuries depending on the pollutant—human intake rates are a function of the time having passed since some pulse emission.

Another, slightly earlier European Commission release called EUSES (European Union System for the Evaluation of Substances [29]) provides a framework including application software for the modelling of both fate and health impact of toxic substances.

6.3 Modelling of Dose and Body Burden

After determining the ambient concentration of a toxic substance in the everyday environment and the food of a person (so-called *exposure media*), the next step consists in determining the corresponding dose. The definition of dose is relatively clear, either as the intake of a substance per time unit by an organism or as the fraction of the intake that is actually resorbed into the body tissue. However, the notion of dose-response functions is used for different kinds of concepts. In some cases it refers to relationships linking exposure medium concentrations or individual exposure directly to response while omitting all intermediate steps. In other cases, the term relates to more elaborate models calculating the body burden of a substance before moving on to its actual health effects. The following paragraphs seek to outline the linkage between exposure, dose and body burden together with the corresponding approaches in the literature.

From Exposure Medium Concentration to Daily Dose

Calculating the individual dose from exposure medium concentrations involves two steps. First, the daily individual exposure is determined by multiplying it with the contact rate. For exposure via drinking water and the food chain, the contact rate equals the amount of a product consumed every day. Standard food baskets serve as a source of information for these data [29]. For inhalation, the contact rate is obtained by multiplying it with daily exposure time and respiration rate ($\sim 20\,\mathrm{m}^3$/day for adults [143]) of an individual. In the case of occupational exposure, for instance, the daily exposure time is easily determined as the time spent at work. This type of reasoning is analogous to the considerations on presence modelling regarding acute structural failure. The modelling and data in Section 5.2 can be directly transferred to the present context. In the case of dermal contact with toxic substances, the exposure duration is multiplied with the exposed skin surface. All these approaches require realistic data and scenarios. In order to avoid lengthy investigations and calculations, this need can be covered by software packages such as the above mentioned EUSES [29]. It is based on the physical properties and behaviour of a standardised and representative person.

Once that individual exposure has been calculated in terms of daily amounts, dose follows in a second step as the product of the individual exposure with a resorption factor. This factor expresses the fraction of the ingested, inhaled or dermally encountered amount of a substance actually absorbed into the body tissue. Depending on the substance, resorption factors can be very high and close to 1, as in the case of brominated flame retardants [44]. The dose, i.e. the resorbed intake, can be expressed in absolute terms, but is usually indicated as amount per body mass unit.

From Daily Dose to Body Burden

Ultimately, it is not the daily amount of a substance added to the body tissue which ultimately determines the toxic consequences, but the actual tissue concentration. This value, called *body burden*, is equal to the daily resorbed intake plus the amount that has remained from the intakes on previous days. In order to determine the body burden, one needs to know the rate at which a specific substance is metabolised and excreted by the organism. Typically, the accumulation effect arising whenever intake occurs at a higher rate than metabolism is expressed in terms of a substance's half life. Many substances exerting chronic toxicity (heavy metals, persistent organic pollutants etc.) accumulate in lipid tissue, lungs or other slowly renewing parts of the body and can have half lives of several years.

6.4 Effect Modelling

Relationships linking a toxic input to the corresponding health consequences—i.e. changes in morbidity and mortality—are commonly referred to as *dose-response* relationships. However, many of the models found in the literature establish a direct link between exposure and health response. Other more biologically orientated models bridge only the last step, i.e. the one between body burden and response while some others again, can in fact be termed dose-response relationships in the strict sense. How many steps of the causal chain in Figure 6.1 are taken at once depends on the method as well as on the depth of the investigation.

6.4.1 Methods for the Establishment of Effect Models

Principally, there are two ways of establishing toxic effect models: animal tests and epidemiological studies. Both can be interpreted as a substitute to something that would bring the most accurate results but cannot even be thought of because of apparent ethic reasons: direct tests on humans. From this point of view however, it is easy to outlay the strengths and shortcomings of the two approaches: *Epidemiological* studies are based on the 'right' species (i.e. humans), but do not permit direct control over exposure or dose. This is the case, because humans must not be exposed to toxic substances on purpose. Therefore, researchers have to content themselves with the exposure values that are part of people's everyday lives and over which they do not have any control. *Animal tests*, on the other hand, allow direct testing including exposure scenarios ranging from very low to very high doses and intrusive measurements on living animals. Furthermore, animals may be killed for the sake of measurements on body tissue level. However, animal tests investigate the 'wrong' species which leads to significant conversion uncertainties when transferring the test results to humans.

Animal Tests

The major strength of animal tests as a toxicological investigation method lies in their ability to cover each step of the causal chain from ambient exposure to final health effects separately (compare Figure 6.1). Unlike with epidemiological analyses, this permits establishing an actual biological model instead of investigating a 'black box' from the outside. Toxic inputs can be directly controlled and disturbing influences can be easily eliminated under laboratory conditions. As mentioned above, the problem with animal test results lies in the transferability upon other species, specifically upon human beings. A commonly applied *inter*species uncertainty factor is 10, where toxicokinetic and toxicodynamic uncertainties[2] each contribute a factor of $\sqrt{10} = 3.2$. This factor is usually calculated by dividing the 95^{th} by the

[2] *Toxicokinetics* describes the resorption and distribution processes of a substance in the body (see Section 6.3), while *toxicodynamics* treats its effects on tissue level.

50^{th} percentile [93]. Some studies [15] argue in favour of a higher value in the order of 65 for comparisons between mammals, e.g. between mice or rats and humans.

Another effect is *intra*species uncertainty. Here, a factor of $3.2 \cdot 3.2 = 10$ is equally common, as in the interspecies case. Pelekis et al. [93] show that a toxicokinetic model describing the properties of the main organs and the blood flow between them by means of distributed quantities can help to reduce the toxicokinetic uncertainty significantly.

Epidemiological Analysis

Epidemiological investigations take advantage of the fact that people are exposed to many harmful substances in their everyday lives. When it is the goal to establish generic exposure-response or dose-response relationships for humans without the uncertainties of interspecies conversion, comparing populations that live with different exposure levels is the only ethically possible approach. Certain population groups are exposed more strongly to a substance because of their job (e.g. at industrial facilities), in other cases it is the population of a whole city which has to endure higher air pollution levels than the population of some other city. Generally, such comparisons cut the causal chain short and treat the sum of the biological processes lying in between as a black box. Other than under laboratory conditions, it is impossible to eliminate distorting influences (so-called *confounders*) from people's everyday environment. This problem is solved by taking population samples that are sufficiently large to compare 30-year old persons with 30-year old persons, women with women, smokers with smokers and so forth. Including data not only from many people, but also from as many different populated areas as possible helps to eliminate errors caused by confounders, e.g. when the emission of one substance is strongly correlated to with that of some other similar substance. This approach is followed e.g. by Pope et al. [96] for exposure with particulate matter.

Another more easily implemented approach in epidemiology consists in regarding one single population and analysing how far temporal changes in mortality correlate with temporal changes in exposure levels. Yet, it has to be seen that this type of investigation yields values for *acute* mortality only while chronic mortality effects are inevitably omitted. Acuteness does not refer to the severity of disease, but to the immediate effect of a heightened exposure level for which reason many authors refer to it as *daily* mortality (e.g. [116]). Acute cases mostly involve persons of weak physical constitution, whose remaining live expectancy is a few months or years. However, for many substances, especially those accumulating in the body (particulate matter, persistent organic pollutants (POPs), heavy metals) it is the chronic effects which account for the greatest loss of life years in a population. Besides, chronic mortality always includes acute mortality, whereas acute mortality values do not contain any information on chronic mortality. Information on acute morbidity and mortality may be of help to public health authorities having to allocate extra health care resources on days with heightened general exposure. Yet, for purposes of risk valuation, the limitation to acute effects is not appropriate.

Because the concentration of a substance in the body and not that in the ecosystem ultimately causes the undesirable effects, the modelling of accumulative effects based on epidemiological data is a delicate task. Some studies (e.g. [96]) do not account for any sort of accumulation. This can mean that there is actually no accumulative effect. However, it could also be understood as the tacit assumption that all exposed persons have been permanently exposed from birth up to the present moment at some present concentration level. Although this assumption can give realistic figures on the quantitative effect of some pollutant on public health, it provides little help in assessing the effect of a countermeasure. In a strict sense, the assessment of a change in pollution levels would only be correct for a complete turn-over of the population, which is estimated to take 110 years (see Chapter 2). This is the period of time, in which every living person will have been born *after* the exposure step, while everybody living today will have deceased. Admittedly, results will be close to reality much sooner, i.e. after a few decades. This however, is still too long when investigating the immediate effects of a measure.

As an alternative, one may analyse the correlation between frequency of health effects and 'accumulated exposure', i.e. exposure level times accumulated exposure time. Although this is only rudimentary substitute for the accumulation effect as is described in the previous section ('From Dose to Body Burden'), it certainly provides additional accuracy (compare Section 6.4.2). Weighting exposure years in the distant past less then more recent ones is a further refinement and complies with the metabolisation effect reducing the body burden. Equally, a latency effect can be included by not counting the exposure years acquired during the latency period. Both the simple and the more advanced approach to epidemiological accumulation modelling are demonstrated in the BEIR IV commission report [89] for the case of radon gas.

6.4.2 Mathematical Representation of Effects Models

A change in concentration dx_q of some substance q leads to a change in the number of people who fall ill dN_{ill} Hereby, the concentration x_q could be measured at exposure medium, individual exposure, daily dose or tissue level. Morbidity ν_b (or its incremental change $d\nu_b$) is the rate or probability of falling ill with some disease b and is defined as $\nu_b = N_{ill,b}/N_{pop}$. Apparently, its value depends on the choice of the reference population N_{pop}. In the case of general threats such as air pollution, everybody in a certain area is at risk, i.e. $N_{pop} = N_{PAR}$. Other risks, e.g. those from occupational exposure concern only a certain section of the general population. In this case, health statistics may refer either to the respective population group or to population as a whole.

As mentioned before, an incremental concentration change dx_q will not lead to an immediate change in morbidity rates for several reasons. For one, there are delays due to the transport process through the ecosystem (Section 6.2). Then, the combined effect of intake rate and metabolism rate determining the accumulation of a substance in the body causes another delay termed latency (Section 6.3 and present section). Once a person has fallen ill with disease b, there is a chance that she or he may die, if b is life-threatening. This can be expressed by a conditional probability $P\{D|ill_b\}$ together with the expected time lying in between the outbreak of the disease and death. Another more direct and more common possibility is to indicate the yearly probability of death due to disease b, i.e. mortality μ_b, directly as a function of concentration x_q, i.e. without taking any detour via the precondition of falling sick. Here, delay and latency effects have to be considered analogously.

Representation of Effects upon Mortality

There is a number of models going to different levels of precision. The most simple cases seem to imply the following assumptions:

- The toxic input (for which the mortality effect is calculated) is represented by the exposure medium concentration.
- Accumulative effects are neglected *or* exposure is assumed to be permanent (lifelong) and constant (which can be equivalent in terms of mathematical modelling, see Section 6.4.1).
- The input-response relationship is linear (Note that, in cases where the input-response relationship is relative, linearity is a necessity. Relativity here, means that a change in concentration Δx is linked to a change in mortality $\Delta\mu$, instead of linking an absolute x to an absolute μ.)

This type of model has the mathematical form

$$\begin{aligned}\mu_b(a, dx_q) &= \mu_b(a)(1 + \eta\, dx_q) \\ &= \mu_b(a) RR(dx_q)\end{aligned} \tag{6.1}$$

Here, $\mu_b(a)$ denotes the age-dependent background mortality due to disease b and η is the slope factor linking a change in x_q to a proportional change in $\mu_b(a)$. Actually, age a is not the only demographic parameter μ_b depends on. Other parameters such as sex, education, body mass index and various behaviour parameters (smoking, dangerous job or leisure activities) play an important role as well. The multiplier expressing the augmentation of $\mu_b(a)$ is also referred to as risk ratio RR. When regarding the effect of dx_q on all-cause background mortality, the following relation holds:

$$\begin{aligned}\mu(a, dx_q) &= \mu(a) + \mu_b(a)\, \eta\, dx_q \\ &= \mu(a) + \mu_b(a)(RR(dx_q) - 1)\end{aligned} \tag{6.2}$$

In (6.2), it is necessary to subtract $\mu_b(a)$ once, because the latter has already been considered in the all-cause mortality rate $\mu(a)$. When comparing this model with the mortality schemes in (2.52) and (2.54), it can be seen as a special case of the proportional scheme.

More sophisticated models leave out one or several of the simplifying assumptions listed above. Introducing 'accumulated exposure time' T_{cum} is a simple way of accounting for accumulative effects of many toxic substances. At the same time this extension also means moving from exposure media concentration to individual exposure level because T_{cum} is corrected for time fractions in daily life spent out of contact with the toxic substance (contact model):

$$\mu(a, dx_q) = \mu(a) + \mu_b(a) T_{cum}\, \eta\, dx_q$$

or

$$= \mu(a) + \mu_b(a)(RR(dx_q) - 1) \tag{6.3}$$

Table 6.1: Some exposure-response functions for changes in mortality rates $d\mu_b$

substance q	**death cause** b	**risk model**	**slope factor** η	**unit of slope factor**
arsenic [142]	lung cancer	$RR = 1 + \eta x T$	$3 \cdot 10^{-4}$	per 1 μg/m^3 & 1 work yr
arsenic [142]	leukemia	$RR = \exp[\eta x]$	$3 \cdot 10^{-5}$	per 1 μg/m^3
radon [89]	lung cancer	$RR = 1 + \eta x T$	0.013	per 1 WLM
$PM_{2.5}$[a] [96]	cardiopulm. disease	$RR = 1 + \eta x$	0.009	per 1 μg/m^3
$PM_{2.5}$ [96]	lung cancer	$RR = 1 + \eta x$	0.014	per 1 μg/m^3
$PM_{2.5}$ [96]	all causes	$RR = 1 + \eta x$	0.006	per 1 μg/m^3

[a] particulate matter $< 2.5\mu$m

$$\begin{aligned}\mu(a, dx_q) &= \mu(a) + \mu_b(a)(\exp[T_{cum}\,\eta\,dx_q] - 1)\\ &= \mu(a) + \mu_b(a)(RR(dx_q) - 1)\end{aligned} \tag{6.4}$$

An exponential input-response relationship with $\eta\,dx_q < 1$ as in (6.4) is a common alternative to the linear relation in (6.3). Table 6.1 includes some exposure-response values for different models and substances. Further mortality information can be derived from the morbidity indications in Table 6.1, if this information is combined with the probability of death given sickness (lung cancer: 90%, leukemia: 70% [33]).

Further refinement is achieved by calculating the actual body burden, i.e. the concentration at body level as a function of daily dose and the half time of the substance taken in. Here, the elimination of a substance from the body can be modelled by counting exposure years from far back in the past by only half as it is done by the BEIR IV committee study [89] on radon gas.

In order to calculate the effect of the described mortality changes upon life expectancy, it is possible to linearise for small changes, as previously demonstrated in Sections 2.5.2 and 4.3. For (6.2), the linearisation coefficient $J_\omega(T_\mu)$ is obtained in analogy to (4.19) from

$$\begin{aligned}E_A\left[\frac{de_d(a)}{e_d(a)}\right] &\approx \int_0^{a_u} \frac{-\frac{d}{d\omega}\,e_d(a,\omega)\big|_{\omega=0}\cdot\omega}{e_d(a)}\,h(a,n)da\\ &= -\int_0^{a_u - T_\mu} \frac{\int_{a+T_\mu}^{a_u}\exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right]\cdot\int_{a+T_\mu}^t \mu_b(\tau)\eta\,d\tau\,dt}{\int_a^{a_u}\exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right]dt}\,h(a,n)da\cdot\omega\\ &= J_\omega(T_\mu)\,\omega\end{aligned} \tag{6.5}$$

with $\omega = dx_q$. The linearisation coefficient for (6.3) follows from

Table 6.2: Average durations per case of different types of disabilities

source	disability outcome b	duration D_b [yrs]
	ischaemic heart disease hospital admission (HA)	0.038 (14 days)
Hofstetter/	respiratory hospital admission (HA)	0.038 (14 days)
Hammitt [49]	respiratory emergency room visit (ERV)	0.033[a] (12 days)
	neurocognitive development deficits	$\equiv e(0)$

[a]Obviously, this number refers to the whole period of disability and not just to the short time spent in the emergency room.

$$
\begin{aligned}
E_A\left[\frac{de_d(a)}{e_d(a)}\right] &\approx \int_0^{a_u} \frac{-\frac{d}{d\omega}\, e_d(a,\omega)\big|_{\omega=0}\cdot\omega}{e_d(a)}\, h(a,n)da \\
&= -\int\limits_0^{a_u-T_\mu} \frac{\int_{a+T_\mu}^{a_u}\exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right]\cdot\int_{a+T_\mu}^t \mu_b(\tau)\,\eta\cdot(\tau-a-T_\mu)\,d\tau\,dt}{\int_a^{a_u}\exp\left[-\int_a^t \mu(\tau)+\gamma(\tau^*)d\tau\right]dt}\, h(a,n)da\cdot\omega \\
&= J_\omega(T_\mu)\,\omega
\end{aligned}
\tag{6.6}
$$

where T_{cum} enters as $(\tau - a - T_\mu)$.

Another concept frequently encountered in toxicology (e.g. [142]) is the so-called *unit risk* (UR). It expresses the additional probability of dying, if the exposure medium concentration of a given substance is increased by one unit (e.g. per $1\,\mu\text{g/m}^3$). However, it is important to notice that the unit risk is not a change in the yearly probability of dying (i.e. mortality rates), but relates to the lifetime risk instead. Therefore, it can be a useful measure of the danger posed by an immission for illustrative purposes, but not for calculating changes in life expectancy as in Chapter 2.

Representation of Effects upon Morbidity

As outlined in Chapter 3, there are different ways of expressing non-fatal health consequences (disabilities), when the aim is to describe their impact upon lifetime utility. One way is to express the occurrence rate, i.e. (age-dependent) morbidity $\nu_b(a)$. This number alone is of little use as long as it is not combined with the typical duration D_b of a case of disease b. Therefore, this number equally needs to be investigated. An alternative consists of expressing morbidity impacts by their prevalence, i.e. the proportion of lifetime lost due to disability b per inhabitant and year. Actually, these two ways of proceeding are just two ways of expressing one phenomenon, since (age-dependent) prevalence $P_b(a)$ can be seen as the product of $\nu_b(a)$ and D_b see (3.1)).

In contrast to mortality changes, morbidity values and their impact upon (disability-adjusted) life-expectancy are much easier to model. This is mainly due to the fact that falling sick with disease b at age a is *not* conditional on not having fallen sick at all previous ages. Therefore, a morbidity model does not need to include all-cause background morbidity rates[3].

[3]Note, that the absence of a disease-specific background mortality term ν_b can lead to nominally lower

Table 6.3: Some exposure-response functions for acute changes in ν_b and P_b

substance q	**disability outcome** b	**risk model**	**slope factor**	**unit of s.f.**
	ERVs[a] for COPD[b]		$7.2 \cdot 10^{-6}$	
	ERVs for asthma		$12.9 \cdot 10^{-6}$	
PM_{10}[c] [30]	Hospital admissions for COPD	$d\nu_b = \eta\, dx_q$	$2.27 \cdot 10^{-6}$	per 1 μg/m^3
	Hosp. adm. for respir. infections		$1.87 \cdot 10^{-6}$	
	Hosp. adm. for childhood croup		$29.1 \cdot 10^{-6}$	
PM_{10} [30]	RADs[d]	$dP_b = \eta'\, dx_q$	$0.137 \cdot 10^{-3}$	per 1 μg/m^3
	Symptom days[e] (SDs)		$1.27 \cdot 10^{-3}$	

[a]Emergency room visits
[b]Chronic obstructive pulmonary disease
[c]PM_{10} concentrations can be estimated from $PM_{2.5}$ concentrations by multiplying the latter with 1.67 [32].
[d]Restricted activity days converted into prevalence values according to (3.2)
[e]same as *d*

Table 6.4: Some exposure-response functions for chronic changes in ν_b and P_b

substance q	**disability outcome** b	**risk model**	**slope factor**	**unit of s.f.**
	chronic bronchitis / adults		$0.70 \cdot 10^{-3}$	
PM_{10} [30]	respiratory illness / adults	$dP_b = \eta'\, dx_q$	$0.95 \cdot 10^{-3}$	per 1 μg/m^3
	chronic bronchitis / children		$1.61 \cdot 10^{-3}$	
	chronic cough / children		$2.07 \cdot 10^{-3}$	
arsenic [33]	lung cancer		$1.5 \cdot 10^{-3}$	
cadmium [33]	lung cancer		$1.8 \cdot 10^{-3}$	
chromium [33]	lung cancer	$d\nu_b = \eta\, dx_q$	0.012	per 1 μg/m^3
lead [33]	neurodevelopmental deficits		1 IQ point	
benzene [33]	leukemia		$2.2 \cdot 10^{-6}$	

In mathematical terms, this approach can be expressed simply as

$$d\nu_b = \eta\, dx_q \tag{6.7}$$

and

$$dP_b = \eta\, dx_q\, D_b = \eta'\, dx_q \tag{6.8}$$

Slope factors are denoted by η and η', respectively. As outlined in Section 6.4.1, the absence of an accumulative term basically means that concentration values are understood as constant throughout the lifetime of an exposed individual with all the shortcomings associated with such a model. A comparison between Tables 6.3 and 6.4 shows that chronic impacts are more important than acute ones.

values of η in Table 6.3 than in Table 6.1, although, in absolute terms, morbidity effects are more frequent than mortality outcomes, of course.

Chapter 7

Examples

The Examples chapter is meant to illustrate as many of the concepts from the previous chapters as possible. Table 7.1 shows which of the concepts are covered by which example.

7.1 Debris Flood Protection

The official website of Whatcom County presents an analysis on the debris flood hazards posed by Canyon Creek [52]. Whatcom County is located in Washington State, USA, at the Canadian border next to Vancouver. The study investigates the annual probabilities of debris flood events of different magnitudes, as well as their economic consequences and the effectiveness of defensive measures and property buy-outs. Debris floods consist of water carrying 15 to 35% of stones and organic debris and should not be confused with debris flows or mud flows carrying a much higher amount of sediment. The study does not contain a quantitative analysis of expected loss of life and therefore does not address the question of acceptable decision-making with respect to such consequences. The present example seeks to perform these two steps. However, it is emphasised that the example is solely based on information that can be derived from [52]. It is exclusively meant to illustrate the methodology from the previous chapters of the thesis. The intention is definitively *not* to give any recommendations whatsoever to the decision makers in charge.

Table 7.1: Coverage of concepts in the Examples chapter *(§ = section number)*

concept	**§7.1**	**§7.2**	**§7.3**	**§7.4**	**§7.5**
WTP for mortality	✓	✓	✓	✓	✓
WTP for morbidity			✓		
acute failure	✓	✓			
chronic impacts			✓	✓	✓
latency			(✓)	✓	✓
cost-benefit optimisation		✓			
differentiable WTP		✓			

Figure 7.1: Hazard zoning at the alluvial fan of Canyon Creek, from Jakob et al. [52]

Figure 7.1 depicts the basic situation at the Canyon Creek alluvial fan. In Zone 1 (dark area), velocity reaches 4.5 m/s and flow depth reaches 2.5 m in case of a 500-year event. Maximum boulder diameters are estimated at 0.6 m. In the hatched area, velocities of 7.5 m/s and depths of 4 m can be expected. Loss of life is expected only under the conditions predicted for Zone 1, including the hatched area. There are six private cabins in the zone (two of which are in the hatched area) and six rental cabins in a separate resort.

Presence model. In methodological accordance with Chapter 5, the first step consists in establishing the presence model. It is assumed that people principally make year-round use of the cabins, skiing in the nearby Mt Baker resort in winter and hiking or fishing during the warmer half of the year. For the private cabins, presence is difficult to model, since these buildings serve as secondary residences. It is assumed that families use their cabin every fourth weekend plus an additional two months of permanent residence and/or rental to other persons. Multiplying with the average U.S. household size of 2.6 leads to $2.6 \cdot (2/7 \cdot 13/52 + 2/12) = 0.62$ persons expected to use the cabin on an average day. 12 out of 24 hours a day are spent for sleeping, eating and indoor activities, leading to an expected presence of $N_{PAR} = 0.62/2 = 0.31$ persons per private cabin at any given moment.

Occupancy is expected to be higher in the separate rental cabins, which are expected to be in use no less than half of the time during the year. Tendentially, groups will be larger in order to cover the high rental rates per cabin. Therefore, average occupancy is expected to be three persons in the small cabins (4 units) and eight in the larger cabins which are designed to accommodate 8–10 persons (2 units). Again, indoor time is assumed to be 12 hours, which

Table 7.2: Estimation of loss-of-life in case of a 500-year event (current situation)

group	N_{PAR}	P_Q (awake)	P_Q (asleep)	k	$N_{D\|F}$
private/hatched	0.6	0.84	0.23	0.8	0.3
private/Zone 1	1.2	0.84	0.23	0.5	0.3
rental/Zone 1	7.0	0.45	0.05	0.5	2.9
total	**8.8**	**-**	**-**	**-**	**3.5**

leads to an expected presence of $N_{PAR} = 3 \cdot 1/2 \cdot 1/2 = 0.75$ persons per small rental cabin and $N_{PAR} = 2.0$ persons per large cabin. Multiplying with the number of cabins (private and rental), 8.8 persons are expected to be present in Zone 1 at any given time, as illustrated by Table 7.2.

Probability of successful escape. Because of the high flow velocities, a potential debris flood is classified as 'sudden' in the sense of Section 5.3.1, i.e. it is impossible to outrun the flood waters *after* having been reached by them. Pre-warning (direct warning) is excluded since there are no monitoring devices or nearby residents at the point of debris flood initiation upstream in the canyon. In accordance with [74] as well as the sources in Table 5.11, average reaction time T_{rea} is estimated as 30 s during wake time and 120 s during sleeping time. It is modelled by a triangular distribution as $\mathsf{T}(15, 30, 45)$ and $\mathsf{T}(90, 120, 150)$, respectively. Of the 12 hours spent indoors, four are spent awake and eight asleep. Escape paths are in the order of 50 m from the private cabins, but they range between 100 and 200 m from the separate rental cabin resort. Escape speed is modelled by a uniform (rectangular) distribution $\mathsf{U}(1.0, 1.5)$ in [m/s] units. Required escape time T_Q is obtained by inserting these values in (5.9). Available escape time T_W is hard to estimate, because it depends not only on the water speed, but also on the audibility (and visibility) of the approaching flood; for flash floods in narrow valleys—which corresponds well with the present case—McClelland & Bowles [74] propose a value between one and four minutes. Therefore, a uniform distribution $T_W = \mathsf{U}(60, 240)$ [s] will be used.

The probability $P(Q|W)$ of successful escape given warning follows from inserting the above values in (5.5). The results are displayed in Table 7.2. The probability $P(W_{prc})$ of perceiving the approaching flood is assumed to equal 0.9 at times awake and 0.5 at night. The probability $P(W_{dc})$ of deciding to flee given perception is set to 1. Inserting this information in (5.4) and (5.3) yields the probability of successful escape P_Q.

Conditional probability of death. The probability of being killed in case of unattempted or unsuccessful warning is the most difficult to estimate, because very little quantitative research has been undertaken in this direction. For people caught by the flood after having left their cabin, inserting in (5.19) shows that it is practically impossible not to be carried away under the expected flow conditions: in fact, the water is carrying a high percentage of debris which further exacerbates the situation. The Dutch *Standard Method* [140] provides critical flow velocities for the collapse of houses: The resistance of log walls is possibly higher than that of brick walls (1-2 m/s) but lower than that of concrete walls (6-8 m/s). Therefore, cabins in Zone 1 have a certain likelihood of withstanding the pressure, whereas those in the 'hatched area' are unlikely to remain intact. Wooden houses are prone to buoyancy and are likely to be swept along. Because of the lack of more detailed models, the probability of dying inside a collapsed building or outside during escape is roughly estimated to equal 0.8 in the 'hatched area' and 0.5 in the remaining part of Zone 1.

Finally, the expected number of lives lost in case of a debris flood under current circumstances (no mitigation measures) follows as

$$N_{D|F} = N_{PAR} \cdot \left[\frac{1}{3}(1 - P_Q(\text{awake})) + \frac{2}{3}(1 - P_Q(\text{asleep}))\right] \cdot k \tag{7.1}$$

As a result, 3.5 lives are expected to be lost in case of an event, as displayed in Table 7.2.

Acceptability analysis. The present analysis deals exclusively with 500-year events because Jakob et al. [52] limit the detailed part of their study on this particular recurrence period. They base this choice on a simple qualitative estimate stating that risk—in terms of probability times consequences—is highest in this case, as compared to shorter or longer recurrence periods. In their report, they operate with a planning horizon of $t_s = 50$ years.

For the United States, the socio-economic key data from Table 2.5 are $G_\Delta = 2.42 \cdot 10^6$ PPP US\$, $\zeta = 1.8\%$ and $\gamma = \rho + \varepsilon\zeta = 4.5$. For a complete elimination of the 3.5 annually expected fatalities and the above-mentioned planning horizon, the willingness to pay criterion (4.16) limits the maximum investment to 530,000 PPP US\$.

The report recommends a a property buy-out in the danger zone and quantifies the total cost as 1 million US\$. This cost exceeds the calculated WTP by almost 100%. on the other hand, it needs to be stated that there is a lot of uncertainty in the present analysis; changes in the planning horizon or in the fatality estimation would significantly reduce the discrepancy. WTP and buy-out costs are in the same order of magnitude, so that it is probably not justified to call a buy-out an unreasonable decision.

Protective berms are an alternative measure. They cost approximately the same amount, but need permanent maintenance. Besides, their effectivity is not garantueed.

7.2 Structural Collapse due to Earthquake Action

The present example is rather academic and cannot be considered as an actual case study. However, it helps to illustrate some concepts from Chapter 4, most notably the case of a differentiable WTP criterion and its interaction with reliability-based cost-benefit optimisation. This type of analysis has been performed on previous occasions by Rackwitz, Streicher and other authors [112, 126, 127].

The example considers the general case of a single occupancy house in an earthquake-prone region. According to Sánchez-Silva & Rackwitz [112], the cost of protecting a residential building against major earthquakes accounts for 20 or 30% of the total building cost. The objective function for economic optimisation follows from (B.9) as

$$Z(p) = \frac{b}{\gamma} - C(p) - C_X(p)\frac{\lambda P_X(p)}{\gamma + \lambda P_X(p)} - C_F(p)\frac{\lambda P_F(p)}{\gamma + \lambda P_F(p)} \tag{7.2}$$

Here, p is the design parameter and λ denotes the rate of disturbances. P_X is the probability of major damage in case of a disturbance, i.e. an earthquake and $C_X(p)$ is the corresponding damage cost. P_F and C_F are the probability and cost of total collapse. The yearly benefit b of owning a house can be derived from average rental prices. If the home owner uses the house him- or herself, the market rental price can be interpreted as the amount he or she saves due to home-ownership.

Earthquake loads can be expressed in different ways, in terms of ground acceleration, ground velocity or ground deplacement. For the present example, peak ground velocity (PGV) is chosen as e.g. in Kanda & Nishijima [59]. From the data in [59], a possible distribution of PGV in a strongly earthquake-prone area is estimated as $f_V(v) \approx 20000 \cdot (v + 25)^{-4}$, with v in [cm/s]. The response spectral velocity S is assumed to have the same mean value as PGV and a coefficient of variation of 0.8, see [112]. Earthquake resistance R is assumed to have a mean of p (which is the design parameter) and a coefficient of variation of 0.2. For a log-normal model, the same simple approach as in Sánchez-Silva & Rackwitz [112] can be used in order to determine the probability of failure as

$$P_F(p|v) = \Phi\left[\frac{-\ln\left[\frac{p}{v}\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right]}{\sqrt{\ln\left[(1+V_S^2)(1+V_R^2)\right]}}\right] \tag{7.3}$$

and further

$$P_F(p) = \int_0^\infty \Phi\left[\frac{-\ln\left[\frac{p}{v}\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right]}{\sqrt{\ln\left[(1+V_S^2)(1+V_R^2)\right]}}\right] f_V(v)\, dv \tag{7.4}$$

Here, p is the design parameter. If P_F is an annual probability (as in the present case), then $\lambda = 1$. According to [59], the critical velocity for major damage (repair costs 30% of building costs) can be estimated as 50 cm/s lower than that for collapse. Therefore

$$P_X(p|v) = \Phi\left[\frac{-\ln\left[\frac{p}{v+50}\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right]}{\sqrt{\ln\left[(1+V_S^2)(1+V_R^2)\right]}}\right] \tag{7.5}$$

In case of major damage, repair is assumed to take three months during which the building does not yield any benefit. In case of collapse and reconstruction, the outage is assumed to last a whole year (see Table 7.3).

The recompensation H_D for a fatality—or simply the loss human capital—depends on the socio-economic situation of the country. Since the present example is very general, it is proposed to use some typical values for industrialised countries instead of choosing one specific

Table 7.3: Calculation of costs (in [PPP US$])

cost item	calculation
construction cost $C(p)$	$C_0 + C_1(p)$
basic construction cost C_0	150,000
earthquake protection cost $C_1(p)$	$150 \cdot p^{1.3}$
cost of major damage C_X	$0.25 \cdot b + 0.3 \cdot C(p)$
cost of collapse C_F	$b + C(p) + N_{D\|F}\, H_D$
cost of lost human capital H_D	975,000
yearly benefit b	18,000

country from Table 2.5. The following values will be used for the determination of H_D as well as for the WTP criterion: $g = 25{,}000$ PPP US$, $q = 0.14$, $\zeta = 0.015$, $\gamma = 0.045$, $e_0 = 78$ years, $J_\Delta = 14.5$, $G_\Delta = 2.6 \cdot 10^6$ PPP US$. From (4.28), the loss of human capital ensues as $H_D \approx \frac{1}{2} \cdot 78 \cdot 25,000 = 975{,}000$ PPP US$.

The expected number of fatalities in case of failure $N_{D|F}$ is estimated as follows: The average household size in Europe, North America and Japan is $N_{pop} = 2.6$. The household size and its effect upon the final result are studied in Figure 7.2. As in Example 7.4, the average inhabitant is expected to be present 15.4 out of 24 hours so that $N_{PAR} = 2.6 \cdot 15.4/24 = 1.7$. In a two-storey building, the probability of escape is $P_Q = 0.25$, because people have a 50% chance of escaping from the ground floor, whereas successful escape from the first floor is highly unlikely [18]. The probability of death (conditional on not having escaped successfully) is $k \approx 0.2$ for low buildings of masonry of wood. Therefore, the expected number of fatalities in case of failure is expected to be

$$N_{D|F} = 1.7 \cdot (1 - 0.25) \cdot 0.2 = 0.26 \tag{7.6}$$

With this information, it is possible to optimise the cost-benefit relation (7.2) with respect to p. Maximal profit $Z_{\max}$ is obtained for a resistance of $p^* = 176$ cm/s.

From a socio-economic point of view, the acceptability limit can be determined by inserting in (4.16) and deriving after p:

$$\frac{dC(p)}{dp} \geq -\frac{1 - \exp[-\gamma t_s]}{\exp[\gamma] - 1} \frac{\zeta}{\exp[\zeta t_s] - 1} G_\Delta\, N_{D|F}\, \frac{d(\lambda P_F(p))}{dp} \tag{7.7}$$

Resolving (7.7) yields a lower acceptable limit of $p_{\lim} = 41$ cm/s. Apparently, the economically optimal solution p^* leads to much safer values than this, so that the WTP criterion does not become a limiting factor. This phenomenon has been remarked by Rackwitz on previous occasions, e.g. in [104]. Figure 7.2 illustrates the interplay of cost-benefit analysis and WTP limit (left part) and the effect of different household sizes N_{pop} upon acceptable and optimal safety levels (right part).

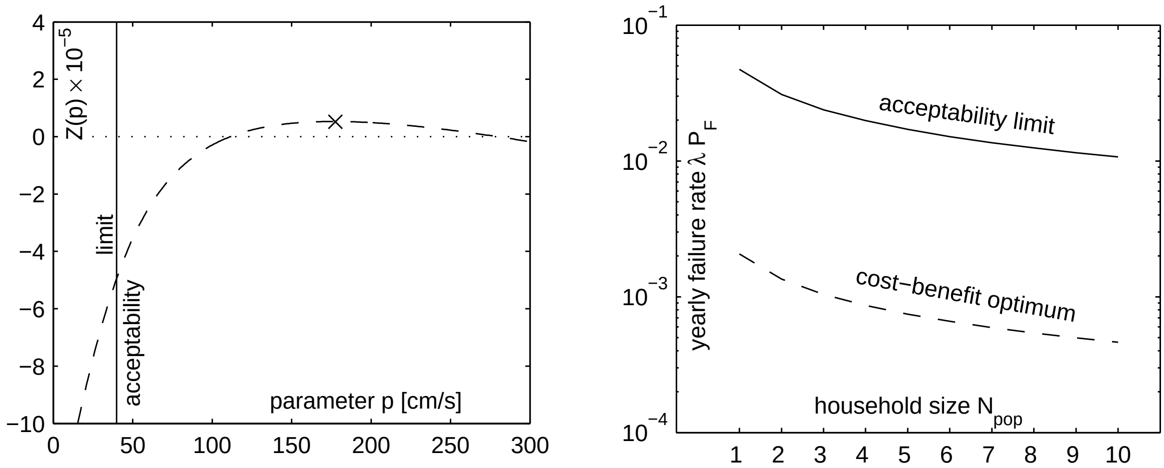

Figure 7.2: Effect of design parameter p upon profitability $Z(p)$ (left) and parameter study for the effect of household size N_{pop} (right)

7.3 Particulate Matter

Although exposure to airborne particulate matter is not an issue of civil engineering, it is a very suitable application for risk acceptance criteria such as the WTP criterion and has previously been discussed by Pandey & Nathwani [91]. Particulate matter has a major impact both upon morbidity and mortality, which is well investigated in quantitative terms [96, 117]. The dose-response relationships are documented in Tables 6.1 and 6.4.

Combustion processes, as they occur in transportation, industry and heating, set free small pieces of soot, which can cause chronic bronchitis, cardio-pulmonary disease and lung cancer. Exposure is expressed in μg/m^3 of particles with less than 2.5 or 10 μm diameter. In the United States, most polluted cities face $PM_{2.5}$ concentrations of about 30 μg/m^3, whereas those in the least polluted ones range at approximately 10 μg/m^3 [96]. This corresponds to a gap of 20 μg/m^3 that can presumably be bridged by adequate mitigation measures.

For the present example, we regard a $PM_{2.5}$ reduction of just 1 μg/m^3 (instead of 20) in order to assess the amount of money that may be invested into the corresponding measure without violating the affordability criterion. The society under consideration is that of the United States, with $\bar{e}_d = 21.6$, $\bar{e}_{DA,d} = 19.4$, $\bar{\mu} = 0.0087$, $g = 30{,}000$ US\$ and $q = 0.19$. The linearisation constant for age-proportional changes in mortality is determined as $J_\delta(0) = 15.5$ for no latency and $J_\delta(15) = 7.43$ for a latency period of 15 years.

The all-cause effect upon mortality corresponds to $\delta = -0.006$ for a 1 μg/m^3 change [96], which leads to a loss of life of 0.010 years (3.7 days). Inserting into criterion (2.56) leads to

$$C_\mu = -dg \leq -\frac{30000}{0.19} \cdot 7.43 \cdot 0.0087 \cdot (-0.006) = 61\,\text{US\$} \tag{7.8}$$

as the maximum affordable investment per inhabitant and year for the proposed exposure reduction. Without latency, the value would rise to 128 US$.

In a next step, the morbidity effects of changes in particulate matter exposure need to be considered. Assuming that children are affected in the same way as adults is an underestimation of the actual impact and leads to overly conservative WTP limits. Under such an assumption, one can use $dP_b = (0.70 + 0.95) \cdot 10^{-3} = 1.65 \cdot 10^{-3}$ as the all-cause effect[1] of a $1\,\mu\text{g/m}^3$ concentration change of PM_{10}. The effects upon chronic bronchitis and respiratory illness prevalences are added [30].

A $1\,\mu\text{g/m}^3$ change of $PM_{2.5}$ corresponds to a PM_{10} change of 1.67 [32], so that $dP_b = 1.67 \cdot 1.65 \cdot 10^{-3} = 2.76 \cdot 10^{-3}$. The severity weight s_b of chronic bronchitis is given as 0.14 in Table 3.1. For this age-independent model, the change in damage-adjusted, age-averaged and discounted life expectancy is obtained as $d\bar{e}_{DA,d} = dP_b \cdot s_b \cdot \bar{e}_{DA,d} = 2.76 \cdot 10^{-3} \cdot 0.14 \cdot 19.4 = 0.075$ years or 2.7 days. Treatment costs are estimated as 600 US$/year. By inserting dP_b into the morbidity criterion, Eq. (3.20), one obtains:

$$C_\nu = -dg \leq 2.76 \cdot 10^{-3} \cdot \left[\frac{30000 \cdot 0.14}{0.19 \cdot 0.88} + 600 \right] = 71\,\text{US\$} \tag{7.9}$$

The overall amount per inhabitant and year that can reasonably be invested into a $PM_{2.5}$ reduction by $1\,\mu\text{g/m}^3$ is now obtained as

$$C = C_\mu + C_\nu \leq 61 + 71 = 132\,\text{US\$} \tag{7.10}$$

The results show that morbidity has a significant impact upon disability-adjusted life expectancy and thus upon the maximum affordable investment for air pollution mitigation measures.

7.4 Indoor Radon

The present example is a slightly adapted version of an example previously presented by Lentz & Rackwitz [64]. Radon (Rn) is a radioactive inert gas seeping from the underground as a natural process. According to Wahrendorf [142] it is one of the carcinogenics with the most detailed epidemiological evidence. Concentrations of the energy potential from alpha decay are usually given in so-called work levels (WL). Under radioactive equilibrium conditions typical for indoor situations the concentration in pico-Curie per litre is estimated as $1\text{WL} \approx 200$ pCi/l [28]. Accumulated exposure is commonly expressed in work level months, which is the exposure to 1WL during 170 hours, i.e., 1 WLM = 170 WLh = $3.5 \cdot 10^{-3}$ Jh/m^3.

Radon tends to accumulate in houses, leading to a rise in lung cancer rates. There are different countermeasures such as the installation of a ventilation system or the sealing of the foundation and other entry points. For a single occupancy house, the U.S. EPA [28] estimates the average cost of measures reducing concentrations to an unproblematic value of 2 pCi/l or lower as 1200 US$, with a typical range of values between 800 and 2500 US$. The EPA advocates action for indoor exposure levels of 4 pCi/l or higher and recommends considering measures for a level of 2 to 4 pCi/l. The average outdoor level in the U.S. is 0.4 pCi/l, while the average indoor level amounts to 1.3 pCi/l.

[1] Values from Table 6.4, prevalence of chronic bronchitis and respiratory illness prevalences added.

Presence time in a residential building is estimated as follows: At night time including breakfast time (9.5h/d) 100% of the residents are present, during working time including commuting (9.5h/d) 20% and, finally, 80% are at home during evening leisure time (5h/d). On weekends, (indoor) presence may be higher or lower depending on the season, but is comparable to that on week days on average (compare Section 5.2). Therefore, the time spent at home by an average citizen amounts to 15.4 h/d = 5621 h/yr so that a concentration of 1 WL leads to an exposure dose of $5621/170 \approx 33$ WLM/yr. Correspondingly, 1 pCi/l leads to $33/200 \approx 0.17$ WLM/yr. The BEIR IV committee (Biological Effects of Ionizing Radiation) of the U.S. National Research Council investigates the quantitative effect of radon on lung cancer mortality [89]. From the different approaches followed in [89], we choose one simple analysis using the model in (6.3) so that

$$\mu_{\text{Rn}}(a) = \mu(a) + \mu_{\text{LC}}(a)\, x_{\text{Rn}}\, \eta\, T_{cum} \tag{7.11}$$

The slope factor is given as $\eta = 0.0134$ per WLM, corresponding to $0.0134 \cdot 0.17 = 0.0023$ per pCi/l and years of exposure. Latency time T_μ amounts to 5 years. Age-dependent lung cancer background mortality $\mu_{\text{LC}}(a)$ for the U.S. is given in [89] and overall background mortality $\mu(a)$ in [144]. x_{Rn} is radon concentration. Inserting (7.11) in (6.6) yields the demographic constant J_{Rn} linking concentration changes to changes in life expectancy:

$$E_A\left[\frac{de_d(a)}{e_d(a)}\right] \approx J_{\text{Rn}}(T_\mu)\, dx_{\text{Rn}} = 4.64 \cdot 10^{-4} \cdot dx_{\text{Rn}} \tag{7.12}$$

Regardless of the present value for x_{Rn}, the reduction measures proposed by the EPA above lead to a rest concentration of $x_{\text{Rn, rest}} = 2$ pCi/l (or less) in a building. When inserting the results into the risk acceptance criterion (2.56), the safety investment is given as a constant (= 1200 US$) but has to be corrected by the average U.S. household size of 2.6 persons, i.e. 460 US$/person. Criterion (2.56) operates with yearly payments, so that the investment has to be annualised. For a planning horizon $t_s = 10$ yrs (existence of building or of radon reduction system e.g., ventilation) and a real discount rate $\gamma = 4.8\%$, a one time investment of 460 US$ corresponds to $460 \cdot (\exp[0.048] - 1)/(1 - \exp[-0.048 \cdot 10]) = 59$ US$/yr $= -dg$. The values for q and g are the same as in the preceding example. After a simple rearrangement of (2.56), the criterion yields the minimum concentration for which radon reduction is indicated in terms of loss of lifetime utility:

$$x_{\text{Rn, cr}} \geq \frac{dg\, q}{J_{\text{Rn}}\, g} + x_{\text{Rn, rest}} \tag{7.13}$$

In the U.S. (where the protection measure is suggested), socio-economic parameters are $g =$ 30,000 PPP US$and $q = 0.19$. $x_{\text{Rn, rest}}$ is the radiation level, under which no action is suggested, i.e. $x_{\text{Rn, rest}} = 2.8$ pCi/l, for the above-mentioned case of $C = 1200$ US$ per repair and $N_{pop} = 2.6$ persons per household. Figure 7.3 displays a study for other parameters, including the upper and lower values for protection measures suggested by EPA. Expressed in (undiscounted) life expectancies, a change from 2.8 to 2.0 pCi/l corresponds to approximately three extra life weeks. Given that the considerable variations of as a function of $x_{\text{Rn, cr}}$ with household size, actual investment cost for a specific house and time horizon, it can be argued

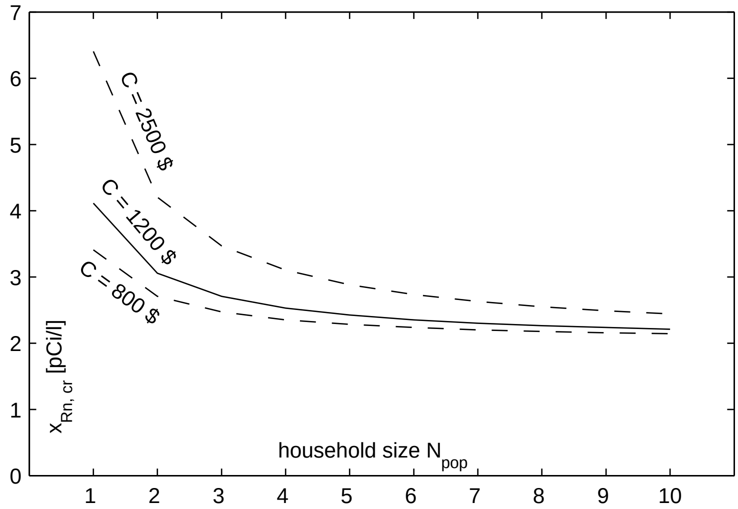

Figure 7.3: Parameter study of the critical radon level $x_{\mathrm{Rn,\,cr}}$ for which protection measures are recommendable

that the 4 pCi/l limit recommended by the EPA is a reasonable value. This is surprising because this recommendation does not appear to be based on any utility-based risk acceptability criterion. For a given building, introducing the actual parameters into criterion (7.13) derived from the WTP criterion will yield case-specific acceptable limit value.

7.5 Brominated Flame Retardants

Due to their chemical properties resembling 'solid fuel', synthetic materials play an important role in the formation process of full-scale apartment or building fires. These materials can be found in structural components or in household equipment, such as upholstered furniture, mattresses, electronic equipment or the padding underneath carpets. There are different types of flame retarding agents on the market, but in the case of synthetic materials only brominated flame retardants (BFRs) are applicable.

Wade et al. [141] show that upholstered items including mattresses are the first items ignited in 8% of all New Zealand residential fires. However, these 8% lead to 26% of all residential fire victims. In an average year, 22 persons lose their lives due to residential fires. For a total population of 4 million, this corresponds to a yearly death risk of $5.5 \cdot 10^{-6}$. Wade et al. show that changing the fire safety regulation and making flame retardant treatment mandatory for upholstered furniture leads to a drop to 15 fatalities, i.e. 7 victims less per year. This corresponds to a drop in raw mortality by $d\mu = \Delta = 1.75 \cdot 10^{-6}$. By assuming that people replace their furniture once in 15 years , they calculate yearly costs of 30 NZ$ ≈ 22 PPP US$ per household. Their estimate of approximately 1.2 million households for the whole of New Zealand leads to an average household size of 3.3 persons, which appears comparatively high for an industrialised country (typical values around 2.5 persons/household).

At this point, it is interesting to take the basic numbers and assumptions from [141] and apply the WTP criterion (2.48) to them. GDP per capita equals 21,800 PPP US$. Personal income disposable for consumption is given as g =16,700 PPP US$. Using life tables from the Human Mortality Database [144] and a discount rate of $\zeta = 1.2\%$, the age-averaged discounted life expectancy for New Zealanders amounts to $\bar{e}_d = 21.0$ years. The work time fraction w is equal to 0.114, so that $q = 0.19$.

Although changing fire proofing regulations leads to an immediate change in safety costs, it takes some time until untreated furniture is replaced by items containing flame retardants. The 15 year cycles estimated above correspond to a replacement rate of $k = 6.7\%$ per year. Using the exponential penetration growth model given in [141], the fraction of flame retarded furniture items after t years is calculated as $R(t) = 1 - \exp[-kt]$. Correspondingly, the change in mortality has the form

$$d\mu(t) = -\Delta\,(1 - \exp[-kt]) \tag{7.14}$$

In the present case, only two options will be considered: The mentioned regulation on flame protection is either prescribed or not. Therefore, it appears reasonable to calculate the result in a direct way as in (2.51) instead of establishing a general demographical constant as in (2.53). A relative change in life age-averaged discounted life expectancy $\bar{e}_d$ follows as

$$\begin{aligned} E_A\left[\frac{de_d(a)}{e_d(a)}\right] &= \int_0^{a_u} \frac{e_d(a, d\mu(t))}{e_d(a)} h(a,n)\,da - 1 \\ &= \int_0^{a_u} \frac{\int_a^{a_u} \exp\left[-\int_a^t \mu(\tau) - \Delta\,(1 - \exp[-k\tau^*]) + \gamma(\tau^*)\,dt\right] dt}{\int_a^{a_u} \exp\left[-\int_a^t \mu(\tau) + \gamma(\tau^*)\,dt\right] dt} h(a,n)\,da - 1 \\ &= 1.70 \cdot 10^{-5} \end{aligned} \tag{7.15}$$

Then, inserting into the WTP criterion (2.48) yields

$$-dg \leq \frac{g}{q}\, E_A\left[\frac{de_d(a)}{e_d(a)}\right] = 1.5\,\text{PPP US\$} \tag{7.16}$$

When dividing the 22 PPP US$ yearly safety investment by the average household size, we obtain $-dg = 6.7 > 1.5$ PPP US$, which violates the WTP criterion.

This result is in good accordance with that of Wade et al. [141] themselves, who use 2.6 mill. NZ$ as an estimate for the value of a human life as opposed to the 9.8 mill. NZ$ paid per life saved in the course of the proposed regulation. However, [141] considers not only the negative effect upon dg (due to fire safety expenditures), but also the positive effect (due to a reduction in destroyed homes and fire brigade actions). This effect equals 0.82 NZ$ or 0.57 PPP US$ per person and year once all furniture has been replaced and proportionally less at earlier points in time, following the same scheme as in (7.14) for the mortality reduction effect. The concept in criterion (7.16) does not consider time dependent dg values. However, it is obvious that even if the full final positive effect were taken into account from the beginning, $-dg = 6.7 - 0.57$= 6.13 PPP US$ would still exceed the acceptable value of 1.5 PPP US$ by far.

BFRs are also known for their toxicity causing neurodevelopmental deficits as well as hypothyroidism [79]. Among the group of of polybrominated diphenyl ethers (PBDEs), partially brominated ones such as penta- and octa-BDEs are more harmful than fully brominated deka-BDEs. In the above-mentioned case of furniture upholstering, toxicity did not require closer attention after the cost of applying BFRs turned out violate the WTP criterion. However, the situation can be very different when considering electronic devices used in households and offices. First, the plastic enclosure of TV sets or personal computers (being the most inflammable part) have a much lower mass than that of a typical furniture item. Moreover, BFRs amount only to a few percentage points of the plastic enclosure mass, as opposed to upholstery, where they contribute up to 30% of total mass. Therefore, the amount of flame retardants required and the associated costs are considerably smaller. Second, electronic devices can ignite spontaneously due to faults in their electric components or due to overheating.

In the United States, the use of BFRs is much more common than in the Europe due to stricter fire safety regulations in the U.S. as well as to enduring toxicological concerns in the EU, leading to a recent ban of some BFR types. It is difficult to estimate the overall number of lives saved by using BFRs in the U.S. In 2002, building fires are estimated to have caused 2980 deaths [135]. According to Muir & Alaee [79], manufacturers claim that BFRs prevented 2600 additional deaths in 1994, which is probably an exaggeration. At the current state of toxicological knowledge it is hard to quantify the consequences of BFRs to humans. Muir & Alaee performed an analysis comparing the cost of purchasing BFRs as well as the socio-economic costs of IQ losses with its life saving effects. By assuming a relatively high WTP of 2 to 5 mill. US$ they found out that costs exceed benefits by far, even if the life-saving effect of BFRs were as high as indicated by the manufacturers.

Chapter 8

Conclusions

8.1 Results

The thesis presents an approach for acceptability analysis and the corresponding human consequence models as an integrated unit.

The central concept and criterion is the implied willingness to pay (WTP) in order to avert potential fatalities. It allows to assess the acceptability of risk and the affordability of risk mitigation measures. The WTP is derived from the personal utility individuals obtain from consuming (disposable income) and being alive (life expectancy) and from the relative weight of the two factors. This derivation is undertaken in three different ways: by means of classical socio-economic utility theory as applied in health-policy analysis, by means of the so-called life quality method and by means of direct empirical evidence.

However, the life quality method provides the only rational derivation for quantifying the elasticity of marginal consumption, which is a key parameter in the WTP concept. In order to implement the WTP approach, it was necessary to deal with a number of preliminary questions, including population statistics and various questions from economics such as production theory and discounting. The choice of the discount rate directly influences intergenerational equality and sustainability in general. Therefore, generation-adjusted discounting is proposed in the context of acceptable decision making.

Apart from potential fatalities (mortality), engineering decisions equally influence the prevalence of non-fatal human consequences (morbidity). These consequences include all types of disabilities, i.e. disease and injury. The WTP criterion is extended by adapting its main components, i.e. disposable income and life expectancy. Income is affected by therapy costs as well as loss of work time. Life expectancy can be replaced by the concept of disability-adjusted life expectancy (DALE). This concept is commonly used in social medicine and weights future life years by the severity of potential disabilities. In the WTP criterion, the influence of morbidity is frequently outweighed by mortality. However, the morbidity influence can become comparable or even dominating in case of chronic or permanent disabilities.

For practical applications the routine application of the WTP criterion is significantly facilitated by linearising the higher-order relation between changes in mortality and changes in life expectancy. In case of morbidity and its effect upon DALE, matters proved less complex. Changes in mortality and morbidity themselves depend upon changes in the failure rate (structures) or emission rate (toxic impacts). This link is established by human consequence models. In the case of acute failure of civil engineering facilities (e.g. collapse, dam failure, tunnel fire), a generalised methodology provides numerous advantages. The expected number of fatalities is determined by multiplying the expected number of people at risk (a sub-quantity of the total population at a given location) the probability of successful escape and the conditional probability of death (if escape has not been successful).

Apart from accelerating the analysis by introducing a standard procedure, it also improves the (often sparse) data situation. In the case of people at risk (presence model) and probability of successful escape, the approach allows to transfer data from one event type (e.g. collapse) to another (e.g. flood). Only the task of estimating the conditional probability of death is untransferrable due to the unique mechanisms involved. This type of synthesis was not necessary in the case of continuous toxic impacts and their consequences due to the greater amount of previous research. Here, the main task consisted in surveying and systematising some of the vast existing knowledge from environmental sciences and toxicology in order to make it more easily accessible for engineering decision problems.

The examples at the end of the thesis comprise two kinds of investigations: On one hand, it assesses existing safety recommendations from different countries with respect to their affordability. On the other hand, there are more academic examples aiming to illustrate those points which are not sufficiently covered in the chosen real-world examples.

One of the main achievements of the thesis certainly consists in the joint treatment of acceptability criteria and human consequence modelling. The discussion and extension of the WTP concept alone is of little help in practical applications as long as the corresponding human consequence models are not developed parallelly. Both aspects and their numerous points of contact were dealt with extensively. In fact, the whole of Chapter 4 (Application of the WTP Criterion) is dedicated to illustrating this interplay.

With respect to originality of work, it can be said that the innovative contributions of the thesis are mostly concentrated in Chapters 3, 4 and 5. This includes the extension of the WTP from mortality to morbidity (Chapter 3) which has had some vague predecessors but without a similar formulation, as well as the establishment of a generalised methodology for estimating loss-of-life (Chapter 5) or the consideration of latency in the WTP criterion (Chapter 4). Chapter 6 prepares toxicological information in order to make it accessible for engineering decision making.

8.2 Outlook

Open questions persist mostly at those points where the thesis deals with other fields of research such as economics or environmental sciences. One of the most prominent points is the discrepancy between utility-theoretic and empirical values for the WTP in order to avert one fatality ('value of a statistical life' or VSL). Presumably, empirical values are higher because they implicitly consider the human aversions linked to risk perception. The question whether to consider this effect or not is basically a philosophical and political choice. However, it can be said that accounting for disaster aversion may lead to decisions that do not meet the criterion of affordability.

It has not been possible to find a second analytical or empirical approach in order to verify the value for the elasticity of marginal consumption brought forward by the life quality method. This issue is central to the WTP criterion, yet it is far beyond the realm of engineering decision-making. With respect to consumption, it would be of equal interest to investigate the age-dependent consumption path of average individuals.

As for human consequence modelling, data are still sparse, especially in the case of acute failure of civil engineering facilities. These gaps can only be filled by continuing analysis of actual disasters. In the case of toxicological consequence models—which are beyond the civil engineering scope—it was equally observed that surprisingly few exposure-response relations have been developed to a point that permits their usage in quantitative decision analysis.

Symbols and Abbreviations

Latin Characters

a	... age
a_u	... assumed maximum attainable age (upper bound in lifetable calculations)
A	... set of all possible ages a
	... severity (extent) of an event (on a magnitude, intensity or damage scale)
	... technology factor in Cobb-Douglas production function
b	... one of several types of disability (disease or injury)
B	... benefit
c	... consumption
C	... cost
d	... *index:* discounting
	... *number:* water depth
D	... *index:* loss of life, death (*event*)
	... *number:* damage
D_b	... average duration of disability b
$e(0), e_0$	... life expectancy at birth
$e(a)$	... (remaining) life expectancy at age a
E	... expected value
$\bar{e}_d$	... age-averaged life expectancy
$\bar{e}_d$	... age-averaged discounted life expectancy
F	... failure (*event*)
g	... average yearly income
G	... WTP in order to avert one fatality (also referred to as so-called 'value of a statistical life' or VSL)
$h(a,n)$	... age distribution of a population
H	... damage cost
H_D	... compensation payment in case of a fatality
i	... one of several sub-areas
$i(a)$	... age-dependent payments and reseceipts to and from a life insurance
$I(a)$	... probability of surviving up to age a, adjusted for disabilities
j	... one of several individuals
J	... linearisation coefficient in LQI criterion
k	... condtional probability of being killed (i.e. given no successful escape)
	... wealth
K	... capital

l	... leisure time (total amount of life not spent in paid work)
L	... life time utility
L_w, L_q	... life quality index (LQI)
n	... population growth rate
N	... number of persons
N_{PAR}	... number of persons at risk
N_D	... expected yearly number of fatalities
$N_{D\|F}$	... expected number of fatalities given failure
p	... safety-relevant design parameter of a facility
	... labour productivity
	... pressure (in explosions)
$P_{\{index\}}(.)$	... probability of {index}, depending on (.)
$P(.)$	... probability of (.)
P_b, P_ι	... prevalence of disability b / of disability belonging to severity class ι
q	... *index:* one of several death or disability causes (toxic substance, heat etc.)
	... *exponent:* exponent of utility function
Q	... successful escape (*event*)
r	... failure rate (of a facility), also written as r
	... auxiliary exponent in the LQI derivation
R	... resistance
s	... severity weight of a disability;
	... auxiliary exponent in the LQI derivation;
	... number of time interval in iterative approach
S	... load
$S(a)$	... probability of surviving up to age a
$S_d(a)$	... discounted probability of surviving up to age a
t	... time
t_s	... planned service life (of a facility)
T	... time span, period
T_{cum}	... accumulation time
T_{rea}	... reaction time
T_ν, T_μ	... latency period (for morbidity and mortality effetcs, respectively)
T_Q	... required escape time
T_W	... available escape time
u	... speed of an escaping person
$u(.)$	... utility of (.)
v	... speed of a flood wave
V	... volume (of a building)
w	... average fraction of lifetime spent at (paid) work
W	... auxiliary quantity for solving a vectorial WTP equation
W_0	... existence of a warning (*event*)
W_{dc}, W_{DC}	... perception of a warning (conditional/uncond.), given its existence (*event*)
W_{dc}, W_{DC}	... decision to escape (conditional/uncond.), given warning and perception (*event*)
W_0	... existence of a warning (*event*)
x	... distance

x_q	... concentration of toxic substance q
y	... *index:* yearly, per year
	... *number:* distance (in escape modelling)
Y	... output of an economy
z	... mortality scheme
$z_{\{index\}}$	... sub-factor of k
Z	... profit (objective function)

Greek Characters

α	... exponent in Cobb-Douglas prodution function;
	... exponent in disaster aversion function
β	... exponent in Cobb-Douglas prodution function
χ	... calendar year (period)
δ	... constant in proportional mortality scheme
Δ	... constant in additive mortality scheme
η	... slope factor in exposure-response relationships
ε	... elasticity of marginal consumption
$\gamma(t)$	... discount rate (continuous discounting)
$\gamma'(t)$	... discount rate (yearly discounting)
$\phi(.)$	... density of standard normal distribution function
$\Phi(.)$	... standard normal distribution function
ι	... severity class of a disability
κ	... cost per time unit
Λ	... labour
λ	... Poissonian rate
μ	... raw (background) mortality
	... mean value
$\mu(a)$	... age-dependent (background) mortality
$\nu(a)$	... age-dependent morbidity
ϑ	... birth year
ρ	... pure time preference rate
τ	... substitute for t in integrals
τ^*	... $= \tau - a$, see footnote 2 (p. 11)
ζ	... economic growth rate

Abbreviations

CBA	... cost-benefit analysis
DALE	... disability-adjusted life expectancy
FID	... fractional incapacitating dose
LOL	... loss of life
LQI	... life quality index

PAR	… persons at risk
VSL	… so-called 'value of a statistical life' (*preferred terminology:* 'WTP in order to avert one fatality' → see symbol G)
WTP	… willingness to pay
WTA	… willingness to accept a payment

Bibliography

[1] *Grundgesetz der Bundesrepublik Deutschland (Basic Law for the Federal Republic of Germany)*, 1949. http://www.gg-artikel.de/.

[2] *Haftpflichtgesetz (German Liability Act)*, 1978.

[3] K.J. Arrow. Intergenerational equity and the rate of discount in long-term social investment. Technical Report 97-005, Stanford University, Department of Economics, 1997.

[4] W.B. Arthur. The economics of risk to life. *The American Economic Review*, 71(1):54–64, 1981.

[5] R.E. Barlow and F. Proschan. *Mathematical Theory of Reliability.* Wiley, New York, 1965.

[6] S. Bayer. Generation-adjusted discounting in long-term decision-making. *International Journal of Sustainable Development*, 6(1):133–145, 2003.

[7] S. Bayer and D. Cansier. Intergenerational discounting: A new approach. Tübinger Diskussionsbeitrag 145, University of Tübingen, Department of Economics, Tübingen, 1998.

[8] T. Bedford. Keynote lecture: Multi-attribute utility theory and FN-criteria. In K. Kołowrocki, editor, *Advances in Safety and Reliability – Proc. ESREL '05*, pages 157–166, London, 2005. Taylor & Francis Group.

[9] H. Bielenski, G. Bosch, and A. Wagner. *Working Time Preferences in 16 European Countries.* European Foundation for the Improvement of Working and Living Conditions, Office for Official Publications of the European Communities, Luxembourg, 2002.

[10] L.C. Boer. Behaviour by motorists on evacuation of a tunnel. TNO Report TM-02-C034, TNO Human Factors, Soesterberg (NL), 2002.

[11] British Health & Safety Executive. *Reducing risks, protecting people. HSE's decision-making process*, 2001. http://www.hse.gov.uk/risk/theory/r2p2.pdf.

[12] I.N. Bronstein, K.A. Semendjajew, G. Musiol, and H. Mühlig. *Taschenbuch der Mathematik (in German).* Verlag Harri Deutsch, Thun and Frankfurt a.M., 5^{th} edition, 2000.

[13] J. Broome. *Counting the Cost of Global Warming.* White Horse Press, Cambridge, U.K., 1992.

[14] C.A. Brown and W.J. Graham. Assessing the threat to life from dam failure. *Water Resources Bulletin*, 24(6):1301–1309, 1988.

[15] E.J. Calabrese. and L.A. Baldwin. A toxicological basis to derive a generic interspecies uncertainty factor. *Environmental Health Perspectives*, 102(1):14–17, 1994.

[16] C.W. Churchman. *Challenge to Reason.* McGraw-Hill Book Company, New York, 1968.

[17] CIA. *CIA Factbook.* http://www.cia.gov/cia/publications/factbook/, viewed in 2006.

[18] A. Coburn and R. Spence. *Earthquake Protection.* John Wiley & Sons Ltd., Chichester, 2nd edition, 2002.

[19] A.W. Coburn, R.J.S. Spence, and A. Pomonis. Factors determining human casualty levels om earthquakes: Mortality prediction in building collapse. In *Proc. 10th World Conference on Earthquake Engineering*, pages 5989–5994, Rotterdam, 1992. Balkema.

[20] D.L. Costa and M.E. Kahn. Changes in the value of life, 1940–1980. NBER Working Paper Series No. 9396, National Bureau of Economic Research, Cambridge, MA, 2002. http://www.nber.org/papers/w9396/.

[21] D.R. Cox. *Renewal Theory.* Methuen, London, 1962.

[22] M.L. Cropper and F.G. Sussman. Valuing future risks to life. *Journal of Environmental Economics and Management*, 19:160–174, 1990.

[23] A. de Blaeij, R.J.G.M. Florax, P. Rietveld, and E. Verhoef. The value of statistical life in road safety: A meta-analysis. Tinbergen Institute Discussion Paper TI 2000-089/3, Tinbergen Institute, Amsterdam, 2000.

[24] M.L. DeKay and G.H. McClelland. Predicting loss of life in cases of dam failure and flash flood. *Risk Analysis*, 13(2):193–205, 1993.

[25] DIN Deutsches Institut für Normung e.V., Berlin. *Eurocode – Basis of structural design. German version EN 1990:2002*, 2002.

[26] O. Ditlevsen. Life quality index revisited. *Structural Safety*, 26(4):443–451, 2004.

[27] M.E. Durkin and H.O. Murakami. Casualties, survival and entrapment on heavily damaged buildings. In *Proc. 9th World Conference on Earthquake Engineering*, pages 977–982. Elsevier, 1988.

[28] Environmental Protection Agency (EPA). *Radon-Specific Publications and Resources.* http://www.epa.gov/radon/pubs/index.html, viewed in 2005.

[29] European Comission, National Institute of Public Health and the Environment (RIVM), the Netherlands. *EUSES – The European Union System for the Evaluation of Substances*, 1996.

[30] European Comission, DG XII, Brussels. EXTERNE *Core Project: Methodology*, 1995. http://www.externe.info/reportex/vol2.pdf.

[31] European Comission, DG XII, Brussels. EXTERNE *Core Project: Extension of the Accounting Framework*, 1997. http://externe.jrc.es/meth2.pdf.

[32] European Comission, DG XII, Brussels. EXTERNE *Core Project: Methodology 1998 Update*, 1999. http://www.externe.info/reportex/vol7.pdf.

[33] European Comission, DG XII, Brussels. *An applied integrated environmental impact assessment framework for the European Union (*GREENSENSE*)*, 2003.

[34] European Industrial Relations Observatory. *Working Time Developments. Annual Update*, 2001.

[35] J.M. Evans, D.C. Lippoldt, and P. Marianna. Trends in working hours in OECD countries. report no. 45, OECD, Paris, 2001.

[36] M.H. Faber. Aspects of sustainable decision making for design and maintenance. In *Proc. AMAS Course on Reliability-Based Design and Optimisation RBO '03*, pages 157–176, Warsaw, 2003. Centre of Excellence, The Polish Academy of Science.

[37] M.H. Faber and K. Nishijima. Aspects of sustainability in engineering decision analysis. In *Proc.* 9^{th} *ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability*. ASCE, 2004.

[38] G. Feichtinger. *Bevölkerungsstatistik (in German)*. Walter de Gruyter, Berlin, 1973.

[39] R. Friedrich and P. Bickel, editors. *Environmental External Costs of Transport*. Springer, Berlin, 2001.

[40] Deutsches Zentrum für Luft- und Raumfahrt (DLR). Pressemitteilung zu akuten Schlafstörungen durch Nachtfluglärm (in German). press release, march 2004.

[41] E.-S. Georgescu. Assessment of evacuation possibilities of apartments in multistoried buildings during earthquakes or subsequent fires, in view of earthquake preparedness. In *Proc.* 9^{th} *World Conference on Earthquake Engineering*, volume 7, pages 687–692, 1988.

[42] H.U. Gerber. *Life Insurance Mathematics*. Springer Verlag, Berlin, 1995.

[43] R. Gerlagh and B.C.C. van der Zwaan. The effects of ageing and an environmental trust fund in an overlapping generations model on carbon emission reductions. *Ecological Economics*, 36(2):311–326, 2001.

[44] H.J. Geyer, K.-H. Schramm, P.-O. Darnerud, M. Aune, E.A. Feicht, K.W. Fried, B. Henkelmann, D. Lenoir, P. Schmid, and T.A. McDonald. Terminal elimination half-lives of the brominated flame retardants TBBPA, HBCD, and lower brominated PBDEs in humans. *Organohalogen Compounds*, 66:3867–3872, 2004.

[45] W.J. Graham. A procedure for estimating loss of life caused by dam failure. Technical Report DSO-99-06, U.S. Bureau of Reclamation, Dam Safety Office, Denver, CO, 1999.

[46] D.N.D. Hartford, H. Assaf, and I. Kerr. The reality of life safety consequence classification. Technical report, BC Hydro, Burnaby, BC, Canada, 2000.

[47] A.M. Hasofer and R. Rackwitz. Time-dependent models for code optimization. In R.E. Melchers and M.G. Stewart, editors, *Proc. ICASP '99*, pages 151–158, Rotterdam, 2000. Balkema.

[48] O. Hayashi. Individual differences between leaders in a simulated fire. *Journal of Natural Disaster Science*, 14(1):1–15, 1992.

[49] P. Hofstetter and J.K. Hammitt. Selecting human health metrics for environmental decision-support tools. *Risk Analysis*, 22(5):965–983, 2002.

[50] S. Hurd. The impact of COPD on lung health worldwide. *Chest*, 117(2 Suppl.), 2000.

[51] E.J. Hutchinson and P.J.G. Pearson. An evaluation of the environmental and health effects of vehicle exhaust catalysts in the United Kingdom. *Environmental Health Perspectives*, 112(2):132–141, 2004.

[52] M. Jakob, H. Weatherly, and M. Currie. Canyon creek alluvial fan risk assessment. Technical report, Kerr Wood Leidal Associates Ltd., Burnaby, BC, Canada, 2003. http://www.co.whatcom.wa.us/publicworks/pdf/riverflood/canyon_creek_final.pdf.

[53] I. Jakobovits. *Jewish Medical Ethics*. Bloch, New York, 1975.

[54] M. Johannesson, P.-O. Johansson, and K.-G. Löfgren. On the value of a statistical life: Blips versus parametric changes. *Journal of Risk and Uncertainty*, 15:221–239, 1997.

[55] P.-O. Johansson. Is there a meaningful definition of the value of a statistical life? *Journal of Health Economics*, 20:131–139, 2001.

[56] S.N. Jonkman. A method for the estimation of casualties caused by accidents in tunnels. In *Proc. ESREL 2004*, pages 2666–2672, London, 2004. Springer-Verlag.

[57] S.N. Jonkman, P.H.A.J.M. van Gelder, and J.K. Vrijling. Loss of life models for sea and river floods. In B. Wu et al., editor, *Proc. Flood Defense 2002*, pages 196–206, New York, 2002. Science Press.

[58] S.N. Jonkman, P.H.A.J.M. van Gelder, and J.K. Vrijling. An overview of quantitative risk measures for loss of life and economic damage. *Journal of Hazardous Materials*, A99:1–30, 2003.

[59] J. Kanda and K. Nishijima. Scope of insurance premium for residential houses against seismic risk in japan. In *Proc. IFED '04*, 2004. http://www.ifed.ethz.ch/events/Forum04/Kanda_paper.pdf.

[60] I. Kant. *Grundlegung zur Metaphysik der Sitten (in German)*. Johann Friedrich Hartknoch, Riga, 1786.

[61] T. Karvonen, A. Hepojki, J. Kotola, and H.-K. Huhta. The use of physical models in dam-break flood analysis. RescDam Project, Appendix 2, Helsinki University of Technology, 2000.

[62] P.J. Kelly. *Utilitarianism and Distributive Justice: Jeremy Bentham and the Civil Law.* Clarendon Press, Oxford, 1990.

[63] O. Kübler. *Applied Decision-Making in Civil Engineering.* PhD thesis, Swiss Federal Institute of Technology Zurich (ETH), 2006. Dissertation ETH No. 16394.

[64] A. Lentz and R. Rackwitz. Consequences and acceptability of long-term exposure to toxic substances in air, water and soil. *accepted by International Journal of Risk Assessment and Management*, 2006. pre-published in Proc. IFED '04, http://www.ifed.ethz.ch/events/Forum04/Lentz_paper.pdf.

[65] W. Lenzen. Der Wert des Lebens – Möglichkeit und Grenzen eines pragmatischen Ansatzes (in German). In *The Role of Pragmatics in Contemporary Philosophy*, pages 352–367. Hölder-Pichler-Tempsky, Vienna, 1998.

[66] N. Lind and D. Hartford. Probability of human instability in a flooding: A hydrodynamic model. In *Proc. ICASP 8*, pages 1151–1156, Rotterdam, 2000. Balkema.

[67] N.C. Lind. Target reliabilities from social indicators. In *Proc. ICOSSAR93*, pages 1897–1904, Rotterdam, 1994. Balkema.

[68] N.C. Lind. Three citeria of acceptable risk. *Structural Concrete*, 4(1):13–18, 2003.

[69] A.D. Lopez, J. Salomon, O. Ahmad, C.J.L. Murray, and D. Mafat. Life tables for 191 countries: Data, methods and results. GPE Discussion Paper 9, World Health Organisation, Geneva, 2001.

[70] R. Lutter, J.F. Morrall, and W.K. Viscusi. The cost-per-life-saved cutoff for safety enhancing regulations. *Economic Inquiry*, 37(4):599–608, 1999.

[71] A. Maddison. *Monitoring the World Economy 1820–1992.* OECD, Paris, 1995.

[72] N.G. Mankiw. *Macroeconomics.* Worth Publishers, New York, 3rd edition, 2003.

[73] A. Markandya and M. Pavan. *Green Accounting in Europe – Four Case Studies.* Kluwer Academic Publishers, Dordrecht (NL), 1999.

[74] D.M. McClelland and D.S. Bowles. Life-loss estimation: What can we learn from case histories? In *Proc. Australian Committee on Large Dams (ANCOLD) Annual Meeting*, 1999.

[75] R.C. Merton. *Continuous-Time Finance.* Blackwell, 1990.

[76] W.P.M. Merx. The consequences of explosion effects on humans. In *Methods for the determination of possible damage to people and objects from releases of hazardous materials*, number CPR 16E ('Green Book'). Directorate-General of the Dutch Ministry of Social Affairs and Employment, Committee for the Prevention of Disasters, The Hague, 1989.

[77] T.R. Miller. Variations between countries in values of statistical life. *Journal of Transport Economics and Policy*, 34(2):169–188, 2000.

[78] J.R. Mrozek and L.O. Taylor. What determines the value of life? a meta-analysis. *Journal of Policy Analysis and Management*, 21(2):253–270, 2002.

[79] T. Muir and M. Alaee. Costs and benefits of brominated flame retardants (bfrs) and alternatives. *Organohalogen Compounds*, 58:237–240, 2002.

[80] T. Muir and M. Zegarac. Societal costs of exposure to toxic substances: Economic and health costs of four case studies that are candidates for environmental causation. *Environmental Health Perspectives*, 109(Suppl. 6):885–903, 2001.

[81] H.O. Murakami. A simulation model to estimate human loss of collapsed buildings in an earthquake. In *Proc. 10th World Conference on Earthquake Engineering*, pages 5969–5974, Rotterdam, 1992. Balkema.

[82] H.O. Murakami. Chances of occupant survival and SAR operation in the buildings collapsed by the Great Hanshin Earthquake. In *Proc. 11th World Conference on Earthquake Engineering*, volume 7, pages 681–685. Elsevier, 1996.

[83] C.J.L. Murray and A.D. Lopez. Regional patterns of disability-free life expectancy and disability-adjusted life expectancy: Global burden of disease study. *The Lancet*, 349:1347–1352, 1997.

[84] J.S. Nathwani, N.C. Lind, and M.D. Pandey. *Affordable Safety by Choice: The Life Quality Method*. Institute of Risk Research, University of Waterloo, Ontario, Canada, 1997.

[85] J.S. Nathwani and J. Narveson. Three principles for managing risk in the public interest. *Risk Analysis*, 15(6):615–626, 1995.

[86] W.D. Nordhaus. *Managing the Global Commons*. MIT Press, Cambridge, MA, 2004.

[87] OECD. *OECD Statistics Portal*. http://www.oecd.org, viewed in 2006.

[88] OECD, Paris. *OECD Employment Outlook*, 2005.

[89] Committee on the Biological Effects of Ionizing Radiation (BEIR IV). *Health Risks of Radon and Other Internally Deposited Alpha-emitters – BEIR IV*. National Academy Press, Washington D.C., 1988.

[90] M.D. Pandey. A discussion of derivation and calibration of the life-quality index. In G. Augusti, G.I. Schuëller, and M. Ciampoli, editors, *Proc. 9th Int. Conf. Structural Safety and Reliability (ICOSSAR)*, pages 1–8, Rotterdam, 2005. Millpress.

[91] M.D. Pandey and J.S. Nathwani. Canada wide standard for particulate matter and ozone: Cost-benefit analysis using a life-quality index. *Risk Analysis*, 23(1):55–68, 2003.

[92] M.D. Pandey and J.S. Nathwani. Life quality index for the estimation of societal willingness-to-pay for safety. *Structural Safety*, 26(2):181–199, 2004.

[93] M. Pelekis, M.J. Nicolich, and J.S. Gauthier. Probabilistic framework for the estimation of the adult and child toxicokinetic intraspecies factors. *Risk Analysis*, 23(6):1239–1255, 2003.

[94] R.J.M. Perenboom, L.M. van Herten, H.C. Boshuizen, and G.A.M. van den Bos. Trends in disability-free life expectancy. *Disability and Rehabilitation*, 26(7):377–386, 2004.

[95] M. Persson. Quantitative risk analysis procedure for the fire evacuation of a road tunnel – an illustrative example. Report 5096, Department of Fire Safety Engineering, Lund University, Lund, Sweden, 2002.

[96] C.A. Pope, R.T. Burnett, M.J. Thun, E.E. Calle, D. Krewski, K. Ito, and G.D. Thurston. Lung cancer, cardiopulmonary mortality and long-term exposure to fine particulate air pollution. *Journal of the American Medical Association*, 287(9):1132–1141, 2002.

[97] C. Price. *Time, Discounting & Value*. Blackwell, Cambridge, U.K., 1993.

[98] D. Proske. *Katalog der Risiken (In German)*. Self-publisher, Dresden, 2004.

[99] A. Rabl. Discounting of long-term cost: What would future generations prefer us to do? *Ecological Economics*, 17:137–145, 1996.

[100] R. Rackwitz. Optimization – The basis of code making and reliability verification. *Structural Safety*, 22(1):27–60, 2000.

[101] R. Rackwitz. Discounting for optimal and acceptable technical facilities. In *Proc. ICASP 9*, pages 725–734, 2003.

[102] R. Rackwitz. Risk perception and rational risk control and management in our natural and technical environment. Memorandum to the Joint Committee on Structural Safety (JCSS), 2003.

[103] R. Rackwitz. Optimal and acceptable technical facilities involving risks. *Risk Analysis*, 24(3):675–695, 2004.

[104] R. Rackwitz. The philosophy behind the life quality index and empirical verification. Updated Memorandum to the Joint Committee on Structural Safety (JCSS), February 2005.

[105] R. Rackwitz, A. Lentz, and M. Faber. Sustainable civil engineering infrastructures by optimization. *Structural Safety*, 27(3):187–229, 2005.

[106] F.P. Ramsey. A mathematical theory of saving. *The Economic Journal*, 38:543–559, 1928.

[107] A. Reil-Held. *Einkommen und Sterblichkeit in Deutschland: Leben Reiche länger? (in German)*. Universität Mannheim, Institut für Volkswirtschaftslehre und Statistik, Mannheim, 2000.

[108] P. Reiter. RescDam – loss of life by dam failure. the rescdam lol method and its application to Kyrkösjärvi dam in Sejnäjoki. RescDam Project, Appendix 4, PR Water Consulting Ltd., 2001.

[109] S. Rosen. The value of changes in life expectancy. *Journal of Risk and Uncertainty*, 1:285–304, 1988.

[110] E. Rosenblueth and E. Mendoza. Reliability optimization in isostatic structures. *Journal of the Engineering Mechanics Division, ASCE*, 97, EM6:1625–1642, 1971.

[111] P.A. Samuelson and D.N. Nordhaus. *Economics.* McGraw-Hill International Editions, Singapore, 13th edition, 1989.

[112] M. Sánachez-Silva and R. Rackwitz. Implications of the life quality index in the seismic design of infrastructure systems. In D.M. Frangopol, E. Brühwiler, M.H. Faber, and B. Adey, editors, *Life-Cycle Performance of Deterioration Structures (Proc. LCC '03)*, pages 263–273, Reston, VA, 2003. ASCE.

[113] J. Schabacker, M. Bettelini, and C. Rudin. Cfd study of temperature and smoke distribution in a railway tunnel with natural ventilation system. In *Proc. 3rd International Conference on Tunnel Fires and Escape from Tunnels*, 2001.

[114] T.C. Schelling. Intergenerational discounting. *Energy Policy*, 23(4/5):395–401, 1995.

[115] G. Schurz. *Der Wert des menschlichen Lebens (in German).* Number 6 in IPS-Preprints. University of Salzburg, Department of Philosophy, Salzburg, 1995.

[116] J. Schwartz. Air pollution and daily mortality in Birmingham, Alabama. *American Journal of Epidemiology*, 137(10):1136–1147, 1993.

[117] J. Schwartz. Particulate air pollution and chronic respiratory disease. *Environmental Research*, 62:7–13, 1993.

[118] D.S. Shepard and R.J. Zeckhauser. Survival versus consumption. *Management Science*, 30(4):423–439, 1984.

[119] P. Singer. *Practical Ethics.* Cambridge University Press, 1990.

[120] P. Slovic. Perception of risk. *Science*, 236(17):280–285, 1987.

[121] Society of Fire Protection Engineering. *Engineering Guide to Human Behavior in Fire*, 2002.

[122] Society of Fire Protection Engineering. *SFPE Handbook of Fire Protection Engineering*, 3rd edition, 2002.

[123] R.M. Solow. *Growth Theory.* Clarendon Press, Oxford, 1970.

[124] R.J.S. Spence, A. Pomonis, D.J. Dowrick, and W.J. Cousins. Estimating human casualties in earthquakes: The case of Wellington. In *Proc. 6th SECED Conference on Seismic Design Practice into the Next Century*, pages 277–286, Rotterdam, 1998. Balkema.

[125] Statistisches Bundesamt, Wiesbaden. *Volkswirtschaftliche Gesamtrechnungen. Revidierte Ergebnisse 1970 bis 2003 (in German)*, 2004.

[126] H. Streicher. *Zeitvariante zuverlässigkeitsorientierte Kosten-Nutzen-Optimierung für Strukturen unter Berücksichtigung neuer Ansätze für Erneuerungs- und Instandhaltungsmodelle (in German).* PhD thesis, Technical University of Munich, Department of Civil Engineering and Surveying, 2004.

[127] H. Streicher, A. Joanni, and R. Rackwitz. Cost-benefit optimization and risk acceptability for existing, aging and maintained structures. *submitted to Structural Safety*, 2005.

[128] D.F. Sullivan. A single index for mortality and morbidity. *HSMHA Health Reports*, 86(4):347–354, 1971.

[129] V.M. Trbojevic. Risk criteria in EU. In K. Kołowrocki, editor, *Advances in Safety and Reliability – Proc. ESREL '05*, pages 1945–1952, London, 2005. Taylor & Francis Group.

[130] UN World Commission on Environment and Development ('Brundtland Comission'). *Our Common Future*, 1987.

[131] United Nations Development Programme (UNDP). *Human Development Report 2001. Making new technologies work for human development*. Oxford University Press, Oxford, 2001.

[132] United Nations Development Programme. *Human Development Report 2000*, 2000. http://hdr.undp.org/reports/global/2000/en/.

[133] United Nations Organisation (UNO). *Universal Declaration of Human Rights*, 1948. http://www.unhchr.ch/udhr/lang/eng.htm.

[134] U.S. Federal Emergency Management Agency (FEMA). HAZUS® *99 SR2 Technical Manual*, 1999. http://www.fema.gov/hazus/dl_sr2.shtm.

[135] U.S. Fire Administration/National Fire Data Center, FEMA. *Fatal Fires*, 2005. topical fire research series, Vol. 5 – Issue 1, http://www.usfa.fema.gov/.

[136] D. Usher. An imputation to the measure of economic growth for changes in life expectancy. In *National Accounting and Economic Theory: The Collected Papers of Dan Usher*. Edward Elgar Publishing Ltd., U.K., 1994.

[137] W.K. Viscusi and J.E. Aldy. The value of a statistical life: A critical review of market estimates throughout the world. NBER Working Paper Series No. 9487, National Bureau of Economic Research, Cambridge, MA, 2003. http://www.nber.org/papers/w9487/.

[138] W.K. Viscusi, W.A. Magat, and J. Huber. Pricing environmental health risks: Survey assessments of risk-risk and risk-dollar trade-offs for chronic bronchitis. *Journal of Environmental Economics and Management*, 21:32–51, 1991.

[139] W.K. Viscusi and M.J. Moore. Rates of time preference and valuations of the duration of life. *Journal of Public Economics*, 38:297–317, 1989.

[140] N. Vrisou van Eck and M. Kok. Standard method for predicting damage and casualties as a result of floods. Technical report, HKV Consultants, Lelystad, NL, 2001.

[141] C.A. Wade, M. Duncanson, D. O'Dea, and C.R. Duncan. Costs and benefits of regulating fire safety performance of upholstered furniture in nz. Research Report 35, New Zealand Fire Service Commission, 2003.

[142] J. Wahrendorf and H. Becher. *Quantitative Risikoabschätzung für ausgewählte Umweltkanzerogene (in German)*. Umweltbundesamt, Erich Schmidt Verlag GmbH, Berlin, 1990.

[143] R. Wijesekera, C. Halliwell, S. Hunter, and S. Harrad. A preliminary assessment of UK human exposure to poybrominated diphenyl ethers PBDEs. *Organohalogen Compounds*, 55:139–242, 2002.

[144] J.R. Wilmoth and V. Shkolnikov. *Human Mortality Data Base*. University of California, Berkeley and Max Planck Institute for Demographic Research, Rostock. http://www.mortality.org, viewed in 2005.

[145] World Bank. *World Development Indicators Database*. http://www.worldbank.org/, viewed in 2005.

[146] World Health Organisation. *WHO Research Tools*. http://www.who.int/research/en/, viewed in 2005.

[147] G.L. Yeo and C.A. Cornell. Building-specific seismic fatality estimation methodology. In *Proc. ICASP 9*, pages 765–771, 2003.

[148] D.T. Yung and N. Bénichou. How design fires can be used in fire hazard analysis. *Fire Technology*, 38(3):231–242, 2003.

[149] R. Zeckhauser. Time as the ultimate source of utility. *The Quarterly Journal of Economics*, 87(4):668–675, 1973.

Appendix A

Selected Conventional Acceptability Criteria

Most of the acceptability criteria currently at use do not take economic considerations or utility theory into account. In Section 1.2, these criteria have been summarised 'conventional' criteria. The present appendix provides a few example for this type of criterion.

Eurocode 1990:2002 [25] prescribes minimum values for the reliability of a building, depending on the potential number of people at risk. In Table A.1, reliability during a one-year period is indicated through a reliability index β. The corresponding yearly failure probability is obtained as $r = \Phi(-\beta)$.

The British Health & Safety Executive (HSE) provides regulations on maximum risks for general applications (industry etc.) [11]. Decisions that lead to one additional death in one million potentially affected persons per year are deemed 'broadly acceptable'. This corresponds to a mortality change of $d\mu = 10^{-6}$, which is very low compared to a typical (raw) background mortality of $\mu \approx 10^{-2}$. Apart from broadly acceptable risk levels, HSE defines a limit for 'tolerable' risks. In case of potential industrial accidents, the tolerability limit is set to $d\mu = 10^{-3}$ for workers at the respective facility. For the general population, it is lower by one order of magnitude, i.e. $d\mu = 10^{-4}$. Note that these numbers cannot be directly compared to the Eurocode numbers in Table A.1, which indicates failure rates instead of fatality rates. The relationship between the two is described in Section 4.2.

Another approach consists in constructing so-called FN diagrams. Here, the maximum tolerable frequency is plotted against the number of fatalities involved. In a recent publication, Trbojevic [129] compiled tolerability limits from different European standards (see Figure A.1).

Table A.1: Minimum reliabilities from Eurocode 1990:2002 [25] (yearly values)

reliability class	example	min. reliability index β	max. failure rate r
high (RC 3)	concert halls etc.	5.2	$1.0 \cdot 10^{-9}$
medium (RC 2)	residential or office bld.	4.7	$1.3 \cdot 10^{-6}$
low (RC 1)	agricultural buildings	4.2	$1.3 \cdot 10^{-5}$

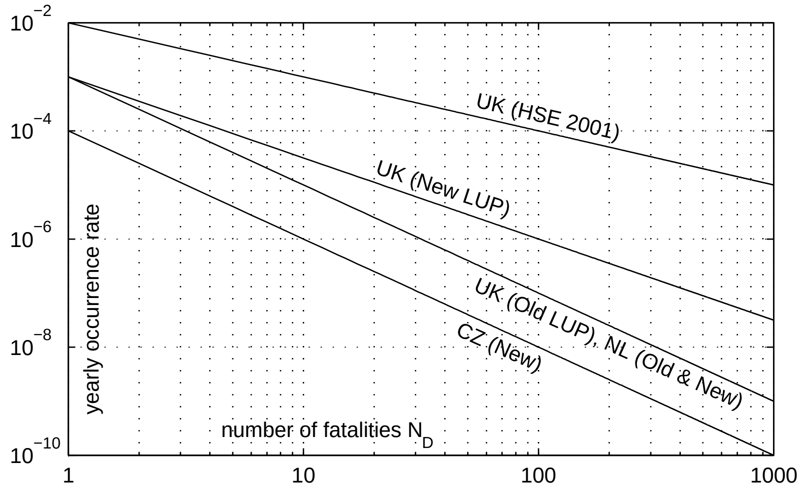

Figure A.1: FN curves derived from different regulations, after Trbojevic [129]

In many cases, a factor for *disaster aversion* is included in the regulations. According to this phenomenon, one accident with ten fatalities is less preferable than ten accidents causing one fatality each. Section 2.4 provides a brief introduction and discussion of the issue. Disaster aversion is usually quantified by raising the number of fatalities N_D to the mentioned factor, i.e. $(N_D)^\alpha$. When plotting the FN function on a double logarithmic scale as in Figure A.1, $-\alpha$ can be interpreted as the slope factor. FN criteria do not differentiate whether an accident with N_D fatalities occurs within a small or a large reference population. In this respect, the approach is not consistent with individual tolerability levels as those by HSE quoted above. Trbojevic presumes that the inconsistencies 'might perhaps be the reason why FN criteria are not officially used' [129].

Appendix B

Cost-benefit Analysis under Reliability Conditions

B.1 Overview

Cost-benefit analysis (CBA) is applied in order to assess the profitability of a project. For the case of civil engineering structures, Rosenblueth & Mendoza's [110] objective function has been introduced in (1.1) as

$$Z(p) = B(p) - C(p) - D(p) \tag{B.1}$$

where $Z(p)$ stands for profit, $B(p)$ for benefit (i.e. revenues), $C(p)$ for building costs and $D(p)$ for expected damage costs due to possible failure events. The design parameter p corresponds to some physical quantity such as the cross section of the reinforcement steel in a concrete structure. It decreases expected damage costs $D(p)$, but increases building costs $C(p)$. This effect is depicted in Figure B.1. Here, $B(p) = B$ is used as an assumption. This is permissible if down-times after a failure are short in relation to the time between failure events. If there is more than just one safety-relevant design parameter, this can be expressed by a design vector $\mathbf{p} = \{p_1, p_2, \ldots, p_n\}^{\mathrm{T}}$.

The objective function (B.1) can be used for two purposes,

1. to determine the parameter values p for which the structure is profitable or at least cost-neutral, i.e. $Z(p) \geq 0$.
2. to determine the optimal parameter value p^* yielding maximum profit $Z(p^*)$ (*cost-benefit optimisation*).

There are a number of different scenarios governing the application of the objective function. Rackwitz et al. [105] name the following ones:

- The facility is given up after service or failure.
- The facility is systematically replaced after failure.

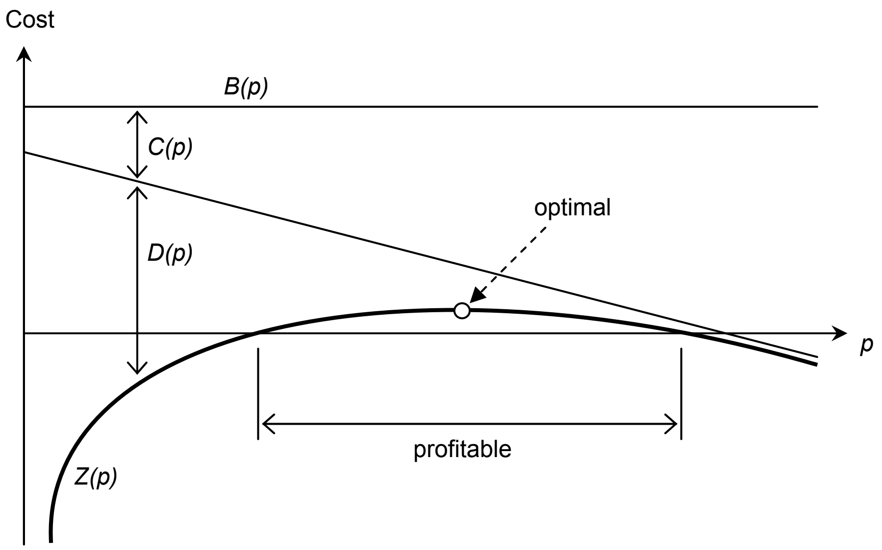

Figure B.1: Profitability as a function of reliability, after [16]

- The facility is repaired after deterioration.
- The facility is renewed because of obsolescence.

With respect to the scope of this thesis, the case of systematic replacement after failure is the most relevant one because it potentially involves consequences for life and limb. In this respect, deterioration and obsolescence are of secondary interest in the present context. The solutions for these cases are given in [105, 127, 126], but will not be reproduced here. The same applies to the strategy of giving up facilities after failure which is excluded at the beginning of Section 4.2.2.

Furthermore, [105] distinguishes between the case in which a facility fails during the construction phase or never (time-invariant reliability problem) and the case where failure can occur at a random point in time during service life (time-variant reliability problem. The following derivations focus on the latter case.

B.2 CBA for Systematic Replacement after Failure

If a structure fails, the economic damage includes the costs of reconstruction $C(p)$ as well as external damages H. The latter includes lost property of third parties and compensation costs for fatalities. The probability of such a failure event is expressed by a probability density function $f_n(t, p)$ describing the time until the n^{th} failure. The timespans between the successive failure events are assumed to be independent of each other. Theoretically, the number of possible events is infinite. In order to assess the cost of all future failure events in present values, it is necessary to discount each of them by a discount rate γ. In consequence, the expected damage is obtained as

Table B.1: Laplace transforms of some common probability density functions, from [100]

name	**density function** $f(t)$	**Laplace transform** $f^*(\gamma)$
deterministic	$\delta(a)$	$\exp[-a\gamma]$
uniform	$\frac{1}{b-a}$	$\frac{\exp[-a\gamma]-\exp[-b\gamma]}{\gamma(b-a)}$
exponential	$\lambda \exp[-\lambda t]$	$\frac{\lambda}{\gamma+\lambda}$
gamma	$\frac{\lambda^k}{\Gamma(k)} t^{k-1} \exp[-\lambda t]$	$\left(\frac{\lambda}{\gamma+\lambda}\right)^k$
normal	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{t-m}{\sigma}\right)^2\right]$	$\exp\left[\frac{1}{2}\gamma(\gamma\sigma^2 - 2m)\right]$ [a]

[a]two-sided Laplace transform

$$D(p) = (C(p) + H) \sum_{n=1}^{\infty} \int_0^{\infty} \exp[-\gamma t] f_n(t,p)\, dt \tag{B.2}$$

The integral in (B.2) can be read as the Laplace transform of another function, i.e.

$$f_n^*(\gamma,p) = \int_0^{\infty} \exp[-\gamma t] f_n(t,p)\, dt \tag{B.3}$$

Given the fact that ordinary renewal process can be written as the following convolution

$$f_n(t,p) = \int_0^{t} f_{n-1}(t-\tau,p) f(\tau,p)\, d\tau \tag{B.4}$$

it is possible to apply the rules describing Laplace transforms of convolutions:

$$\begin{aligned} f_n^*(\gamma,p) &= f_{n-1}^*(\gamma,p) f^*(\gamma,p) = f_{n-2}^*(\gamma,p)(f^*(\gamma,p))^2 = \dots \\ &= (f^*(\gamma,p))^n \end{aligned} \tag{B.5}$$

Inserting in (B.2) leads to

$$\begin{aligned} D(p) &= (C(p) + H) \sum_{n=1}^{\infty} (f^*(\gamma,p))^n \\ &= (C(p) + H) \frac{f^*(\gamma,p)}{1 - f^*(\gamma,p)} \\ &= (C(p) + H) h^*(\gamma,p) \end{aligned} \tag{B.6}$$

where $h^*(\gamma,p)$ is the Laplace transform of the renewal density. Table B.1 contains the Laplace transforms for some common distribution functions.

Construction costs can be separated into two components, basic costs C_0 that need to be afforded regardless of the safety level and safety-dependent costs $C_1(p)$:

$$C(p) = C_0 + C_1(p) \tag{B.7}$$

After the facility has been commissioned, the owner receives a permanent flow of revenues. As a simplifying assumption, benefits per time unit are modelled as time-independent, i.e. $b(t) = b$. Theoretically, the strategy of systematical reconstruction involves infinite service lives $t_s \to \infty$. This leads to a total benefit of

$$B = \int_0^\infty b \exp[-\gamma t] \, dt = \frac{b}{\gamma} \tag{B.8}$$

The objective function in (B.1) can now be recomposed as

$$Z(p) = \frac{b}{\gamma} - C(p) - (C(p) + H)h^*(\gamma, p) \tag{B.9}$$

Hasofer and Rackwitz [47] remarked that a project can only be profitable, if

$$\gamma < \beta - h^*(\gamma, p)\left(1 + \frac{H}{C(p)}\right) \tag{B.10}$$

with $\beta = b/C(p)$ is fulfilled. If the facility is built by a private owner, γ is dictated by the capital market (market rate). If the facility is publicly owned, γ is subject to the considerations in Section 2.3. It is obvious that the choice of a high discount rate will prevent many public projects from being realised.

Another constraint on discount rates is imposed by the mathematics in (B.2) to (B.10): The formulations are only defined for $\gamma > 0$ and do not converge otherwise.

Appendix C

Selected Models for Loss-of-Life Estimation

C.1 The HAZUS Presence Model

In the technical manual of FEMA's HAZUS programme [134], chapter 13 deals extensively with the question of presence modelling (apart from a less in-depth estimation on death probabilities in building collapse). Similarly to the models of Reiter [108] and Hartford et al. [46] described in Section 5.2, the day is subdivided into three time blocks. However, HAZUS uses commuting (i.e. 5 p.m.) instead of leisure time at home as the third block apart from working time and sleeping time. Table C.1 is a reproduction of the original table in the manual and forms the quintessence of the HAZUS presence model. The model is generally adapted to a typical U.S. situations. Therefore, specific values should be carefully reviewed before transferring them to other countries with different settlement structures. Nevertheless, the methodology as such appears very elaborate and can be applies to almost any situation.

The model is based upon three different quantities which need to be multiplied in order to obtain the number of people present at a given location and a given time:

1. The size of the respective population group. Most people belong to several groups (residential population, working population, commuters etc.), so that all groups taken together include more than 100% of the total population. The total population in an area is obtained from census data. From there, the size of the different groups is estimated; indicated as text in brackets in Table C.1.

2. The proportion of the population located within a building (as opposed to those outside, i.e. in front of the building); indicated as numbers in brackets in Table C.1.

3. The proportion of the population realising their affiliation to a population group at a given time. People permanently belong to several groups, but can only realise these affiliations one at a time (e.g. *either* residential affiliation *or* commercial affiliation *or* affiliation to commuting etc.); indicated as numbers without brackets in Table C.1.

Table C.1: Distribution of people in census tract, from HAZUS [134]

occupancy	**2:00 a.m.**	**2:00 p.m.**	**5:00 p.m.**
indoors			
residential	(0.999)0.99(NRES)	(0.70)0.75(DRES)	(0.70)0.5(NRES)
commercial	(0.999)0.02(COMW)	(0.99)0.98(COMW) + (0.80)0.20(DRES) + 0.80(HOTEL) + (0.80)VISIT	0.98[0.50(COMW) + 0.10(NRES) + 0.70(HOTEL)]
educational		(0.90)0.80(AGE_16) + 0.80(COLLEGE)	(0.80)0.50(COLLEGE)
industrial	(0.999)0.10(INDW)	(0.90)0.80(INDW)	(0.90)0.50(INDW)
hotels	0.999(HOTEL)	0.19(HOTEL)	0.299(HOTEL)
outdoors			
residential	(0.001)0.99(NRES)	(0.30)0.75(DRES)	(0.30)0.5(NRES)
commercial	(0.001)0.02(COMW)	(0.01)0.98(COMW) + (0.20)0.20(DRES) + (0.20)VISIT + 0.50(1–PRFIL) 0.05(POP)	0.02[0.50(COMW) + 0.10(NRES) + 0.70(HOTEL)] + 0.50(1–PRFIL) [0.05(POP) + 1.0(COMM)]
educational		(0.10)0.80(AGE_16) + 0.20(COLLEGE)	(0.20)0.50(COLLEGE)
industrial	(0.001)0.10(INDW)	(0.10)0.80(INDW)	(0.10)0.50(INDW)
hotels	0.001(HOTEL)	0.01(HOTEL)	0.001(HOTEL)
commuting			
commuting in cars	0.005(POP)	(PRFIL)0.05(POP)	(PRFIL)0.05[(POP) + 1.0(COMM)]
commuting using other modes		0.50(PRFIL)0.05(POP)	0.50(PRFIL) [0.05(POP) + 1.0(COMM)]

The abbreviations in Table C.1 are defined in [134] as follows:

POP ... census tract population taken from census data
DRES ... daytime residential population inferred from census data
NRES ... nighttime residential population inferred from census data
COMM ... people commuting inferred from census data
COMW ... people employed in the commercial sector
INDW ... people employed in the industrial sector
AGE_16 ... number of people of 16 years of age and under, inferred from census data (used as a proxy for the portion of the population located in schools)
COLLEGE ... number of students on college and university campuses in the census tract inferred from square footage for default values (1 student per 130 ft^2 [12 m^2] of occupancy EDU2)
HOTEL ... number of people staying in hotels in the census tract inferred

		from square footage for default values (1 person per 400 ft^2 [37 m^2] of occupancy RES4)
PRFIL	...	factor representing the proportion of commuters using automobiles, inferred from profile of the community (0.60 for dense urban, 0.80 for less dense urban or suburban, and 0.85 for rural). The default is 0.80.
VISIT	...	number of regional students who do not live in the study area, visiting the census tract for shopping and entertainment. Default is set to zero.

C.2 Graham's Model for Dam Failure

Graham's approach [45] is characterised by a certain reluctance to establish mathematical relationships in human consequence estimation. This reluctance is founded on his bad experiences with empirically derived and overly generalising equations. In this respect, he refers to two models that are briefly described in [45] and in [57]. One of these models was authored by Brown and himself [14], the other by DeKay & G. McClelland[1] [24]. Both models directly calculate the fatality rate or lethality P_D from the available warning time T_W and the number of people at risk N_{PAR}. In both cases, P_D decreases with increasing N_{PAR} values. This is a statistical effect that occurrs if a large area is not subdivided into sufficiently small sub-areas: The larger the affected area the more it becomes likely to find sub-areas which are more or less unaffected, especially in wide flood plains. Major events such as the landslide into the Vajont reservoir in Italy causing around 1300 deaths were not included in the analysis of either [14] or [24] and would be systematically underestimated in consequence.

In order to circumvent these and other inconsistencies, Graham introduces a table instead of one or several mathematical relationships (Table C.2). In the table, the respective influencing parameters do not enter directly, but by means of different categories. They display the combined effect of a certain flood magnitude with a certain warning time and a certain flood severity understanding in the population: In this way, the effect of each parameter is considered in the result, although the final effect of each variable taken alone remains unknown. It can be said that the quantitative effect of each parameter is considered *implicitly* only. The values in Table C.2 are derived from a number of historical events, which can be subsumed under the respective categories.

In order to assess the category of the flood severity in Table C.2 (low, medium and high), Graham does not restrict himself to one single criterion. As a basis, he uses a damage-scale comparable to the intensity measure in earthquake assessment. The subdivision into medium and low severity relies on two further criteria: One is related to the water depth, the other puts the momentary water discharge and the momentary flood width in relation to the average yearly discharge. With respect to warning time T'_W, Graham proceeds in a similar way (see Table C.3 for the specific case of earthfill dams).

[1]Not to be mistaken for D. McClelland, the author of [74] who is also cited in this thesis, equally with respect to dam failure consequences.

Table C.2: Recommended estimation of fatality rate P_D, from Graham [45]

flood severity	warning time T'_W	flood severity understanding	fatality rate P_D suggested	fatality rate P_D sugg. range
high	no warning	not applicable	0.75	0.30–1.00
	15 to 60 minutes	vague	Use the values shown above and apply to the number of people who remain[a] in the the dam failure floodplain after warnings are issued.	
		precise		
	more than 60 minutes	vague		
		precise		
medium	no warning	not applicable	0.15	0.03–0.35
	15 to 60 minutes	vague	0.04	0.01–0.08
		precise	0.02	0.005–0.04
	more than 60 minutes	vague	0.03	0.005–0.06
		precise	0.01	0.002–0.02
low	no warning	not applicable	0.01	0.0–0.02
	15 to 60 minutes	vague	0.007	0.0–0.015
		precise	0.002	0.0–0.004
	more than 60 minutes	vague	0.0003	0.0–0.0006
		precise	0.0002	0.0–0.0004

[a]No guidance is provided on how many people will remain in the floodplain.

Table C.3: Recommended estimation of time of warning for earthfill dams, from Graham [45]

cause of failure	special considerations	time of failure	When would dam failure warning be initiated? observers at dam[a]	no observers at dam[b]
overtopping	drainage area $< 260\,\text{km}^2$	day	0.25 h before dam fails	0.25 h after fw reaches PAR
		night	0.25 h after dam fails	1 h after fw[c] reaches PAR[d]
	drainage area $> 260\,\text{km}^2$	day	2 h before dam fails	1 h before dam failure
		night	1–2 h before dam fails	0–1 h before dam failure
piping	full reservoir, normal weather	day	1 h before dam fails	0.25 h after fw reaches PAR
		night	0.5 h before dam fails	1 h after fw reaches PAR
seismic	immediate failure	day	0.25 h after dam fails	0.25 h after fw reaches PAR
		night	0.5 h after dam fails	1 h after fw reaches PAR
	delayed failure	day	2 h before dam fails	0.5 h before fw reaches PAR
		night	2 h before dam fails	0.5 h before fw reaches PAR

[a]'Observers at dam' means that a dam tender lives on high ground and within sight of the dam or the dam is visible from the homes of many people or the dam crest serves as a heavily used roadway. These dams are typically in urban areas.

[b]'No Observers at dam' means that there is no dam tender at, the dam is out of sight of nearly all homes and there is no roadway on the dam crest. These dams are usually in remote areas.

[c]Floodwater.

[d]People at risk.

C.3 Coburn & Spence's Model for Earthquake-induced Collapse

For a given class of buildings b, Coburn & Spence [18, 19] express the number of lives lost due to collapse as

$$Ks_b = D5_b[M1_b\, M2_b\, M3_b(M4_b + M5_b)] \tag{C.1}$$

However, the rationale given in [19] seems to indicate that Coburn and Spence must have actually had the following relationship in mind[2]:

$$Ks_b = D5_b[M1_b\, M2_b\, M3_b(M4_b + M5_b(1 - M4_b))] \tag{C.2}$$

The symbols are defined as

$D5$	... total number of collapsed buildings (i.e. damage level 5 according to the 'Cambridge Definition': 'More than one wall collapsed or more than half of the roof dislodged or failure of structural members to allow fall of roof or slab.')
$M1$	... inhabitants per building
$M2$	... occupancy at the time of the earthquake [%]
$M3$	... proportion of people entrapped [%]
$M4$	... proportion of people killed in the collapse [%]
$M5$	... post-collapse mortality [%]

When comparing this methodology to the one introduced in Chapter 5, the quantities corresponds to $D5 \cdot M1 \cdot M2 = N_{PAR}$ and $M3 \cdot (M4 + M5) = (1 - P_Q) \cdot k = P_D$.

[2] see p. 5994, table for $M5$: '$M5$ (as % of $M3 - M4$)')