Extended Orthogonal STBC for OFDM with Partial Channel Knowledge at the Transmitter

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Abstract— **Orthogonal space-time block codes (STBC) constitute a simple way of exploiting transmit diversity. If no channel knowledge is available at the transmitter the use of diversity can increase performance significantly. However, if some partial channel state information (CSI) is available, such as knowledge of the transmit correlation matrix, adapting transmission to this knowledge provides additional perfomance gains. In such case, adaptivity can be introduced by using a unitary eigenbeamformer with beams pointing along the directions of the eigenvectors of the transmit correlation matrix and applying a convenient power loading along the resulting beams. While for Rayleigh-fading channel eigenbeamforming has been shown to be optimum in terms of ergodic capacity, so far, no closed solution for the optimum power loading has been found. In the work at hand, orthogonal STBC are combined with eigenbeamforming in an orthogonal frequency-division multiplexing (OFDM) context and an optimum power loading is found, where optimality refers to an upperbound of pairwise error probability (PEP). The resulting signaling scheme can be viewed as an extension of STBC to OFDM with partial channel knowledge. The solution represents a very interesting trade-off between transmit diversity and antenna gain.**

I. Introduction

The deployment of antenna arrays at both sides of a wireless communication link proves highly beneficial. Inherently, the resulting multiple-input multiple-output (MIMO) channel has more capacity than any of the single-input single-output (SISO) channels between single pairs of array elements. Signal processing at the receiver benefits from antenna gain and the possibility to efficiently combat interference by means of spatial filtering. The transmitter can resort to the exploitation of transmit diversity in order to improve reliability if no channel state information (CSI) is available or adapt the transmit signal to the channel as much as the available knowledge allows, which results in an efficient use of transmit power due to spatial signal shaping and thus permits the reduction of interference. Due to the fading nature of wireless channels, all those are indispensable features if the goal of deploying bandwidth-efficient wireless networks supporting high-rate multimedia services is to be reached.

Multipath propagation in the wireless channel causes temporal inter-symbol interference (ISI), which increases the processing burden required at the receiver to recover the transmitted signals. While narrowband signals are hardly affected by multipath, frequency selectivity poses a serious impediment to the efficient detection of broadband signals. OFDM circumvents this problem by introducing a cyclic prefix between consecutive symbols. If the cyclic prefix is longer than the channel delay profile the frequency selective channel can be effectively decomposed in a number of parallel flat-fading channels, which dramatically eases the detection of the transmitted signals.

Aiming at the design of broadband wireless systems, the combination of OFDM and MIMO techniques offers a good trade-off between bandwidth efficiency and complexity. Moreover, the flat-fading property of single frequency channels in OFDM makes possible a simple extension of MIMO approaches, mostly conceived using flat-fading channels as a model, to frequency selective channels.

Orthogonal STBC have been introduced [1] as a means to exploit transmit diversity. They are characterized by their easy implementation and low detection complexity, which different from space-time trellis codes does not esentially increase with the size of the modulation alphabet employed. An extension of orthogonal STBC to OFDM is rather straightforward. The scheme can either be applied in a subcarrier basis, coding along the time, or replacing the time by the frequency axis, thus coding along frequency. In both cases, the optimality of a widely linear receiver will depend on the ratio between the lenght of a code block and the coherence time or bandwidth of the channel respectively.

While extensive literature exists on the topic MIMO OFDM that considers either perfect, e.g. [2], [3], [4], or no channel knowledge at all, e.g. [5], [6], [7], to date, hardly any publication can be found that considers partial CSI in a multicarrier context as it is understood here, namely as knowledge of the transmit channel correlation matrix. However, partial CSI at the transmitter is not difficult to obtain and proves very advantageous, especially in spatially correlated channels [8].

In this work, orthogonal STBC are extended to OFDM with partial CSI. Channel knowledge is taken advantage of by applying eigenbeamforming and conveniently distributing the available transmit power along the resulting beams. The optimum power distribution is found that minimizes an upper bound of PEP and simulation curves are shown that back the use of PEP as a design criterion. The remaining of the paper is structured as follows. Section II introduces the system model and some notation. In Section III an upper bound of PEP is derived for the system model introduced in Section II. In Section IV the extension of STBC to OFDM with partial CSI is described and an optimum distribution of transmit power is found using the upper bound derived in Section III. Section V shows some simulation results and finally, in Section VI conclusions are drawn.

II. System Model

Given a zero-mean complex Gaussian distributed MIMO channel with M_t transmit antennas, M_r receive antennas and L delay paths the relationship between vector $\boldsymbol{y}_k^{(b)} \in \mathbb{C}^{M_r \times 1}$ of receive signals and a vector $\boldsymbol{x}_k^{(b)} \in \mathbb{C}^{N_e \times 1}$ of signals transmitted over subcarrier $k \in \{0, \ldots, N-1\}$ during OFDM block $b \in \{1, \ldots, B\}$ might be written as

$$\boldsymbol{y}_k^{(b)} = \boldsymbol{H}_k \boldsymbol{U}_k \boldsymbol{P}_k^{1/2} \boldsymbol{x}_k^{(b)} + \boldsymbol{n}_k^{(b)} \tag{1}$$

where $\boldsymbol{n}_k^{(b)} \in \mathbb{C}^{M_r \times 1}$ is a white Gaussian distributed noise vector, $\boldsymbol{H}_k \in \mathbb{C}^{M_r \times M_t}$ is the channel matrix at subcarrier k, which is assumed to be approximately constant for the duration of B OFDM blocks, $\boldsymbol{U}_k \in \mathbb{C}^{M_t \times N_e}$ is an arbitrary matrix with orthonormal columns, $\boldsymbol{P}_k^{1/2} \in \mathbb{R}^{N_e \times N_e}$ is a diagonal matrix that assigns a certain transmit power to each signal component and $\mathrm{E}\{|\mathsf{x}_{k,n}^{(b)}|^2\} = 1$. $N_e \leq M_t$ is the number of spatial dimensions of the signal to be transmitted over the channel. If $N_e = M_t$ and power is uniformly distributed over spatial components it is easily shown that, under the assumption of no CSI at the transmitter, this transmit structure is capacity achieving.

The relationship between the channel matrices in the frequency domain and the channel matrices corresponding to each delay path in the time domain is given by [9]

$$\boldsymbol{H}_k = \sum_{\ell=1}^{L} \boldsymbol{H}_\ell^{(t)} e^{j2\pi n_\ell k/N}.$$

where n_ℓ is the delay of temporal path ℓ. If an uncorrelated scattering (US) assumption holds for the channel, i.e. the fading processes of different delay paths are mutually uncorrelated, then the transmit covariance matrix in the frequency domain is found to be

$$\boldsymbol{R} = \mathrm{E}\left\{\boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{H}_k\right\} = \sum_{\ell=1}^{L} \boldsymbol{R}_\ell^{(t)}, \tag{2}$$

which turns out to be independent of frequency. Therefore, the conclusion can be drawn that partial knowledge as considered here provides information about the spatial structure of the channel but not about its spectral characteristic. In [10] it has been shown that, assuming complex Gaussian distributed entries in the channel matrices and spatial decorrelation of the fading process at the receiver, for a system model as given by Eqn. 1 the optimum matrix $\boldsymbol{U}_k$ has the eigenvectors of the transmit covariance matrix of the channel as columns. This result can be readily extended to channels showing spatial correlation at the receiver. As a consequence, index k is henceforth left out and $\boldsymbol{U}$ will be referred to as eigenbeamforming matrix. Furthermore, due to the fact that no information about the spectral characteristic of the channel is available at the transmitter there is no reason for a frequency dependent allocation of transmit power. Accordingly, a unique power allocation matrix $\boldsymbol{P}$ will be applied to all subcarriers. Taking into account these observations and arranging all B OFDM blocks in a matrix format our system model can be compactly expressed as

$$\boldsymbol{Y}_k = \boldsymbol{H}_k \boldsymbol{U} \boldsymbol{P}^{1/2} \boldsymbol{X}_k + \boldsymbol{N}_k,$$

where

$$\boldsymbol{Y}_k = [\ \boldsymbol{y}_k^{(1)} \quad \boldsymbol{y}_k^{(2)} \quad \cdots \quad \boldsymbol{y}_k^{(B)}\ ],$$
$$\boldsymbol{X}_k = [\ \boldsymbol{x}_k^{(1)} \quad \boldsymbol{x}_k^{(2)} \quad \cdots \quad \boldsymbol{x}_k^{(B)}\ ],$$
$$\boldsymbol{N}_k = [\ \boldsymbol{n}_k^{(1)} \quad \boldsymbol{n}_k^{(2)} \quad \cdots \quad \boldsymbol{n}_k^{(B)}\ ].$$

For purposes of clarity, the following notation is introduced that will be used for the derivation of an upper bound of PEP in the next section. $\tilde{\boldsymbol{A}} = \boldsymbol{I}_L \otimes \boldsymbol{A}$, $\omega_{k,\ell} = e^{-j2\pi k n_\ell/N}$ and $\boldsymbol{h}_{\ell,m}^{\mathrm{T}} = [\boldsymbol{H}_\ell^{(\mathrm{t})}]_{m,\bullet}$, where $[\boldsymbol{A}]_{r,\bullet}$ denotes the rth row of matrix $\boldsymbol{A}$.

III. Pairwise Error Probability

Considering all dimensions of a transmitted signal, which can be expressed in matrix form as $\boldsymbol{X} = [\ \boldsymbol{X}_0 \quad \boldsymbol{X}_1 \quad \cdots \quad \boldsymbol{X}_{N-1}\ ]$, a maximum-likelihood detection of the signal at the receiver yields

$$\hat{\boldsymbol{X}} = \arg\min_{\boldsymbol{X}} \sum_{k=0}^{N-1} \left\| \boldsymbol{Y}_k - \boldsymbol{H}_k \boldsymbol{U} \boldsymbol{P}^{1/2} \boldsymbol{X}_k \right\|^2. \tag{3}$$

The probability that in Eqn. 3 $\hat{\boldsymbol{X}}$ is equal to $\boldsymbol{X}'$ having transmitted $\boldsymbol{X}$ conditioned on a particular channel realization is called PEP. For our system model this probability can be expressed in closed form using the Gaussian error function [5],

$$P_{\hat{\mathsf{X}}}(\hat{\boldsymbol{X}} = \boldsymbol{X}' | \boldsymbol{X}; \boldsymbol{H}) = Q\left(\sqrt{\frac{d^2(\boldsymbol{X}, \boldsymbol{X}'; \boldsymbol{H})}{2\sigma^2}}\right), \tag{4}$$

where σ^2 is the variance of the complex-valued noise process, $\boldsymbol{H}$ stands for a particular channel realization and $d(\boldsymbol{X}, \boldsymbol{X}', \boldsymbol{H})$ is the distance of the two transmit signals at the receiver, which is given by

$$d^2(\boldsymbol{X}, \boldsymbol{X}'; \boldsymbol{H}) = \sum_{k=0}^{N-1} \left\| \boldsymbol{H}_k \boldsymbol{U} \boldsymbol{P}^{1/2} \boldsymbol{\Delta}_k \right\|^2, \tag{5}$$

with $\boldsymbol{\Delta}_k = \boldsymbol{X}' - \boldsymbol{X}$. Since at the transmitter no information about the instantaneous channel realization is available, it is appropriate to calculate the expectation of Eqn. 4 over all possible channel realizations, which leads to the ergodic pairwise error probability. However, for this expectation no closed solution has been found. Fortunately, Eqn. 4 can be tightly upperbounded using the Chernoff bound for the Gaussian error function as

$$P_{\hat{\mathsf{X}}}(\hat{\boldsymbol{X}}=\boldsymbol{X}'|\boldsymbol{X};\boldsymbol{H}) \leq e^{-\frac{d^2(\boldsymbol{X},\boldsymbol{X}';\boldsymbol{H})}{4\sigma^2}}, \qquad (6)$$

whose mean value will give an upper bound for the ergodic pairwise error probability. In order to arrive to a meaningful expression for the expected value of this upper bound some algebraic manipulations on Eqn. 5 are needed, viz.

$$\begin{aligned}
d^2(\boldsymbol{X},\boldsymbol{X}';\boldsymbol{H}) &= \\
&= \sum_{m=1}^{M_r} \boldsymbol{h}_m^{\mathrm{T}} \left(\sum_{k=0}^{N-1} \boldsymbol{\Omega}_k \boldsymbol{U}\boldsymbol{P}^{1/2}\boldsymbol{\Delta}_k\boldsymbol{\Delta}_k^{\mathrm{H}}\boldsymbol{P}^{1/2}\boldsymbol{U}^{\mathrm{H}}\boldsymbol{\Omega}_k^{\mathrm{H}} \right) \boldsymbol{h}_m^* \\
&= \sum_{m=1}^{M_r} \boldsymbol{h}_m^{\mathrm{T}} \tilde{\boldsymbol{U}}\tilde{\boldsymbol{P}}^{1/2} \underbrace{\sum_{k=1}^{N} \boldsymbol{\Omega}_k'\boldsymbol{\Delta}_k\boldsymbol{\Delta}_k^{\mathrm{H}}\boldsymbol{\Omega}_k'^{,\mathrm{H}}}_{\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}^*} \tilde{\boldsymbol{P}}^{1/2}\tilde{\boldsymbol{U}}^{\mathrm{H}}\boldsymbol{h}_m^* \qquad (7) \\
&= \sum_{m=1}^{M_r} \boldsymbol{h}_m^{\mathrm{T}} \tilde{\boldsymbol{U}}\tilde{\boldsymbol{P}}^{1/2}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}^*\tilde{\boldsymbol{P}}^{1/2}\tilde{\boldsymbol{U}}^{\mathrm{H}}\boldsymbol{h}_m^* \\
&= \sum_{m=1}^{M_r} \boldsymbol{z}_m^{\mathrm{T}} \underbrace{\boldsymbol{R}^{(\mathrm{t}),\mathrm{H}/2}\tilde{\boldsymbol{U}}\tilde{\boldsymbol{P}}^{1/2}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}^*\tilde{\boldsymbol{P}}^{1/2}\tilde{\boldsymbol{U}}^{\mathrm{H}}\boldsymbol{R}^{(\mathrm{t}),1/2}}_{\boldsymbol{M}} \boldsymbol{z}_m^* \\
&= \sum_{m=1}^{M_r} \boldsymbol{z}_m^{\mathrm{T}}\boldsymbol{M}\boldsymbol{z}_m^* \qquad (8)
\end{aligned}$$

where,

$$\boldsymbol{\Omega}_k = [\ \omega_{k,1} \ \cdots \ \omega_{k,L}\ ]^{\mathrm{T}} \otimes \boldsymbol{I}_{M_t},$$

$$\boldsymbol{\Omega}_k' = [\ \omega_{k,1} \ \cdots \ \omega_{k,L}\ ]^{\mathrm{T}} \otimes \boldsymbol{I}_{N_e},$$

$$\boldsymbol{h}_m^{\mathrm{T}} = [\ \boldsymbol{h}_{1,m}^{\mathrm{T}} \ \cdots \ \boldsymbol{h}_{L,m}^{\mathrm{T}}\ ],$$

$\boldsymbol{R}^{(\mathrm{t})} = M_r \mathrm{E}\{\mathbf{h}_m^*\mathbf{h}_m^{\mathrm{T}}\}$ and $\boldsymbol{R}^{(\mathrm{t})} = \boldsymbol{R}^{(\mathrm{t}),1/2}\boldsymbol{R}^{(\mathrm{t}),\mathrm{H}/2}$. For these last equalities the assumption has been made that all MISO channels that make up the MIMO channel exhibit the same statistics and thus $\mathrm{E}\{\boldsymbol{h}_m^*\boldsymbol{h}_m^{\mathrm{T}}\}$ does not depend on m. Also, it has been implicitely assumed that $\boldsymbol{R}^{(\mathrm{t})}$ is a full-rank matrix, which will be true with probability tending to 1 from an algebraic point of view. Note that the components of $\boldsymbol{z}_m$ realizations of statistically independent, Gaussian distributed random variables. If we additionally assume spatially decorrelated fading processes at the receiver then,

$$\mathrm{E}\{\boldsymbol{z}_m^*\boldsymbol{z}_{m'}^{\mathrm{T}}\} = \frac{\delta_{m,m'}}{M_r}\boldsymbol{I} \ .$$

Considering this property of vectors $\boldsymbol{z}_m$, the expectation of the upper bound in Eqn. 6 can be easily computed and yields

$$P_{\hat{\mathsf{X}}}(\hat{\boldsymbol{X}}=\boldsymbol{X}'|\boldsymbol{X}) \leq \left(\prod_{r=1}^{\rho} \left(1 + \frac{\lambda_r(\boldsymbol{M})}{4\sigma^2 M_r} \right) \right)^{-M_r} \qquad (9)$$

Here $\lambda_r(\boldsymbol{M})$ are the eigenvalues of matrix $\boldsymbol{M}$ and ρ its rank. Finally Eqn. 9 can be rewritten in compact form as a determinant,

$$P_{\hat{\mathsf{X}}}(\hat{\boldsymbol{X}}=\boldsymbol{X}'|\boldsymbol{X}) \leq \left(\det\left(\boldsymbol{I} + \frac{\boldsymbol{M}}{4\sigma^2 M_r} \right) \right)^{-M_r} .$$

This expression provides us with a figure of merit that depends on all relevant design parameters at the transmitter, e.g. modulation alphabet, signalling scheme, transmit power, etc., and includes the channel knowledge available at the transmitter, i.e. the transmit covariance matrix. For high values of SNR and depending on the geometry of the transmit signals, pairwise error probability might provide a good estimation of the bit error rate if the union bound is used. But even for low SNR pairwise error probability will show a certain correlation with the expected bit error rate. This is the main motivation for using pairwise error probability as a design measure. Following this idea the following criterion for the transmitter design can be set up,

$$\max_{\boldsymbol{G},\,\boldsymbol{P}\,:\,\mathrm{tr}\{\boldsymbol{P}\}\,=\,P_T} \left\{ \min_{\boldsymbol{X},\,\boldsymbol{X}'} \det\left(\boldsymbol{I} + \frac{\boldsymbol{M}}{4\sigma^2 M_r} \right) \right\} \ , \qquad (10)$$

where P_T is the transmit power budget per subcarrier. When no CSI is available at the transmitter, we can still perform some transmitter design considering the region of high SNR values. In that case the identity matrix can be neglected, $N_e = M_t$, $\boldsymbol{P} = \frac{P_T}{M_t}\boldsymbol{I}$ and the design problem reduces to

$$\max_{\boldsymbol{G}}\{\min_{\boldsymbol{X},\,\boldsymbol{X}'} \det(\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}^*)\} \ ,$$

which is the criterion that implicitely inspires many approaches that do not consider any channel knowledge at the transmitter (s. Section I).

Solving Eqn. 10 is not a trivial problem. Moreover, the result would yield the optimum code for a given transmit covariance matrix, i.e. the search for the optimum code should be repeated every time the channel statistics change. Here we will focus on a less ambitious goal. Given a particular signaling scheme such as orthogonal space-time block codes the optimum power allocation matrix $\boldsymbol{P}$ will be identified that maximizes the criterion set up above. Note that in this case the signalling scheme employed determines matrix factor $\boldsymbol{G}$ and the maximization of Eqn. 10 is simply carried out over the choice of $\boldsymbol{P}$.

IV. Extended STBC

Orthogonal space-time block codes [1] are characterized by their easy implementation and low detection complexity, which different from space-time trellis codes does not essentially increase with the size of the modulation alphabet employed. Given a number S of symbols s_s drawn from a certain modulation alphabet $\mathcal{A}$, these symbols are arranged in a code block $\boldsymbol{C}_i \in \mathbb{C}^{M_t \times C}$ according to the following construction rule,

$$\boldsymbol{C}_i = \sum_{s=1}^{S} (\boldsymbol{A}_s s_s + \boldsymbol{B}_s s_s^*) \ ,$$

where $\boldsymbol{A}_s \in \mathbb{R}^{M_t \times C}$ and $\boldsymbol{B}_s \in \mathbb{R}^{M_t \times C}$ are chosen so that for any set of symbols s_s

$$\boldsymbol{C}_i \boldsymbol{C}_i^{\mathrm{H}} = \sum_{s=1}^{S} |s_s|^2 \boldsymbol{I} \ .$$

For $M_t = 2$ it is possible to find a code such that $S = C$ whereas for $M_t > 2$, it necessarily holds $S < C$, which implies a rate penalty of S/C.

Aiming at maximizing Eqn. 10 over the choice of $\boldsymbol{P}$, we first consider $B = 1$ and two transmit signals $\boldsymbol{X} = [\ \boldsymbol{C}_1 \ \ \boldsymbol{C}_2 \ \ \cdots \ \ \boldsymbol{C}_K\ ]$ and $\boldsymbol{X}' = [\ \boldsymbol{C}_1' \ \ \boldsymbol{C}_2' \ \ \cdots \ \ \boldsymbol{C}_K'\ ]$ with $K = N/C$. Observing Eqn. 7 it is easy to note that, for any realization of the channel and an arbitrary matrix $\boldsymbol{P}$, the minimum distance of two transmitted signals $\boldsymbol{X}$ and $\boldsymbol{X}'$ at the receiver is reached when these only differ in one code block. This necessarily implies that the mean of the upper bound is maximized for this choice of transmit signal pairs and consequently the minimum for the determinant in Eqn. 10 is reached. Now, substituting this pair of transmit signals in Eqn. 10 we obtain,

$$\boldsymbol{G}^{\mathrm{T}} \boldsymbol{G}^{*} = \sum_{k=(i-1)C}^{iC-1} \boldsymbol{\Omega}_k \boldsymbol{\Delta}_k \boldsymbol{\Delta}_k^{\mathrm{H}} \boldsymbol{\Omega}_k^{\mathrm{H}} \ ,$$

which if $(n_L - n_1)(C-1)/N \ll 1$ can be approximated as

$$\boldsymbol{G}^{\mathrm{T}} \boldsymbol{G}^{*} \approx \boldsymbol{\Omega}_{(i-1)C} \left(\sum_{k=(i-1)C}^{iC-1} \boldsymbol{\Delta}_k \boldsymbol{\Delta}_k^{\mathrm{H}} \right) \boldsymbol{\Omega}_{(i-1)C}^{\mathrm{H}}$$

It can be easily shown that further minimization of the determinant in Eqn. 10 is now reached if the two differing code blocks $\boldsymbol{C}_i$ and $\boldsymbol{C}_i'$ do in turn differ in only one symbol being the distance between both symbols, e.g. s_s and s_s', the minimum distance $d_{\min}$ between any two symbols in $\mathcal{A}$. Considering this and replacing in the last expression we get,

$$\boldsymbol{G}^{\mathrm{T}} \boldsymbol{G}^{*} \approx d_{\min}^2 \boldsymbol{\Omega}_{(i-1)C} \boldsymbol{\Omega}_{(i-1)C}^{\mathrm{H}} \ . \tag{11}$$

The condition $(n_L - n_1)(C-1)/N \ll 1$ is equivalent to the condition $(C-1)\Delta f \ll 1/(n_L - n_1)\Delta t$, where Δf and Δt stand for the frequency and time sampling interval of the OFDM system respectively. $1/(n_L - n_1)\Delta t$ is a measure of the coherence bandwidth of the channel. Thus this condition demands a coherence bandwidth much larger than the span of a block code in the frequency domain, which considering typical values of block length and coherence bandwidth is a realistic assumption. Now, using Eqn. 11, $\boldsymbol{M}$ can be rewritten as

$$\boldsymbol{M} = d_{\min}^2 \boldsymbol{R}^{(\mathrm{t}),\mathrm{H}/2} \boldsymbol{\Omega}_{(i-1)C} \boldsymbol{U} \boldsymbol{P} \boldsymbol{U}^{\mathrm{H}} \boldsymbol{\Omega}_{(i-1)C}^{\mathrm{H}} \boldsymbol{R}^{(\mathrm{t}),1/2}$$

and applying the identity $\det(\boldsymbol{I} + \boldsymbol{A}\boldsymbol{B}) = \det(\boldsymbol{I} + \boldsymbol{B}\boldsymbol{A})$ to the determinant in Eqn. 10 we obtain

$$\det\left(\boldsymbol{I} + \frac{1}{4\sigma^2 M_r} d_{\min}^2 \boldsymbol{P} \boldsymbol{U}^{\mathrm{H}} \boldsymbol{\Omega}_{(i-1)C}^{\mathrm{H}} \boldsymbol{R}^{(\mathrm{t})} \boldsymbol{\Omega}_{(i-1)C} \boldsymbol{U} \right) \ ,$$

where, if decorrelation between different temporal paths is assumed, $\boldsymbol{R}^{(\mathrm{t})}$ becomes block diagonal and

$$\boldsymbol{R} = \boldsymbol{\Omega}_{(i-1)C}^{\mathrm{H}} \boldsymbol{R}^{(\mathrm{t})} \boldsymbol{\Omega}_{(i-1)C} \ ,$$

which is the covariance matrix in the frequency domain (cf. Eqn. 2). Since $\boldsymbol{U}$ is the matrix of eigenvectors of this covariance matrix the optimization problem of Eqn. 10 symplifies to

$$\max_{\boldsymbol{P}\,:\,\mathrm{tr}\{\boldsymbol{P}\} = P_T} \left\{ \det\left(\boldsymbol{I} + \frac{1}{4\sigma^2 M_r} d_{\min}^2 \boldsymbol{P} \boldsymbol{\Lambda} \right) \right\} \ ,$$

where $\boldsymbol{\Lambda}$ is the matrix of eigenvalues of $\boldsymbol{R}$. The solution to this optimization problem is the well-known waterfilling distribution of transmit power

$$P_n = \max\left\{ \nu - \frac{4\sigma^2 M_r}{\lambda_n d_{\min}^2}, 0 \right\} \ ,$$

where ν is chosen to satisfy the constraint $\mathrm{tr}\{\boldsymbol{P}\} = P_T$. Following the same reasoning it is straightforward to show that exactly the same result is obtained if we consider $B = C$ and transmit signals $\boldsymbol{X} = [\ \boldsymbol{C}_0 \ \ \boldsymbol{C}_1 \ \ \cdots \ \ \boldsymbol{C}_{N-1}\ ]$. Interestingly, this result coincides with the one presented in [11] for a spatially correlated, flat fading MISO channel using an upper bound of symbol error probability as criterion.

Finally, for the sake of fairness it should be mentioned that shortly before the acceptance of this paper the authors have become aware of [12], where assuming partial CSI at the transmitter optimum linear precoding for space-time codes has been investigated. Our result turns out to be a particular case of the more general result obtained there. This somehow undermines the originality of the result itself. However, its derivation departing from a multicarrier context offers insights that are not provided by [12].

V. Simulation Results

In order to verify the optimality of the power distribution derived in the previous section some simulations have been carried out. The OFDM scheme employed has 1024 subcarriers and 135 Mhz bandwidth and the modulation alphabet is 8PSK. We have two transmit antennas and one receive antenna, thus, the STBC corresponds to the well-known Alamouti scheme.

Channel A, which is the first of the two channels we will consider, consists of two temporal paths. The second path exhibits a delay of 60 samples and an attenuation of 3 dB with respect to the first. The matrix of eigenvalues is given by

$$\boldsymbol{\Lambda}_A = \mathrm{diag}\{1.9628 \quad 0.0372\} \ .$$

From this eigenvalue profile we note that the channel is strongly correlated. In Fig. 1 BER simulation results are depicted for three different power distributions. Uniform power distribution, which is optimum if no CSI is available, a waterfilling distribution that maximizes the capacity of the average channel [8] (cap wf) and the waterfilling distribution derived in the previous section (pw wf). Solid curves were obtained using an ML detector at the receiver. Dashed lines correspond to the simple widely linear receiver structure for orthogonal STBC.

Due to the frequency selectivity of the channel, the curves corresponding to the simple linear detection show an error floor for high values of SNR. The waterfilling approaches exploit the partial channel knowledge and benefit from antenna gain whereas standard Alamouti tries to exploit a diversity that in this case the channel does not offer. As a consequence, we observe a performance loss with respect to the waterfilling approaches that amounts to 3 dB at 10% BER. Though slightly, still, a performance gain of the PEP-based waterfilling can be observed with respect to the capacity-based waterfilling. However, the essential difference of both approaches is that while capacity-based waterfilling only adapts to the channel statistics, PEP-based waterfilling adapts to both channel statistics and modulation alphabet employed.

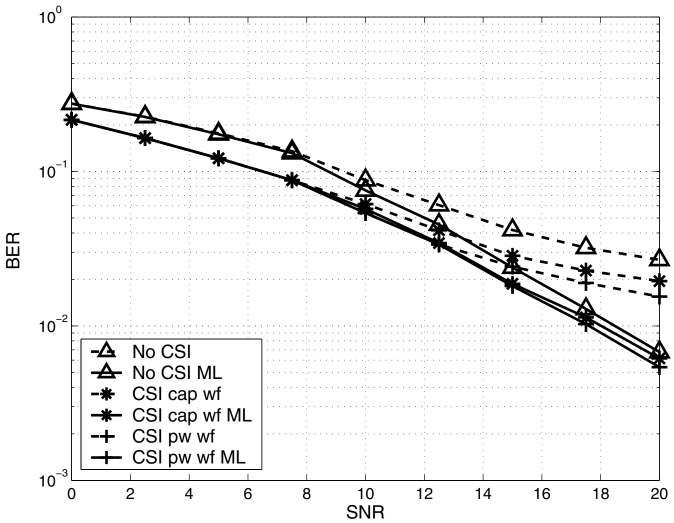

Fig. 1. Performance curves for channel A

Channel B consists of 24 temporal paths with a maximum delay of 184 samples and an exponential profile with a decay rate of 0.5 dB per tap. The matrix of eigenvalues is given by,

$$\mathbf{\Lambda}_B = \mathrm{diag}\{1.7180 \quad 0.2820\} \; .$$

From this eigenvalue profile we note that channel B shows a weaker spatial correlation than channel A. In the curves (cf. Fig. 2), this translates into a reduced performance gap between waterfilling approaches and standard Alamouti. The reason for that is the increase in spatial diversity that channel B offers compared to channel A, from which standard Alamouti profits. Furthermore, due to the stronger frequency selectivity of channel B, performance degradation of linear receivers becomes somewhat larger.

VI. Conclusion

In the present work we have extended orthogonal STBC to OFDM with partial CSI at the transmitter. To this end, based on an upper bound of PEP, the optimum transmit power distribution has been found that allows the transmitter to

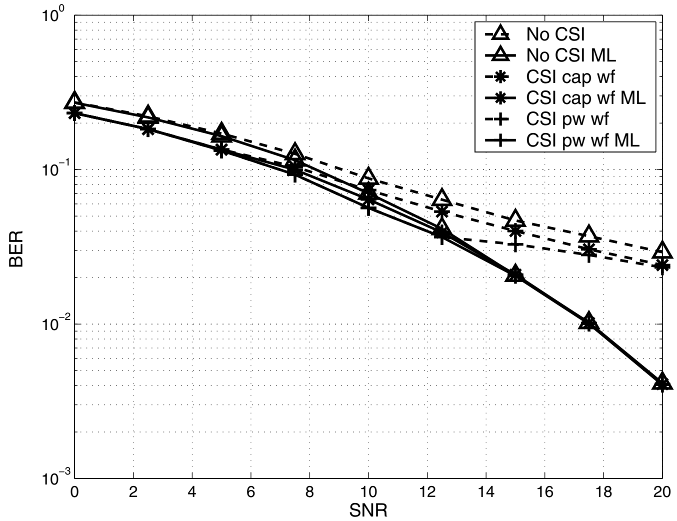

Fig. 2. Performance curves for channel B

adapt to the channel statistics choosing operational points that smoothly range from standard Alamouti (completely uncorrelated channel) to beamforming (completely correlated channel). Simulation results were shown that verify the optimality of our solution.

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