REDUCED COMPLEXITY TRANSMIT WIENER FILTER BASED ON A KRYLOV SUBSPACE MULTI-STAGE DECOMPOSITION

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ABSTRACT

The *multi-stage transmit Wiener filter* (MSTxWF) is presented, an approach to reducing the complexity of the *transmit Wiener filter* (TxWF). The MSTxWF is found by applying the multi-stage decomposition known from the receive *multi-stage Wiener filter* (MSWF) to the TxWF. Complexity reduction is achieved by truncating the decomposition. We show that the resulting reduced rank MSTxWF can be interpreted as an approximation of the TxWF in a Krylov subspace, allowing for an efficient computation of the MSTxWF with the Lanczos algorithm. The reduced rank MSTxWF shows near-optimum performance for relatively low rank, making it an interesting alternative to eigenspace-based methods for complexity reduction.

1. INTRODUCTION

Cellular mobile communication systems exhibit a highly asymmetric structure: A high-complexity *base station* (BS) serves a number of low-complexity *mobile stations* (MSs). In the downlink of such asymmetric systems, linear transmit processing (also linear pre-equalization or linear precoding) provides a means for achieving high performance wireless transmission while maintaining low complexity at the receivers. Transmit processing requires at least partial *channel state information* (CSI) at the transmitter. We assume the transmitter to have full CSI, a valid assumption in time division duplex systems if the coherence time of the channel is large enough. The *transmit Wiener filter* (TxWF) [1, 2] is the optimum linear transmit filter in terms of *mean squared error* (MSE). However, the computation of the TxWF is complex and may be an obstacle for the implementation even at the BS. Therefore, approaches that reduce complexity by computing the TxWF in a lower dimensional subspace are of great interest.

In receive processing, the vector *multi-stage Wiener filter* (MSWF) introduced by Goldstein et al. in [3] provides excellent reduced rank performance (e.g. [4, 5]). In this paper, the multi-stage concept is applied to transmit processing. First, we develop the full rank vector MSTxWF in order to find an algorithm for computing a subspace basis. Analogous to the results of the MSWF presented in [6], we show that under certain conditions this basis spans a Krylov subspace whose basis vectors can be computed with the Lanczos algorithm. In a second step, the reduced-rank MSTxWF is developed, i.e. complexity is reduced by approximating the TxWF in a lower-dimensional Krylov subspace.

We present simulation results for the reduced rank MSTxWF in a frequency-selective multi-user scenario and compare the performance of the MSTxWF with the performance of eigenspace-based TxWF approximations.

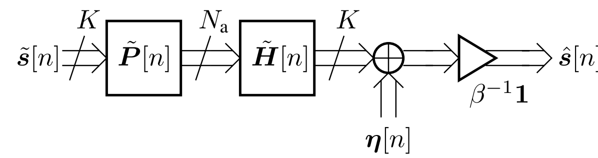

Fig. 1. MIMO FIR System Model

2. SYSTEM MODEL

A MIMO system with N_a antennas at the transmitter and K single antenna receivers is considered, as depicted in Fig. 1. The only processing performed at the receivers is a scaling by a factor β^{-1} (gain control). The transmitted signal, given by the convolution of $\tilde{\boldsymbol{s}}[n]$ with the FIR transmit filter $\tilde{\boldsymbol{P}}[n] = \sum_{l=0}^{L} \tilde{\boldsymbol{P}}_l \delta[n-l]$, propagates over the frequency-selective channel $\tilde{\boldsymbol{H}}[n] = \sum_{q=0}^{Q} \tilde{\boldsymbol{H}}_q \delta[n-q]$ and is perturbed by complex AWGN $\boldsymbol{\eta}[n]$. After defining

$$\boldsymbol{p}_k = \left[\boldsymbol{e}_k^\mathrm{T}\tilde{\boldsymbol{P}}_0^\mathrm{T}, \ldots, \boldsymbol{e}_k^\mathrm{T}\tilde{\boldsymbol{P}}_L^\mathrm{T}\right]^\mathrm{T} \in \mathbb{C}^{N_\mathrm{a}(L+1)},$$
$$\boldsymbol{s}_k[n] = \left[\tilde{s}_k[n], \ldots, \tilde{s}_k[n-Q-L]\right]^\mathrm{T},$$

where $\boldsymbol{e}_i$ denotes the i-th column of the identity matrix $\mathbf{1}$, the k-th element of the estimate $\hat{\boldsymbol{s}}[n]$ can be written as

$$\hat{s}_k[n] = \sum_{i=1}^{K} \boldsymbol{p}_i^\mathrm{T} \boldsymbol{H}_k \boldsymbol{s}_i[n] + \eta_k[n],$$

with an appropriately constructed block-toeplitz matrix $\boldsymbol{H}_k$. We assume that the data symbols are uncorrelated, i.e. $\mathrm{E}\left[\tilde{s}_k[n]\tilde{s}_i^*[n+m]\right] = \sigma_s^2\delta[k-i]\delta[m]$. Under this assumption, the average transmit power is given by

$$\mathrm{E}\left[\left\|\tilde{\boldsymbol{P}}[n] * \tilde{\boldsymbol{s}}[n]\right\|_2^2\right] = \sigma_s^2 \sum_{i=1}^{K} \boldsymbol{p}_i^{\mathrm{T}} \boldsymbol{p}_i^*.$$

3. TXMF AND TXWF

We shortly review the *transmit matched filter* (TxMF) and the TxWF, as both transmit filters are needed in the derivation of the MSTxWF. The TxMF can be found by maximizing the real part of the correlation $\sum_{k=1}^{K}\mathrm{E}\left[\hat{s}_k[n]\tilde{s}_k^*[n-\nu]\right]$ under the constraint that the average transmit power does not exceed E_{tr}, where ν denotes the latency time [7]. After defining $\boldsymbol{r}_{0,k} = \boldsymbol{H}_k \boldsymbol{e}_{\nu+1}$, the filter vector corresponding to the k-th data stream is given by

$$\boldsymbol{p}_{\mathrm{MF},k}^{\mathrm{T}} = \sqrt{\frac{E_{\mathrm{tr}}}{\sigma_s^2 \sum_{i=1}^{K} \boldsymbol{r}_{0,i}^{\mathrm{H}} \boldsymbol{r}_{0,i}}}\, \boldsymbol{r}_{0,k}^{\mathrm{H}}.$$

The TxWF minimizes the MSE

$$\sigma_{\boldsymbol{\varepsilon}}^2 = \sum_{k=1}^{K} \mathrm{E}\left[\left|\tilde{s}_k[n-\nu] - \hat{s}_k[n]\right|^2\right]$$

under an average transmit power constraint [1]:

$$\begin{aligned} \{\boldsymbol{p}_{\mathrm{WF},1}^{\mathrm{T}}, \ldots, \boldsymbol{p}_{\mathrm{WF},K}^{\mathrm{T}}, \beta_{\mathrm{WF}}\} &= \underset{\{\boldsymbol{p}_1^{\mathrm{T}},\ldots,\boldsymbol{p}_K^{\mathrm{T}},\beta\}}{\operatorname{argmin}} \ \sigma_{\boldsymbol{\varepsilon}}^2 \\ \text{s.t.} \quad \sigma_s^2 \sum_{i=1}^{K} \boldsymbol{p}_i^{\mathrm{T}} \boldsymbol{p}_i^* &\leq E_{\mathrm{tr}}. \end{aligned} \tag{1}$$

With $\boldsymbol{R}_0 = \sum_{i=1}^{K} \boldsymbol{H}_i \boldsymbol{H}_i^{\mathrm{H}} + \sum_{i=1}^{K} \sigma_{\eta_i}^2 / E_{\mathrm{tr}} \mathbf{1}$, the TxWF solution reads as

$$\begin{aligned} \boldsymbol{p}_{\mathrm{WF},k}^{\mathrm{T}} &= \beta_{\mathrm{WF}} \boldsymbol{p}_{0,k}^{\mathrm{T}}, \quad \text{where} \\ \boldsymbol{p}_{0,k}^{\mathrm{T}} &= \boldsymbol{r}_{0,k}^{\mathrm{H}} \boldsymbol{R}_0^{-1}, \\ \beta_{\mathrm{WF}} &= \sqrt{E_{\mathrm{tr}}} / \sqrt{\sigma_s^2 \textstyle\sum_{i=1}^{K} \boldsymbol{p}_{0,i}^{\mathrm{T}} \boldsymbol{p}_{0,i}^*}. \end{aligned} \tag{2}$$

Here, $\boldsymbol{R}_0$ is a $N \times N$ matrix, where $N = N_{\mathrm{a}}(L+1)$, resulting in a complexity for the computation of $\boldsymbol{p}_{0,k}^{\mathrm{T}}$ of $\mathrm{O}(N^3)$. For large N, the complexity involved in the computation of the TxWF may be prohibitive. Note that in contrast to receive processing, $\boldsymbol{R}_0$ is not a covariance matrix of any signal — it is only defined to keep notation simple.

4. FULL RANK MULTI-STAGE TXWF

In order to reduce computational complexity, we seek an approximation of each $\boldsymbol{p}_{\mathrm{WF},k}$ in a subspace $\mathcal{S}_k \subset \mathbb{C}^N$. Analogous to the vector MSWF [3], an algorithm for computing a subspace basis can be found by a stage-wise decomposition of $\boldsymbol{p}_{0,k}$. For simplicity, we first consider the case $K = 1$. This allows us to drop the index k and write $\boldsymbol{p}_0$ instead of $\boldsymbol{p}_{0,k}$. In the first stage, by choosing a vector $\boldsymbol{q}_1 \in \mathbb{C}^N$ and a blocking matrix $\boldsymbol{B}_1 \in \mathbb{C}^{N-1\times N}$ satisfying $\mathrm{span}(\boldsymbol{B}_1^{\mathrm{T}}) = \mathrm{null}(\boldsymbol{q}_1^{\mathrm{H}})$, $\mathbb{C}^N$ is partitioned into two orthogonal subspaces $\mathrm{span}(\boldsymbol{q}_1)$ and $\mathrm{span}(\boldsymbol{B}_1^{\mathrm{T}})$. Accordingly, the decomposition of $\boldsymbol{p}_0$ can be written as

$$\boldsymbol{p}_0 = \alpha_1 \left(\boldsymbol{q}_1 - \boldsymbol{B}_1^{\mathrm{T}} \boldsymbol{p}_1\right), \tag{3}$$

with $\alpha_1 \in \mathbb{C}$ and $\boldsymbol{p}_1 \in \mathbb{C}^{N-1}$. Now, we choose

$$\boldsymbol{q}_1 = \boldsymbol{r}_0^* / \left\|\boldsymbol{r}_0\right\|_2. \tag{4}$$

By choosing $\boldsymbol{q}_1$ such that $\mathrm{span}(\boldsymbol{q}_1) = \mathrm{span}(\boldsymbol{p}_{\mathrm{MF}})$, we ensure that the reduced rank MSTxWF performs at least as good as the TxMF. Plugging Eqn. (2) and Eqn. (4) into Eqn. (3) and solving for $\boldsymbol{p}_1$, we find that for this choice of $\boldsymbol{q}_1$, $\boldsymbol{p}_1$ is independent of α_1 and is given by

$$\boldsymbol{p}_1^{\mathrm{T}} = \boldsymbol{r}_1^{\mathrm{H}} \boldsymbol{R}_1^{-1}, \tag{5}$$

with $\boldsymbol{r}_1^{\mathrm{H}} = \boldsymbol{q}_1^{\mathrm{T}} \boldsymbol{R}_0 \boldsymbol{B}_1^{\mathrm{H}}$ and $\boldsymbol{R}_1 = \boldsymbol{B}_1 \boldsymbol{R}_0 \boldsymbol{B}_1^{\mathrm{H}}$. Comparing Eqn. (2) with Eqn. (5) reveals the recursive characteristic of the decomposition: At the i-th stage, $\boldsymbol{p}_i$ is decomposed into $\boldsymbol{p}_i = \alpha_{i+1}\left(\boldsymbol{q}_{i+1} - \boldsymbol{B}_{i+1}^{\mathrm{T}} \boldsymbol{p}_{i+1}\right)$ with

$$\begin{aligned} &\boldsymbol{q}_{i+1} = \boldsymbol{r}_i^* / \left\|\boldsymbol{r}_i\right\|_2, \quad \mathrm{span}(\boldsymbol{B}_{i+1}^{\mathrm{T}}) = \mathrm{null}(\boldsymbol{q}_{i+1}^{\mathrm{H}}), \\ &\boldsymbol{p}_{i+1}^{\mathrm{T}} = \boldsymbol{r}_{i+1}^{\mathrm{H}} \boldsymbol{R}_{i+1}^{-1}, \\ &\boldsymbol{r}_{i+1}^{\mathrm{H}} = \boldsymbol{q}_{i+1}^{\mathrm{T}} \boldsymbol{R}_i \boldsymbol{B}_{i+1}^{\mathrm{H}}, \quad \boldsymbol{R}_{i+1} = \boldsymbol{B}_{i+1} \boldsymbol{R}_i \boldsymbol{B}_{i+1}^{\mathrm{H}}, \\ &\alpha_{i+1} = \boldsymbol{r}_i^{\mathrm{H}} \boldsymbol{q}_{i+1}^* \left(\boldsymbol{q}_{i+1}^{\mathrm{T}} \boldsymbol{R}_i \boldsymbol{q}_{i+1}^* - \boldsymbol{r}_{i+1}^{\mathrm{H}} \boldsymbol{R}_{i+1}^{-1} \boldsymbol{r}_{i+1}\right)^{-1}. \end{aligned}$$

Applying the decomposition N-times, we find

$$\boldsymbol{p}_0 = \alpha_1 \boldsymbol{q}_1 + \sum_{i=2}^{N} (-1)^{i+1} \left(\prod_{k=1}^{i} \alpha_k\right) \left(\prod_{k=1}^{i-1} \boldsymbol{B}_k^{\mathrm{T}}\right) \boldsymbol{q}_i. \tag{6}$$

Motivated by Eqn. (6), define the filters

$$\boldsymbol{t}_i = \left(\prod_{k=1}^{i-1} \boldsymbol{B}_k^{\mathrm{T}}\right) \boldsymbol{q}_i. \tag{7}$$

With Eqn. (7) we have found an iterative algorithm for computing a basis $\boldsymbol{T}^{(D)} = [\boldsymbol{t}_1, \ldots, \boldsymbol{t}_D]$ of a D-dimensional subspace of $\mathbb{C}^N$. For $i > 1$, using the definition of $\boldsymbol{q}_i, \boldsymbol{r}_i$ and $\boldsymbol{R}_i$, we can write

$$\boldsymbol{t}_i = \frac{1}{\left\|\boldsymbol{r}_{i-1}\right\|_2} \left(\prod_{k=1}^{i-1} \boldsymbol{B}_k^{\mathrm{T}}\right) \left(\prod_{k=i-1}^{1} \boldsymbol{B}_k^*\right) \boldsymbol{R}_0^* \boldsymbol{t}_{i-1}. \tag{8}$$

Moreover, it can be shown that $\boldsymbol{t}_i$ and $\boldsymbol{t}_j$ are $\boldsymbol{R}_0^*$-conjugate for $|i-j| > 1$:

$$\boldsymbol{t}_i^{\mathrm{H}} \boldsymbol{R}_0^* \boldsymbol{t}_j = 0, \quad |i-j| > 1. \tag{9}$$

The computation of the basis $\boldsymbol{T}^{(D)}$ can be further simplified if we require the basis vectors to be orthonormal. Plugging Eqn. (7) into $\boldsymbol{t}_i^{\mathrm{H}}\boldsymbol{t}_j = \delta[i-j]$, it follows that orthonormality is achieved if the blocking matrices satisfy $\boldsymbol{B}_i^*\boldsymbol{B}_i^{\mathrm{T}} = \mathbf{1}$. In this case, $\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{B}_i^*$ defines an orthogonal projector onto $\mathrm{null}(\boldsymbol{q}_i^{\mathrm{H}})$. But $\mathbf{1}-\boldsymbol{q}_i\boldsymbol{q}_i^{\mathrm{H}}$ also defines an orthogonal projector onto $\mathrm{null}(\boldsymbol{q}_i^{\mathrm{H}})$. Thus, from the uniqueness of projectors it can be concluded that $\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{B}_i^* = \mathbf{1} - \boldsymbol{q}_i\boldsymbol{q}_i^{\mathrm{H}}$. Plugging this result into Eqn. (8) and using Eqn. (9) finally yields

$$\boldsymbol{t}_i = \frac{\left(\mathbf{1} - \sum_{k=i-2}^{i-1}\boldsymbol{t}_k\boldsymbol{t}_k^{\mathrm{H}}\right)\boldsymbol{R}_0^*\boldsymbol{t}_{i-1}}{\left\|\left(\mathbf{1} - \sum_{k=i-2}^{i-1}\boldsymbol{t}_k\boldsymbol{t}_k^{\mathrm{H}}\right)\boldsymbol{R}_0^*\boldsymbol{t}_{i-1}\right\|_2}, \tag{10}$$

with $\boldsymbol{t}_1 = \boldsymbol{r}_0^*/\left\|\boldsymbol{r}_0\right\|_2$. In contrast to Eqn. (7), it is no longer necessary to explicitly compute the blocking matrices. In addition, the above algorithm is the well-known Lanczos algorithm [8] for computing an orthonormal basis of the Krylov subspace of a Hermitian matrix $\boldsymbol{R}_0^*$ and a vector $\boldsymbol{r}_0^*$. Based on this result, we can conclude that the orthonormal filters $\boldsymbol{t}_1, \ldots, \boldsymbol{t}_D$ constitute a basis of the D-dimensional Krylov subspace $\mathrm{span}([\boldsymbol{r}_0^*, \boldsymbol{R}_0^*\boldsymbol{r}_0^*, \ldots, (\boldsymbol{R}_0^*)^{D-1}\boldsymbol{r}_0^*])$.

For $K > 1$, the algorithm outlined in Eqn. (10) is carried out for each of the K data streams, resulting in K sets of basis vectors $\boldsymbol{T}_1^{(D)}, \ldots, \boldsymbol{T}_K^{(D)}$. Thus, in a multi-user scenario, we have K vector MSTxWFs in parallel. In the following, this configuration is termed as parallel vector MSTxWF.

5. REDUCED RANK MULTI-STAGE TXWF

After having computed the bases $\boldsymbol{T}_k^{(D)}$, the D-stage reduced rank MSTxWF for each k is found by computing the TxWF in $\mathrm{span}(\boldsymbol{T}_k^{(D)})$:

$$\begin{aligned}\{\check{\boldsymbol{p}}_{\mathrm{WF},1}, \ldots, \check{\boldsymbol{p}}_{\mathrm{WF},K}, \beta_{\mathrm{WF}}^{(D)}\} &= \underset{\{\check{\boldsymbol{p}}_1,\ldots,\check{\boldsymbol{p}}_K,\beta\}}{\mathrm{argmin}}\ \check{\sigma}_{\boldsymbol{\varepsilon}}^2 \\ \text{s.t.}\quad \sigma_s^2\sum_{i=1}^{K}\check{\boldsymbol{p}}_i^{\mathrm{T}}\boldsymbol{T}_i^{(D),\mathrm{T}}\boldsymbol{T}_i^{(D),*}\check{\boldsymbol{p}}_i^* &\leq E_{\mathrm{tr}},\end{aligned} \tag{11}$$

where

$$\check{\sigma}_{\boldsymbol{\varepsilon}}^2 = \sigma_{\boldsymbol{\varepsilon}}^2\Big|_{\boldsymbol{p}_1=\boldsymbol{T}_1^{(D)}\check{\boldsymbol{p}}_1,\ldots,\boldsymbol{p}_K=\boldsymbol{T}_K^{(D)}\check{\boldsymbol{p}}_K}.$$

With $\check{\boldsymbol{r}}_{0,k} = \boldsymbol{T}_k^{(D),\mathrm{T}}\boldsymbol{r}_{0,k}$ and $\check{\boldsymbol{R}}_{0,k} = \boldsymbol{T}_k^{(D),\mathrm{T}}\boldsymbol{R}_0\boldsymbol{T}_k^{(D),*}$, the solution of Problem (11) is given by

$$\begin{aligned}\check{\boldsymbol{p}}_{\mathrm{WF},k}^{\mathrm{T}} &= \beta_{\mathrm{WF}}^{(D)}\check{\boldsymbol{r}}_{0,k}^{\mathrm{H}}\check{\boldsymbol{R}}_{0,k}^{-1}, \\ \beta_{\mathrm{WF}}^{(D)} &= \sqrt{E_{\mathrm{tr}}}\Big/\sqrt{\sigma_s^2\textstyle\sum_{i=1}^{K}\check{\boldsymbol{r}}_{0,i}^{\mathrm{H}}\check{\boldsymbol{R}}_{0,i}^{-2}\check{\boldsymbol{r}}_{0,i}}.\end{aligned}$$

Note that $\check{\boldsymbol{R}}_{0,k}$ is a $D \times D$ matrix. As a result, the parallel MSTxWF can be computed by inverting K matrices of dimension $D \times D$. A more efficient computation is possible by exploiting the fact that only the first entry in $\check{\boldsymbol{r}}_{0,k}$ is non-zero and that, according to Eqn. (9), the matrices $\check{\boldsymbol{R}}_{0,k}$ are tri-diagonal. Still, it is important to note that the main complexity lies in the computation of the basis vectors $\boldsymbol{t}_{i,k}$ and the matrices $\check{\boldsymbol{R}}_{0,k}$, since D matrix-vector multiplications $\boldsymbol{R}_0\boldsymbol{t}_{i,k}^*$ of $\mathrm{O}(N^2)$ have to be performed for each k. Thus, the overall complexity of a D-stage parallel MSTxWF implementation is of $\mathrm{O}(KDN^2)$. The complexity of the TxWF solution is of $\mathrm{O}(N^3)$. As a result, a reduction in complexity is achieved if the number of stages D required to obtain a close-enough approximation of the full-rank solution is smaller than N/K.

In the parallel MSTxWF implementation, for each k a reduced-rank solution is computed using a separate set of basis vectors. Obviously, we can collect the bases $\boldsymbol{T}_k^{(D)}$ into $\tilde{\boldsymbol{T}}^{(D)} = [\boldsymbol{T}_1^{(D)}, \ldots, \boldsymbol{T}_K^{(D)}]$ and compute each reduced-rank solution in $\mathrm{span}(\tilde{\boldsymbol{T}}^{(D)})$. In the following, this solution will be termed as joint vector MSTxWF. As this work is focused on the parallel vector MSTxWF, we use the joint vector MSTxWF mainly as a performance measure for the parallel implementation.

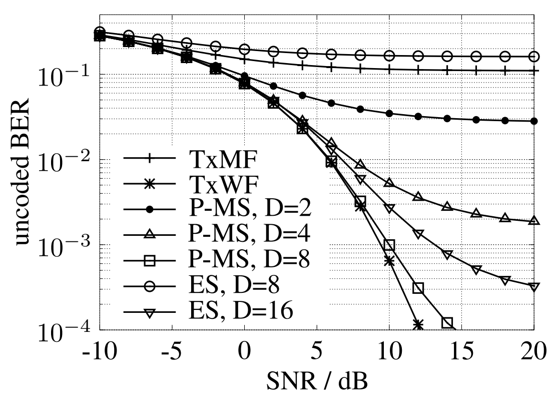

Fig. 2. Parallel MSTxWF versus ESTxWF

6. SIMULATION RESULTS

We present *uncoded bit error rate* (uncoded BER) results for the parallel MSTxWF and compare its performance with the performance of the TxMF, the TxWF, the joint vector MSTxWF and the *eigenspace TxWF* (ESTxWF). Motivated by the principal component analysis [9], the ESTxWF is computed by approximating the TxWF in the subspace spanned by the eigenvectors of $\boldsymbol{R}_0$ that correspond to the DK largest eigenvalues. A system with $N_{\mathrm{a}} = 4$ antenna elements at the receiver and $K = 3$ receivers is considered. In order to keep the overall system as simple as possible, we do not employ CDMA for multi-user separation. QPSK modulation is used for transmission. The channels to the K receivers have an exponential power delay profile with

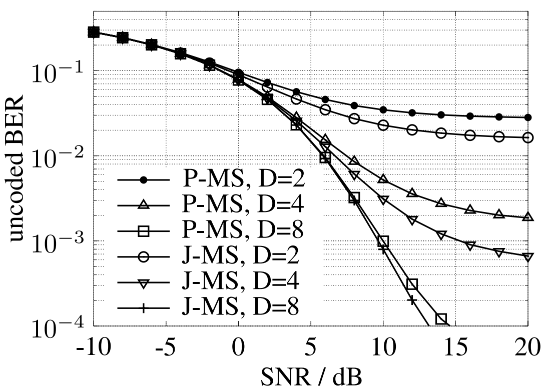

Fig. 3. Parallel MSTxWF versus Joint MSTxWF

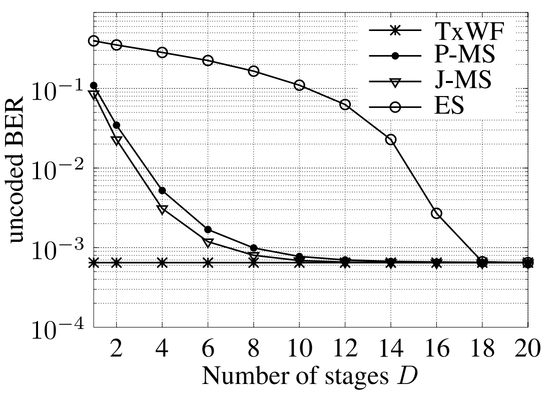

Fig. 4. Reduced Rank Convergence, SNR = 10 dB

$Q + 1 = 6$ paths. We assume temporally and spatially uncorrelated Rayleigh fading. The transmit filters are of order $L = 15$, a fixed latency time $\nu = 5$ is used. The results are the mean of 10000 channel realizations, 100 symbol vectors were transmitted per realization. The (transmit) SNR is defined as SNR $= 10\log_{10}(E_{\text{tr}}/(K\sigma^2_{\eta_k}))$dB, where the noise power $\sigma^2_{\eta_k}$ is the same for all receivers.

In Fig. 2 it can be observed that the *parallel MSTxWF* (P-MS) clearly outperforms the ESTxWF. While the 2-stage MSTxWF performs substantially better than the TxMF, the 8-stage ESTxWF shows worse BER performance than the TxMF. Compared to the TxMF, the 8-stage ESTxWF provides worse performance at higher computational complexity. In contrast to the ESTxWF, the 8-stage MSTxWF provides close to optimum performance for a BER larger or equal to 10^{-3}. Note that $N/K = 64/3$. Thus, in the BER range under consideration, the MSTxWF can provide optimum performance at significantly reduced complexity.

The parallel and the joint vector implementation of the MSTxWF are compared in Fig. 3. The parallel MSTxWF saturates at a higher BER. Nevertheless, of particular interest are the combinations of SNR range and number of stages D in which the deviation from the optimum solution is relatively small. For these combinations of interest, the difference between both MSTxWF implementations is negligible.

In Fig. 4, the BER is plotted versus the number of stages D for a fixed transmit SNR of 10 dB. Again, the results show the superiority of the Krylov subspace-based methods over the eigenspace approach.

We can conclude that in BER ranges of practical interest, the MSTxWF provides an excellent trade-off between BER performance and computational complexity.

7. REFERENCES

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