Sum-Rate Maximizing Decompositon Approaches for Multiuser MIMO-OFDM

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***Abstract*— In the work at hand, decomposition approaches are investigated for the downlink of a multiuser MIMO setting. The focus is on approaches that use successive encoding to eliminate part of the interference between users or information streams.**

We start reviewing the well known zero-forcing with successive encoding (ZF-SE) approach and generalize the basic idea behind this technique to arrive at a block ZF-SE that can be applied to the case of users with multiple antennas exploiting their cooperation capability.

Aiming at a maximization of the achievable sum-rate we elaborate on the ZF-SE and block ZF-SE approaches to come up with the ZF-SE with successive allocation method (ZF-SESAM) and the cooperative ZF-SE with successive allocation method (CZF-SESAM), respectively. This two approaches proceed successively selecting at each step a user to which the next spatial dimension is assigned. Specifically, we propose a largest gain criterion for this selection and provide some rationale for that.

Using an OFDM transmission scheme, Tomlinson-Harashima precoding and a common bit loading algorithm we compare the sum-rate achieved by the different approaches. Finally, some simulation results show that in a large MIMO-OFDM system with a moderate number of users even the weakest user can profit from the increase in sum-rate if compared to a maximally fair system, where the same number of dimensions is assigned to every user.

I. Introduction

One of the most challenging problems to face when operating a MIMO-OFDM scheme in the downlink of a mobile communication system is how to efficiently deal with the many dimensions available. Indeed, if the mobile station or access point has perfect channel knowledge, performance of the system will greatly depend on how this information is used to assign dimensions to users and allocate power and information over these assigned dimensions. In such systems, joint optimization of weighting vectors, number of information streams assigned to each user and power allocation is either mathematically intractable or leads to computationally expensive solutions (e.g. [1]). A typical simplification of the design problem consists of presuming a certain number of streams for each user and optimizing weighting vectors and power allocation (cf. [2]). In spite of the optimality loss, the resulting iterative algorithms remain certainly involved.

Alternatively, system design can be simplified if, first, the multiuser multiple-input multiple-output (MIMO) channel is decomposed into a set of scalar subchannels so that interference between them is effectively eliminated and, then, power allocation upon this set of virtually decoupled channels is optimized. Although it will normally lead to a loss of optimality, elimination of interference might be beneficial if, as a consequence, the design problem becomes tractable or if the complexity reduction justifies the optimality loss [3].

Sticking to this last class of approaches, in the work at hand, we first review the ZF-SE technique described in [4] and propose a block ZF-SE scheme as a natural extension of this technique to the case of receivers with multiple antennas.

Aiming at a maximization of the sum-rate, we choose a successive implementation of the ZF-SE approach, where, at each step, a user, to which the next spatial dimension is assigned, is selected according to some specified criterion. We call the resulting algorithm ZF-SE with successive allocation method (ZF-SESAM). For the selection of users a largest gain criterion is proposed and we provide some rationale for that. Replacing the blockwise allocation of dimensions of the block ZF-SE by a successive allocation of single dimensions to users and specifying some criterion for the assignation at each step of a new dimension to a certain user, the cooperative ZF-SE with successive allocation method (CZF-SESAM) is obtained that was reported in [3]. In [3] it was observed that using a largest gain criterion to assign a new subchannel to some user, the achievable sum-rate practically reached the Sato upper bound on sum-capacity of the downlink multiuser MIMO system. Here, we observe that this criterion is simply an extension of the one proposed for the ZF-SESAM algorithm.

To show performance we compare the sum-rate achieved by the different approaches in a practical implementation

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of a MIMO-OFDM system that uses Tomlinson-Harashima precoding to suppress known interference [5], [6] and a standard loading algorithm [7]. Furthermore, since a sum-rate criterion might pose some fairness concerns, we present some simulation results showing that in a moderately loaded OFDM system the weakest user profits from the application of the sum-rate maximizing CZF-SESAM if compared with the weakest user of maximally fair OFDMA and ZF-SE approaches, which is a consequence of the large superiority of the former in terms of sum-rate.

The remaining of the paper is structured as follows. In Section II, the system model is introduced that is used along this paper. In Section III the ZF-SE approach is reviewed and we present the block ZF-SE scheme for multiuser MIMO channels. In Section IV, the degrees of freedom offered by the ZF-SE and block ZF-SE approaches are optimized to obtain the sum-rate maximizing ZF-SESAM and CZF-SESAM algorithms. In Section V the different approaches are applied to the downlink of a MIMO-OFDM systems and fairness issues are also discussed in this section. Finally, in Section VI the essential content of this paper is summarized and conclusions are drawn.

A. Notation

In the following, vectors and matrices are denoted by lower case bold and capital bold letters, respectively. We use $(\bullet)^*$ for complex conjugation, $(\bullet)^{\mathrm{T}}$ for matrix transposition and $(\bullet)^{\mathrm{H}}$ for conjugate transposition. The identity matrix of dimension q is denoted by $\boldsymbol{I}_q$ and $\boldsymbol{e}_s$ denotes its sth column.

II. System Model

For the downlink we consider the usual system model

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n},$$

with

$$\begin{aligned} \boldsymbol{y} &= [\ \boldsymbol{y}_1^{\mathrm{T}} \ \cdots \ \boldsymbol{y}_K^{\mathrm{T}}\ ]^{\mathrm{T}}, \\ \boldsymbol{n} &= [\ \boldsymbol{n}_1^{\mathrm{T}} \ \cdots \ \boldsymbol{n}_K^{\mathrm{T}}\ ]^{\mathrm{T}}, \\ \boldsymbol{H} &= [\ \boldsymbol{H}_1^{\mathrm{T}} \ \cdots \ \boldsymbol{H}_K^{\mathrm{T}}\ ]^{\mathrm{T}}, \end{aligned}$$

where K is the number of users, $\boldsymbol{H}_k \in \mathbb{C}^{r_k \times t}$ is the channel matrix, $\boldsymbol{y}_k \in \mathbb{C}^{r_k}$ the received signal and r_k the number of receive antennas of user k, $\boldsymbol{n}_k \in \mathbb{C}^{r_k}$ is a realization of a zero-mean circularly symmetric complex Gaussian distributed random variable $\mathbf{n}_k$ representing noise with covariance matrix $\mathrm{E}\{\mathbf{n}_k \mathbf{n}_k^{\mathrm{H}}\} = \boldsymbol{I}_{r_k}$, and t the number of transmit antennas at the base station. This model describes transmission over a particular subcarrier of an ideal OFDM system without intercarrier or intersymbol interference. We assume that to all subcarriers this model applies.

III. Downlink Decomposition Methods

A. ZF-SE

This algorithm has been conceived for application to a setting with single antenna non-cooperative receivers, i.e. $r_1 = \cdots = r_K = 1$ [8] [4]. In this setting each row $\boldsymbol{h}_k^{\mathrm{T}}$ of the composite downlink channel matrix $\boldsymbol{H}$ represents the channel of an individual user k.

Ordering the users according to a predefined permutation function π there are two basic methods to obtain the transmit weighting vectors that achieve the effective decomposition of the channel. The first method consists of permuting the rows of matrix $\boldsymbol{H}$ according to the function π and performing an LQ decomposition of this permuted matrix. The result of such decomposition is a lower triangular matrix $\boldsymbol{L} \in \mathbb{C}^{K \times \mathrm{rank}(\boldsymbol{H})}$ and a matrix with orthonormal row vectors $\boldsymbol{Q} \in \mathbb{C}^{\mathrm{rank}(\boldsymbol{H}) \times t}$.

Assuming that the rank of any $m \times t$ submatrix of $\boldsymbol{H}$ is equal to $\min\{m, t\}$, the transmit weighting vector for user $1 \leq \pi(j) \leq K$ is chosen to be $\boldsymbol{v}_{\pi(j)} = \boldsymbol{q}_j^*$, $0 \leq j \leq t$, where $\boldsymbol{q}_j^{\mathrm{T}}$ denotes the jth row of matrix $\boldsymbol{Q}$. If $K > t$ users $\pi(j > t)$ do not get any spatial dimension and therefore are not served by the base station.

It is easy to show that no interference is caused by users $\pi(i > j)$ on user $\pi(j)$, i.e. $\boldsymbol{h}_{\pi(j)}^{\mathrm{T}} \boldsymbol{v}_{\pi(i>j)} = 0$, which corresponds to the entries in the upper part of $\boldsymbol{L}$. On the contrary, in general users $\pi(i < j)$ will cause interference to user $\pi(j)$ according to the non-zero entries in the lower part of $\boldsymbol{L}$. However, this interference can be neutralized by successive encoding users in the order given by π and taking into account known interference in the coding of signals [9] [10]. The gains of the effectively decoupled subchannels are given by the diagonal entries of matrix $\boldsymbol{L}$.

In the second method the precoding vectors are successively computed. In the jth step of this algorithm, the transmit weighting vector for user $\pi(j)$ is computed. To this end, first, a projector matrix $\boldsymbol{T}_j$ is computed corresponding to the subspace complementary to that spanned by the weighting vectors of previous users,

$$\boldsymbol{T}_j = \boldsymbol{I}_t - \sum_{i<j} \boldsymbol{v}_{\pi(i)} \boldsymbol{v}_{\pi(i)}^{\mathrm{H}}.$$

Then, within this subspace the orthonormal vector is chosen whose inner product with the channel $\boldsymbol{h}_{\pi(j)}^{\mathrm{T}}$ yields the largest gain. This vector turns out to be the conjugate transpose of the projection of the channel vector $\boldsymbol{h}_{\pi(j)}^{\mathrm{T}}$ into the subspace spanned by $\boldsymbol{T}_j$, i.e.

$$\boldsymbol{v}_{\pi(i)} = \frac{\boldsymbol{T}_j \boldsymbol{h}_{\pi(j)}^*}{\|\boldsymbol{T}_j \boldsymbol{h}_{\pi(j)}^*\|}.$$

For the first user, $\boldsymbol{T}_1 = \boldsymbol{I}_t$. For user $\pi(t+1)$, $\boldsymbol{T}_{t+1} = \mathbf{0}$, i.e. only the first t users can be served as we saw above.

The two methods described in this section are equivalent.

B. Block ZF-SE

For the general case with $r_k \geq 1$, considering a predefined ordering of users given by a function π, the principle followed in the second method described above can also be applied with certain modifications.

In general, a user is able to efficiently process as many spatial dimensions as the number of receive antennas of that user. Therefore, rather than computing a single transmit weighting vector, now at each step of the algorithm a block of transmit weighting vectors is computed. Specifically, at step j the transmit weighting vectors for user $\pi(j)$ can be computed as follows. First, a projector matrix $\boldsymbol{T}_j$ is computed corresponding to the subspace complementary to that spanned by the weighting vectors of previous users,

$$\boldsymbol{T}_j = \boldsymbol{I}_t - \sum_{i<j} \boldsymbol{V}_{\pi(i)} \boldsymbol{V}^{\mathrm{H}}_{\pi(i)},$$

where $\boldsymbol{V}_{\pi(i)} \in \mathbb{C}^{t \times L(i)}$ is a matrix with $L(i)$ orthonormal columns which are used as transmit weighting vectors for user $\pi(i)$.

Then within this subspace, rather than chosing a single transmit weighting vector, we select a basis of vectors "matching" the matrix $\boldsymbol{H}_{\pi(j)}$ as much as possible. To this end, proceeding analogously to the ZF-SE approach, we first project this matrix into the subspace spanned by the rows of $\boldsymbol{T}_j$. Then, we compute the singular value decomposition (SVD) of the projected channel,

$$\boldsymbol{H}_{\pi(j)} \boldsymbol{T}_j = \boldsymbol{U}_{\pi(j)} \boldsymbol{\Lambda}_{\pi(j)} \boldsymbol{V}^{\mathrm{H}}_{\pi(j)}.$$

The conjugate transpose of the matrix of right singular vectors is taken as the matrix of transmit weighting vectors of user $\pi(j)$, and in order to simplify transmission and detection the channel is decomposed by using the conjungate transpose of the left singular vectors as weighting vectors at the receiver. The number $L(j)$ of dimensions assigned to user $\pi(j)$ is equal to the dimension of matrix $\boldsymbol{\Lambda}_{\pi(j)}$, which is a diagonal square matrix with non-zero entries in the main diagonal. Note that further dimensions can be assigned only if $\sum_{i=1}^{j} L(i) < t$.

As in the ZF-SE approach, users $\pi(i > j)$ will not cause any interference on user $\pi(j)$, which follows from

$$\begin{aligned} \boldsymbol{H}_{\pi(j)} \boldsymbol{V}_{\pi(i>j)} &= \\ \boldsymbol{H}_{\pi(j)} \boldsymbol{T}_j \boldsymbol{T}_{i>j} \boldsymbol{V}_{\pi(i>j)} &= \\ \boldsymbol{U}_{\pi(j)} \boldsymbol{\Lambda}_{\pi(j)} \boldsymbol{V}^{\mathrm{H}}_{\pi(j)} \boldsymbol{T}_{i>j} \boldsymbol{V}_{\pi(i>j)} &= \mathbf{0}. \end{aligned}$$

Users $\pi(i > j)$ will in general cause interference to user $\pi(j)$ which can be neutralized encoding users successively as explained above. Finally, the gains of the effectively decoupled scalar subchannels over which information is transmitted is given by the diagonal entries of matrices $\boldsymbol{\Lambda}_{\pi(j)}$.

Note that this algorithm is equivalent to a SVD of the channel for the single user MIMO case and to the ZF-SE approach for multiuser case with single receive antennas. In essence, this approach is the same as the SO-THP approach recently and independently proposed in [11] with a predefined ordering of users.

IV. Sum-Rate Optimization

A. ZF-SESAM

In general, the gains of the subchannels resulting from the application of the ZF-SE approach will be different for different orderings of users π and so the achievable sum-rate. Aiming at a maximization of the sum-rate the algorithm sketched in Table I is considered that is basically the ZF-SE algorithm described above but where, rather than assuming a predefined ordering of users, the user to which a spatial dimension is assigned is selected in execution time based on a particular selection rule $\mathcal{O}$. Here, we propose a largest gain criterion as selection rule, i.e.

$$\mathcal{O}\{\cdot\} = \operatorname*{argmax}_{k}\{\cdot\}. \quad (1)$$

```
initialization :  j = 1,  T_1 = I_t
repeat :
   1.  g_k = |h_k^T T_j|  ∀k
   2.  k_0 = O{g_k},  π(j) = k_0
   3.  T_{j+1} = T_j − T_j h*_{π(j)} h^T_{π(j)} T_j^H / g^2_{π(j)},  j = j + 1
until  j > K or T_j = 0
```

TABLE I
ZF-SESAM Algorithm

The following two theorems [12] provide some rationale for this proposal.

Theorem 1: Let $\mathcal{S}_j \subset \{1, \dots, K\}$ be the set of first j selected users and let $C_{\mathcal{S}_j}$ be the sum rate achieved by these users. Selecting the next user according to the proposed method yields the maximum capacity increment $\Delta C = C_{\mathcal{S}_{j+1}} - C_{\mathcal{S}_j}$, where $\mathcal{S}_{j+1} = \mathcal{S}_j \cup \{\pi(j+1)\}$.

While the first theorem tells which user must be chosen to get the largest capacity increment, the second gives the optimum order for the selection of an additional pair of two arbitrary users and therefore states the optimality of this ordering for the two-user case.

Theorem 2: Let $\mathcal{S}_j \subset \{1, \dots, K\}$ be the set of first j selected users and $k_1, k_2 \in \{1, \dots, K\} \setminus \mathcal{S}_j$ so that $\|\boldsymbol{h}^{\mathrm{T}}_{k_1} \boldsymbol{T}_{j+1}\| \geq \|\boldsymbol{h}^{\mathrm{T}}_{k_2} \boldsymbol{T}_{j+1}\|$. Define $\mathcal{S}_{j+2} = \mathcal{S}_j \cup \{k_1, k_2\}$ and the ordering functions π and π' so that $\pi(i) = \pi'(i)\ \forall i \leq j$, $\pi(j+1) = \pi'(j+2) = k_1$ and $\pi(j+2) = \pi'(j+1) = k_2$. $C_{\mathcal{S}_{j+2}}$ is maximized by choosing the encoding order defined by π.

B. CZF-SESAM

Aiming at a maximization of sum rate there are two major drawbacks of the block ZF-SE approach to be noted. First, as described above, at each step, the algorithm assigns to a certain user as many subchannels as the rank of its projected channel matrix. This is clearly suboptimum if, for instance, some of the subchannels are weak. In that case, contribution of these subchannels to the sum rate might be negligible while they may impose severe constraints on subchannels of subsequently encoded users. The second drawback is the predefined encoding order.

The first drawback can be overcome if, instead of performing a blockwise assignation of spatial dimensions, at each step, only a scalar subchannel is assigned to a particular user. The second drawback can be mitigated by specifying some rule according to which the selection of the user is made, to which a new dimension is assigned. The resulting cooperative ZF-SE with successive allocation method was reported in [3] and is scketched in Table II.

```
initialization :   j = 1,   T_1 = I_t
repeat :
    1.  H_k^j = H_k T_j  ∀k
    2.  H_k^j = U_k^j Λ_k^j V_k^{j,H}  ∀k
    3.  (k_0, s_0) = O{λ_{k,s}^j},  π(j) = (k_0, ℓ(k_0))
            v_{π(j)} = V_{k_0}^j e_{s_0},  u_{π(j)} = U_{k_0}^j e_{s_0}
    4.  T_{j+1} = T_j − v_{π(j)} v_{π(j)}^H,   j = j + 1
until   j > Σ_k r_k  or  T_j = 0
```

TABLE II

CZF-SESAM ALGORITHM

Note that now $\pi(j)$ does not denote a user but an assigned scalar subchannel, which is labelled by the pair $(k, \ell(k))$, where k indicates the user the channel has been assigned to and $\ell(k)$ identifies that specific subchannel among all subchannels assigned to user k. Each subchannel $\pi(j)$ is characterized by a transmit weighting vector $\boldsymbol{v}_{\pi(j)}$ and a receive weighting vector $\boldsymbol{u}_{\pi(j)}$ that are selected from the matrices of right and left singular vectors of the projected matrices.

Consistently with the allocation rule chosen for the ZF-SESAM algorithm we also choose a largest gain criterion for the CZF-SESAM,

$$\mathcal{O}\{\cdot\} = \underset{s,k}{\operatorname{argmax}}\{\cdot\}. \quad (2)$$

V. SIMULATION RESULTS

To evaluate performance, simulations have been carried out of a MIMO-OFDM system with $N = 1024$ subcarriers, an access point with $t = 4$ transmit antennas and $K = 10$ mobile units with $r_k = 2$ receive antennas each. The channels are spatially almost uncorrelated and the power delay profiles are in all cases circa 180 samples long. Block fading is assumed, in which for the transmission of each OFDM symbol a new independent channel realization is drawn for each user. The entries of the channel matrices are distributed according to a complex-valued zero-mean Gaussian distribution with unit variance.

The CZF-SESAM and ZF-SESAM algorithms are applied to each subcarrier selecting the spatial dimensions according to (2) and (1), respectively, which produces an optimized allocation of subcarriers to users as a by-product. For the block ZF-SE and the ZF-SE algorithms the ordering of users is predefined so that over the whole spectrum all users receive the same number of space-frequency dimensions. Beside these already described approaches, for comparison purposes, the following schemes have been considered. An OFDMA scheme that serves on each subcarrier the user whose matrix exhibits the largest Frobenius norm (OFDMA dynamic). An OFDMA scheme that assigns to every user the same amount of subcarriers without taking into account channel knowlegde (OFDMA static). In both OFDMA schemes a SVD of the channel matrices is performed and transmission occurs over the singular values. A zero-forcing scheme that arbitrarily selects two users per carrier and inverts the resulting channel matrix, so that both interuser interference and cross-talk between antennas of a same user is linearly suppressed (ZF). The block zero-forcing decomposition scheme presented in [13], which is applied to two arbitrary users per subcarrier (Block ZF).

Over the set of effectively decoupled subchannels obtained from all these approaches the rate maximizing greedy bit and power loading algorithm described in [7] is employed considering square QAM constellations and a target symbol error rate (SER) of 0.1. SE techniques employ Tomlinson-Harashima precoding to cancel known interference [8].

Fig. 1 shows average sum-rate over SNR, which is defined as the ratio of transmit power per subcarrier and noise variance at the receive antenna elements. The best performance is delivered by CZF-SESAM, which profits from both, channel sensitive allocation of space-frequency dimensions and cooperation between antenna elements belonging to a same user. ZF-SESAM profits from the first but not from the second. We observe that dynamic allocation makes a large difference as the gap between successive allocation methods and predefined allocation methods is quite significant. Basically, for this practical implementation, the curves reflect very faithfully the behavior of the capacity curves presented in [3].

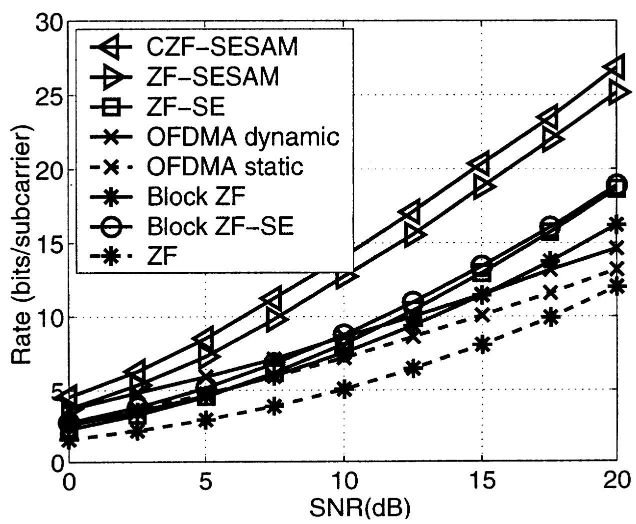

Fig. 1. Average sum rate for a multiuser setting with weakly spatially correlated Rayleigh fading Gaussian channels. $t = 4$, $r_k = 2$, $K = 10$, $N = 1024$, SER $= 0.1$.

It is not surprising that techniques that use channel knowledge to perform an optimized allocation of space and frequency resources to users can achieve a higher sum-rate. Moreover, since this allocation entirely depends on the channel it could result in a very uneven distribution of resources among users. This will especially be the case if the users observe channels with dissimilar gains in average. However, if the channel gains observed by the users are similar, as occurs in the setting studied here, even users being disfavored by a dynamic allocation of dimensions can benefit in terms of rate from the total increase in sum-rate if compared with maximally fair approaches such as OFDMA static or ZF-SE with even allocation. For an OFDM symbol we define the relative rate gain of the most disfavored user in a certain approach with respect to the rate of the most disfavored user in an approach of reference as,

$$g = \frac{R_{\min} - R_{\mathrm{r}}}{R_{\mathrm{r}}} \times 100,$$

where $R_{\min}$ is the minimum rate achieved by a user with a certain scheme that uses dynamic resource allocation and R_{r} is the minimum rate achieved by a user in the reference approach that in this case are either OFDMA static or the ZF-SE with even allocation. The average of this figure over a number of channel realizations is displayed in Figs. 2 and 3. We observe that almost for all values of SNR even the weakest user profits from the increase in sum-rate accomplished by the CZF-SESAM approach.

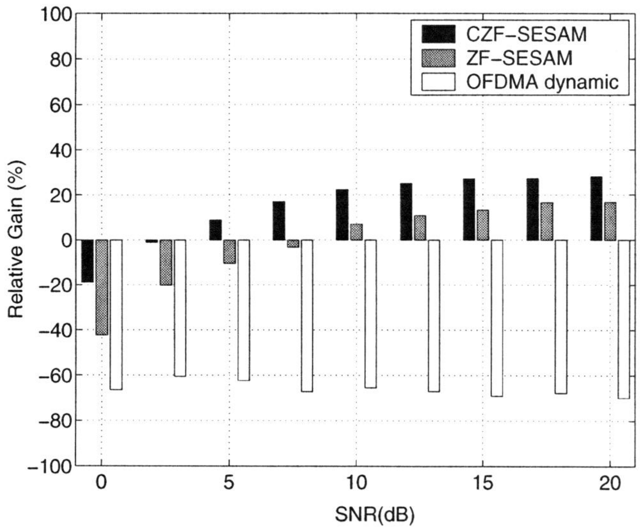

Fig. 2. Relative gains of most disfavored user referred to *OFDMA static*. $t = 4$, $r_k = 2$, $K = 10$, $N = 1024$, $\mathrm{SER} = 0.1$.

VI. Conclusions

A generalization of the ZF-SE approach for users with multiple antennas has been presented. Both ZF-SE and its generalization have been optimized aiming at a maximization of the sum-rate. Performance evaluation of these and other decomposition approaches has been done using a MIMO-OFDM system, Tomlinson-Harashima precoding and a standard loading algorithm. Even if the optimized approaches do not consider any fairness aspects in the allocation of space-frequency dimensions, for a moderate system load and equal average channel gains, simulation results have shown that even the most disfavored user is able to benefit from the resulting increase in sum-rate.

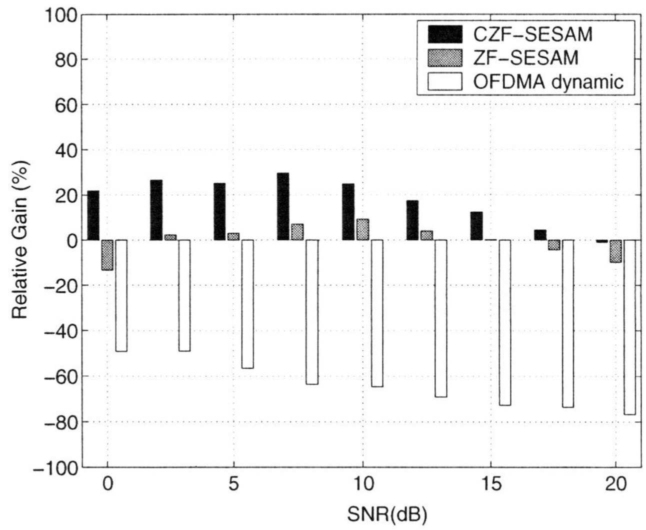

Fig. 3. Relative gains of most disfavored user referred to *ZF-SE*. $t = 4$, $r_k = 2$, $K = 10$, $N = 1024$, $\mathrm{SER} = 0.1$.

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