Impact of Imperfect Channel Knowledge on Transmit Processing Concepts

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Abstract— **Common transmit processing concepts are either based on complete, partial, or no channel state information (CSI). But the quality of CSI is crucial for a fair comparison. Therefore, we derive an explicit expression of the bit error probability (binary modulation) for the transmit matched filter, beamforming, and Alamouti space-time block code in case of linear estimation and prediction of the channel coefficients or a delay of the channel estimates. Uncorrelated as well as correlated frequency flat channels are considered. Based on the analytical expressions a comparison for these three representatives for all three types of CSI is made. The break even point between the three schemes is computed numerically, which can serve as a switching point between the concepts.**

I. Introduction

The performance improvement of space-time processing for wireless communications is widely accepted [1]. But exploiting the spatial domain of the channel increases computational complexity for signal processing, which is a problem at the mobile terminal in particular. Thus, it is explored to equip the base station with multiple antennas and exploit the additional degrees of freedom for equalization, diversity, and antenna gain with signal processing at the transmitter, which is more likely to provide the resources for the additional complexity in the downlink.

The choice of a transmit processing (precoding) concept depends on the *channel state information* (CSI) *available* at the transmitter in principle, i.e., whether uplink and downlink channels are reciprocal or a feedback channel is provided from the receiver to the transmitter. Moreover, the *quality* of the CSI available in the current environment is crucial for selecting the appropriate concept. We distinguish *three main concepts* based on the channel state information at the transmitter, which is required for their design: (1) Processing relying on complete CSI, e.g., linear precoding with a transmit matched filter (TxMF), transmit zero forcing (TxZF), and transmit Wiener filter [2], [3], (2) using partial (average) CSI, which is the second order moments of the channel coefficients, e.g., beamforming [4], [5], (3) precoding without CSI, e.g., Alamouti space-time block codes (STBC) [6].

In the sequel we assume a time-division duplex (TDD) link with M antennas at the transmitter and one antenna at the receiver with a reciprocal up- and downlink channel. Thus, all three concepts can be applied theoretically. To increase the data rate in the downlink an asymmetric slot configuration can be used, i.e., more transmission time is spent for the downlink, which results in a significant delay between the last uplink slot available for channel estimation and the downlink slots. The resulting performance degradation is significant for schemes relying on full CSI [7]. Depending on the speed of the mobile it is advantageous for the transmitter to switch to a concept based on partial or no CSI. Previous contributions considered, for example, switching between different spatial signaling schemes (space-time coding and spatial multiplexing) for MIMO channels based on bounds for the probability of symbol error not considering degradations due to delay and channel estimation, e.g., [8].

We analyze and compare representatives of the three transmit processing concepts described above for frequency flat fading channels introduced in Section II: the TxMF [2], beamforming [4], and Alamouti STBC [6]. To compare performance in time-variant channels it is necessary to take into account the effect of imperfect CSI as a major source of degradation of the TxMF: the analysis is done for linear channel estimators and linear prediction at the transmitter (Section III).

Previously, the effect of channel estimation errors on coherent detection for single antenna systems was studied in [9] and on STBC in [10], where it results in intersymbol interference. Pairwise error probability and cut-off rate were investigated for the TxMF and STBC in uncorrelated channels [11]. A comparison of STBC with beamforming was presented in [12] for uncorrelated channels, which also considers channel coding, but does not include channel estimation. Moreover, an optimistic gain of beamforming was assumed in [12] instead of a more accurate analysis considering channel correlations.

In Section IV we derive the (uncoded) bit error probability (BEP) for BPSK modulation analytically and explicitly for correlated and uncorrelated Rayleigh fading channels taking into account channel estimation errors and the delay of the channel estimates based on results from [13], [14]. The BEP derivations for the TxMF and beamforming are valid for arbitrary M, whereas the Alamouti scheme is designed for $M{=}2$. Note that all three approaches provide the same raw data rate since Alamouti STBC is a rate one code. Results are also applicable for analysis of the TxMF with a feedback channel as long as the complex Gaussian error model for the CSI is valid.

The performance comparison (Section V) based on the derivations is done for $M{=}2$. The gains for perfect CSI are discussed, before considering the delay between channel estimation and the downlink slot at the transmitter for the TxMF and channel estimation at the receiver for beamforming

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and Alamouti STBC. The break even points between the concepts, which represent the switching points, are computed numerically for symmetric/asymmetric slot configurations and different mobile velocities.

Our contributions are: i) The BEP analysis of the Tx-MF for imperfect CSI based on a linear channel estimator/predictor or a simple delay. ii) Comparison of transmit processing/precoding representatives for all three categories of CSI under realistic assumptions. iii) Illustration of the break even point for different TDD system configurations.

Throughout the paper, $\hat{a}$ denotes an estimate of a, '$\otimes$' the Kronecker product, $\boldsymbol{I}_M$ the $M \times M$ identity matrix, and $\boldsymbol{e}_i$ the i-th column of $\boldsymbol{I}$. The transpose, complex conjugate transpose, and complex conjugate of a matrix is written as $\boldsymbol{A}^{\mathrm{T}}$, $\boldsymbol{A}^{\mathrm{H}}$, and $\boldsymbol{A}^*$ respectively. Deterministic variables as well as realizations of a random variable $\mathbf{a}$ are written as $\boldsymbol{a}$.

II. Transmit Processing Concepts

The receive signal $\mathsf{x}[n]$ of a (downlink) data channel for a link with M transmit and 1 receive antenna is described by

$$\mathsf{x}[n] = \boldsymbol{h}^{\mathrm{T}} \boldsymbol{p} s[n] + \mathsf{n}[n], \;\; n \in \mathbb{Z} \tag{1}$$

with channel $\boldsymbol{h} \in \mathbb{C}^M$, linear transmit filter $\boldsymbol{p} \in \mathbb{C}^M$, BPSK data symbols $s[n] \in \{+\sqrt{P_{\mathrm{s}}}, -\sqrt{P_{\mathrm{s}}}\}$, and white additive complex Gaussian noise $\mathsf{n}[n] \sim \mathcal{N}_{\mathrm{c}}(0, \sigma_{\mathrm{n}}^2)$.

A. Transmit Matched Filter

Generally, the goal of linear precoding based on perfect knowledge of $\boldsymbol{h}$ is to exploit additional spatial degrees of freedom keeping the receiver simple. For frequency flat channels the TxMF [2], which is equivalent to transmit zero forcing in this case, *maximizes cross-correlation* between the decision variable $\mathsf{d}[n] = \mathsf{x}[n]$, passed to the decision device, and $\mathsf{s}[n]$ (Figure 1) constrained by the available transmit power P_{T}:

$$\boldsymbol{p}_{\mathrm{MF}} = \arg\max_{\boldsymbol{p}} |\mathrm{E}[\mathsf{s}[n]^* \mathsf{d}[n]]|^2 \text{ s.t.: } \mathrm{E}[\|\boldsymbol{p}\,\mathsf{s}[n]\|_2^2] = P_{\mathrm{T}} \tag{2}$$

$$\boldsymbol{p}_{\mathrm{MF}} = \sqrt{P_{\mathrm{T}}/P_{\mathrm{s}}}\; \boldsymbol{h}^* / \|\boldsymbol{h}\|_2. \tag{3}$$

The instantaneous SNR $\|\boldsymbol{h}\|_2^2 P_{\mathrm{T}}/\sigma_{\mathrm{n}}^2$ at the receiver for detection of one symbol for perfect CSI yields an average SNR for many channel realization of $\overline{\mathrm{SNR}} = \mathrm{trace}(\boldsymbol{R}_{\mathbf{h}}) P_{\mathrm{T}}/\sigma_{\mathrm{n}}^2$. In practice only an estimate $\hat{\boldsymbol{h}}$ is available for $\boldsymbol{p}_{\mathrm{MF}}$ (cf. Sec. III).

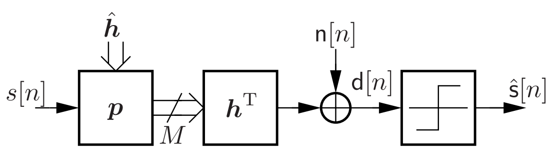

Fig. 1. Transmit matched filter for data channel.

B. Beamforming

If the channel $\boldsymbol{h}$ is considered a random vector $\mathbf{h} \sim \mathcal{N}_{\mathrm{c}}(\mathbf{0}, \boldsymbol{R}_{\mathbf{h}})$ with eigenvalue decomposition (EVD) of its covariance matrix $\boldsymbol{R}_{\mathbf{h}} = \mathrm{E}[\mathbf{h}\mathbf{h}^{\mathrm{H}}] = \boldsymbol{U} \boldsymbol{\Lambda} \boldsymbol{U}^{\mathrm{H}}$ and a diagonal $\boldsymbol{\Lambda}$ with eigenvalues $\lambda_m \geq \lambda_{m+1}$ $m \in \{1, \ldots, M\}$, the filter $\boldsymbol{p}$ can be optimized based on the average (long-term) channel properties described by $\boldsymbol{R}_{\mathbf{h}}$. The optimum beamformer *maximizes the SNR* for fixed available power P_{T} [4]:

$$\boldsymbol{p}_{\mathrm{B}} = \arg\max_{\boldsymbol{p}} \mathrm{E}[|\mathbf{h}^{\mathrm{T}} \boldsymbol{p}|^2] = \arg\max_{\boldsymbol{p}} \boldsymbol{p}^{\mathrm{T}} \boldsymbol{R}_{\mathbf{h}} \boldsymbol{p}^* \quad \text{s.t.: } \mathrm{E}[\|\boldsymbol{p}\,\mathsf{s}[n]\|_2^2] = P_{\mathrm{T}}. \tag{4}$$

It is given by the eigenvector $\boldsymbol{u}_1 = \boldsymbol{U} \boldsymbol{e}_1$ of $\boldsymbol{R}_{\mathbf{h}}$ corresponding to the largest eigenvalue λ_1

$$\boldsymbol{p}_{\mathrm{B}} = \sqrt{P_{\mathrm{T}}/P_{\mathrm{s}}}\; \boldsymbol{u}_1^*. \tag{5}$$

Compared to the TxMF with full channel state information the receiver is more complex for this concept [5]. Here, it simply estimates the resulting channel coefficient c to correct the channel phase

$$\mathsf{d}[n] = \hat{c}^* \mathsf{x}[n], \;\; c = \boldsymbol{u}_1^{\mathrm{H}} \boldsymbol{h}. \tag{6}$$

We assume perfect knowledge of $\boldsymbol{R}_{\mathbf{h}}$, as the spatial channel properties change slowly enough compared to $\boldsymbol{h}$, which allows for its accurate estimation [15], [16]. The average SNR at the receiver for perfect CSI is $\overline{\mathrm{SNR}} = \lambda_1 P_{\mathrm{T}}/\sigma_{\mathrm{n}}^2$.

C. Alamouti Space-Time Block Code

For $M = 2$ the Alamouti STBC *maximizes diversity without rate loss* and without requiring any CSI for flat channels. The receive signal $\mathbf{x}[k]^{\mathrm{T}} = [\mathsf{x}[k], \mathsf{x}[k+1]]$ is

$$\mathbf{x}[k]^{\mathrm{T}} = \boldsymbol{h}^{\mathrm{T}} \boldsymbol{S}[k] + \mathbf{n}[k]^{\mathrm{T}}, \;\; k = 2n, \tag{7}$$

where the symbols $s[n]$ are arranged in blocks of two symbols with block index k:

$$\boldsymbol{S}[k] = \begin{bmatrix} s[k] & s[k+1] \\ -s[k+1]^* & s[k]^* \end{bmatrix}. \tag{8}$$

To satisfy the transmit power constraint we have $P_{\mathrm{s}} = P_{\mathrm{T}}/2$. The receiver decodes the STBC based on estimates of the channel $\boldsymbol{h} = [h_1, h_2]^{\mathrm{T}}$

$$\begin{bmatrix} \mathsf{d}[k] \\ \mathsf{d}[k+1] \end{bmatrix} = \begin{bmatrix} \hat{h}_1^* & \hat{h}_2 \\ \hat{h}_2^* & -\hat{h}_1 \end{bmatrix} \begin{bmatrix} \mathsf{x}[k] \\ \mathsf{x}[k+1]^* \end{bmatrix} = \\ = \|\boldsymbol{h}\|_2^2 \begin{bmatrix} \mathsf{s}[k] \\ \mathsf{s}[k+1] \end{bmatrix} + \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} \mathsf{n}[k] \\ \mathsf{n}[k+1]^* \end{bmatrix}, \tag{9}$$

where perfect channel estimates are assumed for the second equation. The channel is assumed constant over at least one block. The SNR at the decision device for perfect CSI for one channel realization is $\mathrm{SNR} = \|\boldsymbol{h}\|_2^2 P_{\mathrm{T}}/(2\sigma_{\mathrm{n}}^2)$ and on average for many channels $\overline{\mathrm{SNR}} = \mathrm{trace}(\boldsymbol{R}_{\mathbf{h}}) P_{\mathrm{T}}/(2\sigma_{\mathrm{n}}^2)$.

III. Channel Estimation and Prediction

In TDD links we have the possibility to estimate the channel $\boldsymbol{h}$ from the uplink and use it to design $\boldsymbol{p}$ for the next downlink slot due to the reciprocity of the channel. Thus, the channel estimate will be outdated by at least one slot depending on the *slot structure*. For example, an asymmetric slot structure with period $P = 4$ is (downlink slot: '↓'; uplink slot: '↑')

↓	↓	↓	↑	↓	↓	↓	↑	↓	↓	↓	↑	↓	↓	↓

For $P = 2$ the assignment between up- and downlink transmission time is symmetric.

For slot ℓ the N_p *uplink pilot symbols* $s_\mathrm{p}[n]$ are received with M antennas as

$$\mathsf{x}'_{\mathrm{p},\ell}[n] = \boldsymbol{h}[\ell] s_\mathrm{p}[n] + \mathsf{n}'_{\mathrm{p},\ell}[n] \in \mathbb{C}^M, n \in \{1,\ldots,N_\mathrm{p}\}, \quad (10)$$

which can be written more concisely as (Figure 2)

$$\mathsf{x}_\mathrm{p}[\ell] = \boldsymbol{S}_\mathrm{p} \boldsymbol{h}[\ell] + \mathsf{n}_\mathrm{p}[\ell] \in \mathbb{C}^{MN_\mathrm{p}} \quad (11)$$

with $\mathsf{x}_\mathrm{p}[\ell] = [\mathsf{x}'_{\mathrm{p},\ell}[1]^\mathrm{T},\ldots,\mathsf{x}'_{\mathrm{p},\ell}[N_\mathrm{p}]^\mathrm{T}]^\mathrm{T}$. They are arranged in $\boldsymbol{S}_\mathrm{p} = \boldsymbol{s}_\mathrm{p} \otimes \boldsymbol{I}_M$ with $\boldsymbol{s}_\mathrm{p} = [s_\mathrm{p}[1],\ldots,s_\mathrm{p}[N_\mathrm{p}]]^\mathrm{T}$ and power $P_\mathrm{p} = P_\mathrm{T}$. For simplicity—without loss of generality—we assume $\boldsymbol{R}_{\mathsf{n}_\mathrm{p}} = \sigma_\mathrm{n}^2 \boldsymbol{I}_{MN_\mathrm{p}}$. The channel $\boldsymbol{h}[\ell]$ is assumed constant over one slot and correlated over time (slots).

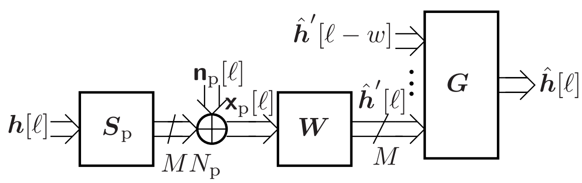

Fig. 2. Uplink channel estimation and linear prediction.

The *downlink pilot signal* for the STBC is (11) with $\mathsf{x}_\mathrm{p}[\ell] \in \mathbb{C}^{N_\mathrm{p}}$ and $2N_\mathrm{p}$ pilot symbols $\boldsymbol{S}_\mathrm{p} \in \mathbb{C}^{N_\mathrm{p}\times 2}$ chosen as $\boldsymbol{S}_\mathrm{p}^\mathrm{H}\boldsymbol{S}_\mathrm{p} = \boldsymbol{I}_2 N_\mathrm{p} P_\mathrm{p}$. The power in the pilot sequence is $P_\mathrm{p} = P_\mathrm{T}/2$. The slot index is ℓ.

In the case of *beamforming* we exchange $\boldsymbol{S}_\mathrm{p}$ by $\boldsymbol{s}_\mathrm{p}\boldsymbol{p}^\mathrm{T}$ in (11) with $P_\mathrm{p} = P_\mathrm{T}$ to obtain $\hat{c}[\ell]$.

In all cases the channel $\boldsymbol{h}[\ell]$ in slot ℓ can be estimated with a linear estimator $\boldsymbol{W}$ as

$$\hat{\mathsf{h}}'[\ell] = \boldsymbol{W}\mathsf{x}_\mathrm{p}[\ell] = \boldsymbol{W}\boldsymbol{S}_\mathrm{p}\boldsymbol{h}[\ell] + \boldsymbol{W}\mathsf{n}_\mathrm{p}[\ell], \quad (12)$$

where the matrix $\boldsymbol{B} = \boldsymbol{W}\boldsymbol{S}_\mathrm{p}$ determines the bias. For white noise the (unbiased) *ML channel estimator* is given as

$$\boldsymbol{W}_\mathrm{ML} = \frac{1}{N_\mathrm{p}P_\mathrm{p}} \boldsymbol{S}_\mathrm{p}^\mathrm{H} \quad (13)$$

under the assumptions from above and $\boldsymbol{S}_\mathrm{p}^\mathrm{H}\boldsymbol{S}_\mathrm{p} = \boldsymbol{I}_M N_\mathrm{p} P_\mathrm{p}$, i.e. $\boldsymbol{B} = \boldsymbol{I}_M$. The analysis below is valid for all linear estimators $\boldsymbol{W}$ (e.g. Wiener filter), where $\boldsymbol{B}$ has to be adjusted accordingly.

With $\boldsymbol{G} = \boldsymbol{g}^\mathrm{H} \otimes \boldsymbol{I}_M$ we describe which channel estimate $\hat{\mathsf{h}}'[\ell-\mu]$ ($\mu > 0$ modeles a delay or out-dating of the channel estimate) or which combination (linear prediction) of previous estimates is available in slot ℓ, i.e.

$$\hat{\mathsf{h}}[\ell] = \boldsymbol{G}(\boldsymbol{I}_{w+1} \otimes \boldsymbol{B})\boldsymbol{h}_\mathrm{t}[\ell] + \boldsymbol{\varepsilon}[\ell] \quad (14)$$

with $\boldsymbol{h}_\mathrm{t}[\ell] = [\boldsymbol{h}[\ell]^\mathrm{T},\ldots,\boldsymbol{h}[\ell-w]^\mathrm{T}]^\mathrm{T} \in \mathbb{C}^{M(w+1)}$. The noise $\boldsymbol{\varepsilon}[\ell] \sim \mathcal{N}_\mathrm{c}(\boldsymbol{0}, \boldsymbol{R}_{\boldsymbol{\varepsilon}})$ in the estimate is

$$\boldsymbol{\varepsilon}[\ell] = \boldsymbol{G}(\boldsymbol{I}_{w+1} \otimes \boldsymbol{W})\mathsf{n}_{\mathrm{p,t}}[\ell] \quad (15)$$

$$\text{with } \boldsymbol{R}_{\boldsymbol{\varepsilon}} = \sigma_\mathrm{n}^2 \boldsymbol{G}(\boldsymbol{I}_{w+1} \otimes \boldsymbol{W}\boldsymbol{W}^\mathrm{H})\boldsymbol{G}^\mathrm{H} = \boldsymbol{I}_M \|\boldsymbol{g}\|_2^2/\gamma_\mathrm{p} \quad (16)$$

$$\begin{aligned}\boldsymbol{R}_{\hat{\mathsf{h}}} &= \boldsymbol{G}(\boldsymbol{I}_{w+1} \otimes \boldsymbol{B})\boldsymbol{R}_{\mathsf{h}_\mathrm{t}}(\boldsymbol{I}_{w+1} \otimes \boldsymbol{B}^\mathrm{H})\boldsymbol{G}^\mathrm{H} + \boldsymbol{R}_{\boldsymbol{\varepsilon}} \\ &= \boldsymbol{g}^\mathrm{H}\boldsymbol{R}_\mathrm{a}\boldsymbol{g}\,\boldsymbol{B}\boldsymbol{R}_{\mathsf{h}}\boldsymbol{B}^\mathrm{H} + \boldsymbol{R}_{\boldsymbol{\varepsilon}} \quad (17)\end{aligned}$$

assuming $\boldsymbol{R}_{\mathsf{h}_\mathrm{t}} = \boldsymbol{R}_\mathrm{a} \otimes \boldsymbol{R}_{\mathsf{h}}$, i.e., same autocorrelation properties for all channel coefficients (e.g. a small array). The normalized autocorrelation properties of the channel coefficients $\mathsf{h}_m[\ell]$ are given as $r[\mu] = \mathrm{E}[\mathsf{h}_m[\ell]\mathsf{h}_m[\ell-\mu]^*]/\mathrm{E}[|\mathsf{h}_m|^2]$ in $\boldsymbol{r} = [r[0],\ldots,r[w]]^\mathrm{T}$ and $\boldsymbol{R}_\mathrm{a}$, which is Hermitian and Toeplitz with first row equal to $\boldsymbol{r}^\mathrm{T}$. The ratio of pilot symbol power and noise variance at the receiver is $\gamma_\mathrm{p} = N_\mathrm{p}P_\mathrm{p}/\sigma_\mathrm{n}^2$.

For example, if the estimate of the μ-th previous (uplink) slot is used for the TxMF we set $\boldsymbol{g} = \boldsymbol{e}_{\mu+1}$ and if $\boldsymbol{g} = \boldsymbol{e}_1$ the estimate from the current slot is available as for Alamouti STBC and beamforming.

Particularly for the TxMF *linear prediction* of the channel improves performance considerably [7], where $\boldsymbol{G} = \boldsymbol{g}^\mathrm{H} \otimes \boldsymbol{I}_M$ predicts the channel in the temporal domain only. The LMMSE predictor

$$\boldsymbol{g} = \left(\boldsymbol{T}\left[\boldsymbol{R}_\mathrm{a} + \boldsymbol{I}_{w+1}/\left(\gamma_\mathrm{p}\mathrm{E}[|\mathsf{h}_m|^2]\right)\right]\boldsymbol{T}\right)^\dagger \boldsymbol{T}\boldsymbol{r} \quad (18)$$

for $\boldsymbol{B} = \boldsymbol{I}_M$ is the solution of the Wiener-Hopf equation, where $\boldsymbol{T}$ is the diagonal (selection) matrix with ones at positions, for which a channel estimate is available, and zeros elsewhere [7]. The pseudo inverse is denoted by '†'.

IV. Bit Error Probability Analysis

A. Transmit Matched Filter

1) Equivalence of TxMF and RxMF: Previous results [14] for the BEP of the receive MF (RxMF) can be used for analysis of the TxMF, as both are equivalent in performance for spatially white noise, flat channels, and PSK modulation. For the TxMF we can write $\mathsf{x}[n]$ with (3) and (1) equivalently as

$$\mathsf{d}_\mathrm{eq}[n] = \mathsf{x}_\mathrm{eq}[n] = \sqrt{P_\mathrm{T}/P_\mathrm{s}}\,\hat{\boldsymbol{h}}^\mathrm{H}\boldsymbol{h}s[n] + \|\hat{\boldsymbol{h}}\|_2 \mathsf{n}[n]. \quad (19)$$

Comparing it with the receive MF for M antennas

$$\mathsf{d}[n] = \hat{\boldsymbol{h}}^\mathrm{H}\mathsf{x}[n] = \hat{\boldsymbol{h}}^\mathrm{H}\boldsymbol{h}s[n] + \hat{\boldsymbol{h}}^\mathrm{H}\mathsf{n}[n] \in \mathbb{C}^M \quad (20)$$

we observe that the noise $\mathsf{n}_\mathrm{eq}[n] = \|\hat{\boldsymbol{h}}\|_2\mathsf{n}[n]$ and $\hat{\boldsymbol{h}}^\mathrm{H}\mathsf{n}[n]$ have the same distribution conditioned on $\hat{\boldsymbol{h}}$ if $\boldsymbol{R}_{\mathsf{n}} = \sigma_\mathrm{n}^2\boldsymbol{I}_M$, i.e., $\mathrm{E}[|\mathsf{n}_\mathrm{eq}[n]|^2] = \mathrm{E}[|\hat{\boldsymbol{h}}^\mathrm{H}\mathsf{n}[n]|^2] = \|\hat{\boldsymbol{h}}\|_2^2\sigma_\mathrm{n}^2$ and $\mathrm{E}[\mathsf{n}_\mathrm{eq}[n]] = \mathrm{E}[\hat{\boldsymbol{h}}^\mathrm{H}\mathsf{n}[n]] = 0$. The BEP for TxMF can be calculated via the RxMF if the same $\hat{\boldsymbol{h}}$ is used, as they are equivalent in performance for each channel realization and, thus, also have the same performance on average.

2) Derivation of Average BEP: The real part of the decision variable $\mathsf{d}[n]$ for the RxMF can be written as a quadratic form

$$\begin{aligned}\mathsf{d}_\mathrm{R}[n] &= \mathrm{Re}\,(\mathsf{d}[n]) = \mathrm{Re}\left(\hat{\mathsf{h}}^\mathrm{H}\boldsymbol{U}\boldsymbol{U}^\mathrm{H}\mathsf{x}[n]\right) \\ &= 1/2\left(\hat{\mathsf{h}}_\mathrm{d}^\mathrm{H}\mathsf{y}[n] + \mathsf{y}[n]^\mathrm{H}\hat{\mathsf{h}}_\mathrm{d}\right) = 1/2\,\mathsf{v}^\mathrm{H}\boldsymbol{C}\mathsf{v} \quad (21)\end{aligned}$$

in $\mathsf{v} = [\hat{\mathsf{h}}_{\mathrm{d},1}, \mathsf{y}_1[n],\ldots,\hat{\mathsf{h}}_{\mathrm{d},M},\mathsf{y}_M[n]]^\mathrm{T}$ and $\boldsymbol{C} = \boldsymbol{I}_M \otimes [\boldsymbol{e}_2,\boldsymbol{e}_1]$. To obtain explicit results later, we introduced $\boldsymbol{U}\boldsymbol{U}^\mathrm{H} = \boldsymbol{I}_M$ to decorrelate the channel coefficients in (21), i.e., the new equivalent channel is $\mathsf{h}_\mathrm{d} = \boldsymbol{U}_\mathrm{d}^\mathrm{H}\mathsf{h}$ with diagonal covariance matrix $\boldsymbol{R}_{\mathsf{h}_\mathrm{d}} = \boldsymbol{\Lambda}$ [14]. The transformed signal is

$$\mathsf{y}[n] = \boldsymbol{U}^\mathrm{H}\mathsf{x}[n] = \mathsf{h}_\mathrm{d}s[n] + \mathsf{n}_\mathrm{d}[n] \quad (22)$$

with $\mathbf{n}_{\rm d}[n] = \boldsymbol{U}_{\rm d}^{\rm H}\mathbf{n}[n]$ and $\boldsymbol{R}_{\mathbf{n}} = \boldsymbol{R}_{\mathbf{n}_{\rm d}} = \boldsymbol{R}_{\mathbf{n}_{\rm p}} = \sigma_{\rm n}^2\boldsymbol{I}_M$.

As the coefficients in $\boldsymbol{h}_{\rm d}$ are uncorrelated, i.e., one element does not contain information about other coefficients, temporal prediction as introduced above with $\boldsymbol{G} = \boldsymbol{g}^{\rm H}\otimes\boldsymbol{I}_M$ is sufficient and $\boldsymbol{B} = \operatorname{diag}([b_1,\ldots,b_M])$ is diagonal in general. The covariance matrix $\boldsymbol{R}_{\mathbf{v}} = \operatorname{diag}(\boldsymbol{R}_{\mathbf{v}_1},\ldots,\boldsymbol{R}_{\mathbf{v}_M})$ of $\mathbf{v}$ is block diagonal with

$$\boldsymbol{R}_{\mathbf{v}_m} = \begin{bmatrix} \mathrm{E}[|\hat{\mathsf{h}}_{\mathrm{d},m}[\ell]|^2] & \mathrm{E}[\hat{\mathsf{h}}_{\mathrm{d},m}[\ell]\mathsf{y}_m[n]^*] \\ \mathrm{E}[\hat{\mathsf{h}}_{\mathrm{d},m}[\ell]\mathsf{y}_m[n]^*]^* & \mathrm{E}[|\mathsf{y}_m[n]|^2] \end{bmatrix} \quad (23)$$
$$= \begin{bmatrix} b_m^2\lambda_m\boldsymbol{g}^{\rm H}\boldsymbol{R}_{\rm a}\boldsymbol{g} + \frac{1}{\gamma_{\rm p}}b_m^2\|\boldsymbol{g}\|_2^2 & b_m\lambda_m\boldsymbol{g}^{\rm H}\boldsymbol{r}s[n] \\ b_m\lambda_m\boldsymbol{g}^{\rm H}\boldsymbol{r}s[n] & \lambda_m P_{\rm s} + \sigma_{\rm n}^2 \end{bmatrix},$$

where r_{vij} denotes the ij-th element of $\boldsymbol{R}_{\mathbf{v}_m}$.

Turin [13] showed that the characteristic function of $\mathsf{d}_{\rm R}[n]$ (21), a quadratic form in zero mean Gaussian random variables, can be written in terms of the eigenvalues ξ_m^+ and ξ_i^- of $1/2\,\boldsymbol{R}_{\mathbf{v}_m}[\boldsymbol{e}_2,\boldsymbol{e}_1]$

$$\psi(\omega) = \mathrm{E}[\mathrm{e}^{\mathrm{j}\omega\mathsf{d}_{\rm R}}] = \prod_{m=1}^{M}\frac{1}{(1-\mathrm{j}\omega\xi_m^+)(1-\mathrm{j}\omega\xi_m^-)}. \quad (24)$$

We introduce the parameters $\alpha_1 = \boldsymbol{g}^{\rm H}\boldsymbol{r}/\|\boldsymbol{g}\|_2$ and $\alpha_2 = \boldsymbol{g}^{\rm H}\boldsymbol{R}_{\rm a}\boldsymbol{g}/\|\boldsymbol{g}\|_2^2$ describing the impact of delayed channel estimates and prediction on the BEP.

The probability density function $\mathrm{p}(d_{\rm R}|s = +\sqrt{P_{\rm s}})$ of the decision variable $\mathsf{d}_{\rm R}[n]$ conditioned on a transmitted data symbol $s[n] = +\sqrt{P_{\rm s}}$ is given by the inverse Fourier transform of the characteristic function. After a partial fraction expansion of the characteristic function (24) and for distinct eigenvalues $\xi_m^{\pm}$ [14] it is

$$\mathrm{p}(d_{\rm R}|s = +\sqrt{P_{\rm s}}) = \sum_{m=1}^{M}\frac{A_m}{\xi_m^+}\mathrm{e}^{-d_{\rm R}/\xi_m^+}\,\Theta(d_{\rm R}) + \frac{B_m}{-\xi_m^-}\mathrm{e}^{-d_{\rm R}/\xi_m^-}\,\Theta(-d_{\rm R}) \quad (25)$$

$$A_m = \prod_{\substack{i=1\\ i\neq m}}^{M}\left(1-\frac{\xi_i^+}{\xi_m^+}\right)^{-1}\prod_{i=1}^{M}\left(1-\frac{\xi_i^-}{\xi_m^+}\right)^{-1}$$
$$B_m = \prod_{i=1}^{M}\left(1-\frac{\xi_i^+}{\xi_m^-}\right)^{-1}\prod_{\substack{i=1\\ i\neq m}}^{M}\left(1-\frac{\xi_i^-}{\xi_m^-}\right)^{-1} \quad (26)$$

with $\Theta(d) = 1$, if $d > 0$, and $\Theta(d) = 0$, elsewhere. The eigenvalues are distinct if λ_m are distinct (the case $\lambda_M = 0$ can be handled reducing M by one), b_m and $\boldsymbol{g}^{\rm T}\boldsymbol{r}$ nonzero, and $\gamma_{\rm d} \neq \left([\lambda_m + 1/(\alpha_2\gamma_{\rm p})]\alpha_2/\alpha_1^2 - \lambda_m\right)^{-1}$. The probability of a bit error $\mathrm{P_b}$ for equally probable transmitted symbols and a maximum a posteriori decision device is (compare e.g. [17])

$$\mathrm{P_b} = \int_{-\infty}^{0}\mathrm{p}(\delta|s[n] = +\sqrt{P_{\rm s}})\,\mathrm{d}\delta = \sum_{m=1}^{M}B_m$$
$$= \sum_{m=1}^{M}\left(\prod_{i=1}^{M}1-\frac{\xi_i^+}{\xi_m^-}\prod_{\substack{i=1\\ i\neq m}}^{M}1-\frac{\xi_i^-}{\xi_m^-}\right)^{-1}, \quad (27)$$

which depends only on the ratio of eigenvalues, e.g.,

$$\frac{\xi_m^+}{\xi_i^-} = \frac{b_m\lambda_m}{b_i\lambda_i}\frac{1+\sqrt{\left(1+\frac{1}{\gamma_{\rm d}\lambda_m}\right)\left(1+\frac{1}{\gamma_{\rm p}\lambda_m\alpha_2}\right)\frac{\alpha_2}{\alpha_1^2}}}{1-\sqrt{\left(1+\frac{1}{\gamma_{\rm d}\lambda_i}\right)\left(1+\frac{1}{\gamma_{\rm p}\lambda_i\alpha_2}\right)\frac{\alpha_2}{\alpha_1^2}}}. \quad (28)$$

For equal eigenvalues ξ_m^+ (ξ_m^-), i.e. $\boldsymbol{U} = \boldsymbol{I}_M$, $\lambda = \lambda_m$, $b = b_m\ \forall m$, the BEP is [18]

$$\mathrm{P_b} = \frac{1}{2}\left[1-\rho\sum_{m=0}^{M-1}\binom{2m}{m}\left(\frac{1-\rho^2}{4}\right)^m\right] \quad (29)$$

$$\rho = \frac{\mathrm{E}[\mathsf{x}_m[n]\hat{\mathsf{h}}_m]}{\sqrt{\mathrm{E}[|\mathsf{x}_m[n]|^2]\mathrm{E}[|\hat{\mathsf{h}}_m|^2]}}$$
$$= \left(\left(1+\frac{1}{\gamma_{\rm d}\lambda}\right)\left(1+\frac{1}{\gamma_{\rm p}\lambda b^2\alpha_2}\right)\frac{\alpha_2}{\alpha_1^2}\right)^{-1/2} \quad (30)$$

for real valued $\boldsymbol{g}^{\rm H}\boldsymbol{r}$ as above.

B. Beamforming

The BEP for beamforming (4) assuming estimates $\hat{c}[\ell]$ as discussed in Section III and perfect knowledge of $\boldsymbol{R}_{\mathbf{h}}$, is a special case of (27) for $M = 1$.

C. Alamouti STBC

From (9) the real part of the decision signal for symbol $s[k]$ is the quadratic form

$$\mathsf{d}_{\rm R}[k] = \mathrm{Re}\left(\hat{\mathsf{h}}_1^*\mathsf{x}[k] + \hat{\mathsf{h}}_2\mathsf{x}^*[k+1]\right)$$
$$= \mathrm{Re}\Big((\hat{\mathsf{h}}_1^*\mathsf{h}_1 + \hat{\mathsf{h}}_2^*\mathsf{h}_2)s[k] + (\hat{\mathsf{h}}_1^*\mathsf{h}_2 - \hat{\mathsf{h}}_2\mathsf{h}_1^*)s[k+1]$$
$$+\hat{\mathsf{h}}_1^*\mathsf{n}[k] + \hat{\mathsf{h}}_2\mathsf{n}[k+1]\Big) = 1/2\ \mathbf{w}^{\rm H}\boldsymbol{A}\mathbf{w}. \quad (31)$$

For imperfect estimates $\hat{\boldsymbol{h}}$ we observe two additional sources of BEP degradation (besides additive noise): phase error in the resulting factor in front of $s[k]$ and intersymbol interference from $s[k+1]$.

The BEP analysis is similar to our procedure above and presented as a special case in [10]. The characteristic function $\psi(\omega)$ of (31) in $\mathbf{w} = [\hat{\mathsf{h}}_1,\hat{\mathsf{h}}_2,\mathsf{x}[k],\mathsf{x}[k+1]]^{\rm T}$ and with $\boldsymbol{A} = [\boldsymbol{e}_2,\boldsymbol{e}_1]\otimes\boldsymbol{I}_2$ is given in terms of the eigenvalues ξ_m of $1/2\,\boldsymbol{R}_{\mathbf{w}}\boldsymbol{A}$ (24). In contrast to above there is no equivalence transform to achieve a block diagonal structure of $\boldsymbol{R}_{\mathbf{w}}$. Thus, the 4 eigenvalues have to be computed numerically from

$$\boldsymbol{R}_{\mathbf{w}} = \begin{bmatrix}\boldsymbol{R}_{\hat{\mathbf{h}}} & \boldsymbol{R}_{\hat{\mathbf{h}}\mathbf{x}} \\ \boldsymbol{R}_{\hat{\mathbf{h}}\mathbf{x}}^{\rm H} & \boldsymbol{R}_{\mathbf{x}}\end{bmatrix} \in \mathbb{C}^{4\times 4} \quad (32)$$

with $\boldsymbol{R}_{\mathbf{x}} = \boldsymbol{S}[k]^{\rm T}\boldsymbol{R}_{\mathbf{h}}\boldsymbol{S}[k]^* + \sigma_{\rm n}^2\boldsymbol{I}_2$, $\boldsymbol{R}_{\hat{\mathbf{h}}\mathbf{x}} = \mathrm{E}[\hat{\mathbf{h}}[\ell]\mathbf{x}[k]^{\rm H}] = \boldsymbol{g}^{\rm H}\boldsymbol{r}\boldsymbol{B}\boldsymbol{R}_{\mathbf{h}}\boldsymbol{S}[k]^*$, and Eqn. (17). With 4 distinct eigenvalues ξ_m the result is similar to (27)

$$\mathrm{P_b} = \sum_{\substack{m=1\\ \xi_m<0}}^{4}\prod_{\substack{i=1\\ i\neq m}}^{4}\left(1-\frac{\xi_i}{\xi_m}\right)^{-1} \quad (33)$$

Transmit Processing	Antenna/SNR gain	Diversity gain
Tx Matched Filter	$2\,(=M)$	$2\,(=M)$
Beamforming	$M\lambda_1/\mathrm{trace}(\boldsymbol{R}_{\mathbf{h}})$	1
Alamouti STBC	1	2

TABLE I

ANTENNA GAIN AND DIVERSITY GAIN OF TRANSMIT PROCESSING SCHEMES FOR $M=2$ ANTENNAS COMPARED TO A SINGLE ANTENNA.

Channel scenario:	λ_1	λ_2	standard deviation
correlated	0.9998	0.0002	$1°$
semi-correlated	0.89	0.11	$60°$
uncorrelated	0.5	0.5	—

TABLE II

EIGENVALUES OF $\boldsymbol{R}_{\mathbf{h}}$ FOR EVALUATED SCENARIOS AND CORRESPONDING STANDARD DEVIATION (ANGLE SPREAD) OF THE ZERO MEAN (BROADSIDE) LAPLACE DISTRIBUTION OF THE ANGLE OF DEPARTURE.

with coefficients of the partial fraction expansion as in (26). For multiple equal eigenvalues a partial fraction expansion of the characteristic function of $\mathrm{d}_{\mathrm{R}}[n]$ (24) for multiple poles is needed, which is tedious but straight forward.

V. PERFORMANCE COMPARISON

A. Perfect Channel State Information

The gain in average SNR (antenna gain) at the receiver (Table I) results in a left-shift of the BEP (Figure 3) compared to the case of 1 antenna at the transmitter. For beamforming it depends on the channel correlations. Thus, we consider 3 scenarios as given in Table II. The diversity gain, i.e., the asymptotic slope of the BEP w.r.t. γ_{d} [17], is independent of channel correlations. But the SNR for which this slope is achieved is shifted to higher SNR the more the channel is correlated (Figure 3): e.g. the break even point between Alamouti STBC and beamforming is at an SNR of $39.3\,\mathrm{dB}$ for the correlated scenario, whereas the BEP of Alamouti STBC is always lower for no correlations. Having complete CSI the TxMF achieves the maximum gains for all scenarios. These well known facts hold for perfect CSI only.

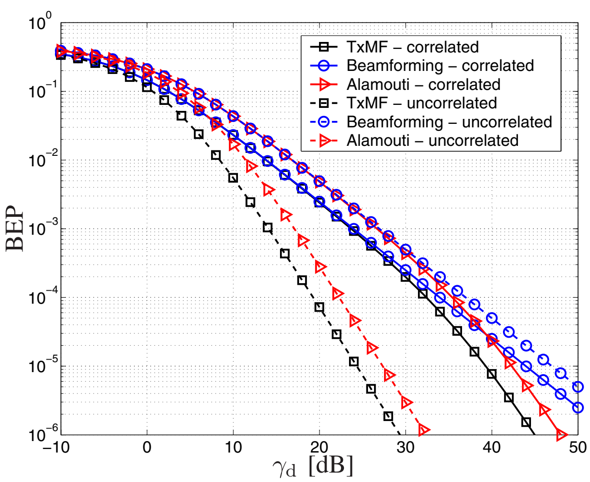

Fig. 3. BEP for perfect CSI and uncorrelated/correlated channels.

Channel scenario	correlated	semi-correlated	uncorrelated
Break even BEP	$4.77 \cdot 10^{-3}$	0.0374	0.3505
Break even γ_{d}	$40.42\,\mathrm{dB}$	$11.50\,\mathrm{dB}$	$-2.56\,\mathrm{dB}$

TABLE III

BREAK EVEN POINT BETWEEN BEAMFORMING AND ALAMOUTI STBC WITH ML CHANNEL ESTIMATION ($N_{\mathrm{p}}=20$) AND WITHOUT PREDICTION.

B. Imperfect Channel State Information

For the comparison equal power $P_{\mathrm{s}}{=}P_{\mathrm{p}}$ in pilot and data channel, $N_{\mathrm{p}}{=}20$ pilot symbols, and a temporal channel correlation function $r[\mu]$ from a Jakes power spectrum [19] with maximum Doppler frequency f_{d} are assumed.

The BEP degradation for beamforming ($\approx 0.2\,\mathrm{dB}$) and Alamouti STBC ($\approx 0.5\,\mathrm{dB}$) considering imperfect CSI is small compared to the TxMF, whose BEP saturates due to the delay between the previous uplink slot and the current downlink slot (Figure 4). For $P{=}4$ we compute the BEP for the worst-case slot, which experiences a delay of 3 slots ($\boldsymbol{g}=\boldsymbol{e}_3$). For a carrier frequency of $2\,\mathrm{GHz}$ and a slot period of $1/1500\,\mathrm{s}$ as in TDD UMTS [20] $f_{\mathrm{d}}=0.001$ corresponds to a velocity of $1\,\mathrm{km/h}$ and $f_{\mathrm{d}}=0.093$ to $75\,\mathrm{km/h}$. With LMMSE prediction (cf. Eqn. 18) the TxMF BEP for $f_{\mathrm{d}}=0.037$ ($30\,\mathrm{km/h}$) can be improved to be superior to Alamouti STBC for a wide SNR range.

For these system parameters Figure 5 shows the break even BEP for velocities between $1\,\mathrm{km/h}$ and $100\,\mathrm{km/h}$ ($f_{\mathrm{d}}=0.001$ to 0.124), i.e., for a desired BEP larger than the break even BEP the TxMF requires a lower γ_{d} for a given maximum Doppler frequency. For uncorrelated channels the region of superior operation (area above the graph) of the TxMF is larger than for correlations. An asymmetric downlink configuration ($P{=}4$) reduces this region considerably compared to $P{=}2$. With LMMSE prediction based on the channel estimates from the previous 10 uplink slots this region is extended significantly again. The break even point at 0.5 (e.g. the saturation level of $P{=}4$ for large f_{d}) occurs, when Alamouti STBC always outperforms the TxMF (no cross-over point at finite γ_{d} in dB).

For uncorrelated channels the SNR degradation can be computed explicitly from (30). Moreover, an optimization of the power distribution between pilot and data channel is possible, similarly to [21] for maximum ratio combining at the receiver. From (28) and (30) we note that an $\alpha_2 \geq 1$ describes a gain achieved from channel prediction, whereas α_2/α_1^2 is a measure of the degradation from the delay and the prediction error, as increasing this ratio has the same effect as decreasing γ_{p}. For a delay of the channel estimate by μ slots we have $\alpha_2=1$ and $\alpha_1=r[\mu]$: the smaller $r[\mu]$ the larger the degradation. For full correlation over time $r[\mu]=1$ there would be no degradation since $\alpha_1=1$.

The break even points between beamforming and Alamouti STBC in Table III state below which γ_{d} or equivalently above which BEP beamforming is superior to the STBC for different scenarios. As expected partial CSI (beamforming) should be exploited at the transmitter for more correlated channels.

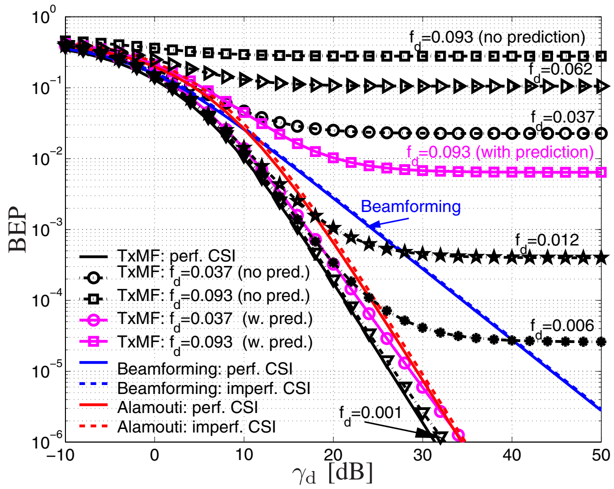

Fig. 4. BEP for imperfect CSI and semi-correlated channel without and with LMMSE prediction ($N_\mathrm{p} = 20$, $P = 4$; dashed-dotted line with markers: $\boldsymbol{g} = \boldsymbol{e}_3$; solid line with markers: LMMSE predictor with $w = 10P$).

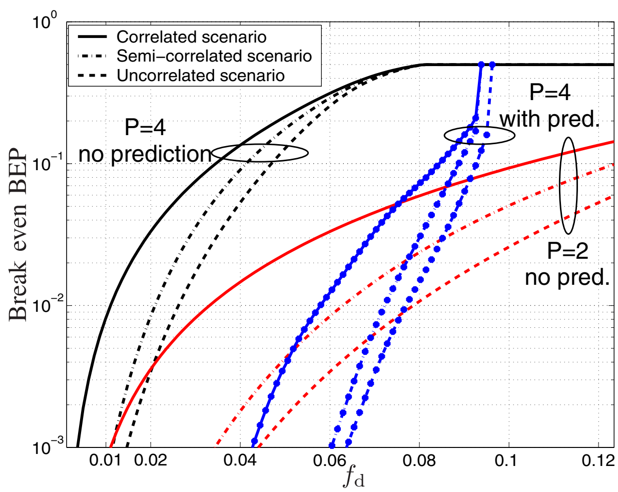

Fig. 5. Break even point of TxMF and Alamouti STBC (BEP vs. maximum normalized Doppler frequency f_d, $N_\mathrm{p} = 20$; with prediction: $w = 10P$; without predition: $\boldsymbol{g} = \boldsymbol{e}_3$).

VI. Conclusions

The quality of available channel state information plays a key role when deciding for a transmit processing concept. We compared three precoding schemes for flat fading channels: the transmit matched filter, beamforming, and Alamouti STBC. Their uncoded bit error probability is derived analytically for BPSK, correlated and uncorrelated Rayleigh fading, and general complex Gaussian distributed errors in channel state information.

The superior performance of the transmit matched filter due to its full channel state information in case of no channel estimation errors degrades when considering time-variant channels: its optimality region mainly depends on the slot configuration in the downlink, the speed of the mobile (maximum Doppler frequency), and is significantly improved by linear prediction. The results show that linear prediction is a key ingredient to make transmit processing based on complete CSI possible in time-variant channels.

The explicit BEP expressions allow further insights about the influence of temporal channel correlations, channel estimation, and linear prediction. For a switching between the concepts the knowledge about their break even point is necessary, which can be computed numerically based on our derivations of the BEP in the case of binary modulation. A derivation of the BEP for higher order modulation, correlated channels, and channel estimation errors would be important and remains a topic for future research.

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