Tutorial: MIMO and Transmit Processing

Part II: Performance Criteria, Optimization, and Overview of Current Research

Michael Joham

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Munich University of Technology
Institute for Circuit Theory and Signal Processing

Outline

1 Receive Processing

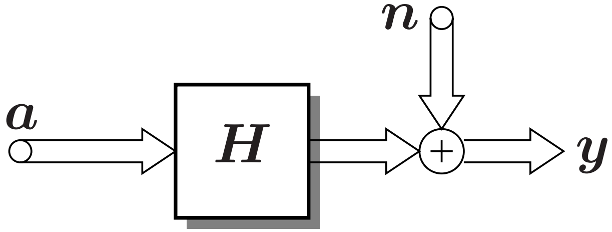
2 Transmit Processing

– Complete Channel State Information

– Partial Channel State Information

– No Channel State Information

- Maximum-Likelihood (ML) Detector
- Linear Receive Filters
- Decision Feedback Equalization (DFE, V-BLAST)
- Lattice-Reduction-Aided Detector



Principle: Maximization of the probability that an assumed symbol vector $\boldsymbol{a} \in \mathbb{A}^{N_\mathsf{T}}$ leads to the received signal $\boldsymbol{y} \in \mathbb{C}^{N_\mathsf{R}}$.

Optimization:

$$\tilde{\boldsymbol{a}}_{\mathsf{ML}} = \underset{\boldsymbol{a}\in\mathbb{A}^{N_\mathsf{T}}}{\operatorname{argmax}}\, \mathsf{p}_{\boldsymbol{y}|\boldsymbol{a}}(\boldsymbol{y}|\boldsymbol{a})$$

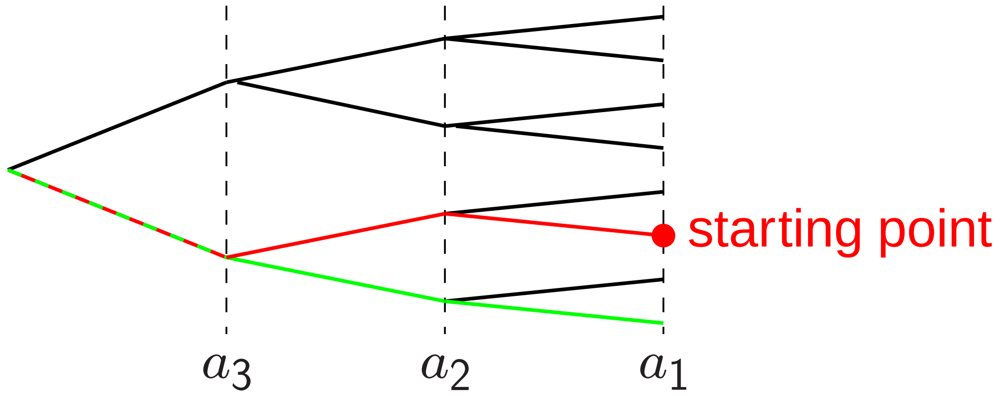
$$\tilde{\boldsymbol{a}}_{\mathsf{ML}} = \underset{\boldsymbol{a}\in\mathbb{A}^{N_\mathsf{T}}}{\operatorname{argmin}}\, (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{a})^{\mathsf{H}} \boldsymbol{R}_{\boldsymbol{n}}^{-1} (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{a}) \quad \text{for} \quad \boldsymbol{n} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \boldsymbol{R}_{\boldsymbol{n}})$$

Complexity: $\mathsf{O}\left(N_\mathsf{R} N_\mathsf{T} |\mathbb{A}|^{N_\mathsf{T}}\right)$

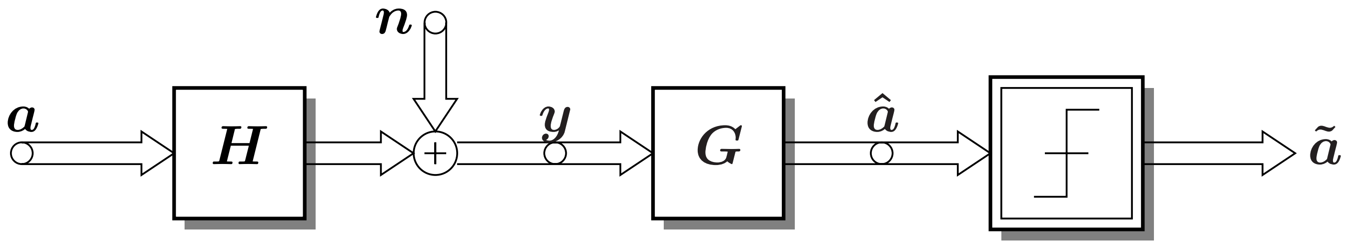
Principle: search in a tree for the point in a lattice nearest to the received signal $\boldsymbol{y}$

Starting Point: e. g. with DFE

Complexity: polynomial in N_T on average
not polynomial in the worst case



[Fincke et al. 1985], [Schnorr et al. 1994], [Viterbo et al. 1999], [Agrell et al. 2002], [Vikalo et al. 2002], . . .



Principle: split into linear estimator $\boldsymbol{G} \in \mathbb{C}^{N_\mathrm{T} \times N_\mathrm{R}}$ and symbol-by-symbol quantizer

Optimization:

$$\boldsymbol{G}_\mathrm{WF} = \underset{\boldsymbol{G}}{\operatorname{argmin}} \,\mathrm{E}\left[\|\boldsymbol{a} - \hat{\boldsymbol{a}}\|_2^2\right]$$

$$\tilde{a}_i = \underset{a_i \in \mathbb{A}}{\operatorname{argmin}} \,|a_i - \hat{a}_i|^2$$

Complexity:

filter computation: $\mathrm{O}\left(N_\mathrm{R}^3\right)$
filtering: $\mathrm{O}\left(N_\mathrm{R} N_\mathrm{T}\right)$
quantization: $\mathrm{O}\left(N_\mathrm{T} |\mathbb{A}|\right)$

[Lucky '65], [Shnidman '67], [Kaye et al. '70], [Lupas et al. '89], [Madhow et al. '94], [Klein et al. '96], . . .

- Minimization of **Mean Square Error (MSE)**:

  $\Rightarrow$ Wiener filter (WF), zero-forcing filter (ZF)

- Maximization of **Signal-to-Noise-Ratio (SNR)**:

  $\Rightarrow$ matched filter (MF), zero-forcing filter (ZF)

- Minimization of **Mean Output Energy (MOE)**:

  $\Rightarrow$ minimum variance distortionless response (MVDR)

- Maximization of **Signal-to-Interference-and-Noise-Ratio (SINR)**:

  $\Rightarrow$ eigenfilter

- Minimization of **Bit Error Probability** zero-forcing filter delivers sufficient statistic:

$$\hat{\boldsymbol{a}}_{\mathsf{ZF}} = \boldsymbol{G}_{\mathsf{ZF}}\boldsymbol{y} = \boldsymbol{a} + \left(\boldsymbol{H}^{\mathsf{H}}\boldsymbol{R}_{\boldsymbol{n}}^{-1}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^{\mathsf{H}}\boldsymbol{R}_{\boldsymbol{n}}^{-1}\boldsymbol{n}$$

ML criterion:

$$\tilde{\boldsymbol{a}}_{\mathsf{ML}} = \underset{\boldsymbol{a}\in\mathbb{A}}{\operatorname{argmin}} \left(\hat{\boldsymbol{a}}_{\mathsf{ZF}} - \boldsymbol{a}\right)^{\mathsf{H}} \left(\boldsymbol{H}^{\mathsf{H}}\boldsymbol{R}_{\boldsymbol{n}}^{-1}\boldsymbol{H}\right)\left(\hat{\boldsymbol{a}}_{\mathsf{ZF}} - \boldsymbol{a}\right)$$

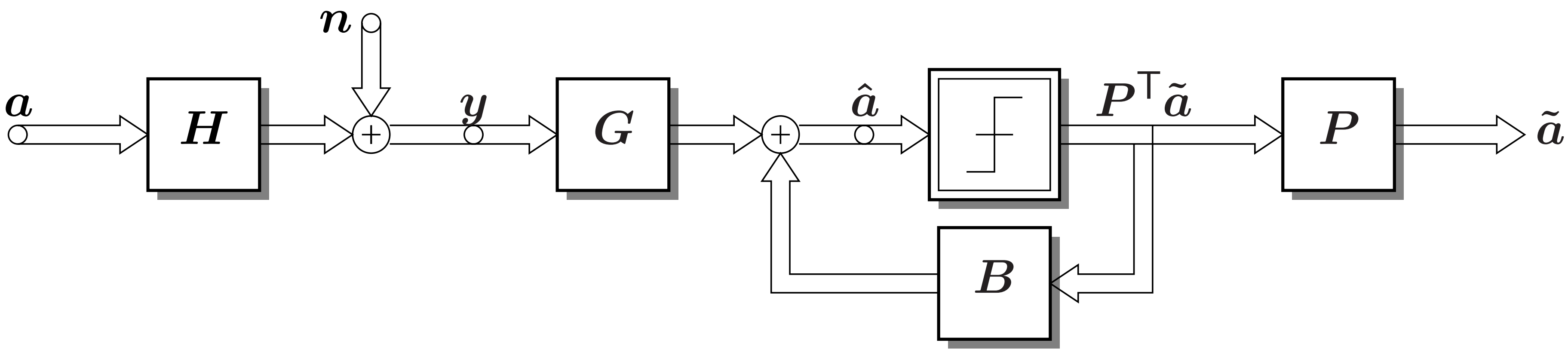
if $\boldsymbol{H}^{\mathsf{H}}\boldsymbol{R}_{\boldsymbol{n}}^{-1}\boldsymbol{H}$ diagonal:

$$\begin{aligned}\tilde{\boldsymbol{a}}_{\mathsf{ML}} &= \underset{\boldsymbol{a}\in\mathbb{A}}{\operatorname{argmin}} \|\hat{\boldsymbol{a}}_{\mathsf{ZF}} - \boldsymbol{a}\|_2^2 = \left[\underset{a_1\in\mathbb{A}}{\operatorname{argmin}} \left|a_1 - \hat{a}_{\mathsf{ZF},1}\right|^2, \ldots, \underset{a_{N_{\mathsf{T}}}\in\mathbb{A}}{\operatorname{argmin}} \left|a_{N_{\mathsf{T}}} - \hat{a}_{\mathsf{ZF},N_{\mathsf{T}}}\right|^2\right]^{\mathsf{T}} \\ &= \tilde{\boldsymbol{a}}_{\mathsf{ZF}}\end{aligned}$$

$\Rightarrow$ symbol-by-symbol quantization only optimal, if $\boldsymbol{H}^{\mathsf{H}}\boldsymbol{R}_{\boldsymbol{n}}^{-1}\boldsymbol{H}$ diagonal

special case: $\boldsymbol{R}_{\boldsymbol{n}} = \sigma_{\boldsymbol{n}}^2\boldsymbol{I}$

$\Rightarrow$ symbol-by-symbol quantization only optimal, if columns of $\boldsymbol{H}$ orthogonal



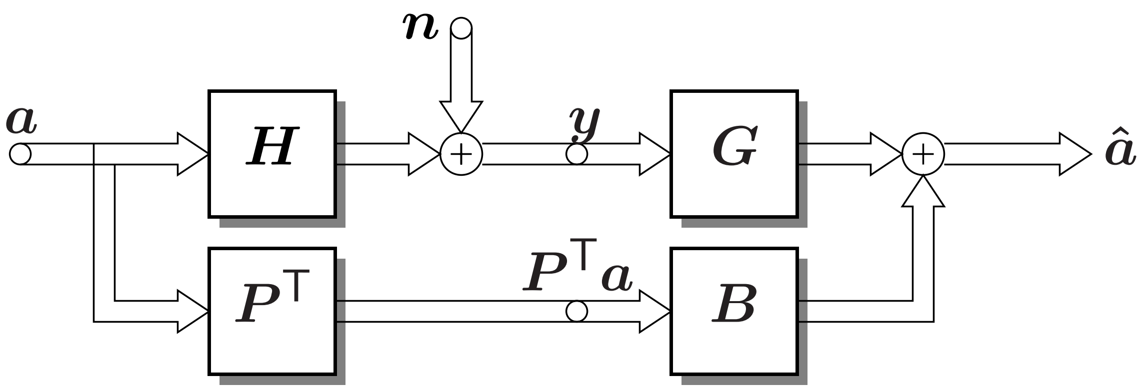
Principle: use already quantized symbols for interference reduction

Feedforward Filter: $\boldsymbol{G}$ suppresses the interference of symbols not already quantized

Feedback Filter: $\boldsymbol{B}$ for realizability: lower triangular and zero main diagonal

Ordering: permutation matrix $\boldsymbol{P} = \sum_{i=1}^{N_{\mathsf{T}}} \boldsymbol{e}_{b_i}\boldsymbol{e}_i^{\mathsf{T}}$ defines detection order

Assumption: symbol-by-symbol quantization delivers transmitted symbols



Criterion: minimization of mean square error

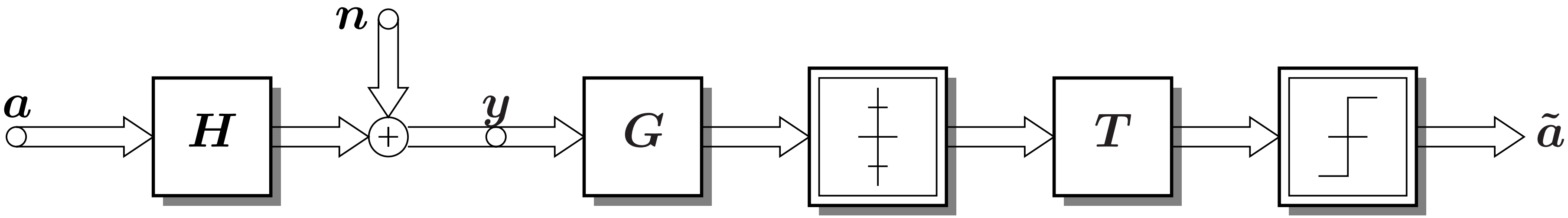
Optimierung: $\{\boldsymbol{G}_{\mathrm{WF}}, \boldsymbol{B}_{\mathrm{WF}}, \boldsymbol{P}_{\mathrm{WF}}\} = \underset{\{\boldsymbol{G},\boldsymbol{B},\boldsymbol{P}\}}{\operatorname{argmin}} \; \mathrm{E}\left[\left\|\boldsymbol{P}^{\mathsf{T}}\boldsymbol{a} - \hat{\boldsymbol{a}}\right\|_2^2\right]$

s. t.: $\boldsymbol{P}$: permutation matrix and
$\boldsymbol{B}$: lower triangular matrix with zero main diagonal

Ordering: with V-BLAST ordering algorithm

Complexity:
filter computation: $\mathrm{O}\left(N_{\mathrm{R}}^3\right)$
filtering, quantization: $\mathrm{O}\left(N_{\mathrm{R}} N_{\mathrm{T}}\right)$, $\mathrm{O}\left(N_{\mathrm{T}}|\mathbb{A}|\right)$

[Wolniansky et al. 1998], [Hassibi 2000], [Wübben et al. 2003], [Böhnke et al. 2003], [Kusume et al. 2004]



Principle: decomposition of channel matrix into one part with (nearly) orthogonal columns and another part with integer entries

quantization after equalization of part with orthogonal columns
$\Rightarrow$ close to optimal

Criterion: minimization of mean square error

Complexity:
filter computation: $\mathrm{O}\left(N_\mathrm{R}^4\right)$ or not polynomial in N_R
filtering: $\mathrm{O}\left(N_\mathrm{R} N_\mathrm{T}\right)$
quantization: $\mathrm{O}\left(N_\mathrm{T}|\mathbb{A}|\right)$

[Yao et al. 2002], [Windpassinger et al. 2003], [Wübben et al. 2004]

- Complete Channel State Information
  - realization of channel matrix completely known

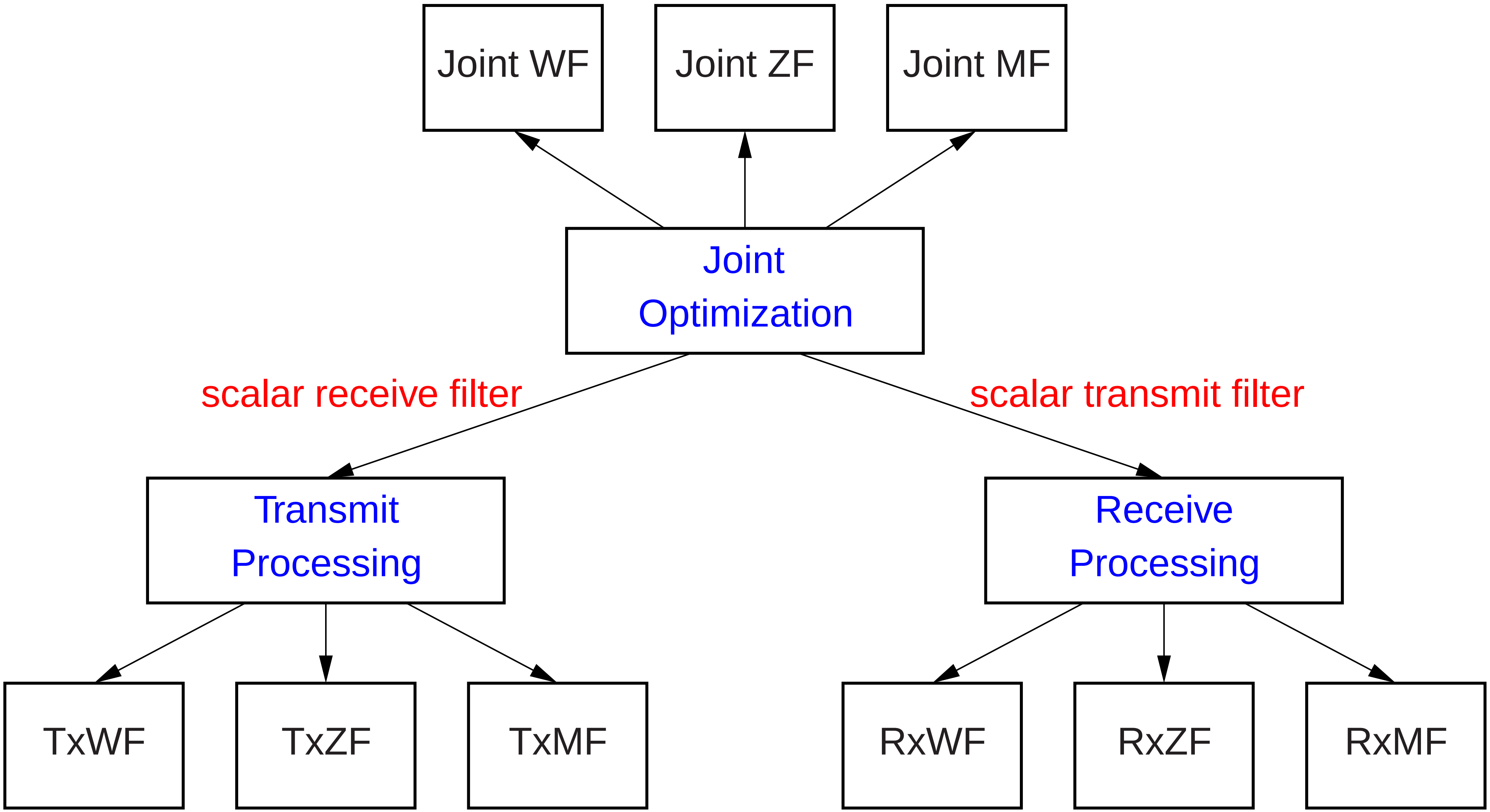
    time division duplex systems (calibration) or
    feedback channel
  - robust design necessary
- Partial Channel State Information
  - only statistics (e. g. covariance matrix) of channel known

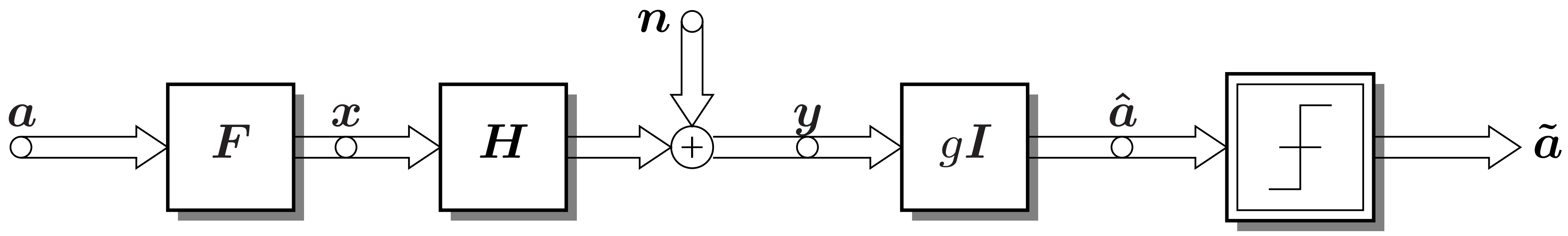
    frequency division duplex systems (frequency gap, calibration) or
    feedback channel
- No Channel State Information
  - only transmit processing independent of channel properties

- Complete CSI
  - linear transmit filters
  - Tomlinson Harashima precoding
  - vector precoding
  - lattice-reduction-aided precoding
  - minimization of bit error probability

- Partial CSI
  - linear transmit filters

- No CSI
  - receive processing


Joint WF
Joint ZF
Joint MF
Joint Optimization
scalar receive filter
scalar transmit filter
Transmit Processing
Receive Processing
TxWF
TxZF
TxMF
RxWF
RxZF
RxMF



Principle: “predistortion” with $\boldsymbol{F} \in \mathbb{C}^{N_\mathsf{T} \times N_\mathsf{R}}$: channel acts as equalizer

correction of amplitude with scalar estimator $g \in \mathbb{C}$

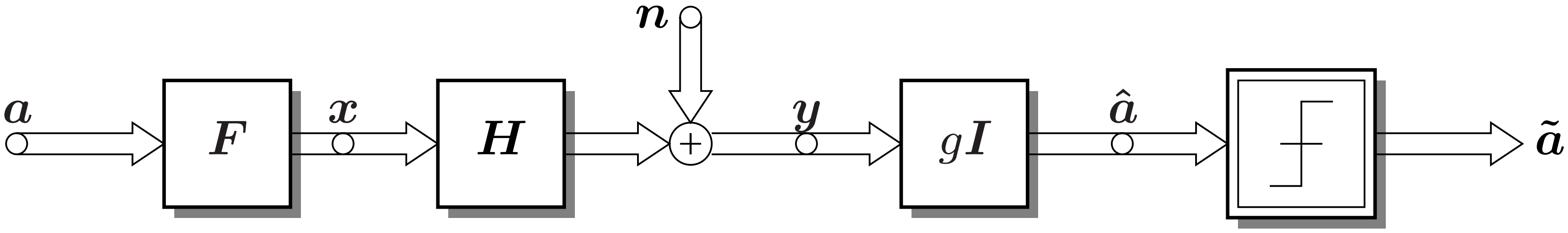
$\Rightarrow$ joint optimization

Transmitter: limitation of average transmit power: $\mathrm{E}\left[\|\boldsymbol{x}\|_2^2\right] \leq P_{\mathsf{tr}}$

Receiver: symbol-by-symbol quantization

$\Rightarrow$ no cooperation of receivers necessary (multiuser systems)

very simple



Criterion: maximization of signal-to-noise-ratio

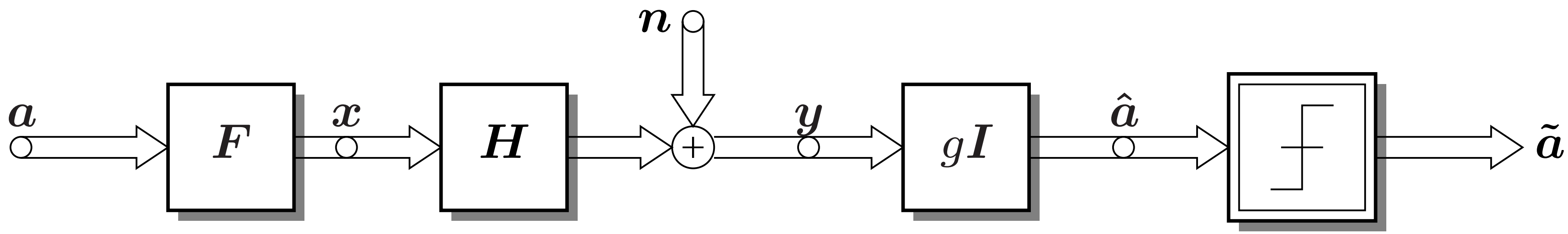
Optimization:
$$\{\boldsymbol{F}_{\mathrm{MF}}, g_{\mathrm{MF}}\} = \underset{\{\boldsymbol{F},g\}}{\operatorname{argmax}} \frac{\left|\mathrm{E}\left[\boldsymbol{a}^{\mathrm{H}}\hat{\boldsymbol{a}}\right]\right|^2}{\mathrm{E}\left[\|\boldsymbol{a}\|_2^2\right]\mathrm{E}\left[\|g\boldsymbol{n}\|_2^2\right]}$$

s. t.: $\mathrm{E}\left[\|\boldsymbol{x}\|_2^2\right] \leq P_{\mathrm{tr}}$

Complexity: filter computation: $\mathrm{O}\left(N_{\mathrm{R}}N_{\mathrm{T}}\right)$
filtering: $\mathrm{O}\left(N_{\mathrm{R}}N_{\mathrm{T}}\right)$

Extension: receive matched filter (eigenprecoder)

[McIntosh et al. ’70], [Esmailzadeh et al. ’93], [Choi et al. ’01], [Wang et al. ’99], [Irmer et al. ’01], . . .



Criterion: minimization of mean square error with complete interference suppression

Optimization:

$$\{\boldsymbol{F}_{\mathrm{ZF}}, g_{\mathrm{ZF}}\} = \underset{\{\boldsymbol{F},g\}}{\operatorname{argmin}} \operatorname{E}\left[\|\boldsymbol{a} - \hat{\boldsymbol{a}}\|_2^2\right]$$

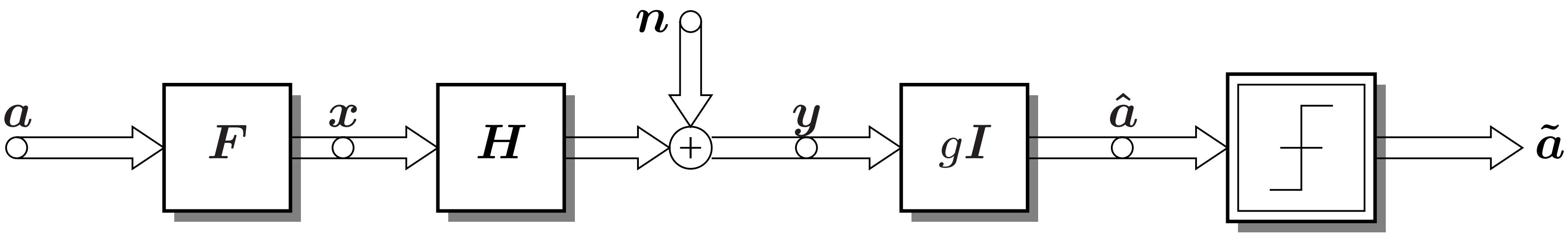
s. t.: $\hat{\boldsymbol{a}}|_{\boldsymbol{n}=\mathbf{0}} = \boldsymbol{a}$ and $\operatorname{E}\left[\|\boldsymbol{x}\|_2^2\right] \leq P_{\mathrm{tr}}$

Complexity:

filter computation: $\mathrm{O}\left(N_{\mathrm{R}}^2 N_{\mathrm{T}}\right)$

filtering: $\mathrm{O}\left(N_{\mathrm{R}} N_{\mathrm{T}}\right)$

[Vojčić et al. ’98], [Montalbano et al. ’98], [Brandt-Pearce et al. ’00], [Baier et al. ’00], [Joham et al. ’00], [Noll Barreto et al. ’01], [Walke et al. ’01], . . .



Criterion: minimization of mean square error

Optimization: $\{\boldsymbol{F}_{\mathrm{WF}}, g_{\mathrm{WF}}\} = \underset{\{\boldsymbol{F},g\}}{\operatorname{argmin}} \,\mathrm{E}\left[\|\boldsymbol{a} - \hat{\boldsymbol{a}}\|_2^2\right]$

s. t.: $\mathrm{E}\left[\|\boldsymbol{x}\|_2^2\right] \leq P_{\mathrm{tr}}$

Convergence: for high SNR: $\boldsymbol{F}_{\mathrm{WF}} \rightarrow \boldsymbol{F}_{\mathrm{ZF}}$
for low SNR: $\boldsymbol{F}_{\mathrm{WF}} \rightarrow \boldsymbol{F}_{\mathrm{MF}}$

Complexity: filter computation: $\mathrm{O}\left(N_{\mathrm{R}}^2 N_{\mathrm{T}}\right)$
filtering: $\mathrm{O}\left(N_{\mathrm{R}} N_{\mathrm{T}}\right)$

[Karimi et al. 1999], [Choi et al. 2002], [Joham et al. 2002], [Peel et al. 2003], [Berenguer et al. 2005]

Other Criteria:

– minimization of transmit power
  guaranteed signal-to-interference-and-noise-ratio
  [Visotsky et al. ’99], [Tse et al. ’02], [Boche et al. ’02], [Wiesel et al. ’04]

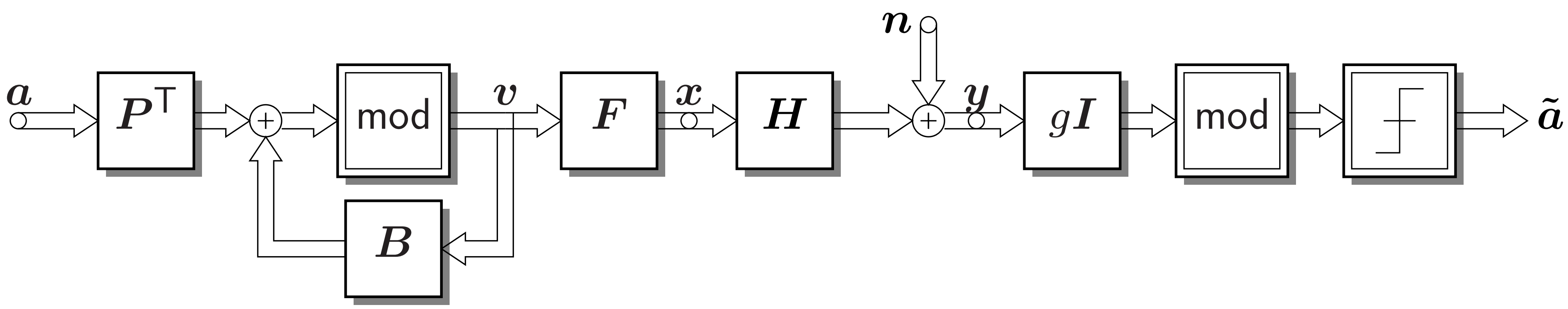
– minimization of bit error probability
  [Hjørungnes et al. 2005]

Extensions:

– different weights at the receivers
  [Schubert et al. 2005], [Hunger et al. 2005]

– prediction of channel impulse response
  [Visotsky et al. 2001], [Guncavdi et al. 2001], [Dietrich et al. 2003]

– robust design
  [Rey et al. 2002], [Dietrich et al. 2003], [Abdel-Samad et al. 2003], [Palomar et al. 2004]



Principle: use already precoded symbols for interference reduction

Feedback Filter: for realizability: lower triangular and zero main diagonal

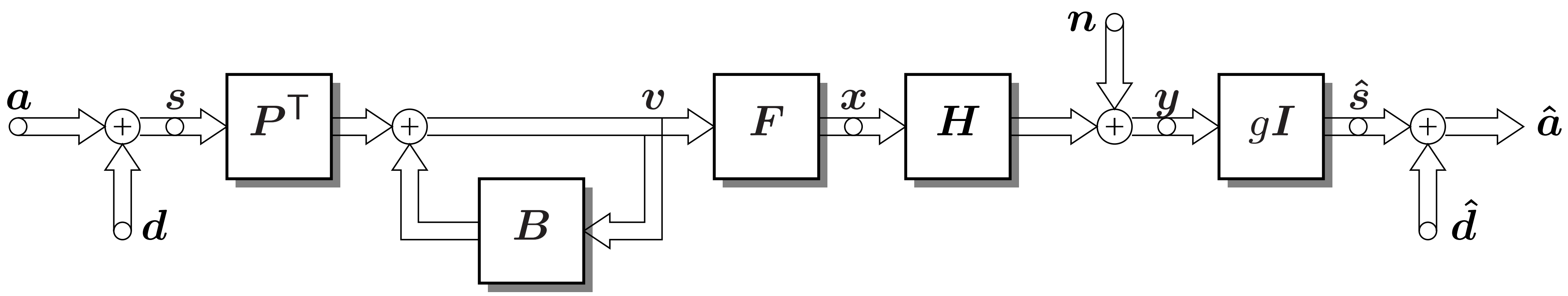
Ordering: permutation matrix $\boldsymbol{P}$ defines precoding order

Modulo:

$$\mathrm{mod}(x) = x - \left\lfloor \frac{\mathrm{Re}(x)}{\tau} + \frac{1}{2} \right\rfloor \tau - \mathrm{j} \left\lfloor \frac{\mathrm{Im}(x)}{\tau} + \frac{1}{2} \right\rfloor \tau$$

$$= x + d(x) \quad \text{with} \quad d(x) \in \tau\mathbb{Z} + \mathrm{j}\,\tau\mathbb{Z}$$

Assumption: statistics of signal $\boldsymbol{v}$ are known



Criterion: minimization of mean square error

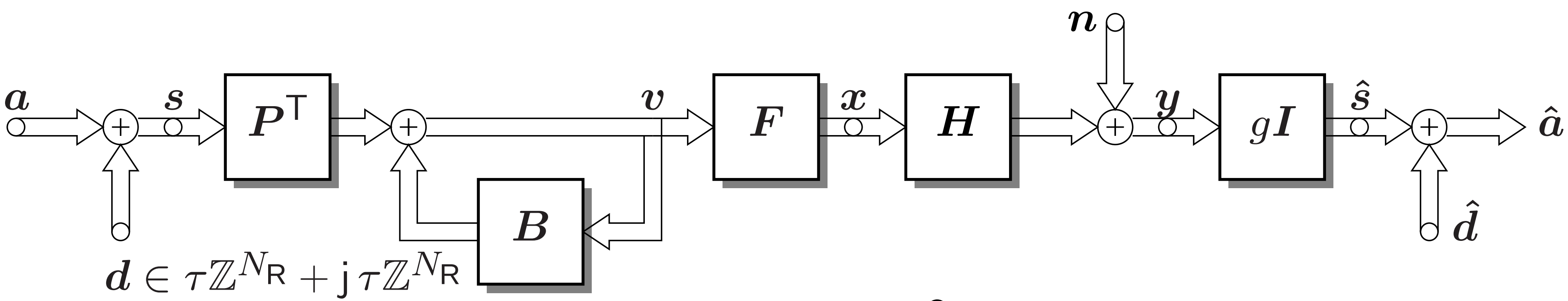
Optimization: $\{\boldsymbol{F}_{\mathrm{WF}}, \boldsymbol{B}_{\mathrm{WF}}, g_{\mathrm{WF}}, \boldsymbol{P}_{\mathrm{WF}}\} = \underset{\{\boldsymbol{F},\boldsymbol{B},g,\boldsymbol{P}\}}{\operatorname{argmin}} \; \mathrm{E}\left[\|\boldsymbol{s}-\hat{\boldsymbol{s}}\|_2^2\right]$

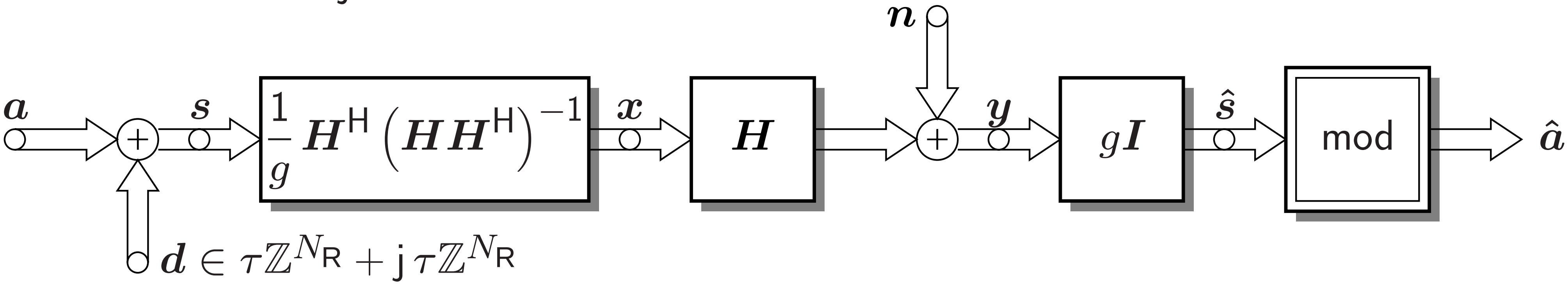
s. t.: $\boldsymbol{P}$: permutation matrix, $\mathrm{E}\left[\|\boldsymbol{x}\|_2^2\right] \leq P_{\mathrm{tr}}$ and
$\boldsymbol{B}$: lower triangular matrix with zero main diagonal

Ordering: similar to V-BLAST ordering algorithm

Complexity: filter computation: $\mathrm{O}\left(N_{\mathrm{R}}^2 N_{\mathrm{T}}\right)$
filtering: $\mathrm{O}\left(N_{\mathrm{R}} N_{\mathrm{T}}\right)$

[Tomlinson 1971], [Harashima et al. 1972], [Fischer et al. 1994/2002], [Ginis et al. 2000], [Schubert et al. 2002], [Liu et al. 2003], [Joham et al. 2004]





Principle: use ambiguity due to modulo operator at receiver for transmit power minimization

Optimization:

$$\boldsymbol{d}_{\mathsf{vec}} = \underset{\boldsymbol{d} \in \tau\mathbb{Z}^{N_\mathsf{R}} + \mathrm{j}\,\tau\mathbb{Z}^{N_\mathsf{R}}}{\operatorname{argmin}} \left\| \boldsymbol{H}^{\mathsf{H}}\left(\boldsymbol{H}\boldsymbol{H}^{\mathsf{H}}\right)^{-1}(\boldsymbol{a}+\boldsymbol{d}) \right\|_2^2$$

[Peel et al. 2003], [Shi et al. 2004], [Fischer et al. 1995]

Optimization:

$$\boldsymbol{d}_{\mathsf{vec}} = \underset{\boldsymbol{d}\in\tau\mathbb{Z}^{N_\mathsf{R}}+\mathrm{j}\,\tau\mathbb{Z}^{N_\mathsf{R}}}{\operatorname{argmin}} \left\| \boldsymbol{H}^\mathsf{H}\left(\boldsymbol{H}\boldsymbol{H}^\mathsf{H}\right)^{-1}(\boldsymbol{a}+\boldsymbol{d})\right\|_2^2$$

$$= \underset{\boldsymbol{d}\in\tau\mathbb{Z}^{N_\mathsf{R}}+\mathrm{j}\,\tau\mathbb{Z}^{N_\mathsf{R}}}{\operatorname{argmin}} \left\| \boldsymbol{H}^\mathsf{H}\left(\boldsymbol{H}\boldsymbol{H}^\mathsf{H}\right)^{-1}\boldsymbol{d}-\boldsymbol{a}'\right\|_2^2$$

compare to: $\tilde{\boldsymbol{a}}_{\mathsf{ML}} = \underset{\boldsymbol{a}\in\mathbb{A}^{N_\mathsf{T}}}{\operatorname{argmin}} \|\boldsymbol{H}\boldsymbol{a}-\boldsymbol{y}\|_2^2$

$\Rightarrow$ like sphere decoder: „maximum likelihood" at transmitter

Transmit Power: weight g follows from $\mathrm{E}\left[\|\boldsymbol{x}\|_2^2\right]$

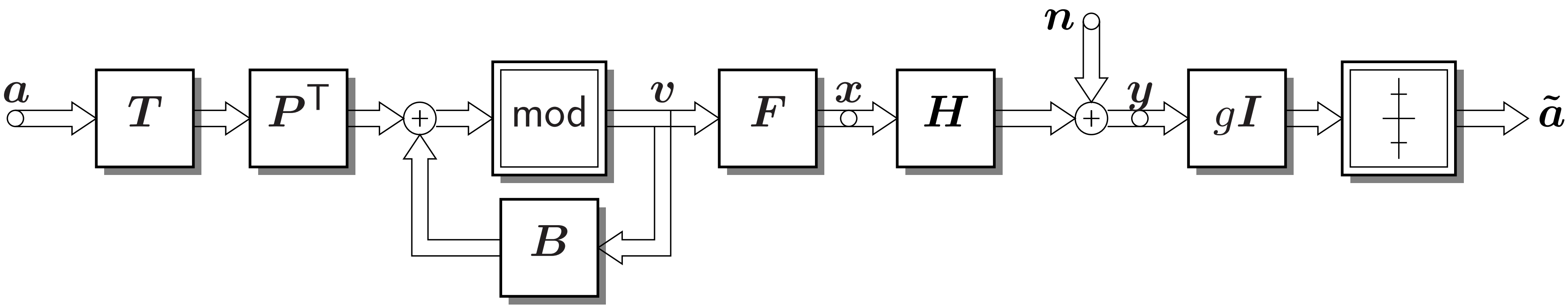
Complexity: not polynomial in N_R

Alternatively:

– transformation with $\boldsymbol{H}^\mathsf{H}\left(\boldsymbol{H}\boldsymbol{H}^\mathsf{H}+\zeta\boldsymbol{I}\right)^{-1}$ instead of $\boldsymbol{H}^\mathsf{H}\left(\boldsymbol{H}\boldsymbol{H}^\mathsf{H}\right)^{-1}$
[Peel et al. 2003]

– lattice-reduction-aided detector instead of sphere decoder
[Windpassinger et al. 2004]

– division into groups
[Meurer et al. 2004]



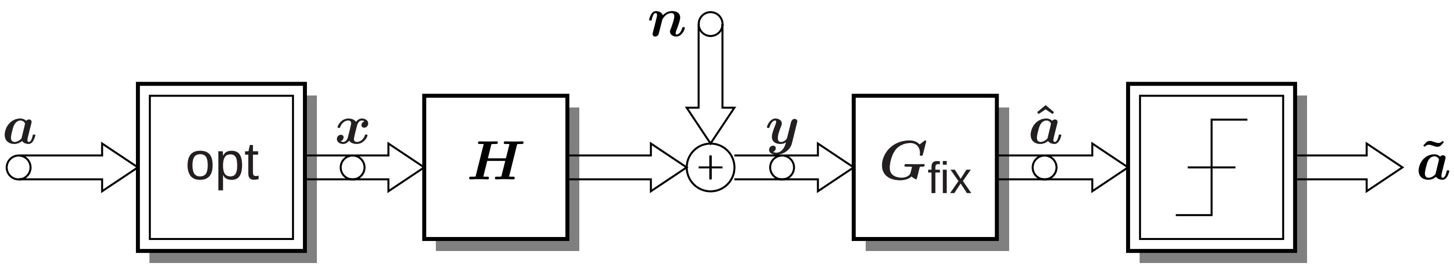
Principle: decomposition of channel matrix into integer part and another part with (nearly) orthogonal rows

– equalization of part with orthogonal rows with precoding
– inversion of part with integer entries in front of modulo operator

Criterion: minimization of mean square error

Complexity: filter computation: open
filtering $\mathcal{O}\left(N_\mathsf{R} N_\mathsf{T}\right)$

[Windpassinger et al. 2003]

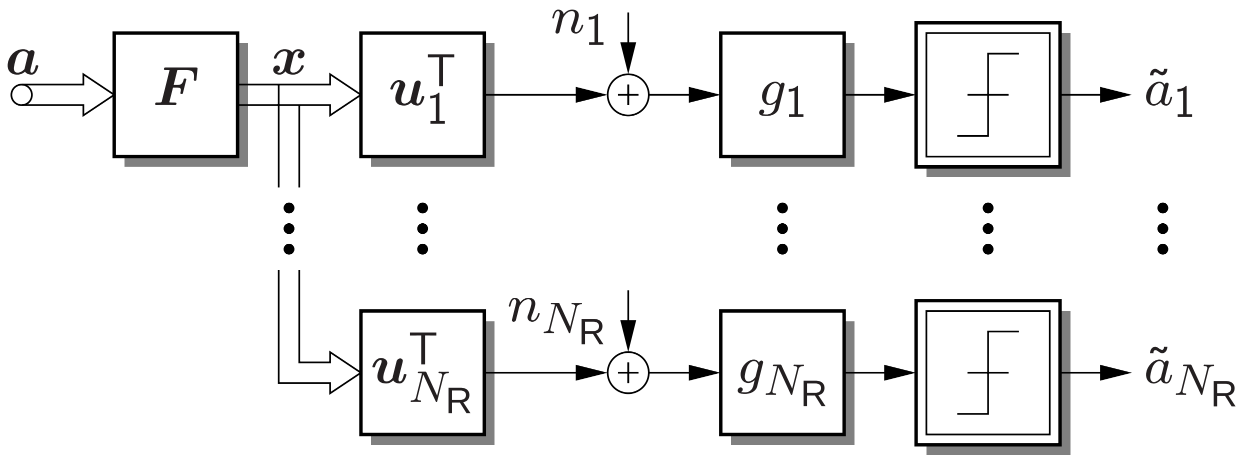


Principle: minimization of bit error probability by appropriate choice of transmit signal for given transmit power

Optimization: $\boldsymbol{x}_{\text{minBEP}} = \underset{\boldsymbol{x}}{\text{argmin}}\, \text{P}_{\text{b}}(\boldsymbol{x}) \qquad \text{s. t.:} \quad \|\boldsymbol{x}\|_2^2 = P_{\text{tr}}$

Complexity: not polynomial

[Irmer et al. 2003], [Weber et al. 2003]



Principle:	formulation of a power equivalent model application of methods for full CSI
CSI:	covariance matrices of channels: $\boldsymbol{R}_{\boldsymbol{h}_k} = \mathrm{E}\left[\boldsymbol{h}_k \boldsymbol{h}_k^{\mathsf{H}}\right]$ equivalent channel $\boldsymbol{u}_k$: dominant eigenvector of $\boldsymbol{R}_{\boldsymbol{h}_k}$
Receiver:	matched filter or correction of phase

[Montalbano et al. ’99], [Forster et al. ’00], [Joham et al. ’02], [Simeone et al. ’04], [Dietrich et al. ’05]

Principle: maximization of minimal SINR for given transmit power or minimization of transmit power for given SINRs

CSI: covariance matrices of channels: $\boldsymbol{R}_{\boldsymbol{h}_k} = \mathsf{E}\left[\boldsymbol{h}_k \boldsymbol{h}_k^{\mathsf{H}}\right]$

Solution:

– division into power scaling and normalized vector

– iterative algorithm to find the normalized vectors via a duality of uplink and downlink

– computation of power scaling via couple matrix

[Gerlach et al. ’96], [Montalbano et al. ’98], [Farsakh et al. ’98], [Rashi-Farrokhi et al. ’98], [Bengtsson et al. ’99], [Tse et al. ’02], [Boche et al. ’02]

Principle: symbol is spread over space and time:

$$\boldsymbol{S} = \sum_{q=1}^{Q} \boldsymbol{A}_q \, \mathrm{Re}(a_q) + \mathrm{j} \, \boldsymbol{B}_q \, \mathrm{Im}(a_q) \in \mathbb{C}^{N_\mathsf{T} \times T} \quad \text{mit} \quad \boldsymbol{A}_q, \boldsymbol{B}_q \in \mathbb{R}^{N_\mathsf{T} \times T}$$

CSI: not necessary at transmitter

Specal Cases:
– (orthogonal) space-time block codes
– spatial multiplex

Receiver:
– sphere decoder
– V-BLAST

[Wittneben 1993], [Foschini 1996], [Tarokh et al. 1998], [Wolniansky et al. 1998], [Alamouti 1998], [Hassibi et al. 2002]

- Point-to-Multipoint MIMO Systems:
  $\Rightarrow$ transmit processing

- Complete Channel State Information:
  - linear transmit filters
  - Tomlinson Harashima precoding
  - robust design

- Partial Channel State Information:
  - linear transmit filters