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***Abstract*— A novel MSE balancing paradigm is presented which is based on the unweighted sum-MSE minimization with individual power constraints in conjunction with an adaptive power allocation. By means of a counter-example, we show that even the two-user MSE region may be nonconvex in case of multi-antenna users which is in contrast to the predominant opinion. Any algorithm utilizing the weighted sum-MSE minimization hence cannot achieve the complete MSE region for any channel realization. This resulting gap due to the nonconvex dent is closed by the presented approach. For the unweighted sum-MSE minimization with individual power constraints which represents the core of the balancing algorithm, we come up with an extremely fast converging alternating optimization outperforming all hitherto existing approaches.**

I. Introduction

Balancing is known, on the one hand, from rate applications where a common rate, namely the *symmetric capacity*, is allocated to every user [1] and on the other hand from SINR balancing for single antenna receivers [2], [3]. In the latter case, SINR balancing is basically equivalent to balancing the individual users' *mean square errors* (MSEs). While there exists only a single balancing level for single antenna receivers, multiple levels arise when MIMO receivers are allowed. As a consequence, only local optimality is ensured in general by most of the existing algorithms covering this topic. Interestingly, only few contributions focus on the multi-antenna case. In [4], the authors tackle the multi-antenna problem by means of an *alternating optimization* (AO) switching between uplink and downlink. A weighted sum-MSE minimization approach is given in [5]. Both approaches need not necessarily converge to the global optimum. In this paper, we present a new balancing paradigm which is applicable not only to mean square error, but also to rate applications. It is based on adaptive individual power constraints and optimizes either the *un*weighted sum-MSE or the *un*weighted sum-rate. Therefore, this part of the algorithm has a very advantageous convex structure.

II. System Model and Dual Uplink Problem

We focus on the downlink of a cellular system where K users are served by a base station with N transmit antennas. Based on a new sort of duality theory in [3], [5], we construct an equivalent problem formulation in the virtual dual uplink featuring the same MSE region under a *sum-power* constraint as the downlink. This dual uplink has the nice property that common inverses arise in the individual users' mean square error expressions leading to a reduced complexity during the optimization. Instead of balancing the MSEs in the downlink, we solve the balancing problem in the virtual dual uplink and afterwards transform the solution to the downlink making use of the uplink-to-downlink conversion rules from the duality in [3], [5].

Let $\boldsymbol{T}_k \in \mathbb{C}^{r_k \times B_k}$ denote the precoding matrix of user k in the dual uplink that maps its B_k streams onto its r_k antennas. Then, the transmit covariance matrix reads as

$$\boldsymbol{Q}_k = \boldsymbol{T}_k \boldsymbol{T}_k^{\mathrm{H}} \in \mathbb{C}^{r_k \times r_k}, \tag{1}$$

where we assumed that the zero-mean symbol vector $\boldsymbol{s}_k \in \mathbb{C}^{B_k}$ has pairwise uncorrelated unit-variance entries. With $\boldsymbol{H}_k \in \mathbb{C}^{N \times r_k}$ representing the channel matrix of user k, $\boldsymbol{\eta} \in \mathbb{C}^N$ being the uncorrelated noise with variance σ_η^2 of each entry, and $\boldsymbol{G}_k \in \mathbb{C}^{B_k \times N}$ denoting the receive filter, the mean square error ε_k of user k in the virtual uplink can be expressed as

$$\varepsilon_k = \mathrm{tr}\left(\mathbf{I}_{B_k} - \boldsymbol{T}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{X}^{-1} \boldsymbol{H}_k \boldsymbol{T}_k\right), \tag{2}$$

where we optimally chose $\boldsymbol{G}_k = \boldsymbol{T}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{X}^{-1}$ to be the MMSE receiver and

$$\boldsymbol{X} = \sigma_\eta^2 \mathbf{I}_N + \sum_{\ell=1}^{K} \boldsymbol{H}_\ell \boldsymbol{T}_\ell \boldsymbol{T}_\ell^{\mathrm{H}} \boldsymbol{H}_\ell^{\mathrm{H}} \tag{3}$$

is the covariance matrix of the received signal. Note that the dual uplink has a *sum-power* constraint $\sum_{k=1}^K \|\boldsymbol{T}_k\|_{\mathrm{F}}^2 = \sum_{k=1}^K \mathrm{tr}(\boldsymbol{Q}_k) \leq P_{\mathrm{Tx}}$ as well as the downlink which follows from the MSE duality in [3].

III. Problem Statement and Optimality Conditions

The MSE balancing problem is based on equating the weighted mean-square errors of all users at a minimum common level with a fixed amount of available power. This approach is already known from rate balancing in [1] and single-antenna SINR balancing in [2], [5]. For the MSE, however, the weights $w_1, \ldots, w_K \in \mathbb{R}_{+,0}$ may not be chosen arbitrarily, since feasibility of the problem cannot always be guaranteed. In particular, the MSE ε_k of user k cannot exceed B_k and cannot drop below a minimum possible value $\varepsilon_{\mathrm{min},k} > 0$. Thus, not all weight ratios $w_k/w_\ell = \varepsilon_k/\varepsilon_\ell$ are supported. The balancing problem can be stated as

$$\min_{\{\alpha, \boldsymbol{T}_1, \ldots, \boldsymbol{T}_k\}} \alpha \quad \text{s.t.:} \ \frac{\varepsilon_k}{w_k} = \alpha \ \text{and} \ \sum_{k=1}^{K} \|\boldsymbol{T}_k\|_{\mathrm{F}}^2 \leq P_{\mathrm{Tx}}. \tag{4}$$

For the single antenna case with $r_k = B_k = 1\ \forall k$, the cost function α becomes redundant as the constraints $\varepsilon_k = w_k\alpha$ have only a single solution under the assumption of MMSE receivers and the complete dissipation of the maximum power. This follows from the fact that due to the structure of the MSE expression in (2) not all MSEs can simultaneously decrease under a fixed sum-power. In the MIMO case, however, several balanced levels α may exist and we seek for the smallest one.

From the KKT conditions of the nonconvex problem (4), we can infer that any candidate for a local minimum, i.e., any stationary point of (4) complies with the KKT conditions of the *weighted* sum-MSE minimization problem

$$\min_{\{\boldsymbol{T}_1,\dots,\boldsymbol{T}_K\}} \sum_{k=1}^{K} \mu_k\varepsilon_k \quad \text{s.t.:} \quad \sum_{k=1}^{K} \|\boldsymbol{T}_k\|_{\mathrm{F}}^2 \leq P_{\mathrm{Tx}}. \tag{5}$$

Unfortunately, the weights $\boldsymbol{\mu} = [\mu_1,\dots,\mu_K]^{\mathrm{T}} \in \mathbb{R}_{+,0}^K$ are not known in advance as they correspond to the Lagrangian multipliers of (4) ensuring the equality of the MSE levels. This problem structure suggests finding the solution for (4) by means of a *weighted* sum-MSE minimization where the weights $\boldsymbol{\mu}$ are adaptively chosen to satisfy the MSE equality as in [5] similarly to the rate-balancing algorithm in [1], where the weighted sum-rate is maximized in combination with an ellipsoid method. However, the structure of the MSE region sometimes prevents the possibility of reaching every point on the border of the MSE region by means of the weighted sum-MSE minimization problem. More precisely, we give a counterexample to the statement made in [6] that the MSE region is convex in case of multi-antenna receivers. The proof presented in [6] testifies that any two points on the border of the MSE region that are connected by a line with -45° slope have no nonconvex dent between them. Nonetheless, convexity of the MSE region does not follow from this, all possible slopes have to be checked instead to show convexity. Evidently, any standard weighted sum-MSE minimization algorithm may fail to find the balanced level (even if the optimum weights were known in advance) if the optimum individual MSEs should turn out to lie on such a nonconvex segment. Algorithms as in [5] adapting the weights $\boldsymbol{\mu}$ according to a subgradient approach which increases the weight for the sum-MSE minimization of a specific user if its MSE level is too high or decreases it if its MSE is too small, therefore tend to oscillate between two points if the balanced level lies on such a dent. As mentioned before, the desired balanced level, i.e., the desired MSE tuple fulfills the KKT conditions of (5), even in a dent. Yet, such an MSE tuple represents a stationary point (local minimum or saddle-point) of the Lagrangian associated to the optimization in (5) which is not globally optimum and hence, extremely hard to find by an iterative algorithm. In order to come up with a convex MSE region for multi-antenna receivers, *time-sharing* would have to be applied to obtain the convex hull of the nonconvex region. But then, computationally complex ellipsoid methods would have to be employed to find a balanced level on the convex hull part which has curvature zero in at least one dimension.

In [4], the authors propose an alternating optimization approach repeatedly switching between uplink and downlink as the receive filters in the respective link are MMSE receivers. However, a *stream-wise* MSE duality is used although the total MSEs *per user* are balanced leading to a high complexity for switching from one link to the other in the alternation.

IV. A Novel Balancing Paradigm: Individual Power Constraints and Unweighted Optimization

The complete MSE region including its interior consists of the union of all individual MSE tuples from (2) achieved by all positive semidefinite transmit covariance matrices $\boldsymbol{Q}_k \succcurlyeq \mathbf{0}$ satisfying the *sum-power* constraint $\sum_{k=1}^{K} \operatorname{tr}(\boldsymbol{Q}_k) \leq P_{\mathrm{Tx}}$. Equivalently, we can say that the region is spanned by the union of all MSE tuples generated by covariance matrices $\boldsymbol{Q}_k \succcurlyeq \mathbf{0}$ with *individual* power constraints $\operatorname{tr}(\boldsymbol{Q}_k) \leq p_k$. Clearly, the union is also taken over all nonnegative powers $\boldsymbol{p} = [p_1,\dots,p_K]^{\mathrm{T}}$, summing up to at most P_{Tx}, i.e., $\sum_{k=1}^{K} p_k \leq P_{\mathrm{Tx}}$. While almost all balancing algorithms apply the weighted sum-MSE/sum-rate optimization in order to achieve balanced levels (e.g. [1], [5]), we come up with the following new paradigm.

A. The Basic Idea

Instead of adapting the weights of a weighted sum-MSE/sum-rate optimization under a sum-power constraint, we impose *individual* power constraints to the covariance matrices and adapt the distribution of the available power over the respective users. The structure of the covariances is then optimized by the slightly restricting property that they follow from the *unweighted* sum-MSE minimization with *individual* power constraints. Obviously, this reduced set of precoders will fail to cover the complete MSE region. However, this approach has several advantages: First, the unweighted sum-MSE minimization problem is a convex minimization (in contrast to the weighted sum-MSE minimization), for which fast and efficient solutions exist, see for example the fixed point Picard iteration in [7], [8]. In addition, we will present a novel, computationally even less complex method to solve this problem in the following. Second, the MSE region obtained from the union of the MSE tuples generated by the unweighted sum-MSE minimization and all individual power distributions touches the complete (possibly nonconvex) MSE region at least $K+1$ times. Once, when all MSE weights are identical, and K times, when the complete power P_{Tx} is allocated to a single user. Moreover, the curvature of both regions is almost the same around the touching point where all weights are identical. So at least in the local vicinity, this approximation is very good. Simulation results reveal that, depending on the channel and the current signal-to-noise ratio, there is a very good match between the complete MSE region and the one obtained by the sum-MSE minimization with different individual power allocations. Third, no step-size adaptations are required for the update of the individual power distribution $\boldsymbol{p}$ to the users as this would be the case for the weights $\boldsymbol{\mu}$ in a weighted sum-MSE minimization.

B. Unweighted Sum-MSE Minimization with Individual Power Constraints

From the definition of the individual MSEs in (2), we can express the sum MSE ε which is jointly convex in all covariance matrices via

$$\varepsilon = \sum_{k=1}^{K} \varepsilon_k = \sum_{k=1}^{K} B_k - N + \sigma_\eta^2 \operatorname{tr}(\boldsymbol{X}^{-1}), \tag{6}$$

where the total receive covariance matrix $\boldsymbol{X}$ is defined in (3). The optimization reads as

$$\min_{\{\boldsymbol{Q}_1,\ldots,\boldsymbol{Q}_K \succcurlyeq \mathbf{0}\}} \varepsilon \quad \text{s.t.:} \quad \operatorname{tr}(\boldsymbol{Q}_k) \leq p_k \ \forall k. \tag{7}$$

It can easily be shown that each user has to transmit with full power p_k. In [9], the authors treat the individual links from each user to the base station as single-user point-to-point systems with colored noise (interference plus noise) and iteratively optimize one covariance matrix after another keeping all others fixed. While this procedure is provably optimum for the rate maximization with individual power constraints [10], it's suboptimum for sum-MSE minimization since the KKT conditions look different. Indeed, they read as

$$\sigma_\eta^2 \boldsymbol{H}_k^{\mathrm{H}} \check{\boldsymbol{X}}^{-2} \boldsymbol{H}_k = \lambda_k \mathbf{I}_{r_k} - \check{\boldsymbol{\Lambda}}_k \quad \text{and} \quad \check{\boldsymbol{Q}}_k \check{\boldsymbol{\Lambda}}_k = \mathbf{0} \ \forall k, \tag{8}$$

where $\check{\boldsymbol{\Lambda}}_k$ ensures that $\check{\boldsymbol{Q}}_k$ is positive semidefinite and is positive semidefinite itself, and λ_k is the Lagrangian multiplier associated to the power constraint of user k. Checked variables denote that they fulfill the KKT conditions. After several transformations, we find under the assumption of a full rank solution ($\check{\boldsymbol{\Lambda}}_k = \mathbf{0}$)

$$\begin{aligned}\check{\boldsymbol{Q}}_k =& \lambda_k \left[\left(\boldsymbol{H}_k^{\mathrm{H}} \check{\boldsymbol{X}}_k^{-1} \boldsymbol{H}_k \right)^{-1} \sigma_\eta^2 \boldsymbol{H}_k^{\mathrm{H}} \check{\boldsymbol{X}}_k^{-2} \boldsymbol{H}_k \left(\boldsymbol{H}_k^{\mathrm{H}} \check{\boldsymbol{X}}_k^{-1} \boldsymbol{H}_k \right)^{-1} \right]^{\frac{1}{2}} \\ &- \left(\boldsymbol{H}_k^{\mathrm{H}} \check{\boldsymbol{X}}_k^{-1} \boldsymbol{H}_k \right)^{-1}\end{aligned} \tag{9}$$

which is optimum only if $\check{\boldsymbol{Q}}_k \succ \mathbf{0}$, and λ_k is chosen such that $\operatorname{tr}(\check{\boldsymbol{Q}}_k) = p_k$. Note that the right hand side does not depend on $\check{\boldsymbol{Q}}_k$ since

$$\boldsymbol{X}_k = \boldsymbol{X} - \boldsymbol{H}_k \boldsymbol{Q}_k \boldsymbol{H}_k^{\mathrm{H}} = \sigma_\eta^2 \mathbf{I}_N + \sum_{\ell \neq k} \boldsymbol{H}_\ell \boldsymbol{Q}_\ell \boldsymbol{H}_\ell^{\mathrm{H}}$$

does not depend on $\boldsymbol{Q}_k$. By means of the matrix inversion lemma, complexity can drastically be reduced: First, we can replace $\boldsymbol{X}_k$ by $\boldsymbol{X}$ in the first summand of (9) as this does not change its value. Second, we can exploit the equality

$$(\boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{X}_k^{-1} \boldsymbol{H}_k)^{-1} = (\boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{X}^{-1} \boldsymbol{H}_k)^{-1} - \boldsymbol{Q}_k$$

to transform the second summand and therefore never have to compute all K inverses $\boldsymbol{X}_k^{-1}$, $k \in \{1, \ldots, K\}$.

Under the assumption of a rank-one precoder, the covariance matrix reads as $\check{\boldsymbol{Q}}_k = \check{\boldsymbol{u}}_k \check{\boldsymbol{u}}_k^{\mathrm{H}}$ with $\check{\boldsymbol{u}}_k$ being the only nonzero column of the precoder $\check{\boldsymbol{T}}_k$. The optimality condition

$$\left(\frac{1}{p_k} \mathbf{I}_{r_k} + \boldsymbol{H}_k^{\mathrm{H}} \check{\boldsymbol{X}}_k^{-1} \boldsymbol{H}_k \right)^{-1} \boldsymbol{H}_k^{\mathrm{H}} \check{\boldsymbol{X}}_k^{-2} \boldsymbol{H}_k \cdot \check{\boldsymbol{u}}_k = \gamma \cdot \check{\boldsymbol{u}}_k \tag{10}$$

reveals that $\check{\boldsymbol{u}}_k$ is the eigenvector with norm $\sqrt{p_k}$ corresponding to the largest eigenvalue γ of $(p_k^{-1} \mathbf{I}_{r_k} + \boldsymbol{H}_k^{\mathrm{H}} \check{\boldsymbol{X}}_k^{-1} \boldsymbol{H}_k)^{-1}$. Thus, we can completely solve the nonlinear KKT conditions of problem (7) by means of the *alternating optimization* (AO) technique for the extremely interesting case where at most $r_k = 2$ antennas are deployed at the mobile terminals. For mobile terminals with more than two antennas, the algorithm in [9] could be used, since the case where the optimum precoding covariance matrix has neither rank one nor full rank might occur.

The individual power constraints allow for an AO [11], [12], [13], where the covariance matrices are optimized separately one after another keeping all other covariances fixed. During iteration $n+1$, the optimum covariance matrix $\boldsymbol{Q}_1^{(n+1)}$ of user 1 follows from (9) if the result is positive definite or otherwise from (10) (each without the check sign) where $\boldsymbol{Q}_2^{(n)}, \ldots, \boldsymbol{Q}_K^{(n)}$ from the previous iteration n are held constant. Since (9) or (10) return the optimum covariance $\boldsymbol{Q}_1^{(n+1)}$ for fixed interference, this procedure is called *minimization mapping*. Depending on whether $\boldsymbol{Q}_1^{(n+1)}$ is already taken into account for the computation of $\boldsymbol{Q}_2^{(n+1)}, \ldots, \boldsymbol{Q}_K^{(n+1)}$ or not, we distinguish two cases: *Parallel* connection uses all other covariances $\{\boldsymbol{Q}_1^{(n)}, \ldots, \boldsymbol{Q}_K^{(n)}\} \setminus \boldsymbol{Q}_\ell^{(n)}$ from the previous iteration n for the update of $\boldsymbol{Q}_\ell^{(n+1)}$ exploiting the fact that the inverse of $\boldsymbol{X}^{(n)}$ has to be computed only once per iteration and not K times. *Serial* connection incorporates the updates $\boldsymbol{Q}_1^{(n+1)}, \ldots, \boldsymbol{Q}_{\ell-1}^{(n+1)}$ for the computation of $\boldsymbol{Q}_\ell^{(n+1)}$ and therefore features a better performance than the parallel connection at the price of a higher computational complexity.

C. Power Updating Rule

The simple heuristic power update rule for iteration $n+1$

$$\begin{aligned} p_k^{\prime(n+1)} &= p_k^{(n)} \frac{\varepsilon_k^{(n)}}{w_k} \\ p_k^{(n+1)} &= P_{\mathrm{Tx}} \frac{p_k^{\prime(n+1)}}{\sum_{i=1}^{K} p_i^{\prime(n+1)}} \end{aligned} \tag{11}$$

features good convergence properties and is equivalent to the Picard fixed point iteration with subsequent power normalization in the single antenna case. In the first step, user k experiences a larger power gain than user ℓ if its individual level $\varepsilon_k^{(n)}/w_k$ is larger than the level $\varepsilon_\ell^{(n)}/w_\ell$ of user ℓ. In the optimum, the individual levels of all users are identical and therefore, all users experience the same power gain. In a second step, all powers are rescaled to meet the sum power constraint.

D. The Complete Algorithm

In order to find balanced MSE levels, we alternately iterate the sum-MSE minimization for fixed power allocations in Section IV-B and the power updating rule in Section IV-C, until all MSEs are balanced. The iterative part in Section IV-B need not run to completion, a single iteration suffices because of the fast convergence. Both parallel and serial connection can be implemented.

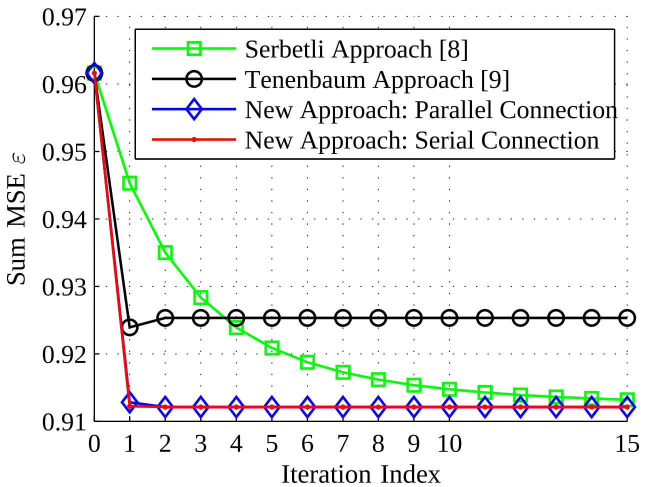

Fig. 1. Sum MSE versus iteration number for different unweighted sum-MSE minimization algorithms with $K = 4$ users, $N = 8$ receive antennas, and $r = 2$ transmit antennas per user. $\sigma_\eta^2 = 1$ and $\boldsymbol{p} = [5, 10, 20, 40]^{\mathrm{T}}$.

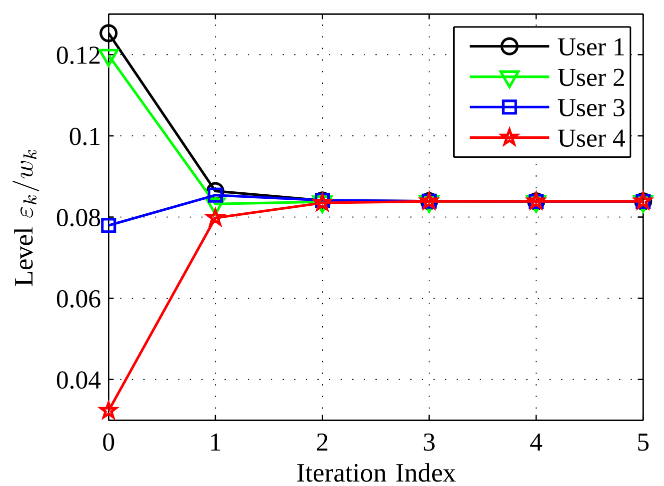

Fig. 2. Speed of convergence for the balancing algorithm with $K = 4$ users, $N = 8$ receive antennas, and $r = 2$ transmit antennas per user. $\sigma_\eta^2 = 1$, $P_{\mathrm{Tx}} = 75$, and $\boldsymbol{w} = [1, 2, 3, 4]^{\mathrm{T}}$.

V. Simulation Results

Different algorithms for the unweighted sum-MSE minimization with individual power constraints are compared in Fig. 1, where the sum-MSE ε is plotted versus the number of iterations in a $K = 4$ users scenario with $N = 8$ antennas at the base station and a power allocation vector $\boldsymbol{p} = [5, 10, 20, 40]^{\mathrm{T}}$. Every user is equipped with $r = 2$ antennas. The square marker curve corresponds to the fixed point approach of Serbetli et al. in [8], which has a rather slow convergence to the global optimum. This originates from the circumstance that no minimization mapping is applied during the iterations and instead of the covariances, the precoders are optimized. The single-user approach of Tenenbaum et al. in [9] (circle marker) features a quick convergence. However, it is suboptimum as the transmit covariance matrix of every user is optimized treating the link between the respective user and the base station as a single-user point-to-point system with colored noise due to interference. In contrast to rate applications, this procedure is suboptimum when applied to MSE minimization. Our new covariance based algorithm presented in Section IV-B features excellent convergence properties: Only a single iteration is required to reach the global optimum when the alternating optimization uses serial connection (point marker). Even more surprisingly, the parallel connection (rhomb marker) also exhibits the extremely fast convergence. Its performance per iteration almost coincides with the one of the serial connection but the complexity per iteration is much lower for the parallel connection.

When the unweighted sum-MSE minimization algorithm from Section IV-B is combined with the power updating rule from Section IV-C, we obtain the complete balancing algorithm. Its performance is displayed in Fig. 2. Again, $K = 4$ two-antenna receivers are served by a base station with $N = 8$ antennas. Given the same total power $P_{\mathrm{Tx}} = 75$ as in Fig. 1, the individual levels ε_k/w_k are plotted over the iteration index for a weight vector $\boldsymbol{w} = [1, 2, 3, 4]^{\mathrm{T}}$. After three iterations, a common balancing level $\alpha \approx 0.834$ has been reached for all users. In every iteration, an unweighted sum-MSE minimization as in Fig. 1 is executed in combination with the power adaptation from Section IV-C.

Fig. 3 shows the boundary of the MSE region obtained by the weighted sum-MSE minimization algorithm (solid line) by sweeping different weights and the boundary obtained from the unweighted sum-MSE minimization with different individual power constraints (dashed curve). We observe that both curves almost merge, so we have to face only a slight performance degradation when we restrict the covariances to result from the unweighted sum-MSE minimization. The AO approach of Shi et al. [4] is dotted and performs slightly worse when being close to the origin and slightly better for small MSEs of user one.

While the MSE region in Fig. 3 is convex, this need not be the case for all channel realizations and transmit powers. A counter-example is depicted in Fig. 4, where a part of the MSE region entailing a nonconvex dent is shown. MSE pairs that can be achieved by means of the weighted sum-MSE region are represented by the discontinuous curve. Due to the nonconvexity, there is a gap between $\varepsilon_1 \approx 0.53$ and $\varepsilon_1 \approx 1.12$, i.e., the weighted sum-MSE minimization cannot return a point inside this interval. The true MSE region has been obtained by evaluating the MSE expressions (2) of all users for an exhaustive number of random covariance matrices in conjunction with setting accumulation points inside the nonconvex gap in order to verify that it is not possible to find MSE pairs lying on the convex hull that is plotted by the dashed curve. Utilizing the presented balancing algorithm yields the curve with the circle marker. In fact, the nonconvex gap is closed and MSEs inside the gap can be reached that are infeasible for the weighted sum-MSE minimization algorithm. We observe a perfect match between the true MSE region

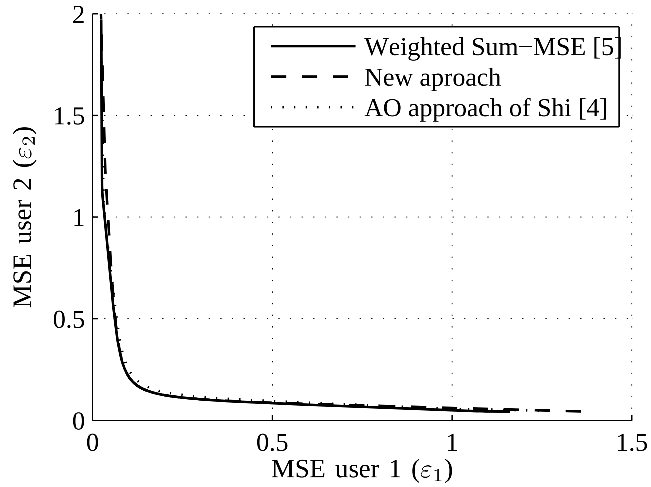

Fig. 3. MSE region obtained from weighted sum-MSE minimization and unweighted sum-MSE minimization with individual power constraints for $K = 2$ users, $N = 4$ receive antennas, and $r = 2$ transmit antennas per user. $P_{\mathrm{Tx}} = 75$ and $\sigma_\eta^2 = 1$.

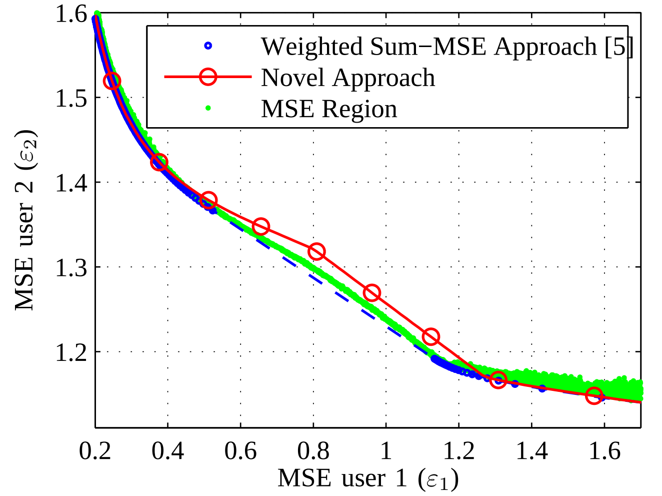

Fig. 4. Points of MSE region that can be obtained by the weighted sum-MSE minimization (dark points) and their convex hull (dashed line). $K = 2$ users, $N = 3$ receive antennas, $r = 2$ transmit antennas per users, $P_{\mathrm{Tx}} = 10$, and $\sigma_\eta^2 = 1$.

and the one obtained by the presented balancing algorithm for MSEs outside the nonconvex gap. At the price of a slight performance degradation, MSE pairs with $0.53 \leq \varepsilon_1 \leq 1.12$ can be achieved.

VI. Conclusion

In this paper we introduced a novel concept for the MSE balancing problem which does not utilize the weighted sum-MSE approach to obtain points lying on the border of the MSE region. Instead, adaptively regulated individual power constraints are used to control the MSE tuple that results from the *unweighted* sum-MSE minimization problem. For latter optimization, we came up with a quickly converging algorithm for the case where the individual users have up to two receiving antennas. This algorithm outperforms all hitherto existing ones in terms of speed of convergence and closes the nonconvex gap the weighted sum-MSE approach inherently suffers from. In addition, a counter-example falsified the common assumption that the two-user MSE region is convex in case of multi-antenna receivers.

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