OPPORTUNISTIC EIGENBEAMFORMING: EXPLOITING MULTIUSER DIVERSITY AND CHANNEL CORRELATIONS

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ABSTRACT

Multiuser diversity is an inherent form of diversity present in any time-varying system with several users. An opportunistic scheduler has to be used in order to exploit this type of diversity. With multiple antennas at the transmitter, opportunistic beamforming increases the dynamic range of the effective channel in spatially correlated scenarios. Moreover, multiuser diversity can also be combined with other transmit schemes that have proven to be effective in correlated channels, such as eigenbeamforming. We refer to the joint use of eigenbeamforming with an opportunistic scheduler as *opportunistic eigenbeamforming*. In this work we show that the available multiuser diversity with opportunistic eigenbeamforming is larger than the one achieved when opportunistic beamforming is employed using the proportional fair scheduler under different degrees of correlation in the channel. In the present work, we have considered a single cell scenario.

1. INTRODUCTION

In third generation wireless systems such as WCDMA, the ever increasing demand for high data rate in the downlink has been addressed by including a high-speed shared channel through the *High Speed Downlink Packet Access* (HSDPA) [1]. In such multiuser systems, the spectral efficiency is improved by exploiting *multiuser diversity* [2]. Multiuser diversity is inherent in the downlink of a system, which actually represents a point-to-multi-point link. However, in order to exploit multiuser diversity feedback of the *signal to noise ratio* (SNR) or partial *channel state information* (CSI) from each user is required. Furthermore, an opportunistic scheduler, such as the *proportional fair scheduler* (PFS) [3], that takes into account the partial CSI, is required at the transmitter in order to serve the users.

Among others factors, the degree of multiuser diversity depends on the dynamic range of the channel fluctuations. An approach for the downlink that increases the dynamic range with the use of multiple antennas at the transmitter is called *opportunistic beamforming* [4].

It has been shown in [5] that combining transmit diversity schemes, traditionally designed for point-to-point links, with an opportunistic scheduler under partial CSI feedback reduces the degree of available multiuser diversity compared to a system with no point-to-point link diversity at all. However, as it has been stated in [6], proper use of spatial diversity does not really reduce the available multiuser diversity. Moreover, when high mobility is present among the users, multiuser diversity suffers due to the use of outdated feedback in the opportunistic scheduler [7]. The previous results motivate us to consider combining point-to-point link transmitting schemes with an opportunistic scheduler in a point-to-multi-point link in order to exploit multiuser diversity.

Opportunistic beamforming produces gain in several scenarios but it has been shown that this scheme achieves a higher gain in correlated channels [4]. However, there is a point-to-point link scheme termed eigenbeamforming [8, 9] that has proven to be effective in correlated channels as well. Furthermore, in [10] it was shown how eigenbeamforming outperforms opportunistic beamforming in correlated channels for different degrees of spatial correlation. In this work, we investigate how eigenbeamforming combined with multiuser diversity can exploit not only spatial correlations in a channel but also the correlation that exists between time slots. We refer to the scheme that uses eigenbeamforming to exploit multiuser diversity as *opportunistic eigenbeamforming*. In the work at hand, it is shown not only that opportunistic eigenbeamforming is able to make better use of the spatial correlations but that it is also more robust to outdated feedback. We focus on the downlink of a multiuser system, i.e. a point-to-multi-point link.

In Section 2, an overview of the proportional fair scheduler is presented. Section 3 describes the channel model that will be utilized in this work. The concept of opportunistic beamforming is discussed in Section 4. Meanwhile, Section 5 defines the opportunistic eigenbeamforming approach by explaining how it can be combined with multiuser diversity. The results and analysis of our work are given in Section 6. Finally, the conclusions of this papers are presented in Section 7.

2. PRELIMINARIES

In order to exploit multiuser diversity in the downlink of a system two requirements are needed. On the one hand, each user must be able to track and estimate his channel magnitude through a common downlink pilot and then feed back its partial CSI to the base station. On the other hand, with this partial CSI, the base station must have the ability to schedule transmission among the users as well as to adapt the data rate to the fed back partial CSI. Nevertheless, the above mentioned requirements are present in the designs of many third generation systems such as IS-856 [13].

Multiuser diversity can only be exploited through the use of an opportunistic scheduler, for which we will consider the *proportional fair scheduler* [3] (PFS). Let us define the supported data rate for user k at time slot n as $R_k[n]$. When the PFS is employed, the base station transmits to the user with the largest current supported data rate compared to its own average rate, i.e. the user k^*

$$k^*[n] = \underset{k}{\text{argmax}} \frac{R_k[n]}{T_k[n]}, \tag{1}$$

where $T_k[n]$ is the average throughput of user k at time slot n. Through this scheduling principle, the statistically weaker users will not suffer at the expense of the stronger user as they do not have to wait to have the best channel or largest supported data rate $R_k[n]$ to be served. In this sense, the user with the best *relative channel* is served. Moreover, the average throughput T_k is updated as follows:

$$T_k[t+1] = \begin{cases} (1 - \frac{1}{t_c})T_k[n] + \frac{1}{t_c}R_k[n] & k = k^*[n], \\ (1 - \frac{1}{t_c})T_k[n] & k \neq k^*[n], \end{cases} \tag{2}$$

where $k^*[n]$ refers to the user served in time slot n and t_c is a time constant.

The proportional fair scheduler can be tuned to achieve different fairness and delay performances. To this end, let us define the forgetting factor f as the inverse of the time constant t_c ($f = \frac{1}{t_c}$). Then, the forgetting factor ranges from 0 to 1 and it represents the percentage of how much weight the served data rate $R_{k^*}[n]$ for time slot n has on the average throughput $T_{k^*}[n]$ for user $k^*[n]$. The PFS achieves the best delay performance when the forgetting factor approaches 1. In this case, the PFS approaches the *round robin scheduler* and no multiuser diversity can be exploited with this setting. Meanwhile, when the forgetting factor in the PFS approaches 0, the PFS now approaches the the *greedy scheduler* (GS), thus achieving the maximum multiuser diversity of the system but at the expense of increased delay on the weaker users. Hence, the degree of multiuser diversity that can be exploited from the system can be tuned with the forgetting factor f in the PFS.

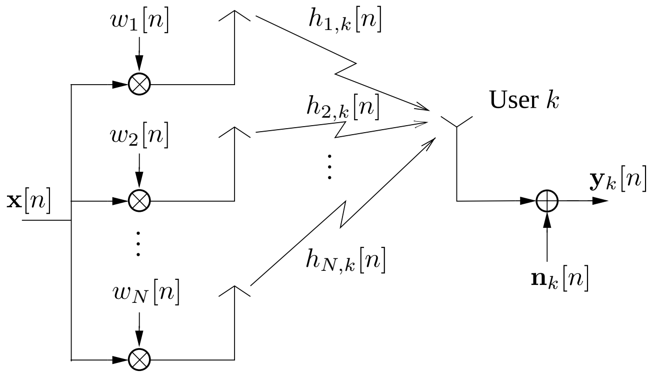

Fig. 1. MISO Channel Model for user k

3. CHANNEL MODEL AND CORRELATIONS

Let us now introduce the channel model that will be employed. We will consider a flat fading downlink of a multiuser system with K users, i.e. a point-to-multi-point link. The base station has a *uniform linear array* (ULA) with N identical transmit antennas while the receiver at each user has only one antenna, thus we have a *multiuser mutiple-input single-output* (MU-MISO) system as shown in Fig. 1 for user k. Let us define $\mathbf{x}[n] \in \mathbb{C}^P$ as the vector of P transmitted symbol for time slot n, $h_{m,k}[n] \in \mathbb{C}$ as the complex channel gain from antenna m to the kth user for time slot n, $\mathbf{n}_k[n] \in \mathbb{C}^P$ as the additive white noise at the receiver k for time slot n, and $\mathbf{y}_k[n] \in \mathbb{C}^P$ as the received signal at user k for time slot n. In our model, we assume that $h_{n,k}[n]$ are complex Gaussian distributed random variables with unit variance, i.e. we assume Rayleigh fading.

Furthermore, we assume that each channel $h_{m,k}[n]$ is composed of B unresolvable subpaths. We suppose that the directions of departure of each of the B subpaths for each user are distributed over a given angle spread δ with a certain mean angle of departure θ_k per user k. This mean angle θ_k per user is taken to be uniformly distributed over $[0, 2\pi]$. Furthermore, a far field assumption is made so that the narrow band signals delay caused by the geometry of ULA can be expressed as a phase shift. Therefore, the mth element of the steering vector of the antenna array is given by $\mathrm{e}^{-\mathrm{j}(m-1)2\pi d \sin\psi_{k,b}}$, where d and $\psi_{k,b}$ are the distance between antennas given in wavelengths of the signal and the angle of departure of the bth subpath of the kth user, respectively. We denote the channel vector for user k as $\mathbf{h}_k[n] = [h_{1,k}[n], h_{1,k}[n], \ldots, h_{N,k}[n]]^{\mathrm{T}}$, where $(\bullet)^{\mathrm{T}}$ represents the transpose operator. Then, assuming a distance between antennas of $d = 1/2$ and based on the geometry of the ULA we can model the channel vector $\mathbf{h}_k[n]$, for user k as follows:

$$\mathbf{h}_k[n] = \mathbf{A}_{\mathrm{Tx},k} \cdot \boldsymbol{\phi}_k[n], \tag{3}$$

where $\boldsymbol{\phi}_k[n] \in \mathbb{C}^B$ whose elements are zero mean independent complex Gaussian random variables with variance equal to $1/B$ in order to have $\mathrm{E}\left\{|h_{m,k}|^2\right\} = 1$. Furthermore, we have that $\mathbf{A}_{\mathrm{Tx},k}$ is the transmit array steering matrix given by the Vandermonde matrix:

$$\mathbf{A}_{\mathrm{Tx},k} = \begin{pmatrix} 1 & \cdots & 1 \\ \mathrm{e}^{-\mathrm{j}\pi \sin\theta_{k,1}} & \cdots & \mathrm{e}^{-\mathrm{j}\pi \sin\theta_{k,B}} \\ \vdots & & \vdots \\ \mathrm{e}^{-\mathrm{j}\pi(M-1)\sin\theta_{k,1}} & \cdots & \mathrm{e}^{-\mathrm{j}\pi(M-1)\sin\theta_{k,B}} \end{pmatrix}, \tag{4}$$

where $\mathbf{A}_{\mathrm{Tx},k} \in \mathbb{C}^{N\times B}$.

If $\mathbf{h}_k$ is generated as shown in (3), then the resulting elements of $h_{m,k}[n]$ are still complex Gaussian random variables with zero mean and unit variance. Therefore, the $h_{m,k}$, $m = 1,\dots,N$, are Rayleigh distributed with unit variance and some correlations among them for each user k.

Moreover, we have that the spatial transmit correlation matrix of the channel vector of each user k is given by:

$$\mathbf{C}_k = \mathrm{E}\left\{\mathbf{h}_k \cdot \mathbf{h}_k^{\mathrm{H}}\right\} = \frac{1}{B}\cdot \mathbf{A}_{\mathrm{Tx},k}\mathbf{A}_{\mathrm{Tx},k}^{\mathrm{H}} \in \mathbb{C}^{N\times N}, \tag{5}$$

where $(\bullet)^{\mathrm{H}}$ denotes the conjugate transpose or Hermitian operator $(\bullet)^{*,\mathrm{T}}$. This spatial correlation matrix $\mathbf{C}_k$ depends especially on the angle spread δ of the path to user k among where the B unresolvable paths are located. For a small angle spread δ ($\delta \approx \sin\delta$) and with a large number of scatterers located on a ring around each user terminal, the spatial correlation between antennas m and p, i.e. the m,p element $\mathrm{E}\left\{h_{m,k}h_{p,k}^*\right\}$ of the matrix $\mathbf{C}_k$, can then be approximated by [11]:

$$\mathrm{J}_0\left(2\pi(p-m)\,d\,\delta\cos(\theta_k)\right)\,\mathrm{e}^{-\mathrm{j}2\pi(p-m)d\sin(\theta_k)}, \tag{6}$$

where $\mathrm{J}_0(\bullet)$ denotes the Bessel function of the first kind of order zero.

Furthermore, we assume the channel to have a temporally correlated block fading, which means that $h_{m,k}[n]$ remains constant for time slot n. As for the temporal correlation, we assume a Jakes power density spectrum, which results in a temporal auto-correlation function of $h_{m,k}[n]$ for antenna m, $m = 1,\dots,N$, and user k that reads as follows [14]:

$$\mathrm{E}\left\{h_{m,k}[n]\cdot h_{m,k}^*[n+\Delta t]\right\} = \mathrm{J}_0(2\pi f_{\mathrm{n}}\Delta t). \tag{7}$$

Here, f_{n} and Δt denote the normalized Doppler frequency, and the difference in time slots, respectively. The normalized Doppler frequency is given by $f_{\mathrm{n}} = \frac{f_{\mathrm{carrier}}\cdot v}{f_{\mathrm{slot}}\cdot \mathrm{c}}\cos\beta$, where f_{carrier}, v, f_{slot}, c, and β are the carrier frequency, the speed of the user, the frequency of the slots, the speed of light, and the angle between the direction of the user and the path to the antenna m, respectively. We assume that $\beta = 0$ for every k.

The multiple antennas at the base station shown in Fig. 1 will be used for beamforming rather than transmit diversity. In this case, the corresponding MISO system for each user can be described by an equivalent SISO system. However, when considering the rest of the users we now have a multi-point-to-point link. Let us denote the beamforming vector applied at the base station, as shown in Fig. 1, as $\mathbf{w}[n] = [w_1[n], w_2[n],\dots,w_N[n]]^{\mathrm{T}} \in \mathbb{C}^N$, where $|w_m[n]| \in [0,1]$ and $\arg(w_m[n]) \in [0,2\pi]$, for $m = 1,\dots,N$, are the power allocation and phase allocation on each antenna m, respectively. In order to preserve the transmit power, we must satisfy $\sum_{m=1}^{N}|w_m[n]|^2 = 1$, i.e. the vector $\mathbf{w}[n]$ has unit norm. Therefore, we then have that the received signal $\mathbf{y}_k[n]$ for user k, shown in Fig. 1, reads as follows:

$$\mathbf{y}_k[n] = \mathbf{w}^{\mathrm{T}}[n]\cdot\mathbf{h}_k[n]\cdot\mathbf{x}[n] + \mathbf{n}_k[n] \tag{8}$$

$$= h_k[n]\cdot\mathbf{x}[n] + \mathbf{n}_k[n], \tag{9}$$

where $h_k[n] = \mathbf{w}^{\mathrm{T}}[n]\cdot\mathbf{h}_k[n]$ is the equivalent channel seen by user k.

4. OPPORTUNISTIC BEAMFORMING

When applying *opportunistic beamforming* (OB) in correlated channels, the dynamic range of the resulting equivalent channel $h_k[n]$ is larger than that of the original channels $h_{m,k}[n]$, $m = 1,\dots,N$, for each user k. Let us denote the random unit norm vector that is applied at the base station as $\mathbf{w}_{\mathrm{ob}}[n]$. Just as in the case of a single antenna at the base station, the users must track their equivalent channel $h_k[n]$ and feed back to the base station their received $|h_k[n]|^2$ or their supported data rate $R_k[n]$ resulting from the beamforming vector $\mathbf{w}_{\mathrm{ob}}[n]$. Afterwards, the base station decides which user to transmit to based on the scheduling policy. If the PFS is used, the base station transmits to the best relative user applying $\mathbf{w}_{\mathrm{ob}}[n]$ at the transmit antennas. For opportunistic beamforming to be effectively employed in a correlated channel, the random beams $\mathbf{w}_{\mathrm{ob}}[n]$ must match the distribution of the channel.

Since the magnitude and phase of each of $h_{m,k}$ are independent, then the magnitude and phase of the beamforming vector $\mathbf{w}_{\mathrm{ob}}[n]$ can be generated separately. Let us consider the correlated channel model described in Section 3 with the $h_{m,k}$, $m = 1,\dots,N$, being Rayleigh distributed. Then, in order to match the distribution of the channel one could generate the magnitudes $|w_{\mathrm{ob},n}|$, $m = 1,\dots,N$ of the vector $\mathbf{w}_{\mathrm{ob}}[n]$ by taking the magnitudes of the elements of an isotropically distributed vector. Nevertheless, we still require a distribution of the angles $\theta_m = \arg(|w_{\mathrm{ob},n}|)$ of the elements of $\mathbf{w}_{\mathrm{ob}}[n]$.

Taking a look at the approximation given in (6) of the elements of $\mathbf{C}_k$ for small angle spread, it can be seen that the phase of the elements in each of the columns of $\mathbf{C}_k$ are multiples of the term $-2\pi d\sin\theta_k$, which is the same per column. This constant phase shift per column is a result of the geometry of the ULA. Considering the spatial correlations one needs only to transmit over the strongest beam to user k. Therefore, only one angle of departure $\theta[n]$ is required, to transmit over one beam to each user, instead of N independent angles [4, 12]. Assuming that the distance between adjacent antennas given in wavelengths is $d = \frac{1}{2}$, then the allocated phase $\theta_m[n]$ would be given by:

$$\theta_m[n] = (m-1)\pi \sin(\theta[n]), \tag{10}$$

for each antenna m, $m = 1, \ldots, N$. Notice that assuming that the angle of depature $\theta[n]$ is uniformly distributed over $[0, 2\pi]$ does not lead to a uniform distribution of the angle $\theta_m[n]$, for $m = 1, \ldots, N$. The fact that only one angle needs to be varied can explain why opportunistic beamforming performs better under correlated fading. In uncorrelated channel, opportunistic beamforming needs to select appropiately N angles $\theta_m[n]$ in order to coherently beamform a user. However, in a correlated channel it is easier to achieve the maximum rate through coherent beamforming since only one angle instead of N needs to be selected appropriately. The angle spread δ used in this paper will still be considered small as $\delta \approx \sin\delta$. Therefore, the random beams applied by opportunistic beamforming will have the structure mentioned above in (10).

5. OPPORTUNISTIC EIGENBEAMFORMING

A transmitting scheme that efficiently makes use of the fading correlations in point-to-point links is eigenbeamforming [8, 9]. Eigenbeamforming takes advantage of the spatial correlations present at the base station by tranmitting over the strongest beam to a given user. To this end, eigenbeamforming requires partial CSI at the transmitter, which in this case refers to the principal eigenvector of the spatial correlation matrix $\mathbf{C}_k$ of the channel for each user k. However, the receiving user can not exactly calculate $\mathbf{C}_k$ given by (5) and instead a *long-term* correlation matrix $\mathbf{C}_{\text{LT},k}$ is used as an estimate. How this long-term correlation matrix is estimated will be described later. Let us then denote the sorted eigenvalue decomposition of the correlation matrix $\mathbf{C}_{\text{LT},k}$ as follows:

$$\mathbf{C}_{\text{LT},k} = \mathbf{V}_k \mathbf{\Lambda}_k \mathbf{V}_k^{\text{H}} = \sum_{i=1}^{N} \lambda_{i,k} \mathbf{v}_{i,k} \mathbf{v}_{i,k}^{\text{H}}, \tag{11}$$

where $\mathbf{v}_{1,k}$ is the principal eigenvector of $\mathbf{C}_k$, i.e. the eigenvector belonging to the largest eigenvalue $\lambda_{1,k}$ of $\mathbf{C}_{\text{LT},k}$. Under eigenbeamforming, the beam vector $\mathbf{w}_{\text{eb},k}[n]$ applied at the transmitting base station for user k would then be $\mathbf{w}_{\text{eb},k}[n] = \mathbf{v}_{1,k}^*$. Contrary to opportunistic beamforming, in opportunistic eigenbeamforming there is a beamforming vector for every user, since each user has his own distinct principal eigenvector. By applying this power and phase allocation at the base station, the data for user k is transmitted over the strongest beam available in the channel to user k. This in average increases the throughput of the point-to-point link under the correlations present in the channel [8, 9].

In [10], it was shown how eigenbeamforming can be combined with multiuser diversity. We refer to this combination as *opportunistic eigenbeamforming* (OEB). In opportunistic eigenbeamforming the users must feed back their principal eigenvector to the base station. This can be done over several time slots with a given feedback rate. For the users to calculate this principal eigenvector, they first require to track and estimate their channels $h_{m,k}[n]$, $m = 1, \ldots, N$, for user k. To this end, the base station must send separate pilot signals on each antenna m for $m = 1, \ldots, N$. Once the receiving users have estimated their channels they proceed to calculate a *short-term* correlation matrix $\mathbf{C}_{\text{ST},k}$ with the current channel conditions:

$$\mathbf{C}_{\text{ST},k}[n] = \mathbf{h}_k[n] \cdot \mathbf{h}_k^{\text{H}}[n], \tag{12}$$

for each user k. This short-term correlation matrix is used to update the long-term correlation matrix $\mathbf{C}_{\text{LT},k}$ at time slot T as follows:

$$\mathbf{C}_{\text{LT},k}[n] = \frac{1}{T} \cdot \sum_{t=1}^{T} \mathbf{C}_{\text{ST},k}[n]. \tag{13}$$

Let us now assume that the base station has the principal eigenvector $\mathbf{v}_{1,k}$ for each user k. When combining eigenbeamforming with multiuser diversity the base station must decide to which user to transmit based on some fed back partial CSI. Even though, that for opportunistic eigenbeamforming the individual links $h_{m,k}$, for $m = 1, \ldots, N$, are required for updating $\mathbf{C}_{\text{LT}}$, a good estimate of the individual links is not required at each time slot for choosing the best user. At each time slot each user must feedback what their equivalent channel h_k from (9) would be, if they were served by transmitting over their strongest beam with the beamforming vector $\mathbf{w}_{\text{eb},k}[n] = \mathbf{v}_{1,k}^*$ applied at the base station. Based on the Karhunen-Loève expansion we can write the channel vector of user k as follows:

$$\mathbf{h}_k = \sum_{i=1}^{N} \xi_{i,k} \cdot \mathbf{v}_{i,k}, \tag{14}$$

where $\xi_{i,k}$, $i = 1, \ldots, N$ are complex Gaussian random variables with variance $\lambda_{i,k}$. If the beamforming vector $\mathbf{w}_{\text{eb},k}[n] = \mathbf{v}_{1,k}^*$ is applied at the transmitter then the equivalent channel is given from (14) as:

$$h_k = \mathbf{w}_{\text{eb},k}^{\text{T}} \cdot \mathbf{h}_k = \mathbf{v}_{1,k}^{\text{H}} \cdot \mathbf{h}_k = \xi_{1,k}. \tag{15}$$

In order to determine h_k, the receivers do not need to measure the individual links $h_{m,k}$, for $m = 1, \ldots, N$. Instead, they just need to measure $\xi_{1,k}$ which represents the equivalent channel h_k seen by user k when applying $\mathbf{w}_{\text{eb},k}[n]$ at the base station. The equivalent channel h_k is still just one complex number as in the case of opportunistic beamforming. Moreover, the users feed back the magnitude of h_k or the supported data rate $R_k[n]$, described in Section 2, for this channel h_k. Upon reception of all the supported data rates from all the users, the base station decides to which user to transmit by employing an opportunistic scheduler. In case the proportional fair scheduler is employed, the base station transmits to the best relative user.

Therefore, in opportunistic eigenbeamforming the channel is tracked, through the aid of the pilot signals transmitted from the base station, for two purposes. On the one hand, these pilots are used to estimate the channels $h_{m,k}[n]$, for $m = 1, \ldots, N$. These individual links are required by the eigenbeamforming scheme in order to calculate the short-term correlation matrix which is then used to update the long-term correlation matrix from where their current principal eigenvector for each user is estimated. On the other hand, the channel is also tracked in order to estimate the equivalent channel $h_k = \xi_{1,k}$ for each user under the assumption that the base station transmits over their strongest current beam.

6. COMPARISON: OB VRS OEB

6.1. Simulation Setup

To evaluate the performance of opportunistic beamforming and opportunistic eigenbeamforming in correlated channels with outdated feedback, let us consider the downlink of a single cell with a base station with a ULA constituted of $N = 4$ transmit antennas with a distance $d = \frac{1}{2}$ wavelengths between antennas and with only one antenna at each receiver. Thus, each point-to-point link constitutes a MISO system as depicted in Fig 1. Furthermore, we have the overall downlink system represented as a point-to-multi-point link where we assume there are a maximum of $K = 64$ users with the same normalized Doppler frequency f_{n} and angle spread δ. The carrier frecuency is $f_{\text{c}} = 2$ GHz. We assume that the channels $h_{m,k}$, for $m =, 1, \ldots, N$ for user k are block correlated Rayleigh flat fading with unit variance as described in Section 3. Moreover, the average SNR at the receiver is 0 dB and there are 1500 time slots transmitted per second.

The effect of the outdated feedback is represented as follows. We consider the existence of a training phase at time slot n where the magnitude of the equivalent channel $h_k[n]$ given by (9) or (15) is measured by user k for opportunistic beamforming and opportunistic eigenbeamforming, respectively. The users are served through the proportional fair scheduler with different forgetting factors f. We assume no processing delay and consider that the feedback required to exploit the multiuser diversity by the PFS is fed back during time slot $n + 1$, while the actual transmission to the best relative user is done in time slot $n + 2$. Therefore, the equivalent channel $h_k[n]$ that is measured is based on the $h_{m,k}[n]$, while the actual channels when the selected user k is served are $h_{m,k}[n+2]$, for $m = 1, \ldots, N$. Hence, the resulting *loop delay* is 2 slots.

In addition, the correlation matrix among the transmit antennas is given by (5) but we use the approximation that of each of the elements of this matrix is given by (6). This approximation is valid since we consider small angle spreads such that $\delta \approx \sin \delta$, then the random beam used for opportunistic beamforming will be directed only over one beam by randomly varying a single angle θ as explained at the end of Section 4. Moreover, the auto-correlation among the time slots is given by a Jakes model described in (7).

Furthermore, when considering opportunistic eigenbeamforming we assume that the long-term correlation matrix $\mathbf{C}_{\text{LT},k}$ has been estimated over a large number of time slots T as given by (13). In addition, we assume that the base station has available the principal eigenvector $\mathbf{v}_{1,k}$ of the long-term correlation matrix for each user $k = 1, \ldots, K$. This is done through some feedback depending on how fast the channel changes. However, if we assume that $\mathbf{C}_{\text{LT},k}[n] = \mathbf{C}_{\text{ST},k}[n]$ at each time slot n and that the users can feedback their principal eigenvector at each time slot, then the base station has available instantaneous channel state information. If this is the case the base station can perform coherent beamforming to the best relative user. With such a theoretic case the maximum rate can be achieved and it serves as an upper bound for opportunistic eigenbeamforming. We will refer to this scheme in the following as *opportunistic coherent beamforming* (OCB).

To depict the corresponding delay performance for different degrees of multiuser diversity achieved through distinct forgetting factors in the proportional fair scheduler, let us define the *outage delay* D_{out} which is related to a probability p_{out} as follows:

$$\text{Prob}\{D < D_{\text{out}}\} = 1 - p_{\text{out}}, \tag{16}$$

where p_{out} is the *outage probability* that a given delay D is larger than D_{out}. The delay D is given in number of time slots. In the simulation we set $p_{\text{out}} = 2\%$. Each forgetting factor in the PFS corresponds to a certain delay performance represented through the outage delay D_{out}.

Regarding the degrees of correlation in the channel, we will consider angle spreads up to $40°$. As for the normalized Doppler frequency the maximum speed treated is 80 km per hour.

6.2. Analysis and Results

In the following, the figure of merit that we will consider is the average sum throughput of the system. Furthermore, we assume that the supported data rate or throughput for user k is given by the Shannon equation $R_k[n] = \log_2(1 + \text{SNR}[n])$, where $\text{SNR}[n] = |h_k[n]|^2/\sigma_k^2$ with σ_k^2 as the variance of the noise at the receiving user k for which we have assumed is equal to unity for every user. In order to observe the gain in multiuser diversity with increasing number of users, we have plotted in Fig. 2 the average sum throughput as a function of the number of users for the three opportunstic schemes detailed in the previous sections: opportunistic beamforming (OB), opportunistic eigenbeamforming (OEB) and opportunistic coherent beamforming (OCB). These results correspond to a speed of 35 kmph with several angle spreads. In addition, the users are served through the PFS with a forgetting factor of 0.001.

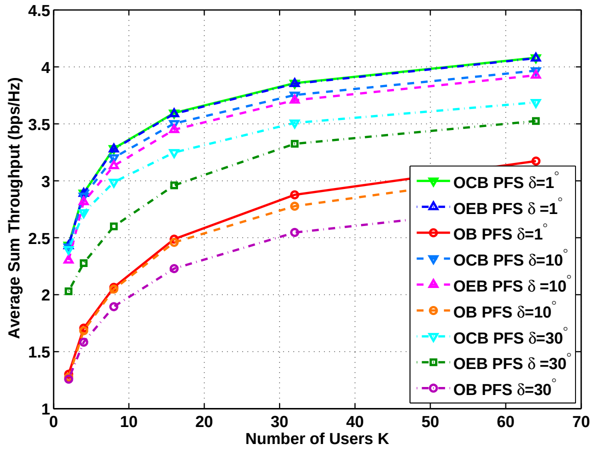

Fig. 2. Multiuser Diversity Gain

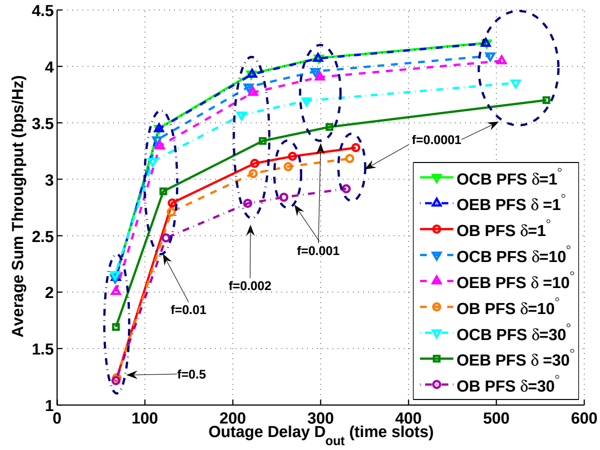

Fig. 4. Multiuser Diversity Gain Tradeoff: Outage Delay

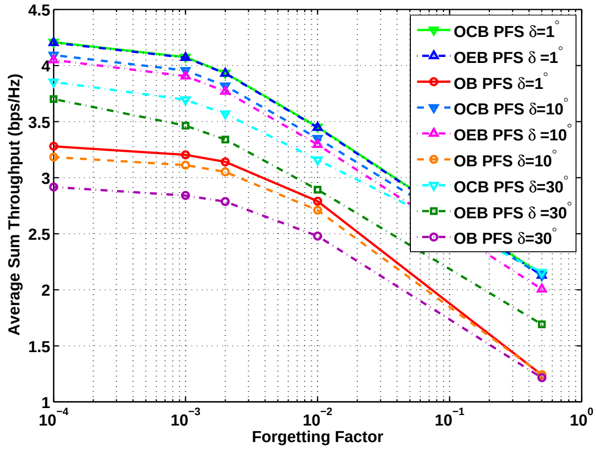

Fig. 3. Multiuser Diversity Gain Tradeoff: Forgetting Factors

To evaluate the peformance of the proportional fair scheduler under different forgetting factors, Fig. 3 depicts the average sum throughput for a set of $K = 64$ users as a function of the forgetting factor and Fig. 4 shows the average sum throughput but now as a function of the outage delay D_{out} with a outage probability set to $p_{\text{out}} = 2\%$. Every forgetting factor from Fig. 3 translates into an outage delay in Fig. 4. From Fig. 4, the tradeoff between multiuser diversity and delay can be observed.

Moreover, in each of the previous figures, Figs. 2–4, it can also be seen how opportunistic eigenbeamforming outperforms opportunistic beamforming for different degrees of correlation (different angle spreads) in the channel. As the angle spread decreases the degree of correlation increases and the performance of opportunistic eigenbeamforming basically matches the one of opportunistic coherent beamforming. For each case, the maximum possible achieved performance is obtained through opportunistic coherent beamforming and is represented as an upper bound on the average sum throughput. The opportunistic eigenbeamforming scheme still outperforms opportunistic beamforming also for different values of the forgetting factor. When the delay performance is considered, it can be seen that for a given outage delay, the average sum throughput achieved with opportunistic eigenbeamforming is higher than compared to opportunistic beamforming. These results agree with the ones presented in [10]. Nevertheless, we will now proceed to evaluate the impact of the temporal correlations in the channel and the effect of the outdated feedback on the proportional fair scheduler for different users' velocities under different degrees of correlation.

When different speeds for the users are taken into account, one must consider the effect of the outdated feedback, since the channel that was tracked is no longer the same at the moment a user is served. It might turn out that the selected user is no longer the best user. In Fig. 5, the effect of the outdated feedback can be observed for the different opportunistic schemes treated so far. The results presented in this figure correspond to angle spread $\delta = 1°$ and $\delta = 30°$. In addition, PFS 1 and PFS 2 refer to the proportional fair scheduler with a forgetting factor $f = 0.001$ and $f = 0.002$, respectively. For low speeds, the degree of multiuser diverstiy increases up to a maximum value as the speed of the users increases. This can be explained from the fact that there is a larger degree of multiuser diversity when the channel fluctuations are faster. When there is fast fading, the dynamic range of the channel fluctuations over the latency time scale t_c increases, thus increasing the available multiuser diversity. Notice also that this increase is relatively larger for OCB and OEB as compared to OB, since opportunistic beamforming is already inducing faster channel fluctuations through the use of the random beam at the transmitter. After reaching maximum sum

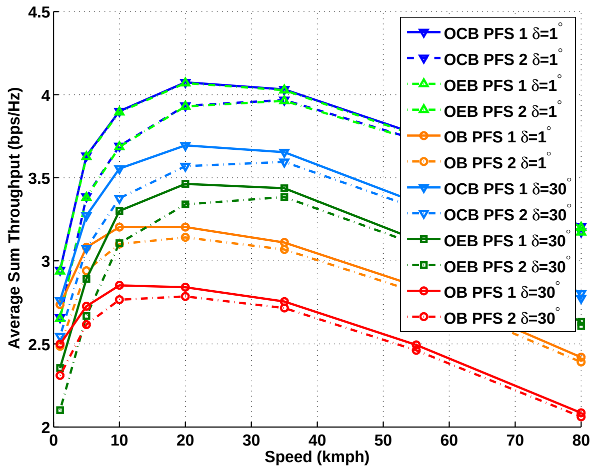

Fig. 5. Outdated Feedback: Average Sum Throughput vrs Speed of the Users

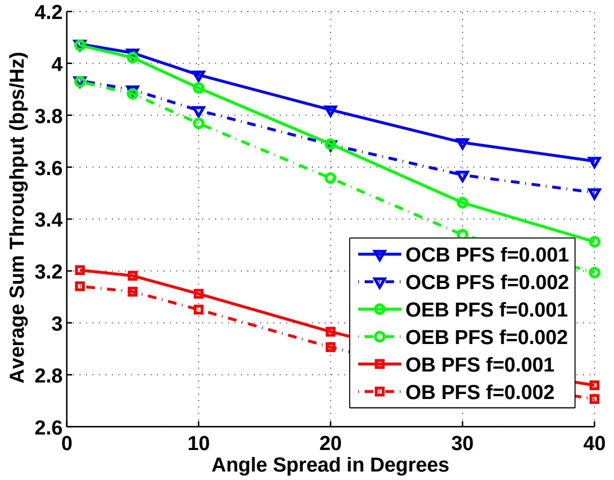

Fig. 6. Degree of Correlation: Average Sum Throughput vrs Angle Spread

throughput, the degree of multiuser diversity decreases as the speed of the users increases for all of the schemes since they suffer from the effect of the outdated feedback and in fact now it incurs in a loss. Moreover, we have that PFS 1 outperforms PFS 2 since PFS 1 has a smaller forgetting factor.

In order to evaluate the degree of multiuser diversity as a function of the degree of correlation, Fig. 6 depicts the average sum throughput as a function of the angle spread. These results correspond to a speed of 35 kmph with the PFS using two forgetting factors, $f = 0.001$ and $f = 0.002$. As the angle spread increases, the degree of correlation decreases and so the multiuser diversity available in the system. When there is a fully correlated channel, all the power of the channel is allocated over only one eigenmode of the channel. However, as the angle spread increases, i.e. the spatial correlation in the channel decreases, the condition of the spatial correlation matrix decreases since the power of the channel is distributed over all the eigenmodes. This means that the throughput achieved through coherent beamforming of a user with full correlation would be in average larger than the throughput achieved through coherent beamforming of a user with a less correlated channel. This would explain the decrease in performance as the angle spread increases for opportunistic eigenbeamforming, since the eigenvalue corresponding to the principal eigenvector is now smaller as compared to when the angle spread is smaller. In addition, we have that OB is outperformed by OEB because OB does not always transmit on the strongest eigenmode of the channel as OEB does. In the limit, when we have a fully uncorrelated channel, where the condition of the spatial correlation matrix of the channel is equal to 1, we would have that the performance of OB is the same as that of OEB.

Furthermore, one can also analyze the performance of opportunistic eigenbeamforming relative to opportunistic beamforming. To this end, let us define the following ratio:

$$\eta(\delta, K) = \frac{S_{\mathrm{OEB}}(\delta, K)}{S_{\mathrm{OB}}(\delta, K)} \tag{17}$$

where S_{OEB} and S_{OB} are the sum throughput achieved with the PFS ($f = 0.001$) for OEB and OB, respectively. This relative gain η is a function of the number of users, speed of the users and of the angle spread. For a speed of 35 kmph, Fig. 7 depicts this ratio as function of the angle spreads for different number of users. It can be seen from this figure that as the number of users increase the gain of OEB over OB decreases. This can be explained as follows. As the number of users increases the probability that the random beam generated by OB actually matches the complex conjugate of the eigenbeam of a certain user increases. In the limit, when $K \to \infty$, one can expect that the performance of OEB is the same as that of OB. The multiuser diversity gain is further reduced as the correlation available in the channel decreases, i.e. the angle spread increases.

7. CONCLUSION

Opportunistic schedulers exploit the multiuser diversity inherent in a multiuser system. Through the use of opportunistic beamforming the degree of multiuser diversity is increased in correlated channels. Nevertheless, an efficient transmit schemes for point-to-point correlated links can be employed to achieve an even greater gain. We have shown that combining eigenbeamforming with an opportunistic scheduler, such as the proportional fair scheduler, increases the degree of multiuser diversity. This concept, which we term opportunistic eigenbeamforming, not only outperforms opportunistic beamforming for different degrees of spatial correlations in a channel, but also at different speeds of the users. Opportunistic

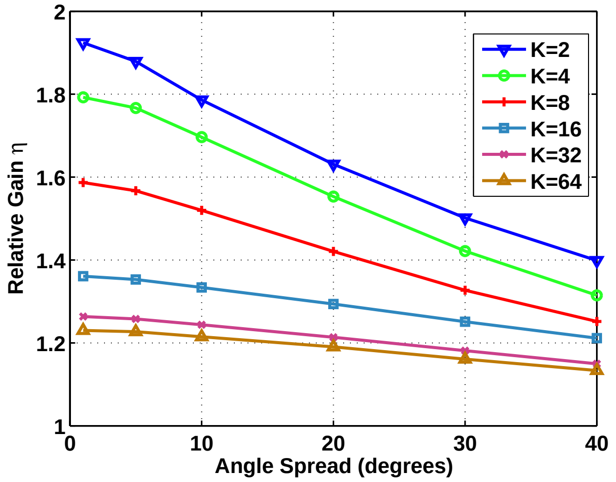

Fig. 7. Gain of OEB over OB: Different Number of Users and Angle Spreads

eigenbeamforming is more robust to outdated feedback that results from the speed of the users. The larger achievable sum throughput of opportunistic eigenbeamforming over opportunistic beamforming is a result of having more partial CSI of each user at the base station. This partial CSI corresponds to the largest eigenvector of each user which must be fed back from each user. However, the feedback of this eigenbeam is not comparable with the feedback required to exploit multiuser diversity in a TDMA system. This additional partial CSI can be fed back at a much slower rate than the SNR feedback required by an opportunistic scheduler to serve a user at each time slot.

In addition, the existing tradeoff between the multiuser diversity gain and the delay performance provided through different settings of the forgetting factor in the proportional fair scheduler was also shown. For all the forgetting factors and the corresponding values of the outage delays, opportunistic eigenbeamforming achieves a higher average sum throughput as compared to opporuntistic beamforming. Furthermore it was shown how opportunistic eigenbeamformer comes close to the upper bound of the average sum throughput, achieved through opportunistic coherent beamforming, when the angle spread is very small. As the angle spread increases the power of the channel is distributed over all the eigenmodes of the channel, thus decreasing the multiuser diversity gain that can be extracted with OCB, OEB, and OB. However, for any angle spread OEB still outperforms OB. Meanwhile, as the number of users increases and the angle spread increases, the perfromance of the opportunistic beamforming and opportunistic eigenbeamforming converge.

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