Pulse Shaping with Bireciprocal Wave Digital Lattice Filters

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***Abstract*— In this work we show that *bireciprocal wave digital lattice filters* (BWDLF) can be used very favourably as pulse shaping filters in digital communications. A new approach, at least in this context, is presented, how to cope with the phase distortions which are caused by the nonlinear phase response of BWDLFs. This approach is characterized by minimizing the energy of the intersymbol interference with a finite impulse response (FIR) filter instead of trying to equalize directly the nonlinear phase with an appropriate allpass filter.**

The simulation results demonstrate that the filter orders and required operations per time interval at both the transmitter and receiver can be reduced compared with conventional solutions. As a new option the effort can be split asymmetrically between transmitter and receiver.

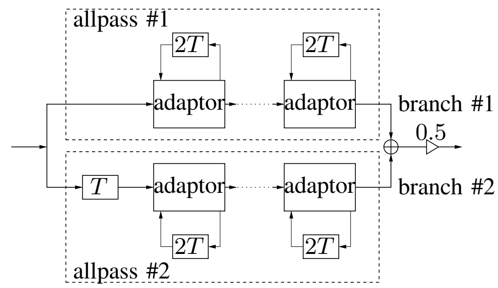

Fig. 1. Bireciprocal WDLF of degree N

I. Introduction

The transmission of digital information over band-limited channels necessitates the design of appropriate transmit signals. In order to accomplish the band-limitation as well as avoiding intersymbol interference in the case of flat communication channels, the first *Nyquist condition* has to be satisfied [1].

The so-called *raised cosine spectrum* (RC), for example, has this desirable spectral property. The raised cosine spectral characteristic can be approximated in a practical FIR filter design, where the receiver filter is matched to the transmitter filter. The result are two identical filters with a *root raised cosine spectrum* (RRC), used in the standardization of the 3G/UMTS radio access scheme.

Wave digital filters (WDF) as introduced by A. Fettweis [2] are derived from classical analog lossless two-ports. Therefore, they adopt very advantageous properties which lead to a small coefficient sensitivity in the passband, optimum scaling and stability under very general conditions. Those features allow an easy and efficient implementation in hardware or software. Moreover, as infinite impulse response (IIR) filters they offer smaller transition bandwidths than FIR filters of the same order.

Wave digital filters can be implemented as lattice filters as in Fig 1. If the wave digital lattice filters are designed with a *bireciprocal characteristic function* $C(\psi) = 1/C(1/\psi)$, where ψ is the complex frequency of the analog reference filter, further benefits are obtained making WDLFs attractive for pulse shaping applications.

The magnitude response of a cascade of two identical BWDLFs, which is the squared magnitude response of one BWDLF, is odd symmetric around $f_{\mathrm{s}}/4$, where f_{s} is the sampling frequency [3]. Therefore, the Nyquist criterion for the magnitude response is perfectly satisfied. Please note, that this holds true even independently of the applied coefficient truncation.

Moreover, the stopband attenuation is directly related to the passband attenuation. As the passband attenuation is very robust against fluctuations of the coefficients, the same is true for the stopband attenuation.

On the other hand — like all IIR filters — wave digital filters cannot show a linear phase response. In previous works and applications the nonlinear phase responses of IIR filters have frequently been equalized by appropriate allpass filters. Another way of removing the undesired impact of the nonlinear phase response of the wave digital lattice filters is proposed for the application of pulse shaping. The solution is oriented towards the idea that the phase response is not the proper criterion to be aimed at. Therefore, we minimize the main objective, the energy of the intersymbol interference, which results in a least squares solution for continuous operation.

In the following it is shown that bireciprocal wave digital lattice filters combined with an FIR filter, which serves as equalizing filter, can be used for the purpose of pulse shaping. The baseband transmission of BPSK symbols $a_n \in \{-1, 1\}$, which are shaped by the BWDLF with impulse response $g[n]$, over an additive white Gaussian noise channel (AWGN) is depicted in Fig. 2. At the receiver the received signal is also filtered by the same BWDLF used at the transmitter for pulse shaping.

In order to rate this approach we compare the performance of our proposed solution with conventionally used FIR root raised cosine filters.

II. Equalizer

Very important for the system is the design of the FIR equalizer. The design criterion is to minimize the energy of the intersymbol interference ϵ at the sampling instances. The intersymbol interference is measured as the error between the impulse response of the entire system chain in Fig. 2 and the desired unit-pulse sequence $\boldsymbol{e}_k$. The delay k between a_m and $\hat{a}_m$ has strong influence on the system performance and has to be optimized by simulation. The energy is given by

$$\epsilon^2 = ||\hat{\boldsymbol{a}} - \boldsymbol{e}_k||_2^2, \tag{1}$$

with $\hat{\boldsymbol{a}} = [\ldots, \hat{a}_{m-1}, \hat{a}_m, \hat{a}_{m+1}, \ldots]^{\mathrm{T}}$.

The equalizing FIR filter $w[n]$ is designed to minimize the error. As the filter coefficients need not be adaptable, they only have to be determined once.

This equalizer can be implemented using two different approaches. In Fig. 3 a *fractionally spaced equalizer* (FSEQ), which operates at the double of the symbol rate $f_{\mathrm{s}} = 2f_{\mathrm{sym}}$, is used, whereas in Fig. 4 the equalizing FIR filter operates at the symbol rate f_{sym} and is called *symbol spaced equalizer* (SSEQ).

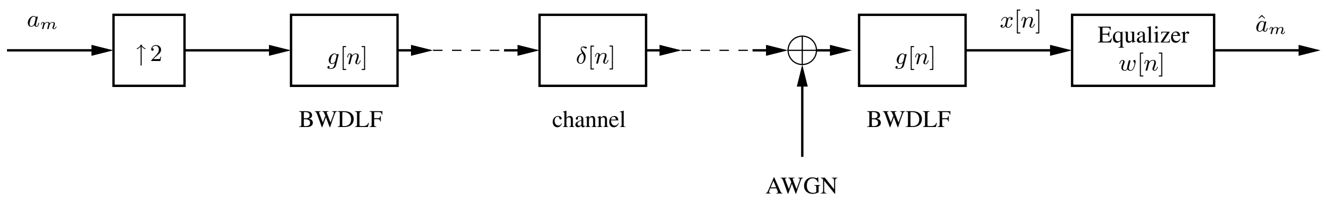

Fig. 2. Pulse shaping system with BWDLFs including AWGN channel and equalizer (FIR)

A. Fractionally Spaced Equalizer

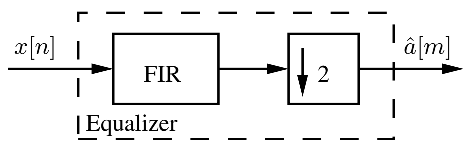

Fig. 3. Fractionally spaced equalizer

The steps for finding the optimum FSEQ in terms of minimum intersymbol interference are as follows:

The FSEQ in Fig. 3 operates at the sampling rate $f_{\mathrm{s}} = 2f_{\mathrm{sym}}$. Because of the infinite but also decaying impulse response of the cascade of both BWDLFs $h[n] = x[n]\big|_{\boldsymbol{a}=\boldsymbol{e}_0}$, a certain threshold value can be determined from when further samples can be neglected, because of being small compared with the maximum sample value.

From this approach the number $M+1$ of relevant samples is obtained, which have to be considered in the optimization process. The continuous operation of the system leads to a convolution of the impulse response $h[n]$ of both BWDLFs and the equalizing FIR filter of length $N_{\mathrm{fs}}+1$. This convolution can be considered as a matrix-vector-multiplication between a matrix $\boldsymbol{H}'$ of dimension $(N_{\mathrm{fs}}+M+1) \times (N_{\mathrm{fs}}+1)$ and the filter vector $\boldsymbol{w}$ of dimension $(N_{\mathrm{fs}}+1)$. The matrix $\boldsymbol{H}'$ is approximately a Toeplitz matrix because the triangle in the lower left corner only contains the elements $h[M+1], h[M+2], \ldots, h[M+N_{\mathrm{fs}}]$, which are very small as mentioned above.

The samples at the filter output are then decimated to the symbol rate by the factor 2. Each second sample is eliminated by this decimation and its value, which is the result of the convolution process described above, is completely irrelevant. Therefore, we can simplify the right hand side of the linear equation system. Each second element of the right hand side vector is arbitrary and the remaining elements are supposed to yield the unit-pulse sequence $\boldsymbol{e}_k$ at the symbol rate for avoiding intersymbol interference.

This means for the matrix-vector-multiplication from above, that the matrix $\boldsymbol{H}'$ becomes the matrix $\boldsymbol{H}$ by removing every second row. The result is a linear equation system

$$\boldsymbol{H}\boldsymbol{w} \stackrel{!}{=} \boldsymbol{e}_k, \tag{2}$$

with the $(N_{\mathrm{fs}}+M+1)/2 \times (N_{\mathrm{fs}}+1)$ dimensional matrix

$$\boldsymbol{H} = \begin{bmatrix} h[0] & 0 & 0 & \cdots & 0 \\ h[2] & h[1] & h[0] & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h[M+N_{\mathrm{fs}}] & h[M+N_{\mathrm{fs}}-1] & \cdots & \cdots & h[M] \end{bmatrix}, \tag{3}$$

which has to be solved.

If $M \geq N_{\mathrm{fs}}+1$, which is usually satisfied, because the filter order N_{fs} should be small and the impulse response of the BWDLF cascade doesn't decay so fast, $\boldsymbol{H}$ is a rectangular matrix and there is no exact solution for the overdetermined linear equation system in Eq. 2. Instead, we have to solve it in the least squares sense with the Moore-Penrose pseudoinverse $\boldsymbol{H}^{\dagger}$

$$\boldsymbol{w} = \boldsymbol{H}^{\dagger}\boldsymbol{e}_k. \tag{4}$$

B. Symbol Spaced Equalizer

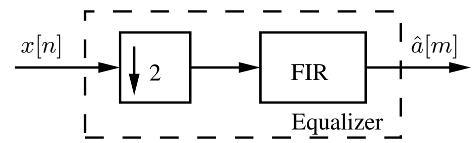

Fig. 4. Symbol spaced equalizer

In contrast the SSEQ of order N_{ss} gets the already decimated samples at the symbol rate f_{sym}. This means that the output of the SSEQ is supposed to be the unit-pulse sequence $\boldsymbol{e}_k$ for avoiding intersymbol interference. The optimum delay k of the pulse has to be found by simulation. As before a certain threshold value determines the number of relevant sample values $M+1$, which have to be equalized. Therefore, we get a linear equation system as in Eq. 2. The difference is the matrix $\boldsymbol{H} \in \mathbb{R}^{((M+1)/2+N_{\mathrm{ss}}) \times (N_{\mathrm{ss}}+1)}$

$$\boldsymbol{H} = \begin{bmatrix} h[0] & 0 & 0 & \cdots & 0 \\ h[2] & h[0] & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h[M+N_{\mathrm{ss}}] & h[M+N_{\mathrm{ss}}-2] & \cdots & \cdots & h[M] \end{bmatrix}. \tag{5}$$

The cascade of BWDLF and FIR filter at the receiver works at least approximately as the matched filter to the transmit BWDLF which will be adumbrated in the next sections.

III. Matched Filter Condition

By considering a flat AWGN channel between transmitter and receiver we have to satisfy the matched filter condition in the z-domain

$$H_{\mathrm{R}}(z) = z^{-N} H_{\mathrm{T}}(z^{-1}) \tag{6}$$

with the transmit filter $H_{\mathrm{T}}(z)$ and the receive filter $H_{\mathrm{R}}(z)$ in order to maximize the signal-to-noise ratio (SNR) at the sampling instances at the receiver. N is chosen such that the SNR is maximized. With Eq. 6 the cascade of both filters is characterized by the following equation:

$$H_{\mathrm{T}}(z)H_{\mathrm{R}}(z) = H_{\mathrm{T}}(z)H_{\mathrm{T}}(z^{-1})z^{-N}. \tag{7}$$

When it is assumed that the transmit filter is a causal linear phase filter with

$$H_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T}) = |H_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T})| \exp(-\mathrm{j}\,\omega \frac{N}{2} T), \tag{8}$$

then the well-known separation of the pulse shaping filter into two identical filters $H_{\mathrm{T}}(z)$ at the transmitter and $H_{\mathrm{R}}(z) = H_{\mathrm{T}}(z)$ at the receiver is obtained.

This assertion of identical filters at the transmitter and the receiver is no longer valid when the assumption is abandoned that $H_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T})$ is a linear phase filter. If, for example, the nonminimum phase part of the above linear phase transmit filter is removed, a minimum phase transmit filter with the same magnitude response remains:

$$H'_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T}) = |H_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T})| \exp(-\mathrm{j}\,\varphi(\omega)), \tag{9}$$

where $\varphi(\omega) = -\arg(H'_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T}))$ is an odd, nonlinear $2\pi/T$-periodic function of ω.

Using Eq. (6), Eq. (9), $|H_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T})| = |H_{\mathrm{T}}(\mathrm{e}^{-\mathrm{j}\,\omega T})|$ and replacing N by N', it holds true for the the matched receive filter $H'_{\mathrm{R}}(\mathrm{e}^{\mathrm{j}\,\omega T})$:

$$\begin{aligned} H'_{\mathrm{R}}(\mathrm{e}^{\mathrm{j}\,\omega T}) &= H'_{\mathrm{T}}(\mathrm{e}^{-\mathrm{j}\,\omega T}) \exp(-\mathrm{j}\,\omega N'T) \\ &= |H_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T})| \exp(\mathrm{j}\,\varphi(\omega)) \exp(-\mathrm{j}\,\omega N'T). \end{aligned} \tag{10}$$

It is interesting here that the transmit filter and the receive filter are no longer identical. They differ by an allpass

$$H_{\mathrm{AP}}(\mathrm{e}^{\mathrm{j}\,\omega T}) = \exp(-\mathrm{j}\,(\omega T - 2\varphi(\omega))). \tag{11}$$

This means that it is not sufficient to apply two identical nonlinear phase filters at the transmitter and the receiver, if the matched filter condition is to be satisfied. An additional filter with allpass characteristic $H_{\mathrm{AP}}(z)$ is necessary for this purpose. An additional FIR filter as depicted in Fig. 2 is applied, but to minimize the energy of the intersymbol interference and not to satisfy the matched filter condition explicitly. In Sec. V the assumption that this FIR filter also contributes in approximating the matched filter condition is affirmed.

In Eq. (9) the minimum phase property is not really demanded, although assumed in the text above. This means that the results are valid for arbitrary phase responses. Therefore, the linear phase response is contained as a special case where the matched receive filter equals the transmit filter.

If the transfer function of the cascade of the transmit filter and the matched receive filter for the linear phase transmit filter is considered by multiplying $H_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T})$ in Eq. (8) by $H_{\mathrm{R}}(\mathrm{e}^{\mathrm{j}\,\omega T})$, we get

$$H_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T}) H_{\mathrm{R}}(\mathrm{e}^{\mathrm{j}\,\omega T}) = |H_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T})|^2 \exp(-\mathrm{j}\,\omega NT), \tag{12}$$

and for the minimum phase transmit filter by multiplying $H'_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T})$ in Eq. (9) by $H'_{\mathrm{R}}(\mathrm{e}^{\mathrm{j}\,\omega T})$ in Eq. (10), we get

$$H'_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T}) H'_{\mathrm{R}}(\mathrm{e}^{\mathrm{j}\,\omega T}) = |H_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T})|^2 \exp(-\mathrm{j}\,\omega N'T). \tag{13}$$

This result can certainly also be obtained by evaluating Eq.(7) for $z = \mathrm{e}^{\mathrm{j}\,\omega T}$ and the respective N

$$H_{\mathrm{T}}(z) H_{\mathrm{R}}(z)\big|_{z=\mathrm{e}^{\mathrm{j}\,\omega T}} = |H_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T})|^2 \exp(-\mathrm{j}\,\omega NT). \tag{14}$$

Therefore, it is concluded that if $H_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T}) H_{\mathrm{R}}(\mathrm{e}^{\mathrm{j}\,\omega T})$ satisfies the Nyquist criterion, also $H'_{\mathrm{T}}(\mathrm{e}^{\mathrm{j}\,\omega T}) H'_{\mathrm{R}}(\mathrm{e}^{\mathrm{j}\,\omega T})$ satisfies it, only the optimum delay N can be different.

IV. Hardware-efficient Realization

A hardware-efficient realization in the context of these investigations aims at filters, which need only few multiplications per time unit and which are very robust against the quantization or small wordlength of their coefficients.

A. Conventional RRC Solution

The conventional pulse shaping solution with RRC FIR filters both at the transmitter and the receiver loses the property of raised cosine impulse responses that the sample values at multiples of the symbol rate are zero. Therefore, each coefficient leads to a multiplication. But, nevertheless, some complexity reduction is possible:

- The RRC FIR filters have a symmetric impulse response. Therefore, there are only $\lfloor N/2 \rfloor + 1$ distinct coefficients, which correspond to multiplications, for a filter degree of N.
- The upsampling operation before the RRC filtering at the transmitter means that every second input value is zero. Therefore, a complexity reduction can be reached by using a two-branch polyphase structure, which avoids the multiplications with zero-valued input samples. This means that the $N+1$ multiplications need not be executed at the sampling rate f_{s} but only at the symbol rate f_{sym}.
- The same is true for the receiver. The downsampling operation by two after filtering means that values are discarded, which have been calculated before. A polyphase structure avoids this calculation in advance and also leads to $N+1$ multiplications at the symbol rate f_{sym} instead of the sampling rate f_{s}.
- In the case of upsampling and downsampling by a factor of two and two-branch polyphase decompositions the polyphase components still have a symmetric impulse response. Therefore, only $\lfloor N/2 \rfloor + 1$ multiplications per symbol are required both at the transmitter and the receiver.

B. BWDLF Solution with FIR Equalizer

The BWDLFs can be implemented very efficiently in terms of multipliers, because for an N-th degree filter we need only $(N-1)/2$ multipliers which correspond to adaptor coefficients. Moreover, the degree of BWDLFs compared with FIR RRC filters is not very demanding because they provide a very good stopband attenuation due to their infinite impulse response.

The multiplications performed inside the adaptors are only carried out at the symbol rate f_{sym}. The sampling rate f_{s} at the output of the BWDLF at the transmitter is achieved by interleaving the outputs of the two allpass branches. Therefore, BWDLFs inherently include a polyphase decomposition.

Again, the same is true for the BWDLF at the receiver as long as the decimation follows directly the BWDLF which holds true for the SSEQ but not for the FSEQ. Only $(N-1)/2$ (SSEQ) resp. $(N-1)$ (FSEQ) multiplications per symbol are necessary.

As mentioned above the BWDLFs are very robust against coefficient quantizations because of their relation to lossless reference filters. Their odd symmetry property is even absolutely independent of coefficient values. Therefore, the coefficient wordlength can be relatively short. In addition the multipliers can be replaced by shift-and-add structures due to the constant coefficients.

If an FIR SSEQ is used in order to avoid intersymbol interference, $N_{\mathrm{ss}} + 1$ multiplications per symbol duration are necessary, because no symmetry properties can be utilized in general.

If an FSEQ is used, the following downsampling by two can be used to apply a polyphase decomposition. Therefore, $N_{\mathrm{fs}} + 1$ multiplications per symbol are necessary. Again, no symmetry properties can be used.

V. Simulation Results

The reference system, with which the results of the BWDLF pulse shaping are compared, uses windowed RRC pulse shaping filters both at the transmitter and the receiver with a rolloff factor of $\rho = 0.22$. The impulse response of the RRC filters is truncated after a duration of 10 symbols which corresponds to 21 coefficients at a sampling rate which is twice the symbol rate.

Taking the results of Section IV into account 22 multiplications per symbol period are necessary for the transmit and the receive filter altogether. This result is also given in the third row of Tab. I.

The cascade of identical RRC filters at both the transmitter and the receiver doesn't either yield zero intersymbol interference at the sampling instances because of the truncated impulse response in the time-domain. However, phase distortions don't occur because of the symmetrical impulse responses of the RRC filters.

The BWDLFs are designed as filters with an elliptic transfer characteristic [4], which is comparable with an RRC filter with a rolloff factor $\rho = 0.22$ according to the stopband attenuation and stopband region. The magnitude responses of both filters are depicted as solid and dotted lines in Fig. 5.

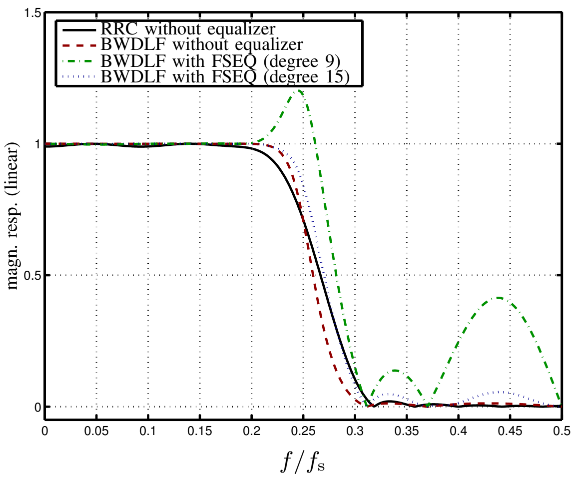

Fig. 5. Magnitude responses

Fig. 5 shows that the 5th degree bireciprocal WDLF already has a better stopband attenuation than the 20th degree root-raised-cosine FIR filter.

	RRC	BWDLF	
		fractionally spaced degree 9	symbol spaced degree 14
squared error	8.1228e-05	2.0842e-15	7.6103e-05
magn. error	2.2593e-02	6.1749e-08	2.3598e-02
mult./symbol	11+11=22	2+4+10=16	2+2+15=19

TABLE I

COMPARISON RRC AND BWDLF APPROACH

Now, the results in terms of the squared error and the magnitude error caused by conventional RRC pulse shaping and the proposed BWDLF pulse shaping with FIR equalizer are compared (Tab. I). The results for the BWDLF pulse shaping solution are given both for the fractionally (degree 9) and the SSEQ (degree 14). The squared error is the energy of the intersymbol interference, whereas the magnitude error is the sum of the magnitude of the difference between the output symbols and the desired unit-pulse sequence. This measure is closely related with the maximum eye-opening. Taking again the results of Sec. IV into accout the 9th order FSEQ and the 14th order SSEQ need 10 resp. 15 multiplications per symbol (Tab. I), additionally to the 2 resp. 4 multiplications per symbol of the 5th degree BWDLFs. The errors of the BWDLF proposal are comparable or even smaller in spite of less computational complexity as listed in Tab. I.

Interesting is also the magnitude response of the equalizing FIR filter in the BWDLF approach. As mentioned in Sec. I the Nyquist criterion for avoiding intersymbol interference is perfectly satisfied by the magnitude response of the cascade of both BWDLFs. Therefore, the FIR equalizer is mainly expected to equalize the phase distortions due to the nonlinear phase response of the BWDLFs, although the filter design criterion has not been explicitly the phase equalization. This assumption can be confirmed by the solid line in Fig. 6 representing the magnitude response of the SSEQ which is approximately an allpass. The FSEQ of degree 9 drawn as dashdotted line shows the allpass characteristic only until $f_s/4$. Its degrees of freedom are not sufficient for forcing an allpass response above $f_s/4$, too. This frequency region is the stopband region of the BWDLF. Only higher order filters are able to approximate an allpass response over the whole frequency range. The dotted line (degree 15) in Fig. 6 is already closer to an allpass function. Please note, that although the SSEQ and FSEQ in Fig. 6 operate at different sampling rates, they have been depicted in only one figure. The dashdotted and dotted

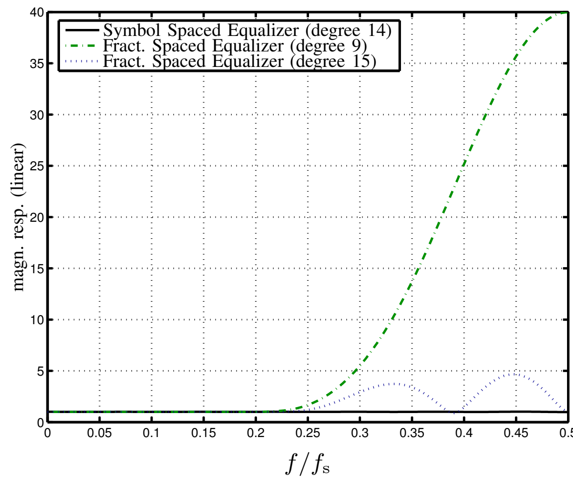

Fig. 6. Magnitude responses of fractionally and symbol spaced equalizer

lines in Fig. 5 belong to the FSEQs of Fig. 6 and represent the overall magnitude response of receive BWDLF and FSEQ. The strong amplification in frequency regions above $f_s/4$ is also evident here. If the channel is AWGN and has, therefore, high frequency components, a higher order FSEQ may be necessary in order to avoid the noise enhancement.

VI. CONCLUSION

Both BWDLFs and root raised cosine FIR filters generate some intersymbol interference. With BWDLFs the nonlinear phase makes absolute absence of intersymbol interference impossible. In contrast the RRC FIR lacks in odd symmetry of the magnitude response because of the necessary truncation in the time domain. The advantage of the pulse shaping with BWDLFs is that the filters need less coefficients than the corresponding FIR filters. Moreover, the wordlength of the BWDLF coefficients can be short due to the good sensitivity properties of BWDLFs. The complexity can be split up asymmetrically between transmitter and receiver. If transmitter and receiver cannot be equipped equally, for example the base and mobile stations of a mobile radio system, the effort of the equalizing filter can be shifted to the more powerful unit.

REFERENCES

[1] J. G. Proakis, *Digital Communications*. McGraw-Hill, Inc. 1995.
[2] A. Fettweis, "Wave Digital Filters: Theory and Practice," *Proc. IEEE*, vol. 74, no. 2, pp. 270–327, Feb. 1986.
[3] J. A. Nossek and H.-D. Schwartz, "Wave Digital Lattice Filters with Applications in Communication Systems," *Proc. IEEE Int. Symp. Circuits and Systems*, no. 2, pp. 845–848, May 1983.
[4] L. Gazsi, "Explicit Formulas for Lattice Wave Digital Filters," *IEEE Trans. Circuits Syst.*, vol. 32, no. 1, pp. 68–88, Jan. 1985.