

# A Novel Decomposition Technique for Multiuser MIMO

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**Abstract**—In the work at hand, a novel decomposition technique for multiuser multiple-input multiple-output (MIMO) systems is presented that combines the zero-forcing with successive encoding (ZF-SE) technique, proposed for the downlink of multiuser systems with single receive antennas, and the singular value decomposition (SVD) technique, which decomposes the single user channel while preserving capacity. The number of spatial dimensions allocated to each user as well as the encoding order of the allocated subchannels are two degrees of freedom of this technique which can be exploited for system design. The technique results in a set of virtually decoupled scalar subchannels upon which power allocation can be performed according to any criterion of interest. Reciprocity between the downlink and uplink channels allows the application of this technique to compute transmit and receive weighting vectors that together with a successive decoding strategy also achieve the decomposition of the uplink or multiple access channel. Thereby, the weighting vectors are the transpose of those applied to the dual downlink and the decoding order turns out to be the reversed encoding order. Exploiting the degrees of freedom offered by the method to maximize sum rate, it is observed that this technique practically reaches the Sato upper bound on sum rate capacity of the downlink MIMO channel. An asymptotical analysis of the approach for large number  $r$  of receive antennas per user shows that the growth in sum rate is proportional to  $\log(r)$  if cooperation is exploited and only proportional to  $\log(\log(r))$  if each antenna is considered to be a individual no cooperating receiver.

## I. INTRODUCTION

Multiple antennas at the base station and user terminals of future wireless communication systems allow to spatially multiplex signals intended for different users in the downlink and to spatially separate signals simultaneously transmitted by a number of users in the uplink. In such systems, joint optimization of weighting vectors, number of information streams assigned to each user and power allocation is either mathematically intractable or leads to computationally expensive solutions (e.g. [1]). A typical simplification of the design problem consists of presuming a certain number of streams

for each user and optimizing weighting vectors and power allocation (cf. [2], [3]). In spite of the optimality loss, the resulting iterative algorithms remain certainly involved.

Alternatively, system design can be simplified if, first, the multiuser multiple-input multiple-output (MIMO) channel is decomposed into a set of scalar subchannels so that interference between them is effectively eliminated and, then, power allocation upon this set of virtually decoupled channels is optimized. Although it will normally lead to a loss of optimality, elimination of interference might be beneficial if, as a consequence, the design problem becomes tractable or if the complexity reduction justifies the optimality loss.

In the work at hand we present a decomposition technique primarily conceived for the downlink of multiuser MIMO systems [4]. The method successively allocates spatial dimensions to users taking care that at each step the new subchannel does not interfere with previously established subchannels. By contrast, the method does not guarantee that previous subchannels do not cause any interference on the new one. However, this remaining interference can be neutralized by successively encoding the streams of information transmitted over the allocated subchannels, and taking into account the knowledge of previously encoded streams [5]. If applied to a system with single receive antennas, the method converges to a zero-forcing with successive encoding approach [6]. If applied to a single user setting, in which all receive antennas can cooperate, it results in a singular value decomposition of the channel matrix. For any given downlink or broadcast channel we also show that a decomposition of the dual uplink or multiple access channel is achieved by simply applying the transpose of the weighting vectors computed for the downlink and reversing the encoding order to obtain the corresponding decoding order. The gains of the resulting subchannels in the uplink are the same of those in the downlink.

In an independent work, [7], a transmission scheme for the downlink of a multiuser MIMO system has been recently

proposed that is related to ours. However, the possibility of assigning more than one spatial dimensions to a same user is not considered. Despite this fact, the asymptotic results on achievable sum rate presented there are also applicable to our approach. According to these results, for a large number of users  $K$ , the achievable sum rate increases linearly with the number of transmit antennas  $t$  and is proportional to  $\log(\log(K))$ .

The remaining of the paper is structured as follows. In Section II, the system model is introduced that is used along this paper. Section III describes the decomposition algorithm applied to the downlink. In Section IV the decomposition algorithm is applied to the uplink and by exploiting the relationship between down- and uplink. In Section V some comments are provided concerning the degrees of freedom of the method and how to make use of these. Section VI provides a comparison between our method and other state-of-the-art methods in terms of achievable sum rate. Finally, in Section VII the essential content of this paper is summarized and conclusions are drawn.

#### A. Notation

In the following, vectors and matrices are denoted by lower case bold and capital bold letters, respectively. We use  $(\bullet)^*$  for complex conjugation,  $(\bullet)^T$  for matrix transposition and  $(\bullet)^H$  for conjugate transposition. The identity matrix of dimension  $q$  is denoted by  $\mathbf{I}_q$  and  $e_s$  denotes its  $s$ th column.

### II. SYSTEM MODEL

For the downlink we consider the usual system model

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n},$$

with

$$\begin{aligned} \mathbf{y} &= [\mathbf{y}_1^T \ \cdots \ \mathbf{y}_K^T]^T, \\ \mathbf{n} &= [\mathbf{n}_1^T \ \cdots \ \mathbf{n}_K^T]^T, \\ \mathbf{H} &= [\mathbf{H}_1^T \ \cdots \ \mathbf{H}_K^T]^T, \end{aligned}$$

where  $K$  is the number of users,  $\mathbf{H}_k \in \mathbb{C}^{r_k \times t}$  is the channel matrix,  $\mathbf{y}_k \in \mathbb{C}^{r_k}$  the received signal and  $r_k$  the number of receive antennas of user  $k$ ,  $\mathbf{n}_k \in \mathbb{C}^{r_k}$  is a realization of a zero-mean circularly symmetric complex Gaussian distributed random variable  $\mathbf{n}_k$  representing noise with covariance matrix  $\mathbb{E}\{\mathbf{n}_k \mathbf{n}_k^H\} = \mathbf{I}_{r_k}$ , and  $t$  the number of transmit antennas at the base station. The transmit signal  $\mathbf{x} \in \mathbb{C}^t$  is given by

$$\mathbf{x} = \sum_{j=1}^J \mathbf{v}_{\pi_e(j)} s_{\pi_e(j)},$$

where index  $j$  indicates the order in which signals  $s_{\pi_e(j)}$  are encoded and  $J$  is the rank of matrix  $\mathbf{H}$ , which is typically equal to the number of transmit antennas  $t$ . The function  $\pi_e$  maps each index  $j$  to a subchannel label  $(k, \ell)$ , where  $k$  is the user to which the  $j$ th subchannel is assigned and  $\ell$  identifies that specific subchannel within the set of  $L(k)$  subchannels assigned to user  $k$ . Each subchannel is characterized by a unit-norm transmit weighting vector  $\mathbf{v}_{\pi_e(j)}$  and a unit-norm receive

weighting vector  $\mathbf{u}_{\pi_e(j)} \in \mathbb{C}^{r_k}$ . Correspondingly, the received signal over subchannel  $\pi_e(j) = (k, \ell)$  is given by the product  $\mathbf{u}_{\pi_e(j)}^H \mathbf{y}_k$ .

Assuming reciprocity, in the uplink the vector  $\mathbf{r} \in \mathbb{C}^t$  of signals received at the base station is given by

$$\mathbf{r} = \sum_{k=1}^K \mathbf{H}_k^T \mathbf{t}_k + \mathbf{w},$$

where  $\mathbf{w} \in \mathbb{C}^t$  is a realization of a zero-mean circularly symmetric complex Gaussian distributed random variable  $\mathbf{w}$  representing noise with covariance matrix  $\mathbb{E}\{\mathbf{w}\mathbf{w}^H\} = \mathbf{I}_t$ . The vector  $\mathbf{t}_k \in \mathbb{C}^{r_k}$  of signals transmitted by user  $k$  is given by

$$\mathbf{t}_k = \sum_{n=u_d(n)} \mathbf{g}_{\pi_d(n)} q_{\pi_d(n)},$$

where index  $n$  indicates the order in which transmit signals  $q_{\pi_d(n)}$  are decoded at the base station. The function  $\pi_d$  maps each index to a subchannel label  $(k, m)$ , where  $k$  is the user to which the  $n$ th subchannel is assigned and  $m$  identifies that specific subchannel within the set of  $M(k)$  subchannels assigned to user  $k$ . The function  $u_d(n)$  returns the user to which the  $n$ th channel is assigned. Each subchannel is characterized by a unit-norm transmit weighting vector  $\mathbf{g}_{\pi_d(n)}$  and a unit-norm receive weighting vector  $\mathbf{f}_{\pi_d(n)}$ . Correspondingly, the received signal over subchannel  $\pi_d(n) = (k, m)$  is given by the product  $\mathbf{f}_{\pi_d(n)}^H \mathbf{r}$ .

### III. DOWNLINK

Here, an algorithm is proposed that at each step computes a pair of transmit and receive weighting vectors so that no interference is caused on previously established subchannels. As encoding order is chosen to be equal to the order in which subchannels are established, interference caused by any channel on subsequent subchannels is known when coding information for transmission over these subchannels and therefore can be efficiently neutralized [5], [8].

The algorithm works as follows. After having established the first  $j-1$  spatial subchannels, the projection matrix  $\mathbf{T}_j$  is computed as

$$\mathbf{T}_j = \mathbf{T}_{j-1} - \mathbf{v}_{\pi_e(j-1)} \mathbf{v}_{\pi_e(j-1)}^H, \quad j = 2, \dots, J$$

with  $\mathbf{T}_1 = \mathbf{I}_t$ . This matrix represents the projector of the subspace defined by the intersection of the kernels of the subchannels already established.

Then, channel matrices of all users are projected into this subspace,

$$\mathbf{H}_k^j = \mathbf{H}_k \mathbf{T}_j, \quad \forall k,$$

and singular value decompositions of all projected channel matrices are performed,

$$\mathbf{H}_k^j = \mathbf{U}_k^j \mathbf{\Lambda}_k^j \mathbf{V}_k^{j,H}, \quad \forall k.$$

At this stage, among the set of potential subchannels one is selected according to any particular design criterion. Denoting by  $\mathcal{O}$  the operator that selects one out of all possible

subchannels we can mathematically write,

$$(k_0, s_0) = \mathcal{O}\{\lambda_{k,s}^j\}, \quad \pi_e(j) = (k_0, \ell(k_0)), \quad (1)$$

$$\mathbf{v}_{\pi_e(j)} = \mathbf{V}_{k_0}^j \mathbf{e}_{s_0}, \quad \mathbf{u}_{\pi_e(j)} = \mathbf{U}_{k_0}^j \mathbf{e}_{s_0},$$

where  $\lambda_{k,s}^j$  is the  $s$ th eigenvalue in the main diagonal of matrix  $\mathbf{\Lambda}_k^j$ , and  $\ell(k_0)$  denotes the number of subchannels provisionally assigned to user  $k_0$ . For allocation of the  $(j+1)$ th spatial subchannel the same procedure is repeated (cf. Table I).

<p>initialization : <math>j = 1, \quad \mathbf{T}_1 = \mathbf{I}_t</math>  repeat :</p> <ol style="list-style-type: none"> <li>1. <math>\mathbf{H}_k^j = \mathbf{H}_k \mathbf{T}_j \quad \forall k</math></li> <li>2. <math>\mathbf{H}_k^j = \mathbf{U}_k^j \mathbf{\Lambda}_k^j \mathbf{V}_k^{j,H} \quad \forall k</math></li> <li>3. <math>(k_0, s_0) = \mathcal{O}\{\lambda_{k,s}^j\}, \quad \pi_e(j) = (k_0, \ell(k_0))</math>  <math>\mathbf{v}_{\pi_e(j)} = \mathbf{V}_{k_0}^j \mathbf{e}_{s_0}, \quad \mathbf{u}_{\pi_e(j)} = \mathbf{U}_{k_0}^j \mathbf{e}_{s_0}</math></li> <li>4. <math>\mathbf{T}_{j+1} = \mathbf{T}_j - \mathbf{v}_{\pi_e(j)} \mathbf{v}_{\pi_e(j)}^H, \quad j = j + 1</math></li> </ol> <p>until <math>j &gt; \sum_k r_k</math> or <math>\mathbf{T}_j = \mathbf{0}</math></p>
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TABLE I  
SUCCESSIVE ALLOCATION ALGORITHM FOR CZF-SESAM

For subchannel  $\pi_e(j)$  interference caused by subchannels  $\pi_e(i > j)$  is forced to zero, i.e.

$$\mathbf{u}_{\pi_e(j)}^H \mathbf{H}_{u_e(j)} \mathbf{v}_{\pi_e(i>j)} = 0, \quad (2)$$

where  $u_e(j)$  is a function that returns the user to which subchannel  $\pi_e(j)$  belongs.

To see this consider the following equalities,

$$\mathbf{u}_{\pi_e(j)}^H \mathbf{H}_{u_e(j)} \mathbf{v}_{\pi_e(i>j)} = \quad (3)$$

$$\mathbf{u}_{\pi_e(j)}^H \mathbf{H}_{u_e(j)} \mathbf{T}_{i>j} \mathbf{v}_{\pi_e(i>j)} = \quad (4)$$

$$\mathbf{u}_{\pi_e(j)}^H \mathbf{H}_{u_e(j)} \mathbf{T}_j \mathbf{T}_{i>j} \mathbf{v}_{\pi_e(i>j)} = \quad (4)$$

$$\lambda_{k_0, s_0}^j \mathbf{v}_{\pi_e(j)}^H \mathbf{T}_{i>j} \mathbf{v}_{\pi_e(i>j)} = 0 \quad (5)$$

In (3) we make use of the fact  $\mathbf{v}_{\pi_e(i>j)}$  lies within the subspace spanned by  $\mathbf{T}_{i>j}$ . In (4) we consider the fact that the image of  $\mathbf{T}_{i>j}$  is within the subspace spanned by  $\mathbf{T}_j$ . Finally, in (5) we note that  $\mathbf{u}_{\pi_e(j)}^H$  is a left singular vector of  $\mathbf{H}_{u_e(j)} \mathbf{T}_j$  with  $\mathbf{v}_{\pi_e(j)}$  as corresponding right singular vector, which, by construction, happens to be perpendicular to the subspace spanned by  $\mathbf{T}_{i>j}$ .

By contrast, interference caused by subchannels  $\pi_e(i < j)$  is, in general, not eliminated by the choice of transmit weighting vectors. Note that in this case  $\mathbf{T}_j \mathbf{T}_{i<j} = \mathbf{T}_j$  and, therefore, the step (4) in the above derivation does not hold. This interference can be neutralized by coding.

An exception occurs when  $u_e(i) = u_e(j)$  with  $i \neq j$ . In such case it can be shown that subchannels  $\pi_e(j)$  and  $\pi_e(i)$  are entirely decoupled. Indeed, assuming  $i > j$ , we observe

$$\begin{aligned} 0 &= \mathbf{u}_{\pi_e(j)}^H \mathbf{H}_{u_e(j)} \mathbf{v}_{\pi_e(i)} \\ &= \mathbf{u}_{\pi_e(j)}^H \mathbf{H}_{u_e(j)} \mathbf{T}_i \mathbf{v}_{\pi_e(i)} \\ &= \mathbf{u}_{\pi_e(j)}^H \mathbf{u}_{\pi_e(i)} \lambda_{k_0, s_0}^i, \end{aligned}$$

which shows that  $\mathbf{u}_{\pi_e(i)}$  and  $\mathbf{u}_{\pi_e(j)}$  are necessarily orthogonal. On the other hand, interference caused by subchannel  $\pi_e(j)$  on subchannel  $\pi_e(i)$  is given by

$$\begin{aligned} \mathbf{u}_{\pi_e(i)}^H \mathbf{H}_{u_e(i)} \mathbf{v}_{\pi_e(j)} &= \\ \mathbf{u}_{\pi_e(i)}^H \mathbf{H}_{u_e(i)} \mathbf{T}_j \mathbf{v}_{\pi_e(j)} &= \\ \mathbf{u}_{\pi_e(i)}^H \mathbf{u}_{\pi_e(j)} \lambda_{k_0, s_0}^j &= 0, \end{aligned} \quad (6)$$

which as it has been shown is equal to zero due to orthogonality of the receive weighting vectors. In fact, the algorithm results in a singular value decomposition if applied to a single user scenario.

Effective transmission of information occurs over each of the allocated scalar subchannels whose gain is given by

$$g_{\pi_e(j)}^{\text{BC}} = \mathbf{u}_{\pi_e(j)}^H \mathbf{H}_{u_e(j)} \mathbf{v}_{\pi_e(j)}.$$

Over this set of virtually decoupled channels allocation of transmit power can be chosen in order to optimize system performance.

#### IV. UPLINK

Considering the outcome of the decomposition algorithm in the downlink the following choice of parameters leads also to a virtual decomposition of the MIMO multiuser channel in the uplink,

$$\pi_d(n) = \pi_e(J + 1 - n), \quad (7)$$

$$\mathbf{g}_{\pi_d(n)} = \mathbf{u}_{\pi_e(J+1-n)}^*, \quad (8)$$

$$\mathbf{f}_{\pi_d(n)} = \mathbf{v}_{\pi_e(J+1-n)}^*. \quad (9)$$

Now, receive signals are successively decoded and the decoding order is obtained by reversing the encoding order as indicated in (7). The receive weighting vectors of the downlink are used to weight the transmit signal in the uplink (8) and the downlink transmit weighting vectors are used in the uplink as receive weighting vectors (9).

Subchannel  $\pi_d(1)$  does not see interference from other subchannels as

$$\begin{aligned} \mathbf{u}_{\pi_e(J>i)}^H \mathbf{H}_{u_e(J>i)} \mathbf{v}_{\pi_e(J)} &= \\ \mathbf{f}_{\pi_d(1)}^H \mathbf{H}_{u_d(i>1)}^T \mathbf{g}_{\pi_d(i>1)} &= 0. \end{aligned}$$

Similarly, subchannel  $\pi_d(2)$  does not see any interference from subchannels  $\pi_d(i > 2)$  but, in general, it might see some from subchannel  $\pi_d(1)$ . However, this interference can be suppressed if information transmitted over subchannel  $\pi_d(1)$  is detected and subtracted from the received signal prior to

detection of information transmitted over subchannel  $\pi(2)$ . The process can be successively repeated until all streams are detected. The result is a set of virtually decoupled subchannels with gain,

$$g_{\pi_d(n)}^{\text{MAC}} = \mathbf{f}_{\pi_d(n)}^{\text{H}} \mathbf{H}_{u_d(n)}^{\text{T}} \mathbf{g}_{\pi_d(n)} = \mathbf{u}_{\pi_e(J+1-n)}^{\text{H}} \mathbf{H}_{u_e(J+1-n)} \mathbf{v}_{\pi_e(J+1-n)} = g_{\pi_e(J+1-n)}^{\text{BC}}.$$

Over the set of subchannels assigned to each particular user, available transmit power can be allocated so as to optimize performance of that user. Note that now no overall optimization of power allocation is possible as transmitters do not cooperate.

## V. DEGREES OF FREEDOM

As already mentioned, this decomposition method offers two degrees of freedom that can be exploited for system design. These are the encoding or decoding order and the number of dimensions assigned to each user. Both come into play in (1) at the time of selecting a new subchannel.

For systems without predefined settings, a natural choice of  $\mathcal{O}$  seems to be

$$\underset{(k,s)}{\operatorname{argmax}} \{ \lambda_{k,s}^j \}, \quad (10)$$

i.e. it seems natural to try to collect at each step as much energy out of the channel as possible. In this case the allocation of subchannels is entirely determined by the channel itself. If a certain degree of fairness among users is desired the same criterion could be used and some constraints might be introduced such as a maximum number of dimensions per user or no contiguous assignments to a same user.

In systems with predefined settings, such as the number of subchannels per user or the modulation alphabets employed for transmission, letting the channel decide as in (10) will surely lead to a conflict between the predefined settings and the resulting allocation. In such a case subchannel allocation should be performed taking into account the system requirements. How to combine system requirements and channel condition to come up with a convenient selection rule  $\mathcal{O}$  is an interesting issue which is beyond the scope of this work.

In the next section we use (10) as selection rule and compare the achievable sum rate of this decomposition method in the downlink with that of other state-of-art transmission approaches. Some rationale for the choice of this criterion to maximize sum rate is provided in [4].

## VI. SIMULATION RESULTS

First, we briefly describe the approaches compared in the figures below. The Sato bound is an upperbound on sum rate capacity of broadcast channels. For the Gaussian broadcast channels considered here, this bound is achieved applying a successive encoding of users and optimum transmit covariance matrices, which can be computed with the algorithm reported in [1]. CZF-SESAM stands for cooperative zero-forcing with successive encoding and successive allocation method and corresponds to the decomposition method presented in this paper using (10) as selection rule at each step. The only

difference of ZF-SESAM with respect to CZF-SESAM is that the former considers each receive antenna as a non-cooperative receiver, i.e. CZF-SESAM reduces to ZF-SESAM if all receivers had single antennas. The Block-ZF-SE approach can be obtained from our approach if instead of using (10) as selection rule an arbitrary ordering of the users is predefined and according to that ordering each user is assigned as many subchannels in contiguous steps as the dimensionality of its channel matrix allows. In essence, the Block-ZF-SE approach is the same as the SO-THP in [9] but substituting THP by an optimum coding strategy and with arbitrary ordering of users. ZF-SE is just the method reported in [10]. This is the same approach as ZF-SESAM but substituting (10) by a random subchannel allocation. Block-ZF denotes the multiuser MIMO decomposition technique independently reported in [11], [12] and [13], which linearly suppresses interuser interference. With ZF a linear zero-forcing approach is meant that uses the column vectors of  $\mathbf{H}^{-1}$  normalized to norm one as transmit weighting vectors. This approach linearly suppresses both interuser interference and cross-talk between receive antennas. Finally, TDMA-CSIT and "TDMA no CSIT" are time division multiplexing approaches, in which only one user is served in a time slot. "TDMA no CSIT" does not assume any channel knowledge at the transmitter whereas TDMA-CSIT assumes perfect channel state information at the transmitter. In both approaches the user to be served is randomly selected.

Fig. 1 shows average sum capacity curves for a Rayleigh distributed channel with  $t = 4$  transmit antennas,  $K = 2$  users and  $r_1 = r_2 = 2$  antennas at each receiver. The entries in the composite channel matrix  $\mathbf{H}$  have been assumed to be mutually uncorrelated with variance equal to one. The horizontal axis represents the ratio between transmit power and noise variance, which is assumed to be equal for all receive antennas.

Successive encoding techniques converge to the Sato bound at high values of transmit power. This is in agreement with the result reported in [6] stating that if the composite channel matrix  $\mathbf{H}$  has full row rank, the sum rate capacity achieved by a ZF-SE approach converges to the capacity of the single user MIMO channel represented by the same matrix. Performance of CZF-SESAM overlaps with the Sato bound. Block-ZF-SE and ZF-SESAM show a similar performance with minimal losses with respect to CZF-SESAM due to no optimization of encoding order and no use of cooperation, respectively. As ZF-SE makes use of none of these two degrees of freedom, its performance loss is larger, but still slight.

Linear zero-forcing techniques show a significant performance loss with respect to successive encoding approaches. This is due to the larger number of orthogonality constraints imposed on the choice of transmit weighting vectors as compared to successive encoding techniques, which neutralize part of the interference by coding. The performance gap observed between the Block-ZF and ZF approaches is due to the larger number of constraints imposed by the latter as a means to suppress not only interuser interference but also cross-talk between receive antennas of a same user. In spite of these

constraint-induced losses, at high SNR, the slope of curves corresponding to linear approaches is the same as the slope of curves corresponding to successive encoding approaches. This is not surprising as the asymptotic slope is determined by the number of subchannels over which information is transmitted, which, for both linear and non linear approaches, is equal to the rank of the composite channel matrix  $\mathbf{H}$ .

By contrast, in TDMA approaches only one user is served in each time slot, and therefore, a maximum of two spatial dimensions are used, which represents half of the rank of an average realization of the composite matrix. Accordingly, TDMA curves asymptotically grow approximately half as fast as all other curves.

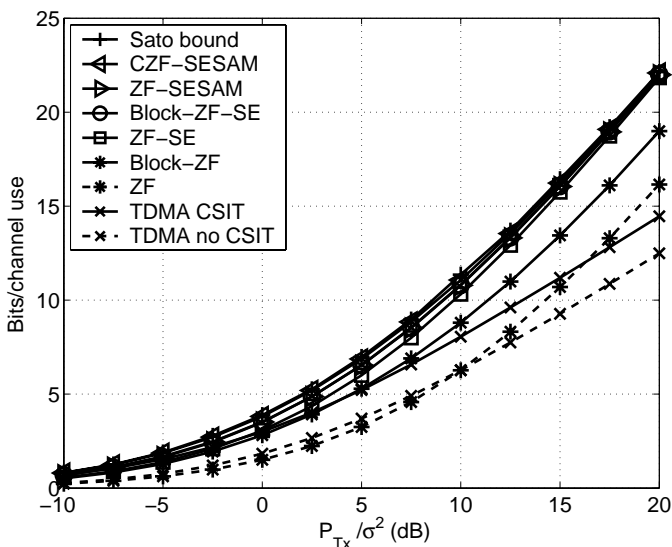


Fig. 1. Sum capacity for a multiuser setting with spatially uncorrelated Rayleigh fading Gaussian channels.  $t = 4$ ,  $r_k = 2$ ,  $K = 10$ .

In Fig. 2 average sum capacity curves are shown for a scenario as described by the settings used in Fig. 1 but where correlation has been introduced between transmit antenna elements. A transmit correlation matrix  $\mathbf{R}_{T_x} = E\{\mathbf{H}^H \mathbf{H}\} / t \sum_k r_k$  has been considered with the following eigenvalue profile,

$$\mathbf{\Lambda} = \text{diag}[0.9141, 0.0845, 0.0014, 0.0000].$$

The practical case of two users being in locations few meters apart from each other that are reached by the base station through quite a narrow bundle of angles of departure matches the setting proposed here.

The asymptotic slope of all curves decay due to the reduced rank of the channel. As particular realizations of composite channel matrices are, at least numerically, not any more full row rank, no convergence of successive encoding approaches can be observed at high SNR. As before, CZF-SESAM overlaps with the Sato upper bound. The losses of all other successive encoding techniques due to no cooperation and no ordering optimization become larger. Ordering is now crucial since the first selected subchannel largely determines

performance as it imposes severe constraints on subsequent subchannels. Cooperation is also important since constructive combination of receive signals raises the gain of the first subchannel. In the light of the simulation results, optimization of encoding order seems to provide more benefit than cooperation between receive antennas. However, the impact of cooperation on capacity increases with increasing number of receive antennas per user as we discuss later in this section. Linear zero-forcing techniques dramatically suffer from the reduced rank of the channel. Finally, it can be observed that the asymptotic slopes of TDMA strategies do not strongly differ from the slopes of successive encoding approaches. Indeed, the number of subchannels is, in this case, mostly limited by the rank of the composite channel, which for most realizations is not larger than one or two, and not by the fact of serving only one user in each time slot.

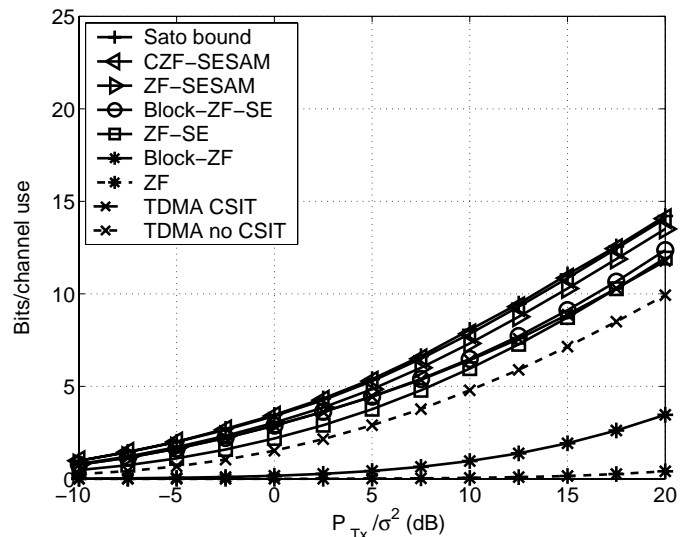


Fig. 2. Sum capacity for a multiuser setting with spatially correlated Rayleigh fading Gaussian channels.  $t = 4$ ,  $r_k = 2$ ,  $K = 10$ .

Fig. 3 shows average sum capacity curves for a Rayleigh distributed channel with  $t = 4$  transmit antennas,  $K = 10$  users and  $r_k = 2$  antennas at each receiver. Entries in the composite channel matrix are assumed mutually independent and with covariance equal to one. Different from the settings of Figs. 1 and 2, now, the number of receive antennas in the system is larger than the number of transmit antennas. This calls for a decision regarding the users to be served and the number of subchannels to be assigned to these users in a particular time slot. This additional degree of freedom is exploited by the two approaches with successive allocation capability, i.e. CZF-SESAM and ZF-SESAM, and yields a significant performance gain with respect to Block-ZF-SE and ZF-SE, which randomly select any two users or four antennas, respectively. CZF-SESAM shows an insignificant loss with respect to the optimum approach. Also moderate is the gain due to receive antenna cooperation of CZF-SESAM with respect to ZF-SESAM as receivers only have two antennas. Linear zero-

forcing approaches are not directly applicable to this setting as the number of orthogonality constraints required to linearly suppress interference exceeds the number of dimensions of the transmit weighting vectors. Nevertheless, these techniques might be endowed with a mechanism to preselect a particular group of receive antennas so as to guarantee their applicability. This possibility has not been considered here and, therefore, curves of linear zero-forcing approaches are not included. The slower asymptotic growth of TDMA strategies with respect to successive encoding strategies can be observed again.

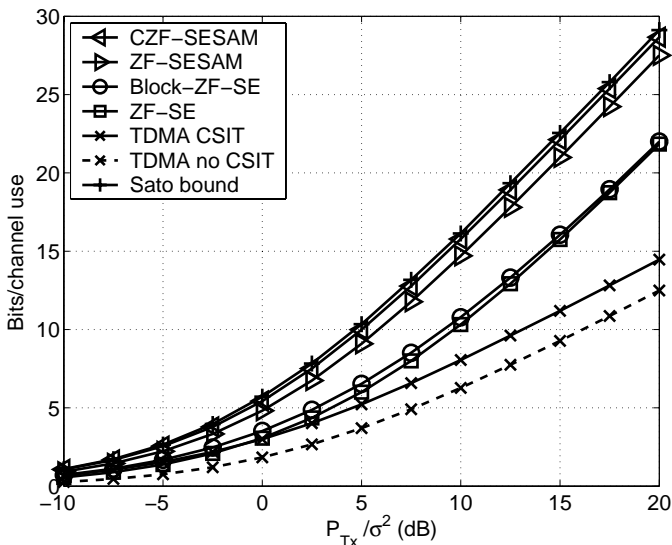


Fig. 3. Sum capacity for a multiuser setting with spatially uncorrelated Rayleigh fading Gaussian channels.  $t = 4$ ,  $r_k = 2$ ,  $K = 10$ .

Fig. 4 shows average sum capacity curves for a scenario as described by the settings used in Fig. 3 but where correlation has been introduced between transmit antenna elements. For these simulations, the following eigenvalue profile of the transmit covariance matrix has been considered,

$$\Lambda = \text{diag}[0.6040, 0.3063, 0.0775, 0.0122].$$

This profile may very well match a scenario in which a group of users located in a same certain area, such as a square or street, are reached from the base station over the same moderately broad bundle of angles of departure.

Again, the moderate rank loss of the channel causes a decay of the asymptotic growth of all approaches. As in Fig. 3, CZF-SESAM shows an insignificant performance loss with respect to the optimum solution and a modest performance gain with respect to ZF-SESAM. Also compared to Fig. 3, the gap between techniques with and without successive allocation capability remains approximately equal while the gap between successive encoding techniques and TDMA strategies diminishes as the asymptotic growth of the latter is not limited by the rank of the composite channel matrix and, as a result, is practically not affected by the rank reduction due to correlation.

As it can be observed in all four plots, our CZF-SESAM approach practically achieves the performance of the optimum solution while it involves significantly less complexity [4].

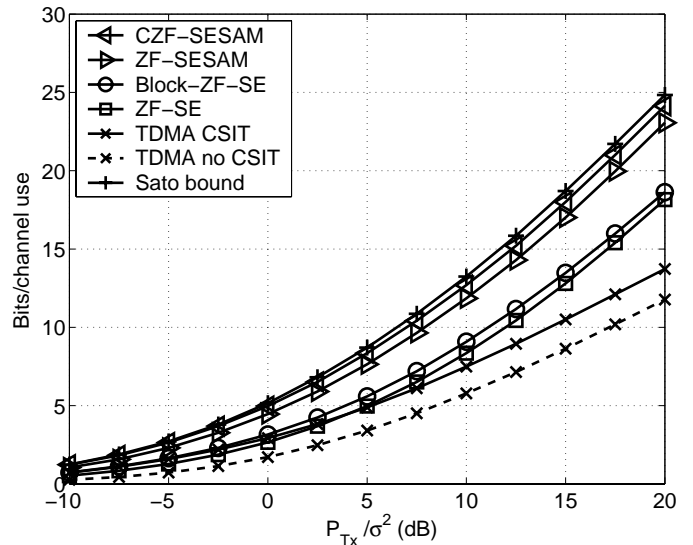


Fig. 4. Sum capacity for a multiuser setting with spatially correlated Rayleigh fading Gaussian channels.  $t = 4$ ,  $r_k = 2$ ,  $K = 10$ .

As already mentioned, in the light of this simulation results the conclusion could be drawn that enabling cooperation between received antennas the achievable gains are almost negligible, however, the following two theorems show that CZF-SESAM and ZF-SESAM exhibit a different asymptotical growth with the number of receive antennas.

*Theorem 1:* Consider a composite channel matrix  $\mathbf{H}$  with i.i.d. zero-mean Gaussian entries. Given a number of transmit antennas  $t$ , a number of users  $K$  and  $r$  receive antennas per user, the average sum-rate achieved by the ZF-SESAM approach grows proportionally to  $\log(\log(r))$  as  $r \rightarrow \infty$ .

*Proof:* This is a direct consequence of equation (40) in [7] and the fact that the ZF-SESAM algorithm considers each antenna as a non-cooperative user. ■

*Theorem 2:* Consider a composite channel matrix  $\mathbf{H}$  with i.i.d. zero-mean Gaussian entries. Given a number of transmit antennas  $t$ , a number of users  $K$  and  $r$  receive antennas per user, the average sum-rate achieved by the CZF-SESAM approach grows proportionally to  $\log(r)$  as  $r \rightarrow \infty$ .

*Proof:* See the Appendix. ■

The asymptotical behavior stated in this two theorems can be visualized in Fig. 5, where sum rate is displayed over the number of receive antennas per user for an SNR = 10 dB. According to this figure, the Sato bound seems to have the same asymptotical growth as the CZF-SESAM approach.

## VII. CONCLUSIONS

A method has been presented to decompose multiuser MIMO channels into a set of virtually decoupled scalar subchannels. In the downlink decomposition comes about by applying weighting vectors at both base station and mobile terminals which are computed in a successive way. At each step, care is taken that the new computed transmit weighting vector does not interfere with previously established subchannels. Interference caused by any transmit weighting vector

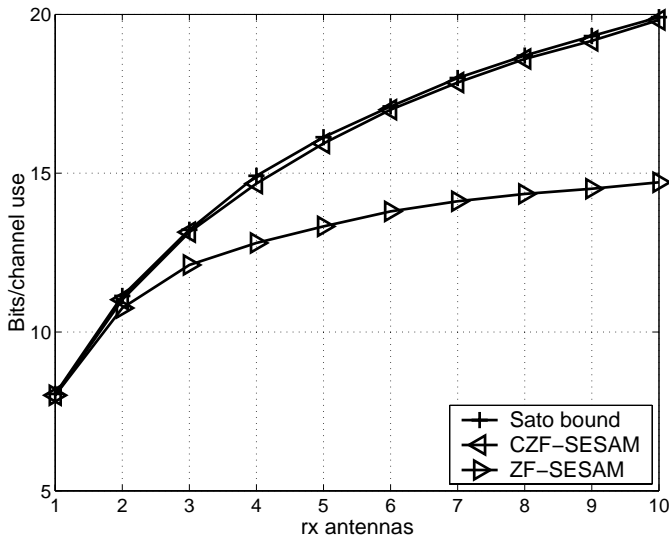


Fig. 5. Sum capacity for a multiuser setting with spatially uncorrelated Rayleigh fading Gaussian channels.  $t = 4$ , SNR = 10 dB,  $K = 2$ .

on subsequently established subchannels is neutralized by performing a successive encoding of the transmit signals. The same weighting vectors decomposing the downlink turn out to achieve the virtual decomposition of the uplink when combined with a successive decoding of the received signals and the decoding order is chosen to be the reverse of the encoding order in the downlink.

The method offers some degrees of freedom that can be optimized for system design. If at each step the strongest subchannel is selected the resulting set of decoupled scalar channels are observed to exhibit nearly the same sum rate capacity as the original channel in the downlink. The advantage of this method with respect to other state-of-the-art techniques has been primarily shown by means of simulation curves under a variety of different settings. Also the impact of cooperating antenna elements at the receivers has been investigated by comparing the performance of our method and a ZF-SE technique with optimized ordering. Asymptotic results show that for large numbers of receive antennas  $r$  the sum-rate achieved by our method grows proportionally to  $\log(r)$  while that achieved by the optimized ZF-SE only grows as  $\log(\log(r))$ .

## APPENDIX

### A. Proof of Theorem 2

As  $r \rightarrow \infty$ , due to the weak law of large numbers, the columns of the channel matrix  $\mathbf{H}_k$  of any user  $1 < k < K$  will be mutually uncorrelated or, from an algebraic point of view, perpendicular. The same holds for columns belonging to matrices of different users.

As a consequence, in the limit, we have a set of tall matrices with orthogonal columns. The left singular vectors of such matrices are given by the column vectors themselves normalized to unity, the right singular vectors are the canonical

basis vectors of dimension  $t$ , and the eigenvalues  $\approx \sqrt{r}\sigma$ , where  $\sigma$  is the standard deviation of the entries in the channel matrices.

The CZF-SESAM algorithm successively selects the right singular vectors as precoding vectors and corresponding left singular vectors of particular users as receive weighting vectors. Note that in this particular asymptotic case the right singular vectors are shared by the channel matrices of all users and the projection performed in each step of the algorithm is equivalent to discarding the right singular vectors already chosen, which does not affect the value of the remaining eigenvalues in the projected matrices. As a consequence, this well structured broadcast channel results in  $t$  subchannels with gain  $\approx \sqrt{r}\sigma$ , and thus the resulting sum-rate is obtained as

$$R \approx t \log \left( \frac{r\sigma^2 \text{SNR}}{t} \right),$$

where SNR is the ratio between the total transmit power and the noise variance at the receivers. Note that  $R$  grows proportionally to  $\log(r)$ .

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