

Effective use of Long-term Transmit Channel State Information in Multi-user MIMO Communication Systems

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Abstract—In this paper, we propose a novel approach to the design of linear transmit processing for the downlink of multi-user multi-input multi-output (MU-MIMO) communication over fading wireless channels. Our purpose is to address the open problem of downlink transmit processing in MU-MIMO systems with only long-term CSI at the transmitter. In contrast to existing schemes, only long-term channel state information is required at the transmitter. For this to work, the MU-MIMO channel must experience correlated fading. The approach consists of two steps: 1) design of transmit processing which can deal with correlated fading channels but is allowed to use full CSI at first, and 2) conversion of the transmit processing to use long-term CSI only. As a solution for the first step, we propose a joint receive and transmit mean square error minimization scheme, while the second step is performed by an innovative subspace-based procedure. Performance evaluation shows the significant potential of the proposed approach.

I. INTRODUCTION

Transmit signal processing for the downlink of multi-user MIMO systems recently has been recognized as effective means of achieving higher bandwidth efficiency. With full channel state information (CSI) available to the transmitter, several algorithms have already been proposed [1], [2]. Solutions for the case when the channel is known only *on average* (*long-term* CSI) however, concentrate mainly on the single receive antenna case [3]. There is still a lack of solution for the multiple receive antenna case.

In this paper, we propose a novel transmit scheme for multiple receive and transmit antennas which makes effective use of long-term CSI in a spatially *correlated fading* multi-user environment. Our approach consists of two steps. To facilitate development, we begin with a transmit processing scheme that requires *full* CSI, but which is suitable for correlated fading channels. We propose to use an algorithm which employs a joint receive and transmit mean square error minimization, since this allows to adapt the number of data streams, which is crucial in a correlated fading environment. In the second step, we propose to use an innovative subspace-based procedure, which converts this transmit processing scheme into one which effectively works with *long-term* CSI only. It should be emphasized that this converting approach is fairly general. It can work with *any* transmit processing scheme which is based on full CSI and turn it into a pure long-term CSI scheme.

II. SYSTEM MODEL

We consider the downlink of a MU-MIMO system depicted in Fig. 1, where K independent receivers are served by a N -

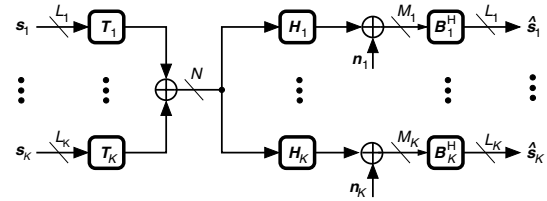


Fig. 1. Multi-user MIMO system with linear transmit and receive processing

antenna transmitter, and the i -th receiver ($i = 1, \dots, K$) is equipped with M_i receive antennas. Each user's frequency flat fading MIMO channel is described by a channel matrix $\mathbf{H}_i \in \mathcal{C}^{M_i \times N}$, where $(\mathbf{H}_i)_{p,q}$ is the complex transmit coefficient from the q -th transmit to the p -th receive antenna of user i . The L_i -dimensional data symbol vector \mathbf{s}_i for the i -th user is linearly transformed by the transmit weight matrix $\mathbf{T}_i \in \mathcal{C}^{N \times L_i}$ before being launched into the channel, producing the received signal which is corrupted by additive, zero-mean, white Gaussian noise $\mathbf{n}_i \in \mathcal{C}^{M_i}$ with power $\sigma_{n_i}^2$ and filtered by the receive matrix $\mathbf{B}_i^H \in \mathcal{C}^{M_i \times L_i}$, which outputs the L_i -dimensional vector

$$\hat{\mathbf{s}}_i = \mathbf{B}_i^H \mathbf{y}_i, \text{ where } \mathbf{y}_i = \mathbf{H}_i \sum_{k=1}^K \mathbf{T}_k \mathbf{s}_k + \mathbf{n}_i \quad (1)$$

is the received signal vector for user i , *before* receive processing. The superscript H refers to the complex conjugate transpose operation. The transmit data symbol vectors \mathbf{s}_i are modeled as i.i.d. complex Gaussian random vectors with zero mean, and unity correlation matrix, (i.e. all data streams of all users are mutually independent and have unity power). The total transmit power is given by $P_T = \sum_{k=1}^K \mathbb{E}_{\mathbf{s}_k} [\|\mathbf{T}_k \mathbf{s}_k\|_2^2] = \sum_{k=1}^K \text{tr} \mathbf{T}_k^H \mathbf{T}_k$, where the symbols tr , $\|\cdot\|_2^2$ and $\mathbb{E}_x[\cdot]$ refer to the trace, squared euclidian norm and expectation with respect to x , respectively. In the following $L_i \leq \min(M_i, N)$ and $K \leq N$ are assumed. Please note, that K is the number of users which are served in the same time slot and frequency band by spatial processing. The total number of users in the communication system can be much larger than K .

III. CHANNEL CAPACITY AND THROUGHPUT

The channel capacity of the downlink of a multi-user MIMO system (sometimes referred to as a broadcast channel) is still an open question [4]. In the following, we will make one additional assumption in order to compute the mutual information.

We assume *single-user coding*, which means that, channel coding is performed at the transmitter for each user *independently* and each user decodes only its dedicated signal. In this case the maximum mutual information $C_i = \max \mathcal{I}(s_i; \mathbf{y}_i)$, between the data vector s_i and the received vector \mathbf{y}_i equals

$$C_i = \log_2 \det \left(\mathbf{I}_{L_i} + \mathbf{T}_i^H \mathbf{H}_i^H \mathbf{R}_i^{-1} \mathbf{H}_i \mathbf{T}_i \right), \text{ where} \quad (2)$$

$$\mathbf{R}_i = \mathbf{H}_i \sum_{k=1, k \neq i}^K \mathbf{T}_k \mathbf{T}_k^H \mathbf{H}_i^H + \sigma_{n_i}^2 \mathbf{I}_{M_i} \quad (3)$$

is the correlation matrix of the noise and the spatial multi-user interference. Due to the assumption of Gaussian signaling and single-user coding, the interference from the received signals of the non-dedicated users are treated as (spatially colored) Gaussian noise. The symbol \mathbf{I}_q refers to the $q \times q$ identity matrix. Under the stated assumptions, we refer to (2) as the channel capacity for the i -th user. Since from the data processing theorem we always have $\mathcal{I}(s_i; \mathbf{y}_i) \geq \mathcal{I}(s_i; \hat{s}_i)$, the receive matrices \mathbf{B}_i are not considered in the capacity calculation. The purpose of including the matrices \mathbf{B}_i in the system model is merely to facilitate a better solution for the transmit matrices \mathbf{T}_i , as will be demonstrated in Section IV.

Due to the randomness of the channel matrices \mathbf{H}_i , the instantaneous channel capacities C_i become random variables. The ergodic capacity \bar{C}_i is defined as the expected value of the instantaneous capacity C_i , i.e.

$$\bar{C}_i = \mathbb{E}[C_i]. \quad (4)$$

This is the supported average data rate if channel coding can be performed over a large number of channel realizations (interleaving) or adaptive coding (e.g. incremental redundancy) is used. On the other hand, if channel codes with *constant* information rates R_i are used without interleaving, the supported average data rate is given by the *throughput*

$$\bar{R}_i = \max_{R_i} \mathbb{E} \left[\begin{array}{ll} R_i & \text{for } R_i < C_i \\ 0 & \text{else} \end{array} \right], \quad (5)$$

since error-free decoding is only possible if $R_i < C_i$.

IV. TRANSMIT PROCESSING WITH FULL CSI

Armed with complete CSI at the transmitter, the proposed joint transmit and receive MMSE processing aims to find the transmit weight matrices \mathbf{T}_i according to the MMSE criterion:

$$(\mathbf{T}_1, \dots, \mathbf{T}_K) = \arg \min_{\substack{\mathbf{T}_1, \dots, \mathbf{T}_K \\ \mathbf{B}_1, \dots, \mathbf{B}_K}} \mathbb{E} \left[\sum_{k=1}^K \|\mathbf{s}_k - \hat{\mathbf{s}}_k\|_2^2 \right]. \quad (6)$$

with the transmit power constraint $\sum_{k=1}^K \text{tr} \mathbf{T}_k^H \mathbf{T}_k = P_T$. The receive matrices \mathbf{B}_i allow us to find a MMSE-solution for the transmit weights which provides the users with given numbers L_i of data streams, which can be less than the number of receive antennas M_i . Proper choice of the L_i is crucial for a

multi-user system in a correlated fading environment, since there is a fundamental tradeoff between the number of data streams and the resulting multi-user interference. To solve (6) we propose the following iterative procedure. For given $\mathbf{T}_1, \dots, \mathbf{T}_K$ the optimum receive matrices become [5]

$$\mathbf{B}_i = \left(\mathbf{H}_i \sum_{k=1}^K \mathbf{T}_k \mathbf{T}_k^H \mathbf{H}_i^H + \sigma_{n_i}^2 \mathbf{I}_{M_i} \right)^{-1} \mathbf{H}_i \mathbf{T}_i, \quad (7)$$

while for given $\mathbf{B}_1, \dots, \mathbf{B}_K$ the optimum \mathbf{T}_i become [6]

$$\mathbf{T}_i = \mathbf{S}_i \sqrt{\frac{P_T}{\sum_{k=1}^K \text{tr} \mathbf{S}_k^H \mathbf{S}_k}}, \text{ where} \quad (8)$$

$$\mathbf{S}_i = \left(\sum_{k=1}^K \mathbf{H}_k^H \mathbf{B}_k \mathbf{B}_k^H \mathbf{H}_k + \beta \cdot \mathbf{I}_N \right)^{-1} \mathbf{H}_i^H \mathbf{B}_i \quad (9)$$

with $\beta = P_T^{-1} \sum_{k=1}^K \sigma_{n_k}^2 \cdot \text{tr} \mathbf{B}_k^H \mathbf{B}_k$. The solution is found by successive application of (7), (8) and (9). To get started we initialize the \mathbf{B}_i with random entries. The iteration terminates, when the Frobenius norm of the change in all \mathbf{T}_i and \mathbf{B}_i drops below some threshold. Please recall, that only the \mathbf{T}_i are needed for capacity calculation. This algorithm will be referred to as the TR-MMSE scheme.

V. LONG-TERM CHANNEL STATE INFORMATION

Let the channel matrices \mathbf{H}_i now be modeled as random variables. We assume the transmitter has knowledge about their *statistical properties*, however *no* knowledge about their actual value. For instance, consider spatially correlated Rayleigh fading channels that can be modeled by [7]

$$\mathbf{H}_i = \left(\text{tr} \mathbf{R}_i^{(\text{Tx})} \right)^{-1/2} \cdot \mathbf{R}_i^{(\text{Rx})1/2} \mathbf{G}_i \mathbf{R}_i^{(\text{Tx})1/2}, \quad (10)$$

where $\mathbf{R}_i^{(\text{Rx})} = \mathbb{E}[\mathbf{H}_i \mathbf{H}_i^H]$ and $\mathbf{R}_i^{(\text{Tx})} = \mathbb{E}[\mathbf{H}_i^H \mathbf{H}_i]$ are the receive and transmit correlation matrices, respectively, and $\mathbf{G}_i \in \mathcal{C}^{M_i \times N}$ are random matrices with i.i.d. zero mean, unity variance Gaussian entries. The transmitter only knows $\mathbf{R}_i^{(\text{Rx})}$ and $\mathbf{R}_i^{(\text{Tx})}$, and the distribution of the random entries of \mathbf{G}_i . We refer to this type of knowledge as *long-term* CSI.

VI. GENERAL CONVERSION TO LONG-TERM CSI USAGE

The transmit weight matrices $\mathbf{T}_i \in \mathcal{C}^{N \times L_i}$ are replaced by matrices $\mathbf{T}_i^{(\text{LT})} \in \mathcal{C}^{N \times L_i^{(\text{LT})}}$, which adapt only to the statistical properties of the channel, instead of its instantaneous value. We propose to choose $\mathbf{T}_i^{(\text{LT})}$ such that the $L_i^{(\text{LT})}$ -dimensional subspace $\text{range}\{\mathbf{T}_i^{(\text{LT})}\}$ captures the maximum signal power, that would have been transmitted *on average* if full CSI had been available. To be more specific, let $\mathbf{P}_i \in \mathcal{C}^{N \times N}$ be the projection matrix onto the subspace $\text{range}\{\mathbf{T}_i^{(\text{LT})}\}$. We choose

$$\mathbf{P}_i = \arg \max_{\mathbf{P}_i} \mathbb{E}_{\mathbf{T}_i} \left[\mathbb{E}_{\mathbf{s}_i} \left[\|\mathbf{P}_i \mathbf{T}_i \mathbf{s}_i\|_2^2 \mid \mathbf{T}_i \right] \right]. \quad (11)$$

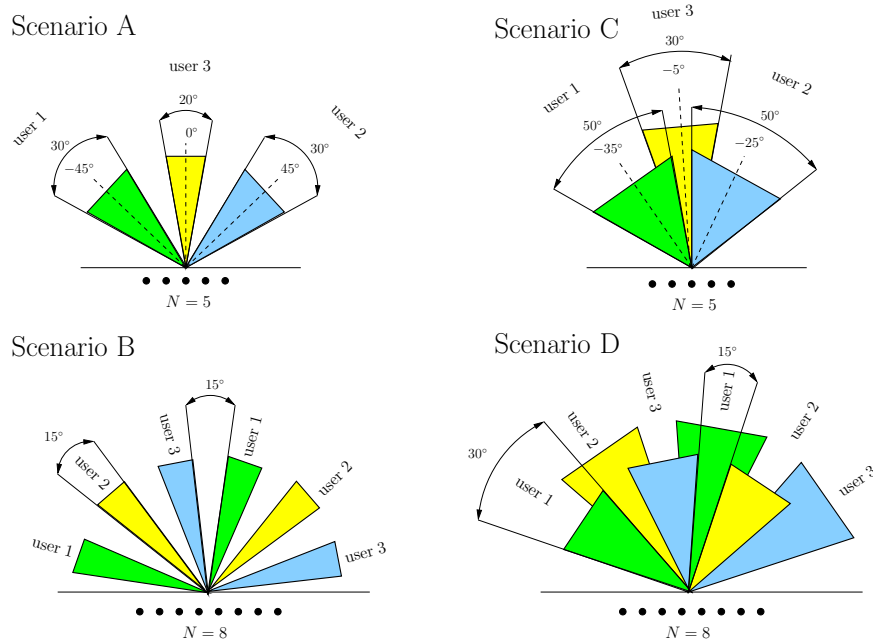


Fig. 2. Definition of four scenarios (A,B,C and D) of spatial transmit correlation. The $K = 3$ users can be reached by the transmitter in different spatial directions (azimuthal angles). Both non-overlapping (A,B) and overlapping angle spreads (C,D) are defined.

Here the matrices \mathbf{T}_i are the transmit weights obtained with full CSI, which become random variables due to the randomness of the channel. The solution can be expressed in terms of the eigenvalue decomposition of the matrix

$$\mathbb{E} [\mathbf{T}_i \mathbf{T}_i^H] = [\mathbf{U}_i \quad \mathbf{U}'_i] \begin{bmatrix} \mathbf{\Lambda}_i & \\ & \mathbf{\Lambda}'_i \end{bmatrix} \begin{bmatrix} \mathbf{U}_i^H \\ \mathbf{U}'_i{}^H \end{bmatrix}, \quad (12)$$

where $\mathbf{U}_i \in \mathcal{C}^{N \times L_i^{(LT)}}$ contains its $L_i^{(LT)}$ dominant eigenvectors. It can be shown that $\mathbf{P}_i = \mathbf{U}_i \mathbf{U}_i^H$, and the long-term transmit matrices become

$$\mathbf{T}_i^{(LT)} = \mathbf{U}_i \mathbf{A}_i, \quad \text{where } \mathbf{A}_i \in \mathcal{C}^{L_i^{(LT)} \times L_i^{(LT)}} \quad (13)$$

can be any invertible matrix, which satisfies

$$\text{tr } \mathbf{A}_i \mathbf{A}_i^H = \text{tr } \mathbf{T}_i^{(LT)} \mathbf{T}_i^{(LT)H} = \text{tr } \mathbb{E} [\mathbf{T}_i \mathbf{T}_i^H]. \quad (14)$$

In this way each user is assigned a transmit power which equals to the average transmit power when using full CSI. Writing $\mathbf{U}_i = [\mathbf{u}_{i,1}, \dots, \mathbf{u}_{i,L_i^{(LT)}}]$, the power which is transmitted in the subspace spanned by $\mathbf{u}_{i,j}$ when full CSI is available is given as $\|\mathbf{u}_{i,j}^H \mathbf{T}_i\|_2^2$. We propose to choose \mathbf{A}_i as

$$\mathbf{A}_i = \mathbf{\Lambda}_i^{1/2} \cdot (1 + \text{tr } \mathbf{\Lambda}'_i / \text{tr } \mathbf{\Lambda}_i)^{1/2}. \quad (15)$$

This not only satisfies (14) but also

$$\frac{\|\mathbf{u}_{i,j}^H \mathbf{T}_i^{(LT)}\|_2^2}{\|\mathbf{u}_{i,k}^H \mathbf{T}_i^{(LT)}\|_2^2} = \frac{\mathbb{E} [\|\mathbf{u}_{i,j}^H \mathbf{T}_i\|_2^2]}{\mathbb{E} [\|\mathbf{u}_{i,k}^H \mathbf{T}_i\|_2^2]}, \quad (16)$$

where $j, k = 1, \dots, L_i^{(LT)}$. This keeps the ratio of transmit powers assigned to each data stream for full and long-term CSI

processing equal on average. Note, that in case $L_i^{(LT)} = L_i$, we have $\mathbf{T}_i^{(LT)} \mathbf{T}_i^{(LT)H} = \mathbb{E} [\mathbf{T}_i \mathbf{T}_i^H]$. Selection of data stream numbers $L_i^{(LT)}$ can be used to optimize system performance, e.g. maximize average mutual information or throughput. The described procedure is fairly general, as *any* transmit processing scheme which is based on full CSI, can be put to work with long-term CSI, without modification.

VII. PERFORMANCE EVALUATION

The performance of the TR-MMSE scheme with full and long-term CSI is investigated by computer simulation. In all experiments we have 3 users which are equipped with 2 receive antennas. For simplicity we assume the noise power is the same for all users and equals σ_n^2 . We use the correlation model from (10) and allow for a rich scattering environment for the mobile *receivers*. This leads to *uncorrelated* fading at the receivers, i.e. all $\mathbf{R}_i^{(Rx)}$ are scaled unity matrices. The *transmitter* however experiences *spatial fading correlation*. In particular we investigate four different scenarios which are depicted in Figure 2. The users can be reached in different spatial directions (azimuthal angles of departure, AoD) with different angle-spread. For instance, scenario C defines an angle spread of 50° around the azimuthal angles of -35° and 25° for the first and second user, respectively, while the third user can be reached with an angle spread of 30° centered around the azimuthal angle of -5° . Note, the large spatial overlap with the first two users, which makes transmit processing based on long-term CSI a difficult task. The number of transmit antennas equals 5 in the scenarios A and C, while 8 transmit antennas are used in scenarios B and D. Omni-directional uniform linear antenna arrays with half-wavelength spacing are assumed throughout. Figures 3 till 6 show the performance

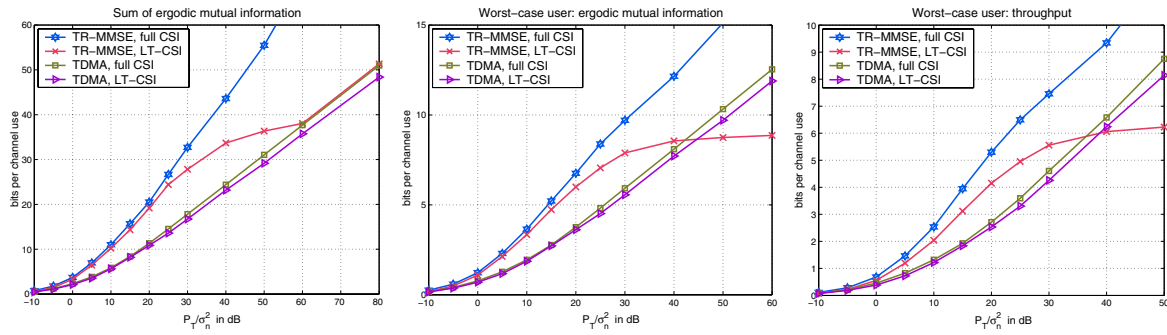


Fig. 3. Scenario A: the Figure shows the sum of the ergodic mutual information for all users (left), the ergodic mutual information for the worst-case user (middle) and the throughput of the worst-case user (right). The TR-MMSE operating with long-term CSI gives better performance than TDMA up to a mutual information of around 8 or a throughput of 6 bits per channel use for the worst-case user. Having only 2 receive antennas per user, these values are fairly large.

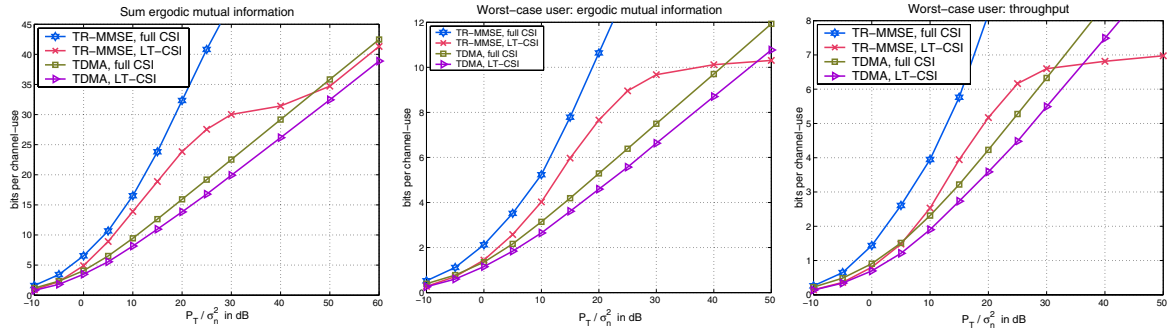


Fig. 4. Scenario B: the Figure shows the sum of the ergodic mutual information for all users (left), the ergodic mutual information for the worst-case user (middle) and the throughput of the worst-case user (right). The difference in performance between full and long-term CSI is larger than in scenario A. However, the TR-MMSE scheme with long-term CSI still outperforms TDMA in a wide range of mutual information and throughput, which is interesting in practice.

of the proposed multi-user MIMO transmit signal processing schemes with respect to three different optimization objectives:

- sum of the ergodic mutual information over all users
- ergodic mutual information of worst-case user
- throughput of worst-case user

We compare the performance of the TR-MMSE scheme with full CSI to the same scheme extended to use long-term CSI only. For both variants the number of data streams for the users are taken out of the set $\{0, 1, 2\}$. From the $3^3 - 1 = 26$ combinations the one which maximizes performance (with respect to one of the three different optimization objectives defined above) is chosen. As a reference we also consider a TDMA approach, where each user is assigned a different time slot.

A fundamental difference between the TR-MMSE scheme using full CSI and the same scheme using long-term CSI can be found in the amount of their ability to avoid multi-user interference in the *high* transmit power regime. While the TR-MMSE scheme with full CSI tends to decouple the users interference-free at high transmit powers, the long-term version always produces a finite amount of interference if it has to serve more than one user. The ergodic mutual information or throughput of the *worst-case* user therefore saturates when only long-term CSI is available. Since TDMA has no multi-user interference it always outperforms the long-term TR-MMSE for sufficiently large transmit powers in terms of the worst-case user mutual information or throughput. In the

low transmit power regime however, the long-term TR-MMSE may offer significant improvement in performance compared to TDMA. Also note, that the *sum* of ergodic mutual information does *not* saturate, for the long-term TR-MMSE scheme, since it is possible to set the number of data streams such, that only one user is served at high transmit power.

Considering the results for scenario A in Figure 3, we can see, that there is almost no loss in performance for the TR-MMSE scheme when switching from full to long-term CSI until fairly large values of both the sum and the minimum of the ergodic mutual information. The performance differs significantly only in the high transmit power regime, due to the multi-user interference problem discussed above. It is interesting to note, that for ergodic mutual information of less than 8 bits per channel use for the worst-case user, the long-term TR-MMSE scheme outperforms the TDMA approach by as much as 10dB in terms of transmit power. A similar improvement can also be observed for the throughput of the worst-case user up to the value of around 6 bits per channel use.

In scenario B the users are slightly more difficult to separate due to the alternating nature and close proximity of the user's AoDs. With the help of 8 transmit antennas, however, the performance of the long-term TR-MMSE is still fairly well, as is shown in Figure 4.

In scenario C we introduce a significant overlap in the AoDs of the users. Note, that user 3 can only be reached within an angle-spread of 10° without interfering with the other two

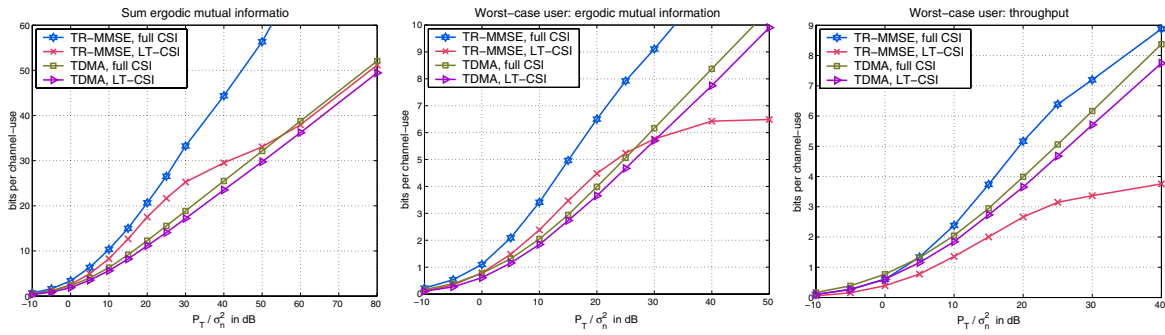


Fig. 5. Scenario C: the Figure shows the sum of the ergodic mutual information for all users (left), the ergodic mutual information for the worst-case user (middle) and the throughput of the worst-case user (right). This is a difficult scenario when only long-term CSI is available, which is due to the combination of large spatial overlap (especially for user 3) and the relatively low number of 5 transmit antennas. Interestingly, in terms of ergodic mutual information the TR-MMSE scheme with long-term CSI still yields better performance than TDMA in a reasonably large range of mutual information. With respect to throughput of the worst-case user however, the TDMA scheme always achieves better performance.

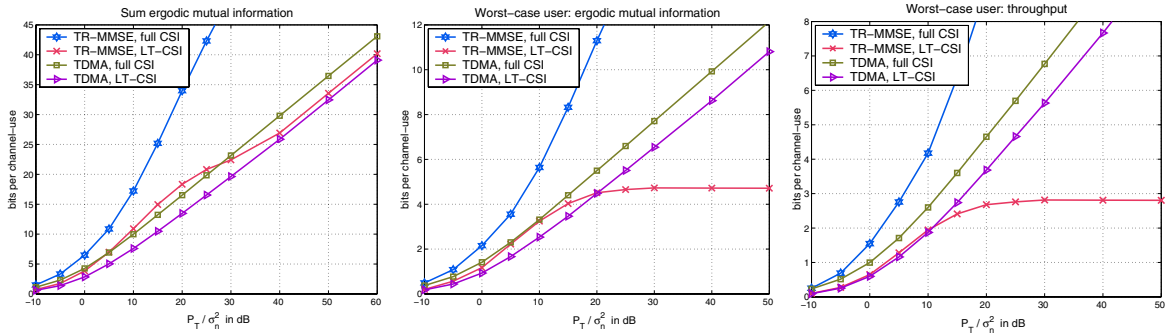


Fig. 6. Scenario D: the Figure shows the sum of the ergodic mutual information for all users (left), the ergodic mutual information for the worst-case user (middle) and the throughput of the worst-case user (right). Similarly to scenario C, this is also a tough situation for the transmitter when only long-term CSI is available. This is because of the large spatial overlap between all users. However, compared to the scenario C, the increased number of 8 transmit antennas allows for higher spatial resolution.

users. Considering a resolution of the 5-antenna uniform linear array of around 25° in bore-side direction, it is quite clear, that this scenario represents a tough situation for the long-term TR-MMSE approach. Interestingly, in terms of ergodic mutual information the TR-MMSE scheme with long-term CSI still yields better performance than TDMA in a reasonably large range of mutual information, as can be seen from the left and middle diagrams in Figure 5. With respect to throughput of the worst-case user however, the TDMA scheme always achieves better performance. This shows the limitations of long-term transmit signal processing. If throughput is the figure of merit, a practical solution would be to assign user 3 to another time or frequency slot.

Scenario D also introduces strong overlap in the alternating AoDs of the users. The larger number of 8 antennas however leads to a better resolution. Figure 6 shows, that for throughputs of the worst case user of less than about 2 bits per channel use, the long-term TR-MMSE scheme offers slight improvement (around 1dB) compared to TDMA. The improvement is somewhat larger (3dB) in terms of ergodic mutual information.

VIII. CONCLUSION

A novel approach for using long-term CSI for the design of linear downlink transmit processing for multi-user MIMO

signaling over correlated fading channels is presented. The approach is based on the conversion of a transmit processing scheme originally designed for full CSI into one that uses long-term CSI only. A simple, yet effective subspace based procedure is introduced, which solves this conversion problem. While applicable for a variety of transmit processing schemes, this procedure works especially well in conjunction with a joint receive and transmit MMSE scheme. Performance evaluation shows capabilities and limitations of the proposed approach.

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