An SDMA system based on spatial covariances

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ABSTRACT

The aim of SDMA is the channel reuse within a mobile radio cell through spatial filtering techniques exploiting the different directions of arrival (DOAs) of the user signals. The performance of high-resolution DOA estimation algorithms may suffer from model errors introduced by diffuse multipath propagation. In this paper, we will present a robust SDMA system concept based on the estimation of spatial covariance matrices rather than of DOAs, therefore being immune to phenomena like diffuse multipath and/or high angular spread.

1. Introduction

The basic idea of SDMA is the RWC (reuse within cell) of a radio channel, incorporated by an FDMA, TDMA or CDMA slot, by $K > 1$ different users.

On the uplink, the spatial separation of K signals can be done by means of the information supplied by an $M \geq K$ element base station antenna array without explicitly estimating the DOAs of the corresponding sources. One way to do this is to exploit the knowledge of the user-specific training sequences to estimate the fast fading uplink channel impulse responses. These channel estimates can then be used by a linear [1] [2] or non-linear [3] [4] data detector yielding estimates for the symbols transmitted by each user.

Since the mobiles are supposed to be equipped with a single antenna only, there is no way of spatial filtering at the mobiles. Therefore, the spatial separation of K users receiving their signals in the same channel has to be done by beamforming at the base station. Unfortunately, in most present mobile radio systems the channel impulse responses estimated on the uplink cannot be directly used for downlink beamforming due to the frequency and/or time gap between the uplink channel and the downlink channel [5] [6].

In this paper we will show, that there are parameters which can be estimated on the uplink and reused for downlink transmission, even if there is both a frequency and a time gap between uplink channel and downlink channel. We will also present an algorithm operating on the uplink receive signals to produce estimates of these parameters without employing explicit direction finding techniques.

The mentioned parameters are the medium-term spatial covariance matrices (SCM) $\boldsymbol{C}_k$ of the users $k = 1 \cdots K$, averaged over the fast fading. In case of a M antenna array the Hermitian $M \times M$ matrix $\boldsymbol{C}$ is defined as:

$$\boldsymbol{C} = \int_{-\pi}^{+\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} |b(\psi,\theta)|^2 \, \boldsymbol{a}(\psi,\theta,\lambda) \, \boldsymbol{a}^H(\psi,\theta,\lambda) \, d\theta \, d\psi \,, \qquad (1)$$

with $b(\psi, \theta, \lambda)$ denoting the channel transfer function of amplitude and phase relevant for the azimuth ψ and the elevation θ. Vector $\boldsymbol{a}(\psi, \theta, \lambda)$ denotes the corresponding array response vector (or "steering vector") for the carrier wavelength λ.

For scenarios with a limited number Q of "discrete" DOAs (1) turns into

$$\boldsymbol{C} = \sum_{q=1}^{Q} |b(\psi_q,\theta_q)|^2 \, \boldsymbol{a}(\psi_q,\theta_q,\lambda) \, \boldsymbol{a}^H(\psi_q,\theta_q,\lambda). \qquad (2)$$

The equation (1) exhibits that the spatial covariance matrix $\boldsymbol{C}$ is in fact a function of the carrier wavelength λ so that in FDD systems the matrix $\boldsymbol{C}(\lambda_{up})$ estimated on the uplink differs from the matrix $\boldsymbol{C}(\lambda_{dl})$ relevant for downlink transmission.

2. Direct Estimation of the SCM during the Signalling Procedure

If it is assured that no mobile but the corresponding user k is transmitting the user-specific spatial covariance matrix $\boldsymbol{C}_k$ could easily be estimated by means of the array receive vectors

$$\tilde{\boldsymbol{x}}_k(t) = \sum_{q=1}^{Q_k} b_{kq}(\psi_q,\theta_q) \, \boldsymbol{a}_{kq} s_k(t-\tau_{kq}) \cdot p_k + \boldsymbol{n}(t) \qquad (3)$$

sampled when only the user k is on air. Q_k denotes the total number of multipath components, p_k the transmit power, τ_{kq} the time delay and $\boldsymbol{n}(t)$ the additive (thermal) noise. By averaging over sufficient time window length that is long enough to compensate the fast fading but within the lognormal distributed fading the expectation value

$$\mathrm{E}\left\{\tilde{\boldsymbol{x}}_k(t)\tilde{\boldsymbol{x}}_k^H(t)\right\}/p_k \approx \boldsymbol{C}_k \tag{4}$$

of the autocorrelated receive vectors of user k normalized towards the transmit power p_k is approximately its spatial covariance matrix (SCM).

These kind of estimation of the SCM is applicable when a user is just booking in during the signalling phase.

Updating of the SCM during the SDMA operation however requires that the system can switch users off freely or that the users are switched of randomly via voice activity detection. When v_k is the probability that a user k is transmitting then the probability that user k is the only one on air is equal to

$$P_k = v_k \cdot \prod_{\substack{i=1 \\ i\neq k}}^{K} (1 - v_i) \quad . \tag{5}$$

Fig.1 shows for different total numbers of users K the

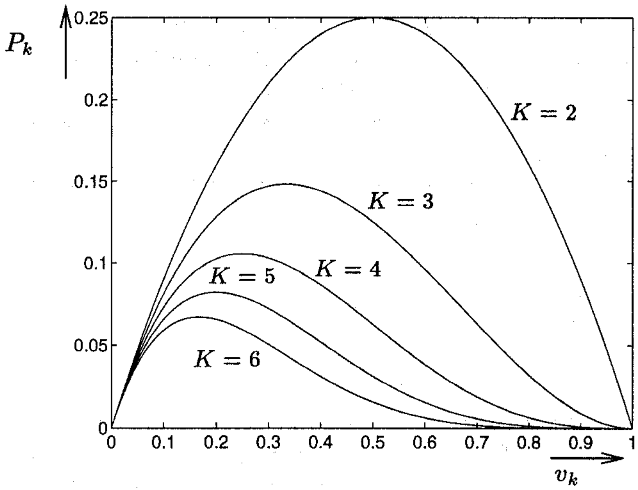

Figure 1: Propability P_k as a function of v_k ($v_1 = v_2 = \ldots$)

resulting probability P_k that user k is transmitting while the other users are silent.

Defining matrix $\tilde{\boldsymbol{X}}_k$ to be composed of N samples of the array receive vectors $\tilde{\boldsymbol{x}}_k(t)$ averaging the burst autocorrelation matrices by a forgetting factor μ

$$\hat{\boldsymbol{C}}_k := (1-\mu)\cdot\hat{\boldsymbol{C}}_k + \mu\cdot\tilde{\boldsymbol{X}}_k\tilde{\boldsymbol{X}}_k^H\cdot\frac{1}{N}\cdot\frac{1}{p_k} \tag{6}$$

that is adapted to the channel properties like fading time variance improves and updates the estimate of the spatial covariance matrix $\hat{\boldsymbol{C}}_k$.

3. Estimation of the SCM under SDMA operation

If the system cannot switch users off or the probability P_k (Fig.1) is very small the user specific sample vectors $\tilde{\boldsymbol{x}}_k$ might be hardly available. In this case it seems to be more advisible to estimate the matrices $\boldsymbol{C}_1\cdots\boldsymbol{C}_K$ by exploiting the knowledge of a-priori known user-specific training sequences or of the detected user data like these means are commonly applied for multi-user detection [1], [2], [3] and [4].

3.1. Estimation by Training Sequences

Assuming that L will be the length of the orthogonal training sequence (e.g. $L = 26$ in the GSM system) and C the channel memory length (e.g. $C = 5$) requires $P = L - C + 1$ roughly synchronized (time offset t_0) samples of the array receive vector $\tilde{\boldsymbol{x}}(t)$:

$$\tilde{\boldsymbol{X}} = [\tilde{\boldsymbol{x}}(t_0+\Delta t)\ \tilde{\boldsymbol{x}}(t_0+2\Delta t)\ \ldots\ \tilde{\boldsymbol{x}}(t_0+P\Delta t)] \tag{7}$$

with sampling period Δt. Now, the matrix

$$\boldsymbol{H} = [\ \boldsymbol{H}_1\quad \boldsymbol{H}_2\quad \ldots\quad \boldsymbol{H}_K\ ] \tag{8}$$

that is composed of the K the $M\times C$ channel impulse response matrices $\boldsymbol{H}_i$ can be easily computed by the pseudo-inverse of matrix

$$\boldsymbol{S} = \left[\begin{array}{ccc} \sqrt{p_1}s_{1,C}\cdots & \sqrt{p_1}s_{1,2C-1}\cdots & \sqrt{p_1}s_{1,L} \\ \vdots\quad\ddots & \vdots & \vdots \\ \sqrt{p_1}s_{1,1}\cdots & \sqrt{p_1}s_{1,C} & \cdots\sqrt{p_1}s_{1,L-C+1} \\ \hline \vdots & \vdots & \vdots \\ \hline \sqrt{p_K}s_{K,C}\cdots & \sqrt{p_K}s_{K,2C-1}\cdots & \sqrt{p_K}s_{K,L} \\ \vdots\quad\ddots & \vdots & \vdots \\ \sqrt{p_K}s_{K,1}\cdots & \sqrt{p_K}s_{K,C} & \cdots\sqrt{p_K}s_{K,L-C+1} \end{array}\right] . \tag{9}$$

$s_{k,j}$ denotes the jth sample of the modulated baseband signal of user k. After the LS-solution

$$\hat{\boldsymbol{H}} = \tilde{\boldsymbol{X}}\cdot\boldsymbol{S}^H\left(\boldsymbol{S}\boldsymbol{S}^H\right)^{-1} \tag{10}$$

the autocorrelation matrix of the channel impulse response estimate $\hat{\boldsymbol{H}}_k$ delivers the requested spatial covariance matrix

$$\hat{\boldsymbol{C}}_k := \hat{\boldsymbol{H}}_k\hat{\boldsymbol{H}}_k^H \quad . \tag{11}$$

Similarly to equation (6) an updating can be executed

$$\hat{\boldsymbol{C}}_k := (1-\mu)\cdot\hat{\boldsymbol{C}}_k + \mu\cdot\hat{\boldsymbol{H}}_k\hat{\boldsymbol{H}}_k^H \quad . \tag{12}$$

3.2. Estimation exploiting Data Detection

When the data detection of the burst is finished the remodulated symbols of this burst can be used to yield better spatial covariance matrix estimates due to the larger sample length $P = L - C + 1$ ($L \approx 150$ in the GSM system).

The procedure is identical to that desribed in the equations from (7) to (12).

4. On the Frequency dependence of spatial Covariance Matrices

The following example will demonstrate that in practice there are good reasons to consider the frequency gap between uplink and downlink negligible. Let us assume we have perfect estimates of the spatial covariance matrices $C_1(\lambda_{up})$ and $C_2(\lambda_{up})$ relevant for the uplink channels of two users $k = 1, 2$ transmitting at the carrier frequency $f_{up} = 890$ MHz. A beamforming procedure similar to the ones presented in [7] and [8] operating on these estimates will yield the beamforming weights $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$, which are optimized in a way that they will cause minimal downlink transmit power at the array while guaranteeing a signal-to-noise-and-interference ratio $\text{SNIR}_1 = \text{SNIR}_2 = 10$ dB for both users receiving on the downlink. Employing these weights for downlink transmission at the carrier frequency $f_{dl} = 960$ MHz (completely ignoring the frequency gap) will cause the signal-to-noise-and-interference ratios SNIR_1^* and SNIR_2^*.

The maximum SNIR loss

$$\Delta\text{SNIR} = \max_{k=1,2} (\text{SNIR}_k - \text{SNIR}_k^*) \tag{13}$$

based on perfectly known spatial covariance matrices is depicted in Fig.2 for a uniform linear array (ULA) and a uniform circular frame array (UCFA) with $M = 8$ antennas and the array spacing $d = 16.2$ cm. In both cases the first user directly looks at the array boresight ($\psi_1 = 0$), whereas the azimuth ψ_2 of the second user varies from 0^0 to 360^0. The channels are characterized by a single DOA per user with the elevation $\theta = 0^0$ and a Gaussian azimuthal distribution with the standard deviations ("angular spreads") $\sigma = 0^0$ (discrete DOA), $\sigma = 5^0$ and $\sigma = 10^0$, respectively.

Fig.2 exhibits that even for the above scenarios with a relative FDD frequency offset of $(f_{dl} - f_{up})/(\frac{1}{2}f_{dl} + \frac{1}{2}f_{up}) = 7.6\%$ (representing the worst case in GSM) the SNIR loss does not exceed 1.7 dB in this scenario. Note that there is the possibility to further reduce the SNIR loss modifying the beamforming weights $\boldsymbol{w}_k$ obtained for the uplink carrier wavelength λ_{up} in a way that they better match the downlink carrier wavelength λ_{dl}, since both wavelengths are known to the system. Such a correction, however, needs to use explicit knowledge of the array geometry.

By comparison with Fig.3 where an array spacing of $d = 15$ cm has been choosen it becomes clear that SNIR loss is the smaller the smaller the aperture of the antenna array is at the expense of uplink and downlink transmit power. The same applies to the comparison between a uniform linear array (ULA) with an aperture of $(M-1)\cdot d$ and a uniform circular array (UCFA) with an aperture of around $\frac{d}{\sin\frac{360^o}{2M}}$.

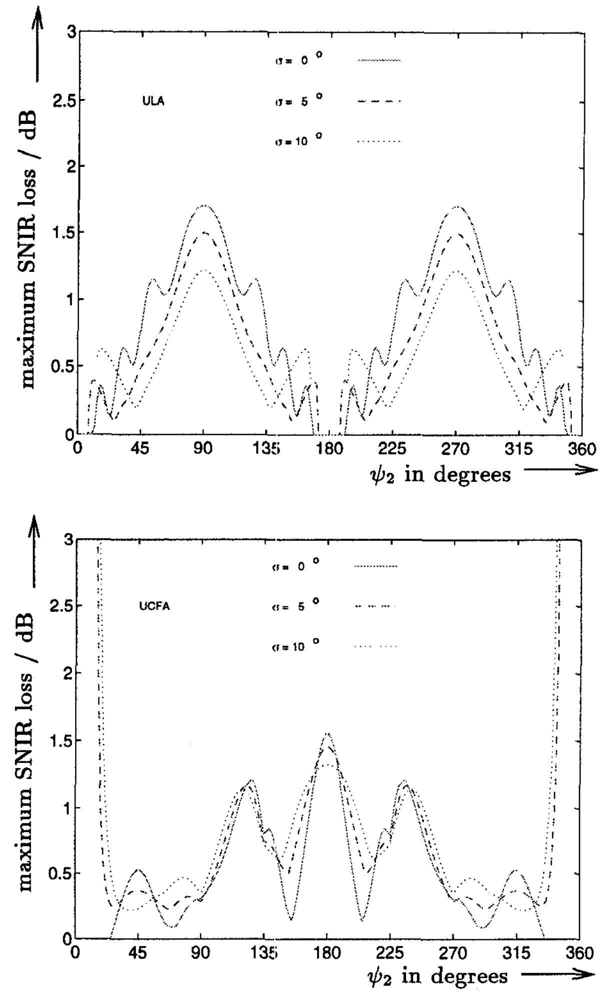

Figure 2: Maximum SNIR loss versus the average azimuth ψ_2 of the second user at the frequency of 965 Mhz, $d = 16.2$ cm

5. Simulations

In the following simulations a Gaussian angular spread of $\sigma = 5$ oder 15^o has been choosen. The azimuthal directions of the 2 users are $\psi_1 = 0^o$, $\psi_2 = 50^o$ and their transmit power 0 dB. The ULA consists of $M = 8$ antennas. The spatial covariance matrices estimated according to section 3.1 at the uplink carrier frequency $f_{up} = 890$ MHz are directly applied to compute the beamforming weights according to [7]. The resulting downlink SNIR measured by the mobiles is evaluated at the downlink carrier frequency $f_{dl} = 965$ MHz.

Fig.4 shows the SNIR as a function of time. Typical are the deep fades due to multipath propagation. The updating of the spatial covariance matrix (12) with a forgetting factor $\mu = 0.02$ improves the SNIR clearly within the first 50 bursts.

In the next picures the averaged SNIR and downlink transmit power are plotted as a function of the relative uplink noise power (mostly caused by intercell and cochannel interference). The SNIRs (Fig.5 and 7) and the downlink transmit power (Fig.6 and 8) keep constant wi-

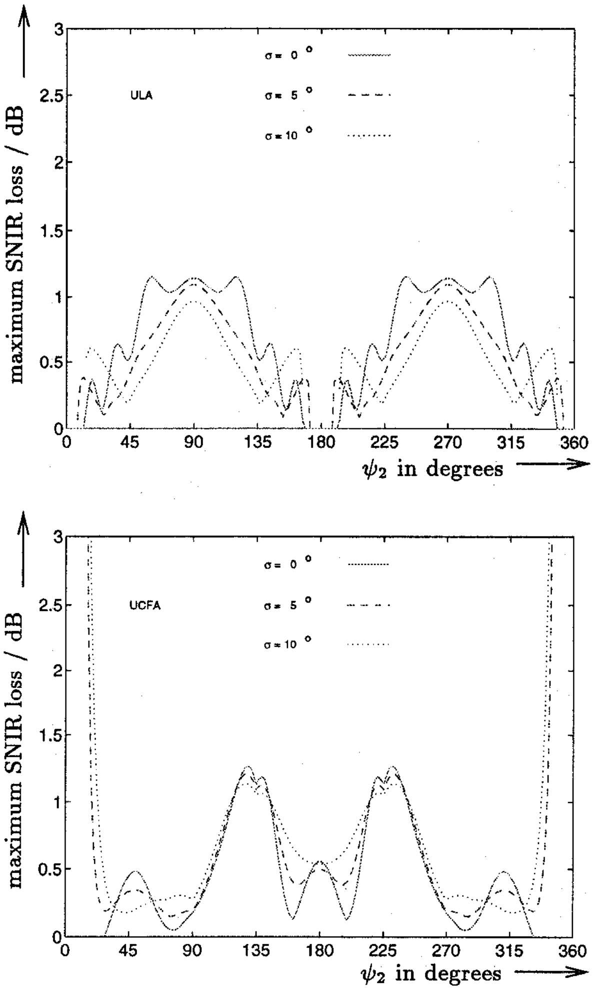

Figure 3: Maximum SNIR loss versus the average azimuth ψ_2 of the second user at the frequency of 965 Mhz, $d = 15$ cm

thin their standard deviation.

By comparison of Figs.5, 6 and Figs.7, 8 it becomes clear that an angular spread of $\sigma = 15^o$ induces a performance degradation, firstly by an increased transmit power and secondly by a breakdown in computing the beamforming weights in case of high noise powers in the uplink.

6. Conclusions

In spite of the performance degradation caused by the FDD frequency offset the presented algorithm seems to be an interesting alternative to direction finding methods because of the following advantages:

- In contrast to the employment of DOA estimation algorithms like Unitary ESPRIT [9], 2D Unitary ESPRIT [10] or Root-MUSIC [11], the estimation of spatial covariance matrices does not imply any restrictions on the array geometry.
- Array calibration melts down to maintaining the reciprocity of each receiver/transmitter chain at the antennas. There is no need to enforce the gain, the phase shift and the angular sensitivity pattern of each antenna to be identical to those of all other antennas in the array.

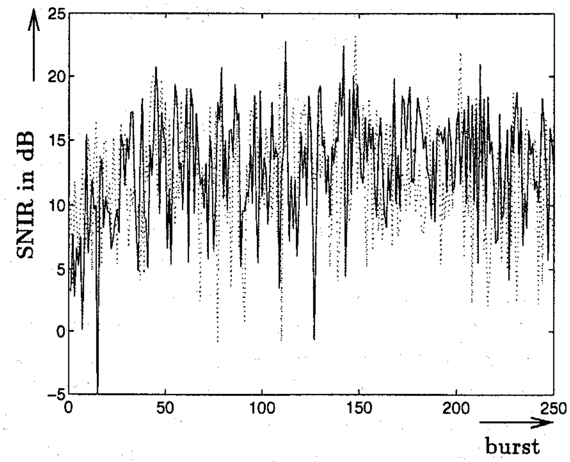

Figure 4: output SNIR as a function of time 2. method, $\sigma = 5°$, $\mu = 0.02$, relative noise power of 0 dB

- DOA estimation algorithms implicitly assume "discrete DOA channels", i.e. discrete spatial power density spectra $|b(\psi, \theta)|^2$. Moreover, even in discrete DOA channels they only work reliably, if the number Q of dominant DOAs does not exceed a certain limit Q_{max}. Therefore, these methods exhibit performance degradations in mobile radio scenarios characterized either by diffuse multipath ($Q \gg Q_{max}$) or by a continuous power density spectrum with a large angular spread.

  On the other hand, the spatial covariance matrix is well defined for any channel situation (see (1)). Hence, beamforming techniques based on these matrices rather than based on DOAs are not impaired by channel situations like the ones described above.

- By generalizing the far-field/narrowband definition (1), spatial covariance matrices can also be used for SDMA applications in near-field and wideband scenarios.

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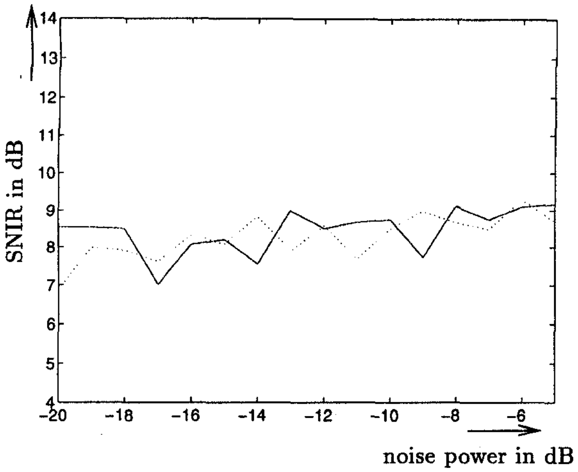

Figure 5: SNIR as a function of noise power, $\sigma = 5^o$

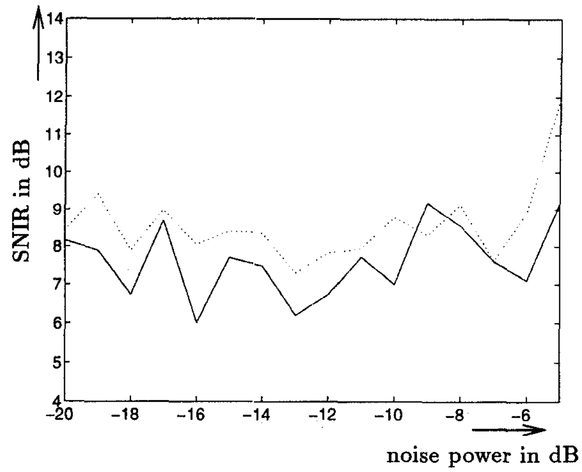

Figure 7: SNIR as a function of noise power, $\sigma = 15^o$

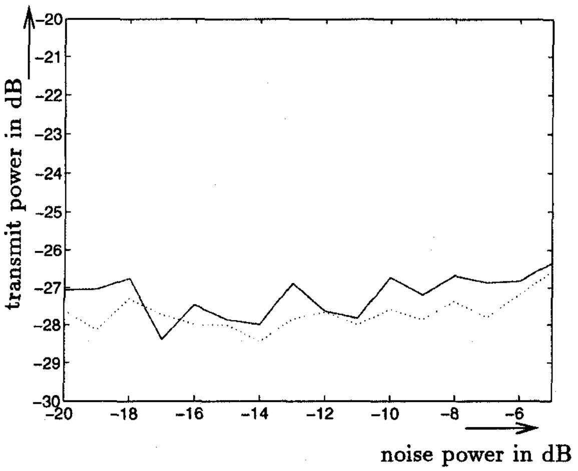

Figure 6: transmit power as a function of noise power, $\sigma = 5^o$

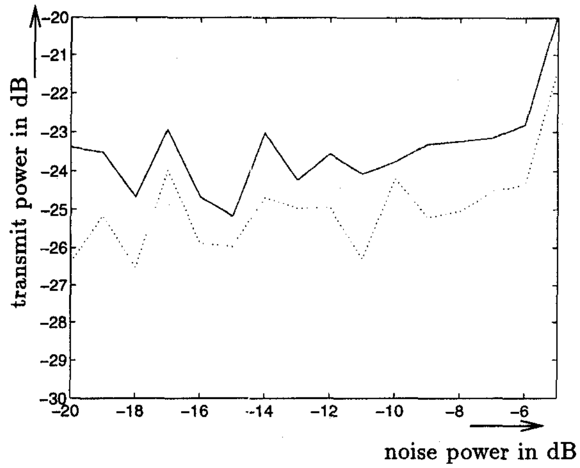

Figure 8: transmit power as a function of noise power, $\sigma = 15^o$

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