CHANNEL ALLOCATION AND DOWNLINK BEAMFORMING IN AN SDMA MOBILE RADIO SYSTEM

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ABSTRACT

On the downlink of an SDMA mobile radio system, co-channel interference has to be kept down by beamforming. In this paper two DOA-based beamforming approaches are presented minimizing array signal power while maintaining given signal-to-noise-and-interference ratios for all users. The linear approach is computationally cheap, so that it is suited for channel allocation as well, quickly evaluating the spatial separability of a specific mobile radio scenario. The nonlinear algorithm yields optimum results but only converges quickly, if provided with a good starting point. Therefore, its basic application is burst-to-burst updating of the beamformer.

1. DATA MODEL

1.1. Channel model

We assume an SDMA mobile radio system with K mobile users in the same cell, time slot and frequency band like the one described in [1]. The basestation of the SDMA cell is equipped with a linear array of M antennae equally spaced at distance d. The distances from all users, reflectors and scatterers to the array are assumed to be much larger than d, so that all waves reaching the array are planar (far-field approximation). Moreover, the bandwidth of all baseband signals received from (rsp. transmitted to) the users is considered to be much smaller than the reciprocal of the maximum time $\Delta t = (M-1) \cdot d/c_0$ that a planar radio wave with velocity c_0 of propagation needs to cover the length of the antenna array (narrow-band approximation).

Throughout this paper, vectors that are always considered to be column vectors, are denoted by lower case bold faced letters and matrices by upper case bold faced letters. All elements $v(i)$ of any vector $\boldsymbol{v}$ and all elements $M(i,k)$ of any matrix $\boldsymbol{M}$ are complex valued. The symbol $(\ldots)^H$ designates the complex conjugate transposition.

On the uplink and on the downlink we consider a discrete direction of arrival (DDA) channel. The signals produced by the K users reach the base station through a limited number of discrete propagation paths. Each of them is supposed to consist of a high number of subpaths produced by reflections, diffractions and scatterings in the close vicinity of the mobile. The subpaths have individual amplitudes, phase shifts and Doppler frequencies [2], but as they are produced in a comparably small area, they all have almost identical directions of arrival (DOAs) and time delays at the base station. Therefore, each discrete propagation path can be defined by a flat-fading complex amplitude b, a DOA θ and a delay τ.

Each user k is assumed to produce Q_k propagation paths. We will regard a medium-term mobile radio channel situation of a duration between 100 ms and 10 s. The properties of the channel parameters are postulated as follows:

- Over the mentioned period, the numbers $Q_1 \ldots Q_K$ of dominant propagation paths as well as the corresponding DOAs do not change.
- The complex amplitudes $b(t)$ of the $Q = \sum_{k=1}^{K} Q_k$ paths are samples of independent, quickly time-variant stochastic processes. Their absolute values $\|b(t)\|$ are Dirac-, Rice- or Rayleigh-distributed [3] [4]. Their phases $arg\,(b(t))$ are assumed to be equally distributed in the range $]-\pi;\pi]$.

1.2. Useful and interference power

Beamforming is done by multiplying the baseband signal $s_k(t)$ bound for user k by a specific complex factor $w_{k,m}$ at each antenna m before modulation and transmission. The vector $\boldsymbol{w}_k$ containing the M complex factors $w_{k,m}$ will be called a beamforming vector. Arran-

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ging the signals transmitted from the M antennae in the vector $\boldsymbol{a}(t)$ yields

$$\boldsymbol{a}(t) = \sum_{k=1}^{K} s_k(t)\boldsymbol{w}_k, \qquad (1)$$

Thus the signal $u(t)$ received by a user through a single path of propagation can be expressed in complex baseband notation as follows:

$$u(t) = b(t)\,\boldsymbol{p}^H \boldsymbol{a}(t-\tau). \qquad (2)$$

The phase shifts of the radio carrier caused by the propagation delays within the array are incorporated by the steering vector

$$\boldsymbol{p} = \begin{pmatrix} \Phi^0 & \ldots & \Phi^{M-1} \end{pmatrix}^H, \;\; \Phi = e^{-2\pi j f_0 d \sin\theta / c_0}, \qquad (3)$$

with the carrier frequency denoted by f_0.
In the following sequel we will use the term $S_{l,k}$ to denote the expectancy of the signal power received by user l and caused by the vector $\boldsymbol{w}_k$. If $k = l$ holds, $S_{l,k}$ will be regarded as useful power, whereas in all other cases $S_{l,k}$ represents interference power. Without constraining the validity of our channel model we will assume $E\{\|s_k(t)\|^2\} = 1$, so that $S_{l,k}$ can be written as

$$S_{l,k} = \sum_{q=1}^{Q_l} E\{\|b_{l,q}(t)\|^2\}\, \boldsymbol{w}_k^H \boldsymbol{p}_{l,q} \boldsymbol{p}_{l,q}^H \boldsymbol{w}_k. \qquad (4)$$

To get (4) in matrix notation, we will define a path vector $\boldsymbol{r}_{l,q}$ corresponding to user l and path q:

$$\boldsymbol{r}_{l,q} = \sqrt{E\{\|b_{l,q}(t)\|^2\}}\, \boldsymbol{p}_{l,q}. \qquad (5)$$

It incorporates the DOA of the path as well as the average power transmitted through it. All Q_l path vectors $\boldsymbol{r}_{l,q}$ of user l are arranged in the path matrix

$$\boldsymbol{R}_l = \begin{pmatrix} \boldsymbol{r}_{l,1} & \ldots & \boldsymbol{r}_{l,Q_l} \end{pmatrix}. \qquad (6)$$

We will also define a $M \times (Q - Q_l)$-matrix

$$\boldsymbol{T}_l = \begin{pmatrix} \ldots & \boldsymbol{R}_n & \ldots \end{pmatrix}, \quad \text{with}\, n = 1 \ldots K \wedge n \neq l, \qquad (7)$$

that is made up by the path vectors of all other users. Finally, the average useful power and the average interference power produced by the weight vector $\boldsymbol{w}_k$ are given by

$$S_{k,k} = \boldsymbol{w}_k^H \boldsymbol{R}_k \boldsymbol{R}_k^H \boldsymbol{w}_k \quad , \quad \sum_{\substack{l=1 \\ l\neq k}}^{K} S_{l,k} = \boldsymbol{w}_k^H \boldsymbol{T}_k \boldsymbol{T}_k^H \boldsymbol{w}_k. \qquad (8)$$

2. CALCULATION OF THE BEAMFORMING VECTORS

In mobile radio channels directly operating on the rapidly changing spatial signatures of the users as done in [5] and [6] is critical. Therefore, only DOA-based beamforming approaches will be presented in the following sequel.

2.1. Acquisition of the channel parameters

As (8) shows, the useful power and the interference power produced by the array are dependent not only on the beamforming vectors $\boldsymbol{w}_k$ but also on the path matrices $\boldsymbol{R}_l$ of the users. These matrices incorporate time variant DDA channel parameters that have to be estimated prior to doing any beamforming. According to (3), (5) and (6), the following DDA channel parameters have to be known:

- The numbers $Q_1 \ldots Q_K$ of dominant propagation paths per user.
- The $Q = \sum_{l=1}^{K} Q_l$ corresponding DOAs.
- The Q expectancies $E\{\|b_{l,q}(t)\|^2\}$ of the squared absolute values of the complex amplitudes.

The authors favor the acquisition of the mentioned parameters during uplink reception at the base station. According to [3], the period from uplink transmission to downlink reception has to be shorter than 100 ms. We assume that during this period the numbers of propagation paths as well as their DOAs do not change decisively.

2.2. Interference minimization

One idea to minimize the total signal power P that has to be produced by the array is to calculate the K weight vectors $\boldsymbol{w}_k$ in a way that each of them produces a maximum ratio of useful power to interference power. According to (8) K Rayleigh quotients

$$\frac{S_{k,k}}{\sum_{\substack{l=1 \\ l\neq k}}^{K} S_{l,k}} = \frac{\boldsymbol{w}_k^H \boldsymbol{R}_k \boldsymbol{R}_k^H \boldsymbol{w}_k}{\boldsymbol{w}_k^H \boldsymbol{T}_k \boldsymbol{T}_k^H \boldsymbol{w}_k} \qquad (9)$$

have to be maximized. In order to find a representative unit vector $\boldsymbol{x}_k = \boldsymbol{w}_k / \|\boldsymbol{w}_k\|$ maximizing (9), first the singular value decomposition (SVD)

$$\boldsymbol{T}_k = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H \qquad (10)$$

is calculated. If $\boldsymbol{T}_k$ has rank M, $\boldsymbol{x}_k$ is a unit eigenvector corresponding to the largest eigenvalue of the matrix

$$\left(\boldsymbol{T}_k\boldsymbol{T}_k^H\right)^{-1} \boldsymbol{R}_k \boldsymbol{R}_k^H = \boldsymbol{U}\left(\boldsymbol{S}\boldsymbol{S}^H\right)^{-1} \boldsymbol{U}^H \boldsymbol{R}_k \boldsymbol{R}_k^H. \qquad (11)$$

Otherwise $\boldsymbol{x}_k$ is element of the null space of $\boldsymbol{T}_k$. An orthogonal base $\bar{\boldsymbol{U}}$ spanning this null space can be easily extracted from $\boldsymbol{U}$ so that $\boldsymbol{T}_k^H \bar{\boldsymbol{U}} = \boldsymbol{0}$ holds. Then $\boldsymbol{x}_k$ is proportional to the product $\bar{\boldsymbol{U}}\boldsymbol{y}$ with $\boldsymbol{y}$ representing an eigenvector corresponding to the largest eigenvalue of

$$\bar{\boldsymbol{U}}^H \boldsymbol{R}_k \boldsymbol{R}_k^H \bar{\boldsymbol{U}}. \tag{12}$$

Setting out from $\boldsymbol{x}_k$ the quotient (9) is maximized by any weight vector $\boldsymbol{w}_k = v_k \boldsymbol{x}_k$, with v_k denoting an arbitrary complex factor. The degrees of freedom represented by the K factors v_k can be exploited to meet further SDMA system requirements. The most important parameter determining the performance of downlink data detection is the signal-to-noise-and-interference ratio ($SNIR$) for each user. We assume a minimum $SNIR$ that has to be maintained for all users in order to keep the expected bit error rate below a given limit. By means of the K a-priori known noise powers N_l each specific for receiver l and the K vectors $\boldsymbol{x}_k$, an equation to determine the coefficients v_k can be stated. By defining the factors

$$u_{l,k} = \frac{S_{l,k}}{v_k^H v_k N_l} = \frac{\boldsymbol{x}_k^H \boldsymbol{R}_l \boldsymbol{R}_l^H \boldsymbol{x}_k}{N_l} \tag{13}$$

the signal-to-noise-and-interferer ratio to be met for each user l is obtained by

$$SNIR = \frac{S_{l,l}}{N_l + \sum\limits_{\substack{k=1 \\ k \neq l}}^{K} S_{l,k}} = \frac{u_{l,l} v_l^H v_l}{1 + \sum\limits_{\substack{k=1 \\ k \neq l}}^{K} u_{l,k} v_k^H v_k}. \tag{14}$$

Therefore, the squared absolute values of the required factors v_k can be calculated by solving the linear equation

$$\boldsymbol{\Psi v} = \boldsymbol{\Psi} \begin{pmatrix} v_1^H v_1 \\ \vdots \\ v_K^H v_K \end{pmatrix} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \tag{15}$$

with the constraint matrix $\boldsymbol{\Psi}$ given by

$$\boldsymbol{\Psi} = \begin{pmatrix} u_{1,1}/SNIR & -u_{1,2} & \cdots & -u_{1,K} \\ -u_{2,1} & \ddots & & -u_{2,K} \\ \vdots & & \ddots & \vdots \\ -u_{K,1} & -u_{K,2} & \cdots & u_{K,K}/SNIR \end{pmatrix}. \tag{16}$$

2.3. Linear improvement of the beamformer

The algorithm presented in the previous chapter is suboptimum for SDMA applications. It rather minimizes interference than the total signal power P that has to be produced by the array. Apart from that there is no guarantee that the constraint matrix $\boldsymbol{\Psi}$ defined in (16) is always invertible or that all elements of the solution $\boldsymbol{v}$ of (15) are positive. In these cases the algorithm does not yield any solution at all.

In order to improve the beamformer, we will set out from the total array signal power being a function $P(\boldsymbol{x}_k)$ of only one unit vector $\boldsymbol{x}_k$. The other $K-1$ vectors yielded by the interference mimization algorithm will be considered constant. It can be proved that the problem of minimizing $P(\boldsymbol{x}_k)$ with respect to $\boldsymbol{x}_k$ automatically leads to the problem of finding a unit eigenvector corresponding to the largest eigenvalue of a matrix $\boldsymbol{\Upsilon}$ of the form

$$\boldsymbol{\Upsilon} = \left(\boldsymbol{I} + \sum_{i=1}^{K} m_i \boldsymbol{R}_i \boldsymbol{R}_i^H \right)^{-1} \left(\sum_{i=1}^{K} n_i \boldsymbol{R}_i \boldsymbol{R}_i^H \right). \tag{17}$$

According to (15) the array power P can be written by means of the inverse $\boldsymbol{\Omega} = \boldsymbol{\Psi}^{-1}$ as follows:

$$P = \sum_{k=1}^{K} \boldsymbol{w}_k^H \boldsymbol{w}_k = \sum_{i=1}^{K} \sum_{k=1}^{K} \Omega(i,k). \tag{18}$$

This consideration leads to a computationally cheap algorithm yielding the real parameters m_i and n_i necessary to compute $\boldsymbol{\Upsilon}$. Unfortunately the parameters m_i and n_i cannot be presented in elegant formulae but only in complicated expressions depending on all elements of $\boldsymbol{\Psi}$ except those of its k-th column. Therefore, the authors will restrict their presentation to the case of all matrices $\boldsymbol{T}_k$ being rank-deficient. In this case $\boldsymbol{\Psi}$ is diagonal and the parameters m_i and n_i can be easily calculated as:

$$m_i = \begin{cases} 0 & \text{if } i = k \\ 1/\Psi(i,i) & \text{else} \end{cases}, \quad n_i = \begin{cases} 1 & \text{if } i = k \\ 0 & \text{else} \end{cases}. \tag{19}$$

The linear optimization yields a new vector $\boldsymbol{x}'_k$ and a new matrix $\boldsymbol{\Psi}'$. Further improvement of the beamformer can be reached by repeating the algorithm for a different index $k' \neq k$. The improvement of the beamformer is most effectively done by starting with the index k that corresponds to the smallest diagonal element $\Psi(k,k)$. Simulations have shown that in most cases the result of the first step is quite close to the global optimum. Improvements reached by repeating the algorithm tend to be comparably marginal. Anyway, a linear improvement step is computionally much cheaper than executing the interference mimization algorithm described in the previous section. The main reason is that the calculation of the largest eigenvalue of a matrix and a corresponding eigenvector can be done much faster than a complete SVD [7].

2.4. Nonlinear improvement of the beamformer

The optimum beamforming algorithm jointly optimizes all weight vectors $\boldsymbol{w}_k$. This optimization problem can be mathematically put as the nonlinear minimization problem

$$\underset{\boldsymbol{w}_1 \dots \boldsymbol{w}_K}{\text{minimize}} \; \{P = \sum_{k=1}^{K} \boldsymbol{w}_k^H \boldsymbol{w}_k\} \tag{20}$$

with the K nonlinear constraints

$$\frac{\boldsymbol{w}_l^H \boldsymbol{R}_l \boldsymbol{R}_l^H \boldsymbol{w}_l}{N_l SNIR} - \sum_{\substack{k=1 \\ k \neq l}}^{K} \frac{\boldsymbol{w}_k^H \boldsymbol{R}_l \boldsymbol{R}_l^H \boldsymbol{w}_k}{N_l} = 1, \; l = 1 \dots K. \tag{21}$$

There are well-known penalty-type algorithms that are able to stably compute the global optimum of problems like (20), e.g. the Augmented-Lagrange-Algorithm or the SQP-Method [8] [9]. But they all have in common a high computational burden, if they are not provided with a starting point close to the optimum solution. In our opininion this disadvantage restricts these methods to only two applications in mobile radio: First improving the solution yielded by the previously presented linear algorithm and second burst-by-burst updating of the beamformer in moderately time-variant mobile radio channels.

3. CHANNEL ALLOCATION

3.1. SDMA power gain

We will consider a hybride FTDMA/SDMA mobile radio system with $K \cdot N$ users. If there are N different FTDMA channels at disposal, each of them has to manage K users by means of SDMA. Provided the carrier frequency does not influence the DDA channel parameters, there are altogether $(K \cdot N)!/(N! \cdot K!^N)$ possible arrangements of users. Any algorithm trying to find an optimum arrangement must be able to tell 'good' sets of users from 'bad' sets. We suggest the SDMA power gain ϱ that will be defined in this sequel as a quantitative criterion to do this kind of evaluation.

The SDMA power gain ϱ measures the amount of signal energy saved by adding SDMA features to a conventional FTDMA system. First we will consider an SDMA system managing K users per FTDMA slot by means of a base station antenna array. Optimum weight vectors $\boldsymbol{w}_1 \dots \boldsymbol{w}_K$ provided the minimum array power P_1 needed to maintain the required SNIRs is according to (20) given by

$$P_1 = \sum_{k=1}^{K} \boldsymbol{w}_k^H \boldsymbol{w}_k. \tag{22}$$

The second case is a single base station antenna. The ratio of the radio power P_{rec} received by user k to the radio power P_{cre} created by the base station can be

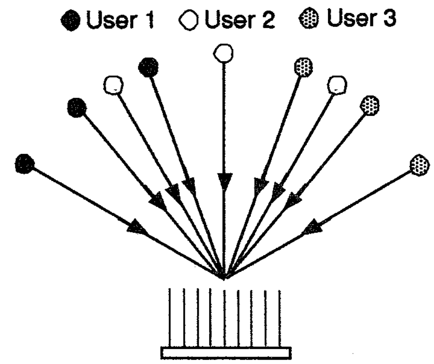

Figure 1: SDMA scenario with good spatial separability

expressed as

$$\frac{P_{rec}}{P_{cre}} = \sum_{q=1}^{Q_k} E\{\|b_{k,q}\|^2\}. \tag{23}$$

Thus, the overall signal power P_2 necessary to supply all users in K different FTDMA channels is given by

$$P_2 = SNIR \sum_{k=1}^{K} N_k / \sum_{q=1}^{Q_k} E\{\|b_{k,q}\|^2\}. \tag{24}$$

Therefore, the SDMA power gain ϱ can be computed as

$$\varrho = \frac{P_2}{P_1} = \frac{SNIR \sum_{k=1}^{K} N_k / \sum_{q=1}^{Q_k} E\{\|b_{k,q}\|^2\}}{\sum_{k=1}^{K} \boldsymbol{w}_k^H \boldsymbol{w}_k}. \tag{25}$$

As optimum beamforming vectors are necessary to calculate ϱ, we have to resort to an estimate $\hat{\varrho}$ for many realtime applications. For the channel allocation problem we suggest to first minimize interference (section 2.2) and then execute $1 \dots K$ linear improvement steps (section 2.3).

3.2. Examples

To illustrate the SDMA power gain ϱ we will compare two concrete SDMA scenarios (see fig. 1 and fig. 2). The first one is characterized by an array of M antennae and by $K = 3$ users, each having $Q_k = 3$ DOAs. All DOAs have a minimum angular distance of 10° to each other. The second scenario has as many antennae, users and DOAs as the first one, but the DOAs are arranged in an unfavorable way. Since user 2 does not have any DOA that exclusively corresponds to him, it is obvious that supplying him with signal energy causes more interference than in the first scenario.

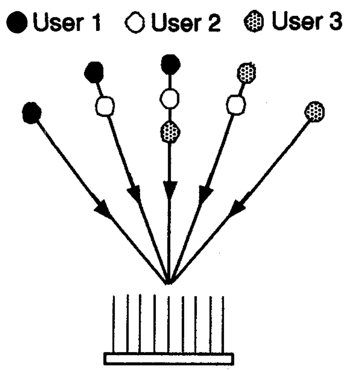

Figure 2: SDMA scenario with bad spatial separability

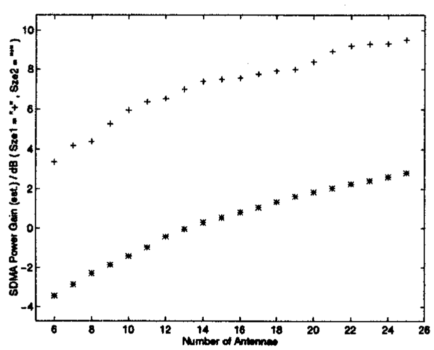

Figure 3: Estimated SDMA power gain for varying numbers of antennae

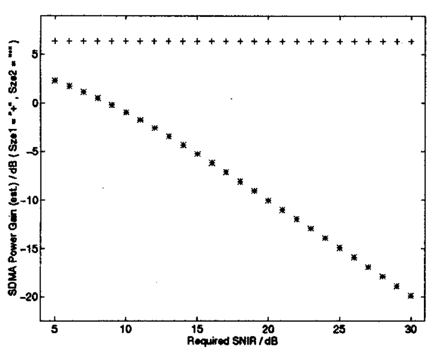

Figure 4: Estimated SDMA power gain for varying SNIR requirements

Executing the interference minimization algorithm and one linear improvement step yields beamforming vectors close to the optimum beamformer. They are used to calculate an estimate $\hat{\varrho}$ of the SDMA power gain. Applying this method to both scenarios, 1 and 2 yielded the results summarized in the two figures 3 and 4. In both simulations all complex amplitudes were equally set to $b_{k,q} = 1\,(k, q = 1\ldots3)$. Every user was subject to the same noise level $N_k = 0\,dB$.

In figure 3 the number M of antennae was varied from 6 to 25 while a constant signal-to-noise-and-interferer ratio of $10\,dB$ was required. In figure 4 the number $M = 11$ of antennae was constant whereas the signal-to-noise-and-interference level for all users ranged from $5\,dB$ to $30\,dB$.

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