Weighted Sum Rate Maximization in the MIMO MAC with Linear Transceivers: Algorithmic Solutions

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***Abstract*—The problem of maximizing weighted sum rate in the MIMO multiple access channel with individual power constraints is considered. The optimum is achieved by successive interference cancellation, where the covariances are found by iterative water-filling. As successive interference cancellation implies long decoding delays, we consider linear approaches with zero-forcing constraints. To avoid the associated non-convex and combinatorial optimization, we allocate successively data streams to users, while keeping transmit filters and user allocations of previous steps fixed. The transmit filters are determined based on two lower bounds for the weighted sum rate. The algorithms converge to the optimum linear solution for infinite transmit powers in many scenarios at low computational complexity.**

I. Introduction

Weighted sum rate maximization constitutes an important problem in wireless communication systems. By varying the weights, different points on the convex hull of the attainable rate region can be achieved. Furthermore, the weights can consider priorities of the different users. In this paper, we focus on the Multiple-Input Multiple-Output (MIMO) Multiple Access Channel (MAC), where each user has an individual power constraint. The optimum covariance matrices for the weighted sum rate maximization in the MIMO-MAC can be obtained with the well-known iterative water-filling algorithm [1], [2]. However, it relies on the principle of Successive Interference Cancellation (SIC), which implies long decoding delays. We therefore use linear filters at the transmitters and the receiver to mitigate the interference in this paper such that all data streams can be decoded in parallel. By additionally introducing zero-forcing constraints, the resulting power allocation reduces to simple scalar water-filling. It seems that the problem of weighted sum rate maximization with linear transceivers has not been covered so far. A different problem in the MAC, namely Signal-to-Interference-and-Noise-Ratio (SINR) balancing, is considered in [3]. In this paper, however, we do not restrict to single stream communication as in [3].
In contrast, for the MIMO broadcast channel, many algorithms aiming at maximizing a weighted sum rate using linear transceivers can be found in the literature. Some of them can be applied straightforwardly to the MAC. When the number of antennas at the base station is larger than the total number of antennas at the user terminals, the concept of Block Diagonalization [4] can be applied to the MAC as well, where individual power constraints must be considered instead of the sum power constraint. When the base station has less antennas, each user cannot get as many data streams as it has antennas with zero-forcing constraints, leading to a combinatorial optimization problem. To approach the optimum solution as close as possible, greedy algorithms are used [5], [6], [7][1] in the broadcast channel, where the data streams are successively allocated to users. The methods from [6], [7] have been proposed for Multiple-Input Single-Output (MISO) systems, the extension to MIMO is possible by fixing the filters at the terminals a priori by the corresponding singular vectors of the channel matrices as proposed in [8]. By furthermore using the pseudo-inverse of the effective channel at the base station as receive filters, the application of these algorithms to the MAC is straightforward. In [5], the filters at the terminals are included into the successive optimization, where in each step the filters at the terminals for data streams allocated in previous steps are kept fixed. These filters are determined based on a lower bound for the weighted sum rate, whose derivation is based on the power allocation in the broadcast channel with a sum power constraint. As the power allocation in the MAC with individual power constraints is different, the bound cannot be used for the problem considered in this paper. We therefore present algorithms in the following that successively allocate data streams to users in the MAC and determine the corresponding transmit filters based on two novel lower bounds for the weighted sum rate. The first bound is tighter, but leads to a more complex optimization problem than the second looser bound. The presented algorithms outperform the straightforward extensions of greedy broadcast algorithms in terms of performance and complexity.

II. System Model

We consider a MIMO MAC with individual power constraints. The channel matrix of user k is denoted as $\boldsymbol{H}_k \in \mathbb{C}^{M_{\mathrm{Rx}} \times M_{\mathrm{Tx},k}}$, where M_{Rx} and $M_{\mathrm{Tx},k}$ are the number of receive antennas and the number of transmit antennas at user k, respectively. We assume that all matrices $\boldsymbol{H}_k$ are perfectly known at the receiver, and the resulting transmit filters can be perfectly signaled to the corresponding terminals.[2] The signal

[1] [7] considers only the case of equal weights for all users.

[2] The problem of feedback of precoders generally arises for the optimized MAC.

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$\hat{\boldsymbol{s}}_k \in \mathbb{C}^{d_k}$ received at the base station from the k-th user is given by

$$\hat{\boldsymbol{s}}_k = \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{T}_k \boldsymbol{P}_k^{\frac{1}{2}} \boldsymbol{s}_k + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{H}_\ell \boldsymbol{T}_\ell \boldsymbol{P}_\ell^{\frac{1}{2}} \boldsymbol{s}_\ell + \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{\eta}, \quad (1)$$

where $\boldsymbol{s}_k \in \mathbb{C}^{d_k}$, $\boldsymbol{G}_k^{\mathrm{H}} \in \mathbb{C}^{d_k \times M_{\mathrm{Rx}}}$ and $\boldsymbol{\eta} \in \mathbb{C}^{M_{\mathrm{Rx}}}$ denote the data vector and the receive filter for user k and the additive white Gaussian noise, respectively. All components of the noise vector have unit variance and are uncorrelated among each other as well as with all signals. $\boldsymbol{T}_k \in \mathbb{C}^{M_{\mathrm{Tx},k} \times d_k}$ denotes the beamforming matrix of user k, which has unit norm columns. The power distribution over the data streams of user k is collected in the diagonal matrix $\boldsymbol{P}_k \in \mathbb{R}^{d_k \times d_k}$, where d_k denotes the number of data streams user k transmits. Furthermore we do not apply SIC at the receiver, that is why all other users are treated as interference in (1). In the following we are considering the problem of maximizing the weighted sum rate $\sum_{k=1}^{K} \mu_k R_k$ with non-negative weights μ_k in the MIMO MAC under individual power constraints, where the k-th user's power limit is denoted as P_k, i.e.,

$$\max_{\{\boldsymbol{T}_k, \boldsymbol{P}_k, \boldsymbol{G}_k\}_{k=1,\ldots,K}} \sum_{k=1}^{K} \mu_k R_k = \max_{\{\boldsymbol{T}_k, \boldsymbol{P}_k, \boldsymbol{G}_k\}_{k=1,\ldots,K}}$$
$$\sum_{k=1}^{K} \mu_k \log_2 \frac{\left| \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{G}_k + \sum_{\ell=1}^{K} \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{H}_\ell \boldsymbol{T}_\ell \boldsymbol{P}_\ell \boldsymbol{T}_\ell^{\mathrm{H}} \boldsymbol{H}_\ell^{\mathrm{H}} \boldsymbol{G}_k \right|}{\left| \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{G}_k + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{H}_\ell \boldsymbol{T}_\ell \boldsymbol{P}_\ell \boldsymbol{T}_\ell^{\mathrm{H}} \boldsymbol{H}_\ell^{\mathrm{H}} \boldsymbol{G}_k \right|},$$
$$\text{s.t. } \mathrm{tr}\left(\boldsymbol{T}_k \boldsymbol{P}_k \boldsymbol{T}_k^{\mathrm{H}} \right) = \mathrm{tr}(\boldsymbol{P}_k) \leq P_k, \forall k. \quad (2)$$

III. Algorithms

Problem (2) constitutes a highly non-convex optimization problem. In order to find a near optimum, but computationally efficient solution to this problem, we propose to make the following simplifications, similar to the ones proposed for the broadcast channel with a sum power constraint in [5].

A. Zero-Forcing

As a first simplifying step we suppress the interference between different users as well as between different data streams of the same user completely by linear transmit and receive signal processing. The filters at the receiver are then given by the pseudo-inverse of the composite channel matrix, i.e.,

$$\begin{bmatrix} \boldsymbol{G}_1^{\mathrm{H}} \\ \vdots \\ \boldsymbol{G}_K^{\mathrm{H}} \end{bmatrix} = \boldsymbol{H}_{\mathrm{comp}}^{+} = \left(\boldsymbol{H}_{\mathrm{comp}}^{\mathrm{H}} \boldsymbol{H}_{\mathrm{comp}} \right)^{-1} \boldsymbol{H}_{\mathrm{comp}}^{\mathrm{H}}, \quad (3)$$

where the composite channel matrix $\boldsymbol{H}_{\mathrm{comp}}$ is given by

$$\boldsymbol{H}_{\mathrm{comp}} = \left[\boldsymbol{H}_1 \boldsymbol{T}_1 \boldsymbol{P}_1^{\frac{1}{2}} \ldots \boldsymbol{H}_K \boldsymbol{T}_K \boldsymbol{P}_K^{\frac{1}{2}} \right] =$$
$$= \hat{\boldsymbol{H}}_{\mathrm{comp}} \, \mathrm{diag}\left(\boldsymbol{P}_1^{\frac{1}{2}}, \ldots, \boldsymbol{P}_K^{\frac{1}{2}} \right), \quad (4)$$

where $\mathrm{diag}(\bullet)$ constructs a diagonal matrix with its arguments. Then the Signal-to-Noise-Ratio (SNR) of the j-th subchannel computes according to

$$\mathrm{SNR}_j = \frac{1}{\left\| \boldsymbol{e}_j^{\mathrm{T}} \boldsymbol{H}_{\mathrm{comp}}^{+} \right\|_2^2} = \frac{p_j}{\left\| \boldsymbol{e}_j^{\mathrm{T}} \hat{\boldsymbol{H}}_{\mathrm{comp}}^{+} \right\|_2^2},$$

where $\boldsymbol{e}_j$ denotes the j-th canonical unit vector and the powers p_j are determined by waterfilling [9] of the P_k over all subchannels of the corresponding user k. The pseudo-inverse exists only, if $d_k \leq M_{\mathrm{Tx},k}$ for all users k and $\sum_{k=1}^{K} d_k \leq M_{\mathrm{Rx}}$. As in most practical scenarios, setting $d_k = M_{\mathrm{Tx},k}, \forall k$ does not fulfill the latter requirement, finding that optimum allocation of data streams to users becomes a combinatorial optimization problem.

B. Successive Allocation

To circumvent the exhaustive search for determining the optimum d_k, we propose to follow a greedy approach [5], [6], [7], [10]. This implies that data streams are allocated successively to users. Thus, we initially set all d_k's to zero, i.e., $d_k = 0, \forall k$. In each step, the user to which a data stream is allocated to and the corresponding transmit filter are determined such that the increase in weighted sum rate becomes maximum, where the direction of the transmit filters and user allocation from previous steps are kept fixed. Mathematically the problem for the allocation of the i-th data stream reads as

$$\{\pi(i), \boldsymbol{t}_i\} = \underset{k, \boldsymbol{t}}{\mathrm{argmax}} \, R_{\mathrm{WSR}}^{(i)}(k, \boldsymbol{t}), \quad \text{s.t. } \boldsymbol{t}^{\mathrm{H}} \boldsymbol{t} = 1, \quad (5)$$

with

$$R_{\mathrm{WSR}}^{(i)}(k, \boldsymbol{t}) =$$
$$\max_{\{p_j\}_{j=1,\ldots,i}} \sum_{j=1}^{i-1} \mu_{\pi(j)} \log_2 \left(1 + \frac{p_j}{\left\| \boldsymbol{e}_j^{\mathrm{T}} \left[\tilde{\boldsymbol{H}}_{\mathrm{comp}}^{(i-1)}, \boldsymbol{H}_k \boldsymbol{t} \right]^{+} \right\|_2^2} \right) +$$
$$+ \mu_k \log_2 \left(1 + \frac{p_i}{\left\| \boldsymbol{e}_i^{\mathrm{T}} \left[\tilde{\boldsymbol{H}}_{\mathrm{comp}}^{(i-1)}, \boldsymbol{H}_k \boldsymbol{t} \right]^{+} \right\|_2^2} \right),$$
$$\text{s.t.} \sum_{j:\pi(j)=k} p_j \leq P_k, \forall k. \quad (6)$$

Note that for notational convenience the columns of $\tilde{\boldsymbol{H}}_{\mathrm{comp}}^{(i-1)}$ are not sorted user-wise as in (4) but stream-wise such that

$$\tilde{\boldsymbol{H}}_{\mathrm{comp}}^{(i-1)} = \left[\boldsymbol{H}_{\pi(1)} \boldsymbol{t}_1, \ldots, \boldsymbol{H}_{\pi(i-1)} \boldsymbol{t}_{i-1} \right].$$

$\pi(i) \in \{1, \ldots, K\}$ is the user which the i-th data stream is allocated to. In case $R_{\mathrm{WSR}}^{(i)}(\pi(i), \boldsymbol{t}_i)$ is smaller than the weighted sum rate obtained in the previous step, the allocation is stopped and the user $\pi(i)$ will not receive a further data stream. Otherwise, $d_{\pi(i)}$ is increased by one and $\boldsymbol{T}_{\pi(i)} = \boldsymbol{t}_i$ in case user $\pi(i)$ receives a data stream for the first time,

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otherwise $\boldsymbol{T}_{\pi(i)}$ is enlarged by $\boldsymbol{t}_i$ as a further column. For $i = 1$, (6) simplifies to

$$R_{\text{WSR}}^{(1)}(k, \boldsymbol{t}) = \mu_k \log_2(1 + P_k \boldsymbol{t}^{\text{H}} \boldsymbol{H}_k^{\text{H}} \boldsymbol{H}_k \boldsymbol{t}).$$

Thus, each user's rate is maximized by taking the left and right singular vector belonging to the principal singular value of that user's channel matrix as receive and transmit vectors, respectively. For $i > 1$, however, Problem (5) is still non-convex.

C. Maximization of a Lower Bound

To simplify (5) further, we propose to use a lower bound for the weighted sum rate, which is derived in the appendix and is given by

$$R_{\text{WSR}}^{(i)}(k, \boldsymbol{t}) \geq R_{\text{WSR,lb}}^{(i)}(k, \boldsymbol{t}) = \left(\sum_{j=1}^{i-1} \mu_{\pi(j)} + \mu_k \right) \log_2 \left(\frac{\sum_{j=1}^{i-1} \mu_{\pi(j)} + \mu_k}{\text{tr}\left(\left(\left[\tilde{\boldsymbol{H}}_{\text{comp}}^{(i-1)}, \boldsymbol{H}_k \boldsymbol{t} \right]^+ \right)^{\text{H}} \boldsymbol{\Omega}_i(k) \left[\tilde{\boldsymbol{H}}_{\text{comp}}^{(i-1)}, \boldsymbol{H}_k \boldsymbol{t} \right]^+ \right)} \right). \tag{7}$$

The matrix $\boldsymbol{\Omega}_i(k)$ is diagonal and given by

$$\boldsymbol{\Omega}_i(k) = \begin{bmatrix} \frac{\mu_{\pi(1)} d_{\pi(1)}(k)}{P_{\pi(1)}} & & & \\ & \ddots & & \\ & & \frac{\mu_{\pi(i-1)} d_{\pi(i-1)}(k)}{P_{\pi(i-1)}} & \\ & & & \frac{\mu_k (d_k + 1)}{P_k} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Omega}_{i-1}(k) & \\ & \alpha_i(k) \end{bmatrix}, \tag{8}$$

where $d_j(k) = \begin{cases} d_j, j \neq k, \\ d_j + 1, j = k. \end{cases}$. As only the denominator in the argument of the logarithm in (7) depends on the transmit filters, we first determine the transmit filters $\boldsymbol{t}_i(k)$ for each user k that maximize the lower bound for the weighted sum rate if the i-th data stream is allocated to user k, i.e.,

$$\boldsymbol{t}_i(k) = \underset{\|\boldsymbol{t}\|_2 = 1}{\text{argmin}} \, \text{tr} \left[\left(\left[\tilde{\boldsymbol{H}}_{\text{comp}}^{(i-1)}, \boldsymbol{H}_k \boldsymbol{t} \right]^+ \right)^{\text{H}} \boldsymbol{\Omega}_{i-1}(k) \left[\tilde{\boldsymbol{H}}_{\text{comp}}^{(i-1)}, \boldsymbol{H}_k \boldsymbol{t} \right]^+ \right].$$

Similar to [11], where the broadcast channel is considered, we use the QR decomposition of $\tilde{\boldsymbol{H}}_{\text{comp}}^{(i-1)} = \boldsymbol{Q}_{i-1} \boldsymbol{R}_{i-1}$, and obtain after some modifications

$$\boldsymbol{t}_i(k) = \underset{\|\boldsymbol{t}\|_2 = 1}{\text{argmax}} \frac{\boldsymbol{t}^{\text{H}} \boldsymbol{H}_k^{\text{H}} (\boldsymbol{I} - \boldsymbol{Q}_{i-1} \boldsymbol{Q}_{i-1}^{\text{H}}) \boldsymbol{H}_k \boldsymbol{t}}{\boldsymbol{t}^{\text{H}} \left(\alpha_i(k) \boldsymbol{I} + \boldsymbol{H}_k^{\text{H}} \boldsymbol{Q}_{i-1} \boldsymbol{R}_{i-1}^{-\text{H}} \boldsymbol{\Omega}_{i-1}(k) \boldsymbol{R}_{i-1}^{-1} \boldsymbol{Q}_{i-1}^{\text{H}} \boldsymbol{H}_k \right) \boldsymbol{t}}, \tag{9}$$

where $\alpha_i(k)$ has been implicitly defined in (8). Problem (9) can be solved via a generalized eigenvalue problem. After determining the transmit filters $\boldsymbol{t}_i(k)$ for each candidate user, these filters are used to compute the achievable weighted sum rates for each user, where the accurate formula from (6) is used, and the user $\pi(i)$ is finally chosen according to

$$\pi(i) = \underset{k}{\text{argmax}} \, R_{\text{WSR}}^{(i)}(k, \boldsymbol{t}_i(k)).$$

To avoid the necessary generalized eigenvalue decompositions, choosing transmit filters $\tilde{\boldsymbol{t}}_i(k)$ according to

$$\tilde{\boldsymbol{t}}_i(k) = = \underset{\|\boldsymbol{t}\|_2 = 1}{\text{argmax}} \boldsymbol{t}^{\text{H}} \boldsymbol{H}_k^{\text{H}} (\boldsymbol{I} - \boldsymbol{Q}_{i-1} \boldsymbol{Q}_{i-1}^{\text{H}}) \boldsymbol{H}_k \boldsymbol{t} \tag{10}$$

maximizes a looser lower bound for the weighted sum rate. That is because the denominator in (9) can be upper bounded by

$$\max_k \left(\alpha_i(k) + \rho_1(\boldsymbol{H}_k^{\text{H}} \boldsymbol{H}_k) \, \text{tr}(\boldsymbol{R}_{i-1}^{-\text{H}} \boldsymbol{\Omega}_{i-1}(k) \boldsymbol{R}_{i-1}^{-1}) \right),$$

which can be derived similarly as Equation (17) in [10], and where $\rho_1(\boldsymbol{A})$ denotes the maximum eigenvalue of the matrix $\boldsymbol{A}$. Equation (10) leads to different transmit filters, but requires the solution of an ordinary eigenvalue problem instead of a generalized eigenvalue problem for each candidate user.

IV. Complexity Reduction

From a computational point of view (generalized) eigenvalue decompositions are still cost-intensive. To avoid explict computations of eigenvalues for all users in each step as required by (9) and (10), a user preselection can be applied similar to the one proposed for weighted sum rate maximization in the broadcast channel in [5]. During user preselection, users are identified, which will not maximize the objective function $R_{\text{WSR}}^{(i)}(k, \boldsymbol{t})$ in step i based on simple computations. For the sake of simplicity, we use the lower bound $R_{\text{WSR,lb}}^{(i)}(k, \boldsymbol{t})$ as an approximation of the objective function $R_{\text{WSR}}^{(i)}(k, \boldsymbol{t})$ for the user preselection. With (9) and $\tilde{\boldsymbol{H}}_{\text{comp}}^{(i-1)} = \boldsymbol{Q}_{i-1} \boldsymbol{R}_{i-1}$, we have

$$\hat{R}_{\text{WSR,lb}}^{(i)}(\rho_1(\boldsymbol{C}_k)) = \max_{\|\boldsymbol{t}\|_2 = 1} R_{\text{WSR},lb}^{(i)}(k, \boldsymbol{t}) = \left(\sum_{j=1}^{i-1} \mu_{\pi(j)} + \mu_k \right) \log_2 \left(\frac{\sum_{j=1}^{i-1} \mu_{\pi(j)} + \mu_k}{\text{tr}\left(\boldsymbol{R}_{i-1}^{-\text{H}} \boldsymbol{\Omega}_{i-1}(k) \boldsymbol{R}_{i-1}^{-1} \right) + \frac{1}{\rho_1(\boldsymbol{C}_k)}} \right),$$

which is a monotonically increasing function in $\rho_1(\boldsymbol{C}_k)$, where $\boldsymbol{C}_k = (\alpha_i(k) \boldsymbol{I} + \boldsymbol{B}_k)^{-1} \boldsymbol{A}_k$, $\boldsymbol{A}_k = \boldsymbol{H}_k^{\text{H}} (\boldsymbol{I} - \boldsymbol{Q}_{i-1} \boldsymbol{Q}_{i-1}^{\text{H}}) \boldsymbol{H}_k$ and $\boldsymbol{B}_k = \boldsymbol{H}_k^{\text{H}} \boldsymbol{Q}_{i-1} \boldsymbol{R}_{i-1}^{-\text{H}} \boldsymbol{\Omega}_{i-1}(k) \boldsymbol{R}_{i-1}^{-1} \boldsymbol{Q}_{i-1}^{\text{H}} \boldsymbol{H}_k$. From [5, Appendix C], $\rho_1(\boldsymbol{C}_k)$ can be bounded as follows:

$$\frac{\text{tr}(\boldsymbol{A}_k)}{\alpha_i(k) + \text{tr}(\boldsymbol{B}_k)} \leq \rho_1(\boldsymbol{C}_k) \leq \text{tr}(\boldsymbol{A}_k). \tag{11}$$

Users ℓ with

$$\hat{R}_{\text{WSR,lb}}^{(i)}(\text{tr}(\boldsymbol{A}_\ell)) < \underbrace{\max_k \hat{R}_{\text{WSR,lb}}^{(i)} \left(\frac{\text{tr}(\boldsymbol{A}_k)}{\alpha_i(k) + \text{tr}(\boldsymbol{B}_k)} \right)}_{\leq \max_k \hat{R}_{\text{WSR,lb}}^{(i)}(\rho_1(\boldsymbol{C}_k))} \tag{12}$$

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will certainly not maximize $\hat{R}^{(i)}_{\mathrm{WSR,lb}}(\rho_1(\boldsymbol{C}_k))$ with respect to k as even with an upper bound for $\rho_1(\boldsymbol{C}_\ell)$ the objective function is smaller than the maximum lower bound for the objective function over all users. Hence users ℓ, for which (12) holds, can be excluded from the allocation with simple computations of traces avoiding the numerically intensive determination of eigenvalues and eigenvectors. Only the remaining users are then considered for the allocation of the next data stream. Despite the conservative bounds in (11), this kind of user preselection turns out to be very effective in practice.

V. Asymptotic Analysis

In this section we will show that in case the channel matrices of all active users have full rank, the columns of the channel matrices of all active users are linearly independent, and the sum of transmit antennas of the active users is equal to the number of receive antennas, the proposed successive algorithms achieve the optimum solution to (2) in the limit of infinite transmit powers $P_k = \alpha_k P$, with $P \to \infty$ and finite α_k. In [12], it is shown that the asymptotically optimum data stream allocation is solely determined by the weights μ_k, i.e., first the user with the highest weights receives as many data streams as it has transmit antennas, then the user with the second highest weight is treated the same way and so forth until the number of allocated data streams is equal to the number of receive antennas. In the asymptotic limit the weighted sum rate in (6) is dominated by the terms depending on P such that

$$R^{(i)}_{\mathrm{WSR}}(k, \boldsymbol{t}) \approx \sum_{j=1}^{i-1} \mu_{\pi(j)} \log_2(P) + \mu_k \log_2(P),$$

as long as $d_j \leq M_{\mathrm{Tx},j}$ for all users j. Consequently in the asymptotic limit the successive allocation proposed in this paper will lead to the same data stream allocation as the optimum. Additionally, in case the assumptions listed above are fulfilled, the asymptotically optimum transmit covariance matrices are scaled identity matrices, i.e., $\boldsymbol{T}_k \boldsymbol{P}_k \boldsymbol{T}_k^{\mathrm{H}} = P_k / M_{\mathrm{Tx},k} \boldsymbol{I}$. As shown in [5] for a similar filter computation in the broadcast channel, the filters $\boldsymbol{t}_i$ and $\boldsymbol{t}_j$ with $j \neq i$ obtained from (9) or (10) are orthogonal to each other, if the corresponding data streams are allocated to the same user, i.e., $\boldsymbol{t}_j^{\mathrm{H}} \boldsymbol{t}_i = 0, \forall \pi(i) = \pi(j), \forall i \neq j$. As we are furthermore considering the case that all active users receive as many data streams as they have transmit antennas, the transmit filters $\boldsymbol{T}_k$ are orthonormal, i.e., $\boldsymbol{T}_k \boldsymbol{T}_k^{\mathrm{H}} = \boldsymbol{I}$. As additionally water-filling converges to an equal power distribution in the asymptotic limit, the transmit covariances obtained from our algorithms are scaled identity matrices and thus asymptotically optimum. To complete the proof of asymptotic optimality for the case described above it remains to show that also the receive filters $\boldsymbol{G}_k$ from (3) are asymptotically optimum. In general, using the MMSE filters $\boldsymbol{G}_k = \sqrt{P_k / M_{\mathrm{Tx},k}} (\boldsymbol{I} + \sum_{i=1}^{K} \boldsymbol{H}_i \boldsymbol{H}_i^{\mathrm{H}} P_i / M_{\mathrm{Tx},i})^{-1} \boldsymbol{H}_k$ at the receiver is optimum with scaled identity matrices as covariance matrices at the transmitters. With infinite transmit powers, first the matrix inversion lemma is applied to the $\boldsymbol{G}_k$, then the identity matrices can be neglected, such that the composite matrix of the receive filters of the active users reads as

$$\begin{bmatrix} \boldsymbol{G}_1^{\mathrm{H}} \\ \vdots \\ \boldsymbol{G}_k^{\mathrm{H}} \end{bmatrix} = \check{\boldsymbol{H}}_{\mathrm{comp}}^{\mathrm{H}} \left(\check{\boldsymbol{H}}_{\mathrm{comp}} \check{\boldsymbol{H}}_{\mathrm{comp}}^{\mathrm{H}} \right)^{-1} = \left(\check{\boldsymbol{H}}_{\mathrm{comp}}^{\mathrm{H}} \check{\boldsymbol{H}}_{\mathrm{comp}} \right)^{-1} \check{\boldsymbol{H}}_{\mathrm{comp}}^{\mathrm{H}} = \check{\boldsymbol{H}}_{\mathrm{comp}}^{+},$$

where, for simplicity reasons, the active users are indexed by $1, \ldots, k$ and

$$\check{\boldsymbol{H}}_{\mathrm{comp}} = \left[\sqrt{\frac{P_1}{M_{\mathrm{Tx},1}}} \boldsymbol{H}_1, \ldots, \sqrt{\frac{P_k}{M_{\mathrm{Tx},k}}} \boldsymbol{H}_k \right],$$

which is due to our initial assumptions invertible. This completes the proof of asymptotic optimality of the proposed successive approaches.

VI. Simulation Results

Fig. 1 shows the weighted sum rates averaged over 1000 channel realizations, where each channel matrix has contained independent identically proper Gaussian distributed entries with zero mean and unit variance. There are $K = 3$ users in the system, each equipped with $M_{\mathrm{Tx},k} = 4$ transmit antennas, $M_{\mathrm{Rx}} = 8$ antennas can be found at the base station. The weight for user 1 is twice as high as the weights for users 2 and 3 such that $\mu_1 = 2$ and $\mu_2 = \mu_3 = 1$. The SNR on the abscissa denotes the ratio of transmit power to noise variance for one terminal, where it is assumed that those values are equal for all terminals. The optimum curve has been obtained with the

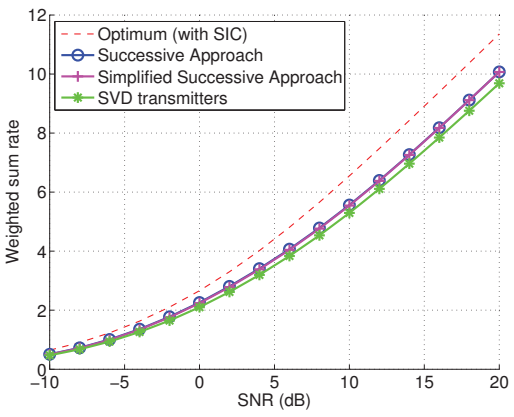

Fig. 1. Average sum rates in a Rayleigh fading environment with $K = 3$ users, $M_{\mathrm{Tx},k} = 4$ transmit antennas at each user and $M_{\mathrm{Rx}} = 8$ receive antennas, $\mu_1 = 2, \mu_2 = \mu_3 = 1$.

iterative water-filling algorithm from [2]. Note that it therefore relies on successive interference cancellation, whereas with our successive approach interference suppression is solely achieved by linear transceivers. Using simplified receivers according to (10) leads to no visible performance losses. When the transmit filters are chosen to be the left singular vectors

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of the channel matrices, i.e. they are not optimized, but the data stream allocation is done in the same greedy manner as described above, slight performance degradations can be observed. However, it should be noted that the complexity of this approach is even higher, as for each data stream allocation, $\sum M_{\mathrm{Tx},k}$ tests for the maximum increase in weighted sum rate are required and, as in the broadcast channel, almost no complexity reductions can be achieved through a user preselection.

Fig. 2 shows the rate regions of the different algorithms in a $K = 2$ user scenario, each equipped with $M_{\mathrm{Tx},k} = 4$ antennas and there are $M_{\mathrm{Rx}} = 8$ antennas at the base station. Each transmitter can use a power of $P_1 = P_2 = 100$, which corresponds to 20dB transmit SNR. The order of performance is the same as in Figure 1. Furthermore a simple Time Division Multiple Access (TDMA) scheme is outperformed clearly by all approaches.

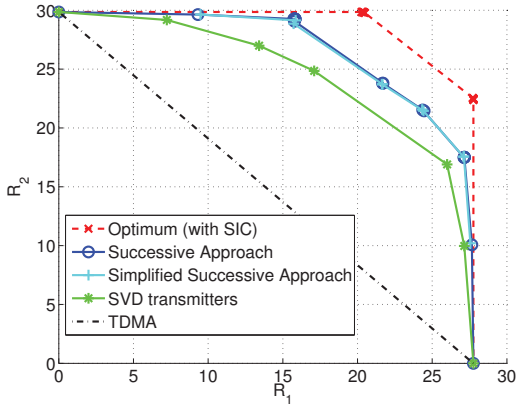

Fig. 2. Rate region for $K = 2$ users, $M_{\mathrm{Tx},k} = 4$ transmit antennas at each user and $M_{\mathrm{Rx}} = 8$ receive antennas, $P_1 = P_2 = 100$.

APPENDIX

We derive a lower bound for the weighted sum rate $R^{(i)}_{\mathrm{WSR}}(\pi(i), \boldsymbol{t}_i) = \sum_{j=1}^{i} \mu_{\pi(j)} \log_2(1 + p_j \lambda_j(\pi(i), \boldsymbol{t}_i))$, where

$$\frac{1}{\lambda_j(\pi(i), \boldsymbol{t}_i)} = \left\| \boldsymbol{e}_j^{\mathrm{T}} \left[\tilde{\boldsymbol{H}}^{(i-1)}_{\mathrm{comp}}, \boldsymbol{H}_{\pi(i)} \boldsymbol{t}_i \right]^{+} \right\|_2^2 .$$

By waterfilling the p_j are given by

$$p_j = \frac{P_{\pi(j)} + \sum_{n:\pi(n)=\pi(j)} \frac{1}{\lambda_n(\pi(i), \boldsymbol{t}_i)}}{d_{\pi(j)}(k)} - \frac{1}{\lambda_j(\pi(i), \boldsymbol{t}_i)}.$$

We assume that all p_j are greater than zero. In the final allocation that will always be the case, as considering a data stream with zero power for the zero-forcing constraints decreases the rate of the other data streams without contributing to the total sum rate. The algorithm therefore has to prevent such a scenario. Using the optimum power allocation we obtain for the weighted sum rate

$$R^{(i)}_{\mathrm{WSR}}(\pi(i), \boldsymbol{t}_i) = \sum_{j=1}^{i} \mu_{\pi(j)}$$

$$\log_2 \prod_{j=1}^{i} \left(\frac{P_{\pi(j)} + \sum_{n:\pi(n)=\pi(j)} \frac{1}{\lambda_n(\pi(i), \boldsymbol{t}_i)}}{d_{\pi(j)}(\pi(i))} \lambda_j(\pi(i), \boldsymbol{t}_i) \right)^{\frac{\mu_{\pi(j)}}{\sum_{\ell=1}^{i} \mu_{\pi(\ell)}}}$$

$$\overset{(a)}{\geq} \sum_{j=1}^{i} \mu_{\pi(j)} \log_2 \left(\frac{\sum_{j=1}^{i} \mu_{\pi(j)}}{\sum_{j=1}^{i} \frac{d_j(\pi(i)) \mu_{\pi(j)}}{\lambda_j(\pi(i), \boldsymbol{t}_i) \left(P_{\pi(j)} + \sum_{k:\pi(k)=\pi(j)} \frac{1}{\lambda_k(\pi(i), \boldsymbol{t}_i)} \right)}} \right)$$

$$\overset{(b)}{\geq} \sum_{j=1}^{i} \mu_{\pi(j)} \log_2 \left(\frac{\sum_{j=1}^{i} \mu_{\pi(j)}}{\sum_{j=1}^{i} \frac{d_j(\pi(i)) \mu_{\pi(j)}}{P_{\pi(j)}} \frac{1}{\lambda_j(\pi(i), \boldsymbol{t}_i)}} \right),$$

where (a) stems from the inequality between weighted harmonic and weighted geometric mean [6, Lemma 1] and (b) holds, since all $\lambda_k(\pi(i), \boldsymbol{t}_i) > 0$. Using the equality $\sum_{j=1}^{i} \rho_j \|e_j^{\mathrm{T}} \boldsymbol{A}\|_2^2 = \mathrm{tr}(\boldsymbol{A}^{\mathrm{H}} \boldsymbol{\Omega}_i \boldsymbol{A})$ with $\boldsymbol{\Omega}_i = \mathrm{diag}(\rho_1, \ldots, \rho_i)$ and replacing $\pi(i)$ with k and $\boldsymbol{t}_i$ with $\boldsymbol{t}$ leads to (7).

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