A Multiport Theory of Communications

Michel T. Ivrlač & Josef A. Nossek

Institute for Circuit Theory and Signal Processing
Technische Universität München, D-80333 Munich, Germany
email: {ivrlac, nossek}@tum.de

***Abstract*—Electromagnetics provides the ground for a *physical* theory of communications, while information theory and signal theory approach the problem from a purely *mathematical* point of view. Nevertheless, the latter theories frequently do refer to physical terms, such as: energy, power, noise, antennas, or waves. It is strange enough, that, at present time, there is no provision being made such that the usage of such terms in information theory is at least consistent with the governing physics. More often than not, this results in less than optimum signal processing solutions and does not contribute to a complete understanding of communication systems. Circuit theoretic channel models can help to bridge the gap between the physics of electromagnetics, and the mathematical world of information theory. The multiport concept makes sure that important physical concepts like energy, power, or noise are captured correctly, and terms such as antennas or waves are applied consistently with their physical meanings in information theory and signal processing. We suggest how to make circuit theoretic channel models and apply them to wireless communication systems which uses multiple antennas at both ends of the link. We thereby show that, in contrast to common belief, arrays of closely spaced antennas actually do support the bandwidth- and power-efficient multi-stream transmission.**

I. Introduction

There is a host of technical and scientific disciplines involved in the analysis and the design of telecommunication systems. In radio communications these include: electromagnetic field theory, radio-frequency engineering, circuit theory and design, signal-, coding- and information theory. The first two disciplines form a part of the *physical* theory of communications, for the laws of nature, like the Maxwell equations or the major conservation laws, play a central role in their concepts and methods. In contrast to that, signal-, coding-, and information theory are *mathematical* theories. As such, they are not based on the laws of nature but rather on definitions and mathematical logic. However, it is only in conjunction with the physical disciplines that one can attempt a complete theory which predictions can be put to the test by experiment. To this end, it is crucial that the mathematical and the physical layers of abstraction are consistent with each other. For instance, the field-theoretic view of antennas [1], [2], is rather different from the typical signal- or information theoretic view [3], [4], which drops many physical concepts, such as impedance or mutual coupling. Nevertheless, it is essential that these different representations are consistent with each other.

It might be surprising that, at present time at least, no provision is made which ensures that all the layers of abstraction actually *are* consistent, or at least, free of any conflict with one another. We can most easily see this when we look at a typical signal-processing channel model: the *vector additive white Gaussian noise* (AWGN) channel. Signals are brought up to the channel through a number of, say, N inputs, and observed at a number of, say, M outputs. The transmit power is *defined* to be proportional to the average squared Euclidean norm of the channel input vector, and the output is perturbed by additive white Gaussian noise. The noiseless input-output relationship is described by the $(M \times N)$ dimensional »channel matrix«, which has got its name due to the fact, that once it is known, the information capacity of the AWGN channel can be computed [5]. Hence, from an information theory point of view, the channel matrix tells all about the channel. The channel input, somehow has to be related with a relevant physical quantity of the communication system: perhaps a voltage or an electric field strength. However, physical power or energy cannot be obtained from just one such quantity, but instead a *conjugated pair* [6] is needed, say voltage and electric current, or electric and magnetic field strength. Hence, in a physical description of the channel, there are twice as many variables (one conjugated pair for each input and each output) than in the information theoretic description. By identifying each conjugated pair with one *port* of an $(M+N)$-port, the noiseless input-output relationship needs an $(M+N) \times (M+N)$ matrix, which connects one half of the port variables with the other half [7]. Because $MN < (M+N)^2$, the channel matrix does not have enough degrees of freedom to capture the complete physics of the channel. Yet, from an information theory point of view, it *has to* tell everything about the channel!

Nevertheless, this conflict can be resolved by making use of the *additional* degrees of freedom which come from the relationship between the information theoretic channel input and output on the one hand, and some of the physical port variables, on the other. By virtue of this relationship, the physical context can be »encoded« into the channel matrix, such that the correct channel capacity can be obtained in the usual way, within the information theory layer of abstraction.

To this end, our approach consists of the introduction of *circuit theoretic multiport models*, which are to serve as the »medium« between physical and purely mathematical layers of abstraction. On the one hand, circuit theory is so close to electromagnetics such that many important elements of high frequency engineering are captured very accurately by equivalent circuits. On the other hand, the port concept of circuit theory allows to abstract from these equivalent circuits again, and work instead with »black-box« multiports, that are completely described as soon as the relationship between the port-variables is known. For linear multiports this relationship is a linear one and yields a simple algebraic multiport description which connects neatly with information theory. Thereby, it keeps important information theoretic aspects of commu-

ISBN 978-3-8007-3211-1 © VDE VERLAG -GMBH · Berlin · Offenbach

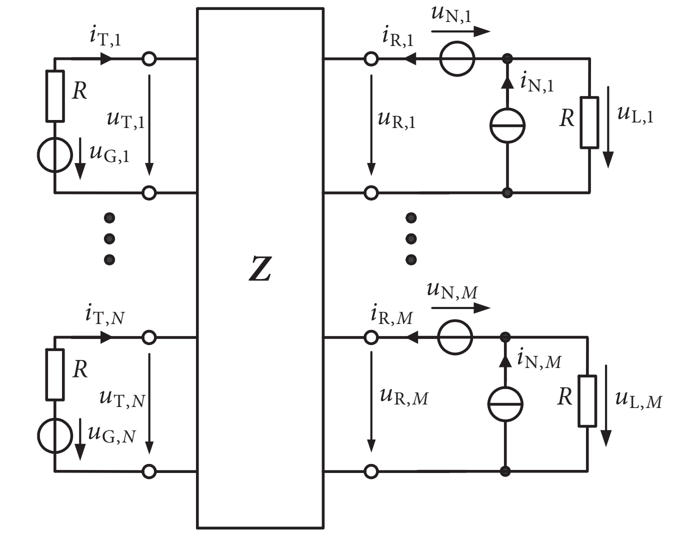

Figure 1. Linear multiport communication system.

nications, like transmit power or noise covariance, consistent with the physics of communications.

In this paper, we present a circuit theoretic multiport model for linear multi-input multi-output (MIMO) communication systems. We apply this multiport model to a wireless MIMO system and analyze its channel capacity as we move the antennas in the arrays closer and closer. While it is a common belief that close antenna spacing reduces the numerical rank of the channel matrix, and, henceforth, finally only permits a single data stream to be transfered, we find that quite the contrary can be true. Under certain conditions, close antenna spacing does *not* impair multi-stream transmission *at all*. This shows that wireless MIMO systems which are based on highly compact antenna arrays have potential for multi-streaming.

II. Multiport Communications

Let us now develop a physical model of a communication system with N inputs, and M outputs. Every input and output is replaced by a *port*, defined by two conjugated variables: complex voltage envelopes, and complex current envelopes. The $(M+N)$-port which results is displayed in Figure 1. The input signals are supplied by N voltage generators, which are modeled as ideal voltage sources with a series resistance R. The voltage envelopes $u_{G,n}$, with the index $n \in \{1, 2, \dots, N\}$, contain the information that has to be transfered over the channel. The voltage generators are connected to the first N ports of the multiport. To the remaining M ports, we connect noise voltage sources $u_{N,m}$, noise current sources $i_{N,m}$, and a termination resistance R, with the index $m \in \{1, 2, \dots, M\}$. These voltage sources and current sources model the system noise which may consist of the noise generated by the low-noise amplifiers and the noise that is generated by the communication multiport itself. We assume the multiport can be described by its impedance matrix $\boldsymbol{Z} \in \mathbb{C}^{(M+N)\times(M+N)} \cdot \mathrm{VA}^{-1}$:

$$\begin{bmatrix} \boldsymbol{u}_\mathrm{T} \\ \boldsymbol{u}_\mathrm{R} \end{bmatrix} = \boldsymbol{Z} \begin{bmatrix} \boldsymbol{i}_\mathrm{T} \\ \boldsymbol{i}_\mathrm{R} \end{bmatrix}, \quad (1)$$

where $\boldsymbol{u}_\mathrm{T} \in \mathbb{C}^{N\times 1} \cdot \mathrm{V}$, and $\boldsymbol{u}_\mathrm{R} \in \mathbb{C}^{M\times 1} \cdot \mathrm{V}$, are the vectors of the complex voltage envelopes appearing across the transmit and the receive ports, while $\boldsymbol{i}_\mathrm{T} \in \mathbb{C}^{N\times 1} \cdot \mathrm{A}$, and $\boldsymbol{i}_\mathrm{R} \in \mathbb{C}^{M\times 1} \cdot \mathrm{A}$, are the vectors of the corresponding complex current envelopes. When we collect all the generator complex voltage envelopes $u_{G,n}$, and all the load complex voltage envelopes $u_{L,m}$ into vectors $\boldsymbol{u}_\mathrm{G} \in \mathbb{C}^{N\times 1} \cdot \mathrm{V}$, and $\boldsymbol{u}_\mathrm{L} \in \mathbb{C}^{M\times 1} \cdot \mathrm{V}$, respectively, we can write the input-output relationship as:

$$\boldsymbol{u}_\mathrm{L} = \boldsymbol{D}\boldsymbol{u}_\mathrm{G} + \underbrace{\boldsymbol{E}\boldsymbol{u}_\mathrm{N} + \boldsymbol{F}\boldsymbol{i}_\mathrm{N} \cdot R}_{\sqrt{R}\,\boldsymbol{\eta}}, \quad (2)$$

where $\boldsymbol{u}_\mathrm{N} \in \mathbb{C}^{M\times 1} \cdot \mathrm{V}$, and $\boldsymbol{i}_\mathrm{N} \in \mathbb{C}^{M\times 1} \cdot \mathrm{A}$, are the vectors of the complex noise voltage and noise current envelopes, respectively, and

$$\boldsymbol{D} = -\widetilde{\boldsymbol{\Gamma}} \left(\mathbf{I}_{M+N} + R^{-1}\boldsymbol{Z}\right)^{-1} \boldsymbol{\Gamma}^\mathrm{T}, \quad (3)$$

$$\boldsymbol{E} = -\widetilde{\boldsymbol{\Gamma}} \left(\mathbf{I}_{M+N} + R^{-1}\boldsymbol{Z}\right)^{-1} \widetilde{\boldsymbol{\Gamma}}^\mathrm{T}, \quad (3a)$$

$$\boldsymbol{F} = \boldsymbol{E} + \mathbf{I}_M, \quad (3b)$$

where $\mathbf{I}_n$, is the $n \times n$, identity matrix, and

$$\boldsymbol{\Gamma} = \left[\, \mathbf{I}_N \;\; \mathbf{O}_{N\times M} \,\right], \quad (4)$$

$$\widetilde{\boldsymbol{\Gamma}} = \left[\, \mathbf{O}_{M\times N} \;\; \mathbf{I}_M \,\right], \quad (4a)$$

with $\mathbf{O}_{m\times n}$, denoting the $m \times n$, all-zeros matrix. The transmit power is defined as the noise-free net power flowing into the N transmit-side ports:

$$P_\mathrm{Tx} = \mathrm{E}\left[\mathrm{Re}\left\{\boldsymbol{u}_\mathrm{T}^\mathrm{H} \boldsymbol{i}_\mathrm{T}\right\} \mid (\boldsymbol{u}_\mathrm{N} = \mathbf{0}, \boldsymbol{i}_\mathrm{N} = \mathbf{0})\right] \quad (5)$$

$$= \frac{1}{4R} \mathrm{E}\left[\boldsymbol{u}_\mathrm{G}^\mathrm{H} \boldsymbol{B} \boldsymbol{u}_\mathrm{G}\right], \quad (5a)$$

where the matrix $\boldsymbol{B} \in \mathbb{C}^{N\times N}$, is given as:

$$\boldsymbol{B} = \boldsymbol{C} + \boldsymbol{C}^\mathrm{H} - \boldsymbol{C}^\mathrm{H}\boldsymbol{C}, \quad (6)$$

with the auxiliary matrix $\boldsymbol{C} \in \mathbb{C}^{N\times N}$, defined as:

$$\boldsymbol{C} = 2\boldsymbol{\Gamma} \left(\mathbf{I}_{M+N} + R^{-1}\boldsymbol{Z}\right)^{-1} \boldsymbol{\Gamma}^\mathrm{T}. \quad (6a)$$

In this paper, we assume for simplicity, that all complex noise voltage envelopes are mutually uncorrelated, and all complex noise current envelopes are mutually uncorrelated, but there may be a correlation between $u_{N,m}$, and $i_{N,m}$:

$$\mathrm{E}\left[u_{N,m}\, i^*_{N,m'}\right] = \delta_{m,m'} \cdot \rho \cdot \sqrt{\mathrm{E}\left[|u_{N,m}|^2\right] \mathrm{E}\left[|i_{N,m}|^2\right]}, \quad \forall m, \quad (7)$$

where $\rho \in \mathbb{C}$, is the correlation coefficient, which we assume is the same for all receiver side ports. The receiver noise covariance matrix is then given by:

$$\boldsymbol{R}_{\boldsymbol{\eta}} = \mathrm{E}\left[\boldsymbol{\eta}\boldsymbol{\eta}^\mathrm{H}\right] \quad (8)$$

$$= 4\sigma^2 \frac{\boldsymbol{F}\boldsymbol{F}^\mathrm{H} + \frac{R_\mathrm{N}}{R}\left(\rho \boldsymbol{E}\boldsymbol{F}^\mathrm{H} + \rho^* \boldsymbol{F}\boldsymbol{E}^\mathrm{H}\right) + \frac{R_\mathrm{N}^2}{R^2} \boldsymbol{E}\boldsymbol{E}^\mathrm{H}}{1 + \frac{R_\mathrm{N}}{R}\left(\rho + \rho^*\right) + \frac{R_\mathrm{N}^2}{R^2}}, \quad (8a)$$

where $\sigma^2 = ¼\,\mathrm{E}[|u_{N,m} + R i_{N,m}|^2]/R \in \mathbb{R}_+ \cdot \mathrm{W}$, denotes the available noise power, and

$$R_\mathrm{N} = \sqrt{\mathrm{E}\left[|u_{N,m}|^2\right] / \mathrm{E}\left[|i_{N,m}|^2\right]}, \quad \forall m, \quad (9)$$

is the so-called noise resistance [8].

ISBN 978-3-8007-3211-1

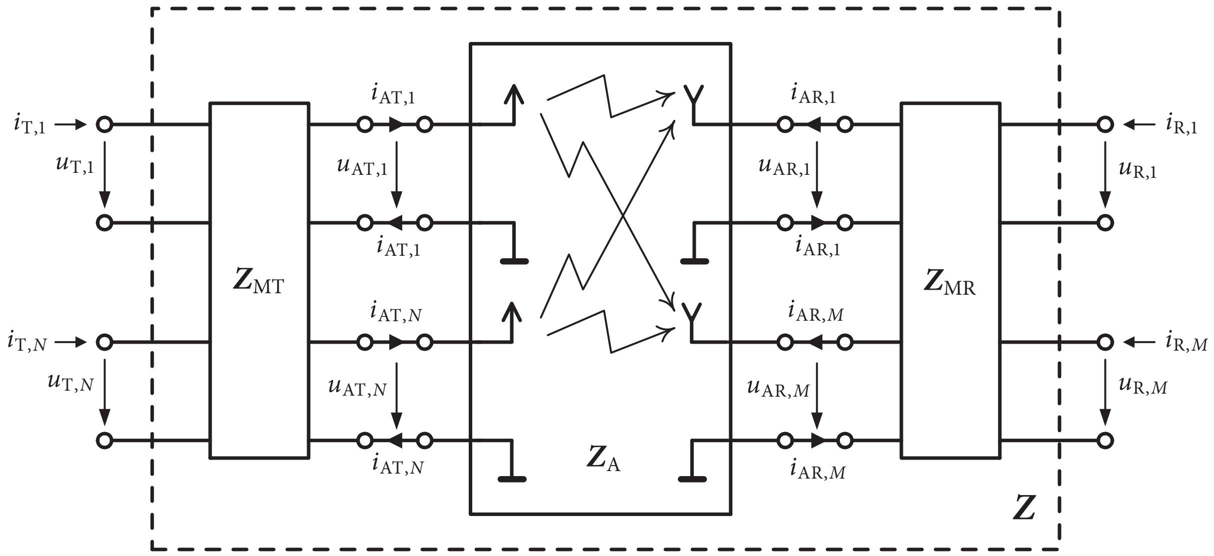

Figure 2. Multiport model of a wireless MIMO communication system with transmit and receive impedance matching networks.

With such a complete physical description of the multiport communication system, the next task is to convert it into the standard information theoretic channel model:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{\vartheta}, \tag{10}$$

$$\mathrm{E}\left[\|\boldsymbol{x}\|_2^2\right] = P_{\mathrm{Tx}}, \tag{10a}$$

$$\mathrm{E}\left[\boldsymbol{\vartheta}\boldsymbol{\vartheta}^{\mathrm{H}}\right] = \sigma_\vartheta^2 \mathbf{I}_M. \tag{10b}$$

Herein, the N-dimensional vector $\boldsymbol{x}$, is the channel input, the M-dimensional vector $\boldsymbol{y}$, is the channel output, while $\boldsymbol{\vartheta}$ denotes the M-dimensional channel noise vector, and $\boldsymbol{H} \in \mathbb{C}^{M\times N}$, is the channel matrix. For a given channel matrix, the channel capacity of the MIMO system can be computed [5].

It is part of the beauty of information theory, that one does *not have to* – and usually doesn't – define what the »channel input«, and »channel output« actually are, that is, how they are related with measurable quantities (physical quantities) of the communication system. By this abstract approach, (10), (10a), and (10b), can be used to model a great variety of communication systems.

In order to successfully apply information theory to a particular communication system – successfully in the sense that the predictions of the theory can stand the test of actual measurement –, one has to »encode« the *physical context* of the system into the channel matrix. It must make a difference in the way how we build the channel matrix, when, one time, our MIMO system is a multi-wire on-chip bus, and another time, a multi-antenna radio communication system, for the governing physics is different for the two. But how does one »encode« the physical context into the channel matrix? To this end, we define the following two bijective transformations:

$$\boldsymbol{x} = \boldsymbol{V}\boldsymbol{u}_{\mathrm{G}}, \tag{11}$$

$$\boldsymbol{y} = \boldsymbol{W}^{-1}\boldsymbol{u}_{\mathrm{L}}, \tag{11a}$$

between the physical port variables $(\boldsymbol{u}_{\mathrm{G}}, \boldsymbol{u}_{\mathrm{L}})$, and the information theoretic input and output variables $(\boldsymbol{x}, \boldsymbol{y})$, where $\boldsymbol{V}$, and $\boldsymbol{W}$ are invertible matrices. When we substitute (11) and (11a) into (10), and compare the result with (2), we see that

$$\boldsymbol{H} = \boldsymbol{W}^{-1}\boldsymbol{D}\boldsymbol{V}^{-1}, \tag{12}$$

must hold. By defining:

$$\boldsymbol{W} = \sqrt{\frac{R}{\sigma_\vartheta^2}}\boldsymbol{R}_{\boldsymbol{\eta}}^{1/2} \in \mathbb{C}^{M\times M} \cdot \sqrt{\mathrm{V}\cdot\mathrm{A}^{-1}}, \tag{13}$$

$$\boldsymbol{V} = \frac{1}{2\sqrt{R}}\boldsymbol{B}^{1/2} \in \mathbb{C}^{N\times N} \cdot \sqrt{\mathrm{A}\cdot\mathrm{V}^{-1}}, \tag{13a}$$

it is made sure that (10a) and (10b) hold true, provided that, firstly: $\boldsymbol{B}^{\mathrm{H}} = \boldsymbol{B} > \boldsymbol{0}$, and secondly $\boldsymbol{R}_{\boldsymbol{\eta}}$ is regular. If this is so, then (10), (10a), (10b), is a circuit theoretic channel model, where the dimensionless matrix $\boldsymbol{H} \in \mathbb{C}^{M\times N}$, as defined in (12), captures all physical properties of the multiport communication system that are of relevance for information theory.

III. Modeling Wireless MIMO Systems

We apply the developed circuit theoretic channel model to the wireless MIMO system shown in Figure 2. It consists of two antenna arrays with N, and M antennas, located at the transmitter and the receiver, respectively, and it is modeled as a linear multiport with impedance matrix $\mathbf{Z}_{\mathrm{A}} \in \mathbb{C}^{(M+N)\times(M+N)} \cdot \mathrm{VA}^{-1}$:

$$\begin{bmatrix} \boldsymbol{u}_{\mathrm{AT}} \\ \boldsymbol{u}_{\mathrm{AR}} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{Z}_{\mathrm{AT}} & \mathbf{Z}_{\mathrm{ATR}} \\ \mathbf{Z}_{\mathrm{ART}} & \mathbf{Z}_{\mathrm{AR}} \end{bmatrix}}_{\mathbf{Z}_{\mathrm{A}}} \begin{bmatrix} \boldsymbol{i}_{\mathrm{AT}} \\ \boldsymbol{i}_{\mathrm{AR}} \end{bmatrix}, \tag{14}$$

where $\boldsymbol{u}_{\mathrm{AT}}$, $\boldsymbol{u}_{\mathrm{AR}}$, $\boldsymbol{i}_{\mathrm{AT}}$, and $\boldsymbol{i}_{\mathrm{AR}}$, denote the vectors of complex voltage and current envelopes of the transmit and receive antenna arrays, respectively. Because antennas are reciprocal, it is true that $\mathbf{Z}_{\mathrm{ATR}} = \mathbf{Z}_{\mathrm{ART}}^{\mathrm{T}}$. However, it is also true, that in radio communications, there usually is a huge attenuation between transmitter and receiver, such that the receiver (almost) does not act back on the transmitter. In the following, we therefore apply the so-called unilateral approximation of (14), which is obtained by setting $\mathbf{Z}_{\mathrm{ATR}} = \mathbf{O}_{N\times M}$, while keeping $\mathbf{Z}_{\mathrm{ART}}$, as it is.

ISBN 978-3-8007-3211-1 © VDE VERLAG -GMBH · Berlin · Offenbach

Let the impedance matching networks be lossless multiports, which are described by:

$$\begin{bmatrix} \boldsymbol{u}_\mathrm{T} \\ \boldsymbol{u}_\mathrm{AT} \end{bmatrix} = \mathrm{j}\begin{bmatrix} \mathbf{O}_N & -\sqrt{R}\,\mathrm{Re}\{\mathbf{Z}_\mathrm{AT}\}^{1/2} \\ -\sqrt{R}\,\mathrm{Re}\{\mathbf{Z}_\mathrm{AT}\}^{1/2} & -\mathrm{Im}\{\mathbf{Z}_\mathrm{AT}\} \end{bmatrix}\begin{bmatrix} \boldsymbol{i}_\mathrm{T} \\ -\boldsymbol{i}_\mathrm{AT} \end{bmatrix},$$

$$\begin{bmatrix} \boldsymbol{u}_\mathrm{R} \\ \boldsymbol{u}_\mathrm{AR} \end{bmatrix} = \mathrm{j}\begin{bmatrix} R_\mathrm{N}\beta\mathbf{I}_M & \sqrt{R_\mathrm{N}\alpha}\,\mathrm{Re}\{\mathbf{Z}_\mathrm{AR}\}^{1/2} \\ \sqrt{R_\mathrm{N}\alpha}\,\mathrm{Re}\{\mathbf{Z}_\mathrm{AR}\}^{1/2} & -\mathrm{Im}\{\mathbf{Z}_\mathrm{AR}\} \end{bmatrix}\begin{bmatrix} \boldsymbol{i}_\mathrm{R} \\ -\boldsymbol{i}_\mathrm{AR} \end{bmatrix},$$

where $\alpha, \beta \in \mathbb{R}$, will be explained in just a moment. Within the domain of the unilateral approximation, the impedance matrix $\boldsymbol{Z}$, of the complete system, composed of the antenna arrays, impedance matching networks, and the medium connecting receiver and transmitter, then becomes:

$$\boldsymbol{Z} = \begin{bmatrix} \mathbf{Z}_\mathrm{T} & \mathbf{O}_{N\times M} \\ \mathbf{Z}_\mathrm{RT} & \mathbf{Z}_\mathrm{R} \end{bmatrix} \in \mathbb{C}^{(M+N)\times(M+N)} \cdot \mathrm{VA}^{-1}, \qquad (15)$$

where

$$\mathbf{Z}_\mathrm{T} = R \cdot \mathbf{I}_N, \qquad (16)$$

$$\mathbf{Z}_\mathrm{R} = R_\mathrm{N}\,(\alpha + \mathrm{j}\beta) \cdot \mathbf{I}_M, \qquad (16a)$$

$$\mathbf{Z}_\mathrm{RT} = \sqrt{RR_\mathrm{N}\alpha} \cdot \mathrm{Re}\{\mathbf{Z}_\mathrm{AR}\}^{-1/2}\,\mathbf{Z}_\mathrm{ART}\,\mathrm{Re}\{\mathbf{Z}_\mathrm{AT}\}^{-1/2}. \qquad (16b)$$

The purpose of the impedance matching networks becomes clear now. Firstly, both transmit side and receive side antenna arrays get *decoupled*, because $\mathbf{Z}_\mathrm{T}$, and $\mathbf{Z}_\mathrm{R}$, are diagonal matrices. Secondly, each transmit side voltage generator is loaded by the impedance R. Since this equals its internal impedance, the generators deliver all their available power (*power matching*). Thirdly, each receive amplifier is driven by a source impedance equal to $R_\mathrm{N}\,(\alpha + \mathrm{j}\beta)$, where α, and β, can be optimized such as to obtain the largest possible signal to noise ratio (SNR) at the output of the amplifiers (*noise matching*) [9]:

$$\alpha = \sqrt{1 - \mathrm{Im}\{\rho\}^2}, \quad \beta = \mathrm{Im}\{\rho\}. \qquad (17)$$

Notice from (16b), that even though the impedance matching network does decouple the antennas, mutual antenna coupling reappears in the transimpedance matrix $\mathbf{Z}_\mathrm{RT}$, by virtue of the *real-parts* of $\mathbf{Z}_\mathrm{AR}$, and $\mathbf{Z}_\mathrm{AT}$. Therefore, **a *de*coupled antenna array is substantially different from an array of *un*coupled antennas.** By defining:

$$\sigma_\vartheta^2 = 8\sigma^2 \cdot \frac{\sqrt{1 - \mathrm{Im}\{\rho\}^2} - \mathrm{Re}\{\rho\}}{R_\mathrm{N}/R + R/R_\mathrm{N} + 2\mathrm{Re}\{\rho\}}, \qquad (18)$$

and $\varphi = \arctan \beta/(\alpha + R/R_\mathrm{N})$, the MIMO channel matrix from (12), becomes:

$$\boldsymbol{H} = \mathrm{e}^{-\mathrm{j}\varphi} \cdot \mathrm{Re}\{\mathbf{Z}_\mathrm{AR}\}^{-1/2}\,\mathbf{Z}_\mathrm{ART}\,\mathrm{Re}\{\mathbf{Z}_\mathrm{AT}\}^{-1/2}. \qquad (19)$$

Notice that the physical noise properties (σ^2, R_N, and ρ), are condensed into the variance σ_ϑ^2. The unimodular term $\mathrm{e}^{-\mathrm{j}\varphi}$, is not important from an information theoretic perspective, for it has no effect on the channel capacity. For a *uniform linear array of isotropic radiators*, it can be shown [10] that:

$$\left(\mathrm{Re}\{\mathbf{Z}_\mathrm{AR}\}\right)_{m,n} = \left(\mathrm{Re}\{\mathbf{Z}_\mathrm{AT}\}\right)_{m,n} = R_\mathrm{r} \cdot \mathrm{sinc}(kd(m-n)), \qquad (20)$$

with wave number $k = 2\pi/\lambda$, where λ is the wave length, and d is the distance between neighboring radiators, while $\mathrm{sinc}(x)$, is the $\sin(x)/x$ - function, and $R_\mathrm{r} \in \mathbb{R}_+ \cdot \mathrm{VA}^{-1}$, is the so-called radiation resistance of the antennas [1]. Being *canonical minimum scattering* antennas [11], the isotropic radiators do *not* interfere with each other as long as there are *no* electric currents flowing through them [12]. As zero port current is exactly the condition one needs to compute the entries of an impedance matrix, the transimpedance matrix $\mathbf{Z}_\mathrm{ART}$, can be readily obtained. One just has to look at antenna pairs formed by one receive side and one transmit side antenna, for the neighboring antennas do not interfere for zero port currents. For two uniform linear arrays in empty space, and aligned with the z-axis, or the z'-axis of Cartesian coordinate systems, respectively, (see Figure 3), we, therefore, obtain the well known relationship:

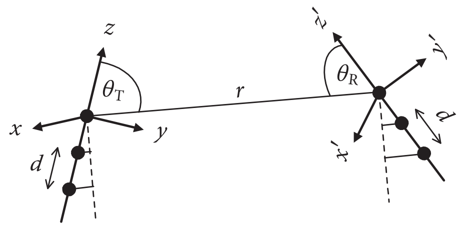

Figure 3. Two arbitrarily oriented uniform linear arrays in free space.

$$\mathbf{Z}_\mathrm{ART} = \zeta \cdot \boldsymbol{a}_\mathrm{R}(\theta_\mathrm{R})\boldsymbol{a}_\mathrm{T}^\mathrm{T}(\theta_\mathrm{T}),$$

where θ_R, and θ_T, are the angles of elevation from which the receiver sees the transmitter, and vice versa (see also Figure 3), while the transmit and receive array steering vectors are defined in the usual way:

$$\boldsymbol{a}_\mathrm{T}(\theta) = \left[\,1\ \ \mathrm{e}^{-\mathrm{j}kd\cos\theta}\ \ \mathrm{e}^{-2\mathrm{j}kd\cos\theta}\ \cdots\ \mathrm{e}^{-(N-1)\mathrm{j}kd\cos\theta}\,\right]^\mathrm{T}, \qquad (21)$$

$$\boldsymbol{a}_\mathrm{R}(\theta) = \left[\,1\ \ \mathrm{e}^{-\mathrm{j}kd\cos\theta}\ \ \mathrm{e}^{-2\mathrm{j}kd\cos\theta}\ \cdots\ \mathrm{e}^{-(M-1)\mathrm{j}kd\cos\theta}\,\right]^\mathrm{T}, \qquad (21a)$$

and $\zeta \in \mathbb{C} \cdot \mathrm{VA}^{-1}$, is a constant (which depends on the distance r, of receiver and transmitter, and the wave length). In a multipath environment, the transimpedance matrix becomes:

$$\mathbf{Z}_\mathrm{ART} = \sum_i \zeta_i \cdot \boldsymbol{a}_\mathrm{R}(\theta_{\mathrm{R},i})\boldsymbol{a}_\mathrm{T}^\mathrm{T}(\theta_{\mathrm{T},i}), \qquad (22)$$

where $\theta_{\mathrm{R},i}$, and $\theta_{\mathrm{T},i}$, are the i-th path's angles of arrival and departure, respectively, and $\zeta_i \in \mathbb{C} \cdot \mathrm{VA}^{-1}$, is the corresponding path coefficient. With (19), (20), and (22), we now have got all the necessary ingredients to model wireless MIMO systems consistently with the governing physics.

IV. Compact Wireless MIMO Systems

Because mutual antenna coupling is strongest at small antenna separation, it can be expected that it has a strong impact on the performance of compact MIMO systems – and indeed, it has. To this end, let there be *two* paths connecting the transmitter to the receiver, say, one direct path in line of sight, and another path via some reflectors. Notice that $\mathbf{Z}_\mathrm{ART}$, as given in (22), will converge to a scaled all-ones matrix, as $d \to 0$, hence, becoming rank deficient. However, what about the rank of the channel matrix $\boldsymbol{H}$? Substituting (22) and (20) into (19), it follows for the case $M = N = 2$, that

$$\det \lim_{d\to 0} \boldsymbol{H} =$$
$$-3\mathrm{e}^{-2\mathrm{j}\varphi}\frac{\zeta_1\zeta_2}{R_\mathrm{r}^2}\left(\cos(\theta_{\mathrm{R},1}) - \cos(\theta_{\mathrm{R},2})\right)\left(\cos(\theta_{\mathrm{T},1}) - \cos(\theta_{\mathrm{T},2})\right).$$

ISBN 978-3-8007-3211-1 © VDE VERLAG -GMBH · Berlin · Offenbach

Interestingly, the channel matrix remains regular, even when $d \to 0$, provided that both of the paths are distinct at the receiver and the transmitter. Therefore, transmission and reception of multiple data streams at the same time inside the same band of frequencies is possible, even when very compact antenna arrays are employed. In order to demonstrate this effect in an even more striking way, consider the special case:

$$\theta_{\mathrm{R},1} = \theta_{\mathrm{T},2} = \pi/2 - \arccos\sqrt{2/3} \approx 55°,$$
$$\theta_{\mathrm{R},2} = \theta_{\mathrm{T},1} = \pi/2 + \arccos\sqrt{2/3} \approx 125°,$$
$$\zeta_1 = \zeta_2 = \zeta.$$

The channel matrix now becomes, as $d \to 0$:

$$\lim_{d\to 0} \boldsymbol{H} = 2\mathrm{e}^{-\mathrm{j}\varphi}\frac{\zeta}{R_\mathrm{r}}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

This intriguing result shows that it is possible to transfer two data streams which are even capable to carry *the same* information rate, despite that $d \to 0$.

To look at the same subject from a slightly different point of view, consider a multi-path environment, which gives raise to be modeled as correlated Rayleigh fading which is independent between receiver and transmitter:

$$\boldsymbol{Z}_\mathrm{ATR} = \frac{1}{\sqrt{\operatorname{tr}\boldsymbol{R}_\mathrm{Tx}}}\boldsymbol{R}_\mathrm{Rx}^{1/2}\boldsymbol{G}\boldsymbol{R}_\mathrm{Tx}^{1/2}, \qquad (23)$$

where $\boldsymbol{R}_\mathrm{Tx} = \mathrm{E}\left[\boldsymbol{Z}_\mathrm{ATR}^\mathrm{H}\boldsymbol{Z}_\mathrm{ATR}\right]$, and $\boldsymbol{R}_\mathrm{Rx} = \mathrm{E}\left[\boldsymbol{Z}_\mathrm{ATR}\boldsymbol{Z}_\mathrm{ATR}^\mathrm{H}\right]$, denote the transmit- and the receive side fading covariance matrices, respectively, while $\boldsymbol{G} \in \mathbb{C}^{M\times N}$, contains independent and identically distributed, zero-mean, unity-variance, circularly symmetric, complex Gaussian random variables. The channel capacity of the MIMO system can be expressed as [5]:

$$C = \sum_{i=1}^{N} C_i, \qquad (24)$$

that is, as the sum of information rates C_i, of up to N data streams, computable as:

$$C_i = \log_2 \max\left(1, \xi\mu_i/\sigma^2\right), \qquad (25)$$

where the constant $\xi \in \mathbb{R}_+ \cdot \mathrm{W}$, is chosen such that

$$\sum_{i=1}^{N} \max\left(0, \xi - \sigma^2/\mu_i\right) = P_\mathrm{Tx}, \qquad (26)$$

is fulfilled. Herein, the $\mu_i \in \mathbb{R}_{0+}$, with $i \in \{1, 2, \ldots, N\}$, denote the eigenvalues of $\boldsymbol{H}^\mathrm{H}\boldsymbol{H}$. Let the transmit and receive side antenna arrays be aligned co-linearly, that is, be aligned in the so-called »end-fire« direction. Let there be a uniform-conical angle spread with an opening angle of 120°, centered around the »end-fire« direction, for both the transmitter and the receiver. For a fixed transmit power, we compute the ergodic channel capacity $\mathrm{E}[C]$, and the ergodic information rates $\mathrm{E}[C_i]$, of the individual data streams, by evaluating (24), and (25), respectively, and averaging over different realizations of Rayleigh fading (that is, different realizations of the matrix $\boldsymbol{G}$). Figure 4 shows the results. As the antenna spacing d, is reduced from half wavelength towards zero, both the ergodic channel capacity $\mathrm{E}[C]$, and the individual ergodic rates $\mathrm{E}[C_1]$, and $\mathrm{E}[C_2]$, actually even *increase* a little bit.

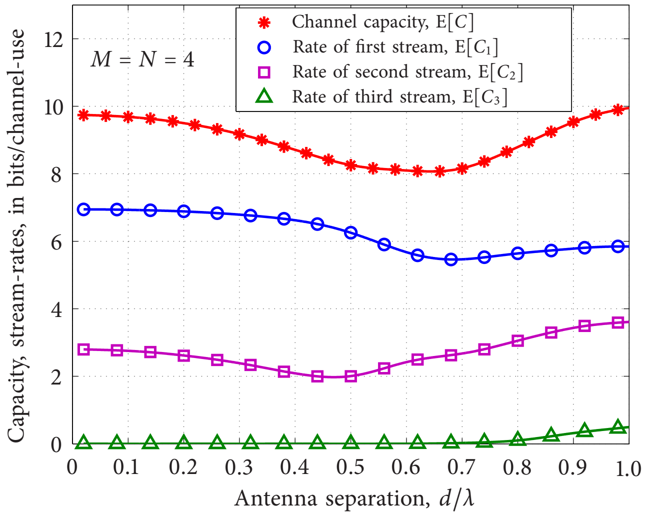

Figure 4. Ergodic channel capacity and stream-rates.

V. Conclusion

A circuit theoretic multiport modeling approach is presented, which has the purpose to ensure consistency among the different layers of abstraction which are used in analysis and design of communication systems. Especially, it is taken care that important terms, such as »energy«, »noise«, or »antenna« are used consistently with the physics that governs the communication system. To demonstrate the utility of the proposed multiport modeling approach, it is shown that MIMO systems composed of very densely packed antenna arrays *can* be used for multi-streaming. This fact is a direct result of the physics of mutual antenna coupling, in conjunction with properly designed impedance matching networks. This result highlights the necessity that channel models are used in communications engineering, which are consistent with physics.

References

[1] S. A. Schelkunoff and H. T. Friis, *Antennas. Theory and Practice.* New York, NY: Wiley, 1952.

[2] P. Russer, *Electromagnetics, Microwave Circuits and Antenna Design for Communications Engineering*, 2nd ed. Nordwood, MA: Artec House, 2006.

[3] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication.* Cambridge University Press, 2005.

[4] J. C. Liberti and T. S. Rappaport, *Smart Antennas for Wireless Communications.* New Jersey: Prentice Hall, 1999.

[5] I. E. Telatar, "Capacity of Multi-Antenna Gaussian Channels," *European Transactions on Telecommunications*, vol. 10(6), pp. 585–596, Nov. 1999.

[6] H. M. Paynter, *Analysis and Design of Engineering Systems.* MIT Press, Cambridge, MA, 1961.

[7] V. Belevitch, "Elementary Applications of the Scattering Formalism to Network Design," *IRE Transactions on Circuit Theory*, vol. 3(2), pp. 97–104, June 1956.

[8] J. Engberg and T. Larsen, *Noise Theory of Linear and Nonlinear Circuits.* New York: Wiley, 1995.

[9] H. Hillbrand and P. Russer, "An Efficient Method for Computer Aided Noise Analysis of Linear Amplifier Networks," *IEEE Transactions on Circuits and Systems*, vol. 23(4), pp. 235–238, June 1976.

[10] H. Yordanov, M. T. Ivrlač, P. Russer, and J. A. Nossek, "Arrays of Isotropic Radiators – A Field-theoretic Justification," in *Proc. ITG/IEEE Workshop on Smart Antennas*, Berlin, Germany, Feb. 2009.

[11] K. Kahn and H. Kurss, "Minimum-Scattering Antennas," *IEEE Transactions on Antennas and Propagation*, vol. AP-13, pp. 671–675, Sept. 1965.

[12] W. Wasylkiwskyj and W. K. Kahn, "Theory of Mutual Coupling Among Minimum-Scattering Antennas," *IEEE Transactions on Antennas and Propagation*, vol. 18(2), pp. 204–216, Mar. 1970.

ISBN 978-3-8007-3211-1 © VDE VERLAG -GMBH · Berlin · Offenbach