Inharmonicity of Sounds from Electric Guitars: Physical Flaw or Musical Asset?

Hugo Fastl, Florian Völk

AG Technische Akustik, MMK, Technische Universität München, Germany
fastl@mmk.ei.tum.de

ABSTRACT

Because of bending waves, strings of electric guitars produce (slightly) inharmonic spectra. One aim of the study was to find out - also in view of synthesized musical instruments - whether sounds of electric guitars should preferably produce strictly harmonic or slightly inharmonic spectra. Inharmonicities of typical sounds from electric guitars were analyzed and studied in psychoacoustic experiments. Strictly harmonic as well as slightly inharmonic spectra were realized and evaluated with respect to audibility of beats, possible differences in pitch height, and overall preference. Strictly harmonic as well as slightly inharmonic spectra produce essentially the same pitch height. Depending on the magnitude of the inharmonicity, beats are clearly audible. Low, wound strings (e.g. E2, A2) usually produce larger inharmonicities than high strings (e.g. E4), and hence beats of low notes are easily detected. The inharmonicity increases with the diameter of the respective string's kernel. Therefore, an unwound G3 string can show larger inharmonicity and more prominent beats than a wound D3 string. In a musical context, i.e. when playing short melodies with synthesized strictly harmonic versus slightly inharmonic string sounds, the latter are somewhat preferred, in particular by players of string instruments. The slight inharmonicity of sounds from electric guitars is not a physical flaw which should be avoided by synthesizing perfectly harmonic sounds, but can be regarded as a musical asset.

I. INTRODUCTION

With modern synthesizers it is possible to realize an almost infinite variety of sounds. On the one hand, with advanced sampling techniques sounds can be produced which resemble to a large extent the sounds radiated from the original musical instruments. Thus, well-known musical sounds of traditional instruments are imitated. On the other hand, by digital synthesis sounds can be produced which go far beyond the possibilities of traditional musical instruments. Sounds can be composed out of many sinusoidal components and the frequencies, amplitudes and phases can be chosen with great freedom. In this way, new, so far "unheard" sounds can be synthesized which can be evaluated in a musical context.

Along these lines, synthetic sounds simulating sounds from electric guitars were studied. Because of bending waves, spectra of sounds from electric guitars are not perfectly harmonic, but show slight inharmonicities. This means that the harmonics of a guitar sound are not literally harmonically related, i.e. they do not form frequency ratios of 1:2:3:4 etc, but are slightly stretched. In this situation, two possibilities could exist:

1) The inharmonicity of the sounds from electric guitars is a physical flaw stemming from the bending waves of the strings which could be avoided by modern electronic means when synthesizing "pure" strictly harmonic sounds.

2) The reputed flaw over the years has become a musical asset, i.e. in a musical context, "imperfect" inharmonic sounds are preferred and need no "improvement".

II. EXPERIMENTS

A. Inharmonicity of guitar strings

The deviation of the sound signals produced by a plucked string from being exactly harmonically is dependent on the inharmonicity of the string, which itself is dependent on the stiffness of the string. The latter is dependent primarily on physical parameters of the considered string-material. If the inharmonicity is given, it is possible to compute the frequency of the n^{th} component for a string, which would have the fundamental frequency without stiffness (cf. Fletcher and Rossing 1998) the following way:

$$f_n = n \cdot f_1 \cdot \sqrt{1 + b \cdot n^2} \quad \text{for} \quad n \in [1; N] \qquad [1]$$

There is a variety of different string combinations on the market, which allows the musician to select the set best fitting his or her personal preferences and musical needs. Table 1 lists the inharmonicities of an often used standard string set (cf. Zollner 2007).

Table 1. Inharmonicities of a typical string set used on electric guitars (after Zollner 2007).

string	inharmonicity
E2	$1.1 \cdot 10^{-4}$
A2	$6.5 \cdot 10^{-5}$
D3	$4.5 \cdot 10^{-5}$
G3	$8.4 \cdot 10^{-5}$
B3	$2.9 \cdot 10^{-5}$
E4	$1.1 \cdot 10^{-5}$

It is obvious that the inharmonicity generally decreases with increasing frequency. The discontinuity in Table 1 between D3 and G3 evolves from the fact that the string set considered here consisted of three wound (E2, A2, D3) and three unwound strings (G3, B3, E4). The inharmonicity of a solid (unwound) string grows with its diameter, whereas the inharmonicity of a wound string depends mainly on its core diameter. For that reason there might occur a discontinuity, dependent on the string set, as it is the case in Table 1.

B. Synthesis of sounds

The essential factor influencing the amplitude of each spectral component within the complex tone produced by a plucked guitar string is, according to Fletcher and Rossing (1998), the point at which the string is plucked. For the synthesis performed in the current study, a position at 20 % of the full length was chosen, which is assumed to best match the average plucking position under standard conditions. This plucking position causes that every fifth harmonic in the spectrum is missing (cf. Fletcher and Rossing 1998). Additionally, a phase jump of 180° appears between every two partial tones

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neighboring a missing harmonic, because at the plucking point, there is a maximum of the amplitude at the plucking time.

The amplitude of the spectral components decreases with time because of different damping-mechanisms. According to Fleischer (2002), the amplitude of each harmonic is decreasing exponentially with a time constant dependent on the order *n* of the considered harmonic, and on the specific complex tone. Each tone/string combination is characterized by a time constant dependent on the physical parameters of the string under the circumstances considered.

Taking into account all of the aforementioned considerations, sound signals of the following structure were synthesized:

$$p(t) = p_1(t) + p_2(t) \qquad [2]$$

With

$$p_1(t) = \begin{cases} \sum_{n=1}^{N} \hat{p}_n \cos(2\pi f_n t) \cdot e^{\frac{-t}{\frac{\tau}{n}}} & \text{for } n \bmod 10 < 5 \\ 0 & \text{otherwise} \end{cases} \qquad [3]$$

$$p_2(t) = \begin{cases} \sum_{n=1}^{N} \hat{p}_n \cos(2\pi f_n t + \pi) \cdot e^{\frac{-t}{\frac{\tau}{n}}} & \text{for } n \bmod 10 > 5 \\ 0 & \text{otherwise} \end{cases} \qquad [4]$$

and

$$\hat{p}_n = \frac{25}{2n^2\pi^2} \sin\frac{n\pi}{5} \qquad [5]$$

This way slightly inharmonic as well as perfectly harmonic sounds could be synthesized by including the appropriate harmonic or inharmonic overtones. In both cases, all overtones up to 20 kHz have been included in the synthesis process.

The resulting signals were filtered by a second order low pass (Tschebyscheff, 3 dB magnification factor at 4 kHz, 12 dB/oct. attenuation), which should approximate the transfer function of an electric guitar and should this way account for the typical tone color of guitar sounds.

To prevent audible clicks at the beginning and the end of the synthesized tones, each tone was switched on and off with Gaussian gating signals. The rise time of the gating signals at the beginnings was selected to 5 ms in order to simulate a realistic plucking of the string. The decay time of the gating signal at the end of the synthesized guitar sound was chosen to 50 ms in order to create an authentic decay.

Two different kinds of stimuli were created according to the procedure described:

Pairs of harmonic versus inharmonic synthesized guitar sounds with a duration of 2 s and a pitch corresponding to each free floating guitar string, respectively.

Pairs of a six melodies with durations between 3 s and 8 s realized with harmonic versus inharmonic guitar sounds.

C. Procedures

Sounds were presented diotically via electrodynamic Headphones (Beyer DT48) with free-field equalizer according to Fastl and Zwicker (2007, p. 7) in a sound attenuation booth. Sound levels as displayed in Table 2 were used for isolated guitar sounds.

Table 2. Presentation levels of the synthesized guitar sounds.

tone	level harmonic tone [dB]	level inharmonic tone [dB]
E2	73.1	73.1
A2	73.1	73.0
D3	72.2	72.2
G3	71.9	71.9
B3	71.5	71.5
E4	70.4	70.4

As can be seen from the data displayed in Table 2, care was taken that the harmonic versus the slightly inharmonic guitar sounds were presented at the same level.

For the short melodies with durations between 3 and 8 s, the levels displayed in Table 3 were used.

Table 3. Presentation levels of the short melodies.

melody	level harmonic melody [dB]	level inharmonic melody [dB]
M1	64.7	64.6
M2	66.1	66.1
M3	64.4	64.4
M4	66.6	66.6
M5	65.2	65.2
M6	68.1	68.1

The data displayed in Table 3 indicate that short melodies synthesized from guitar sounds with harmonic versus slightly inharmonic spectra were presented at the same level. The small deviation of 0.1 dB for the different realizations of melody M1 in practice does not lead to audible loudness differences.

In a first experiment, pairs of inharmonic versus harmonic guitar sounds were presented and subjects had to answer by “yes” or “no” to the question: “Can you hear a difference between the sounds within a pair?” Each pair was presented four times in random order. In addition, pairs where both sounds were either inharmonic or harmonic were presented to check the reliability of the responses.

In a second experiment, again pairs of inharmonic versus harmonic guitar sounds were presented. The question was, whether one of the two sounds within a pair contains more beats. Three alternative responses were allowed: “neither of both sounds”, “the first sound”, “the second sound”.

In a third experiment short melodies realized with inharmonic versus harmonic guitar sounds, and arranged in pairs had to be compared. Again three alternative responses were allowed: “neither of the melodies within a pair is preferred”, “the first melody within a pair is preferred”, “the second melody within a pair is preferred”.

In a fourth experiment subjects had to match the pitch of a pure tone to the pitch of harmonic versus slightly inharmonic guitar sounds using a method of adjustment.

D. Subjects

All subjects who participated in the experiment had normal ability, i.e. their thresholds in quiet were within 20 dB of standard values.

In the first experiment participated 20 subjects, nine of whom play a string instrument, seven play other instruments and, four play no instrument, but like to listen to music.

In the second experiment participated 19 subjects who were the same as in experiment one. However, one subject who plays no instrument could not participate for lack of time.

In the third experiment participated 16 subjects, eight of whom played a string instrument, five another instrument, and three play no instrument, but like to listen to music.

In the fourth experiment participated five subjects who play a string instrument: three guitar and two double bass, one of them with a degree in music.

III. RESULTS

In this paragraph results are given for four sets of experiments according to the following questions:

Can synthesized guitar sounds with inharmonic versus harmonic spectra be distinguished?

Which synthesized guitar sounds contain more beats, inharmonic versus harmonic sounds?

Which type of short melodies is preferred, composed of synthesized inharmonic versus harmonic guitar sounds?

Is the pitch of synthesized inharmonic versus harmonic guitar sounds different?

A. Distinguishing synthesized inharmonic versus harmonic guitar sounds

The results displayed in Figure 1 address the question, whether synthesized inharmonic versus harmonic guitar sounds can be distinguished. The guitar strings E2 to E4 are indicated along the abscissa. On the left ordinate the relative number of correct positive responses is given. The right ordinate shows the inharmonicity of the respective synthesized guitar string sound. Filled columns illustrate psychoacoustic evaluations of audible differences, unfilled columns inharmonicities.

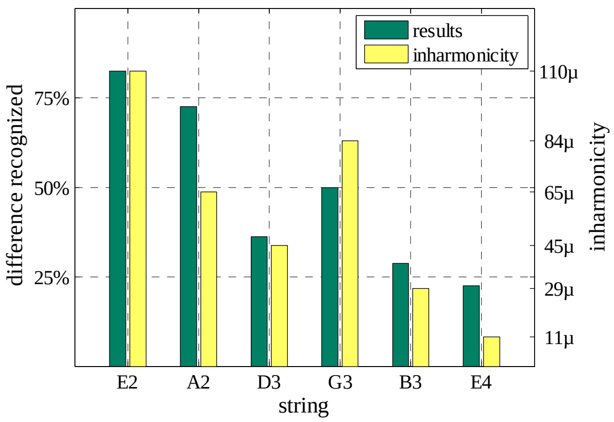

Figure 1. Perceived differences between synthesized harmonic and inharmonic guitar sounds. Along the abscissa, the guitar strings E2 to E4 are indicated. On the left ordinate, the relative number of correct positive responses, on the right ordinate, the inharmonicity of the respective synthesized guitar string is given. Filled columns show psychoacoustic results, unfilled columns inharmonicities.

The data plotted in Figure 1 show that synthesized inharmonic versus harmonic guitar sounds are easily distinguished (about 75 %) at the low strings E2 and A2, but frequently mixed up at the high strings B3 and E4. Closer inspection reveals a steady decrease of the distinction between synthesized inharmonic versus harmonic guitar sounds with the pitch of the respective string, except for G3, where the distinction increases again. When comparing unfilled and filled columns in Figure 1 it becomes clear that this increase is related to a larger inharmonicity of the synthesized sound for the G3. Since E2, A2, and D3 are wound strings, the diameter of their kernel decreases. However, G3 being an unwound string, shows a kernel with larger diameter than (the wound string) D3 and therefore produces more inharmonicity, which is easier detected.

B. Audibility of beats in synthesized inharmonic versus harmonic guitar sounds

The results displayed in Figure 2 address the question, whether synthesized inharmonic or harmonic guitar sounds produce more audible beats. For the strings E2 to E4 the histograms that more beats are audible are plotted. Filled columns indicate responses that guitar sounds with inharmonic spectra produce more beats, unfilled columns suggest that sounds with harmonic spectra would produce more beats.

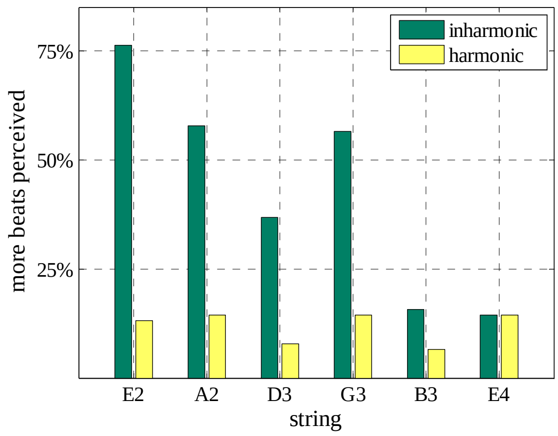

Figure 2. Audibility of beats. Filled columns indicate responses that guitar sounds with inharmonic spectra elicit more beats. Unfilled columns point to the fact that sounds with harmonic spectra would produce more beats.

Except for E4, the highest pitch considered, the synthesized guitar sounds with inharmonic spectra (filled columns) clearly produce more audible beats than the sounds with harmonic spectra. As expected, the audibility of beats is closely related to the inharmonicity of the synthesized sounds. For the inharmonic G3 sound with relatively large inharmonicity (cf. Figure 1) beats are easier detected than for the D3 sound with less inharmonicity. The unfilled columns in Figure 2 indicate that only for about 10 % of the sounds presented, subjects feel that the synthesized guitar sounds with harmonic spectra would produce more beats which is not easily understood in terms of stimulus-response relations. From a physical point of view, harmonic spectra do not produce beats and therefore the (imagined) audibility of beats can be taken as an indication of the accuracy of measurement.

C. Preference of melodies composed from synthesized inharmonic versus harmonic guitar sounds

Six short melodies as illustrated in Figure 3 were realized by synthetic inharmonic as well as harmonic guitar sounds.

Figure 3. Presented melodies.

The melodies M1 through M6 as illustrated in Figure 3 were presented in pairs and rated according to preference. The related results are given in Figure 4 where preference is plotted for the melodies. Filled columns indicate that melodies from synthesized inharmonic guitar sounds were preferred, unfilled columns show that melodies from synthesized harmonic guitar sounds were preferred.

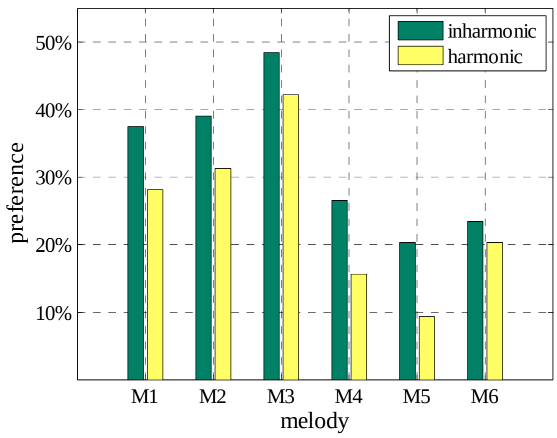

Figure 4. Preferences for the presented melodies. Filled columns indicate that melodies composed of inharmonic components were preferred, unfilled columns show preferences for harmonic spectra.

The results displayed in Figure 4 suggest that melodies composed from synthesized inharmonic guitar sounds (filled columns) are somewhat preferred in comparison to melodies composed from synthesized harmonic guitar sounds (unfilled columns). For melodies which contain more low notes (M2, M3) the preference seems to be somewhat larger than for melodies with predominantly higher notes (e.g. M6). Interested readers might listen to melody M3 in inharmonic as well as harmonic realization, recorded on a CD which goes with the third edition of the book psychoacoustics (Fastl and Zwicker 2007).

D. Pitch of synthesized inharmonic versus harmonic guitar sounds

In Figure 5 pitch matches of a pure tone to synthetic inharmonic as well as harmonic guitar sounds are displayed for the notes E2, A2, and G3, i.e. for the sounds with the largest inharmonicities. Circles indicate data for harmonic sounds, squares for inharmonic sounds. Medians with inter-quartiles are given; dashed lines illustrate the nominal pitches of the respective notes.

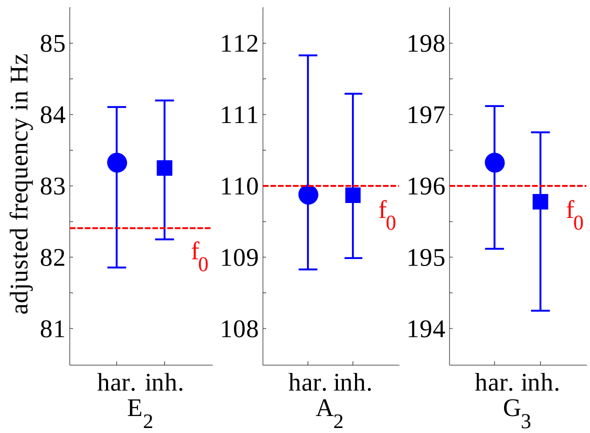

Figure 5. Pitch matching of the pure tones E2, A2, and G3 to their synthesized harmonic as well as inharmonic pendants. Medians with inter-quartiles are given as well as the nominal pitches of the respective notes (dashed lines). Circles represent data for harmonic tones, squares for inharmonic ones.

The data displayed in Figure 5 suggest no influence of the synthesized guitar sounds' harmonicity on pitch (height). For G3 the median of the synthesized inharmonic guitar sound (square) is a little lower than its harmonic counterpart; however, the inter-quartiles overlap completely.

Table 4 allows a comparison of predictions of pitches for synthesized inharmonic versus harmonic guitar sounds according to the model of Terhardt (Terhardt 1979, Terhardt et al. 1982).

Table 4. Predicted pitches according to the model of Terhardt (1979).

Tone	pitch harmonic tone [Hz]	pitch inharmonic tone [Hz]
E2	80.5	80.5
A2	108.2	108.2
D3	145.3	145.3
G3	194.5	194.5
B3	245	245
E4	327.7	327.7

The data displayed in Table 4 indicate – in line with the subjective results plotted in Figure 5 – that slight inharmonicities of sounds from guitars do not influence their pitch.

IV. CONCLUSION

Synthesized inharmonic guitar sounds produce clearly audible beats. The same holds true for original guitar sounds. Nevertheless, the inharmonicity of the guitar sounds does not influence pitch height in comparison to perfectly harmonic sounds. In a musical context, i.e. when playing short melodies with synthesized strictly harmonic versus slightly inharmonic guitar sounds, the latter are somewhat preferred, in particular by players of string instruments. Therefore, the slight inharmonicity of sounds from electric guitars is not a physical flaw which should be avoided by synthesizing perfectly harmonic sounds, but can be regarded as a musical asset.

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