

Fractional Reuse Partitioning for MIMO Networks

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Abstract—Inter-cell interference diminishes the performance of wireless cellular networks, hence interference management by cooperation of basestations should be employed to combat interference and increase spectral efficiency. Contrary to full cooperation, which renders the network into a super-cell with distributed antennas, we investigate a form of weak cooperation: transmission strategies among the basestations are coordinated, which requires a minor overhead, while the users treat interference of other cells as noise. Although our results are not restricted to reuse partitioning, we assume a set of strategies, each corresponding to a different reuse factor, and assign orthogonal resources to each strategy. Basestation cooperation is realized by dynamically adjusting the resource allocation, so-called fractional reuse partitioning, while the capacity achieving single-cell strategies are employed in each cell in order to optimally manage intra-cell interference exploiting all degrees of freedom offered by multiple antennas at the transmitter and receiver. Efficient operation of a cellular communications network requires interference management in order to achieve high data rates including rate assignment matched to the user demands, which we formulate as network utility maximization problem. We put special emphasis on the popular utilities sum-rate and proportional fairness, either with or without additional quality of service constraints. Finally, we illustrate the performance gain of our method by providing system level simulation results for a three sectorized cellular network with nineteen sites.

I. INTRODUCTION

The downlink of a cellular network where information is transmitted with multiple transmit antennas to users equipped with multiple receive antennas (MIMO) is regarded. Inter-cell interference (ICI) can be a severely limiting factor, especially users at the cell edge are affected and might be excluded from network service. A possible solution to completely eliminate ICI is the joint encoding of information over multiple transmitters [1], [2], so-called network MIMO. Joint encoding over geographically distributed antennas renders the network into a super-cell, which is related to the MIMO broadcast scenario [3], [4]. In case full channel state information and all data is available at a central controller, network MIMO can efficiently exploit all spatial degrees of freedom to eliminate ICI. Although the network's performance is no longer limited by interference, there is a huge amount of additional complexity and coordination overhead compared to single cell signal processing. Therefore, methods aiming at elimination or reduction of interference by cooperation of the transmitters, while every user is served by a single cell, are of great interest.

Finding the optimal coordinated transmission strategy is a hard and non-convex problem, where an algorithm to compute the globally optimal solution is only known for the single antenna receiver case [5], however computational complexity prohibits real-time implementation or computing it as reference for larger networks.

A simple method to reduce ICI is to assign non-overlapping frequency bands to neighbouring cells, known as frequency planning. Originally designed for systems intended to provide all users with the same data rate, e.g. phone service, the reduction in spectral efficiency, depending on the frequency reuse factor applied, was accepted. In order to achieve the data rates targeted in the specifications of future networks, that will provide a huge variety of services, a frequency reuse of one is desired. However, in a network with universal reuse, the cell-edge users might be excluded from service due to high inter-cell interference. Hybrid schemes that are a combination of universal reuse and higher reuse factors, so called fractional reuse partitioning, were first introduced in [6] and are suggested for standards like LTE and WiMAX. Additionally the topic is frequently discussed in research literature, for example an elaborate and systematic view on the OFDMA case is given by Ksairi et al. [7], who introduce and motivate fractional reuse partitioning in detail and list the relevant references. In this work we provide a rather abstract treatment of the topic that is valid for a huge manifold of transmission strategies including MIMO.

In short, the main challenge in finding efficient interference management by fractional reuse partitioning is cast by the following two questions:

- How to assign resources to each reuse partition?
- What is the optimal multi-user transmission strategy for each reuse partitions?

In this work we give a universal answer by jointly optimizing resource allocation, transmission strategies, user selection and assignment. Based on a network utility maximization (NUM) problem we provide a general algorithmic framework for optimizing multi-cell MIMO communications. The central idea is to inner approximate the unknown capacity region of the network by an achievable rate region. This allows us to draw general conclusions on the operation of the network independent of the transmission strategies applied, as long as

they constitute an achievable rate region. Our considerations and the algorithm proposed provide valuable insights for designing interference management built on a mix of various strategies. A nice illustration of the algorithm is a competition among the strategies for the available resources, which can be sequentially implemented. We use fractional reuse partitioning to demonstrate our method, other strategies as soft reuse, where the transmission power is limited, cooperation in the spatial domain, or clustered network MIMO can be directly included.

A. System Model

We regard the downlink of a cellular MIMO network with a set of cells \mathcal{S} , $S = |\mathcal{S}|$, where each cell is equipped with N_{tx} antennas. Users \mathcal{K} , $K = |\mathcal{K}|$, are distributed over the area covered and each user has N_{rx} antennas. The channel matrices are given by $\{\mathbf{H}_{ks}\}_{k \in \mathcal{K}, s \in \mathcal{S}} \in \mathbb{C}^{N_{\text{rx}} \times N_{\text{tx}}}$. Further, we assume additive white Gaussian noise $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{N_{\text{rx}}})$ at each receiver $k \in \mathcal{K}$ and a transmit power budget P for each cell. Every user is served with a certain transmission rate, depicted by the rate vector $\mathbf{r} = [r_1, \dots, r_K]^T \in \mathbb{R}_+^K$. Quality of service constraints are expressed by $\mathbf{r} > \mathbf{r}_{\min} \in \mathbb{R}_+^K$. A utility $U(\mathbf{r})$ measures the network performance by mapping the rate vector to a real number, i.e. $U : \mathbb{R}_+^K \mapsto \mathbb{R}$. The algorithm we propose is valid for arbitrary concave utilities, we however put special emphasis on two popular utilities, namely sum-rate,

$$U(\mathbf{r}) = \sum_{k \in \mathcal{K}} r_k,$$

and network-wide proportional fairness in the sense of Kelly [8],

$$U(\mathbf{r}) = \sum_{k \in \mathcal{K}} \log(r_k). \quad (1)$$

Shared resources and interference couple users rates $\mathbf{r} \in \mathcal{C} \subset \mathbb{R}_+^K$, universally modeled by the capacity region \mathcal{C} of the network. The capacity region enfolds all imaginable transmission strategies and therefore all interference management schemes, which allows us to formulate finding effective interference management by solving a network utility maximization (NUM) problem:

$$\text{maximize } U(\mathbf{r}) \text{ subject to } \mathbf{r} > \mathbf{r}_{\min}, \mathbf{r} \in \mathcal{C}.$$

Unfortunately the capacity region of arbitrary interference networks is unknown and using heuristics is necessary. Instead of applying heuristics directly on the operation of the network, the central idea of our framework is to inner approximate \mathcal{C} by an achievable rate region $\mathcal{R} \subseteq \mathcal{C}$, constituted by known transmission strategies, and to solve

$$\text{maximize } U(\mathbf{r}) \text{ subject to } \mathbf{r} > \mathbf{r}_{\min}, \mathbf{r} \in \mathcal{R}. \quad (2)$$

As we will see in the following, this procedure has some profound advantages and allows us to draw general conclusions on the operation of the network and the design of interference management schemes.

We suppose the availability of orthogonal resources, for example frequency bands or time slots, and assume they

experience equal channel conditions. In order to obtain an achievable rate region \mathcal{R} that inner approximates \mathcal{C} , the idea is to apply multiple strategies, for example reuse partitioning where each reuse factor corresponds to a strategy. Given a set of transmission strategies \mathcal{N} , $N = |\mathcal{N}|$, each strategy has an achievable rate region $\mathcal{R}_n \subset \mathbb{R}_+^K$. Exclusive resources are assigned to each strategy, expressed by the fractions $\mathbf{t} = [t_1, \dots, t_N]^T \in \mathcal{T}$ of the total resources available, where $\mathcal{T} = \{\mathbf{t} \geq \mathbf{0} : \|\mathbf{t}\|_1 = 1\}$. In case P is a peak power constraint, data rates scale linearly with the assigned resources and an operation point of the network,

$$\mathbf{r} = t_1 \mathbf{r}'_1 + \dots + t_N \mathbf{r}'_N = [\mathbf{r}'_1, \dots, \mathbf{r}'_N]^T \mathbf{t}, \quad (3)$$

is determined by the resource allocation \mathbf{t} and the rate assignment $\mathbf{r}'_1 \in \mathcal{R}_1, \dots, \mathbf{r}'_N \in \mathcal{R}_N$. The rate region \mathcal{R}_n is achieved in case all available resources are assigned to strategy n . This implies an approximation of the capacity region \mathcal{C} by an achievable rate region $\mathcal{R} \subseteq \mathcal{C}$, such that

$$\begin{aligned} \mathcal{R} &= \{[\mathbf{r}'_1, \dots, \mathbf{r}'_N]^T \mathbf{t} : \mathbf{r}'_1 \in \mathcal{R}_1, \dots, \mathbf{r}'_N \in \mathcal{R}_N, \mathbf{t} \in \mathcal{T}\} \\ &= \text{co}\{\mathcal{R}_1, \dots, \mathcal{R}_N\}. \end{aligned} \quad (4)$$

The challenge in solving the NUM problem is to jointly optimize resource allocation by selecting \mathbf{t} and rate assignment by selecting $\mathbf{r}'_1 \in \mathcal{R}_1, \dots, \mathbf{r}'_N \in \mathcal{R}_N$.

Remark 1 Assuming equal channel conditions among all resources is an unrealistic assumption in case the resources correspond to carriers in an OFDM system with time-varying channels. Subject to the availability of suitable descriptions it might be more reasonable to consider ergodic rate regions, especially as coordinated resource reallocation requires signaling among all cells and might only be allowed on a larger time scale than the rate assignment that usually is adopted quickly to changes of the channel conditions.

II. NETWORK UTILITY MAXIMIZATION – JOINT RESOURCE ALLOCATION AND RATE ASSIGNMENT

In this Section we discuss how resource allocation is jointly optimized with rate assignment. As we propose to solve the NUM problem, given in Equation (2), by a dual method we require that $U(\mathbf{r})$ is a jointly concave function in all parameters, which is a reasonable assumption for so-called *elastic* traffic [9].

A. Concave Utilities

As a first step we revise a general method to solve a NUM problem for a known rate region \mathcal{R} . Thereafter, we present an efficient reformulation for the case that the achievable rate region has the structure given by Equation (4).

As we assume a concave utility $U(\mathbf{r})$ and \mathcal{R} is always convex due to a timesharing argument, the problem is to maximize a concave utility over a convex set and we can find the solution by a dual approach. We dualize an artificially introduced constraint $\mathbf{0} \leq \mathbf{r} \leq \mathbf{c}$, which is interpreted elementwise, by introducing Lagrangian variables $\boldsymbol{\lambda} \in \mathbb{R}_+^K$ and the Lagrangian function is given by

$$L(\mathbf{r}, \mathbf{c}, \boldsymbol{\lambda}^T) = U(\mathbf{r}) - \boldsymbol{\lambda}^T (\mathbf{r} - \mathbf{c}).$$

Evaluating the dual function, $d(\boldsymbol{\lambda})$, which is the supremum of $L(\mathbf{r}, \mathbf{c}, \boldsymbol{\lambda}^\top)$ for fixed $\boldsymbol{\lambda}^\top$, can be decomposed into two sub-problems, commonly named application layer problem¹,

$$\underset{\mathbf{r}}{\text{maximize}} U(\mathbf{r}) - \boldsymbol{\lambda}^\top \mathbf{r} \text{ subject to } \mathbf{r}_{\min} \leq \mathbf{r} \leq \mathbf{r}_{\max}, \quad (5)$$

which is a convex optimization problem, and physical layer problem

$$\underset{\mathbf{c}}{\text{maximize}} \boldsymbol{\lambda}^\top \mathbf{c} \text{ subject to } \mathbf{c} \in \mathcal{R}.$$

For the case where $\mathcal{R} = \text{co}\{\mathcal{R}_1, \dots, \mathcal{R}_N\}$, there exists an elegant reformulation, which was already successfully applied in other scenarios [10], [11]. When evaluating the dual function, solving the physical layer problem,

$$\underset{\mathbf{c}}{\text{maximize}} \boldsymbol{\lambda}^\top \mathbf{c} \text{ subject to } \mathbf{c} \in \text{co}\{\mathcal{R}_1, \dots, \mathcal{R}_N\}, \quad (6)$$

can be accomplished as follows:

Theorem 1: Problem (6) is equivalent to

$$\underset{n \in \mathcal{N}}{\text{maximize}} \left(\underset{\mathbf{c}}{\text{maximize}} \boldsymbol{\lambda}^\top \mathbf{c} \text{ subject to } \mathbf{c} \in \mathcal{R}_n \right).$$

Proof: see [10], [11]

Consequently, it is sufficient to find the maximizer $\mathbf{c}_n^* \in \mathcal{R}_n$ for given $\boldsymbol{\lambda}$ for all strategies $n \in \mathcal{N}$ independently

$$\underset{\mathbf{c}_n}{\text{maximize}} \boldsymbol{\lambda}^\top \mathbf{c}_n \text{ subject to } \mathbf{c}_n \in \mathcal{R}_n, \quad (7)$$

followed by choosing the best strategy

$$\underset{n \in \mathcal{N}}{\text{maximize}} \boldsymbol{\lambda}^\top \mathbf{c}_n^*. \quad (8)$$

From Theorem 1, it is clear that solving problem (6) essentially means solving the weighted sum-rate (WSR) maximization problem independently for all strategies and taking the best one for given dual variables $\boldsymbol{\lambda}$.

The dual problem is

$$\underset{\boldsymbol{\lambda}}{\text{minimize}} d(\boldsymbol{\lambda}) \text{ subject to } \boldsymbol{\lambda} \geq \mathbf{0},$$

and can be solved by a primal-dual algorithm, that iteratively evaluates the dual function and uses the obtained maximizers to update the dual variables. Primal-dual algorithms compute the optimal dual variables $\boldsymbol{\lambda}^*$, but the primal variables found by evaluating the dual function at $\boldsymbol{\lambda}^*$ are in general not feasible to the primal problem. Nevertheless, feasible primal solutions can be constructed by a convex combination of the solutions found in each iteration [12]. The optimal resource allocation coefficients \mathbf{t} for the strategies are automatically retrieved as the coefficients of the convex combination, when recovering the primal solution, see Section II-E for an example.

¹In order to guarantee a finite optimal value of the application layer problem we use an arbitrary loose upper bound \mathbf{r}_{\max} on the user rates.

B. Sum-Rate

In case no QoS constraints are given, the sum-rate optimization is directly expressed by Equation (6), and by Theorem 1, it is clear that the solution is to allocate all resources to the strategy that achieves the highest sum-rate. For various reuse partitions, the strategies considered in this work, this usually means to decide for reuse 1 and serving the strongest users in the network, which matches the intuition. In case QoS constraints are given by \mathbf{r}_{\min} , reducing interference might be crucial for some users and our algorithm can be used to find the optimal strategy mix. In this case, the solution to the application layer sub-problem is

$$r_k = \begin{cases} r_{\max,k} & \lambda_k < 1 \\ r_{\min,k} & \lambda_k \geq 1 \end{cases}.$$

C. Proportional Fairness

For the proportional fairness utility, Equation (1), the application layer sub-problem, Equation (5), has a closed form solution

$$r_k = \min \left\{ \max \left\{ \frac{1}{\lambda_k}, r_{\min,k} \right\}, r_{\max,k} \right\}.$$

For the case where users are served by a single strategy, determined by some not further regarded assignment strategy, we are able to derive a surprising result: the NUM problem, Equation (2), decouples into two separate and independent problems. Assume \mathcal{K}_n are the users assigned to strategy n , as first step, we insert the parametrization of the user rates given in Equation (3), where $r'_{k,n} = 0$ if $k \notin \mathcal{K}_n$, into the proportional fairness utility (1):

$$\begin{aligned} U(\mathbf{r}) &= \sum_{k \in \mathcal{K}} \log(r_k) = \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}_n} \log(t_n r'_{k,n}) \\ &= \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}_n} \log(t_n) + \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}_n} \log(r'_{k,n}) \end{aligned}$$

Now we can see that the NUM problem, given by Equation (2), decouples into two sub-problems. Rate assignment is done by solving

$$\underset{\mathbf{r}'_1, \dots, \mathbf{r}'_N}{\text{maximize}} \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}_n} \log(r'_{k,n}) \text{ subject to } \mathbf{r}'_n \in \mathcal{R}_n \forall n \in \mathcal{N}.$$

As non-overlapping resources are assigned to the user strategies they do not interfere and the rate assignment sub-problem can be solved per strategy, for example by the primal-dual algorithm presented in Section II-A. The resource allocation sub-problem, which we reformulate as

$$\underset{\mathbf{t}}{\text{maximize}} \sum_{n \in \mathcal{N}} |\mathcal{K}_n| \log(t_n) \text{ subject to } \sum_{n \in \mathcal{N}} t_n = 1,$$

has as a closed form solution which we derive in the following. As the problem is convex every KKT point is a global optimum. The Lagrangian function is

$$L(\mathbf{t}, \lambda) = \sum_{n \in \mathcal{N}} |\mathcal{K}_n| \log(t_n) - \lambda \left(\sum_{n \in \mathcal{N}} t_n - 1 \right),$$

and the KKT conditions are

$$\sum_{n \in \mathcal{N}} t_n - 1 = 0,$$

$$\frac{\partial L(\mathbf{t}, \lambda)}{\partial t_n} = \frac{|\mathcal{K}_n|}{t_n} - \lambda = 0, \quad \forall n \in \mathcal{N}.$$

From the KKT conditions, we can directly conclude,

$$\lambda = \sum_{n \in \mathcal{N}} |\mathcal{K}_n| = |\mathcal{K}| \text{ and } t_n = \frac{|\mathcal{K}_n|}{|\mathcal{K}|}.$$

The resource allocation problem has a simple solution given by the cardinality of the users assigned to the strategies. The solution is independent of the instantaneous channel realizations and therefore very attractive for commercially deployable networks. Notable, the resource allocation rule does not depend on the transmission strategy used, which makes it especially valuable.

D. Weighted Sum-Rate Maximization within the Strategies

We assume that intra-cell interference is coordinated when choosing the transmission strategy of each cell, in our case given by the solution of a WSR problem, given by Equation (7). Although well possible, we do not consider further cooperation of the transmitters within each reuse partition. Here we assume each sector egoistically selects its transmission strategy, for the DPC case the transmit covariance matrix for each user and an encoding order. In case the interference plus noise covariance matrix \mathbf{R}_k for every user $k \in \mathcal{K}$ is known, so called interference awareness, we know how to compute the optimal WSR point for each sector [3], [4], [13]. It is clear that the transmit strategies chosen mutually depend on each other, therefore we do not know the inter-cell interference in advance and work with an estimation \mathbf{R}' based on perfect channel knowledge to the interfering cells and assuming each sector to transmit with full power P and a white transmit covariance matrix, therefore

$$\mathbf{R}'_k = \sigma^2 \mathbf{I}_{N_{\text{rx}}} + \sum_{s \in \mathcal{S}_{\text{int}}(k, n)} \frac{P}{N_{\text{tx}}} \mathbf{H}_{ks} \mathbf{H}_{ks}^H, \quad (9)$$

where $\mathcal{S}_{\text{int}}(k, n)$ is the set of cells that cause interference to user k when served in strategy n . Note that solving the weighted sum-rate problems based on an erroneous estimation corresponds to substituting the rate regions $\mathcal{R}_1, \dots, \mathcal{R}_N$ by $\tilde{\mathcal{R}}_1, \dots, \tilde{\mathcal{R}}_N$, which assures convergence of the algorithm. After the transmission strategies are fixed, the transmission rates have to be adapted to the actual interference received by the users. It turns out that the error made by the estimation of the interference, which neither strictly under nor over estimates the impact of the interference, is tolerable. Advanced algorithms, that provide some sort of interference coordination within each pattern are of great interest. However, an implementable and globally optimal algorithm, that jointly optimizes the transmission strategy in all the sectors of the network, which is currently not available, makes reuse partitioning obsolete.

scenario	urban macro-cell
inter site distance	500m
antenna configuration	4x4 MIMO
sectors	3 per site = 57
users	300

Table I
PARAMETERS SYSTEM LEVEL SIMULATIONS

E. Implementation and Coordination Overhead

In this Section, we treat some aspects related to implementation and the required coordination overhead. Although not the best choice with respect to convergence, subgradients can be used to update the dual variables, which in turn allows a nice interpretation of the suggested algorithm suitable for practical implementation. Assume iteration step i and given dual variables λ^i . The optimizers of the dual function are $\mathbf{r}^{i,*}$ and $\mathbf{c}^{i,*}$ and a possible subgradient update is given by

$$\lambda^{i+1} = \left[\lambda^i + \frac{1}{i} (\mathbf{r}^{i,*} - \mathbf{c}^{i,*}) \right]^+$$

In case a subgradient type algorithm is applied, one possibility for primal recovery is to average over the results obtained in each iteration

$$\mathbf{r}_{\text{opt}} = \frac{1}{i} (\mathbf{c}^{1,*} + \mathbf{c}^{2,*} + \dots + \mathbf{c}^{i,*}).$$

Consequently, this means that the solution found in each iteration can be applied instantaneously, while convergence to the optimum is guaranteed. This enables online implementation, and allows to interpret the sequential WSR problems, Equation (7), as a sort of multi-user scheduler. The resource allocation can then be nicely interpreted as a competition among all strategies, Equation (8), where the fraction of resources is adjusted to the frequency of winning the competition. The determination of the maximum WSR points and the update of the dual variables can be calculated locally at each sector, so there is no exchange of user data, channels, or dual variables. The interference estimation can be done at the receivers and are automatically considered by feedback of effective channels. Only the best performing strategy has to be determined network-wide every iteration, which causes a very minor coordination overhead and stops us from labeling our interference management scheme a distributed approach.

III. SIMULATION RESULTS

For the system level simulations we consider a three sectorized network with nineteen sites and wrap-around configuration, see Table I, the channels are generated according to stochastic spatial MIMO channel model with the parameters chosen for the urban macro-cell model. Users are uniformly distributed on the area served and to obtain smooth results we average over 10 drops. We optimize for network-wide proportional fairness and assume as available strategies, reuse

one, reuse three, and Reuse 57². The spectral noise density is -174 dBm/Hz and we gradually increase the transmit power from 0 dBm to 40 dBm. Compared to reuse 1, Figure 1 shows gains for combining reuse 1 + 3, which is however still an interference limited scheme. Additionally including reuse 57, which is able to eliminate interference completely, provides additional gains and increasing performance with increasing transmit power. Finally, it is interesting to investigate the fractions of resource allocated to each strategy, shown in Figure 2, where can notice a shift towards the interference avoiding strategies for higher transmit power. Especially interesting to see is that indeed resources are allocated to the reuse 57 strategy. These resources are further distributed among the sectors such that multiple users can be served interference free. Reducing interference comes at the price of having less sectors active, to find optimal cost-benefit ratio, with respect to a given utility, is essential for efficient operation of wireless networks. Under the assumptions made, our algorithm determines the optimal level of interference protection for each user: no protection (reuse 1), removing the most dominant interference (reuse 3), strict interference avoidance (reuse 57), or combinations thereof.

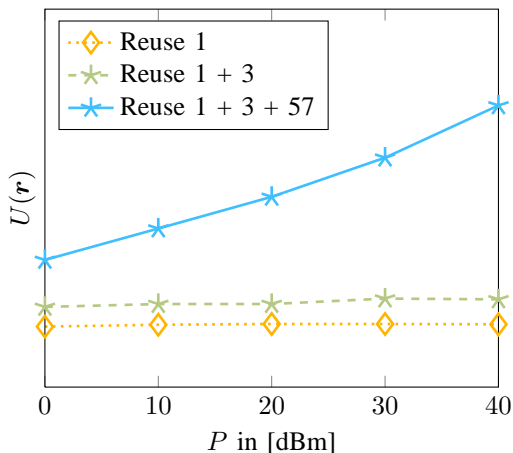


Figure 1. Simulation Results – Proportional Fairness Utility

IV. CONCLUSIONS AND FUTURE WORK

We introduced an algorithm for optimizing multi-cell MIMO communications based on a mix of various strategies implemented by solving a network utility maximization, which we regard as a step towards systematically developing, understanding, and evaluating interference management techniques. We use fractional reuse partitioning as an example to illustrate the outcome of this work and are able to demonstrate gains compared to classical reuse patterns. Indeed, the algorithm presented is more than just a variation of reuse partitioning and allows to include a huge manifold of other coordination

²Although 57 is not a commonly considered reuse factor it is included in order to have at least one strategy that is not interference limited and therefore allows the utility to grow arbitrarily high with increasing transmit power

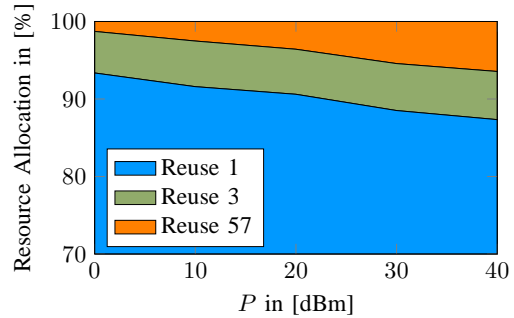


Figure 2. Simulation Results – Resource Allocation

strategies, for example soft reuse with limited transmit power, coordinated beamforming, or clustered network MIMO. Our contribution can be regarded as a concept to evaluate the performance of interference management strategies, by throwing them all into the algorithm and letting them fight for resources. Our algorithm can be used to find the best mix of interference management strategies matched to the utility and scenario, while only causing a marginal coordination overhead.

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