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***Abstract*—Many existing optimization approaches for parallel multiple-input multiple-output (MIMO) broadcast channels with linear transceivers make the assumption that encoding and decoding is performed separately on each carrier. However, unlike in the case of optimal non-linear dirty paper coding, such a carrier-noncooperative transmission was shown to be potentially suboptimal if linear precoding is used. A possible way to design carrier-cooperative transmit strategies is to perform the optimization in an equivalent single-carrier MIMO broadcast channel. In this paper, we show that the careless application of commonly used optimization tools nevertheless leads to carrier-noncooperative solutions, and we demonstrate that a notable gain can be achieved by modifying the algorithms such that they can converge to more general carrier-cooperative strategies.**

I. Introduction

A communication system with a set of orthogonal resources (e.g., carriers) can perform precoding either separately on each resource, or joint precoding across the orthogonal resources can be allowed. Adopting the nomenclature of [1], we call these two types of transmit strategies *carrier-noncooperative* and *carrier-cooperative* transmission, respectively. Note that carrier-cooperative transmission also includes the case where joint precoding is allowed, but not applied. Therefore, it is the more general scheme and includes the other one as a special case: carrier-noncooperative strategies are also valid carrier-cooperative strategies. A mathematical definition of both types of strategies is given along with the system model in Section II.

We study a set of parallel multiple-input multiple-output (MIMO) broadcast channels, i.e., a base station with multiple antennas uses a set of orthogonal carriers to transmit individual data streams to a set of multi-antenna receivers. Though less general, carrier-noncooperative transmission is optimal in this setting, as long as capacity-achieving dirty paper coding (DPC) is applied (e.g., [2]). As DPC is difficult to implement in practice, many researchers have focused on the optimization of linear transceivers. In this case, carrier-noncooperative transmission is no longer optimal, as was shown in our previous works [3], [4]—a property that is often ignored when transmit strategies for linear transceivers are designed (e.g., [5]–[10]).

In this paper, we discuss how carrier-cooperative strategies for parallel MIMO broadcast channels with linear transceivers can be optimized by introducing an equivalent single-carrier MIMO system (cf. Section II). A similar approach was pursued for the single-user case in [1].

The contributions of this paper are threefold. First, we reveal that existing optimization algorithms tend to produce carrier-noncooperative strategies when applied to the equivalent single-carrier system. Second, we discuss that initializing iterative optimization algorithms with random filters is a way to find carrier-cooperative solutions. Finally, by demonstrating a notable gain in numerical simulations, we show that the suboptimality of carrier-noncooperative transmission shown in [3], [4] is not only a theoretic issue, but can have an impact in practice.

Notation: We use $\bullet^*$ for complex conjugate, $\bullet^{\mathrm{T}}$ for transpose, and $\bullet^{\mathrm{H}}$ for conjugate transpose. The matrix $\mathbf{I}_L$ is the identity matrix of size L, $\mathbf{0}$ is the zero vector, and $\boldsymbol{e}_i$ is the i-th canonical unit vector, which has a one as the i-th entry and zeros elsewhere. $|\bullet|$ is used for the cardinality of a set.

II. System Model and Problem Formulation

An M-antenna base station serves K receivers using orthogonal carriers $c \in \{1,\dots,C\}$. The frequency flat channel on carrier c between the M transmit antennas and the N_k receive antennas of user k can be described by a matrix $\boldsymbol{H}_k^{(c),\mathrm{H}} \in \mathbb{C}^{N_k \times M}$, and the corresponding additive Gaussian noise $\boldsymbol{\eta}_k^{(c)}$ is characterized by the covariance matrix $\boldsymbol{C}_{\boldsymbol{\eta}_k^{(c)}} \in \mathbb{C}^{N_k \times N_k}$.

If we restrict the system to perform carrier-noncooperative transmission, the data intended for user k has to be split into up to C streams of data vectors $\boldsymbol{x}_k^{(c)} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{S_k^{(c)}})$ with $S_k^{(c)} \leq \min\{N_k, M\}$. The transmission is then described by

$$\boldsymbol{y}_k^{(c)} = \boldsymbol{H}_k^{(c),\mathrm{H}} \sum_{k'=1}^{K} \boldsymbol{B}_{k'}^{(c)} \boldsymbol{x}_{k'}^{(c)} + \boldsymbol{\eta}_k^{(c)}, \tag{1}$$

where the matrices $\boldsymbol{B}_k^{(c)} \in \mathbb{C}^{M \times S_k^{(c)}}$ are the beamforming matrices (transmit filters), and the vectors $\boldsymbol{y}_k^{(c)} \in \mathbb{C}^{N_k}$ are the received signals. In this case, the only coupling between the carriers is that each receiver achieves a data rate

$$r_k = \sum_{c=1}^{C} \log\det\left(\mathbf{I}_{N_k} + \boldsymbol{R}_k^{(c),-1} \boldsymbol{H}_k^{(c),\mathrm{H}} \boldsymbol{B}_k^{(c)} \boldsymbol{B}_k^{(c),\mathrm{H}} \boldsymbol{H}_k^{(c)}\right) \tag{2}$$

$$\text{with} \quad \boldsymbol{R}_k^{(c)} = \boldsymbol{C}_{\boldsymbol{\eta}_k^{(c)}} + \sum_{j \neq k} \boldsymbol{H}_k^{(c),\mathrm{H}} \boldsymbol{B}_j^{(c)} \boldsymbol{B}_j^{(c),\mathrm{H}} \boldsymbol{H}_k^{(c)}, \tag{3}$$

which is the sum of its rates $r_k^{(c)}$ on each carrier, and the total transmit power p is the sum of per-carrier powers $p^{(c)}$:

$$p = \sum_{c=1}^{C} \sum_{k=1}^{K} \operatorname{trace}\left[\boldsymbol{B}_k^{(c)} \boldsymbol{B}_k^{(c),\mathrm{H}}\right]. \tag{4}$$

To allow carrier-cooperation, we introduce an equivalent single-carrier broadcast channel with block-diagonal channels

$$\boldsymbol{H}_k^{\mathrm{H}} = \mathrm{blkdiag}\left(\boldsymbol{H}_k^{(1),\mathrm{H}}, \dots, \boldsymbol{H}_k^{(C),\mathrm{H}}\right) \in \mathbb{C}^{N_k C \times MC} \tag{5}$$

and block-diagonal noise covariance matrices

$$\boldsymbol{C}_{\boldsymbol{\eta}_k} = \mathrm{blkdiag}\left(\boldsymbol{C}_{\boldsymbol{\eta}_k}^{(1)}, \dots, \boldsymbol{C}_{\boldsymbol{\eta}_k}^{(C)}\right) \in \mathbb{C}^{N_k C \times N_k C} \tag{6}$$

(cf., e.g., [1]). In this setting, each user has only one stream of data vectors $\boldsymbol{x}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{S_k})$ with $S_k \leq C \min\{N_k, M\}$, and the data transmission can be described by the equation

$$\boldsymbol{y}_k = \boldsymbol{H}_k^{\mathrm{H}} \sum_{k'=1}^{K} \boldsymbol{B}_{k'} \boldsymbol{x}_{k'} + \boldsymbol{\eta}_k \tag{7}$$

with $\boldsymbol{y}_k \in \mathbb{C}^{N_k C}$ and $\boldsymbol{B}_k \in \mathbb{C}^{MC \times S_k}$. The rate of user k is

$$r_k = \log\det\left(\mathbf{I}_{N_k C} + \boldsymbol{R}_k^{-1} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{B}_k \boldsymbol{B}_k^{\mathrm{H}} \boldsymbol{H}_k\right) \tag{8}$$

$$\text{with} \quad \boldsymbol{R}_k = \boldsymbol{C}_{\boldsymbol{\eta}_k} + \sum_{j \neq k} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{B}_j \boldsymbol{B}_j^{\mathrm{H}} \boldsymbol{H}_k, \tag{9}$$

and the total power is given by

$$p = \sum_{k=1}^{K} \mathrm{trace}\left[\boldsymbol{B}_k \boldsymbol{B}_k^{\mathrm{H}}\right]. \tag{10}$$

If the beamforming matrices $\boldsymbol{B}_k$ have no special structure, the signals corresponding to the components $x_{k,s},\ s \in \{1, \dots, S_k\}$ of the transmit symbol vector $\boldsymbol{x}_k$ may be spread across several carriers, i.e., when partitioning the received signal $\boldsymbol{y}_k$ into per-carrier signals $\boldsymbol{y}_k^{(c)}$, an influence of $x_{k,s}$ can be found in the received vectors $\boldsymbol{y}_k^{(c)}$ of several carriers. However, if all beamforming matrices match the block-diagonal structure of the channel matrices and the noise covariance matrices, i.e.,

$$\boldsymbol{B}_k = \mathrm{blkdiag}\left(\boldsymbol{B}_k^{(1)}, \dots, \boldsymbol{B}_k^{(C)}\right) \in \mathbb{C}^{MC \times S_k}, \tag{11}$$

the sum in (10) can be decomposed as in (4), and the determinant in (8) factorizes such that the rates r_k can be computed as in (2). In this case, each component of $\boldsymbol{x}_k$ is transmitted only across a certain carrier. Consequently, as was discussed for single-user systems in [1], transmission with block-diagonal beamforming matrices is equivalent to the carrier-noncooperative transmission in (1).[1]

Optimization algorithms for single-carrier MIMO broadcast channels (e.g., weighted sum rate maximization, power minimization, etc.) can be applied to the equivalent single-carrier system, and the possibility of carrier-cooperation is exploited whenever the structure of the resulting transmit filter matrices differs from the block-diagonal structure of the channels. As the globally optimal solution might require carrier-cooperation (a formal proof of this statement is given in [4]), generating such solutions makes sense—even though we cannot expect the algorithms to converge to the globally optimal solution due to the non-convexity of the optimization problems. However, we will observe in the following sections that in many cases, existing optimization algorithms yield block-diagonal beamforming matrices, i.e., carrier-noncooperative solutions.

In the remainder of the paper, we assume that $\boldsymbol{C}_{\boldsymbol{\eta}_k} = \mathbf{I}_{N_k C}$ for all users k. This is without loss of generality since in any other case covered by (6), a whitening filter $\boldsymbol{C}_{\boldsymbol{\eta}_k}^{-\frac{1}{2}}$ could be applied at the receiver yielding a block-diagonal effective channel $\hat{\boldsymbol{H}}_k^{\mathrm{H}} = \boldsymbol{C}_{\boldsymbol{\eta}_k}^{-\frac{1}{2}} \boldsymbol{H}_k^{\mathrm{H}}$ with white noise. Furthermore, we make use of a dual uplink formulation [11] with uplink channel matrices $\boldsymbol{H}_k$, an uplink noise covariance $\mathbf{I}_{MC}$, uplink rates

$$R_k = \log\det\left(\mathbf{I}_{MC} + \boldsymbol{X}_k^{-1} \boldsymbol{H}_k \boldsymbol{T}_k \boldsymbol{T}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}}\right) \tag{12}$$

$$\text{with} \quad \boldsymbol{X}_k = \mathbf{I}_{MC} + \sum_{j \neq k} \boldsymbol{H}_j \boldsymbol{T}_j \boldsymbol{T}_j^{\mathrm{H}} \boldsymbol{H}_j^{\mathrm{H}}, \tag{13}$$

and uplink transmit power

$$P = \sum_{k=1}^{K} \mathrm{trace}\left[\boldsymbol{T}_k \boldsymbol{T}_k^{\mathrm{H}}\right], \tag{14}$$

where $\boldsymbol{T}_k \in \mathbb{C}^{N_k C \times S_k}$ are the uplink beamforming matrices. According to [11], the rates $r_k = R_k$ are achievable in the downlink with a sum transmit power $p = P$. Thus, many optimization problems can also be treated in the dual uplink. The downlink beamforming matrices $\boldsymbol{B}_k$ can be related to linear receive filters $\boldsymbol{V}_k^{\mathrm{H}}$ in the uplink as described in Section III-B and [11], and the beamformers $\boldsymbol{T}_k$ of the dual uplink can be related to receive filters $\boldsymbol{G}_k^{\mathrm{H}}$ that are applied to the received downlink signals $\boldsymbol{y}_k$ [11].

For block-diagonal uplink transmit filters $\boldsymbol{T}_k$, (12) and (14) can be decomposed into per-carrier equations in a similar manner as their counterparts in the downlink. Thus, the discussion of carrier-cooperative and carrier-noncooperative transmission can be performed in the uplink similarly as in the downlink.

III. Iterative Update Methods

In this section, we study iterative methods like gradient-based algorithms and alternating filter updates with respect to their potential to exploit the possibility of carrier-cooperation.

A. Gradient Based Filter Update

Gradient ascent or descent methods are common methods to find suboptimal solutions of non-convex maximization or minimizations problems, respectively. For example, algorithms involving gradient-projection updates of the uplink transmit filters were proposed in [9], [12] for (weighted) sum rate maximization. If the optimization is performed in the dual uplink, the gradient matrices are given by

$$\frac{\partial \sum_{k'=1}^{K} w_{k'} R_{k'}}{\partial \boldsymbol{T}_k^*} = \boldsymbol{A}_k \boldsymbol{T}_k, \tag{15}$$

where the scalars $w_{k'}$ are constant weighting factors,

$$\boldsymbol{A}_k = \frac{1}{\ln 2} \boldsymbol{H}_k^{\mathrm{H}} \left(\sum_{k'=1}^{K} w_{k'} \boldsymbol{X}^{-1} - \sum_{k' \neq k} w_{k'} \boldsymbol{X}_{k'}^{-1} \right) \boldsymbol{H}_k, \tag{16}$$

[1]Transmission is mathematically equivalent to carrier-noncooperative transmission if the transmit covariance matrices $\boldsymbol{B}_k \boldsymbol{B}_k^{\mathrm{H}}$ are block-diagonal, even if the beamforming matrices $\boldsymbol{B}_k$ are not. However, we define carrier-noncooperative transmission in terms of the transmit filters as in [1] since this allows the interpretation that data symbols are not spread across carriers. The results in this paper equivalently hold when defining carrier-noncooperative transmission in terms of block-diagonal transmit covariance matrices.

$\boldsymbol{X}_{k'}$ is defined as in (13), and

$$\boldsymbol{X} = \mathbf{I}_{MC} + \sum_{k=1}^{K} \boldsymbol{H}_k \boldsymbol{T}_k \boldsymbol{T}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}}. \tag{17}$$

Obviously, all matrices $\boldsymbol{A}_k$ match the block-diagonal structure of the channels if all uplink transmit filter matrices $\boldsymbol{T}_k$ do so. However, if at least one of the transmit filters is not block-diagonal, $\boldsymbol{A}_k$ may have an arbitrary structure for all k.

The gradient-projection update is performed by setting each beamforming matrix $\boldsymbol{T}_k$ to a linear combination of the respective old beamforming matrix and the gradient, i.e.,

$$\boldsymbol{T}_k \leftarrow (a_k \mathbf{I}_{N_k C} + b_k \boldsymbol{A}_k) \boldsymbol{T}_k, \tag{18}$$

where the scaling factors a_k and b_k are chosen according to a step size rule and subject to a sum power constraint [9], [12]. Due to the aforementioned behavior of the matrices $\boldsymbol{A}_k$, the update in (18) preserves block-diagonality of the beamforming matrices $\boldsymbol{T}_k$. Thus, the new beamforming matrices after the gradient-projection step correspond to carrier-noncooperative transmission if all old beamforming matrices do so.

B. Alternating Filter Updates

Updating the uplink and downlink receive filters in an alternating manner is another iterative technique. It has been applied to a wide variety of optimization problems such as weighted sum rate maximization [13], [14], sum MSE minimization [13], and power minimization [13]–[15].

For given uplink beamforming matrices, the optimal uplink receive filters in the MMSE sense $\boldsymbol{V}_k^{\mathrm{H}} = \boldsymbol{T}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{X}^{-1} \in \mathbb{C}^{S_k \times MC}$ are computed, where $\boldsymbol{X}$ is defined in (17). Then, an uplink-to-downlink transformation is applied, i.e., the downlink beamforming matrices are chosen according to $\boldsymbol{B}_k \leftarrow \operatorname{diag}\{\alpha_{k,i}\} \boldsymbol{V}_k \boldsymbol{W}_k$, where the scalars $\alpha_{k,i} \in \mathbb{R}$ are chosen as in [11] and the unitary matrix $\boldsymbol{W}_k \in \mathbb{C}^{S_k \times S_k}$ contains the eigenvectors of $\boldsymbol{V}_k^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{T}_k$ [11]. As can be easily verified, all updated downlink beamforming matrices $\boldsymbol{B}_k$ match the block-diagonal structure of the channels if all uplink beamformers $\boldsymbol{T}_k$ do so. The same observation holds in the other direction when computing the optimal downlink receivers and performing the downlink-to-uplink transformation from [11].

In many optimization algorithms, these two updates are alternately repeated until convergence. Usually, additional steps that only affect the transmit powers of the data streams, i.e., the norms of the columns of $\boldsymbol{B}_k$ or $\boldsymbol{T}_k$, are performed in each iteration. As scaling does not destroy block-diagonality, alternating optimization preserves carrier-noncooperative strategies: if a carrier-noncooperative strategy is used in a certain iteration, the same is true for all subsequent iterations.

IV. Common Initializations

The two iterative update methods studied in the last section were shown to preserve carrier-noncooperative strategies. However, if the initial strategy exploits carrier-cooperation, the same might be true for the strategy obtained after convergence, and numerical simulations confirm that this is indeed what happens. Therefore, we now briefly discuss initial choices for the beamforming matrices, which are commonly used in iterative algorithms. For simplicity, we discuss each initialization either in the uplink or in the downlink, but the results accordingly hold in the respectively other domain.

A. Scaled and Truncated Identity Matrices

A quite simple initialization is using (possibly scaled) identity matrices and truncating them to the required rectangular shape of $\boldsymbol{B}_k$ (e.g., [9], [12], [15]). Obviously, this corresponds to a carrier-noncooperative strategy.

B. Singular Value Decomposition

Another possibility is to initialize the columns of the uplink beamforming matrices $\boldsymbol{T}_k$ with the right singular vectors of the uplink channels $\boldsymbol{H}_k$, (e.g., [14]). For block-diagonal $\boldsymbol{H}_k$, each of the singular vectors has non-zero entries only in components corresponding to one of the blocks. This is carrier-noncooperative as the transmit filters $\boldsymbol{T}_k$ can be brought to block-diagonal form by reindexing the data streams appropriately (i.e., by changing the order of the singular vectors).

C. Initialization based on Zero-Forcing

Zero-forcing, i.e., complete suppression of inter-user interference, is a popular method to derive simple suboptimal solutions that perform quite well in many cases (e.g., [7], [10], [16]–[19]). Therefore, zero-forcing solutions can also serve as good initializations for iterative algorithms (e.g., [13], [15]). A possible way to find such zero-forcing solutions in a MIMO system is block-diagonalization [19]. As it is based on a singular value decomposition, the resulting beamformers match the block-diagonal structure of the channels. The same holds for the method proposed in [13], where a Gram-Schmidt orthogonalization was applied to the set of dominant right singular vectors of the channels. Another possibility to find initial zero-forcing strategies is successive allocation, cf. Section V.

D. Random Initialization

Choosing the initial beamforming matrices randomly was proposed, e.g., in [13], [15]. For instance, the entries can be chosen to be i.i.d. circularly symmetric complex Gaussian, or, if it is desired that the S_k columns of each initial beamforming matrix are orthogonal to each other, we can use a (possibly truncated) eigenbasis of $\boldsymbol{Y}\boldsymbol{Y}^{\mathrm{H}}$, where $\boldsymbol{Y}$ is a square random matrix with i.i.d. Gaussian elements [20]. In both cases, the resulting strategy exploits carrier-cooperation almost surely. Among the initializations discussed in this paper, the random initialization is the only one that is not carrier-noncooperative.

V. Successive Allocation Algorithms

A popular way to design zero-forcing strategies (as initialization or to directly apply them for data transmission) is successive allocation (e.g., [16]–[18]). Successive techniques are applicable even if block-diagonalization [19] is impossible due to a total number of receive antennas that exceeds the number of transmit antennas. The methods described in this section choose transmit and receive filters in the downlink in a way that the resulting data streams are interference-free.

In [16] and [17], it was proposed to apply the left singular vectors of the downlink channels as receive filters, yielding an equivalent setting with virtual single-antenna users, whose channels are row vectors. Then, data streams are successively allocated to these virtual users. As the receive filters are the singular vectors of the channels, they match the block-diagonal structure and so do the resulting zero-forcing transmit filters.

The method proposed in [18] computes not only the transmit, but also the receive filters in a successive manner. The receive filter $\boldsymbol{g}_1^{\mathrm{H}}$ of the first allocated data stream is again a left singular vector of $\boldsymbol{H}_{k(1)}^{\mathrm{H}}$, where $k(i)$ is the user corresponding to the i-th data stream. However, for $i > 1$, the receive filter $\boldsymbol{g}_i^{\mathrm{H}}$ of the i-th data stream is computed as generalized eigenvector of the pair of matrices $\boldsymbol{H}_{k(i)}^{\mathrm{H}} \left(\mathbf{I}_{MC} - \boldsymbol{Q}_i \boldsymbol{Q}_i^{\mathrm{H}}\right) \boldsymbol{H}_{k(i)}$ and $\boldsymbol{I}_{N_{k(i)}C} + \boldsymbol{H}_{k(i)}^{\mathrm{H}} \boldsymbol{Q}_i \boldsymbol{L}_i^{-1} \boldsymbol{L}_i^{-\mathrm{H}} \boldsymbol{Q}_i^{\mathrm{H}} \boldsymbol{H}_{k(i)}$, where $\boldsymbol{L}_i$ and $\boldsymbol{Q}_i$ result from the QR decomposition $\boldsymbol{Q}_i \boldsymbol{L}_i^{\mathrm{H}} = \left[\boldsymbol{H}_{k(1)} \boldsymbol{g}_1, \ldots, \boldsymbol{H}_{k(i-1)} \boldsymbol{g}_{i-1}\right]$. As the first receive filter $\boldsymbol{g}_1^{\mathrm{H}}$ matches the block-diagonal structure of the channels, the same is true for the matrices $\boldsymbol{Q}_2$ and $\boldsymbol{L}_2$ and, consequently, for the second filter $\boldsymbol{g}_2^{\mathrm{H}}$. This reasoning can be continued successively, revealing that also with this method, all receive filters are chosen in a way that the resulting strategy is carrier-noncooperative.

VI. Example: Power Minimization

In the following, we exemplarily study the application of an algorithm for MIMO broadcast channels with linear precoding to find carrier-cooperative solutions in parallel broadcast channels, and we demonstrate that a notable gain can be achieved. To this end, we consider the power minimization problem

$$\min_{\boldsymbol{B}_1, \ldots, \boldsymbol{B}_K} \; p \quad \text{s.t.: } r_k \geq \rho_k \; \forall k \tag{19}$$

in parallel vector broadcast channels, i.e., in the case where the receivers have only $N_k = 1$ antenna.[2] We assume that the rate requirements ρ_k are feasible both for carrier-cooperative and for carrier-noncooperative transmission (cf. [3]).

In this setting, the matrices $\boldsymbol{H}_k^{(c),\mathrm{H}}$ are reduced to row vectors $\boldsymbol{h}_k^{(c),\mathrm{H}}$ and instead of the noise covariance matrices $\boldsymbol{C}_{\boldsymbol{\eta}_k^{(c)}}$, we have scalar variances $\sigma_k^{(c),2}$. However, in the equivalent single-carrier system, we still have channel matrices $\boldsymbol{H}_k$ and noise covariance matrices $\boldsymbol{C}_{\boldsymbol{\eta}_k}$. To this equivalent MIMO system, we apply the power minimization algorithm from [15].

The essential steps of this algorithm can be summarized as follows: Each rate requirement ρ_k is divided into per-stream rate targets $\rho_{k,s}$, each corresponding to a component $x_{k,s}$ of the transmit symbol vector of user k.[3] Then, for given spatial directions of the columns of the uplink transmit beamforming matrices, the optimal per-stream transmit powers are computed, i.e., the columns are scaled, such that each stream achieves its target rate. To further reduce the transmit power, the division of the rate requirements into per-stream rate targets is updated with a gradient-projection step. Up to this point, no update of the directions of the filter vectors has been performed, but this is done subsequently by performing one iteration of the alternating filter update method described in Subsection III-B. With the obtained new uplink beamforming matrices, the algorithm proceeds as before by computing new transmit powers and so on.

The algorithm is initialized with a unit norm uplink beamforming vector and a per-stream rate target for each data stream. Note that the initial per-stream rate targets have to be chosen such that a feasible solution for the transmit powers exists with the given directions of the beamforming vectors [15]. In particular, feasibility might be impaired if the uplink beamformers are chosen such that the effective uplink channels $\tilde{\boldsymbol{h}}_{k,s} = \boldsymbol{H}_k \frac{\boldsymbol{T}_k \boldsymbol{e}_s}{\sqrt{\boldsymbol{e}_s^{\mathrm{T}} \boldsymbol{T}_k^{\mathrm{H}} \boldsymbol{T}_k \boldsymbol{e}_s}}$ violate the regularity condition [3]

$$\operatorname{rank}\left[\tilde{\boldsymbol{H}}_{\mathcal{H}}\right] = \min(|\mathcal{H}|, MC) \;\; \forall \mathcal{H} \subseteq \{\tilde{\boldsymbol{h}}_{1,1}, \ldots, \tilde{\boldsymbol{h}}_{k,S_k}\}, \tag{20}$$

where $\tilde{\boldsymbol{H}}_{\mathcal{H}} \in \mathbb{C}^{MC \times |\mathcal{H}|}$ has the vectors in $\mathcal{H}$ as columns.

To obtain a general carrier-cooperative solution, we choose random orthogonal vectors as initial beamformers as discussed in Subsection IV-D, and we set the per-stream rate targets to $\rho_{k,1} = \rho_k$ for the first stream of each user k and to $\rho_{k,s} = 0$ for all other streams, which is a feasible initialization of the rate targets [3], [15]. The sum transmit power of the obtained carrier-cooperative strategy is plotted in Fig. 1 ("carrier-coop"), where the rate requirements are $\rho_k = 2\rho_0$ for half of the users and $\rho_k = \rho_0$ for the remaining users. The simulation results are averaged[4] over 1000 channel realizations, and the channel vectors $\boldsymbol{h}_k^{(c)}$ have i.i.d. circularly symmetric complex Gaussian entries with zero mean and unit variance.

On the other hand, with truncated identity matrices as initial beamforming matrices, the alternating filter updates preserve the block-diagonal structure, and the algorithm converges to a carrier-noncooperative solution. This is equivalent to the suboptimal solution that was obtained in [8] by an algorithm without filter updates, and—as discussed therein—the performance strongly depends on the initial per-stream rate targets. The initial choice for $\rho_{k,s}$ used above might no longer be feasible as the effective channels $\tilde{\boldsymbol{h}}_{k,s} = \boldsymbol{H}_k \boldsymbol{e}_s$ do not satisfy the regularity condition (20) in general [3]. Just like in [8], the algorithm proposed in [3] could be used to find a feasible initialization ("carrier-noncoop, basic init"). Another possibility is to use the per-stream rates resulting from any other heuristic power minimization algorithm as initial per-stream rate targets. For instance, the greedy zero-forcing scheme from [10] could be used to find such initial rate targets ("carrier-noncoop, ZF init"). Note that only the per-stream rates achieved by the zero-forcing scheme are used as initialization here, but not the zero-forcing filters. As can be seen in Fig. 1 and 2, both solutions are outperformed by the carrier-cooperative solution with random initialization of the filters.

[2] Note that the theoretical parts of this work do not assume any restrictions on N_k.

[3] As there is no mapping between streams and carriers in the case of carrier-cooperative transmission, we introduce the stream index s. For carrier-noncooperative transmission, s and the carrier index c are interchangeable.

[4] We use the arithmetic mean in the dB-domain, which is more robust against outliers as it corresponds to the geometric mean in linear scale.

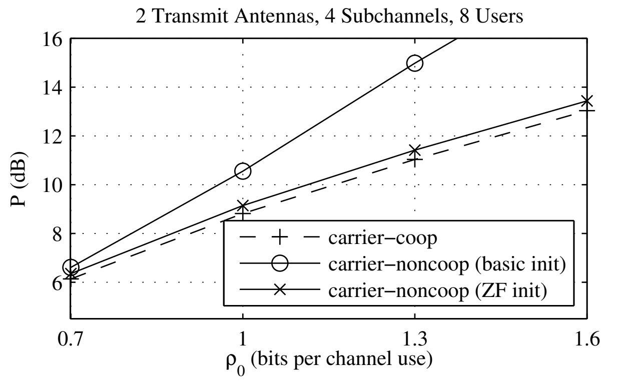

Fig. 1. Average transmit power for different per-user rate requirements.

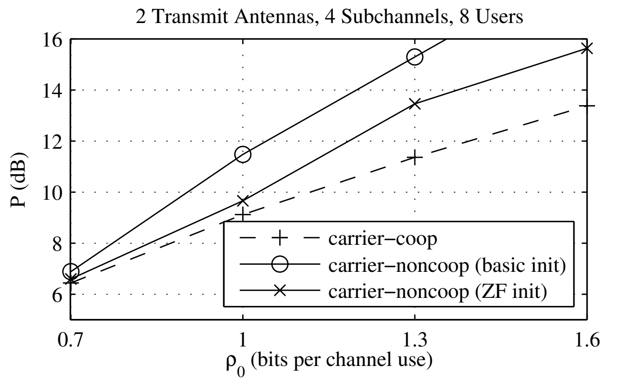

Fig. 2. This plot for a single channel realization shows that there are cases where the gain is significantly larger than on average.

VII. Conclusion

Carrier-noncooperative transmission can be suboptimal in parallel MIMO broadcast channels with linear transceivers [4]. To design carrier-cooperative strategies, we have introduced an equivalent single-carrier MIMO system, and we have studied the application of commonly used optimization techniques to this particular scenario. It turned out that the considered iterative methods lead to carrier-noncooperative solutions if any intermediate result is carrier-noncooperative. Therefore, if a solution that exploits carrier-cooperation is desired, the initial beamforming matrices need to correspond to such a strategy, too. However, this is not fulfilled by most initializations proposed in the literature. If successive allocation is used instead of iterative optimization, the state-of-the-art algorithms also have an inherent restriction to carrier-noncooperative solutions.

These observations reveal a possible reason why carrier-cooperative transmission has not attracted considerable interest in the context of broadcast channels so far: by thoughtlessly applying existing algorithms to a block-diagonal model, solutions that exploit carrier-cooperation are not created by chance.

However, by using random beamforming matrices as the initialization of an iterative optimization algorithm, it is possible to find good carrier-cooperative solutions. Doing so, we were able to present an example where carrier-cooperative transmission indeed outperforms carrier-noncooperative transmission in numerical simulations with random channels (and not only in a constructed channel realization as in [4]).

As random initialization is not completely satisfying from a theoretical point of view, and as the performance of iterative methods might strongly depend on the initialization, deterministic carrier-cooperative initializations that lead to good solutions are an interesting topic for future research. Another open question is how to design successive allocation methods that are able to find good carrier-cooperative solutions.

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