Optimal Event-triggered Control under Costly Observations

Adam Molin and Sandra Hirche

Abstract— **Digital control design is commonly constrained to time-triggered control systems with equidistant sampling intervals. The emergence of more and more complex and distributed systems urges the development of advanced triggering schemes that utilize computational and communication resources efficiently. This paper considers a linear stochastic continuous-time setting, where the design objective is to find an event-triggered controller that optimally meets the trade-off between control performance and resource utilization. This is reflected by imposing a cost penalty on updating the controller by current observations that is added to a quadratic control cost. It is shown that the underlying optimization problem results in an event-triggered controller, where the controller is updated, when the estimation error of the controller exceeds a apriori determined threshold. The controller design is related to linear quadratic Gaussian regulation and to optimal stopping time problems. Contrary to the initial problem, these can be solved by standard methods of stochastic optimal control. Numerical examples underline the effectiveness compared to optimal time-triggered controllers.**

Keywords: Control over communications, event-triggered sampling, optimal stochastic control, jump diffusion process

I. Introduction

Recently, many control problems have appeared, where event-triggered exchange of information is favorable compared to periodic time-triggered schemes. Examples can be found in control over communications [1]–[4], multi-agent systems [5], [6] and distributed optimization algorithms [7]. It is proved in [1] that event-triggered impulse control for scalar stochastic continuous-time systems reduces the state variance significantly compared to a time-triggered minimum variance controller with same average transmission rate. Within an impulsive stochastic continuous-time framework, multiple independent control loops sharing a common digital network are considered in [2]. There, it turns out that an event-triggered scheduling scheme outperforms a time-triggered scheduling scheme in terms of aggregate state variance. Results in [7] indicate that an event-triggered communication scheme for distributed network utility maximization algorithms is scale-free with respect to the network size. In contradistinction, a time-triggered scheme scales poorly with network size. With respect to connectivity maintenance for autonomous mobile agents, it is shown in [5] that event-triggered control strategy reduce the need of communication significantly, while maintaining a certain degree of connectivity.

A. Molin and S. Hirche are with the Institute of Automatic Control Engineering, Technische Universität München, Arcisstraße 21, D-80290 München, Germany; http://www.lsr.ei.tum.de, adam.molin@tum.de, hirche@tum.de

All these problems have in common that they must deal with limited exchange of information between engineering entities. Instead of limiting the transmission rate, this paper introduces a cost that penalizes updating the controller with current observations. While the objective of the motivating examples is to show the benefits of event-triggered control schemes, there is no general specification how to choose optimally the rules, when events should occur. The focus of this paper is a single-loop control system to regulate an Itŏ diffusion process, where sensor measurements are sent over a digital network to the controller. The objective is to find joint optimal controllers and schedulers based on a non-classical cost function penalizing communication exchange between sensor and controller. By scheduling, we refer to as determining the optimal timings of the measurements to be sent over the network. Hence, an event-triggered controller consists of a (i) control policy that applies control inputs from available observations and (ii) a scheduling policy assigning transmission timings. The choice for this cost function is inspired by related work for estimation problems with limited communication capabilities [8], [9]. In [4], [10], optimal controller and event-triggered schedulers are obtained for stochastic continuous-time systems, when limiting the number of transmissions for a finite interval and control inputs remain constant between transmissions. It is shown in [10] that the optimal event-triggered controller can be calculated analytically for the scalar Brownian motion process. Another related problem has been considered in [11], where controllers are derived under Poisson distributed observations. It is shown that the therein proposed problem is related to linear quadratic regulation with an exponentially discounted cost, when the controller is allowed to be time-varying between observation updates.

The main contribution of this paper is to find the optimal event-triggered controller that minimizes the underlying cost function. Built upon results from [12], it is showed within the framework on continuous-time jump-diffusion processes that the solution of the underlying optimization problem is closely related to the linear quadratic Gaussian regulator problem and to optimal stopping time problems. The optimal event-triggered controller is a linear time-variant controller with gains calculated by the Riccati differential equation and a linear state estimator. The event-trigger is a threshold policy of the difference between the actual state and the estimated state at the controller. Optimal policies are obtained by application standards methods of optimal stochastic control, which has significant computational benefits compared to the initial problem. The key innovation are reformulation techniques of the underlying optimization problem in such way that the separation principle of stochastic optimal control is still valid. Besides, its numerical benefits, the obtained results reveal additional properties in terms of optimality of event-triggered controllers that are proposed in the literature [13], [14]. Therefore, the obtained results bridge the gap between stochastic optimal control under costly observations and event-triggered control design.

The remainder of the paper is organized into three sections. Section II introduces the system model and gives the design objective within the linear quadratic Gaussian framework. In section III the initial problem is transformed into the mentioned subproblems and the design procedure of the optimal event-triggered controller is derived. A numerical validation illustrates the efficacy of the proposed approach in section IV.

Notation. In this paper, the operators $\mathrm{tr}[\cdot]$ and $(\cdot)^{\mathsf{T}}$ denote the trace and the transpose operator of a square matrix, respectively. The variable P denotes the probability measure on the abstract sample space denoted by Ω. The expectation operator is denoted by $\mathsf{E}[\cdot]$ and the conditional expectation is denoted by $\mathsf{E}[\cdot|\cdot]$.

II. Problem Description

We consider a stochastic system $\mathcal{P}$ evolving according to the following stochastic differential equation defined as an Itŏ diffusion

$$dx_t = (Ax_t + Bu_t)dt + dw_t. \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times d}$. The variables, x_t and u_t denote the state and the control input and are taking values in $\mathbb{R}^n$ and $\mathbb{R}^d$, respectively, for each time t. The initial state, x_0 is given a priori at scheduler and controller. The variable w corresponds to the vector-valued Brownian motion process in $\mathbb{R}^n$ with zero-mean and normalized variance. In the following, we let $(\Omega, \mathcal{F}, \mathsf{P})$ denote the probability space generated by the Brownian motion process on the interval $[0, T]$. Hence, we restrict our attention to finite horizon problems with horizon T. Additionally, let $\mathcal{F}_t$ be the σ-algebra generated by the random variables w_s with $s \le t$.

The system model is illustrated in Figure 1. It is assumed that the scheduler $\mathcal{S}$ situated at the sensor has complete state information. Based on the observation, the scheduler decides at any time t whether to send a state update x_t to the controller $\mathcal{C}$. The controller situated at the actuator directly manipulates the process $\mathcal{P}$ based on the information it receives from the sensor. We assume that system parameters and statistics are known to both scheduler and controller.

Let the process k be a counting process with $k_0 = 0$. The value of k_t increases by 1, whenever a state update is sent over the network. For convenience, the time index is omitted in the following. Admissible transmission timings τ_k are restricted to be measurable with respect to $\mathcal{F}_t$, i.e. τ_k is non-anticipative. Hence, k is an $\mathcal{F}_t$-adapted process and the transmission timings τ_k are stopping times [15]. In the following the counting process k and the scheduling policy have equal meaning.

Admissible control laws denoted by γ are time-variant functions with the following structure:

$$u_t = \gamma(x_{\tau_k}, \tau_k, t), \quad \tau_k \le t < \tau_{k+1}, \tag{2}$$

where τ_0 is defined to be 0. In case τ_{k+1} is not defined, the upper bound τ_{k+1} is replaced by T. Let $\mathcal{I}_t = \{x_{\tau_k}, \tau_k\}$ denote the available information at the controller at time t. The control laws γ are assumed to be Borel measurable. A crucial assumption at this place is to allow time-varying control inputs that has also been observed in [11] for Poisson distributed observations. This also differs from the analysis in [4], [10], where only constant control inputs are considered between transmission timings.

The design objective is to find admissible policies γ and counting process k that minimize the following cost function

$$J(\gamma, k) = \mathsf{E}\left[\int_0^T x_t^{\mathsf{T}} Q x_t + u_t^{\mathsf{T}} R u_t dt + x_T^{\mathsf{T}} Q_T x_T + \lambda k_T\right]. \tag{3}$$

The weighting matrices Q, Q_T and R are assumed to be positive definite. The weighting factor $\lambda > 0$ refers to the amount of penalizing information exchange between sensor and controller, as $\mathsf{E}[k_T]$ is the average number of transmissions during $[0, T]$.

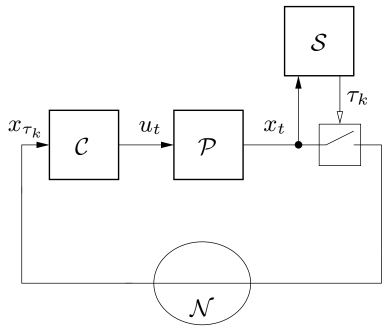

Fig. 1. System model of the networked control system with process $\mathcal{P}$, scheduler $\mathcal{S}$, controller $\mathcal{C}$ and communication network $\mathcal{N}$.

III. Event-triggered controller

Determining the optimal control policy and counting process that minimize (3) directly by given methods is not feasible, as it is an joint optimal control and stopping time problem with different information patterns. An information pattern refers to the information available for the admissible policy. Hence, results from dynamic programming are not directly applicable. Therefore, the subsequent paragraph is dedicated to show various properties of the underlying optimization problem in order to facilitate the calculation of the optimal solution.

First, it should be noticed that the optimal counting process has only a finite number jumps P-almost surely. That is due to the fact that the cost function (3) would be infinite otherwise. In order to cater for well-definedness, we can therefore restrict our attention to counting processes with finite number of jumps in $[0, T]$ P-almost surely.

The following lemma is the central results of this paper:

Lemma 1: The optimal control policy γ^* minimizing (3) has the following structure:

$$u_t = \gamma^*(x_{\tau_k}, \tau_k, t) = -L_t \, \mathsf{E}[x_t|\mathcal{I}_t], \tag{4}$$

where

$$L_t = -R^{-1}B^\mathsf{T}S_t, \tag{5}$$

$$-\frac{dS_t}{dt} = A^\mathsf{T}S_t + S_tA + Q - S_tBR^{-1}B^\mathsf{T}S_t, \quad t \in [0,T] \tag{6}$$

with initial condition $S_T = Q_T$.

Proof: In order to prove the above lemma, we introduce an equivalence class of admissible pairs of control and scheduling policies. The equivalence is denoted by $[(\gamma, k)]$. The equivalence relation is defined as follows: Two pairs (γ_1, k_1) and (γ_2, k_2) are equivalent, when

$$k_1(\omega) \equiv k_2(\omega), \quad \forall \omega \in \Omega. \tag{7}$$

This means transmissions occur at same times for any sample path $\omega \in \Omega$. Besides, we assume that there is an additional side channel transmitting control input u_t to the scheduler $\mathcal{S}$. Hence, past control inputs are part of the information pattern at the scheduler and will be subsequently considered as an additional argument of k. We will later observe that the side channel gives only redundant information to the optimal scheduler.

Let us consider an arbitrary equivalence class $[(\bar{\gamma}, \bar{k})]$ in the following. The set $[(\bar{\gamma}, \bar{k})]$ is parameterized by admissible control laws γ, while the scheduling policy denoted by $\bar{k} \circ \mathcal{T}_{\bar{\gamma}}$ is prespecified by $\bar{k}$ and a transformation $\mathcal{T}_{\bar{\gamma}}$ constructed by $\bar{\gamma}$. The transformation $\mathcal{T}_{\bar{\gamma}}$ is defined as follows

$$\bar{x}_t = x_t + \int_0^t e^{A(t-s)}B(\gamma(\bar{x}_{\tau_s}, \tau_s, s) - u_s)ds.$$

This transformation recalculates the state that results, when $\bar{k}$ and $\bar{\gamma}$ were used and is independent of the applied controls u_t, $t \in [0,T]$. This implies that the transmission times τ_k are independent of u_t, $t \in [0,T]$, which is in accordance with Equation (7), i.e.

$$(\gamma, \bar{k} \circ \mathcal{T}_{\bar{\gamma}}) \in [(\bar{\gamma}, \bar{k})].$$

On the other, it should be noted that the prespecified scheduling law $\bar{k} \circ \mathcal{T}_{\bar{\gamma}}$ is not unique to represent the equivalence class $[(\bar{\gamma}, \bar{k})]$. But as it describes uniquely the behavior of the scheduling law for any sample path within a equivalence class, $\bar{k} \circ \mathcal{T}_{\bar{\gamma}}$ replaces every other valid realization of a prespecified scheduling law representing $[(\bar{\gamma}, \bar{k})]$ without changing the evolution of the system.

Within a given equivalence class $[(\bar{\gamma}, \bar{k})]$ parameterized by control policy γ, we want to find the optimal control policy γ^*. For that reason, we state the following identity taken from Lemma 7.1 in [16] with L_t and S_t chosen as in (5) and (6):

$$\begin{aligned}
&\int_0^T x_t^\mathsf{T}Qx_t + u_t^\mathsf{T}Ru_tdt + x_T^\mathsf{T}Q_Tx_T \\
&\quad = x_0^\mathsf{T}S_0x_0 + \int_0^T (u_t + L_tx_t)^\mathsf{T}R(u_t + L_tx_t)dt \\
&\qquad + \int_0^T \mathrm{tr}[S_t]dt + \int_0^T dw_t^\mathsf{T}S_tx_t + \int_0^T x_t^\mathsf{T}S_tdw_t
\end{aligned} \tag{8}$$

With Equation (8), the minimization of cost function $J(\gamma, \bar{k} \circ \mathcal{T}_{\bar{\gamma}})$ defined in (3) can be written as

$$\begin{aligned}
\min_{\gamma} J(\gamma, \bar{k} \circ \mathcal{T}_{\bar{\gamma}}) &= x_0^\mathsf{T}S_0x_0 + \int_0^T \mathrm{tr}[S_t]dt + \mathsf{E}[\lambda k_T] \\
&\quad + \min_{\gamma} \mathsf{E}\left[\int_0^T (u_t + L_tx_t)^\mathsf{T}R(u_t + L_tx_t)dt\right].
\end{aligned}$$

As the sample paths of the counting process within $[(\bar{\gamma}, \bar{k})]$ are equivalent for any γ, the communication cost, $\mathsf{E}[\lambda k_T]$ does not depend on the control policy chosen and can be excluded from the minimization. With the estimation error $\Delta_t = x_t - \mathsf{E}[x_t|\mathcal{I}_t]$, we write further

$$\begin{aligned}
&\mathsf{E}\left[\int_0^T (u_t + L_tx_t)^\mathsf{T}R(u_t + L_tx_t)dt\right] \\
&\quad = \mathsf{E}\left[\int_0^T \mathsf{E}\left[(u_t + L_tx_t)^\mathsf{T}R(u_t + L_tx_t)|\mathcal{I}_t\right] dt\right] \\
&\quad = \mathsf{E}\left[\int_0^T (u_t + L_t\,\mathsf{E}[x_t|\mathcal{I}_t])^\mathsf{T}R(u_t + L_t\,\mathsf{E}[x_t|\mathcal{I}_t])dt\right] \\
&\qquad + \mathsf{E}\left[\int_0^T \Delta_t^\mathsf{T}L_t^\mathsf{T}RL_t\Delta_tdt\right].
\end{aligned} \tag{9}$$

Last equality holds, because Δ_t and $\mathsf{E}[x_t|\mathcal{I}_t]$ are orthogonal in the corresponding Hilbert space.

Next, we observe that Δ_t is a random variable that does not depend on the control law chosen within the equivalence class for any t. This is due to linearity of the system evolution given by (1) and the fact that the output and the internal state of the scheduler is not controllable by u_t. Therefore, the last term in Equation (9) is constant within an equivalence class and can therefore be omitted from the optimization. The remaining cost function excluding all other constant summands is non-negative due to R being a positive definite matrix. The remaining cost attains 0, when the control law is chosen to be given by

$$u_t = -L_t\,\mathsf{E}[x_t|\mathcal{I}_t].$$

Hence, the optimal controller within a equivalence class is given by Equation (4). Within an equivalence class, the control input u_t can be recalculated at time t by Equation (4), as the scheduling law is fixed. Therefore, the side information channel transmitting the control inputs is not needed by the scheduler.

Finally, by considering any arbitrary equivalence class $[(\bar{\gamma}, \bar{k})]$, we are able to construct a pair $(\gamma^*, \bar{k} \circ \mathcal{T}_{\bar{\gamma}})$ having a cost

$$J(\gamma^*, \bar{k} \circ \mathcal{T}_{\bar{\gamma}}) \leq J(\gamma, \bar{k} \circ \mathcal{T}_{\bar{\gamma}}).$$

Besides, the sets of equivalence classes partition the complete space of admissible solutions. Thus, J attains its minimum globally with γ^* and a optimal scheduling law that is yet to determine. This completes the proof. ∎

Remark 1: It should be noted that in general the resulting augmented output system has the dual effect for a fixed scheduling policy. The term *dual effect* comes from the dual role of the controller, i.e. (i) affecting state evolution and (ii) probing the system to reduce estimation uncertainty [17]. For systems, where the latter property is present, the determination of the controller is in general very difficult. By our construction of the equivalence relation, probing the system by the controller will not reduce estimation uncertainty, when considering scheduler $\bar{k} \circ \mathcal{T}_{\bar{\gamma}}$. This fact facilitates the calculation of the optimal control law and the task of regulating uncertainty within the system is fully taken over by the scheduler.

Remark 2: It should be noted that a crucial assumption of Lemma 1 is that the controller is allowed to be time-varying. For time-invariant control laws, the proposed consideration of equivalence classes does not carry over and other approaches must be considered, e.g. [4], [10].

Remark 3: The additional side channel introduced in the proof should be rather regarded as a technical means to prove above lemma. It enables the use of standard approaches to show that the separation principle holds within an equivalence class. Eventually, any control input resulting from an admissible control input can be recalculated by the scheduler as u_t is an $\mathcal{F}_t$-adapted process.

The remaining issue is to derive the estimator $E[x_t|\mathcal{I}_t]$ and the scheduling policy. It should be noted that the estimator is not necessarily equivalent to a estimator, where transmission timings are predetermined beforehand. A counterexample for this fact is given in [18] for the estimation of counting processes. An intuitive explanation is that not receiving an update can be used as information to improve state estimation at the controller.

In the following, we revise the cost function defined in (3) by taking Lemma 1 into account. Determining the optimal scheduling policy reduces to

$$\min_k \mathsf{E}\left[\int_0^T \Delta_t^\mathsf{T} L_t^\mathsf{T} R L_t \Delta_t + \lambda k_T\right],$$

where $\Delta_t = x_t - \mathsf{E}[x_t|\mathcal{I}_t]$ is the estimation error. At jumps of k, that is at τ_k, the estimation error is zero, i.e.

$$\Delta_{\tau_k} = 0.$$

We observe that estimator and scheduler are coupled in that the scheduler policy affects the estimator design and vice versa. In other words, the state estimate $\mathsf{E}[x_t|\mathcal{I}_t]$ depends on the scheduling policy k. This issue is resolved in the following theorem by stating that the optimal estimator is the equivalent to the state estimator for deterministic transmission timings. Besides, Theorem 1 summarizes the complete design procedure.

Theorem 1: The optimal event-triggered controller minimizing (3) is given by

(a) control policy

$$u_t = -L_t \, \mathsf{E}[x_t|\mathcal{I}_t]$$

with L_t given by Equation (5),

(b) estimator

$$\mathsf{E}[x_t|\mathcal{I}_t] = e^{(A-BL_t)(t-\tau_k)} x_{\tau_k} \quad \text{and} \tag{10}$$

(c) scheduling policy k^* which minimizes

$$J^{\mathrm{E}}(k) = \min_k \mathsf{E}\left[\int_0^T \Delta_t^\mathsf{T} L_t^\mathsf{T} R L_t \Delta_t + \lambda k_T\right], \tag{11}$$

where Δ_t is a jump-diffusion process

$$\begin{cases} d\Delta_t &= A\Delta_t dt + dw_t, \\ \Delta_{\tau_k} &= 0 \end{cases} \tag{12}$$

with initial condition $\Delta_0 = 0$.

Proof:

The structure of the optimal control policy is obtained by Lemma 1. The decision, whether to transmit a state update at time t is given by a Borel measurable set $F_t \in \mathbb{R}^n$. In order to cater for well-definedness, the complementary set $\bar{F}_t = F_t \backslash \mathbb{R}^n$ of this set is assumed to be open and contain the origin for all $t \in [0, T]$. In case of no update is transmitted, this information can be used to improve state estimation, because the controller knows for $t \neq \tau_k$ that Δ_t must be in $\bar{F}_t$. By considering the fact that the Brownian motion process is a Martingale and the sub-level sets

$$D_c = \{\Delta_t | \Delta_t^\mathsf{T} L_t^\mathsf{T} R L_t \Delta_t \leq c\}$$

are point symmetric to the origin, it can be seen that the sets $\bar{F}_t$ are also point symmetric to the origin to be optimal [12]. The remaining degree of freedom is a time-varying bias term $\alpha(\tau_{k_t}, t)$ added to the least-squares estimate which does not incorporate additional knowledge about transmission timings. Due to this fact and [19], the estimator is given by

$$\mathsf{E}[x_t|\mathcal{I}_t] = e^{(A-BL_t)(t-\tau_k)} x_{\tau_k} + \alpha(\tau_{k_t}, t),$$

where α is a Borel-measurable functions of τ_{k_t} and t mapping to $\mathbb{R}^n$. The bias term α comprises the dependence on the scheduling law k.

It turns out that any non-zero bias term α increases the overall cost, as it increases the average estimation error and transmissions occur more likely [12]. Hence, the optimal estimator is equivalent to the optimal estimator for fixed transmission timings. This implies that the optimal state estimator does not depend on the scheduling policy and is given by Equation (10). The estimation error evolution is then given by Equation (12). As Δ_t forms a time-inhomogeneous Markov chain controlled by k_t, the estimation error Δ_t is a sufficient statistic for determining the optimal scheduling policy k by Equation (11), [20]. This completes the proof. ∎

Remark 4: It can be observed from Equations (11) and (12) that the optimal scheduler is a threshold policy

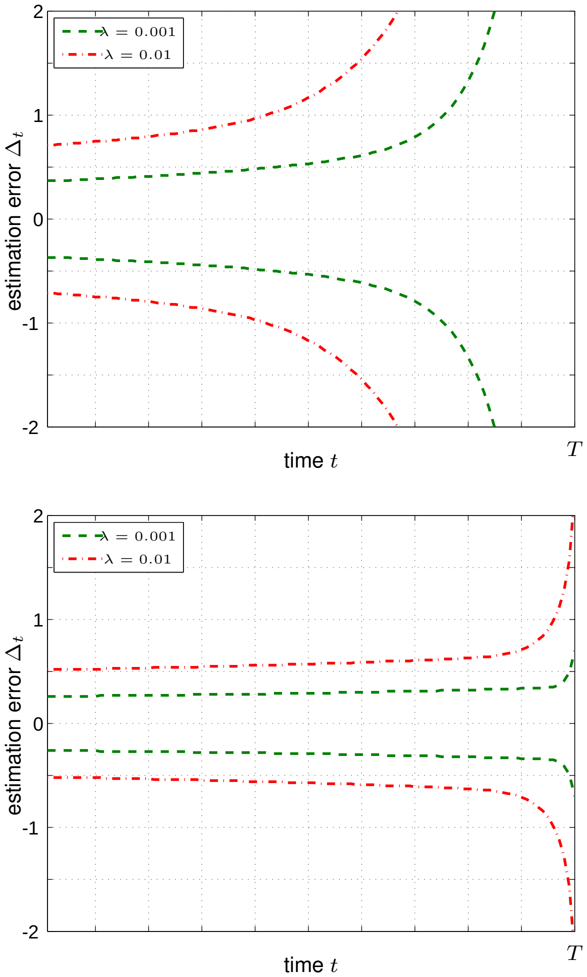

Fig. 2. Optimal event-triggered scheduler with indicated switching thresholds for different parameter sets. The upper graph considers no terminal cost ($Q_T = 0$) of J. The lower graph considers cost J with terminal cost $x^{\mathsf{T}} Q_T x$ with $Q_T = 1$.

depending on the estimation error Δ_t at time t. This observation reflects our intuition that the controller should first be updated, when a certain amount of uncertainty is surpassed. The threshold policy is computationally attractive, as the scheduler at the sensor-side just needs to compute the predicted estimate of the controller and compare it with the current state. If the estimation error exceeds the apriori determined threshold, the scheduler sends the current state to the controller.

Remark 5: Optimization problem (11) can be solved with stochastic dynamic programming by using discrete approximations, which converge to the optimal solution, see [21], [22]. These are used in the subsequent section in order to determine the optimal event-trigger numerically.

IV. Numerical Validation

In order to conduct a numerical comparison with time-triggered controllers, we consider a scalar diffusion process given by (1) with $A = 0$, $B = 1$. The parameters of cost function J in (3) are chosen to be $Q = 1$, $R = 5$ and for the terminal cost we consider $Q_T \in \{0, 1\}$. The communication penalty takes values $\lambda \in [0, 0.1]$. We suppose the optimization horizon is given by $T = 1$.

The optimal event-triggered controller given by Theorem 1 is compared with the optimal time-triggered controller which may be asynchronous. For the time-triggered controller, transmission timings are determined beforehand to minimize the cost function J in (3). It is straight-forward to show that for such type of controllers the optimal solution is given by (a) and (b) of Theorem 1 and optimal transmission timings that are solved by deterministic dynamic programming. Therefore, both optimal event-triggered and time-triggered controller differ solely in the term J^{E} given by (11).

Figure 2 illustrates the optimal event-triggered policy that depends on the estimation error Δ_t. The resetting thresholds are indicated by the dashed lines for $\lambda \in \{0.001, 0.01\}$ and $Q_T \in \{0, 1\}$. In case Δ_t reaches these thresholds, a state update is sent to the controller and Δ_t is set to zero. It is clear that these thresholds increase with growing λ, as transmissions are more costly and occur more sparsely.

In Figure 3 the optimal transmission timings for the time-triggered controller are shown for various parameter sets $\lambda \in \{0.01, 0.001\}$ and $Q_T \in \{0, 1\}$. Transmission timings occur at jumps of k_t. For the case of $\lambda = 0.01$ and $Q_T = 0$, it is optimal not to sent any updates. For the considered settings, there is a similarity between optimal event-triggered and time-triggered controller. When approaching horizon T updates are transmitted less likely for both event-triggered and time-triggered controller. This fact shows up significantly, when the objective function has no terminal cost.

Figure 4 gives a comparison between the optimal solution given by Theorem 1 and the optimal controller that chooses the transmission timings beforehand. As the optimal timings for the optimal time-triggered controller are lower bounds for optimal periodically sampled controllers, the dashed lines in Figure 4 are lower bounds for the optimal periodically sampled controller. It can be observed that the cost J^{E} increases by at least 50%, when optimal transmission timings are determined beforehand instead of using the event-triggered scheduler. For $\lambda > 0.01$ and no terminal cost, we have the special case that it is optimal not to transmit any updates. It is clear that for such setting event-trigger and time-trigger have similar performance.

V. Conclusions

By considering an Itǒ diffusion process, this paper solves the problem of jointly optimize the control and scheduling policy of a non-classical cost function that incorporates communication costs between sensor and controller. It is showed that the underlying problem can be transformed into tractable subproblems related to linear quadratic regulation and optimal stopping times. These subproblems can be solved by standard numerical methods. Numerical examples demonstrate the efficacy of the proposed approach compared to the optimal time-triggered controller. Future work includes the extension to the infinite horizon case with discounted cost, where ergodicity issues are to be considered, extension

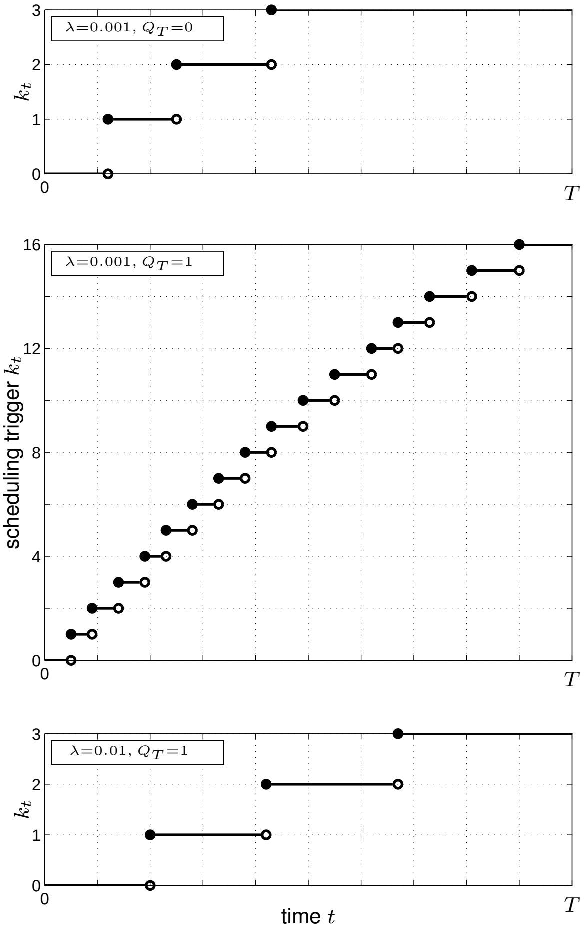

Fig. 3. Optimal time-triggered scheduling sequences for various parameter sets. Update transmissions occur at jumps of k.

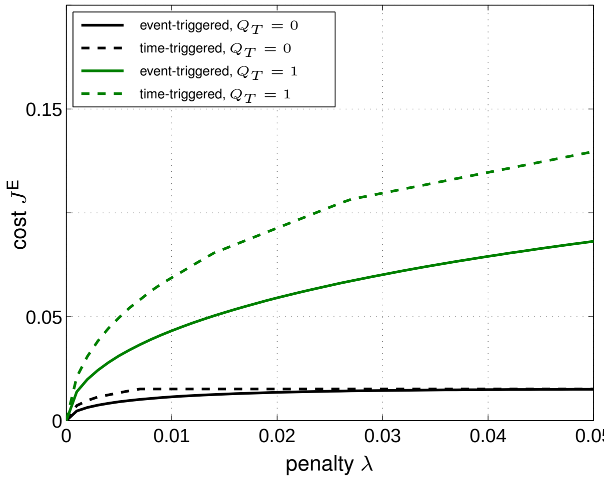

Fig. 4. Comparison of optimal event-triggered controller and optimal time-triggered controller with respect to cost J^{E} given by (11).

to partial observations at the sensor-side, non-ideal communications and extension to multi-terminal settings.

VI. ACKNOWLEDGMENTS

This work was supported in part by the German Research Foundation (DFG) within the Priority Program SPP 1305 "Control Theory of Digitally Networked Dynamical Systems".

REFERENCES

[1] K. Åström and B. Bernhardsson, "Comparison of Riemann and Lebesgue sampling for first order stochastic systems," *Decision and Control, 2002, Proceedings of the 41st IEEE Conference on*, vol. 2, pp. 2011–2016, Dec. 2002.

[2] A. Cervin and T. Henningsson, "Scheduling of event-triggered controllers on a shared network," in *Proc. 47th IEEE Conference on Decision and Control*, (Cancun, Mexico), 2008.

[3] T. Henningsson, E. Johannesson, and A. Cervin, "Sporadic event-based control of first-order linear stochastic systems," *Automatica*, vol. 44, pp. 2890–2895, Nov. 2008.

[4] M. Rabi and J. S. Baras, "Level-triggered control of a scalar linear system," *Control and Automation, 2007. MED 07. Mediterranean Conference on*, pp. 1–6, 27-29 June 2007.

[5] J. S. Baras, P. Hovareshti, and S. Perumal, "Event Triggered Distributed Collaborative Control," in *Proc. European Control Conf. (ECC)*, (Budapest, Hungary), Aug. 2009.

[6] D. V. Dimarogonas and K. H. Johansson, "Event-triggered Cooperative Control," in *Proc. European Control Conf. (ECC)*, (Budapest, Hungary), Aug. 2009.

[7] P. Wan and M. D. Lemmon, "Distributed Network Utility Maximization using Event-triggered Barrier Methods," in *Proc. European Control Conf. (ECC)*, (Budapest, Hungary), Aug. 2009.

[8] Y. Xu and J. Hespanha, "Optimal communication logics in networked control systems," *Decision and Control, 2004. CDC. 43rd IEEE Conference on*, vol. 4, pp. 3527–3532 Vol.4, Dec. 2004.

[9] Y. Xu and J. P. Hespanha, "Communication logic design and analysis for networked control systems," in *Current trends in nonlinear systems and control* (L. Menini, L. Zaccarian, and C. T. Abdallah, eds.), Boston: Birkhäuser, 2006.

[10] M. Rabi, K. H. Johansson, and M. Johansson, "Optimal stopping for event-triggered sensing and actuation," *Decision and Control, 2008 47th IEEE Conference on*, Dec. 2008.

[11] M. Ades, P. E. Caines, and R. P. Malhame, "Stochastic optimal control under poisson-distributed observations," *Automatic Control, IEEE Transactions on*, vol. 45, no. 1, pp. 3–13, Jan 2000.

[12] A. Molin and S. Hirche, "On LQG joint optimal scheduling and control under communication constraints," in *Proc. 48th IEEE Conference on Decision and Control*, (Shanghai, China), 2009.

[13] C. Ramesh, H. Sandberg, and K. H. Johansson, "LQG and medium access control," in *1st IFAC Workshop on Estimation and Control of Networked Systems*, 2009.

[14] J. K. Yook, D. M. Tillbury, and N. R. Soparkar, "Trading Computation for Bandwidth: Reducing Communication in Distributed Control Systems Using State Estimators," *IEEE Trans. on Control Systems Technology*, vol. 10, pp. 503–518, July 2002.

[15] B. Øksendal, *Stochastic Differential Equations - An Introduction with Applications*. Springer, 6th ed., 2003.

[16] K. J. Åström, *Introduction to Stochastic Control Theory*. Dover Publications, 2006.

[17] Y. Bar-Shalom and E. Tse, "Dual effect, certainty equivalence, and separation in stochastic control," *Automatic Control, IEEE Transactions on*, vol. 19, pp. 494–500, Oct 1974.

[18] M. Rabi, G. V. Moustakides, and J. S. Baras, "Adpative sampling for linear state estimation," *SIAM journal on control and optimization*, Apr. 2009. Submitted.

[19] A. Jazwinski, *Stochastic processes and filtering theory*. Academic Press, 1970.

[20] W. Fleming and R. Rishel, *Deterministic and stochastic optimal control*. Springer New York, 1975.

[21] X. D. Koutsoukos, "Optimal control of stochastic hybrid systems based on locally consistent markov decision processes," *International Journal of Hybrid Systems*, vol. 4, pp. 301–318, 2005.

[22] H. J. Kushner and P. Dupuis, *Numerical Methods for Stochastic Control Problems in Continuous Time*. Springer, 2nd ed., 2001.