# A Method to Convert Near-perfect into Perfect Reconstruction FIR Prototype Filters for Modulated Filter Banks

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***Abstract*— In this contribution we propose a simple method of obtaining a perfect reconstruction (PR) FIR prototype filter starting from a given near-perfect reconstruction (NPR) prototype filter for complex modulated filter banks. Our method consists of two steps. First, we calculate the product of a pair of polyphase components that fulfill the PR conditions, and second, we search for spectral factors that maximize the correlation coefficients between the impulse response of the new polyphase components and of the original ones. Our preliminary numerical results show that the price to be payed to satisfy the PR conditions is some loss in the spectral characteristics.**

## I. Introduction

Filter banks (FB) are signal processing systems that find application in various fields. Our main interest lays in the application to communication systems, for example, multicarrier systems or frequency (subchannel) domain equalization [1]. Because the signals in those applications are complex, mainly built by the I and Q components, it is necessary to employ complex filter banks [2], [3].

Its simplified design and efficient implementation make the complex modulated filter banks based on the discrete Fourier transform (DFT) very attractive. These FBs have some variations in their structure and are also known by the names modified DFT (MDFT) [2], exponentially modulated FB (EMFB) [3] and in the specific case of multicarrier communication systems as FB based multicarrier (FBMC) [4] or Offset-QAM orthogonal frequency division multiplexing (OFDM/OQAM) [5].

The concept of perfect reconstruction (PR) [1] is very important in the theory of filter banks [2]. It basically defines what conditions are necessary to allow that any signal passing through the complete FB, without any further processing, remains with its amplitude and phase undistorted. It resembles the definition of orthogonal lossless networks from the classical circuit theory. Another important concept is the near-perfect reconstruction (NPR), when some amplitude and phase changes are allowed, similar to the well known problem of intersymbol interference in communication systems.

In most applications of FBs, the PR property will be destroyed by the subchannel processing or by the transmission channel. However, besides the guarantee of keeping the signals undistorted through the FB processing, the PR property also allows efficient and robust implementations. One good example is the deployment of lattice structures to perform the FIR filtering. They reduce the sensitivity to coefficient quantization, between other advantages, and allow very efficient realization, including the filtering without multiplications. Those are important characteristics when the processing has to be implemented in dedicated hardware or ASICs, specially in todays and future wideband wireless, wired and optical communications systems.

There exist many methods to design optimum PR prototype filters following various criteria, e.g. minimization of the prototype stopband energy (least-squares, LS), minimization of the peaks in the stopband (minimax), peak constrained LS (PCLS) or maximization of the time-frequency localization (TFL) (see [6], [7] and the references therein). They are often computationally intensive and involve some complicated optimization method.

To the best of our knowledge there are no contributions in the current literature considering the conversion of a prototype filter that constitutes an NPR FB into a new prototype that satisfy the PR constraints. In some cases an NPR prototype can be calculated with very simple methods and with closed forms, like in the case of a root raised cosine or by the frequency sampling method. Even some kind of optimization can be considered to obtain the NPR prototype. One drawback is that the FBs using those prototypes cannot employ a lattice structure, for example.

This work is organized as follows. In section II we briefly introduce the FB structure we are interested in. In section III we describe our proposed method to convert the NPR prototype into a PR one. We show two numerical examples in section IV and we draw the conclusions in section V.

## II. Complex Modulated Filter Banks

A general overview of a maximally decimated FB is shown in Fig. 1. We have used the definitions $z_1 = e^{sT/M}$ and $z = e^{sT}$, where $s = \sigma + j\omega$ is the complex frequency variable. The set of filters $H_k(z_1)$ that decompose the wideband signal

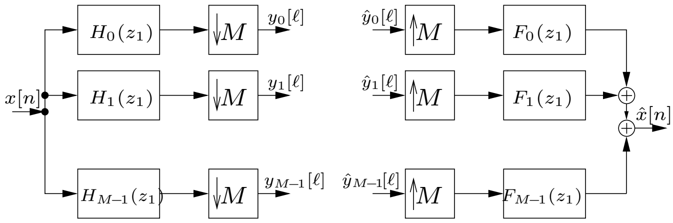


Fig. 1. Basic complex modulated filter bank structure

$x[n]$ into narrowband signals $y_k[m]$ is called analysis FB (AFB) and the filters $F_k(z_1)$ that synthesize the wideband signal $\hat{x}[n]$ from the narrowband signals $\hat{y}_k[m]$ are called synthesis FB (SFB). Both AFB and SFB filters are obtained by exponentially modulating a linear phase FIR prototype zero phase filter $p[n]$ of length $N = KM$.

$$h_k[n] = f_k[n] = p\left[n - \frac{N-1}{2}\right] W_M^{k\left(n-\frac{N-1}{2}\right)}, \quad (1)$$

$$H_k(z_1) = F_k(z_1) = z_1^{-\frac{N-1}{2}} P(z_1 W_M^k), \quad (2)$$

where $W_M = e^{j\frac{2\pi}{M}}$ and $k = 0, ..., M-1$. It is worth noting that the signals $x[n]$ and $\hat{x}[n]$ have a sample rate of $\frac{M}{T}$ and the signals $y_k[\ell]$ and $\hat{y}_k[\ell]$ of $\frac{1}{T}$. We restrict our attention to the specific case where $N$ is even.

Both AFB and SFB can be implemented in an efficient structure by deploying some important identities for multirate processing and by performing the complex modulation by means of the IDFT and DFT [2]. Fig. 2 depicts the efficient realization of the AFB, while in Fig. 3 the low complexity SFB is shown. Note that the same prototype is used for the AFB as well as the SFB.

Afterwards, let us introduce the transfer functions $G_m(z)$, $m = 0, \ldots, M-1$, as the type-1 polyphase components of length $K$ of the prototype filter [1], which are derived from the relation

$$H_0(z_1) = \sum_{m=0}^{M-1} z_1^{-m} G_m(z_1^M), \quad (3)$$

and can be calculated in the time domain as

$$g_m[n] = h_0[nM + m], \quad m = 0, ...M-1. \quad (4)$$

The delay $T_d$ is specifically necessary in the case of trans-multiplex application, e.g. multicarrier systems. In the case of $N = KM$ this delay is $T_d = \frac{T}{2}$ or $\frac{M}{2}$ samples in the higher sampling rate.

The blocks $\mathcal{O}_k$ perform a $\frac{T}{2}$ staggering of the real and imaginary parts of the low rate signals $\hat{y}_k[\ell]$, while the blocks $\mathcal{O}'_k$ perform a de-staggering to generate the signals $y_k[\ell]$. Figs. 4 and 5 depict the internal structure of $\mathcal{O}_k$ and $\mathcal{O}'_k$ for $k$ even. In the cases of $k$ odd the roles of the real and imaginary parts are interchanged. These blocks are necessary to guarantee the construction of alias-free and cross-talk-free DFT FBs or, in other words, PR FBs [2].

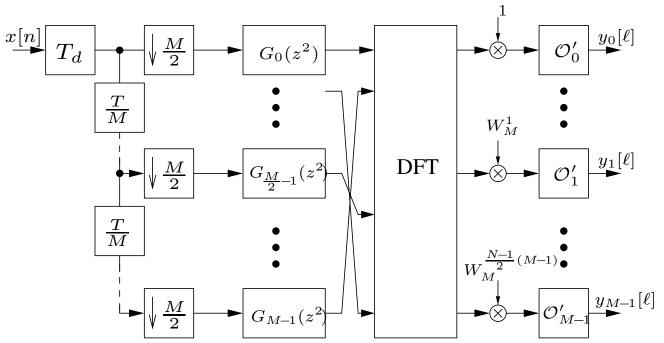


Fig. 2. Efficient realization of the analysis FB

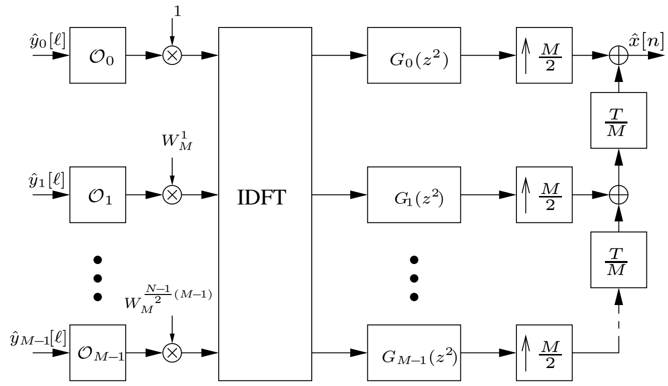


Fig. 3. Efficient realization of the synthesis FB

## III. NPR TO PR CONVERSION

It is known from the theory of FBs that the PR is guaranteed if the following constraints are satisfied [1], [2]

$$G_m(z) G_m^*(z) + G_{m+\frac{M}{2}}(z) G_{m+\frac{M}{2}}^*(z) = \frac{2}{M}, \quad (5)$$

for $m = 0, ..., \frac{M}{2} - 1$ and where $G_m^*(z)$ is the paraconjugate of the transfer function $G_m(z)$. Eq. (5) resembles the Feldtkeller equation for lossless two-ports.

Because we assume a linear phase prototype and if we consider the relation between the polyphase components $m$ and $M - m - 1$ we get

$$G_m(z) G_{M-m-1}(z) + G_{m+\frac{M}{2}}(z) G_{\frac{M}{2}-m-1}(z) = \frac{2}{M} z^{-K+1}. \quad (6)$$

If (6) is written for NPR prototypes yields to the following result

$$\hat{G}_m(z) \hat{G}_{M-m-1}(z) + \hat{G}_{m+\frac{M}{2}}(z) \hat{G}_{\frac{M}{2}-m-1}(z) = D(z) \quad (7)$$

where $D(z)$ is a symmetric polynomial of order $2K - 2$ that has very small coefficients apart from the middle one.

To satisfy the PR constraints we can subtract evenly the interference part of $D(z)$ from both products on the left hand side of (7). The new polyphase components of the PR prototype are obtained then from

$$G_m(z) G_{M-1-m}(z) = \hat{G}_m(z) \hat{G}_{M-m-1}(z) - \frac{1}{2}\Gamma(z), \quad (8)$$

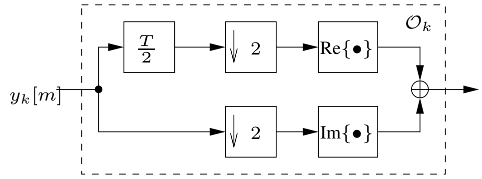


Fig. 4. Real and Imaginary parts de-staggering, $k$ even

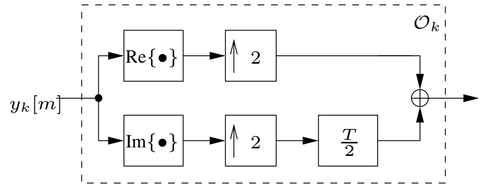


Fig. 5. Real and Imaginary parts staggering, $k$ odd

where

$$\Gamma(z) = D(z) - \frac{2}{M} z^{-K+1}. \tag{9}$$

We could have written a second equation for the second product at the left hand side of (7), but for symmetrical prototypes the two equations are equal and (8) already gives two of the polyphase components. Mathematically, we can make the variable substitution $m' = \frac{M}{2} - 1 - m$ in (8) and get

$$G_{\frac{M}{2}-1-m'}(z) G_{\frac{M}{2}+m'}(z) = \hat{G}_{\frac{M}{2}-1-m'}(z) \hat{G}_{\frac{M}{2}+m'}(z) - \frac{1}{2}\Gamma(z), \tag{10}$$

where $m' = 0, .., \frac{M}{2} - 1$. Consequently, the sum of (8) and (10) satisfy the PR constraints (6).

Then, the new $m$ and $M-m-1$ polyphase components are the spectral factors of the product $G_m(z)G_{M-m-1}(z)$. Each of them has $2(K-1)$ zeros and they have to be combined in groups of $K-1$ zeros. Consequently, the maximum number of combinations of zeros is given by $\frac{(2K-2)!}{(K-1)!(K-1)!}$. However, only a few of the combinations are valid, first because the complex zeros can only appear in conjugate pairs to end-up in a prototype with real coefficients, and if one zero is inside the unit circle, its inverse cannot appear in the same spectral factor, otherwise the products will not be symmetric.

We propose the following method to search for the best spectral decomposition. After obtaining the product $G_m(z)G_{M-m-1}(z)$ and finding its zeros, the valid combinations have to be determined. We search the solution for $G_m(z)$ that maximizes the correlation factor between it and the original polyphase component $\hat{G}_m(z)$.

We define $\mathbf{g}_m$ and $\hat{\mathbf{g}}_m$ as the vectors with the coefficients of $G_m(z)$ and $\hat{G}_m(z)$ or, in other words, with their impulse responses. Then we choose the combination of zeros that maximizes

$$r_m = \frac{\mathbf{g}_m^T \hat{\mathbf{g}}_m}{||\mathbf{g}_m^T||_2 ||\hat{\mathbf{g}}_m||_2}. \tag{11}$$

The remaining zeros are allocated to $G_{M-m-1}(z)$.

In the applications we are interested in, and due to complexity reasons, $K$ is a small number between 3 and 5, making it straightforward to perform the search.

## IV. Numerical results

In all examples presented here, we consider $M = 8$ and $K = 4$. The length of the polyphase components does not depend on the number of subchannels, making it simple to extend the result to FBs with a higher number of subchannels.

In our first example, we took an approximation of a root raised cosine (RRC) filter with a roll-off factor $\rho = 1$. The coefficients are calculated with a closed form. In Fig. 6, we have plotted the impulse response of both original NPR RRC filter and the obtained PR prototype. A significant change can be observed specially in the side-lobes.

To evaluate the spectral performance, Fig. 7 shows the magnitude of the frequency response of the original filter, the obtained prototype and another PR prototype optimized in the least-squares sense and obtained with the method described [8]. We can see that there is a significant loss in the stop-band attenuation when our method is employed. But also the optimized PR has some peaks in that band that are higher then the NPR. That is part of the price payed to satisfy the PR constraints.

The second example we considered was an NPR prototype obtained with the frequency sampling approach of [9]. Fig. 8 depicts the magnitude of the frequency response for this second example. Again, we see an increase in some of the stop-band lobes.

We should observe that because the criterion utilized to choose the polyphase components that satisfy the PR conditions was the correlation between the impulse responses before and after the conversion, the spectral characteristics have been neglected. To evaluate to influence of the results obtained on the performance of an FBMC system, for example, other aspects of the whole system have to be taken account, like the spectral mask, the noise level, the channel impulse response, the choice of the equalizer, etc.

It is worth noting that we have not considered in any of the results showed here the effect of coefficient quantization. This is an essential issue to be taken into account when realizing computationally efficient modulated FBs. The NPR prototypes cannot be realized in low sensitivity structures like the lattice form, that can only be employed for PR prototypes.

## V. Conclusion

In this contribution we described a simple method of obtaining PR prototypes for modulated filter banks starting from any given NPR prototype. The method is based on finding new polyphase components that satisfy the PR constraints and have the most correlated impulse response to the original ones.

It is important to note that in applications where the computational complexity is critical, the coefficients of the polyphase components will have to be represented in short word lengths. In this case all the transfer functions will be changed and, by recalling that the direct form has a much

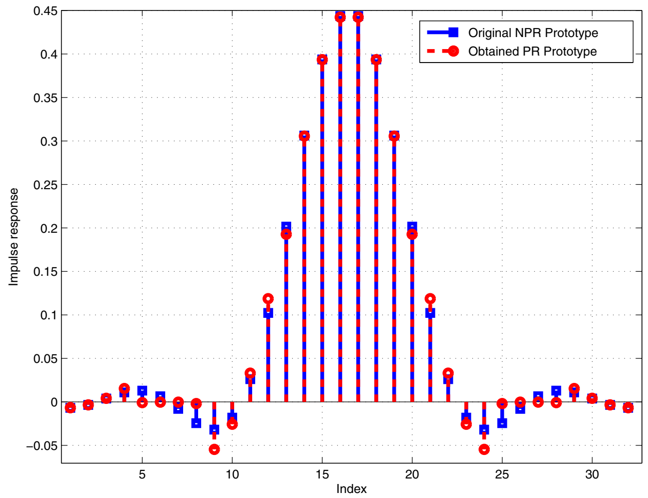


Fig. 6. Impulse response of the obtained PR prototype and original RRC NPR prototype

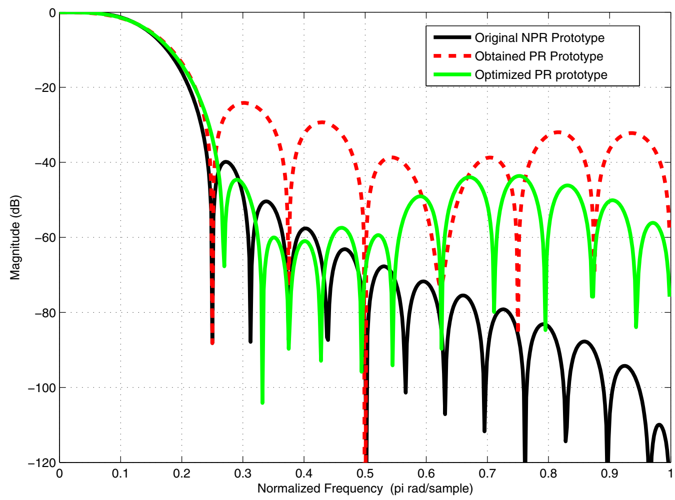


Fig. 8. Frequency response of the obtained PR prototype, original FS NPR prototype and optimized PR prototype

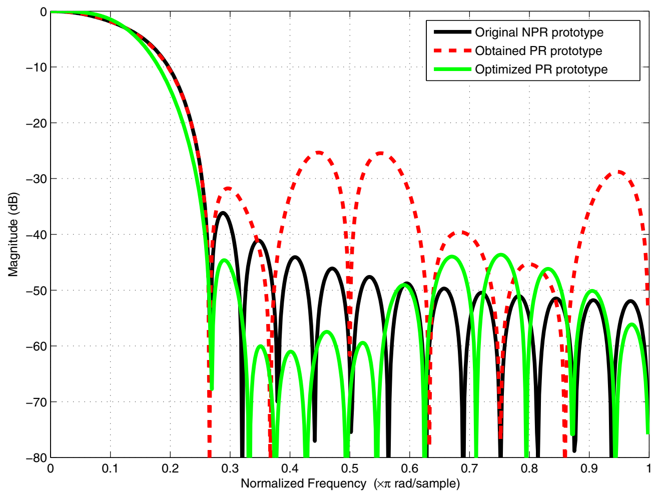


Fig. 7. Frequency response of the obtained PR prototype, original RRC NPR prototype and optimized PR prototype

higher sensitivity to coefficient quantization than the lattice form, it is expected a greater change in the frequency response of the NPR prototypes than the PR ones.

In the numerical results we see that our proposed method still needs some improvements to achieve better spectral characteristics. Some other way of looking for the best combination of zeros can be tested, like also looking into the spectral characteristics instead of only comparing the impulse response. We also intend to consider the coefficient quantization issue in future works.

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