MULTIUSER FEEDBACK DESIGN WITH MULTIPLE RECEIVE ANTENNAS

Mario H. Castañeda and Josef A. Nossek

Institute for Circuit Theory and Signal Processing
Technische Universität München, 80290 Munich, Germany

ABSTRACT

In a frequency division duplex system with a base station equipped with multiple antennas, the base station relies on feedback of the channel state information of the users in order to perform user selection and transmit beamforming in the downlink. Nonetheless, it is usually assumed that the feedback link is noise-free, i.e. the base station has access to error-free feedback. In practice, however, the feedback link is the uplink of the system and is subject to fading, possible multiuser interference and noise. In this work, with a given amount of resources reserved for the feedback of all the users, we tackle the system feedback design considering feedback errors. We take into account that the base station can detect the feedback of several users simultaneously by employing receive beamforming. In addition, we propose to exploit multiuser diversity in the feedback link in order to minimize the feedback error probability. We also present an orthogonal pilot design which enables the base station to estimate the feedback (uplink) channels of the users.

1. INTRODUCTION

Multiuser diversity is an inherent form of diversity present in time-varying systems with several users. For instance, in a single isolated cell, with M antennas at the base station and $K > M$ single-antenna active users, the base station can select a set of at most M users and transmit to them in the downlink employing transmit beamforming. In order to perform user selection and transmit beamforming, the base station must know the *channel state information* (CSI) of the K users in the downlink. In a *frequency division duplex* (FDD) system, the transmit CSI for the downlink is obtained with a limited feedback of B bits per user in the uplink [1].

With B bits per user, the required feedback load of the system increases linearly with the number of users. Despite most of the current literature presume a per-user limited feedback, an overall *system* limited feedback is actually more appropriate. This point has been recently discussed in [2] and [3]. In [2] a constraint on the total number of feedback bits is considered, while [3] considers a constraint on the total amount of *channel uses* reserved for the feedback of the K users. Both instances lead to a tradeoff between the attainable degree of multiuser diversity and the users' feedback quality.

Most of the prior works dealing with limited feedback usually also assume error-free feedback. This assumption, nonetheless, is impractical in FDD systems since the feedback link (uplink) is subject to fading, possible multiuser interference and noise. For a given amount of resources allocated for the system feedback, we investigate in this work the system feedback design by taking into account feedback errors. We consider that the system feedback load consists of T_{F} channel uses reserved for the feedback of all the users as in [3]. For this purpose, we take into account that the base station can detect the feedback of several users simultaneously by employing *receive* beamforming. Moreover, multiuser diversity can also be exploited in the feedback link by deciding which group of users should relay their feedback at the same time in order to minimize the feedback error probability.

The presented multiuser feedback design is transparent to the information being fed back with the B bits per user and is also independent of the user selection and transmit strategy employed in the downlink. To complement the feedback design, we present in addition an orthogonal pilot design which enables the base station to estimate the feedback (uplink) channels of the K users. To this end, this paper is orgnaized as follows. The system model and the uplink channel estimation is discussed in Section 2, while Section 3 deals with the feedback detection. The feedback design given a system feedback consisting of T_{F} channel uses is treated in Section 4. Some numerical results are also shown in Section 4. Finally, Section 5 concludes the paper.

2. UPLINK CHANNEL ESTIMATION

We focus on the feedback link of an FDD system in a single cell with M antennas at the base station and $K \geq M$ single-antenna users. The feedback link which is the uplink of the FDD system represents a *multiuser single-input multiple-output* (MU-SIMO) channel. The available transmit power at each of the K users is taken to be P_{UL}, while σ_v^2 is the variance of the AWGN at each receive antenna at the BS in the uplink. The AWGN is assumed to be independent over the antennas. The SIMO uplink channel of user k is denoted as $\boldsymbol{h}_k \in \mathbb{C}^M$ where the elements of $\boldsymbol{h}_k \,\forall\, k$ are i.i.d. zero-mean unit-variance complex Gaussian random variables. Hence, with the channel of user k as $\boldsymbol{h}_k = [h_{k,1}, \cdots, h_{k,M}]^{\mathrm{T}}$, we have that $\mathrm{E}\left[|h_{k,m}|^2\right] = 1$ for $m = 1, \ldots, M$.

As in [4], we assume the users report their feedback un-

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coded using QPSK symbols in the uplink. The feedback is detected with receive beamforming, for which the base station must know the uplink channels of the users. Hence, before presenting the feedback detection and design, we discuss the estimation of the uplink channels of the K users.

We can perform the channel estimation per receive antenna m for $m = 1, \cdots, M$, since for each user the uplink channels over the antennas are uncorrelated. Let us focus first on the estimation of the scalar uplink channel $h_{k,m}$. To this end, we assume that only user k is present and that it transmits a pilot sequence consisting of T_{UL} channel uses (symbols). Stacking the T_{UL} pilot symbols of user k's pilot sequence into a vector $\boldsymbol{p}_k \in \mathbb{C}^{T_{\text{UL}}}$, the received signal at the m-th antenna of the BS during the training phase is

$$\boldsymbol{x}_{m,\text{T}} = \sqrt{P_{\text{UL}}}\, \boldsymbol{p}_k\, h_{k,m} + \boldsymbol{v}_{m,\text{T}}, \tag{1}$$

where $\boldsymbol{x}_{m,\text{T}} \in \mathbb{C}^{T_{\text{UL}}}$ represents the T_{UL} received symbols available for the estimation of the scalar channel $h_{k,m}$ and in addition, $\boldsymbol{v}_{m,\text{T}}[n] \in \mathbb{C}^{T_{\text{UL}}}$ is the AWGN during the training phase at the m-th antenna of the BS. Assuming zero-mean and unit-variance pilot symbols, we have that $\operatorname{tr}(\boldsymbol{p}_k\boldsymbol{p}_k^{\text{H}}) = \|\boldsymbol{p}_k\|_2^2 = T_{\text{UL}}$. We obtain a *minimum means square error* (MMSE) estimate of the channel $\hat{h}_{k,m}$ as follows

$$\hat{h}_{k,m} = \boldsymbol{q}_k^{\text{H}}\boldsymbol{x}_{m,\text{T}}, \tag{2}$$

where

$$\boldsymbol{q}_k = \underset{\boldsymbol{q}_k}{\operatorname{argmin}}\ \mathrm{E}\left[|h_{k,m} - \boldsymbol{q}_k^{\text{H}}\boldsymbol{x}_{m,\text{T}}|^2\right] = \frac{\sqrt{P_{\text{UL}}}}{\sigma_v^2 + P_{\text{UL}}T_{\text{UL}}}\,\boldsymbol{p}_k. \tag{3}$$

From (2) and (3), the estimate of the channel from user k to the m-th antenna is given by

$$\hat{h}_{k,m} = \frac{\sqrt{P_{\text{UL}}}}{\sigma_v^2 + P_{\text{UL}}T_{\text{UL}}}\,\boldsymbol{p}_k^{\text{H}}\boldsymbol{x}_{m,\text{T}}. \tag{4}$$

The scalar uplink channel $h_{k,m}$ can be written as

$$h_{k,m} = \hat{h}_{k,m} + e_{k,m}, \tag{5}$$

where $e_{k,m}$ is the estimation error, which is a Guassian random variable with zero mean and variance σ_e^2. Moreover, the channel estimate $\hat{h}_{k,m}$ is a complex Gaussian random variable with zero mean and variance $(1-\sigma_e^2)$. The variance of the estimation error is given by

$$\sigma_e^2 = \mathrm{E}\left[|h_{k,m} - \boldsymbol{q}_k^{\text{H}}\boldsymbol{x}_{k,m}|^2\right] = \frac{1}{1 + \frac{P_{\text{UL}}}{\sigma_v^2}T_{\text{UL}}}, \tag{6}$$

where the second step follows after plugging (3) and taking the expectation over the noise and the channel. The variance of the estimation error does not depend on the user k and on the antenna m since the channels are independent over the users and the antennas.

Let us stack the channel estimates $\hat{h}_{k,m}$ from user k to each receive antenna m into a vector

$$\hat{\boldsymbol{h}}_k = \left[\hat{h}_{k,1}, \cdots, \hat{h}_{k,M}\right]^{\text{T}}, \tag{7}$$

and with $\boldsymbol{e}_k = [e_{k,1}, \cdots, e_{k,M}]^{\text{T}}$, we can write the uplink SIMO channel vector for user k as

$$\boldsymbol{h}_k = \hat{\boldsymbol{h}}_k + \boldsymbol{e}_k. \tag{8}$$

Since the channels over the antennas are uncorrelated the covariance matrix of the error is

$$\mathrm{E}\left[\boldsymbol{e}_k\boldsymbol{e}_k^{\text{H}}\right] = \sigma_e^2\,\mathbf{1}_M, \tag{9}$$

which is the same among all the users, since the assumed statistics of the users are i.i.d. The channel estimate $\hat{\boldsymbol{h}}_k$ is the MMSE estimate of $\boldsymbol{h}_k$, and we have that $\hat{\boldsymbol{h}}_k$ is independent of $\boldsymbol{e}_k$, with zero mean and covariance matrix $(1-\sigma_e^2)\mathbf{1}_M$. If the channels accross the antennas are correlated, a spatial filter $\boldsymbol{T}_k \in \mathbb{C}^{M\times M}$ can be applied to the estimated channel vector $\hat{\boldsymbol{h}}_k$ in order to obtain a refined estimate $\hat{\hat{\boldsymbol{h}}}_k = \boldsymbol{T}_k\hat{\boldsymbol{h}}_k$ by taking into account the covariance matrix of the channel of user k, i.e. $\boldsymbol{C}_k = \mathrm{E}\left[\boldsymbol{h}_k\boldsymbol{h}_k^{\text{H}}\right]$, such that

$$\boldsymbol{T}_k = \underset{\boldsymbol{T}_k}{\operatorname{argmin}}\ \mathrm{E}\left[\|\boldsymbol{h}_k - \boldsymbol{T}_k\hat{\boldsymbol{h}}_k\|^2\right]. \tag{10}$$

In the single-user and noise free case, we require only $T_{\text{UL}} = 1$ to estimate the scalar channel at each receive antenna. However, in the multiuser case with K users, we need to set $T_{\text{UL}} \geq K$ in order to estimate K channels. This can be performed with dedicated *orthogonal* pilot sequences by imposing the following constraint over the pilot sequences $\boldsymbol{p}_k$ for all the users $k = 1, \cdots, K$:

$$\boldsymbol{p}_k^{\text{H}}\boldsymbol{p}_j = 0 \quad \text{for} \quad j \neq k. \tag{11}$$

Such a set of K orthogonal pilot sequences of length T_{UL} can easily be obtained by taking K orthogonal basis vectors of a T_{UL}-dimensional complex space. Let us denote the K orthogonal basis vectors as $\boldsymbol{p}'_1, \ldots, \boldsymbol{p}'_K$, such that the K orthogonal pilot sequences are obtained as

$$\boldsymbol{p}_k = \frac{T_{\text{UL}}}{\|\boldsymbol{p}'_k\|_2}\,\boldsymbol{p}'_k, \tag{12}$$

such that $\|\boldsymbol{p}_k\|_2^2 = T_{\text{UL}}$ holds for $k = 1, \ldots, K$.

With K users each sending one dedicated pilot sequence, we need to rewrite (1) as

$$\boldsymbol{x}_{m,\text{T}} = \sum_{k=1}^{K}\sqrt{P_{\text{UL}}}\,\boldsymbol{p}_k\,h_{k,m} + \boldsymbol{v}_{m,\text{T}}. \tag{13}$$

However, the estimate $\hat{h}_{k,m}$ is still given as (4), since the pilot sequences among the users are orthogonal. As long as the pilot sequences $\boldsymbol{p}_k$ for $k = 1, \ldots, K$ satisfy (11), the expressions derived previously with (1) still hold. The channel estimate $\hat{\boldsymbol{h}}_k$ of user k is given by (7), where the elements $\hat{h}_{k,m}$ of the estimated channel for $m = 1, \ldots, M$ are obtained as shown in (4) by using its dedicated pilot sequence $\boldsymbol{p}_k$.

Given a packet length of T channel uses in the uplink, the K users can each send its dedicated orthogonal pilot sequence during the training phase consisting of T_{UL} channel uses at the beginning of the packet as shown in Figure 1. Consider the minimum value for the pilot sequences, i.e. $T_{\text{UL}} = K$ channel uses. With the orthogonal pilot design, as K increases the base station can estimate the uplink channels of all the K users with an increasing training phase. The channel estimation of the K users is achieved with two benefits: with no pilot contamination irrespective of K and with a decreasing variance of the estimation error σ_e^2 (c.f. (6)). Notwithstanding, this comes at the price of an increase in overhead.

After the training phase, the base station can detect the feedback of the K users based on the estimated channel of the K users during the feedback phase of T_{F} channel uses at the end of the packet as depicted in Figure 1. However, the channel estimation of the K users does not really impose an additional overhead for the feedback detection if multiuser diversity is to be exploited in the uplink data transmission of the users. We point out that the base station would need to estimate the uplink channels of the K users *anyhow* in order to perform user selection in the uplink. With a packet length of T, where T_{UL} and T_{F} channel uses are reserved for the training phase and feedback phase, respectively, we have the remaining $T - T_{\text{UL}} - T_{\text{F}}$ channel uses available for the uplink data transmission of at most M users as depicted in Figure 1.

3. FEEDBACK DETECTION IN THE UPLINK

As stated before, T_{F} channel uses are reserved for the feedback of the K users. The base station with its M receive antennas can detect the feedback of at most M users simultaneously using *receive* beamforming based on the estimated channels of all the users. The feedback load per user consists of B bits and considering without loss of generality only even values of B, each user can feed back its B bits uncoded using $\frac{B}{2}$ QPSK symbols. For a given K and a system feedback load consisting of an airtime of T_{F} channel uses, where at each channel use at most M users can each feed back one QPSK feedback symbol, each user can report back at most

$$B = 2\left\lfloor \frac{T_{\text{F}} M}{K} \right\rfloor \quad \text{feedback bits.} \tag{14}$$

For the time being, we consider that the K users are somehow ordered into T_{F} groups with at most M users per group such that each user is a member of exactly $\frac{B}{2}$ groups. We will discuss in Section 4 how this can be done in a clever manner.

Let us denote the ℓ-th group as $\mathcal{G}_\ell$ for $\ell = 1, \ldots, T_{\text{F}}$, where $|\mathcal{G}_\ell| \leq M$. The users in the group $\mathcal{G}_\ell$ are the users which feed back one QPSK symbol from its feedback load during the ℓ-th time instance of the feedback phase. The users in group $\mathcal{G}_\ell$ are denoted as $\mathcal{G}_\ell = \{\pi_\ell(1), \pi_\ell(2), \cdots, \pi_\ell(|\mathcal{G}_\ell|)\}$, where $\pi_\ell(n) \in \{1, 2, \cdots, K\}$, for $n = 1, 2, \ldots, |\mathcal{G}_\ell|$. For example for $M = 4$, $K = 10$, $T_{\text{F}} = 8$, B given as (14) and with the assignment of the users to group $\mathcal{G}_\ell$ for $\ell = 1, \ldots, 8$ as shown in Figure 1, we have for $\ell = 4$ that $|\mathcal{G}_4| = 4$ and $\pi_4(1) = 2$, $\pi_4(2) = 4$, $\pi_4(3) = 7$ and $\pi_4(4) = 10$.

The base station can detect one QPSK feedback symbol from each user in group $\mathcal{G}_\ell$ by performing MMSE receive

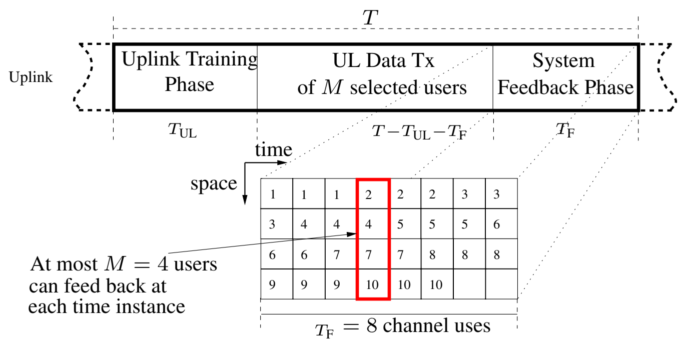

Fig. 1. System Limited Feedback Design.

beamforming at time instance ℓ based on the estimated uplink channels of the users in group $\mathcal{G}_\ell$ for $\ell = 1, \ldots, T_{\text{F}}$.

To this end, we define the following matrices

$$\begin{aligned}
\boldsymbol{H}[\ell] &= \left[\boldsymbol{h}_{\pi_\ell(1)}, \boldsymbol{h}_{\pi_\ell(2)}, \cdots, \boldsymbol{h}_{\pi_\ell(|\mathcal{G}_\ell|)}\right] \in \mathbb{C}^{M \times |\mathcal{G}_\ell|} \\
\hat{\boldsymbol{H}}[\ell] &= \left[\hat{\boldsymbol{h}}_{\pi_\ell(1)}, \hat{\boldsymbol{h}}_{\pi_\ell(2)}, \cdots, \hat{\boldsymbol{h}}_{\pi_\ell(|\mathcal{G}_\ell|)}\right] \in \mathbb{C}^{M \times |\mathcal{G}_\ell|} \\
\boldsymbol{E}[\ell] &= \left[\boldsymbol{e}_{\pi_\ell(1)}, \boldsymbol{e}_{\pi_\ell(2)}, \cdots, \boldsymbol{e}_{\pi_\ell(|\mathcal{G}_\ell|)}\right] \in \mathbb{C}^{M \times |\mathcal{G}_\ell|},
\end{aligned}$$

such that

$$\boldsymbol{H}[\ell] = \hat{\boldsymbol{H}}[\ell] + \boldsymbol{E}[\ell], \tag{15}$$

where the columns of $\boldsymbol{H}[\ell]$, $\hat{\boldsymbol{H}}[\ell]$ and $\boldsymbol{E}[\ell]$ correspond to the channel, estimated channel and estimation error of the users in group $\mathcal{G}_\ell$, respectively.

We collect the unit-amplitude QPSK feedback symbols of the users in group $\mathcal{G}_\ell$ in the vector $\boldsymbol{s}[\ell] \in \left\{\pm\frac{\sqrt{2}}{2} \pm \mathrm{j}\,\frac{\sqrt{2}}{2}\right\}^{|\mathcal{G}_\ell|}$. The estimated feedback symbols $\hat{\boldsymbol{s}}[\ell]$ at the ℓ-th time instance of the feedback phase are obtained as follows with the receive filter $\boldsymbol{W}[\ell] \in \mathbb{C}^{|\mathcal{G}_\ell| \times M}$

$$\hat{\boldsymbol{s}}[\ell] = \boldsymbol{W}[\ell]\left(\sqrt{P_{\text{UL}}}\boldsymbol{H}[\ell]\boldsymbol{s}[\ell] + \boldsymbol{n}[\ell]\right) \tag{16}$$

$$= \boldsymbol{W}[\ell]\left(\sqrt{P_{\text{UL}}}\hat{\boldsymbol{H}}[\ell]\boldsymbol{s}[\ell] + \sqrt{P_{\text{UL}}}\boldsymbol{E}[\ell]\boldsymbol{s}[\ell] + \boldsymbol{n}[\ell]\right), \tag{17}$$

where $\boldsymbol{n}[\ell] \in \mathbb{C}^M$ is the AWGN at the base station at the ℓ-th time instance of the feedback phase. The filter $\boldsymbol{W}[\ell]$ is chosen such that mean square error is minimized, i.e.

$$\boldsymbol{W}[\ell] = \underset{\boldsymbol{W}[\ell]}{\operatorname{argmin}} \ \mathrm{E}\left[\|\boldsymbol{s}[\ell] - \hat{\boldsymbol{s}}[\ell]\|_2^2\right] \tag{18}$$

$$= \frac{\hat{\boldsymbol{H}}^{\mathrm{H}}[\ell]}{\sqrt{P_{\text{UL}}}}\left(\hat{\boldsymbol{H}}[\ell]\hat{\boldsymbol{H}}^{\mathrm{H}}[\ell] + \left(|\mathcal{G}_\ell|\sigma_e^2 + \frac{\sigma_v^2}{P_{\text{UL}}}\right)\mathbf{1}_M\right)^{-1} \tag{19}$$

where the expected value in (18) is taken over the noise and the estimation error $\boldsymbol{E}[\ell]$. The MMSE receiver $\boldsymbol{W}[\ell]$ is based on the estimated channel of the users in group $\mathcal{G}_\ell$ and has been derived by taking into account that the estimation error has zero mean with covariance matrix given in (9) and in addition, is uncorrelated with the signal and noise.

The SINR of the QPSK feedback symbol of the n-th user in the group $\mathcal{G}_\ell$ for $n = 1, \dots, |\mathcal{G}_\ell|$ is $\tilde{\gamma}(\pi_\ell(n)) =$

$$\frac{|\boldsymbol{w}_n^{\mathrm{T}} \hat{\boldsymbol{h}}_{\pi_\ell(n)}|^2}{\sum_{i=1, i\neq n}^{|\mathcal{G}_\ell|} |\boldsymbol{w}_n^{\mathrm{T}} \hat{\boldsymbol{h}}_{\pi_\ell(i)}|^2 + \sum_{i=1}^{|\mathcal{G}_\ell|} |\boldsymbol{w}_n^{\mathrm{T}} \boldsymbol{e}_{\pi_\ell(i)}|^2 + \frac{\sigma_v^2}{P_{\text{UL}}} \|\boldsymbol{w}_n\|_2^2}, \quad (20)$$

where $\boldsymbol{w}_n \in \mathbb{C}^M$ is the n-th row of $\boldsymbol{W}[\ell]$. However, the base station does not know $\tilde{\gamma}_\ell(n)$ since the estimation error $\boldsymbol{e}_{\pi_\ell(i)}$ for $i = 1, \dots, |\mathcal{G}_\ell|$ is unknown. In addition, note that the estimation error remains constant during the feedback phase. We approximate $\tilde{\gamma}_\ell(n)$ with the SINR $\gamma_\ell(n)$ assuming a worst case error by taking the error to be a zero-mean complex Gaussian random variable with variance σ_e^2, i.e.

$$\gamma(\pi_\ell(n)) = \frac{|\boldsymbol{w}_n^{\mathrm{T}} \hat{\boldsymbol{h}}_{\pi_\ell(n)}|^2}{\sum_{i=1, i\neq n}^{|\mathcal{G}_\ell|} |\boldsymbol{w}_n^{\mathrm{T}} \hat{\boldsymbol{h}}_{\pi_\ell(i)}|^2 + \left(|\mathcal{G}_\ell|\sigma_e^2 + \frac{\sigma_v^2}{P_{\text{UL}}}\right) \|\boldsymbol{w}_n\|_2^2}. \quad (21)$$

Using [5], the bit error probability $p_{\text{b}}(\pi_\ell(n))$ for uncoded QPSK with SINR $\gamma(\pi_\ell(n))$ can be approximated as

$$p_{\text{b}}(\pi_\ell(n)) = Q\left(\sqrt{\gamma(\pi_\ell(n))}\right), \quad (22)$$

where $Q(\bullet)$ is the Q-function. $p_{\text{b}}(\pi_\ell(n))$ represents the bit error probability of user $\pi_\ell(n)$ at the ℓ-th time instance. Assuming that one bit error leads to a total feedback loss, the feedback error probability for user k for a given realization of the K users' uplink channels is

$$p_{\text{FE,k}} = 1 - \prod_{\ell \in \mathcal{S}_k} \left(1 - p_{\text{b}}(\pi_\ell(n))\right)^2, \quad (23)$$

where $\mathcal{S}_k$ is the set including the indices of the feedback phase channel uses ℓ during where user $k = \pi_\ell(n)$ relays back one QPSK feedback symbol (2 feedback bits) to the base station.

4. SYSTEM FEEDBACK DESIGN

Finding the optimum ordering of the K users into T_{F} groups with at most M users per group such that each user is a member of exactly $\frac{B}{2}$ groups implies a search over all possible allocations. Such a search is not feasible since for moderate values of K and T_{F} there is a very large number of possibilities. To this end, we propose to use a suboptimal selection scheme in order to allocate the K users into the T_{F} groups.

Step 1) *Allocation Initialization:* We first initialize the possible position of the users in the feedback phase. For this purpose, we define the index $P_{n,\ell}$

$$P_{n,\ell} = \left\lceil \frac{T_{\text{F}}(n-1) + \ell}{\frac{B}{2}} \right\rceil, \quad (24)$$

for $T_{\text{F}}(n-1) + \ell \leq K\frac{B}{2}$ with $1 \leq \ell \leq T_{\text{F}}$ and $1 \leq n \leq M$, where B is given by (14). With such a definition, $P_{n,\ell} \in \{1, \dots, K\}$. Each of the K possible index values appears $\frac{B}{2}$ times for distinct values of ℓ. In addition, let us define $N_\ell = \left\lfloor \frac{K\frac{B}{2} - \ell}{T_{\text{F}}} + 1 \right\rfloor$, which represents the number of users in each group ℓ. We also initialize the set of users and the groups

$$\begin{aligned} \mathcal{T} &= \{1, 2, \dots, K\} \\ \mathcal{G}_\ell &= \{\emptyset\} \quad \text{for} \quad \ell = 1, \dots, T_{\text{F}}. \end{aligned}$$

For example for $M = 4$, $K = 6$, $T_{\text{F}} = 5$, and with $B = 6$ from (14) we would have

$$\begin{bmatrix} P_{1,1} & P_{1,2} & P_{1,3} & P_{1,4} & P_{1,5} \\ P_{2,1} & P_{2,2} & P_{2,3} & P_{2,4} & P_{2,5} \\ P_{3,1} & P_{3,2} & P_{3,3} & P_{3,4} & P_{3,5} \\ P_{4,1} & P_{4,2} & P_{4,3} & & \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 \\ 2 & 3 & 3 & 3 & 4 \\ 4 & 4 & 5 & 5 & 5 \\ 6 & 6 & 6 & & \end{bmatrix}, \quad (25)$$

with $N_1 = N_2 = N_3 = 4$ and $N_4 = N_5 = 3$.

Step 2) *Selecting First User:* The first user we allocate is the user with the smallest estimated channel norm, i.e.

$$\pi_1(1) = \underset{k \in \{1, \dots, K\}}{\operatorname{argmin}} \|\hat{\boldsymbol{h}}_k\|_2^2.$$

This user is allocated to the first group in the feedback phase:

$$\mathcal{G}_1 = \{\pi_1(1)\}.$$

In addition,

$$\left.\begin{array}{c} \pi_{\ell'}(n') = \pi_1(1) \\ \mathcal{G}_{\ell'} = \{\pi_1(1)\} \end{array}\right\} \quad \forall n', \forall \ell' \quad \text{where} \quad P_{n',\ell'} = P_{1,1},$$

such that user $\pi_1(1)$ is assigned to $\frac{B}{2}$ distinct groups. We update the set of remaining users $\mathcal{T}$:

$$\begin{aligned} \mathcal{T} &= \{k \in \mathcal{T}, k \neq \pi_1(1)\} \\ \ell &= 1, \\ n &= 2. \end{aligned}$$

Step 3) *Adding a user:* The next user $\pi_\ell(n)$ is chosen as follows. For each $k \in \mathcal{T}$, we set $\pi_\ell(n) = k$, which implies also

$$\pi_{\ell'}(n') = k \quad \forall n', \forall \ell' \quad \text{where} \quad P_{n',\ell'} = P_{n,\ell}.$$

With the resulting allocation for each $k \in \mathcal{T}$, we compute the average bit error probability over all the groups, where the bit error probability for a given user in a group is given as (22). Out of all the users in $\mathcal{T}$, we select $\pi_\ell(n)$ as the user which minimizes the average bit error probability. Thus, we have

$$\left.\begin{array}{c} \pi_{\ell'}(n') = \pi_\ell(n) \\ \mathcal{G}_{\ell'} = \mathcal{G}_{\ell'} \cup \{\pi_\ell(n)\} \end{array}\right\} \quad \forall n', \forall \ell' \quad \text{where} \quad P_{n',\ell'} = P_{n,\ell}$$

$$\mathcal{T} = \{k \in \mathcal{T}, k \neq \pi_\ell(n)\}.$$

Step 4) *Next iteration or stop:* If $\mathcal{T} = \{\emptyset\}$, we finish the algorithm. If this is not the case, we proceed to allocate the next user. If $N_\ell = n$, we have that $|\mathcal{G}_\ell| = N_\ell$ and we move to the next available position in the next group, i.e.

$$\begin{aligned} \ell &\leftarrow \ell + 1 \\ n &\leftarrow |\mathcal{G}_\ell| + 1, \end{aligned}$$

otherwise simply update

$$n \leftarrow n + 1.$$

Afterwards we go back to Step 3) to allocate the next user.

With this algorithm, we have that for the example resulting in (25), a possible allocation of the users to the groups could be as follows

$$\begin{bmatrix} \pi_1(1) & \pi_2(1) & \pi_3(1) & \pi_4(1) & \pi_5(1) \\ \pi_1(2) & \pi_2(2) & \pi_3(2) & \pi_4(2) & \pi_5(2) \\ \pi_1(3) & \pi_2(3) & \pi_3(3) & \pi_4(3) & \pi_5(3) \\ \pi_1(4) & \pi_2(4) & \pi_3(4) & & \end{bmatrix} = \begin{bmatrix} 4 & 4 & 4 & 6 & 6 \\ 6 & 2 & 2 & 2 & 5 \\ 5 & 5 & 1 & 1 & 1 \\ 3 & 3 & 3 & & \end{bmatrix}$$

We would also have $\mathcal{G}_1 = \{4,6,5,3\}$, $\mathcal{G}_2 = \{4,2,5,3\}$, $\mathcal{G}_3 = \{4,2,1,3\}$, $\mathcal{G}_4 = \{6,2,1\}$, and $\mathcal{G}_5 = \{6,5,1\}$. The users in group $\mathcal{G}_\ell$ feedback one QPSK symbol during the ℓ-th time instance of the feedback phase and the base station detects the feedback with the receive filter given by (19). The users must be informed via the downlink at which time instances they should relay their feedback bits.

Let us now present some numerical results to evaluate the feedback design. We assume that $M = 4$, $T_\mathrm{F} = 30$, $\rho_\mathrm{UL} = \frac{P_\mathrm{UL}}{\sigma_v^2} = 2$ and 4. In Figure 2 we depict the bit error probability resulting from the previously discussed feedback design and ordering of users as a function of K averaged over 1000 realizations. Each user relays B feedback bits, where B is given as a function of K in (14). As a comparison we include the case of a random allocation of the users to the groups, since to the best of our knowledge the problem discussed here has not been addressed yet in the literature. It can clearly be seen how the user ordering can exploit multiuser diversity in order to minimize the bit error probability. With respect to the random allocation, the decrease of the bit error probability can be as large as 40% with the proposed design.

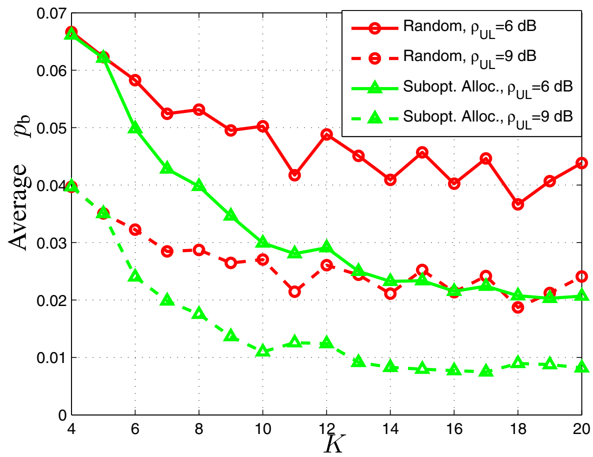

Fig. 2. Average bit error probability vs. K

In the previous example, we have considered a fixed feedback phase duration T_F with a varying number of users K and consequently, a varying number of feedback bits B for each value of K. We will now assume a limited feedback per user of $B = 10$ bits, such that T_F varies with K. With (14) and $M = 4$ we have that $T_\mathrm{F} = \left\lceil \frac{KB}{2M} \right\rceil = \left\lceil \frac{5K}{4} \right\rceil$. From Figure 3, we can also observe how the feedback error probability decreases as more users feed back their CSI, in contrast to the random allocation. Let us recall that this is achieved by allowing at most $M = 4$ users to feed back simultaneously.

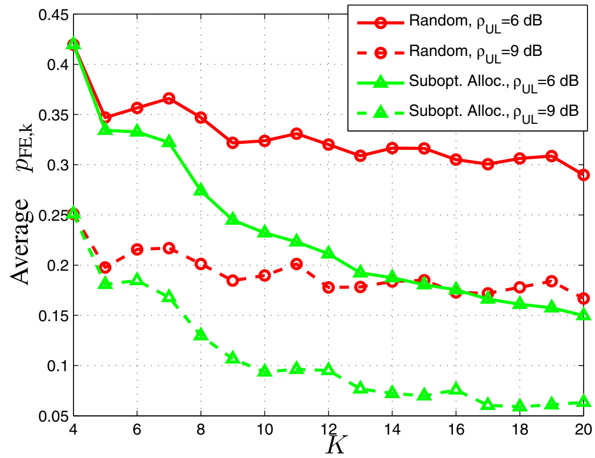

Fig. 3. Average feedback error probability vs. K

5. CONCLUSION

In this paper we have addressed the multiuser feedback design with multiple receive antennas by taking into account feedback errors. We discussed how multiple users can relay their feedback while also minimizing the feedback errors. We have considered that the base station can detect the feedback of several users simultaneously by employing receive beamforming based on the estimated uplink channels. The smallest feedback error would be achieved if only one user fed back at each time instance. However, this comes at the expense of a larger feedback phase. Hence, as future work it is interesting to investigate the tradeoff between feedback errors and the overhead due to the feedback.

6. REFERENCES

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