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A Subjective Logic Based Extensional Approach to Non-Monotonic Reasoning under Uncertainty and its Application to Visual Surveillance

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Abstract

Most of the automated and intelligent visual surveillance systems, so far, have focused on real-time observation and understanding abnormal human or object behaviour. The usual approach is building specific analytic algorithms for well-defined and fixed domains, (e.g. pedestrian detection and tracking, left baggage detection, intrusion detection, etc.), Thereby, the semantic space of the approach is bound to the results and capacity of used algorithms. When this kind of system is used in high-level forensic, such an approach is inherently too limited. This is due to the fact that modeling all situations at development time is very difficult. The difficulty arises not only from the limited vision analytic power on signal level vision information processing but also from the ambiguity inherent in interpreting complex situational context. Human cognition usually accomplishes the interpretation by collecting visual evidences and combining such evidences with contextual knowledge. Such a human activity includes derivation, modification and retraction of different semantic conclusions upon arrival of new information. Thus, it can be regarded as *non-monotonic* reasoning

Bearing such a human ability in mind, the presented work focuses on designing a systematic approach and methodical support for a more intelligent forensic visual surveillance system. Especially, we regard traditional vision analytics as perceptional sources of evidences and focus more on the way of manipulating such evidences to derive more complex semantics of interest. We approach the manipulation from the *epistemological* stance, that is a theory aimed at targeting notions such as knowledge, belief, awareness and propositional attitudes, by means of logical and semantical tools. First, we propose the use of *logic programming* for the basis of knowledge representation and reasoning. Second, we propose

the use of *subjective logic* to capture uncertain epistemic belief about knowledge. Third, upon the proposed subjective logic extended logic programming framework, we present an inference scheme for so-called default reasoning, that can draw plausible reasoning results under incomplete and contradictory information. Forth, we present an approach to model uncertain and ambiguous contextual rules, based on so-called reputation operator in subjective logic. Fifth, we address a problem that usual logic-based frameworks do not allow bidirectional interpretation of a rule. Based on so-called deduction and abduction operators in subjective logic we show that bidirectional reasoning can be done in our framework. Finally, by the use of abductive logic programming with subjective logic, we demonstrate a reasoning framework that can answer most feasible hypotheses to the best of current knowledge-base upon a set of observations that is collected by user. Each of fore-mentioned aspects are presented with case studies from typical public area scenes with visual surveillance data.

Contents

1 Introduction

1.1 High-level Semantic Reasoning in Visual Surveillance and Epistemology

In the security sector of industry, visual surveillance is one of the oldest and most widespread technologies as cameras are quite informative and relatively inexpensive sensors. Since the emergence of the first generation of digital recording technology, it is undergoing a digital revolution, such as compressing video footages, archiving digitized contents, integrating hundreds of cameras, etc. Consequently, as it is getting digitized, we also get huge amount of video information that can not be processed only by human security agents. Therefore, automatic understanding of semantics in surveillance video contents is one of the keys to success. However, this is an extremely difficult task for visual surveillance systems[1]. To address this problem, there has been active research focus and paradigm shift for the last decades as shown in Figure 1.1.

Considering that one of the primary objectives of visual surveillance is to prevent unexpected and potentially dangerous situations for immediate intervention, most of the automated and intelligent visual surveillance systems, so far, have focused on real-time observation and understanding of unusual or dangerous activities. The usual approach is

[1]This is the same for other visual information systems (VIS), that deal with usual broadcasting or consumer contents as more and more visual material (*i.e., images and videos*) is produced [65]. Major researches in VIS have been driven in terms of efficient access for a massive content database, by means of (*semi-*) automated semantic annotation, indexing, content-based retrieval and automated video abstraction (*i.e., shot/scene summarization*), etc [52, 47]. Especially, automated semantic annotation in VIS treats an even wider semantic lexicon of concepts in various content domains, thereby making it even more challenging [162, 16, 25]. A work introduced by Schreer and Izquierdo et al. [147] is one that addressed automatic semantic annotation in the RUSHES [5] european project. In this dissertation, however, we will limit our scope of discussion to visual surveillance contents.

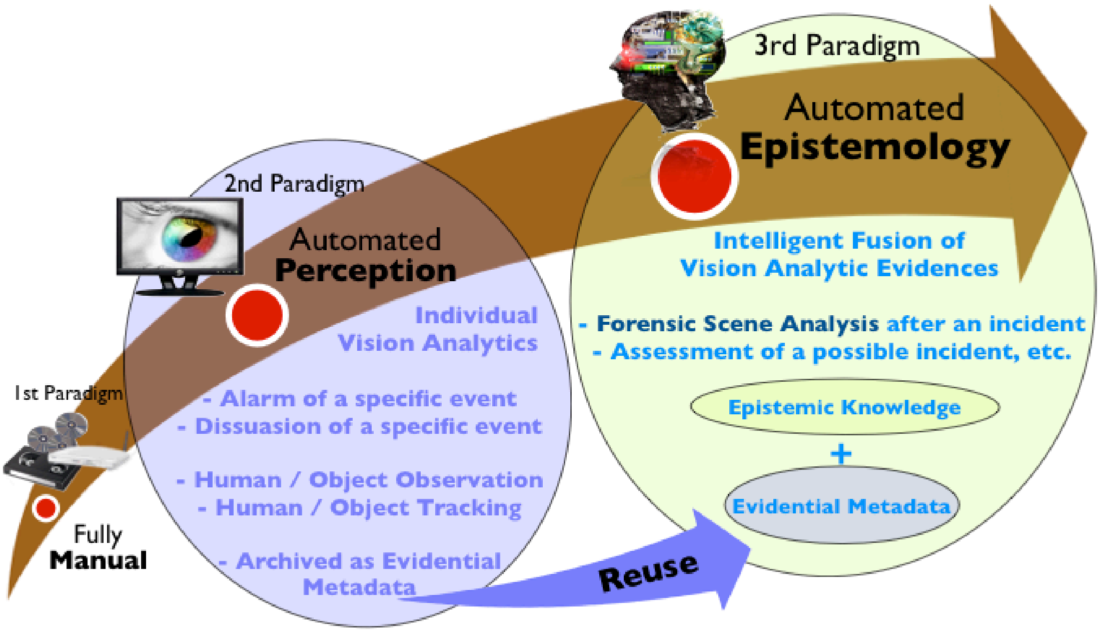


Figure 1.1: Research Spectrum of Intelligent Semantic Analysis in Visual Surveillance and Paradigm Shift.

building specific analytic algorithms for well-defined and fixed domains (e.g. pedestrian detection and tracking, left baggage detection, intrusion detection, etc). Such analytics usually model activities or events as a sequence of a number of *'states'* that are related to visual features (such *'state model'* based approaches are also referred to as *'intensional approach'* [130]). Therefore, in this paradigm the focus is more on *'automated perception'* of specific events as depicted in Figure 1.1 - *'2nd Paradigm'*. Consequently, due to the difficulty of modeling all situations at development time, the semantic space is inherently too limited to the results and capacity of used algorithms.

Following the September 2001 attacks on the United States [134] and then those in London in 2005 [10], it was proven that the primary objective to prevent all potentially possible threats is not always achievable [148, 74] even in cities where a huge number of surveillance cameras was deployed (London is the city the most often cited for the number of cameras deployed in its streets. [74]). Naturally, forensic use of visual surveillance in an intelligent way is gaining increasing attention, not only for real-time observation but also for post incident investigation. Such investigation usually deals with propositional semantic assumptions or queries to be investigated after an incident. However, analyzing such higher level semantics is not a trivial task. The difficulty arises not only from the ambiguity inherent in visual information and limited vision analytic power but also from the *'complex plot'* [148] implied in video scenes in case of planned or intended events.

Humans usually accomplish such investigation by collecting visual evidences and combining the evidences with contextual knowledge. This can be regarded as *'cognitive'* human activity that deal with fusion of *'propositional context knowledge'* and *'belief'* on it. In that sense, it is closely related to *'epistemology'* [2] that is a theory aimed at targeting notions such as knowledge, belief, awareness and propositional attitudes, by means of logical and semantical tools [137, 138]. By this aspect, we advocate that high level semantic analysis of visual surveillance can be regarded as *'epistemic reasoning'* upon available evidences and contextual knowledge as depicted in Figure 1.1 - *'3rd Paradigm'*.

Although it is undergoing the paradigm shift, relatively little work from the *'automated epistemic reasoning'* stance has been done. This dissertation is intended to address this void in semantic analysis research. Thus, the main objectives of this dissertation is as follows.

- Designing a systematic approach for reuse of metadata acquired from vision analytics.
- Introducing a methodical support for epistemic reasoning under incomplete and uncertain knowledge representation and (vision analytic) metadata.
- Studying the feasibility of the proposed systematic approach and methodical support for higher level semantic reasoning scenarios in visual surveillance.

In visual surveillance systems, metadata acquired from vision analytic modules can

[2] Human *'cognition'* consists of many aspects such as psychology, neuroscience, logics, linguistics, sociology, etc., and *'cognitive science'* is the study on these aspects. While *'artificial intelligence'* aims to model some of these aspects for a machine. Among these aspects, *'epistemology'* addresses the question *'How do we know what we know?'* and deals with the means of production of knowledge, as well as skepticism about different knowledge claims. In artificial intelligence, therefore, it is closely related to logics, automated judgement of belief, etc. For more detailed overview, refer to [116, 138, 137].

be regarded as symbolized visual evidences obtained by artificial vision sensors from the 2nd paradigm view. Then, such evidential metadata should be epistemically fused by contextual knowledge. The contextual semantic knowledge can be formal representations in a set of semantic rules, constraints and relations, i.e., in particular, this is the part that has a large degree of overlap between how humans describe what constitutes a situational concept and how it is defined within contextual knowledge modeling formalisms. In other words, recognizing certain high level contextual semantic concept as it occurs becomes a problem of *'explaining'* the observation using the available semantic knowledge. Therefore, the higher level semantic models should contain modeling approaches that do not just aim to define the entire *'state'* space of the event domain as in *'state model (intensional)'* approaches. Instead, we advocate that it should rather enable human-like *'epistemic reasoning'* that can offer a formal way how we derive and guarantee the target semantics of interest. In particular, in such circumstance, knowledge should be able to be defined as a *'modular'* knowledge segments (such knowledge representation guaranteeing modularity of partial knowledge segments is referred to as *'extensional approach'* [130]). Regarding these objectives several fundamental questions can be posed:

- What are the semantic queries that can be answered based on epistemic reasoning under incomplete and restricted evidences ?
- How can one make the *'sensing (automated perception)'* result of available vision analytic modules reusable ?
- Where does the semantic knowledge come from ? How can one represent semantic knowledge in a machine understandable way ? What is the proper formalism for representing such a conceptual knowledge ?
- How humans assess propositional assumptions and contextual concepts along the given evidences and contextual knowledge ? For example, humans tend to assess a proposition in a vague manner such as *'strongly certain'*, *'less certain'*, *'seem to be true but quite not*

sure', etc. How such an epistemic status can be modeled ?

- Can one guarantee that a given contextual knowledge is always sound and complete ? Humans have a capability on reasoning under inconsistent and incomplete knowledge. What are the epistemic aspects that can arise under such a circumstance and how can it be modeled in a machine interpretable way ?
- The results from vision analytic modules usually come with uncertainty, and their performance may vary according to its internal analytic mechanisms and training data. How can one take into account such uncertainty at the time of reuse ?
- What are the practical and pragmatic building blocks given currently available software components in terms of building a system architecture to enable such a reasoning ?

This dissertation addresses some of these questions by focussing on designing a systematic approach and methodical support for a more intelligent higher level reasoning capability.

1.2 Challenges and Requirements

This subsection details characteristics of visual surveillance and derives challenges and requirements in terms of forensic sense of high level semantic queries (refer to [49, 74] for a more general view of challenges in visual surveillance related researches.).

1.2.1 Characteristics of Visual Surveillance Systems

In the sense that *'automated epistemic reasoning'* rely on currently available vision analytic powers and existing infrastructures, it is important to know the characteristics of visual surveillance systems. The environmental nature of a surveillance system can be characterized as follows.

1. Environment : Distributed

2. Processable Semantics :

- Pre-manual annotation of background objects for fixed camera setting
- Realtime processable objects and some of pre-defined atomic behaviors

3. Types of data :

- Alarms, objects and events representable in symbolic form (e.g. text format, etc)
- Various low-level feature vectors extractable from ROIs

4. Size of data : Large scale

5. Access to data : Very rare

6. Semantic complexity of query : Very high and vary from specific to abstract and vague

7. Degree of automation : User interaction in the loop

8. Performance metrics :

- Acceptable accuracy, robustness and response time

1.2.2 Episodic Case Study

Bearing such an environmental characteristics in mind, a top-down approach is taken to derive more concrete requirements on the components and the system itself for complex queries. We analyze real events, e.g. in which the authorites released CCTV footage after closing investigation on the subway bombing attacks in London on 7th july 2005. A question roadmap [10] shows the questions dealt by London Police such as 'Who is the suspect?', 'Was it done by a single person or by a group?', 'Why did they do it?', 'How did they do it?', etc. To be machine supportable, it is important to take into account the semantic granularity of the query. In the example, the *'why'* or *'how'* questions seem much harder than *'who'* questions because the more knowledge about the situation is required. Consequently, queries should have reasonable semantic level to be interpreted by machine. In terms of machine process-ability, the semantic complexity of query is desired to be compositional, that can be tied with other atomic semantic meanings with visual cues and situational context. Figure 1.2 (left) shows steps gathering situational knowledge and how the acquired knowledge influences the iterative reasoning especially for forensic sense of search and retrieval. Figure 1.2 (right) shows one sample of the released footage. Although

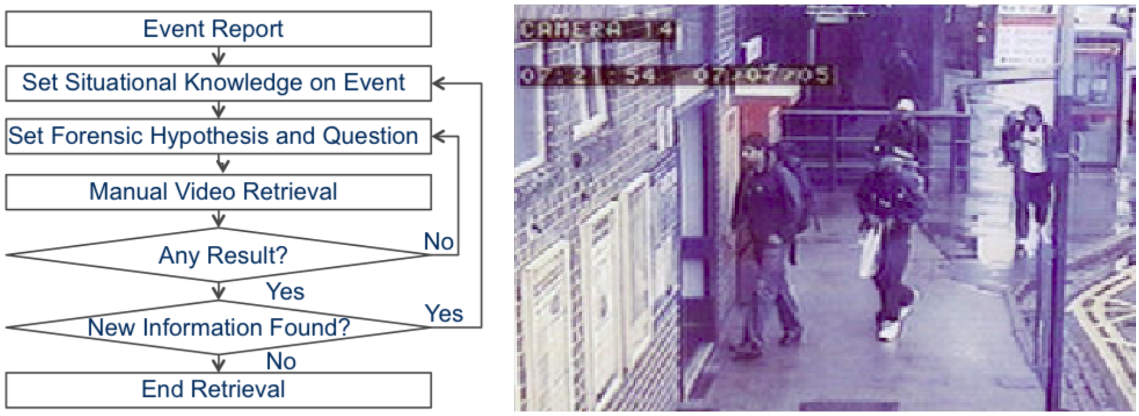


Figure 1.2: Conceptual Workflow of Forensic Reasoning in Case of Video Retrieval (left) and a Released Retrieved-Footage (right) [10].

this sample seems very ordinary, the people in the scene were regarded as suspects of a terror when considered in the situational context. Therefore, interpreting the semantic granularity implied in a question and understanding the situational context knowledge is important to fulfill the forensic reasoning task.

1.2.3 Query Scenario

For a more concrete discussion, we assume a distributed camera network and an unexpected incident not directly captured by a camera due to the sparseness of the network and lack of sufficient vision analytic power. Assuming some clues, let us consider following queries.

- Atomic level of granularity for semantic queries;

  Conjunctive queries that can be seen as existence of some of specific atomic metadata (e.g. shown, detected, etc).

  - *Q1 'scenes in which a person and a red vehicle are shown'*
  - *Q2 'scenes in which a person shown on a given photo is in'*

- Medium level of granularity for semantic queries;

  Compositional queries having relational semantics that may require more contextual information and that may require iterative and active collection of contextual information. (e.g. group of, passing by, carrying, talking to, witness of, theft of, etc).

- *Q3 'group of four people passing by a telephone booth'*
- *Q4 'scenes in which a person is carrying an oil box around the place of the reported incident'*
- *Q5 'scenes one of selected person, among retrieved after Q4, is talking to somebody'*
- *Q6 'probable witnesses of a selected person after Q4'*
- *Q7 'who is the suspect of the event?'*
- *Q8 'who is the most probable witness of the suspect of the event?'*
- *Q9 'scenes in which a person thieve a bag of other persons'*

• Large level of granularity for semantic queries;

Queries that require explanations of an observed or even semantically reasoned event.

- *Q10 'why did the woman detected in a robby suddenly went to outside?'*
- *Q11 'how did the guy reached at the place of incident?'*
- *Q12 'why did he thieved the bag?'*

1.2.4 Requirements - Non-monotonic Reasoning under Uncertainty

In the following, by considering above query scenarios, the most important requirements towards target high level semantic reasoning capability and systems are described.

• **Metadata Representation and Reuse** : it is necessary to exploit as many existing semantic understanding / extraction power and intermediate results as possible to maximize the potential utilization for later query and reasoning processing. As shown in Section 1.2.1 - 2, manual annotation could be also used to denote the concept *'telephone booth'* in *Q3*. Similarly, results from automatic human detection algorithm could be exploited to serve as atomic ingredient for processing more complex concepts such as *'group of people'*, *'witness'*, *'suspect'* and *'theft'* in *Q1-Q9*. This should include low-level features as well, as shown in Section 1.2.1 - 3. Thus, flexible machine-readable means like a suited metadata representation model is required to archive the processing result.

- **Semantic Knowledge Representation** : to cope with the *'complex plot'* at reasoning time, the system requires an efficient way to describe contextual knowledge about scenes. Unlike in the deterministic *'state-model (intensional)'* based approach [130], here the reasoning target is not given at development time. Furthermore, a given problem can not be solved directly by a known set of steps (see *Q6-Q8, Q10-Q12*) and the input data is *'incomplete'* because the available results generated by the analytics can not be fully pre-determined due to variations in the system configuration or settings of the employed analytics. However, considering the complexity of possible semantic knowledge *'model-free'* approach is also not possible. Thus, we rather need a more flexible *'extensional'* semantic inference mechanism that allows *'modularity'* of each knowledge segments [130].

- **Uncertainty Representation and Attachment to Metadata** : usually, intermediate metadata comes with *'uncertainty'* (e.g. trust worthiness of results and errors in the generation of metadata such as false alarms). As mentioned in Section 1.2.1 - 8 this will influence the acceptable accuracy and robustness for a semantically complex queries. Thus, a mechanism is required for dealing with the varying quality to the retrieved results, i.e. to cope with the *'uncertain'* nature of intermediate results captured as metadata from the analytic side.

- **Epistemic Uncertainty Representation and Belief Revision** : we can not guarantee the *'soundness'* and *'completeness'* of given contextual knowledge. There are *'uncertainty'* in knowledge itself. For example, there can be often situational *'contradictions'*, *'counter examples'* or *'stronger knowledge segments'* against given knowledge segments (see *Q7-Q9*). Therefore, we need a mean to assess *'epistemic belief'* to attach to given knowledge segments. Furthermore, derived reasoning results should be able to be revised upon arrival of new evidential data or contextual knowledge. Such capability is often referred to as *'belief revision'* [138]. Especially, the handling of contradictory information is referred to as *'default reasoning'* [143]. Thus, the reasoning capability should cover *'belief revision'* and *'default reasoning'*.

- **Epistemic Meta Reasoning and Abduction** : in the case of highly abstract and vague queries such as *Q10-Q12*, it requires a *'meta-reasoning'* (i.e. reasoning about reasoning [45]) power that can even able to reason about the query itself in an iterative manner. The system should be able to set *'epistemic hypothesis'* by itself and assess if the hypothesis satisfies. Especially, such a reasoning explaining a given observation is known as *'abduction'* [150] (note that, especially, more complicated handling of this aspect is closely related to *'planning'* in the view of artificial intelligent). Therefore, the desired reasoning power should cover *'abduction'* capability.

The requirements for *'belief revision'*, *'default reasoning'*, *'meta-reasoning'* and *'abduction'* can be categorized under the term of *'non-monotonic reasoning'* [35]. Considering *'uncertainty'* both in intermediate metadata and a knowledge segment itself, it can be denoted as *'non-monotonic reasoning under uncertainty'*. Further, considering that the knowledge modeling is preferred to be done in an *'extensional'* way, the requirements can be summarized as *'extensional approach to non-monotonic reasoning under uncertainty'*.

1.3 Proposed Approach

In this section, we present our approach to the requirements designated in the previous section.

First, one of the most important aspects throughout above derived requirements is the representation of uncertainty in terms of both *'uncertainty in vision analytic metadata'* and *'epistemic uncertainty in knowledge representation'*. We employ subjective logic theory [93] that is a relatively new branch of probabilistic logic which comes with a rich set of logical operators. Subjective logic also provides an intuitive representation of human like epistemic belief in a model called *'subjective opinion'*. A subjective opinion not only allows to describe one's belief based on degree of truth or falsity but also allows explicit representation of ignorance about the degree of truth. This aspect is inherited from Dempster Shafer's belief theory [149]. But it also provides a mathematical mapping between the opinion representation and 2nd order Bayesian especially, via a Beta function representation [20]. The intuition behind is that the uncertainty representation of subjective logic covers traditional probability so that we can directly encode traditional bayesian based uncertainty calculation results coming from vision analytic modules (in the sense that most of *'state model'* based approaches rely on Bayesian approach throughout model learning and recognition phase). This aspect matches to the requirements of uncertainty attachment to metadata. Indeed, including ignorance on trustworthiness, we believe that more intuitive and human like *'epistemic status'* can be represented. This aspect matches to the requirement of *'epistemic uncertainty representation'* about a given knowledge segments.

Second, for the semantic knowledge modeling and representation, we exploit logic programing approach. We do not believe that we can derive a *'whole-model'* that can cover most of semantic situation. We also do not believe that a *'model-free'* approach is possible because introducing a learning mechanism on semantic knowledge out of visual surveillance data is again extremely difficult task. Therefore, our choice is to bestow more degree of freedom on modeling semantic knowledge. To achieve this purpose, the most important aspect is that the system need to guarantee a *'modularity'* so that any domain engineers can just concentrate on their partial knowledge segment without considering whole knowledge. Another aspect is that the knowledge representation formalism should provide a mean to describe many types of relations. Traditional *'state-model'* based approaches can be also seen in a logic sense, in that a pair of nodes connected with edge can be seen as *'if node1 then node2'* manner. However, this level of conditional description remains in *'propositional logic'* that only offers one relation *'influence'*. Unlike propositional logic, logic programming usually offers predicate logic that can build various relations. Although there are other technologies for knowledge representation such as *'entity-relation diagram'*, *'semantic network'*, *'ontology and semantic web'* they tend to rely on schematic approach that more fit to data archiving aspect. Therefore, we adopt semantic web and database approach for intermediate metadata archival scheme.

Finally and most importantly, to achieve *'nonmonotonic reasoning'* capability, we introduce a principled reasoning scheme especially for *'default reasoning'*, *'vague propositional rule'* modeling and *'abductive explanation of observation'*. The reasoning mechanism is realized based on logic programming extended with subjective logic based operations on epistemic beliefs (subjective opinions). Concerning uncertainty propagation, we also attempts to provide the ability of interpreting conditional knowledges in a bijective manner. In this way, we give more flexibility and on demand way of using both *'intensional'* and *'extensional'* reasoning. To enable *'default reasoning'*, we introduce principled scheme of handling *'contradictory information'* so that it can derive plausible reasoning result. For the vague propositional rule definition, we introduce subjective reputation function for evaluating trustworthiness of a given proposition. For the semantic explanation scenario, we take both *'deductive'* and *'abductive'* logic programming. Especially, we take both forward-chaining and backward-chaining logic programming engines for *'meta-reasoning'*.

1.4 Key Contributions

We now summarize the key contributions of this dissertation tackling the challenges of high level epistemic semantic reasoning for visual surveillance.

- The idea to combine logic programming and subjective logic theory for flexible knowledge representation, uncertainty handling and epistemic reasoning.
- The study on system architecture for reuse of vision analytic metadata comparing traditional database, ontology, logic programming based technologies.
- The design of methodical support for contradictory information handling using subjective logic theory and its comprehensive comparison with L-Fuzzy set based approaches.
- The idea of using reputation concept for modeling vague propositional knowledge.
- The idea of using deduction and abduction operators of subjective logic for enabling bidirectional interpretation of a conditional proposition.

- The idea of layering abductive and deductive logic programming for logical abduction based diagnostic scene explanation given collected observations.

1.5 Organization of the Thesis

The rest of this dissertation is organized as follows.

In the following Chapter 2, we give a brief historical review of epistemology and logic both in terms of philosophical view and artificial intelligence. Then we review relevant literatures to the problems of automatic interpretation of semantic occurrence in video.

In Chapter 3, we present preliminaries fundamental to understand this dissertation. This will cover subjective logic and logic programming. Except the fundamentals explained in the preliminaries (Chapter 3), the rest of chapters are organized to be self-contained.

In Chapter 4, we present our framework for representing vision analytic metadata, semantic knowledge representation and extension of logic programming component with subjective logic. We discuss the software components, data processing pipeline and system architecture design. We present some of case studies of forensic query scenarios. We conclude the chapter with a discussion of experimental evaluation on query performance on different software component settings.

In Chapter 5, we introduce the problem of handling contradictory information in extensional system especially in terms of so-called default reasoning. We present our approach to expand, refine, and translate epistemic status on contradiction by comparing subjective opinion space with square bilattice. We then models a default reasoning scheme adopting the idea exploited in bilattice system. There, we also compare our approach to L-fuzzy set based approaches. We then presents a case study on handling contradictory information in visual surveillance scenario. Finally, we conclude with discussions especially on the comparison between proposed approach and L-fuzzy set based approaches such as square bilattice.

In Chapter 6 we present our research on vague propositional knowledge modeling and handling. There, we present the specific idea on evaluating a proposition with a reputation function of subjective logic. We also describe a case study on reasoning under vague propositional rules. We conclude the chapter with a discussion of the experimental evaluation of these techniques.

Chapter 7 describes our work on hybrid knowledge modeling and reasoning. First, we contrast intensional and extensional way of interpretation of given conditional rules. Then, we present the idea how to enable intensional handling of conditional rules in an extensional framework using subjective logic. Finally, this chapter presents, in detail, the case study performed for evaluating a public scene based reasoning.

In Chapter 8, we present our framework for logical abduction based scene explanation under collected observations. There, we present the idea of layering deduction on query itself and abduction to get final answer. This chapter also shows how the final answers can be evaluated in terms of default reasoning. Finally, we conclude the chapter with discussions on open issues and future work.

In Chapter 9, we present the conclusions of our work with discussions and summary of contributions and some future research directions.

2 Prior Art and Related Work

In this chapter, we will review related work ranging from a philosophical background of epistemology to knowledge representation researches in artificial intelligence. Thereafter, it follows with reviews of prior art ranging from a semantic multimedia retrieval to automatic interpretation of semantic occurrences in video.

2.1 Epistemology - A Philosophical Background

Epistemology is the branch of philosophy concerned with the nature and scope of knowledge [11, 174]. In the view of philosophical history, since Aristotle, Platon to Descartes and Kant, the majority of philosophy was metaphysics to explain the fundamental nature of being and the world [105]. Since Aristole, epistemology was dealt as a sub topic of metaphysics in the focus on structuring existence of entities in the world. Especially, *'ontology'* received much interests. Ontology is the theory of objects in terms of the criteria which allow one to distinguish between different types of objects and the relations, dependencies, and properties through which they may be described. Since Descartes, epistemology has paid more attention on how humans think and how humans can sure their belief on knowledge. Addressing this problem, we face with following fundamental question, "Is the limitation of one's cognition bounded via sensory experience ? or is it bounded via innated humans rationality ?". Through 17 to 18 centuries, we bestowed the name *'Empiricism'* on the first category and *'Rationalism'* on the other category. These two major trends are compromised by Immanuel Kant in his remarkable work "Die Kritik der reinen Vernunft". Kant thought humans do not know as knowledge exists but believe a certain knowledge as humans know.

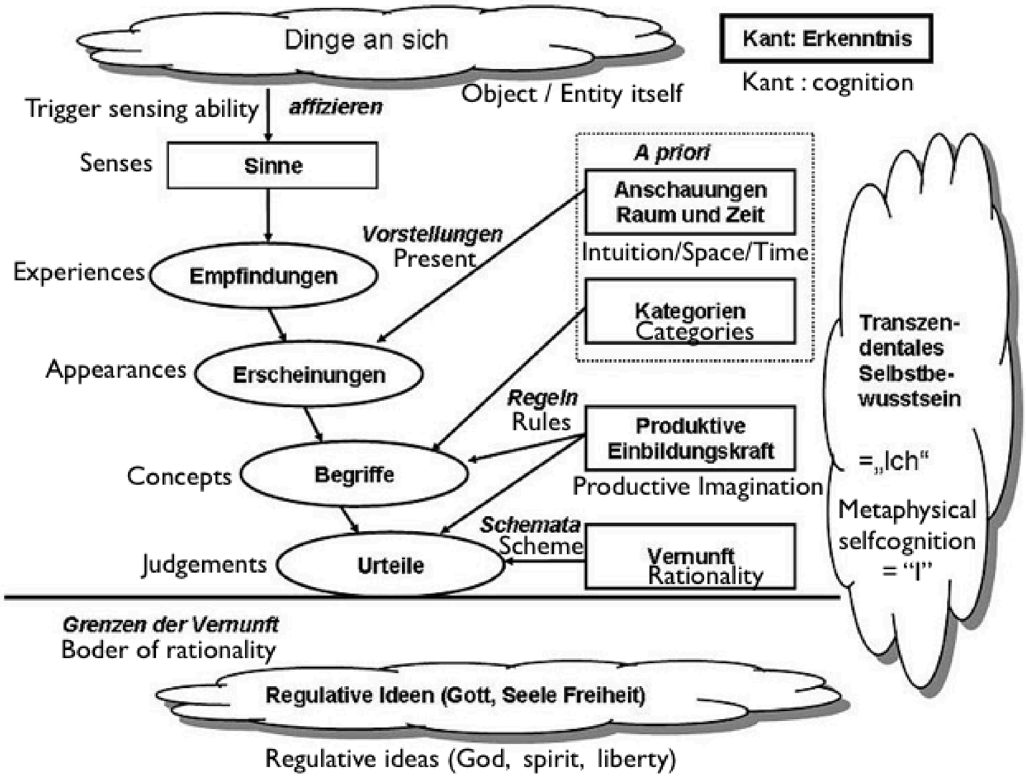


Figure 2.1: Kant's Epistemological Structure of Thinking [4].

Therefore, to him truth is something that we need to find out of structure of human thinking [107]. This idea is depicted in Figure 2.1. In terms of rationalism, he introduces *'a priori innate human rationality'* that serves categories on thinking as posed follows:

- Quantity (Quantität) :

  Unity (Allheit), Plurality (Vielheit), Unity (Einheit)

- Quality (Qualität) :

  Reality (Realität), Negation (Negation), Limitation (Limitation)

- Relation (Relation) :

  Subsistence (Substanz), Causality (Kausalität), Community (Gemeinschaft)

- Modality (Modalität) :

  Possibility (Möglichkeit), Existence (Wirklichkeit), Necessity (Notwerdigkeit)

With the help of such priory categories, he believed that humans can fuse their empirical sensing ability within these categories to extend their knowledge. This aspect is understood as *'judgement'* and *'propositional rules'* are the representation of judgement. Kant also introduces 12 categories of judgement as posed follows :

- Quantity (Quantität) :
  - Universal (allgemeine Urteil) ; every S is P
  - Particular (besondere Urteil) ; some S is P
  - Singular (einzelne Urteil) ; this S is P

- Quality (Qualität) :
  - Affirmative (bejahende Urteil) ; S is P
  - Negation (verneinde Urteil) ; S is not P
  - Infinite (unendliche Urteil) ; S is non P

- Relation (Relation) :
  - Categorical (kategorische Urteil) ; S is P
  - Hypothetical (hypothetische Urteil) ; if A is B then C is D
  - Disjunctive (disjunktive Urteil) ; A is either B or C

- Modality (Modalität) :
  - Problematical (problematische Urteil) ; S seems P
  - Assertoric (assertorische Urteil) ; S is P
  - Apodictic (apodiktische Urteil) ; S must be P

Humans in everyday life face with various situations and events. Then humans think various ways to solve, analyze the facing situations and events. Of course the ways would be very varying, however, as long as their thinking is related on *'judgement'*, it falls into above categories. Categorical judgement (Kategorische Urteil), among others, is the most basic one and every 12 types can be transformed into the categorical judgement except *'Hypothetical'* and *'Disjunctive'*. When the categorical judgment is considered with *'universal'*, *'particular'*, *'affirmative'* and *'negation'*, following four types of basic judgement is possible.

- Universal Affirmative : Every S is P
- Universal Negation : Every S is not P
- Particular Affirmative : Some S is P
- Particular Negation : Some S is not P

Based on above mentioned categories, *'inference'* is the process of deriving *'new judgement'* out of known judgements (namely, *'therefore, new judgement'*). In inference, already known judgements are called *'premises'*. In case there is only one *'premise'* we call it *'direct inference'* and in case of handling *'multiple premises'*, we call it *'indirect inference'*. Indirect inference can fall into following categories.

- Deductive Inference : Deductive inference attempts to show that a conclusion necessarily follows from a set of premises or hypotheses. A deductive argument is valid if the conclusion does follow necessarily from the premises, i.e., if the conclusion must be true provided that the premises are true. A deductive argument is sound if it is valid and its premises are true. Deductive arguments are valid or invalid, sound or unsound, but are never false nor true. Deductive reasoning is a method of gaining knowledge. An example of a deductive argument: 1) All men are mortal, 2) Socrates is a man, 3) Therefore, Socrates is mortal. The first premise states that all objects classified as 'men' have the attribute 'mortal'. The second premise states that 'Socrates' is classified as a man - a member of the set 'men'. The conclusion states that 'Socrates' must be mortal because he inherits this attribute from his classification as a man.
- Inductive Inference : Inductive inference is a kind of empirical reasoning that constructs or evaluates inductive arguments. The premises of an inductive logical argument indicate some degree of support (inductive probability) for the conclusion but do not entail it;

that is, they suggest truth but do not ensure it. Induction is employed, for example, in the following argument: 1) Every life form we know of depends on liquid water to exist. 2.) All life depends on liquid water to exist. Inductive reasoning allows for the possibility that the conclusion is false, even where all of the premises are true. For example: 1) All of the swans we have seen are white, 2) All swans are white. Through many dictionaries inductive inference are also defined as reasoning that derives general principles from specific observations.

- Abductive Inference : Abductive inference is a kind of logical reasoning described by Charles Sanders Peirce as "guessing" [131]. The term refers to the process of arriving at an explanatory hypothesis. Peirce said that to abduce a hypothetical explanation a from an observed surprising circumstance b is to surmise that a may be true because then b would be a matter of course. Thus, to abduce a from b involves determining that a is sufficient (or nearly sufficient), but not necessary, for b. For example, the lawn is wet. But if it rained last night, then it would be unsurprising that the lawn is wet. Therefore, by abductive reasoning, it rained last night. (But note that Peirce did not remain convinced that a single logical form covers all abduction.) Peirce argues that good abductive reasoning from P to Q involves not simply a determination that, e.g., Q is sufficient for P, but also that Q is among the most economical explanations for P. Simplification and economy are what call for the 'leap' of abduction. There has been renewed interest in the subject of abduction in the fields of computer science and artificial intelligence research [111].

In the context of traditional computer vision research on semantic analysis, the majority of the work focusses on *'concept'* level as a result of automated perception. Although the importance of judgements and inference, little attempts have been paid to *'automated judgements and inference'*. In this dissertation, we especially pay more attention to the *'automated inference'* in terms of *'deduction'* and *'abduction'*. To cope with the quality property of the judgement, we also pay attention for the type of *'Particular (besondere*

Urteil)' judgement.

2.2 Knowledge Representation in Artificial Intelligence

In the previous section, we have reviewed that *'epistemology'* is related to *'philosophical ontology'* that tries to encode the relational structure of concepts which one can use to describe and reason about aspects of the world. Inheriting this tradition, today, formal logic also focusses on artificially structured languages and its syntax or semantics. Traditional knowledge representation models developed in the field of artificial intelligence are formal logics [146], semantic networks [141, 84], frames [118, 84] , Description Logic [24, 2] and semantic web [7, 48].

2.2.1 Formal Logics

Logics and semantic networks are widely accepted models for effective knowledge representation. Logics aim at emulating the laws of thought by providing a mechanism to represent statements about the world - the representation language - and a set of rules to deduce new statements from previous ones - the proof theory. The representation language is defined by its syntax and semantics, which specify the structure and the meaning of the statements, respectively. Different logics make different assumptions about what exists in the world (e.g. facts) and on the beliefs about the statements. The most basic logic is Propositional Logic [146]. Propositional logic declaratively deals with pieces of syntax correspond to facts via partial, disjunctive, negated information. Propositional logic is compositional in that meaning of $A \wedge B$ is derived from meaning of A and B. Traditional probability theory can be seen as a propositional logic in the sense that a conditional probability $P(B|A)$ can be considered as *'If B then A'*. Fuzzy logic [176] also deals facts that comes with degree of truth. However, propositional logic has very limited expressive power unlike natural language. The most widely used and understood logic is First-Order Logic (FOL) [146], also known as First-Order Predicate Logic (FOPL). Whereas propositional logic assumes the

Formalism	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true / false / (unknown)
First-order logic	facts, objects, relations	true / false / (unknown)
Temporal logic	facts, objects, relations, times	true / false / (unknown)
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

Table 2.1: Formal Logics and Their Ontological and Epistemological Commitments [146].

world contains facts therefore supports only one abstract relation *'influences'*, first-order logic (like natural language) assumes the world contains (1) Objects, e.g. people, houses, numbers, colors, baseball games, wars, etc. (2) Relations, e.g. red, round, prime, brother of, bigger than, part of, between, etc. (3) Functions, e.g, father of, best friend, one more than, plus, etc. For example, "Brother (Richard, John) $\wedge$ Brother (John, Richard)" means that "Richard is the brother of John and John is the brother of Richard"; $\forall$x King(x) $\implies$ Person(x)" means that "All kings are persons". Logics of various kinds and logical reasoning and representation languages such as Prolog and KL-ONE have been popular tools for knowledge modeling, for example in the definition of *'expert systems'*. The ontological and epistemological commitments of different formal logics are summarized in Figure 2.1.

2.2.2 Semantic Networks, Frames and Description Logic

A semantic network [141, 84] is a directed graph consisting of vertices that represent concepts and edges that encode semantic relations between them. Concepts can be arranged into taxonomic hierarchies and have associated properties (e.g. the state "Bill is a person" could be represented by the chain: Bill Clinton Node - Is-A Arc - Person Node). In spite of their simplicity and support for modular inheritance, semantic networks suffer from limited expressiveness, as they can not represent negation or disjunction, among others. Frames [118, 84] are closely related to semantic networks but represent knowledge in terms of hierarchies of frames containing named slots, together with rules such as type constraints, to define concepts and relationships. It is widely accepted that knowledge in the form of semantic networks, frames, and scripts can be expressed using logics such as first-order logic. Much recent work on knowledge modeling has focused on description logics [24, 2], which have evolved from semantic networks to provide a more rigorous definition of semantics in terms of a (typically restricted) form of first-order logic. Description logics usually provide a syntax that makes it easy to specify categories and perform inference tasks such as subsumption and classification.

2.2.3 The Semantic Web

The term ontology has recently undergone a strong revival largely due to the efforts of the semantic web community [7]. Ontologies are seen in a knowledge management sense as providing an important tool for the representation of semantic information and the automated processing of such information for applications such as data mining, retrieval, and automated discovery and utilization of services by autonomous software agents. The consensus definition of ontology in this context is as a *'formal, explicit specification of a shared conceptualization'* [75], hence the **focus** is on *'knowledge sharing'* and *'sufficiently formal representation'* to allow manipulation by computer.

As shown in Figure 2.2, the semantic web is based on a layered hierarchy in which ontologies provide the underpinning that enables metadata to be interpreted automatically [158, 14]. In addition to the XML and RDF standards for structured document annotation and resource description, attention is now focussed on the new framework for Web Ontology languages (OWL) [76]. It is now recognized that ontologies are the natural vehicle for knowledge representation and interchange at different levels of granularity and across different domains, hence they are to form a vital cornerstone of future generations of the internet.

While the proposed formalisms for the semantic web draw on a rich heritage of work in artificial intelligence and linguistics, they remain limited due to an explicit focus on the description of deterministic resources especially textual data. Many decades of research into knowledge engineering have shown the limitations of methodologies such as semantic

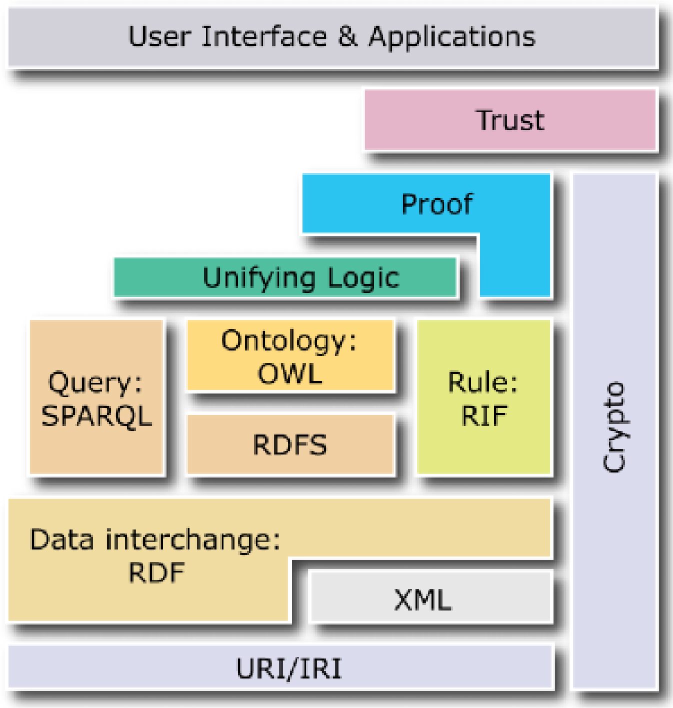


Figure 2.2: Semantic Web Architecture Layer Cake [14].

nets and description logics on which OWL is based. The World Wide Web Consortium (W3C) and the knowledge engineering community have yet to address the enormous challenge of reasoning about non-textual multimedia data and supporting uncertain knowledge representation. Some efforts have been paid to extend ontology language with uncertainty representation capability using for instance fuzzy logic [38, 160, 181]. However, there is no recommended languages for this purpose. So is remained at rule description and logic framework layer. Even though there is several language candidates for describing rules such as SWRL and RIF, they are still a working draft in the W3C and not yet a recommendation. Logic framework layer that is assumed to provides more epistemic answer for the question of why this piece of information is taken or appear to user ? again there is no technology specification at present for this layer. Proof layer is assumed to answer agents about the question of why they should believe the results. At present, again there is no technology recommended by W3C to this layer.

Overall, concerning our purpose of this dissertation, however the semantic web layered architecture gives us an insight on the component for epistemic reasoning framework. Therefore, we regards the layered architecture rather as thorough requirements for intelligent systems. At the level of metadata representation, there have been some efforts on building ontological annotation taxonomies for multimedia description and event description. VERL (Video Event Representation Language) [122, 32] provides taxonomies to annotate instances of the events and aceMedia project [28] aimed at providing semantic taxonomies for multimedia annotation. In spite of such efforts, it is not yet widely accepted for annotation and reasoning due to the lack of handling uncertainties and lack of technical support on logical reasoning and lack of technical means for proof layer as explained above. Indeed, the focus of such efforts are to share semantic annotation in an interoperable manner among systems, rather than focusing on the reasoning power.

2.3 Reasoning under Uncertainty in Artificial Intelligence

Given a knowledge representation, reasoning is the process to draw new knowledge out of observed patterns of facts and their attributes. Such reasoning methods can be categorized into two approaches *'extensional'* and *'intensional'*. These terms can be explained in terms of defining sets. It is well known that there are two fundamental ways of defining sets. First, a finite set may be defined by simply listing its members, as in the concept {Auckland, Hamilton, Wellington}. This is definition by *'extension'*. Alternatively, any set may be defined by stating the properties of its elements. This is definition by *'intension'*. Following this convention, a semantic reasoning is *'extensional'* if it is primarily concerned with enumerating the final result and not other aspects of computation such as the path traversed during computation, the amount of space or time required, etc [161]. In formal logic and its realization such as *'rule-based systems' (a.k.a production systems)*, its primary aim is to find *'extensions'* based on their internal *'search'* mechanism upon input facts.

Conversely, a semantic reasoning is *'intensional'* precisely when it is concerned with such internal details in the given knowledge model [161]. These two classifications are relative more than absolute, as the definition of *'final result'* and *'internal details'* may vary depending on the purpose of the semantic models and reasoning aspect.

While efforts in AI have often relied on formalisms such as mathematical formal logic as a means of representing knowledge and reasoning *'extensionally'* [130], much of the recent work in machine learning [62, 130] can be seen as providing *'intensional'* definitions of concepts in terms of classifiers. Given a training corpus, these methods by themselves defines a mechanism of deciding whether a given example is an instance of a particular concept or not. They do not generally produce an explanation of why it is a member of that category and do not incorporate a description of the properties of the object. In fact many statistical learning paradigms such as support vector machines rely on simplifying and abstracting the classification problem to minimize the amount of information about the problem that needs to be represented while maximizing the separability of the different classes. A purely *'extensional'* representation on the other hand consists of rules or statements whose truth may be assessed independently of the particular extensions involved, i.e. regardless of which element of the modeled class is being considered.

In the following we will briefly review *'intensional approaches'* and *'extensional approaches'* especially in consideration how these approaches handle uncertainty and semantics.

2.3.1 Intensional Approaches

In intensional approaches, also known as *'state-based approaches'*, uncertainty is attached to *'states of affairs'* or subsets of *'possible worlds'*. Such systems treat uncertainty by connectives that combine sets of worlds by set theory operations. For example, the probability $P(A \wedge B)$ is given by the weight assigned to the intersection of two sets of worlds, those in which A is true and those in which B is true, but $P(A \wedge B)$ can not be determined from the individual probabilities $P(A)$ and $P(B)$. In intensional systems, the rules denote elastic constraints about the world. For example, the rule $A \xrightarrow{m} B$ does not describe how an agent reacts to the finding of A, but asserts that the set of worlds in which A and $\neg B$ hold simultaneously has low likelihood and hence should be excluded with probability m. In the Bayesian formalism the rule $A \xrightarrow{m} B$ is interpreted as a conditional probability expression $P(B|A) = m$, stating that among all worlds satisfying A, those that also satisfy B constitute an m percent majority.

2.3.2 Extensional Approaches

Extensional approaches also known as rule-based systems or production-systems treat uncertainty as a generalized truth value; that is, the certainty of a formula is defined to be a unique function of the certainties of its sub formulas. Thus, the connectives in the formula serve to select the appropriate weight-combining function. The certaint-factors calculus used in MYCIN [36] is a well-known example of *'extensional system'*. For example, the certainty of the conjunction $A \wedge B$ is given by some function of the certainty measures assigned to A and B individually. The rules in extensional systems provide license for certain symbolic activities. For example, a rule $A \xrightarrow{m} B$ may mean "If you see A, then you are given the license to update the certainty of B by a certain amount which is a function of the rule strength m". The rules are interpreted as a summary of past performance of the problem solver, describing the way an agent normally reacts to problem situations or to items of evidence.

Consider Figure 2.3 that depicts the combination functions that apply to serial and parallel rules, from which one can form a *'rule network'*. In extensional approach, it is required to define a modular procedure for determining the certainty of a conclusion, given the credibility of each rule and the certainty of the premises (i.e., the roots of the rule network). To complete the calculus we also need to define combining functions for conjunction and negation as well.

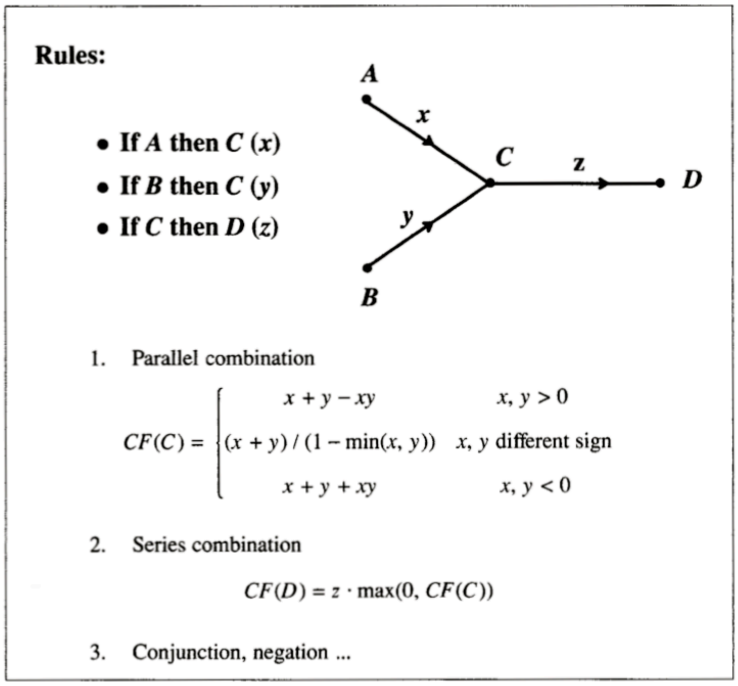


Figure 2.3: An Example of Handling Uncertainty in Extensional Approach - Certainty Combination Function used in MYCIN [130].

2.3.3 Extensional vs. Intensional : Merits and Deficiencies

Extensional approach tend to be computationally efficient but semantically sloppy while intensional approach tend to be semantically clear but computationally clumsy .

In extensional systems, setting mathematical detail aside, it is important to notice that the same combination function applies uniformly to any two rules in such systems, *'regardless of what other rules might be in the neighborhood'*. This is mainly due to the nature of ***'modularity'*** of inference in classical logic. For example, the logical rule "If A then B" has the following procedural interpretation:

- **locality :** If you see A anywhere in the knowledge base, then *'regardless of what other things'* the knowledge base contains, you are given the license to assert B and add it to the database.

- **detachment :** If you see A anywhere in the knowledge base, then regardless of how A was derived, you are given the license to assert B and add it to the database.

When it is considered with uncertainty, the procedural license provided by the rule $A \xrightarrow{x} B$ reads as follows : "If you see the certainty of A undergoing a change δ_A, then regardless of what other things the knowledge base contains and regardless of how δ_A was triggered, you are given an unqualified license to modify the current certainty of B by some amount δ_B, which may depend on x, on δ_A, and on the current certainty of B."

Contrary, in intensional approach such as probability statements, $P(B|A) = p$, does not give us license to do anything. Even if we are fortunate enough to find A true in the database, we still can not assert a thing about B or $P(B)$, because the meaning of the statement is "If A is true and A is the *'only'* thing that you know, then you are given license to attach a probability p to B.". As soon as other facts K appear in the database, the license to assert $P(B) = p$ is automatically revoked, and we need to look up $P(B|A, K)$ instead. The probability statement leaves us totally impotent, unable to initiate any computation, unless we can verify that everything else in the knowledge base is irrelevant. This is why verification of irrelevancy is so crucial in intensional systems.

However, there are known semantic penalties imposed in extensional approaches when relevance considerations are ignored. Such semantic deficiencies are posed :

- **The Limits of Modularity in Retracting Conclusions :** In extensional systems, *'detachment'* can create problems. In deductive logic the following holds true: $A \to B$ and $B \to C \implies A \to C$. In other words, finding evidence for A leads us to conclude C by chaining. Derived evidence B triggers the rule $B \to C$ with the same rigor as would a directly observed proposition. However consider the case, "ground is wet $\to$ it rained" and "sprinkler is on $\to$ ground is wet". In this case, if an extensional system is told that sprinkler is on, it will conclude that it rained. This is incorrect and in fact finding that the sprinkler was on should only reduce the likelihood that it rained.

- **Improper Handling of Bidirectional Inferences :** Plausible reasoning requires that both predictive as well as diagnostic components of reasoning be used. if $A \to B$, then finding B to be true makes A more credible (abductive reasoning). This requires reasoning both ways. Extensional systems do not allow such bidirectional inference i.e. reasoning from both A to B and B to A. To implement this in extensional systems, one has to explicitly specify the reverse rule, possibly risking creation of a cycle that can cause evidence to be cyclically amplified until both cause and effect are completely certain with no apparent factual justification. Removing the predictive component prevents system from exhibiting another important pattern of plausible reasoning called explaining away: if $A \rightarrow B$ and $C \rightarrow B$ and B is true, then finding C is true makes A less credible. To exhibit this kind of reasoning, the system must use bi-directed inferences; from evidence to hypothesis and from hypothesis to evidence. While it might be possible to get around this problem by exhaustively listing all possible exceptions, to restore explaining away (without the danger of circular reasoning), any system that does that sacrifices on principles of modularity. Inversely, any system that updates beliefs modularly and treats all rules equally is bound to defy patterns of plausible reasoning.

- **Improper Treatment of Correlated Sources of Evidences:** Due to *'locality'*, extensional systems do not store information on how a proposition was derived. As a result, they risk treating correlated evidence as independent. Consider a situation where someone hears a piece of news independently from the radio, television as well as the newspapers. Since from his point of view, the sources are independent, his belief in the veracity of the piece of news should be very high. However, if that person were to realize later that all the three sources got their information from the same source, then his belief in the piece of news should decrease. This can never happen in extensional systems as they treat each source of information completely independently of the others.

In this dissertation, we propose an *'extensional reasoning framework'* that comes with remedies for the problem of *'bidirectional reasoning'* and *'limitation of modularity'*. The former was addressed in the sense of interpreting conditional propositions in bi-directional way and the latter was addressed in the sense of *'meta-reasoning'* using *'abductive logic programming'* [50].

2.4 Automatic Interpretation of Semantic Occurrence in Video

This chapter examines some relevant research trends and presents an overview of published work that is related to automatic interpretation of high level semantic occurrence in video sequences. We have chosen to organize semantic models into three different categories : *'pattern recognition methods'*, *'state models'* and *'semantic models'*. *'pattern recognition methods'* do not generally address the representation aspect of semantic models and rather trained and specified by feature vectors of training data. *'state models'* represent domain knowledge in a graphical model. *'semantic models'* uses formal languages for knowledge representation. The *'state models'* and *'semantic models'* categorizations again fall into aforementioned *'intensional'* and *'extensional'* approaches. In the following sections, we will further take a more in-depth look at the three categories of semantic interpretation methods, and explore examples from the literature.

2.4.1 Pattern Recognition Methods - Data Centric Approaches

This class of techniques in this section is not quite semantic models, in the sense that they do not consider the problem of semantic representation. Instead, they focus on the event recognition problem, which is formulated as a traditional pattern-recognition and classification problem. Accordingly, traditional approaches to these problems such as support vector machines, neural networks, nearest neighbor classifiers, etc., are applied to the perception scheme. Minimal semantic knowledge is needed in building the semantic classifiers in this category.

The main advantage of the classifiers in this category is that they are well understood. Usually, they may be fully specified from a set of training data. These approaches are usually simple and straightforward to implement. This simplicity is afforded by excluding semantics (i.e., high-level knowledge about the semantic domain) entirely from the specification of the classifier.

There are many examples of pattern-recognition methods especially for event recognition in the literature including [29, 180, 27, 71] and [151] (nearest neighbor), [136, 63, 135] (Support Vector Machine), [124, 157, 145] (boosting) and [167] (neural networks). A more comprehensive discussion of these approaches and examples can be found in [12, 102, 165, 86].

2.4.2 State Based Models - Intensional Approaches

"State" based semantic models are a class of formalisms that model the state of the video semantics in space and time using semantic knowledge. State models improve on pattern-recognition methods in that they *'intrinsically'* model the structure of the state space of the semantic domain. Such approaches fall into the category of *'intensional'*. Modeling formalisms in this category are also well studied and mathematically well formulated. This allows for efficient algorithms and sound formulations of problems such as parameter learning. In most cases, however, the semantic information associated with the model structure makes this structure difficult to learn from training data. In such approaches, states are considered as symbolic facts and contextual knowledge is represented as a graph structure having state nodes that are connected to each other. In the sense of logic, connected two state nodes can be interpreted as a propositional logic rule that can consider only one relation, *'the causality implication'* (that can be often interpreted as *'influence'*, *'affect'*, *'cause'* in natural language). A piece of propositional knowledge segment should exist within the whole graph structure, thereby, once uncertainty propagation mechanism is learnt, adding additional pieces of knowledge will require restructuring causality influence relation of the whole graph structure. This aspect restricts expressive power and increases the modeling cost. Due to this complexity and lack of modularity, such approaches have been focusing on relatively narrow and specific semantics. State modeling formalisms also include FSMs (Finite-State Machines), Bayesian networks (BNs), hidden Markov models (HMMs) and dynamic BNs (DBNs).

FSMs assume a *'fully observable sequence of states'*, therefore have been used for modeling the temporal aspects of video semantics with less concerns on uncertainty. FSMs appearing in the literature naturally model single thread *'event'* formed by a sequence of states. FSM semantic models are utilized include hand gestures [91], single-actor actions [108] and aerial surveillance [117].

In order to deal with the inherent uncertainty of observations and interpretation that exists in video, semantic models utilizing probability have been proposed. One such event modeling formalism is the BNs (also known as probability network) [90]. In BNs, states are considered as a random variable and they are connected with acyclic edges. Edges represent joint probability between states using the notion of conditional independence. BN semantic models are utilized for indoor surveillance [109] such as left luggage detection, Hongeng et al. [85] considers an activity to be composed of action threads and recognizes activities by propagating constraints and likelihood of event threads using a BN model, Remagnino et al. [144] uses BNs for parking lot surveillance, Wang et al. [170] uses a BN model for primitive activity detection such as *'jay walking'*. One major drawback of BN based semantic models is that they do not have an inherent capacity for modeling temporal composition, which is an important aspect of video semantics. Solutions to this problem include single-frame event classification [37] and choosing abstraction schemes that encapsulate temporal properties of the input [109].

The benefits of a temporal evolution model (like FSM) and a probabilistic model (like BN) are combined within the framework of the hidden Markov model (HMM). HMMs are a class of directed graphical models extended to model the temporal evolution of the state [142]. Due to this aspect, HMMs have become one of the most popular formalisms for modeling video semantics. Makris et al. [113] uses HMM to reason about human behaviors based on trajectory information. Oagale et al. [127] models single-person activities such as *'walking'* and *'kneeling'*. Gong et al. [72] applies a HMM model for airport tarmac surveillance. Oliver et al. [128] use a layered HMM (LHMM) model in the event domain of office surveillance. Due to the markov assumption that the current state depends only on the state at a previous time, the semantics recognized in these works are tend to be mostly a few seconds in length. One drawback of HMM is that as the model topology becomes more complex, the efficient exact algorithms associated with the HMM structure are no longer applicable and must be replaced with approximation algorithms.

Dynamic BNs (DBNs) generalize BNs with a temporal extent. In fact, HMMs are a special case of DBNs in which the structure is restricted to provide efficient algorithms for learning and inference. This, however, often comes at the cost of computational tractability. Approximation techniques are usually used to perform learning and inference. Thus, DBNs in their general form appear less often as semantic modeling formalism in the literature. [120] and [121] apply DBNs for surveillance of people such as *'entering'*, *'passing'*, etc.

Other approaches use a qualitative representation of uncertainty [58], a context representation *'scheme'* for surveillance systems [34], AND/OR tree for the analysis of specific situations [46], or a GMM based scene representation for reasoning upon activities [114].

These extensions to the formalism have attempted to introduce aspects such as hierarchy and uncertainty. These methods have largely been applied to specific event domains and have not been embraced as general solutions.

2.4.3 Semantic Based Models - Extensional Approaches

While semantics such as many types of events can be described as a sequence of a number of states, an interesting subset of semantics are those defined by the semantic relationships between their composing sub-events or sub-semantics. The category of *"compositional high level semantic models"* groups several primitive unit semantics to allow these kinds of relationships to be represented and recognized. These approaches do not aim to define the *'entire state space'* of desired semantic domain as in *'state modeling' (intensional)* approaches. Instead, semantic knowledge is used to define a set of semantic rules, constraints and relations.

This type of approach allows the event model to capture high-level semantics such as long-term temporal dependence (e.g. meeting, eating, etc.), hierarchical semantics (e.g. boss of, employee of, etc.), semantically complex relations (e.g. friends of, thief of) using primitive sub-semantics (i.e. mostly basic events or perceptional object/human recognition results). Thus, approaches in this category are usually applied in domains where the semantics (e.g. events, human activities or human interactions) of interest are relatively complex and a particular semantics has large variance. However, because of the high-level nature of this class of models, they often must be manually specified by a domain expert (*i.e., learning model structure and parameters is generally infeasible*). Generally, the formalisms in this category of event models such as grammars and logic formalisms comes without uncertainty handling mechanism.

To cope with uncertainty, there have been some work on the use of logic programming languages to achieve better expressive power and on the use of different uncertainty handling formalisms to reason under uncertainty. Such logic framework based uncertainty handling approaches can be categorized as *'extensional'*.

Yuri et al. [88] use a stochastic grammar and its parser for parking lot surveillance. Ogale et al. [126] also use a stochastic grammar for human activity recognition. However, in logical sense, their grammar rules corresponds to propositional logic, therefore has much overlap with graphical *'intensional'* models in the sense that it can not really represent predicates as first-order logic. In fact, the achievement of better expressive power in *'extensional'* approaches is mainly due to the first-order predicate logic that logic programming provides. While propositional logic deals with simple declarative propositions, first-order logic additionally covers predicates and quantifiers. Akdemir et al. [13] proposed an ontology based approach for activity recognition, but without uncertainty handling mechanism (In ontology community, Description Logics (DLs) are often used as knowledge representation formalism and DLs are decidable fragments of first-oder-logic [24, 2]). Shet et al. [152] proposed a system that adopts Prolog based logic programming for high-level reasoning. In [155] the same authors extended their system with the bilattice framework [69] to perform the task of detecting humans under partial occlusion based on the output of parts based detectors. Jianbing et al. [110] used rule-based reasoning with Dempster Shafer's Theory [149] for a bus surveillance scenario. Anderson et al. [17] used Fuzzy Logic [177] to

Approach	Ogale / Yuri et al. [126, 88]	Akdemir et al. [13]	Jianbing et al. [110]	Shet et. al [152, 155]	Anderson / Dorado et al. [17, 54]	Our approach [81, 80, 82] [83, 79]
Knowledge Modeling	Rule Based (Prop. L)	Ontology (DL)	Rule Based (FOL)	Rule Based (FOL)	Rule Based (Prop. L)	Rule Based (FOL)
Uncertainty Formalism	Stochastic Grammar	-	Dempster Shafer	Bilattice	Fuzzy Logic	Subjective Logic
Traditional Logic Operators	-	-	-	√	√	√
Arithmetic Operators	-	-	-	-	-	√
Info. Fusion Operators	-	-	√	√	-	√
Extra Operators MT, MP,Rep,etc.	-	-	-	-	-	√
Default Reasoning	-	-	-	√	-	√ Chapter 5
Vague Rule Modeling	-	-	-	-	√	√ Chapter 6
Bidirectional Inference	-	-	-	-	-	√ Chapter 7
Diagnostic Abduction	-	-	-	-	-	√ Chapter 8
Belief Revision	-	-	-	√	-	√ Chapter 5-8

Table 2.2: A Comparison of Previous Extensional Approaches.

model human activity for video based eldercare. Dorado et al. [54] also applied fuzzy rules but directly to low-level features for annotation of broadcasting contents. Therefore, their focus was more on *‘vision based perception’* aspect. They defined propositional logic based rules in form of *‘<condition>* $\rightarrow$ *<action>’*, where *‘<condition>’* expresses the instances of low-level features and *‘<action>’* denotes annotating a word (concept). For example, using the MPEG-7 edge histogram descriptor (EHD), they mapped an EHD to a fuzzy set elements such as V(ertical), H(orizontal), D(iagonal) and N(on)D(irectional) then built a rule such as *‘V is H(igh)* $\wedge$ *H is M(edium)* $\wedge$ *D is L(ow)* $\wedge$ *ND is L(ow)* $\rightarrow$ *label as BUILDING’*. Such V, H, D and ND can be considered as state symbols that can be posed by low-level features. Because their rules are propositional it can be also represented as a graph having such symbols as nodes that are connected to a word symbol. In this sense their approach is similar to *'state base model'*.

In this dissertation we proposed the use of logic programming and subjective logic [93] to encode contextual knowledge with uncertainty handling. Based on our previous work [78, 81, 80, 82, 83, 79], we extend the system to demonstrate bidirectional conditional inference, belief revision, default reasoning and vague rule modeling. Table 2.2 shows a brief comparison of the previously proposed *'extensional'* approaches. The table shows that the coverage of our subjective logic based approach is most broad. For example, while some provides information fusion capability for fusing two contradictory information sources, such as dempster shafer's fusion operator, bilattice's operator and subjective logic's consensus operator, only some of them support default reasoning that handles such contradictory information to draw reasonable decision and belief revision. Indeed, our system also supports modeling vague propositional rules and inference under such vague rules for high level semantic analysis of visual surveillance data (in the sense of linguistic interpretation of the rules, the most similar previous approach to the proposed work would be [17]). Finally, bidirectional inference (as is possible in *'intensional'* approaches) is only supported by subjective logic based approach.

3 Preliminaries

This chapter gives an overview of the fundamental background about subjective logic theory and logic programming that will be discussed throughout this dissertation.

3.1 Subjective Logic Theory

Audun Jøsang introduced subjective logic as a framework for artificial reasoning [92, 93]. Unlike traditional binary logic or probabilistic logic (the former can only consider true or false, and the latter can consider degrees of truth or falseness), subjective logic explicitly represents the amount of 'lack of information (ignorance) on the degree of truth about a proposition' in a model called *'subjective opinion'* [92]. The idea of explicit representation of ignorance is inherited from the Dempster Shafer belief theory [149, 92, 149] and the interpretation of an opinion in bayesian perspective is possible by mapping opinions into beta distributions [93]. Subjective logic also comes with a rich set of operators for the manipulation of opinions. In addition to the standard logical operators, subjective logic provides some operators specific for Dempster Shafer belief theory such as consensus and recommendation. However, unlike Dempster Shafer's evidence fusion rule that is inconsistent with Bayes theorem, it provides an alternative consensus rule with a solid mathematical basis [92]. It is also different from fuzzy logic: While fuzzy logic maps quantitative measure to non-crisp premises called fuzzy sets (e.g. *'fast'*, *'slow'*, *'cold'*, *'hot'*, etc.), subjective logic deals with the uncertain belief itself on a crisp premise (e.g. *'intrusion happened'*, *'accident happened'*, etc.). However, in the sense of interpretation, mapping of an opinion into the linguistic certainty fuzzy set (i.e., *'very certainly true'*, *'less certainly true'*, etc) is also possible. In general, subjective logic is suitable for modeling real situations under partial ignorance on a proposition's being true or false.

Known application areas are trust network modeling, decision supporting, modeling and analyzing Bayesian network, etc. However, to the best of our knowledge, the application of subjective logic in computer vision related domains has been limited to our previous work [78, 81, 80, 82, 83, 79]. In this dissertation, based on our previous work, we demonstrate the capability of default reasoning to handle contradictory information, bidirectional interpretation of conditional rules using abduction and deduction operators and modeling vague rules relying on reputation operator in subjective logic. In this section, we will give a brief introduction to subjective logic theory.

3.1.1 Dempster Shafer Belief Theory

Subjective logic uses theoritical elements from the Dempster-Shafer belief theory [149]. In this section, we give a brief introduction to Dempster Shafer belief theory with an example. Dempster Shafer belief theory deals with a set of hypotheses called the *'frame of discernment'*, conventionally denoted as Θ. The elements of Θ are considered to be mutually exclusive, therefore exactly one element is assumed to be true. To illustrate the frame of discernment let us consider following example shown by Zadeh [178] (The same example was also used by Audun Jøsang in [98, 94]).

Example 1. ***(A Zadeh's Example on Murder Case) .*** Suppose that we have a murder case with three suspects; Peter, Paul and Mary then Figure 3.1 is an example of frame of discernment shown in [94]. If an element is assumed to be true, its supersets are also considered to be true as well. An observer can assign belief mass to one or several states in the powerset of Θ (denoted 2^Θ).

Definition 1. ***(Belief Mass Assignment)*** *[93] . Let Θ be a frame of discernment. A belief mass assignment is a mapping $m_\Theta : 2^\Theta \rightarrow [0,1]$ such that $m_\Theta(x) \geq 0$, $m_\Theta(\emptyset) = 0$ and $\sum_{x \subseteq 2^\Theta} m_\Theta(x) = 1$.*

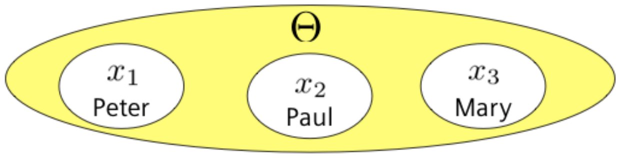


Figure 3.1: An Example of Frame of Discernment Θ containing 3 Elements [94].

For example, if an observer assigned belief mass to a set $x_4 = \{x_1, x_3\}$, it is considered that one of the elements in x_4 is true but the observer is uncertain about which of them is true (i.e., ignorance). In contrast to belief mass, the *'belief'* to a set x should be the sum of belief mass assigned to its subsets. Namely,

Definition 2. ***(Belief Function)*** *[93] . Let Θ be a frame of discernment, and let m_Θ be a BMA on Θ. Then the belief function corresponding with m_Θ is the function $b : 2^\Theta \to [0, 1]$, such that $b(x) = \sum_{y \subseteq x} m_\Theta(y)$, where $x, y \in 2^\Theta$.*

This idea can be expanded to the cases of *'disbelief'* and *'ignorance'* as follows.

Definition 3. ***(Disbelief Function)*** *[93] . Let Θ be a frame of discernment, and let m_Θ be a BMA on Θ. Then the disbelief function corresponding with m_Θ is the function $d : 2^\Theta \to [0, 1]$, such that $d(x) = \sum_{y \cap x = \emptyset} m_\Theta(y)$, where $x, y \in 2^\Theta$.*

Definition 4. ***(Ignorance Function)*** *[93] . Let Θ be a frame of discernment, and let m_Θ be a BMA on Θ. Then the ignorance function corresponding with m_Θ is the function $i : 2^\Theta \to [0, 1]$, such that $i(x) = \sum_{y \cap x \neq \emptyset, y \not\subseteq x} m_\Theta(y)$, where $x, y \in 2^\Theta$.*

By the Definition 1, the sum of the *'belief'*, *'disbelief'* and *'ignorance'* is equal to the sum of the belief masses which sums up to 1. Therefore, $b(x) + d(x) + i(x) = 1$. Given the elements in frame of discernment, the relative number of atomic elements is called *'relative atomicity'* which is formally defined as follows.

Definition 5. ***(Relative Atomicity)*** *[93] . Let Θ be a frame of discernment, and let $x, y \in 2^\Theta$. Then for any given $y \neq \emptyset$, the relative atomicity of x to y is the function $a : 2^\Theta \to [0, 1]$ such that $a(x/y) = \frac{|x \cap y|}{|y|}$, where $x, y \in 2^\Theta, y \neq \emptyset$ (Especially, $a(x/\Theta)$ is simply written as $a(x)$).*

$x \in 2^\Theta$	m_Θ^A	$b^A(x)$	$d^A(x)$	$i^A(x)$	m_Θ^B	Dempster's rule	Non-normalised Dempster's rule
Peter	0.98	0.98	0.01	0.01	0.00	0.490	0.0098
Paul	0.01	0.01	0.98	0.01	0.01	0.015	0.0003
Mary	0.00	0.00	0.99	0.01	0.98	0.490	0.0098
Θ	0.01	1.00	0.00	0.00	0.01	0.005	0.0001
$\emptyset$	0.00	0.00	1.00	0.00	0.00	0.000	0.9800

Table 3.1: An Example of Applying Dempster's Rule shown in [94].

On a frame of discernment, a probability expectation can be calculated by the following definition.

Definition 6. ***(Probability Expectation)*** *[93]* **.** *Let Θ be a frame of discernment with BMA m_Θ, then the probability expectation function corresponding with m_Θ is the function $E : 2^\Theta \rightarrow [0, 1]$ such that, $E(x) = \sum_y m_\Theta(y)a(x/y)$, where $y \in 2^\Theta$.*

In Dempster Shafer belief theory, belief mass assignment m_Θ plays the most fundamental basis for representing belief and calculating probability expectation, etc. In addition, Dempster Shafer belief theory comes with a rule to combine two different belief mass assignments which is defined as follows.

Definition 7. ***(Dempster's Rule)*** *[94]* **.** *Let Θ be a frame of discernment, and let m_Θ^A and m_Θ^B be belief mass assignment on Θ of two observers A and B. Then $m_\Theta^A \oplus' m_\Theta^B$ is a function $m_\Theta^A \oplus' m_\Theta^B : 2^\Theta \rightarrow [0, 1]$ such that, 1. $m_\Theta^A \oplus' m_\Theta^B(\emptyset) = \sum_{y \cap z = \emptyset} m_\Theta^A(y) \cdot m_\Theta^B(z) - K$ and 2. $m_\Theta^A \oplus' m_\Theta^B(x) = \frac{\sum_{y \cap z = x} m_\Theta^A(y) \cdot m_\Theta^B(z)}{1-K}$, for all $x \neq \emptyset$, where $K = \sum_{y \cap z = \emptyset} m^A \Theta(y) \cdot m_\Theta^B(z)$ and $K \neq 1$ in standard Dempster's rule, and where $K = 0$ in the non-normalized version of Dempster's rule.*

Suppose that two observers assigned their belief masses to the frame of discernment shown in Figure 3.1 that its elements are suspect of a murder case. Table 3.1 shows an example of using dempster shafer theory and dempster's rule.

3.1.2 Opinion Model in Subjective Logic

The ideas of Dempster Shafer belief theory can be simplified in subjective logic theory, by restricting the frame of discernment to be binary, i.e., it will only contain (focus on) one particular set and its complement. Such a frame of discernment is called *'focused frame of discernment'*. Figure 3.2 shows examples of focused frame of discernment that can be derived from Figure 3.1. The formal definition of *'focused frame of discernment'* is as follows.

Definition 8. ***(Focused Frame of Discernment)*** *[93] . Let Θ be a frame of discernment and let $x \in 2^{\Theta}$. The frame of discernment denoted by $\tilde{\Theta}^x$ containing only x and $\neg x$(i.e., $\overline{x}$, where $\neg x$ is the complement of x in Θ is then called a focused frame of discernment with focus on x.*

Remember that the belief, disbelief and ignorance functions are also indexed by a specific element in a frame of discernment (i.e., $b(x), d(x), i(x)$, respectively). By this *'focused belief mass assignment'* is defined as follows.

Definition 9. ***(Focused Belief Mass Assignment)*** *[93] . Let Θ be a frame of discernment with belief mass assignment m_Θ where $b(x)$, $d(x)$ and $i(x)$ are the belief, disbelief and ignorance functions of x in 2^{Θ}, and let $a(x)$ be the real relative atomicity of x in Θ. Let $\tilde{\Theta}^x$ be the focused frame of discernment with focus on x. The corresponding focused belief mass assignment $m_{\tilde{\Theta}^x}$ and relative atomicity $a_{\tilde{\Theta}^x}(x)$ on $\tilde{\Theta}^x$ is defined according to:*

$$\begin{cases} m_{\tilde{\Theta}^x}(x) = b(x) \\ m_{\tilde{\Theta}^x}(\neg x) = d(x) \\ m_{\tilde{\Theta}^x}(\tilde{\Theta}^x) = i(x) \end{cases} \qquad \begin{cases} a_{\tilde{\Theta}^x}(x) = \frac{E(x)-b(x)}{i(x)}, fori(x) \neq 0 \\ a_{\tilde{\Theta}^x}(x) = a(x), fori(x) = 0 \end{cases}.$$

Opinion model called *'opinion triangle'* or *'subjective opinion'* in subjective logic is represented based on the concept of focused belief mass assignment as shown in Figure 3.3. The formal definition of subjective opinion model is as follows.

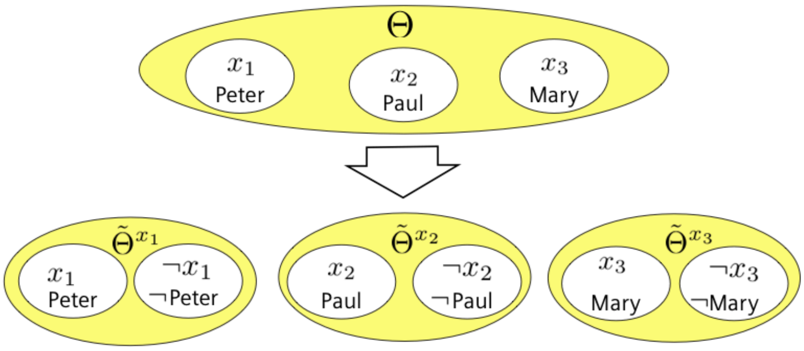


Figure 3.2: Examples of Focused Frame of Discernment Derived from Figure 3.1.

Definition 10. ***(Opinion)*** *[93] . Let $\Theta = \{x, \neg x\}$ be a state space (a.k.a 'Frame') containing x and its complement $\overline{x}$. Let b_x, d_x, i_x represent the belief, disbelief and ignorance in the truth of x satisfying the equation: $b_x + d_x + i_x = 1$ and let a_x be the base rate of x in Θ. Then the opinion of an agent ag about x, denoted by w_x^{ag}, is the tuple $w_x^{ag} = (b_x^{ag}, d_x^{ag}, i_x^{ag}, a_x^{ag})$.*

In the context of the frame of discernment shown in Figure 3.1 and belief mass assignment shown in Table 3.1, the examples of the focused opinions of agent A are $w^A_{Peter} = (0.98, 0.01, 0.01, 1/3)$, $w^A_{Paul} = (0.01, 0.98, 0.01, 1/3)$ and $w^A_{Mary} = (0.00, 0.99, 0.01, 1/3)$. The probability expectation can be also defined on the opinion model in subjective logic.

Definition 11. ***(Probability Expectation)*** *[93] . Let $w_x^{ag} = \{b_x^{ag}, d_x^{ag}, i_x^{ag}, a_x^{ag}\}$ be an opinion about the truth of x, then the probability expectation of w_x^{ag} is defined by: $E(w_x^{ag}) = b_x^{ag} + a_x^{ag} i_x^{ag}$.*

A point inside the triangle represents a (b_x, d_x, i_x) triple. The corner points marked with Belief, Disbelief and Ignorance represent the extreme cases, i.e., full belief $(1, 0, 0)$, full disbelief $(0, 1, 0)$ and no knowledge $(0, 0, 1)$. An opinion lying on base line (the line connecting Belief and Disbelief) is called *'dogmatic opinion'* in the sense that they do not contain any degree of ignorance (*i.e.*, $i_x = 0$). Such dogmatic opinions correspond to traditional traditional probability. The base rate a_x represents the prior knowledge on the tendency of a given proposition p's being true and can be indicated along the base line.

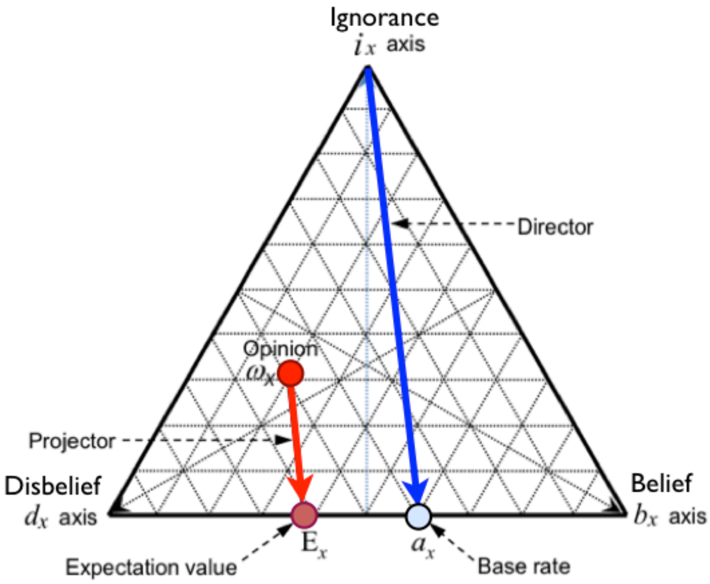
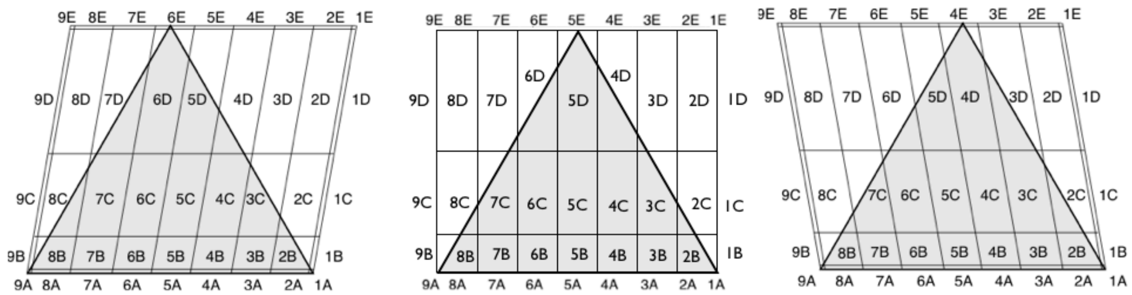


Figure 3.3: Opinion Triangle.

Certainty Categories \ Likelihood Categories		Absolutely Not 9	Very Unlikely 8	Unlikely 7	Somewhat Unlikely 6	Chances about even 5	Somewhat Likely 4	Likely 3	Very Lkely 2	Absolutely 1
Completely Uncertain	E	9E	8E	7E	6E	5E	4E	3E	2E	1E
Very Uncertain	D	9D	8D	7D	6D	5D	4D	3D	2D	1D
Uncertain	C	9C	8C	7C	6C	5C	4C	3C	2C	1C
Slightly Uncertain	B	9B	8B	7B	6B	5B	4B	3B	2B	1B
Completely Certain	A	9A	8A	7A	6A	5A	4A	3A	2A	1A

(a) Linguistic Fuzzy Categories for Opinion Triangle



(b) Fuzzy Categories with a = 1/3 (c) Fuzzy Categories with a = 1/2 (d) Fuzzy Categories with a = 1/2

Figure 3.4: Linguistic Fuzzy Category for Opinion Triangle as a Function of the Base Rate [139].

For example, in the case we toss a balanced coin with the proposition x as *'get head'*, then we put $a_x = 1/2$ however, in the case of biased coin we could set different values. Similarly, in the case we toss a balanced dice we put $a_x = 1/6$ (note that, the original frame of discernment has six elements). Usually, when we consider balanced binomial cases, the default value is $1/2$. The probability expectation E is then formed by projecting the opinion onto the base line, parallel to the base rate projector line (see the blue line) that is built by connecting the a_x point with the Ignorance corner (see the red line).

As human language provides various terms to express various types of likelihood and uncertainty, a linguistic interpretation of an opinion is also possible. Pope et al. [139] showed an example mapping of fuzzy categories to opinion triangle. As shown in Figure 3.4, the stronger opinion we have, the lesser changes of mapping areas are made. Similarly, the weaker opinion we have, we are relying the more on base rate. For example, the area $5A$ can be interpreted linguistically as *'completely certain chances about even'* and the area $2B$ can be interpreted as *'very likely but slightly uncertain'*.

3.1.3 The Correlation between Subjective Opinion and Beta Distribution, The Bayesian Perspective

An interesting property of subjective opinions is their direct mapping to beta distributions. As shown in the Definition 10, subjective opinion's frame Θ is binomial, because it deals with two elements about a proposition x. Namely, x (i.e., x happened, success) and $\overline{x}$ (i.e., x does not happened, failure). Assuming n number of independent observations about the proposition x with a fixed base rate $a_x = \pi$, the conditional probabilistic likelihood that x could happen y times is represented as $f(y|\pi) = {}_nC_y\, \pi^y (1-\pi)^{n-y}, for\ y = 0, ..., n$. Here, we are holding a_x fixed and are looking at the probability distribution of y over its possible discrete integer values of $0, ..., n$. This is referred to as *'binomial distribution'* and noted as $binomial(n, a_x)$. If we look at this same relationship between the probability π and number of successes y, holding y fixed and let π vary over its possible values $0 \le \pi \le 1$. Then we get the likelihood function given by $f(y|\pi) = {}_nC_y\, \pi^y (1-\pi)^{n-y}, for\ 0 \le \pi \le 1$.

Taking a different view, let us assume that we want to know about the probability density of the probability π itself given y successes of observations, namely $g(\pi|y)$. By the Bayes' theorem, we know that *'posterior* $\propto$ *prior* $\times$ *likelihood'*, therefore, $g(\pi|y) \propto g(\pi) \times f(y|\pi)$. This gives us only the shape of the posterior density. To get the actual posterior, we need to normalize this by some constant k, to make sure that the area under the posterior integrates to 1. We find k by integrating $g(\pi) \times f(y|\pi)$ over the whole range. In general we get,

$$g(\pi|y) = \frac{g(\pi) \times f(y|\pi)}{\int_0^1 g(\pi) \times f(y|\pi)\, d\pi}, \tag{3.1}$$

In above Equation 3.1, it requires an integration. However, depending on the prior $g(\pi)$ chosen, we do not always need to do the actual integration numerically. Assume that we do not have any idea beforehand what the π is, therefore, we assign,

$$\textbf{Using a Uniform Prior}: \; g(\pi) = 1, for\ 0 \leq \pi \leq 1. \tag{3.2}$$

Clearly, we see that in this case, the posterior density is proportional to the likelihood therefore, $g(\pi|y) = {}_nC_y\, \pi^y(1-\pi)^{n-y}, for\ 0 \leq \pi \leq 1$. This equation can be slightly changed by introducing the number of successes $\alpha = y + 1$ and number of failures $\beta = n - y + 1$. $g(\pi|\alpha,\beta) = {}_{(\alpha+\beta-2)}C_\alpha\, \pi^{\alpha-1}(1-\pi)^{\beta-1} = \frac{(\alpha+\beta-2)!}{(\alpha-1)!(\beta-1)!}\pi^{\alpha-1}(1-\pi)^{\beta-1}$. Here, each of the factorial part in the coefficient can be generalized to the cases of real values using the property of the *gamma function*[1], $\Gamma(n) = (n-1)!$. Therefore, we get,

$$g(\pi|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\pi^{\alpha-1}(1-\pi)^{\beta-1} = Beta(\pi|\alpha,\beta),\; for\ 0 \leq \pi \leq 1 \tag{3.3}$$

Above Equation 3.3 is the probability density function called *beta distribution* [2] that its shape is only dependent on the indexes α and β. Now, let us consider assigning the beta

[1] The Gamma function is a generalization of the factorial function over integer to real and complex numbers, with its argument shifted down by 1. Formally, it is defined as $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\, dt$.

[2] Note that, the coefficient part of the beta distribution is $\frac{1}{B(\alpha,\beta)}$, where $B(\alpha,\beta)$ is a special function called Beta function that is formally defined as $B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\, dt = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$.

prior to $g(\pi)$ instead, in the step (3.2).

$$\textbf{Using a Beta Prior :}\ g(\pi;\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\pi^{\alpha-1}(1-\pi)^{\beta-1}, for\ 0 \leq \pi \leq 1. \tag{3.4}$$

Then we get, $g(\pi|y) \propto g(\pi;\alpha,\beta) \times \pi^{y}(1-\pi)^{n-y} = k \times \pi^{y+(\alpha-1)}(1-\pi)^{n-y+(\beta-1)}$ again by the Bayes' theorem. To make $g(\pi|y)$ a probability density function that the area under the function integrates to 1, we need to calculate,

$$g(\pi|y) = \frac{k \times \pi^{y+(\alpha-1)}(1-\pi)^{n-y+(\beta-1)}}{\int_0^1 k \times \pi^{y+(\alpha-1)}(1-\pi)^{n-y+(\beta-1)}\,d\pi} = k' \times \pi^{y+(\alpha-1)}(1-\pi)^{n-y+(\beta-1)}. \tag{3.5}$$

In the above Equation 3.5, we can easily recognize that this can be a beta distribution with parameters $\alpha' = y + \alpha$ and $\beta' = n - y + \beta$. Therefore, we get,

$$g(\pi|y) = \frac{\Gamma(n+\alpha+\beta)}{\Gamma(y+\alpha)\Gamma(n-y+\beta)}\pi^{y+\alpha-1}(1-\pi)^{n-y+b-1}, for\ 0 \leq \pi \leq 1. \tag{3.6}$$

Again, the posterior density of π has been easily obtained without having to go through the numerical integration. Furthermore, the posterior is to be the same form as the $Beta(\pi|\alpha,\beta)$ distribution and a production of π to a power times $(1-\pi)$ to another power. When we multiply the beta prior times the binomial likelihood, we just add the exponents of π and $(1-\pi)$, respectively. This makes using $Beta(\pi|\alpha,\beta)$ priors when we have binomial observations particularly easy. Such priors that make their posteriors of the same form, is called *'conjugate family of priors'*. Such *'conjugate priors'* play an important role in bayesian statistics by simplifying numerical computations significantly. Through our discussion, we explained that beta distribution is the conjugate family for the *'binomial'* observation distribution. Another advantage of the beta distribution is that it offers various shapes according to its indexed parameters α and β as shown in Figure 3.6. In fact, the uniform distribution $g(\pi) = 1$ used in (3.2) is a special case of beta distribution that is indexed with $\alpha = 1$ and $\beta = 1$, namely, $Beta(\pi|1,1)$ (see Figure 3.6-(b)). To explain this advantage further, it is worth to remind the fundamental idea of bayesian statistics.

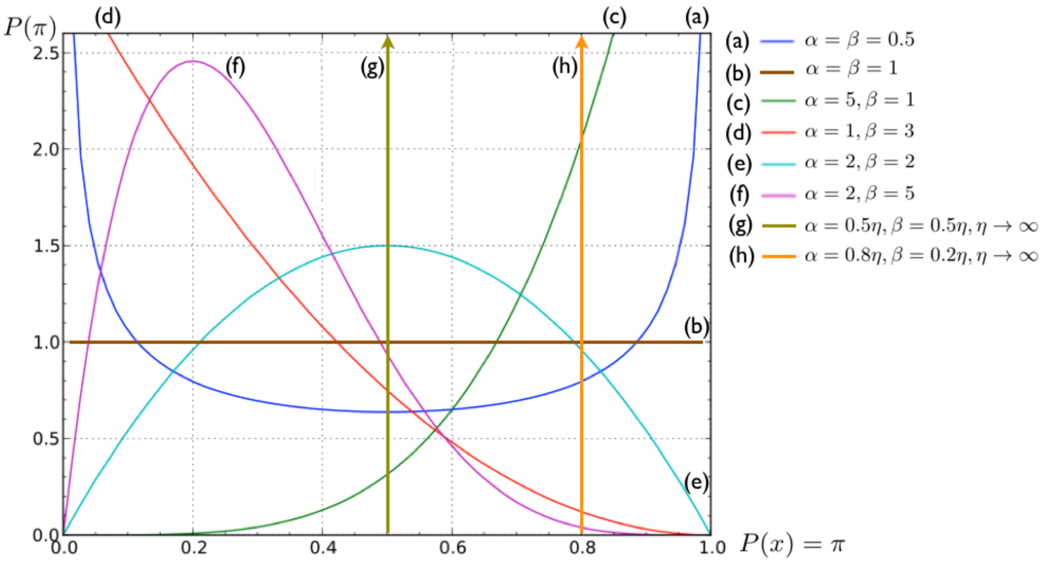


Figure 3.5: Examples of Beta Distributions.

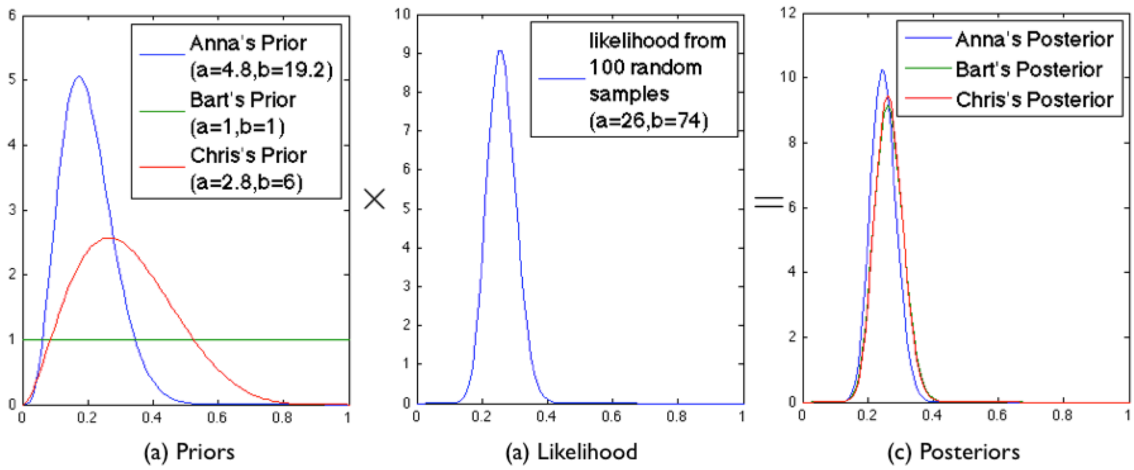


Figure 3.6: Examples of Applying Bayes' Theorem to Different Priors and Given a Likelihood.

The essence of Bayesian statistics, as explained above, is the use of both the *prior* knowledge and the experimental result called *likelihood*, to get the *posterior* knowledge. For example, assume that we have tossed a biased coin 5 times today and got 2 times of head. Then, what is the chance that we would get head if we toss the coin tomorrow ? To infer this, ideally, it would be good if we could appropriately fuse a subjective belief or opinion about the coin and an objective experimental likelihood (2 times of head out of 5 trials). The subjective opinion in bayesian statistics is called *'subjective probability'* [33]. In general, the subjective probability is not a single probability value but a distribution over all possible probability values. Therefore, the characteristics that beta family of distributions can form a various shapes give us high degree of flexibility on modeling the desired prior density. Further, it is also known that some of other types of well known priors can be also approximated to beta distributions [33]. For instance, Alfers et al. introduced a normal (gaussian) approximation for beta and gamma probabilities [9], Teerapabolarn introduced a poisson approximation to the beta binomial distribution [164], etc. (refer to [33] for more details on beta distributions, conjugated priors and bayesian statistics).

In bayesian statistics, however, the effect of the prior we choose will be small when we have a likelihood from enough data. For example, consider three students Anna, Bart and Chris are constructing their prior belief about the proportion of residents in their city who support building a casino (this example is shown in [33]). Based on their subjective beliefs, let us assume that they have chosen weak priors as shown in Figure 3.6 - (a). Then assume that they took a random sample of $n = 100$. Out of the random sample, 26 said they support and 74 said they do not support building a casino. Figure 3.6 - (b) shows this likelihood. Despite starting with different priors, when they are considered with the likelihood, the three posteriors shown in Figure 3.6 - (c) show very similar posteriors. In other words, in general, putting roughly shaped *'subjective prior'* is considered to be enough in case we have reasonable amount of data or reasonably strong likelihood distribution [33].

Audun Jøsang introduced the mapping between beta distribution $Beta(\Theta|\alpha, \beta)$ and subjective opinion $w_x = (b_x, d_x, i_x, a_x)$ [92, 98]. Consider a beta distribution over the frame (state space) Θ and let π be the probability that $x \in \Theta$ could happen. Replacing $r = \alpha - 1$ number of positive observations (success) and $s = \beta - 1$ number of negative observations (failure) in Equation 3.3, regarding the proposition x in the frame Θ, we get,

$$Beta(\pi|r, s) = \frac{\Gamma(r+s+2)}{\Gamma(r+1)\Gamma(s+1)}\pi^r(1-\pi)^s, for\ 0 \leq \pi \leq 1, r \geq 0, s \geq 0. \tag{3.7}$$

In above case, we have considered a default atomicity $a_x = \frac{1}{2}$, so we had $\alpha = r+1 = r+2\cdot\frac{1}{2}$ and $\beta = s+1 = s+2\cdot(1-\frac{1}{2})$. If we consider arbitrary atomicity a_x, the index parameters becomes $\alpha = r + 2 \cdot a_x$ and $\beta = s + 2 \cdot (1 - a_x)$. By this, it becomes as follows,

$$Beta(\pi|r, s, a_x) = \frac{\Gamma(r+s+2)}{\Gamma(r+2a_x)\Gamma(s+2(1-a_x))}\pi^{r+2a_x-1}(1-\pi)^{s+2(1-a_x)-1}. \tag{3.8}$$

Then the probability expectation of Equation 3.8 is known to be as $E(\pi) = \frac{(r+2a_x)}{(r+s+2)}$. Similarly, by the Definition 11, $Ew_x = b_x + a_x i_x$. Then we need to make the two notations of E to be the same (i.e., $E(w_x) = b_x + a_x i_x = E(\pi) = (r + 2a_x)/(r + s + 2)$).

$$\begin{cases} b_x + \cancel{a_x} i_x = r/(r+s+2) + 2\cancel{a_x}/(r+s+2) \\ b_x + d_x + i_x = 1 \end{cases}$$

$\therefore$ by above and *'affinity'* we get,

$$\begin{cases} b_x = r/(r+s+2), \text{increasing function of } r \\ d_x = s/(r+s+2), \text{increasing function of } s \\ i_x = 2/(r+s+2), \text{decreasing function of } (r,s) \\ a_x = \text{base rate of } x \end{cases} \tag{3.9}$$

Now, remember that we have replaced the parameters α and β in Equation 3.7, with $\alpha = r + 2 \cdot a_x$ and $\beta = s + 2 \cdot (1 - a_x)$, by this and above (3.9) we can get parameters r, s, α and β as follows,

1. when $i_x \neq 0$,

$$\begin{cases} r = 2b_x/i_x \to \alpha = 2b_x/i_x + 2 \cdot a_x & s = 2d_x/i_x \to \beta = 2d_x/i_x + 2\cdot(1-a_x) \\ 1 = b_x + d_x + i_x & a_x = \text{base rate of } x \end{cases}$$

$\therefore Beta(\pi|\alpha, \beta) = Beta(\pi|2b_x/i_x + 2 \cdot a_x, 2d_x/i_x \to \beta = 2d_x/i_x + 2 \cdot (1 - a_x))$.

2. when $i_x = 0$, (3.10)

$$\begin{cases} r = 2b_x/i_x \to \alpha = 2b_x\eta & s = 2d_x/i_x \to \beta = 2d_x\eta \quad where,\ \eta \to \infty \\ 1 = b_x + d_x & a_x = \text{base rate of } x \end{cases}$$

$\therefore Beta(\pi|\alpha, \beta) = Beta(\pi|b_x\eta, d_x\eta) = \delta(t - b_x)$

where, $\delta(t - b_x)$ is the Dirac Delta function such that, $\begin{cases} +\infty,\ t = b_x \\ 0,\ t \neq b_x \end{cases}$.

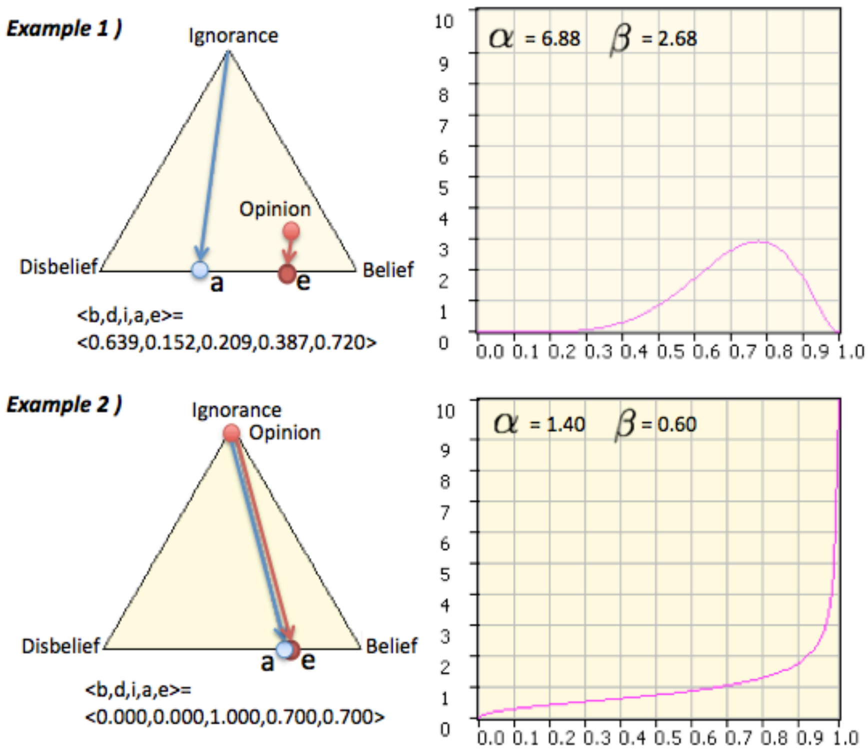


Figure 3.7: Examples of Opinion Triangle and Their Mapping to Beta Distribution.

This means, for example, that an opinion with $b_x = 0$, $d_x = 0$, $i_x = 1$ and $a_x = 0.5$ which maps to $Beta(1, 1)$ is equivalent to a uniform probability density function (see Figure 3.6-(b)). It also means that a dogmatic opinion with $i_x = 0$ which maps to $Beta(b_x\eta, d_x\eta)$ where $\eta \to \infty$ is equivalent to a spike probability density function (i.e., dirac delta) with infinitesimal width and infinite height at b_x (see Figure 3.6-(g) and (h)). Dogmatic opinions in beta distribution perspective means that infinite amount of evidences converges to the ratio of $\frac{\alpha}{\alpha+\beta}$ [3] [98].

In Figure 3.7, example 1) shows an opinion about a proposition of an agent, that can be interpreted as *'seems likely and slightly uncertain true'*, and example 2) shows full ignorance (a.k.a. *'vacous'* opinion) at the time of judgement about a proposition. Assuming base rate to be 0.7 in the example we get expectation value also to be 0.7 and the beta

[3] however, we introduce a bound to avoid the actual numerical computation of infinity value and to visualize the distribution. Currently, we assign a very tiny value such as $i_x = 1 \times 10^{-5}$ for numerical calculation, when $i_x \to 0$.

distribution appears biased towards 'True' though the opinion represents full ignorance.

3.1.4 Operators in Subjective Logic

Subjective logic is a generalization of binary logic and probability calculus. This means that when a corresponding operator exists in binary logic, and the input parameters are equivalent to binary logic TRUE or FALSE, then the result opinion is equivalent to the result that the corresponding binary logic expression would have produced. Table 3.2 provides a brief overview of the main subjective logic operators. Additional operators exist for modeling special situations, such as when fusing opinions of multiple observers. Most of the operators correspond to well-known operators from binary logic and probability calculus, whereas others are specific to subjective logic. For example addition is simply a generalization of addition of probabilities. However, subjective logic also comes with non-traditional logic operators. For example, deduction[95, 100], abduction[95, 139], discounting [99] operators are generalization of Bayesian interpretation of conditional probability and fusion of different sources of observations. Cummulative (average) fusion (unfusion) [99, 95] operators are generalization that is inspired from the dempster shafer's belief fusion rule [149] (this particular operators will be further discussed in Chapter 5).

Apart from the computations on the opinion values themselves, subjective logic operators also affect the attributes, i.e. the subjects, the propositions, as well as the frames containing the propositions. In general, the attributes of the derived opinion are functions of the argument attributes. Following the principle illustrated in Figure 3.8. the derived proposition is typically obtained using the propositional logic operator corresponding to the subjective logic operator. For example, consider two frames of discernment $X = \{healthy, cough, fever, dizzy\}$ and $Y = \{dayworker, nightworker\}$, and a doctor A examining a patient in terms of X and Y respectively. Then the frame composition of X and Y will be the cartesian product of X and Y connected through one of above logical operators, and $f_{pl}(x, y)$ will be one of possible elements in f_{FC}. Similarly, f_{sc} will simply become A because in this case we are considering only one agent A. Therefore,

Subjective Logic Operator	Symbol	Propositional / Binary Logic Operator	Symbol	Subjective Logic Notation
Addition [115]	$+$	XOR	$\cup$	$w_{x\cup y} = w_x + w_y$
Subtraction [115]	$-$	Difference	$\backslash$	$w_{x\backslash y} = w_x - w_y$
Multiplication [97]	$\cdot$	AND	$\wedge$	$w_{x\wedge y} = w_x \cdot w_y$
Division [97]	$/$	UN-AND	$\overline{\wedge}$	$w_{x\overline{\wedge y}} = w_x / w_y$
Comultiplication [97]	$\sqcup$	OR	$\vee$	$w_{x\vee y} = w_x \sqcup w_y$
Codivision [97]	$\overline{\sqcup}$	UN-OR	$\overline{\vee}$	$w_{x\overline{\vee}y} = w_x \overline{\sqcup} w_y$
Complement [93]	$\neg$	NOT	$\overline{x}$	$w_{\overline{x}} = \neg w_x$
Deduction [95, 100]	$\circledcirc$	MP	$\Vert$	$w_{y\Vert x} = w_x \circledcirc w_{y\vert x}$
Abduction [95, 139]	$\overline{\circledcirc}$	MT	$\overline{\Vert}$	$w_{y\overline{\Vert}x} = w_x \overline{\circledcirc} w_{x\vert y}$
Discounting [99]	$\otimes$	Transitivity	$:$	$w_x^{A:B} = w_x^A \otimes w_x^B$
Cumulative Fusion [99]	$\oplus$	n/a	$\diamond$	$w_x^{A\diamond B} = w_x^A \oplus w_x^B$
Cumulative Unfusion [96]	$\ominus$	n/a	$\overline{\diamond}$	$w_x^{A\overline{\diamond}B} = w_x^A \ominus w_x^B$
Average Fusion [99]	$\underline{\oplus}$	n/a	$\underline{\diamond}$	$w_x^{A\underline{\diamond}B} = w_x^A \underline{\oplus} w_x^B$
Average Unfusion [95]	$\underline{\ominus}$	n/a	$\overline{\underline{\diamond}}$	$w_{x\cup y}^{\overline{\underline{\diamond}}} = w_x^A \underline{\ominus} w_x^B$

Table 3.2: Subjective Logic Operators, Notations, and Corresponding Propositional / Binary Logic Operators.

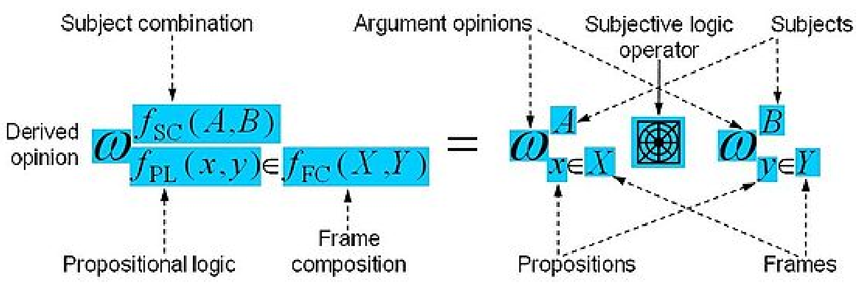


Figure 3.8: Composition of Subjective Logic Operators - A Propositional Logic View.

for instance, $w^A_{healthy\wedge dayworker} = w^A_{healthy} \cdot w^A_{dayworker}$ deals with two focused propositions '*healthy*' and '*dayworker*' in X and Y. Then doctor A would have an opinion about the proposition $healthy \wedge dayworker$ to the amount of opinion that is calculated via subjective logic's multiplication operator '$\cdot$'.

The functions for deriving attributes depend on the operator. Some operators, such as cumulative and averaging fusion, only affect the subject attribute, not the proposition which then is equal to that of the arguments. Fusion for example assumes that two separate argument subjects are fused into one. Other operators, such as multiplication, only affect the proposition and its frame, not the subject which then is equal to that of the arguments. Multiplication for example assumes that the derived proposition is the conjunction of the argument propositions, and that the derived frame is composed as the Cartesian product of the two argument frames. The transitivity operator is the only operator where both the subject and the proposition attributes are affected, more specifically by making the derived subject equal to the subject of the first argument opinion, and the derived proposition and frame equal to the proposition and frame of the second argument opinion.

It is impractical to explicitly express complex subject combinations and propositional logic expressions as attributes of derived opinions. Instead, the trust origin subject and a compact substitute propositional logic term can be used.

Subject combinations can be expressed in a compact or expanded form. For example, the transitive trust path from A via B to C can be expressed as $A : B : C$ in compact form, or as $[A, B] : [B, C]$ in expanded form. The expanded form is the most general, and corresponds directly with the way subjective logic expressions are formed with operators.

3.2 Logic Formalisms and Logic Programming

In this section we provide a brief introduction to fundamentals of logic formalisms and logic programming. In this dissertation the *'extensional'* way of knowledge representation and uncertainty handling are based on the extension of logic programming with subjective logic formalism. While subjective logic also is a logic, it's expressive power is remain within the expressive power of propositional logic as of many *'intensional'* approaches. In this dissertation, to cope with complex semantics, we adopt *'predicate'* logic that is based on first-order logic. In the following, we give a preliminary introduction to propositional logic, first-order predicate logic and logic programming.

3.2.1 Propositional Logic

A propositional logic is the most basic branch of mathematical logic in which the only objects are propositions, that is, objects which themselves have truth values. Variables represent propositions, and there are no *'relations'*, *'functions'*, or *'quantifiers'* except for the constants T and F (representing true and false respectively). Propositional logic comes with four basic connectives $\neg$, $\wedge$, $\vee$ and $\rightarrow$ (representing negation, conjunction, disjunction and implication). A model for propositional logic is just a *'truth function'* on a set of variables. Such a truth function can be easily extended to a truth function on all formulas which contain only the variables are defined on by adding recursive clauses for the usual definitions of connectives. Propositional logic is decidable. There is an easy way to determine whether a sentence is a tautology (i.e., a proposition that is true for all possible interpretations). It can be done using truth tables, since a truth table for a particular formula can be easily produced, and the formula is a tautology if every assignment of truth values makes it true.

Formally speaking, propositional formulae (or propositions) are strings of symbols from a countable alphabet as defined in Definition 12, and formed according to certain rules stated in Definition 13

Definition 12. ***(The Alphabet for Propositional Formulae) .*** *this alphabet consists of:*

(1) A countable set PS of propositional symbols : P_0, P_1, P_2, ...

(2) The logical connectives : $\wedge$ (and), $\vee$ (or), $\rightarrow$ (implication), $\neg$ (not) and sometimes $\equiv$ (equivalence).

(3) Auxiliary symbols : '(', ')' (left and right parenthesis respectively).

Definition 13. ***(Propositional Formulae) .*** *The set $PROP$ of propositional formulae (or propositions) is the inductive closure of the set $PS \cup F$ under the functions $C_\neg$,$C_\wedge$,$C_\vee$,$C_\rightarrow$ and $C_\equiv$, defined as follows : For any two strings A, B over the alphabet of Definition 12,*

$C_\neg(A) = \neg A$, $C_\wedge(A, B) = (A \wedge B)$, $C_\vee(A, B) = (A \vee B)$, $C_\rightarrow(A, B) = (A \rightarrow B)$ and $C_\equiv(A, B) = (A \equiv B)$.

The truth table in propositional logic can be generalized into continuous space by replacing the truth values and logical connectives with for instance, probability values and probabilistic calculation. For instance, given propositional sentences A and B, $\cdot$ for

$\wedge$, $+$ for $\vee$, complement for $\neg$ and $P(B|A)$ for $A \rightarrow B$. Similarly, subjective logic also corresponds to propositional logic sense in that it deals with propositions. In this sense, *'intensional'* approaches that models propositional logic sentences in a graphical model can also be understood as propositional logic that is generalized to handle continuous truth values.

3.2.2 First Order Predicate Logic

In propositional logic, it is not possible to express assertions about elements of a structure. The weak expressive power of propositional logic accounts for its relative mathematical simplicity, but it is a very severe limitation, and it is desirable to have more expressive logics. First-order predicate logic is a considerably richer logic than propositional logic, but yet enjoys many convinient mathematical properties. For example, in propositional logic the proposition *'John is tall'* can not be decomposed into a simple sentence because there is no logical connectives implied in the proposition. In first-order predicate logic, it is allowed to decompose the proposition into predicates and individuals as *'tall (john)'*. First-order predicate logic also allows to handle expressions of generalization using quantificational expressions. For example, propositions *'Every cat is sleeping'*, *'Some girls likes David'* or *'No students are happy'* are possible.

In first-order logic, assertions about elements of structures can be expressed. Technically, this is achieved by allowing the propositional symbols to have arguments ranging over elements of structures. For convenience, we also allow symbols denoting functions and constants.

Following is the formal definition of the syntax of first-order predicate logic.

Definition 14. ***(The Alphabet for First-Order Predicate Formulae) .*** *this alphabet consists of the following sets of symbols:*

(1) Variables : A countably infinite set $V = \{x_0, x_1, x_2, ...\}$

(2) The logical connectives : $\wedge$ *(and),* $\vee$ *(or),* $\rightarrow$ *(implication),* $\neg$ *(not),* $\equiv$ *(equivalence) and quantifiers* $\forall$ *(for all),* $\exists$ *(there exists)*

(3) Auxiliary symbols : '(', ')' (left and right parenthesis respectively).

A set L of nonlogical symbols consisting of:

(i) Function symbols: A (countable, possibly empty) set FS of symbols $f_0, f_1, ...,$ and a rank function r assigning a positive integer $r(f)$ (rank or arity) to every function symbol f.

(ii) Constants: A (countable, possibly empty) set CS of symbols $c_0, c_1, ...,$ each of rank zero.

(iii) Predicate symbols: A (countable, possibly empty) set PS of symbols $P_0, P_1, ...,$ and a rank function r assigning a nonnegative integer $r(P)$ (called rank or arity) to each predicate symbol P.

Definition 15. ***(First Order Predicate Formulae** L) .* *A First Order Formulae L over the alphabet A of Definition 14 is the collection of all WFFs (Well formed formulas) that can be constructed from the alphabet A.*

Based on the formal definition given above, the aforementioned example propositions can be defined as follows :

- John is tall : tall (john)
- Every cat is sleeping : $\forall.X[cat(X) \rightarrow sleeping(X)]$
- Some girls likes david : $\exists.X[girl(X) \rightarrow love(X, david)]$
- No students are happy : $\forall.X[student(X) \rightarrow \neg happy(X)]$

3.2.3 Logic Programming

Logic Programming is, in its broadest sense, the use of declarative mathematical logic for computer programming. However, logic programming, in the narrower sense in which it is more commonly understood, is the use of *'predicate'* logic as both a declarative and procedural representation language (in the sense in which it also supports procedural *'predicate functions'*). Logic Programming considers logic theories of a specific form. The theories, called logic programs, mainly consists of two types of logical formulae, rules and facts.

Rules are of the form '$A \leftarrow f_0, f_1, ..., f_m$' where A is rule head and the right hand side is called body. Each f_i is an atom and ',' represents logical conjunction. Each atom is of the form '$p(t_1, t_2, ..., t_n)$', where t_i is a term and p is a predicate symbol of arity n. Terms could either be variables or constant symbols. In practical implementation of logic programming language, it also supports defining procedural *'predicates functions'* and some of reserved predicates are used as *'predicate functions'* to check equality of the terms (e.g., $eq(t_1, t_2)$ returns true when t_1 and t_2 are the same in CLIPS [1]) or to compare/calculate arithmetic values between terms of predicates (e.g., $+(t_1, t_2)$ the reserved predicate '+' acts as a function returns value of $t_1 + t_2$, similarly, the functional predicate leq, $leq(t_1, t_2)$ returns true in case that the condition $t_1 \leq t_2$ is satisfied in CLIPS). There is one special term, called the anonymous term, for which the underscore (_) character is used. When the character is used, it basically means that it does not care which variable or symbol it is bound to, as long as it is bound to something. Rules of the form '$f \leftarrow$' (denoted by just 'f') is called facts and can serve as an atom when used in a rule body. Negation is represented with the symbol '¬' such that '¬¬A=A'. Both positive and negative atoms are referenced to as literals. Given a rule '*head* ← *body*', we interpret the meaning as *'IF body THEN head'*. Traditionally, a resolved set of facts that matches to a rule is called *'extension'*. In logic programming language based visual surveillance applications as the ones mentioned in Section 2.4.3, rules have been used to define and reason about various contextual events or activities.

3.2.4 Issues on Supporting Non-monotonicity

Classical logic is *'monotonic'* in the following sense: whenever a sentence A is a logical consequence of a set of sentences T , then A is also a consequence of an arbitrary superset of T . In other words, adding information never invalidates any conclusions. However, reasoning under *'uncertain knowledge'* (as of our interest for reasoning in visual surveillance) is different. We often draw plausible conclusions based on the assumption that the world in which we function and about which we reason is normal and as expected. This is far from being irrational. To the contrary, it is the best we can do in situations in which we have only incomplete information. However, as unexpected as it may be, it can happen that our normality assumptions turn out to be wrong. New information can show that the situation actually is abnormal in some respect. In this case we may have to revise our conclusions. Such reasoning, where additional information may invalidate conclusions, is called *'nonmonotonic'*. It has been a focus of extensive studies by the knowledge representation community since the early eighties of the last century. This interest was fueled by several fundamental challenges facing knowledge representation such as modeling and reasoning about rules with exceptions or defaults. Another important class of *'nonmonotonic'* reasoning is *'abduction'* that is to draw plausible explanation supporting given observation. Therefore, according to the given set of observations such reasoning need to be able to rate the plausibility among possible solutions.

To cope with such *'non-monotonic'* behavior, in this dissertation, we adopt subjective logic for uncertainty representation formalism. However, subjective logic itself is remain within the expressive power of propositional logic as of may *'intensional'* approaches. In this dissertation to cope with complex semantics, we adopt *'predicate'* logic that is based on first-order logic. For the flexible manipulation of facts and predicates on the need of arithmetic calculation, etc., we also benefit from procedural handling of them. Therefore, the chosen approach is the use and extension of Logic Programming.

4 Proposed Architecture and Case Study on Forensic Queries

Thus far, we have reviewed fundamental background of this dissertation. The discussions covered paradigm changes in intelligent visual surveillance, technical challenges and requirements toward intelligent visual surveillance system, prior art and related work appear in literature and important preliminaries. Bearing the background of this work in mind, this chapter starts with the architectural aspect of our approach to high level semantic reasoning in visual surveillance system.

4.1 Introduction

In this chapter, we present our conceptual system architecture supporting high-level semantic analysis of visual surveillance data. Taking pragmatic view, efficient semantic analysis of visual surveillance data requires a system oriented approach that optimally combines the individual legacy vision analytic power and its optimal use. Regarding this objective, in Section 1.2.4, we have discussed following main requirements : 1) Metadata representation and reuse. 2) Semantic knowledge representation. 3) Uncertainty representation and attachment to metadata. 4) Epistemic uncertainty representation and belief revision. 5) Epistemic meta reasoning and abduction. Therefore, reminding the discussion, the main motivation of the proposed system architecture is to bring components corresponding to each requirement. In terms of functionality, above requirements can be roughly fall into two parts: *'1) knowledge representation and reuse'* and *'2) epistemic reasoning mechanism'*. In this chapter, we give more focus on *'knowledge representation and reuse'* introducing a *'data model'* for contextual metadata description. Regarding the *'epistemic reasoning mechanism'*, we introduce our framework of *'logic programming with subjective logic'*, and the detailed aspect on particular reasoning mechanisms will be focussed in the rest of chapters.

We first start with discussion on the software components, data processing pipeline and system architecture design. Then we introduce our prototype implementation and present some of case study on forensic query scenarios. We conclude with discussions on *'scalability'* issues by conducting performance comparison between different settings of *'data models'* and *'rule-engines'*.

4.2 Proposed Architecture

Figure 4.1 shows the overall architecture proposed base on the discussed critical requirements. The details are as follows.

- **Metadata Representation and Reuse:** Proper reuse of results generated from each of individual vision analytic modules introduces *'interoperability'* problem among *'heterogeneous vision analytics'*. To resolve this issue, individual vision analytics should share a principled description method. As introduced in Section 2.2.3, there have been remarkable research efforts on the formal description for the sharable knowledge in the field of *'ontological knowledge description'* or *'semantic web'*. Following this recent trend, we also introduce formal description of metadata. The ontological knowledge description on possible metadata plays an important role in providing a machine process-able representation: it provides the basis to enable fetching and collecting meaningful subsets of information from unstructured data sets by giving specific and sharable semantics to terminologies. Description Logic [24] and its encoding format RDF/OWL-DL [76] is a good candidate due the widely available supporting tools (such as OWL-DL reasoners, triple stores and SPARQL query APIs). Along the guideline of metadata ontology, intermediate video analysis results will get packaged as ontological metadata instances.

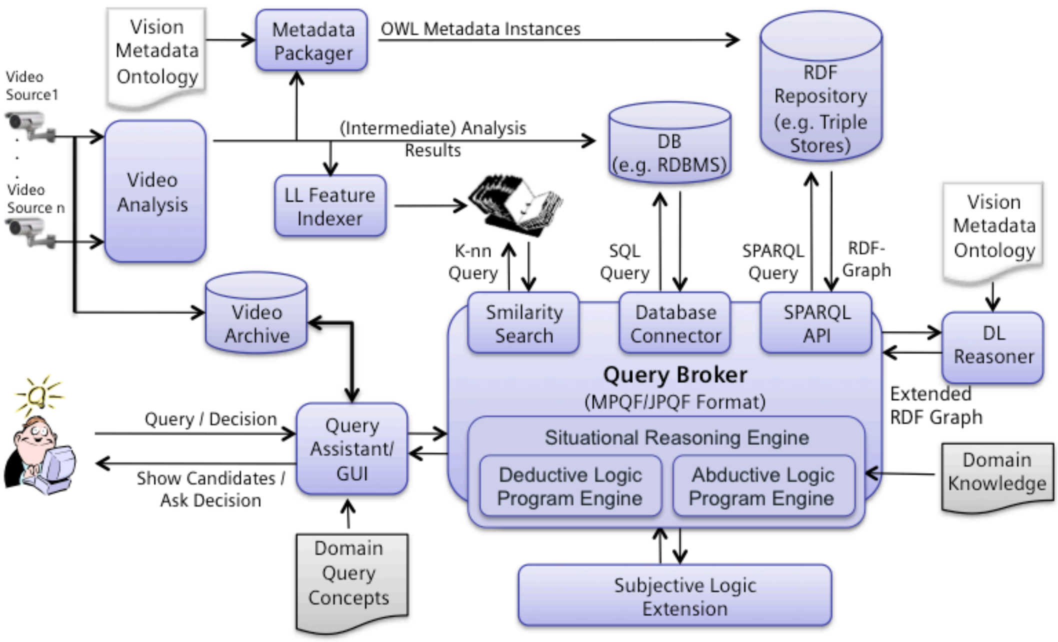


Figure 4.1: Conceptual Architecture of the Proposed Intelligent Forensic Reasoning System.

Packaged metadata instances will get fed into the archival layer, where the low-level feature vectors are indexed in an appropriate index structure [31] and ontological instances will be archived into triple store [103] backed with a relational data base (RDBMS).

- **Semantic Knowledge Representation:** To address non-static behavior that causes difficulties to apply predefined state model based approaches, the logic programming approach [104] is exploited (see Section 2.3 and Section 2.4 for the detailed comparison between *'intensional'* and *'extensional'* approaches). In declarative logic programming, the detailed procedures and instructions are omitted but what the machine should do is described. The steps to solve the given problem are executed by a kind of a runtime system (called rule engine or logical reasoner) that understands how to use the declarative information. Therefore, the models for solving the problems are shifted from a set of procedures to a set of rules. Taking this advantage, contextual knowledge about a visual surveillance scenario can be represented as sets of rules. Each rule can be considered independent therefore, adding/modifying/updating rules can be done flexibly without hitting whole knowledge structure as of state model based approaches (see the *'modularity'* concept in Section 2.3).

- **Uncertainty Representation and Attachment to Metadata** : As discussed in Section 1.2.4, vision analytic metadata usually comes with *'uncertainty'* and therefore influence the acceptable accuracy and robustness for a semantically complex queries. Considering the popularity of *'probability theory (especially, Bayesian probability)'* in state-based models (such as FSMs, BNs, HMMs, DBNs, etc. see Section 2.4.2 for details.), and also considering the popularity of *'state-based models'* in vision analytic modules, the resulting values tend to represent probability of being true about target semantics of vision analytic modules. Therefore, the values normally lay in the interval $[0, 1]$. In logic programming, such metadata can be seen as facts (see Section 3.2.3) and the facts should be fed into rule-engine with the values attached. Our approach is to encode such values as subjective logic's opinion [93]. Subjective logic is compatible to the traditional probability theory but can also handle *'uncertainty about the probability itself'*. This way of *'uncertainty'* representation and attachment will be referred to as *'opinion assignment to metadata'*.

- **Epistemic Uncertainty Representation and Belief Revision:** The logic programming approach, as an inference mechanism, generally works on binary facts which can be seen as either true or false. As discussed above, however, the facts will come with *'subjective opinion'* in our design. In addition, semantic knowledge encoded as rules can not be considered as absolute ones. Rather such rules are also epistemically uncertain. Thus, we need a means to represent the strength or belief on a particular rule as well. As *'opinion assignment to metadata'*, we will also attach an epistemic belief to each rule using subjective logic opinion. This way of *'epistemic uncertainty representation'* will be referred to as *'opinion assignment to rule'*. Besides assigning opinions to information (i.e., metadata in terms of vision analytics, and that can be seen as a fact in terms of logic programming) and rules, an additional step is needed to deal with the uncertain aspect of the gathered information and rules. For example, evaluating the amount of information sources towards the same conclusion, the amount of information sources contradicting to others, reputational quality of a given uncertainty value or errors in the generation of metadata should be dealt with appropriate formalism. Ultimately, it should be also possible to revise current belief upon arrival of new information and aforementioned evaluations. We use subjective logic due to its rich set of logical operators such as consensus, discounting, averaging fusion etc., which are not available in traditional propositional/probabilistic logic (see Chapter 5, Chapter 6 and Chapter 7 for more details).

- **Epistemic Meta Reasoning and Abduction:** As shown in Section 1.2.3, for large level of semantic granularity queries, it requires a *'meta-reasoning'* (i.e., reasoning about reasoning) power that reason about the possible sub queries from a set of observation. Thus, such reasoning should be done by setting possible semantic hypotheses upon given observations and assessing each hypothesis in an iterative manner. This aspect resembles diagnostic reasoning which the so called *'abductive (constraint) logic programming'* [50] plays an important role. Therefore, we embed the capability of *'abductive logic programming'* inside our rule engine. Both normal logic programming based reasoning and abductive logic programming lay in the whole reasoning pipeline to interact each other (see Chapter 8 for the details).

- **Query Representation:** Although it was not explicitly discussed as a requirement in Section 1.2.4, one fundamental aspect of such a high level semantic analysis system should deal with is the method to deliver user intention into a system. Machine-processable query representation such as MPQF [53] can be considered as a candidate format with appropriate query interface. Once a query is delivered, an agent should decide where to access to fetch proper information in case there are several information sources. We name this particular conceptual agent as *'Query Broker'* (and a cooperative project is undergoing on this issue). However, we will leave this particular component out of scope in this dissertation but rather focus on above mentioned items.

4.3 Prototype Implementation

As proof of concept, a demonstrator along the afore introduced architectural approach has been built. In this section, we mainly focussed on metadata representation and inference mechanism upon acquired metadata instance among all requirements. We first describe the conceptual ontological metadata description model, Next the subjective logic extension of logic programming is described.

4.3.1 Ontological Metadata Model

An ideal high-level semantic analysis of visual surveillance system should enable optimal reuse of currently available assets of vision analytic power. In reality, various vision analytics could exist and deployed from varying vendors or legacy systems. In this sense, proper reuse of intermediate results generated from each of individual vision analytic modules introduces *'interoperability'* problem among *'heterogeneous vision analytics'*. To resolve this issue, individual vision analytics should share a principled formal metadata description method. This metadata description should be also considered in the sense of logical reasoning. The logical reasoning framework usually gets list of facts and then returns derived knowledge. Moreover, each fact should be described with specific semantics. However, considering potentially large scale everyday data, we can not feed all of them into the rule engine. Rather, we need to be able to prune out most of the unnecessary metadata except the ones potentially related to the desired reasoning task. Therefore, selecting an appropriate list of facts from a bunch of surveillance metadata also requires understanding semantics of each metadata items. Thus, to maximize the potential use of metadata, we follow an ontological data representation and storing approach that captures dominant concepts and relations employed in our surveillance system. The proposed data model supports SPARQL [168] query for data segment selection.

Six basic ontologies have been created. The first one is formed by data describing the surrounding conditions and environment, the "*Surveillance System Ontology*". The next "*Source Media Ontology*" covers fundamental concepts and relations of possible input data and sensors. The "*Color Model*" represents fundamental concepts and relations concerning color model and is required for the similarity-based sub-query based on the MPEG-7 Color Structure Descriptor [87]. For low-level features, we used slightly modified version of "*MPEG-7 Visual Descriptor Ontology*" from AceMediaProject [156, 133]. While the "*Video Analytics*" ontology models fundamental concepts and relations concerning the applied video analysis algorithms, the "*Domain Top Ontology*" serves basic concepts and relations for metadata representations.

Figure 4.2 shows a partial description of these ontologies (For the ontology description we follow one of well-known knowledge engineering methodology called ORSD (Ontology Requirement Specification Document) [163]). The whole graphical ontology structure is represented in Figure 4.3.

2.2.3.4 Video Analytics Ontology

Domain

- Vision Analytic Module that is used to detect/analyze a primitive concept in a video scene

Goal

- The ontology represents fundamental concepts and relations to depict Vision Analytic Module

Domain and Scope

- Vision Analytic Module including its internal algorithm and intrinsic configuration parameters

Supported Applications

- Metadata generator such as Smart Cameras or Video Analytic Algorithms

Concept Description

1. Analytic Module:

   A software module implementing observation and understanding algorithm on input data from various sensors.

   An analytic module consists of several algorithms
2. Algorithm:

   A series of computational steps.
3. Feature:

   A way of representing prominent aspect of visual entity
4. Configuration:

   The profile of intended status of analytic module

Ontology Packages

Namespace allocation:

In the context of theseus research project

1. The computer vision analytic aspect is represented in the ontology named "Sicoosle_VisionAnalytics.owl"

Concept Description

Formal Definition:

$$\begin{aligned} Algorithm &\sqsubseteq \exists hasConfiguration.AlgorithmConfiguration \\ &\sqsubseteq \exists hasFeature.Feature \\ AnalyticModuleCamera &\sqsubseteq \exists hasAlgorithm.Algorithm \\ &\sqsubseteq \exists hasConfiguration.AnalyticConfiguration \end{aligned}$$

Graphical Representation:

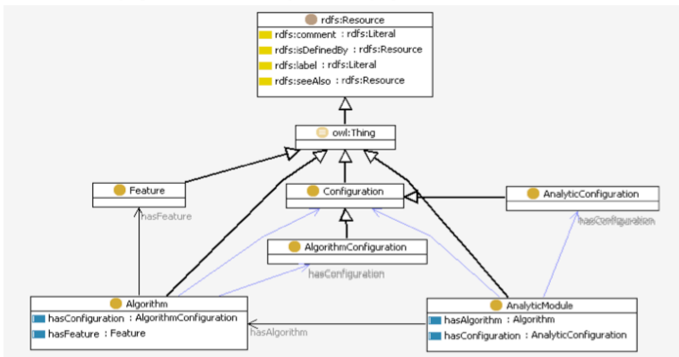


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Figure 4.2: A Piece of the *'Video Analytics'* Ontology Description in ORSD (Ontology Requirement Specification Document) [163] Format.

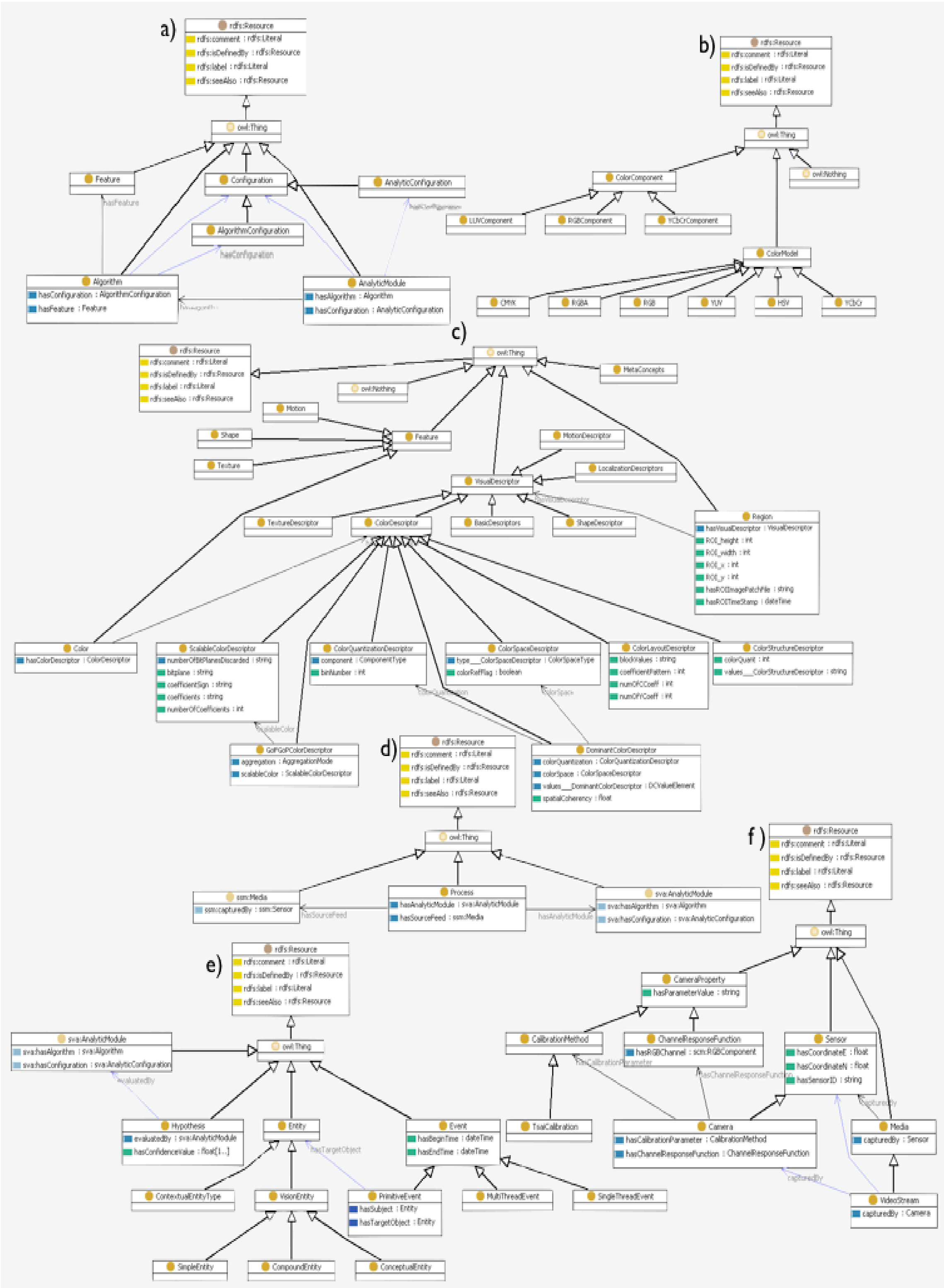

Figure 4.3: Graphical Representation of Ontological Metadata Model: a) Video Analytics, b) Color Model, c) MPEG-7 Visual Descriptor, d) Surveillance System, e) Domain Top and f) Source Media Ontologies respectively.

4.3.2 Logic Programming Extended with Subjective Logic

To extend logic programming with subjective logic, the CLIPS [1] rule engine was used as a basis to provide flexibility for defining complex data structure as well as for providing a rule resolving mechanism. To extend this system, we defined a data structure having the form of '$opinion(agent, proposition, b, d, i, a)$'. This structure can be interpreted as a fact of arity 6 with the following terms, *agent (opinion owner)*, *proposition*, *belief*, *disbelief*, *ignorance*, and *atomicity*. To represent propositions as first-order predicate logic (see Section 3.2.2 for details), we extended the structure so that it can take arity n properties as well. Therefore, given a predicate p the proposition can be described as '$p(a_1, a_2, ..., a_n)$'. In our system, therefore, each fact is represented as the form of '$w^{agent}_{p(a_1,a_2,...,a_n)}$'. Namely, rules are defined with the opinion and predicate logic based proposition structure. Additionally, functions of subjective logic operators taking opinions as parameters were defined. In this way, uncertainty in the form of opinion triangle is attached to rules and facts. This aspect is depicted as follows :

Definition 16. ***(Opinion Assignment) .*** *Given a knowledge base $\mathcal{K}$ in form of declarative language and Subjective Opinion Space O, an opinion assignment over sentences $k \in \mathcal{K}$ is a function $\phi : k \to O$. s.t*

1. $\phi_{fact} : Fact \to O$, *e.g.* $w^{a}_{p(a_1,a_2,..a_n)} = (b, d, i, a)$
2. $\phi_{Rule} : Rule \to O$, *e.g.* $(w^{a_c}_{p_c(a_{c1},..,a_{cn})} \leftarrow w^{a1}_{p_1(a_{11},..,a_{1n})}, .., w^{ai}_{p_n(a_{i1},..,a_{in})}) = (b, d, i, a)$
3. $\phi_{RuleEval} : RuleHead \to (\underset{w^{a_i}_{p_i} \in RuleBody}{\circledast} w^{a_i}_{p_i(a_{ai1},..,a_{in})} = O)$,

   where $\circledast$ indicates one of subjective logic's operators.

   e.g. for a given rule $\quad w^{a_c}_{p_c(a_{c1},..,a_{cn})} \leftarrow w^{a1}_{p_1(a_{11},..,a_{1n})}, .., w^{ai}_{p_n(a_{i1},..,a_{in})}$,

   we interpret it as $\quad w^{a_c}_{p_c(a_{c1},..,a_{cn})} = w^{a1}_{p_1(a_{11},..,a_{1n})} \circledast ... \circledast w^{ai}_{p_n(a_{i1},..,a_{in})} = (b, d, i, a)$
4. $\phi_{inference}$ *denoted* $cl(\phi) : q \to O$, *where* $\mathcal{K} \models q$ *called Closure.*

It is important to note that there are different ways of opinion assignment. While Definition 16 - 2 assigns an opinion to a whole rule sentence itself, Definition 16 - 3 assigns an opinion to the consequence part of the rule (rule head). The assigned opinion is functionally calculated out of opinions in the rule body using appropriate subjective logic operators. Definition 16 - 2 especially plays an important role for prioritizing or weighting rules for default reasoning (see Chapter 5 and [82]). Given the initial opinion assignment by Definition 16 - 1 and Definition 16 - 2, the actual inference is performed by Definition 16 - 3 and Definition 16 - 4, where Definition 16 - 4 is further defined as follows :

Definition 17. ***(Closure) .*** *Given a knowledge base $\mathcal{K}$ in form of declarative language and an opinion assignment ϕ, labeling every sentence $k \in \mathcal{K}$ into Subjective Opinion Space O, then the closure over $k \in \mathcal{K}$, is the opinion assignment function $cl(\phi)(q)$ that labels information q entailed by $\mathcal{K}$ (i.e.$\mathcal{K} \models q$).*

For example, if ϕ labels sentences $\{a, b, c \leftarrow a, b\} \in \mathcal{K}$ as $\phi_{fact}(a)$, $\phi_{fact}(b)$ and $\phi_{Rule}(c \leftarrow a, b)$, then $cl(\phi)$ should also label c as it is information entailed by $\mathcal{K}$. The assignment can be principled by the definition of closure. For example, an opinion assignment to c, in a simple conjunctive sense can be $\phi_{fact}(a) \cdot \phi_{fact}(b) \cdot \phi_{Rule}(c \leftarrow a, b)$, where $\cdot$ represent conjunction in Subjective Logic. In our system, to support the rich set subjective logic operators, we made the specification of Definition 16 - 3 in rule description as follows (note that, most of rule based systems also support describing actions in the head part of a rule) :

$$\begin{aligned}
&\textit{Rule Head (ACTION) :}\\
&\qquad\qquad \textit{Assert new Opinion } w^{a_c}_{p_c(a_{c1},..,a_{cn})},\\
&\qquad\qquad \text{where } w^{a_c}_{p_c(a_{c1},..,a_{cn})} = w^{a1}_{p_1(a_{11},..,a_{1n})} \circledast .. \circledast w^{ai}_{p_n(a_{i1},..,a_{in})}\\
&\leftarrow\\
&\textit{Rule Body :}\\
&\qquad\qquad w^{a1}_{p_1(a_{11},..,a_{1n})}, .., w^{ai}_{p_n(a_{i1},..,a_{in})}
\end{aligned} \tag{4.1}$$

Due to the redundancy that arises when describing rules at the opinion structure level, we will use abbreviated rule formulae as follows :

$$w^{a_c}_{p_c(a_{c1},..,a_{cn})} \leftarrow w^{a1}_{p_1(a_{11},..,a_{1n})} \circledast ... \circledast w^{ai}_{p_n(a_{i1},..,a_{in})} \tag{4.2}$$

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where $\circledast$ indicates one of subjective logic's operators. This way of representing rules, we can build a propositional rules that comprise opinions about a predicate as facts that can also check logical conjunction based existence of involved opinions and finally define resulted predicate with opinion attached by calculating opinion values with subjective logic operators.

4.4 Case Study I

In this section, a case study is presented to apply the described system architecture to a simple forensic retrieval scenario on assumption of wide area city surveillance. The case study is designed to show the whole work-through and end-to-end work-pipeline of the proposed system. In this case study, upon the manually annotated ontological data instances, query *Q5* in Section 1.2.3 is demonstrated.

4.4.1 Scenario Setting for Case Study I

Consider a virtual scenario described below.

> **"Who did this person talk to?"** : *Monitoring guards are watching live captured video sequences in the monitoring room. They were informed that some event happened around the area of camera 7. According to the report, a suspicious person wearing a white T-shirt was seen was witnessed. While browsing the video archive of camera 7, the guards found a person wearing a white T-shirt. They captured the frame, marked the region of the person, and then narrowed down the location where the reported event happened. By loading the query ontology, the system enables the "findTalkTo" query. This query provides an automatic search for additional persons that had been talking to the selected (suspected) person. By clicking the search button, the system starts to search to resolve "TalkTo" relation for the selected person and reports a list of key frames showing instances where the indicated person is talking to other persons. By Clicking on the key frames the referenced video sequences are displayed on a monitor for a final*

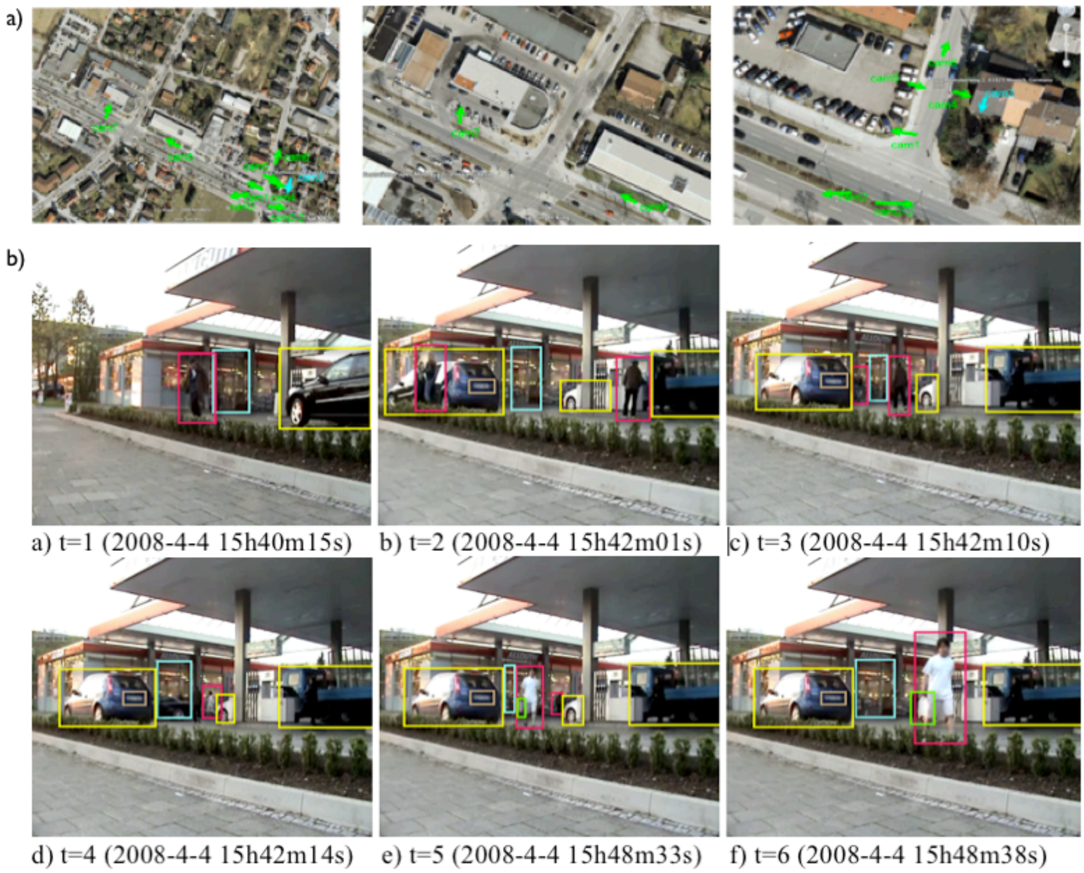

Figure 4.4: Environmental Assumption : a) Locations of Nine Assumed Cameras, b) Some of Frames from Camera 7 mounted in front of a Gas Station.

verification by the guards.

As an environmental assumption for the case study, ordinary video sequences are taken during a day from nine different locations over an area. Figure 4.4 - a) shows the distributed camera setup. Note that our test data has just normal and ordinary video sequences that have no prominent unusual behaviors causing any alarm. For example, Figure 4.4 - b) shows some of frames in our test data captured by camera 7, which is mounted in front of a gas station.

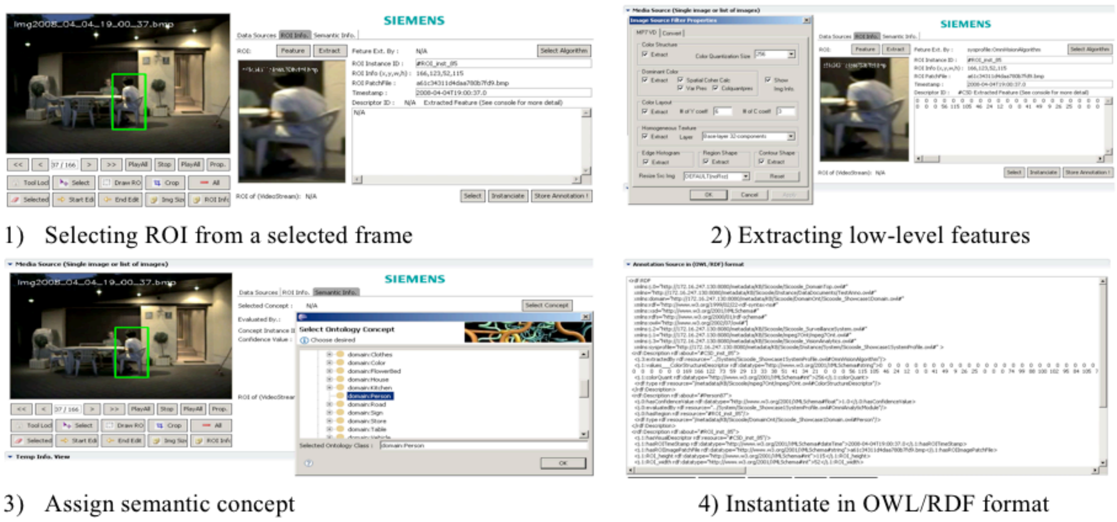

Figure 4.5: Example Usage of Our Annotation Tool.

4.4.2 Metadata Generation

In this first case study, we assume that all persons and all moving objects have been detected and annotated. An annotation tool is made to generate metadata instances along the proposed ontological metadata models. To take into account low-level features, the extraction capability of MPEG-7 visual features is also implemented. Annotation of contents using our manual annotation tool is done by selecting bounding boxes from a given video frames and designating a semantic concept defined in selected domain ontology. At the same time, to incorporate low-level features and scene structure, we use a variation of the MPEG-7 ontology (see Section 4.3.1 and [156, 133]). In order to represent the source of the detection algorithm and related properties (such as, configuration parameters and equipped camera the algorithm is running and so on) the tool allows additionally linking a selected region of interest (ROI) (bounding box) to an appropriate algorithm or cameras, etc. Figure 4.5 shows a simple example of the annotation process with a semantically labeled bounding box and the associated MPEG-7 CSD feature vector.

The generated annotation instances are encoded in RDF/OWL format. When RDF/ OWL technologies are used, the instances and owl concepts can be represented as DIG (Di-

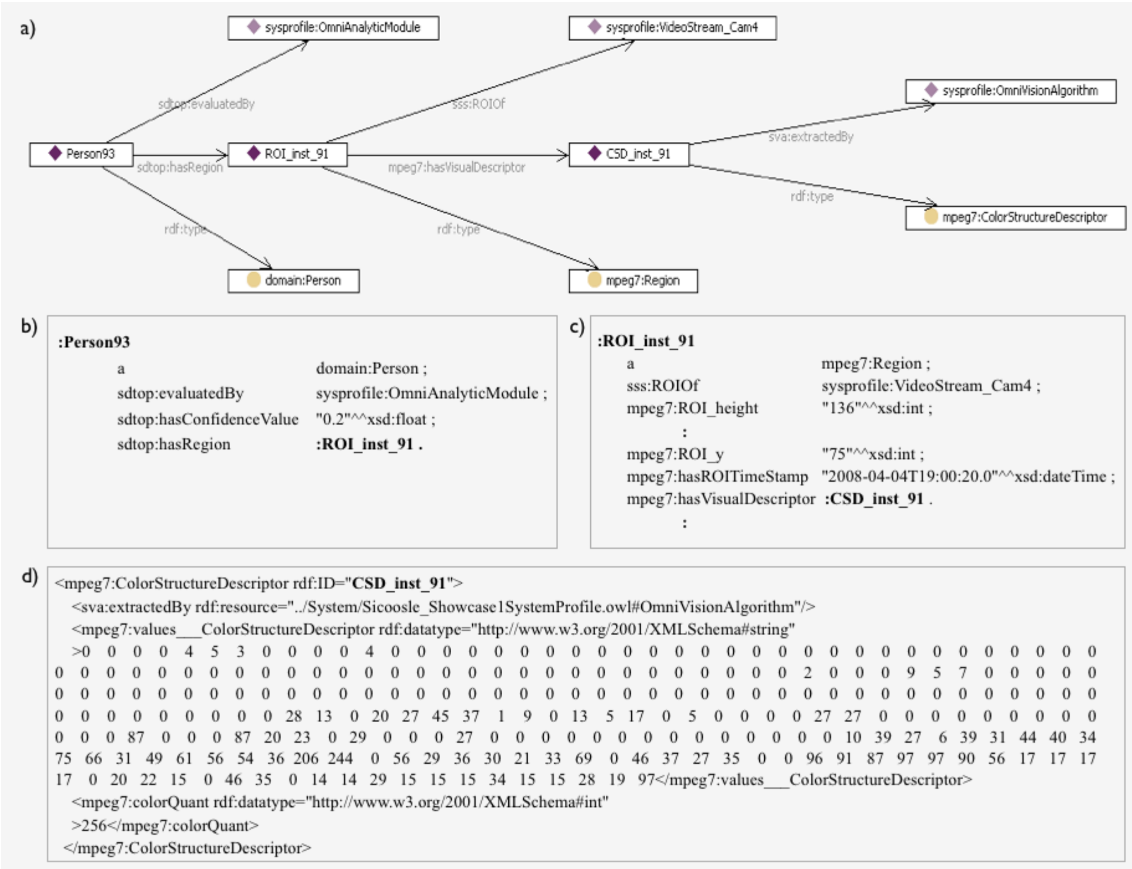

Figure 4.6: Annotation Instance of Figure 4.5 : a) DIG Representation of the Example, b) Person Instance in N3-Triple Notation, c) ROI Instance in N3-Triple Notation and d) Color Structure Descriptor in RDF/OWL Format of MPEG-7 Ontology.

rected Graph) consisting of RDF-triples of the form $< subjective, predicate, objective >$. There are several encoding formats for RDF triples, among them N3-Triple (a.k.a Notation3) [169] is one of the simplest representation for human understanding. Figure 4.6 shows an example of a partial DIG of concepts and instances generated by the annotation tool (ideally, this should be generated by the analytic modules automatically) and its encoding in N3-Triple format. Figure 4.6 can be interpreted as follows:

- the graph shows that there is an instance which is named as Person93 and having the type "person" coming from the "domain" namespace
- It was classified (evaluatedBy) by an algorithm instance "OmniAnalyticModule" defined

in the system profile.

- It belongs to a ROI instance named as ROI_inst_91. The ROI_inst_91 is an instance of the MPEG-7 Region concept.
- The ROI exists in a video stream captured from camera id 4 (also defined in the system profile).
- The ROI also has associated an instance of a visual descriptor named "CSD_inst_91" which is an instance of MPEG-7 ColorStructureDescriptor defined in the MPEG-7 namespace and extracted by an algorithm defined in the system profile.

Once generated, the metadata should be archived into a persistent repository for later use. Because RDF/OWL is used as basis for the knowledge representation, it naturally needs a RDF triple store as data archive. For practical use, the machine should be able to parse such a graph model. There are several infrastructures available for this purpose such as Sesame, Jena and Kaon, etc [175]. In the current system integration the Jena semantic web framework from Hewlet-Packard (HP) is used, which is a Java framework for building Semantic Web application. It provides a programmatic environment for RDF, RDFS, OWL, and SPARQL (note. Jena is open source and grown out of work at the HP labs).

In addition to RDF/OWL archiving, low level feature vectors should be separately indexed for picking similar vectors. As shown in Figure 4.6 - d), low level features usually consist of a number of numeric values. In the above case, the shown CSD (Colour Structure Descriptor) has 256 bins. Indexing such multi-dimensional vectors is however not a trivial task. In textual data indexing as it has been done in RDBMS side, the most common way is building a tree structure that balances the average degree of depth of nodes from the root, by appropriately partitioning the search space. B-tree and its variations (B*-tree, etc) are well known examples of this approach. However, it turns out that the partitioning is meaningless when we deal with high dimensions, because no matter how we try to partition, the

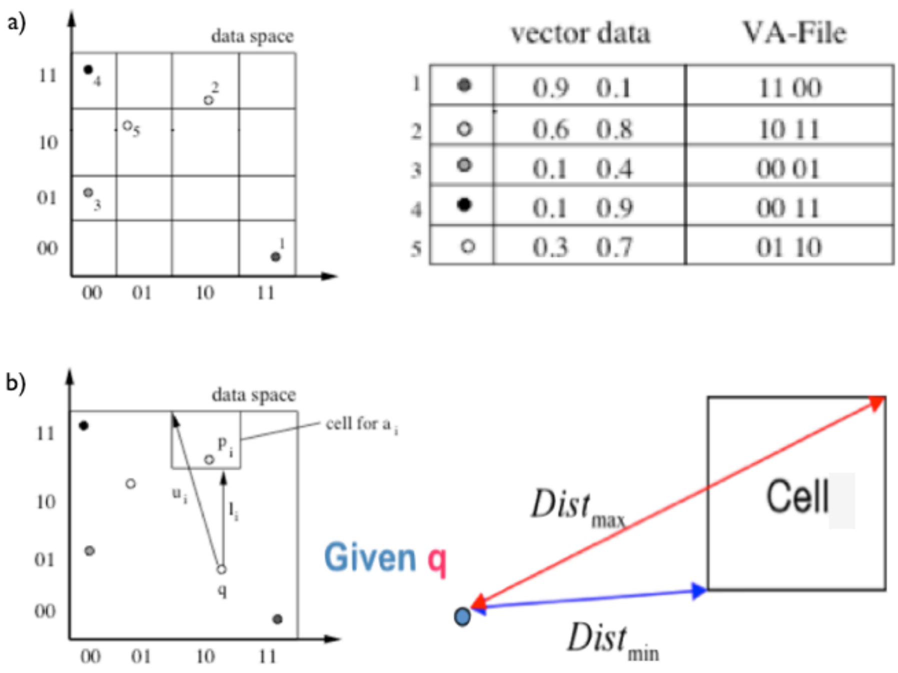

Figure 4.7: Illustrative Example of VA-file : a) VA-file for 2-D Vectors and b) Lower and Upper Bound between a Given Query Vector q and Feature Vectors in a Cell.

curse of dimensionality causes the sparseness problem meaning that the data points in high dimensional space will be dispersed so they cannot be well clustered. Multiple approaches have been proposed so far: dimensionality reduction, vector approximation and locality sensitive hashing technologies are well known examples [31, 129]. At this stage, considering the simplicity on implementation, VA file [171] is used. VA file is based on the assumption that we cannot partition data space that will in turn cause full data search. Therefore, in the VA file approach, the focus is more on reducing I/O time and distance calculation time. By quantizing each bin vectors with several bits, one specific feature vector can be represented by a small number of bits. To cope with K-nearest neighbor search, it also provides an algorithm to calculate distances at I/O time and thereby sort the requested K elements. According to the quantization scheme in VA-file, we allocate appropriate bits (b_j), as follows :

$$b_j = \left\lfloor \frac{b}{d} \right\rfloor + \begin{cases} 1 & if\ j \leq (b\ mod\ d) \\ 0 & otherwise \end{cases} \tag{4.3}$$

$where,\ b\ is\ the\ total\ number\ of\ bits\ and\ d\ is\ the\ number\ of\ dimensions.$

This concept is shown in Figure 4.7 - a) in the case of $b = 2$ and $d = 2$. Once indexed, the search can be done as shown in Figure 4.7 - b). While reading approximated feature vectors, for each feature vector P_i we calculate $Dist_{max}(U_i)$ and $Dist_{min}(l_i)$ as shown above. For K-nearest neighbour search, we first initialize an array with k-index with appropriately large distance values, then with P_i we calculate the lower bound ($Dist_{min}$) if the lower bound is less than values in the array, we set the P_i as a candidate, and repeat this process till we reach the end of the VA index file. Though the VA file approach is described here, again, our ultimate aim is not to find another indexing structure, but to provide knowledge on the index, so that the system can automatically decide what and how to use such an index, when third party index structures are plugged into our system and are made available.

4.4.3 Querying and Retrieval

As described, let us assume that someone informed that something happened somewhere. With this information, we zoom into the reported area and click on one available camera around the place. We also load a domain knowledge description, which will show us all possible queries that we can resolve. Clicking a camera will show us a key frame summary of what happened during the past period, sorted according to the time line. By clicking on one of the images, we can play the corresponding video in the window on the top left side (see Figure 4.8 - 1). While browsing the video sequences, we found a guy who is holding a white box. We were suspicious about the guy and want to know as much as we can (see Figure 4.8 - 2, 3). After capturing the scene (by clicking the capture button), we start composing a query. This includes manually selecting the person by drawing a bounding box, cropping the image and finally extracting a set of low level features for this

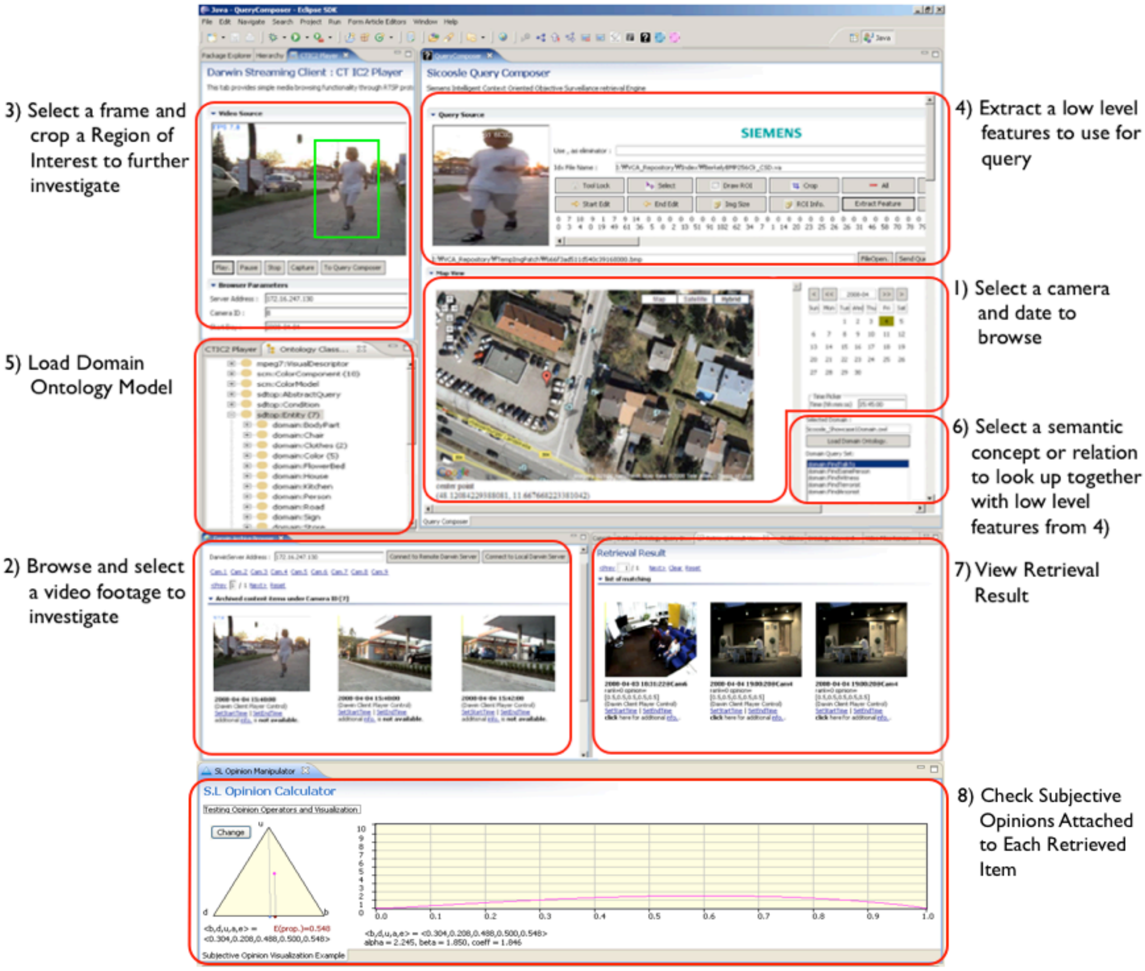

Figure 4.8: Querying and Retrieval Graphical User Interface and Walkthrough.

region of interest (see Figure 4.8 - 4). From the available set of queries we select the query *'FindTalkTo'* and trigger it (see Figure 4.8 - 5, 6). This in turn launches a SPARQL [168] query that is expanded by the DL Reasoner using the Vision Metadata Ontology. The SPARQL query used at this phase is depicted as follows :

- *SELECT DISTINCT ?visionEntity ?am ?type ?ROIx ?ROIy ?ROI_width ?ROI_height ?camID ?timeStamp ?cvstr*

  #(opinion (agentl ?am) (prop ?type) (args ID ?visionEntity CamID ?camID RectInfo "?ROIx ?ROIy ?ROI_width ?ROID_height Time ?timeStamp) #OpinionMapperMethod1(?cvstr)#)

  FROM NAMED

  <http://SicosleOntologyServer/metadata/KB/Sicosle/Instance/DataDocuments/MetaC1:040408:001.owl>

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```
WHERE {
?visionEntity rdf:type sdtop:VisionEntity;
              rdf:type ?type;
              sdtop:hasRegion ?region;
              vao:hasAnalyticModule ?am;
              sdTop:hasConfidenceVale ?cvstr;
FILTER (regex (str(?type), "^ http://SicosleOntologyServer/metadata/KB/Sicosle/DomainOnt/
              Sicosle_Showcase1Domain.owl")).
?region mpeg7:hasROITimeStamp ?timeStamp ;
              mpeg7:ROI_x ?ROIx;
              mpeg7:ROI_y ?ROIy;
              mpeg7:ROI_width ?ROI_width;
              mpeg7:ROI_height ?ROI_height;
             sss:ROIOf ?videoStream .
# FILTER ((?timeStamp > "2005-01-01 T00:00:00Z"^^ xsd:dateTime)).
?videoStream ssm:capturedBy ?cam.
?cam rdf:type ssm:Camera .
?cam ssm:hasSensorID ?camID.
```

The result set is then transformed into a fact list for the CLIPS rule engine [1]. We have also enabled describing hints on this translation in the comments (see the lines with comment symbol '#'). Following shows examples of the translated opinion lists in CLIPS syntax as explained in Section 4.3.2.

- *(opinion (agent humanAnnotator) (prop person) (args ID person96 CamID 4 RectInfo "259 75 58 136" Time 2008-04-04T19:00:20.0) (b 1) (d 0) (i 0) (a 0.5))*
  (opinion (agent humanAnnotator) (prop person) (args ID person93 CamID 4 RectInfo "159 55 50 126" Time 2008-04-04T19:00:21.0) (b 1) (d 0) (i 0) (a 0.5))
  (opinion (agent humanAnnotator) (prop person) (args ID chair4 CamID 4 RectInfo "240 105 31 25" Time 2008-04-04T19:00:21.5) (b 1) (d 0) (i 0) (a 0.5))
  (opinion (agent humanAnnotator) (prop person) (args ID table3 CamID 4 RectInfo "188 42 100 52" Time 2008-04-04T19:00:19.0) (b 1) (d 0) (i 0) (a 0.5))
  :

As the metadata is based on manual annotation, the translated opinion values represent absolute truth (i.e., (1,0,0,0.5)). Once gathered, rules are executed on gathered data segments, to find evidential patterns which satisfy given rules. Following shows a rule that represents *'talk-to'* semantics in the syntax explained in Section 4.3.2.

$$\begin{aligned} & w^{talkto\_rule}_{talkto(append(Ve2,Ve3,Ts2,Ts3),CamID,(duration(Ts2,Ts3)),(dist(Roi2,Roi3)))} \\ & \leftarrow w^{-}_{furniture(Ve1,CamID,Roi1,Ts1)} \circledast w^{-}_{person(Ve2,CamID,Roi2,Ts2)} \circledast w^{-}_{person(Ve3,CamID,Roi3,Ts3)} \\ & \quad \circledast w^{SLSystem}_{distwithin(centerradius,100,Roi1,Roi2,Roi3)} \circledast w^{SLSystem}_{timeholduntil(min,2,Ts1,Ts2,Ts3)} \circledast w^{SLSystem}_{neq(Ve2,Ve3)} \end{aligned} \tag{4.4}$$

In our system, actual representation of the above Rule 4.4 is as follows.

- *(defrule talkto-rule*

  ?op1<-(opinion(prop ?ty1&:(eq ?ty1 furniture))(args ID ?ve1 CamID ?camID RectInfo ?roi1 Time ?ts1))

  ?op2<-(opinion(prop ?ty2&:(eq ?ty2 person))(args ID ?ve2 CamID ?camID RectInfo ?roi2 Time ?ts2))

  ?op3<-(opinion(prop ?ty3&:(eq ?ty3 person))(args ID ?ve3 CamID ?camID RectInfo ?roi3 Time ?ts3))

  (distwithin centerradius 100 ?roi1 ?roi2 ?roi3)

  (timeholduntil min 2 ?ts1 ?ts2 ?ts3)

  (test (neq ?ve2 ?ve3))

  =>

  (bind ?new_op (sl-conjunction$?op1 ?op2 ?op3))

  (modify ?new_op (prop talkto) (args ID (apply str-cat "talkto" ?ve2 ?ve3 ?ts2 ?ts3)

  CamID ?camID Time (duration ?ts2 ?ts3) Dist (dist ?roi2 ?roi3)))

)

Above rule also shows the use of subjective logic's conjunction operator to derive a new opinion. However, due to the manual annotation that gives full belief to every metadata, the rule will also derive an opinion with full belief.

In this step it is e.g. verified in which instance at least two people are appearing in the same image (because one of the rules defines that the presence of two persons is a necessary condition for the *'talk-to'* relation). The result from the rule based filtering will be a largely reduced candidate set. The final check now is on verifying that one of the persons is indeed the same as the initially selected (suspicious) person. This is performed by a similarity match over all the features extracted from the selected ROI. If the distance between the query feature set and the feature set associated to a person in the candidate frame is below a certain threshold then it is assumed that both persons are identical. The remaining candidate instances are presented as a list of key frames. Figure 4.8 - 7) shows scenes with different people around chairs and table are regarded satisfying the given query. Clicking 'view info menu', it also shows visual similarity matching results using $L1-norm$ between features of objects in the retrieved scenes and the extracted query feature vector. In addition to this, ideally, these retrieved items can be clicked to check the attached subjective opinion on being the right answer to the query Figure 4.8 - 8). However, at this stage of case study, we have not considered the uncertainty handling mechanism but rather used straightforward rule matching. The detailed mechanism on uncertainty handling will be discussed in the rest of chapters.

4.5 Case Study II

In this section, we present a case study in applying the proposed system to real traffic surveillance scenes processed with automated vision analytics [1]. Considering that we are not yet facing the use of vision analytic metadata generated from heterogeneous vision analytic systems or vendors, we will focus more on logical reasoning aspect rather than the metadata representation (note that, one of the reasons we use ontological metadata model was to provide machine-interpretable inter-operability between heterogeneous systems by giving a sharable formal semantics to metadata. Refer to the Section 2.2.3, Section 4.2 and Section 4.3.1). Therefore, this case study is to show how logical rules can be used to model contextual semantics implicitly implied in traffic video scenes. For the convenient implementation, the generated metadata is directly archived into a DataBase (MySQL) in form of opinion representation. Total 7 different compositional logical queries are demonstrated.

[1] This case study is conducted in cooperation with SCR (Siemens Corporate Research, Princeton, NJ, US) using SCR's proprietary vision analytic modules. Especially, special credits must go to Dr. Vinay Shet, Dr. Gao Xing and Dr. Vasu Parameswaransu

Figure 4.9: Samples of Traffic Visual Surveillance Data for Case Study 2.

4.5.1 Scenario Setting for Case Study II

Total 5 hours and 30 minutes ordinary tunnel and highway scenes are collected from 8 different video sources. These video sources are processed using SCR's (Siemens Corporate Research) proprietary automated vision analytic modules. The volume of the video footages is 4.64Gb in compressed form. Figure 4.9 shows some sample frames of the traffic video data. The applied vision analytic modules generate total 12 primitive semantics (e.g. trajectory information of vehicles, distances between vehicles, scene with hazard light, occlusion, human appear and disappear, etc.). Upon such semantics, we focus on composing and answering complex queries by the use of spatio-temporal rules. Following shows the compositional queries we applied.

- *'Q1'* Find a Truck following an Emergency Vehicles
- *'Q2'* Find an Emergency Vehicle following another Emergency Vehicle
- *'Q3'* Find a Vehicle going past a Pedestrian
- *'Q4'* Find a Truck going past a Pedestrian
- *'Q5'* Find a Truck passing by a Stopped Vehicle
- *'Q6'* Find a Truck following another Truck
- *'Q7'* Find a Truck passing a vehicle that is Backing up

Vocabulary of Primitive Semantics	Properties of each Vocabulary Item	Total # of instances
humanappear	(CamID, PedID, Tlx, Tly, Brx, Bry, Time, Ef, Ea)	30
humandisappear	(CamID, PedID, Tlx, Tly, Brx, Bry, Time, Ef, Ea)	26
vehicle_trajectory	(CamID, VehID, Time, Ef, Ea)	6397
distance	(CamID, VehID, LaneID, Distance, AssocVehID, Time, Ef, Ea)	19198
hazardlight	(CamID, LaneID, Time, CfgV, Ef, Ea)	899
lostcargo	(CamID, LaneID, Y, Time, Ef, Ea)	3
occlusion	(CamID, Lambda, Time, Ef, Ea)	649
slowvehicle	(CamID, LaneID, Direction, Time, Ef, Ea)	24
speedestimation	(CamID, VehID, LaneID, Direction, Speed, Time, Ef, Ea)	6400
stoppedvehicle	(CamID, LaneID, Y, Time, Ef, Ea)	52
vehicletype	(CamID, VehID, Type, CfgV, Ef, Ea) / Type : 0-small, 1-large	6399
weather	(CamID, Type, Time, Ef, Ea) / Type : 1-normal, 2-shiny	17
Total 5h 30m from 8 different sources (≒ 40m for each source) .Total # of instances =		40094

Table 4.1: Metadata Vocabulary and the Property of Each Primitive Semantic Item.

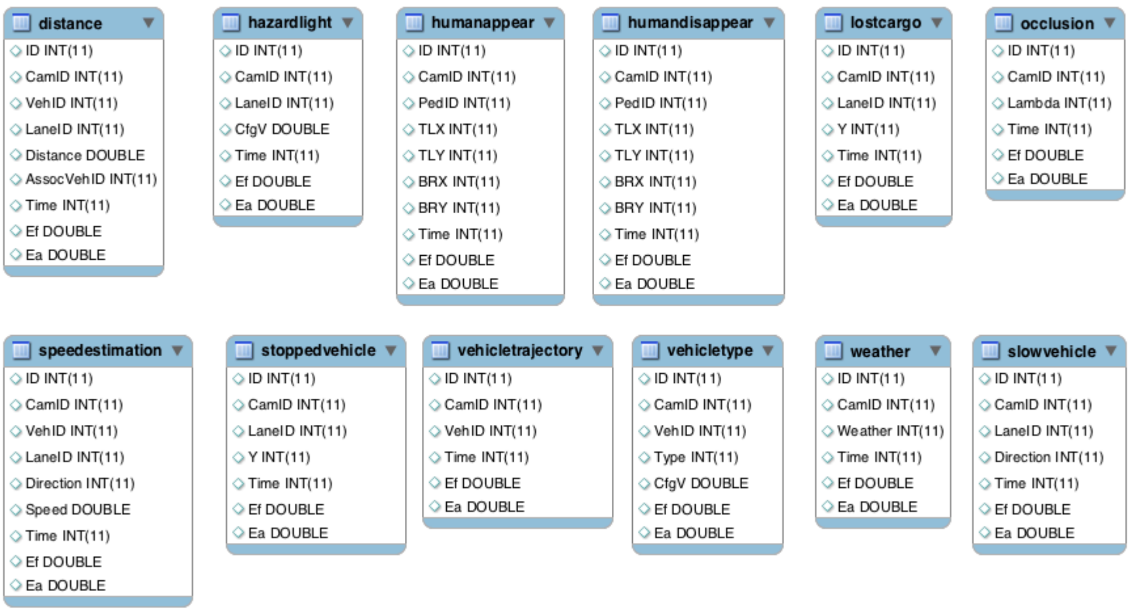

Figure 4.10: Flat-file Style Database Tables in ERD (Entity Relation Diagram) Each Representing Metadata Vocabularies of Vision Analytics.

4.5.2 Metadata Generation

Table 4.1 shows the metadata vocabulary used in this case study. Using the vision analytics available in Siemens (i.e., SCR's proprietary vision analytics), total 12 primitive semantic items are generated with corresponding properties. Archiving of the metadata is done using a flat-file-like database scheme shown in Figure 4.13, which directly maps each semantic item into a corresponding database table. In the vocabulary uncertainty is captured in the $[Ef, Ea]$ pair that satisfies $Ef + Ea = 1$. The value Ef represents amount of truth *'Evidence For'* and the other value Ea represents amount of falsity *'Evidence Against'*. In our system, each metadata item is represented in form of subjective opinion following Definition 16 - 1, as shown in Section 4.3.2. For example, $w^{SCR\_VisionAnalytic}_{humanappear(CamID,PedID,Time)} = (b = Ef, d = Ea, i = 0)$ represents a subjective opinion about *'humanappear'* metadata with the amount of opinion $(b = Ef, d = Ea, i = 0, a = 0.5$ *by default*$)$.

4.5.3 Querying and Retrieval

To be able to answer complex queries *Q1-Q7* discussed in Section 4.5.1, it should be able to augment implicit high level semantics, based on the currently available metadata. For example, the semantics of *'emergency vehicles'* in *'Q1'* is not explicitly defined in the metadata vocabulary. However, combining *'hazardlight'* information with *'vehicle_trajectory'* information, we could compositionally consider an instance of vehicle detection as an *'emergency vehicle'* if there was also a hazard light at the same time of the vehicle detection. Using the logical conjunction operator (*'*$\wedge$*'*) of subjective logic, this can be represented as follows :

$$\begin{aligned} & w^{evrule}_{emergencyVehicle(CamID,VehID,Vht)} \\ & \leftarrow w^{-}_{hazardlight(CamID,\_,Ht)} \wedge w^{-}_{vehicletrajectory(CamID,VehID,Vht)} \\ & \quad \wedge w^{SLSystem}_{test(Vht+25<Ht<Vht+25)} \end{aligned} \tag{4.5}$$

The semantic concept *'truck'* in *'Q1'* is also not explicitly defined in the metadata vocabulary. However, we can also infer the concept *'truck'* by referring to *'vehicletype'* that indicates a large vehicle when the property *'type'* is set with *'1'*. This can be also depicted as follows:

$$\begin{aligned} & w^{truckrule}_{truckVehicle(CamID,VehID,Vht)} \\ & \leftarrow w^{-}_{vehicletype(CamID,VehID,1)} \wedge w^{-}_{vehicletrajectory(CamID,VehID,Vht)} \end{aligned} \tag{4.6}$$

Based on the inferred concept defined by Rule 4.5 and Rule 4.6, the semantics of *'Q1'* can be further defined as follows:

$$\begin{aligned}&w^{q1}_{truckNearAnEmergencyVehicle(CamID,TvID,EmerT)}\\&\quad\leftarrow w^{-}_{emergencyVehicle(CamID,EvID,EmerT)} \wedge w^{-}_{truckVehicle(CamID,TvID,Vht)}\\&\qquad\wedge w^{SLSystem}_{appearEqual(EmerT,Vht,70)}\end{aligned} \tag{4.7}$$

Rule 4.7 represents that it will assert a new opinion on the proposition *'truckNear-AnEmergencyVehicle'*, if there were semantics which can be regarded as *'emergency vehicle'* and *'truck vehicle'* at the same time. To manipulate temporal relations, Allen's temporal relations were modeled as predicate function [15]. These temporal relation covers *'{equal, before, after, meets, meetby, overlaps, overlappedby, starts, startedby, during, contains, finishes, finishedby }'*. For example, the predicate function term *'appearEqual'* corresponds to *'equal'* in allen's temporal logic with acceptable tolerance 0.7 sec. Semantics of *'Q2-Q7'* can be also augmented in the similar way. Table 4.2 shows some of augmented semantics in form of rules and all the rules directly corresponding to the queries *'Q1-Q7'*.

Figure 4.11 shows the prototype interface of the demonstrator. The user interface shows retrieved items for the queries *'Q1-Q7'*. Each items comes with basic information and the item can be played for further examination. The items are ranked along the expectation value of the calculated subjective opinions attached to items. In this case study, however, the uncertainty handling is only based on the *'conjunction'* operator, thereby, represent rather conjunctive probability sense of subjective opinions. (Note that, the rest of this dissertation deal with the detailed aspects on handling uncertainty and this section focusses more on the architectural case study.) As an architectural proof of concept, every queries seem to retrieve reasonable scenes that matches to the intention of the queries. However, for some cases as shown in Figure 4.11 - *'Q4 : Find a truck going past a Pedestrian'*, due to the false alarms of pedestrian metadata, it also retrieved some of wrong items. Therefore, the robuster analytics we have, the better result would be possible.

Query	Semantics / Corresponding Rules
‘-’	Augmented Semantics : $w^{evrule}_{emergencyVehicle(CamID,VehID,Vht)} \leftarrow w^{-}_{hazardlight(CamID,\_,Ht)} \wedge w^{-}_{vehicletrajectory(CamID,VehID,Vht)} \wedge w^{SLSystem}_{test(Vht+25<Ht<Vht+25)}$ (4.5) $w^{truckrule}_{truckVehicle(CamID,VehID,Vht)} \leftarrow w^{-}_{vehicletype(CamID,VehID,1)} \wedge w^{-}_{vehicletrajectory(CamID,VehID,Vht)}$ (4.6) $w^{bvrule}_{backingUpVehicle(CamID,VhID,Vht)} \leftarrow w^{-}_{speedestimation(CamID,VhID,1,Vht)}$ (4.8) $\vdots$
‘Q1’	Find a Truck following an Emergency Vehicles $w^{q1}_{truckNearAnEmergencyVehicle(CamID,TvID,EmerT)} \leftarrow w^{-}_{emergencyVehicle(CamID,EvID,EmerT)} \wedge w^{-}_{truckVehicle(CamID,TvID,Vht)} \wedge w^{SLSystem}_{appearEqual(EmerT,Vht,70)}$ (4.7)
‘Q2’	Find an Emergency Vehicle following another Emergency Vehicle $w^{q2}_{emerVehicleFollowingAnotherEmerVehicle(CamID,EvID2,EmT2)} \leftarrow w^{-}_{emergencyVehicle(CamID,EvID1,EmT1)} \wedge w^{-}_{emergencyVehicle(CamID,EvID2,EmT2)} \wedge w^{SLSystem}_{appearBefore(EmT1,EmT2,80)}$ (4.9)
‘Q3’	Find a Vehicle going past a Pedestrian $w^{q3}_{vehicleGoingPastPedestrian(CamID,VhID,Ht)} \leftarrow w^{-}_{humanappear(CamID,PedID,Ht)} \wedge w^{-}_{vehicletrajectory(CamID,VhID,Vt)} \wedge w^{SLSystem}_{appearBefore(Ht,Vt,80)}$ (4.10)
‘Q4’	Find a Truck going past a Pedestrian $w^{q4}_{truckGoingPastPedestrian(CamID,VhID,Ht)} \leftarrow w^{-}_{humanappear(CamID,PedID,Ht)} \wedge w^{-}_{truckVehicle(CamID,VhID,Vt)} \wedge w^{SLSystem}_{appearBefore(Ht,Vt,80)}$ (4.11)
‘Q5’	Find a Truck passing by a Stopped Vehicle $w^{q5}_{truckPassingByStoppedVehicle(CamID,VhID,St)} \leftarrow w^{-}_{stoppedvehicle(CamID,\_,\_,St)} \wedge w^{-}_{truckVehicle(CamID,VhID,Vt)} \wedge w^{SLSystem}_{appearBefore(St,Vt,80)}$ (4.12)
‘Q6’	Find a Truck following another Truck $w^{q6}_{truckFollowingAnotherTruck(CamID,TvID1,T1)} \leftarrow w^{-}_{truckVehicle(CamID,TvID1,T1)} \wedge w^{-}_{truckVehicle(CamID,TvID2,T2)} \wedge w^{-}_{distance(CamID,TvID1,\_,Dist,TvID2,\_)} \wedge w^{SLSystem}_{geq(Dist,25)} \wedge w^{SLSystem}_{appearBefore(T1,T2,80)}$ (4.13)
‘Q7’	Find a Truck passing a vehicle that is Backing up $w^{q7}_{truckPassingBackingUpVehicle(CamID,VhID1,Vt1)} \leftarrow w^{-}_{truckVehicle(CamID,VhID1,Vt1)} \wedge w^{-}_{backingUpVehicle(CamID,VhID2,Vt2)} \wedge w^{SLSystem}_{appearBefore(Vt1,Vt2,30)}$ (4.14)

Table 4.2: Rules used in Case Study 2.

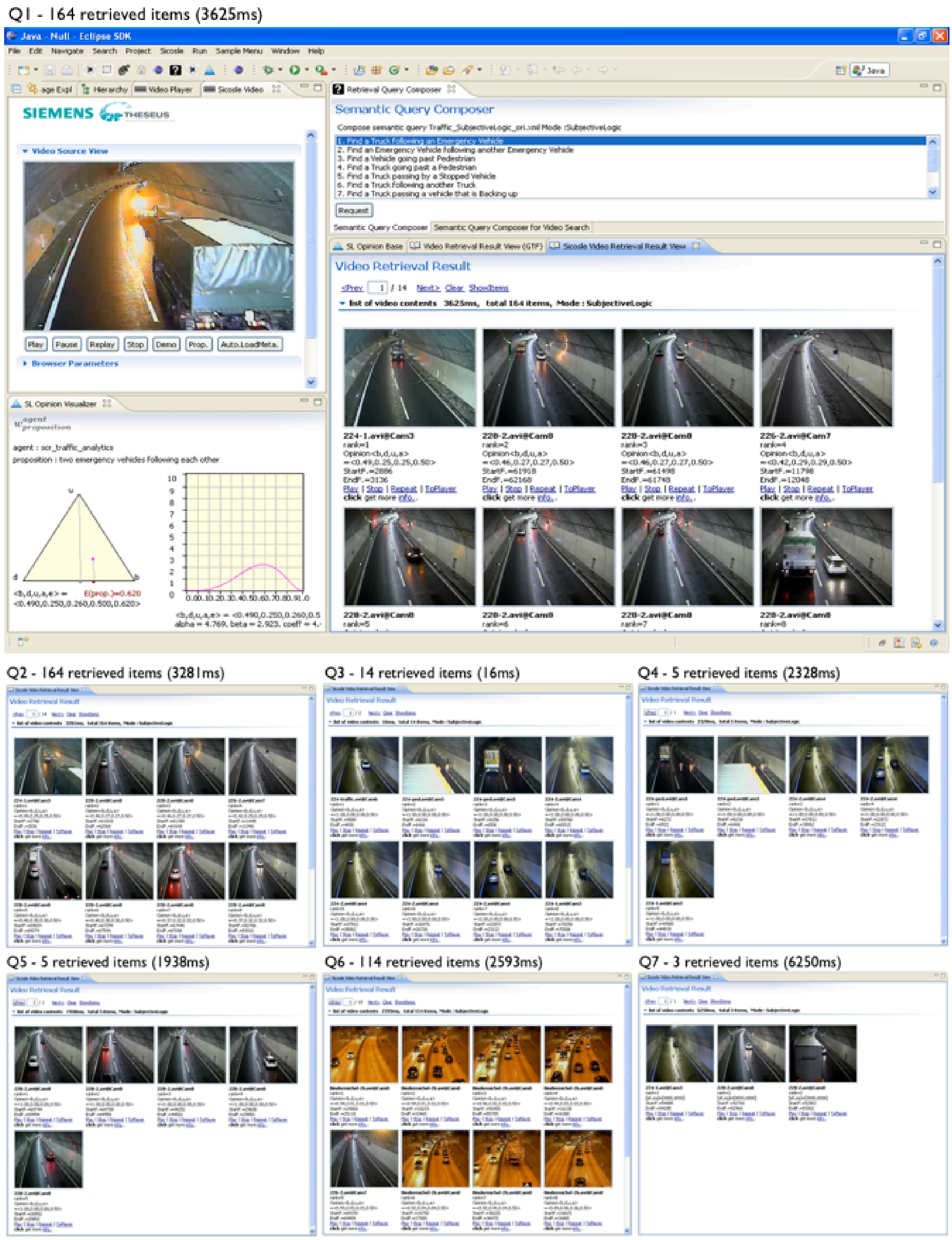

Figure 4.11: Querying and Retrieval Graphical User Interface and Retrieval Results of *'Q1-Q7'* with Sample Items.

4.6 Discussion

In the previous section, we have demonstrated the proposed architecture approach in two different case studies. These case studies especially show the feasibility of logical reasoning for modeling compositional high-level semantics that can be augmented based on available metadata. While the reasoning aspect is focussed, there are however still open issues such as *'Scalability'*, *'Proper Uncertainty Handling'*, etc. In this section we will further discuss the *'Scalability'* issue and the issues around *'Uncertainty Handling'* will be addressed in the rest of this dissertation. In the proposed architectural pipeline, there are two main components to discuss in terms of *'Scalability'*. Namely, *'scalability issues around Ontological instance handling'* and the *'scalability issues around rule-based reasoning'*. This section addresses both of the aspects.

4.6.1 Scalability of Ontological Instance

In Section 4.4, ontological representation of metadata played an important role in the proper selection of metadata segments for further logical reasoning. The selection is done by the use of SPARQL [168] query upon OWL/DL reasoner [175]. Although there are issues such as how to come up with Ontologies that can be agreed by other systems and venders, etc. (*'in the sense of knowledge sharing and interoperability'*), the more essential and natural question here could be the performance in terms of the scalability due to the potentially large scale metadata that visual surveillance systems would produce. It is not only the case of visual surveillance but also the case of many other domains. Therefore, it is important to know the general performance of OWL/DL reasoners.

There have been performance benchmarks of currently available reasoners [67, 172, 173, 30]. The benchmarks have been conducted in terms of many aspects such as size of TBox (*'Size of Knowledge Description'*), size of ABox (*'Size of Instances of TBox'*), loading time of Ontology, complexity of the queries and currently available large scale ontologies (i.e., DBLP database for publications, etc.). In a relatively recent comprehensive

Ontology	Class	Prop.	SubCl.	Equi.	SubPr.	Domain	Range	Functional	C(a)	R(a,b)	Axioms_total
vicodi_0									16942	36711	53876
vicodi_1									33884	73422	107529
vicodi_2	194	10	193	0	9	10	10	0	50826	110133	161182
vicodi_3									67768	146844	214835
vicodi_4									84710	183555	268488
swrc_0									4124	13712	27227
swrc_1									8248	27424	54328
swrc_2									12372	41136	81429
swrc_3									16496	54848	108530
swrc_4									20620	68560	135631
swrc_5	55	41	115	0	0	0	1	0	24744	82272	162732
swrc_6									28868	95984	189833
swrc_7									32992	109696	216934
swrc_8									37116	123408	244035
swrc_9									41240	137120	271136
swrc_10									45364	150832	298237
lubm_1									18128	49336	100637
lubm_2									40508	113463	230155
lubm_3	43	25	36	6	5	25	18	0	58897	166682	337221
lubm_4									83200	236514	477878
wine_0									247	246	719
wine_1									741	738	1721
wine_2									1235	1230	2723
wine_3									1729	1722	3725
wine_4									2223	2214	4727
wine_5	141	13	126	61	5	6	9	6	2717	2706	5729
wine_6									5187	5166	10739
wine_7									10127	10086	20759
wine_8									20007	19926	40799
wine_9									39767	39606	80879
wine_10									79287	78966	161039
Manual Annotation in CaseStudy I	147	264	90	-	-	-	-	-	724	-	-

Table 4.3: Statistics of Test Ontologies used in the Work of Jürgen Bock et al. [30]. The Table Indicates # of TBox Parameters (Class, Property, SubClass, etc.) and # of ABox Parameters (C(a) : Instances of a Class and Instances of a Relation R(a,b)). The Red Boxed Row Shows the Statistics of Our Manual Annotation Used in Case Study I (see Section 4.4) Extracted by TopBraid Composer. '-' Represents the Statistics not Available in TopBraid Composer. The Red Arrow Indicates that Our Annotation Resembles wine_1 Data Set in terms of Statistics.

workbench report done by Jürgen Bock et al. [30], they have conducted a performance benchmark using some of prominent example data sets of RDF/OWL ontology [2].

Table 4.3 shows the statistics of the test ontologies. We have also extracted the statistic information of our annotation using an ontology editing tool called 'TopBraid Composer' [3]. Although not every statistics were available as indicated with '-', the red

[2]Datasets: VICODI (http://www.vicodi.org), SWRC (http://www.ontoware.org/index.html), LUBM (http://swat.cse.lehigh.edu/projects/lubm/index.htm) and WINE (http://www.schemaweb.info/schema/SchemaDetails.aspx?id=62)

[3]http://www.topquadrant.com/products/TB_Composer.html

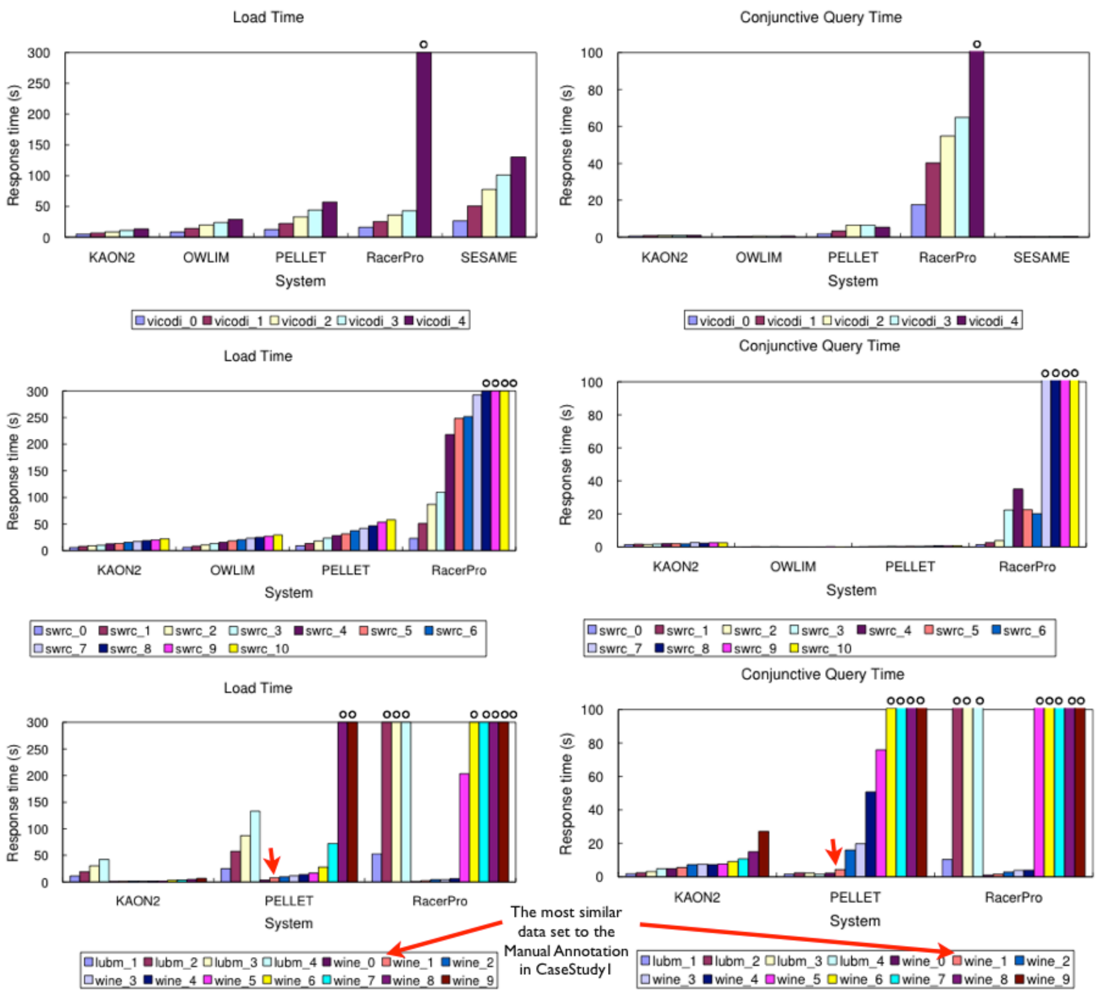

Figure 4.12: Benchmark Result in the Work of Jürgen Bock et al. [30]. The Figures show the Average Load and Query Time with a Particular Reasoner on the Dataset shown in Table 4.3. The '◦' indicates Time-out (> 5 min).

box and the red arrow in Table 4.3 shows that our annotation is quite similar to the wine_1 data set. Therefore, it is expected that our ontology model would also show similar performance aspect as the cases of the wine data set.

The performance evaluation is done with different implementation of reasoners currently available. The reasoners used in their evaluations fall into three groups: tableau algorithm based (RacerPro and Pellet), disjunctive datalog based (KAON2) and standard rule engine based (Sesame and OWLIM) [4].

[4]Reasoners: RacerPro(http://www.RacerPro-systems.com/), Pellet(http://www.mindswap.org/2003/

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Figure 4.12 shows the result of performance evaluation. In the benchmark report, they informed that it was conducted on a Linux 2.6.16.1 system, however, no further information regarding the hardware setting is available. Unfortunately, it shows that many of reasoners except KAON2 becomes *'time-out'* when it is performed with *wine* data sets containing more than 5000 instances. In the case of *wine_ 1* data set on Pellet, as expected, it shows similar performance to the result we experienced with our annotation in the case study I (Including loading and query processing time, it took approximately 20-25 sec on a core2duo T7200 1.8 Ghz, 2Gb Ram Windows XP machine). According to the benchmark, it does not come up with a clear *'winner'* OWL/DL reasoner that performs well for all types of ontologies and reasoning tasks. However, Jürgen Bock et al. [30] have concluded as follows:

- *As general conclusions we can summarize our results in that (1) reasoners that employ a simple rule engine scale very well for large ABoxes, but are in principle very limited to 'lightweight language fragments', (2) classical tableau reasoners scale well for complex TBox reasoning tasks, but are limited with respect to their support for large ABoxes, and (3) the reasoning techniques based on reduction to disjunctive datalog as implemented in KAON2 scale well for large ABoxes, while at the same time they support are rich language fragment. If nominals are important for a given scenario, Pellet is the only reasoner in this benchmark, which has adequate support* [30] *.*

The benchmark shows that the complexity of TBox also influence the ABox performance. To this extent, the word *'lightweight language fragments'* in *(1)* means OWL-Lite that is a restricted subset of OWL-DL in terms of expressivity to keep computational tractability. At this stage of the proof of concept, we have used Pellet reasoner not to be restricted by expressivity on knowledge modeling. However, it seems that a lot more effort should be paid to derive a matured ontological knowledge model in visual surveillance domain, that is not only agreeable by the parties of systems or venders but also balanced in

pellet/index.shtml), KAON2(http://kaon2.semanticweb.org/), Sesame(http://openrdf.org) and OWLIM(http://www.ontotext.com/owlim) terms of TBox and ABox complexity. Therefore, we believe that this should be driven in the sense of *'collaborative standardization'*. Both the *'scalability'* and the *'standardization of ontology model'* hamper the wide acceptance of the ontology related technology not only in the visual surveillance but also in other domains. This seems the reason why many of current data management systems remain in the traditional database technology. However, considering the current active research effort paid in OWL related technologies, we believe that it would come up with improved performance in the near future. To this extent, we believe that the proof of concept (case study I) on the use of ontological metadata shows a step towards intelligent use of metadata for intelligent visual surveillance.

4.6.2 Scalability of Rule Engines

In the proposed system architecture, ontological metadata representation is not only to archive information in a *'sharable'* and *'machine interpretable'* way, but also to provide a mechanism to pick and collect probable amount of partial evidences (called facts in logic programming) which in turn need to be pumped up to a logical reasoner. Therefore, another important focus regarding *'scalability'* is on the rule based logical reasoning engines that are used after collecting evidential facts. Normally, rule engines implement traditional backtracking (e.g., SLD resolution [66]) or forward chaining (e.g., Rete algorithm [64]) algorithms. In such algorithms, facts and rules are loaded in memory space called fact base and rule base respectively. It is known that the size of the facts affects reasoning performance.

In this section, we represent a simple performance evaluation regarding the *'scalability'* of representative rule engines such as CLIPS [1] and Prolog [5]. Although it is not designed for *'automated'* inference, in logical view, it is important to note that the *'relational data model'* is also based on the *'first-order-predicate-logic'* [40, 41] and SQL is the language to access the relational data model. Namely, they operate across the same

[5]For the evaluation, we used SWI-Prolog [8] among many variations due to its GNU-license policy. Comparison of Prolog implementations can be found on (http://en.wikipedia.org/wiki/Comparison_of_Prolog_implementations)

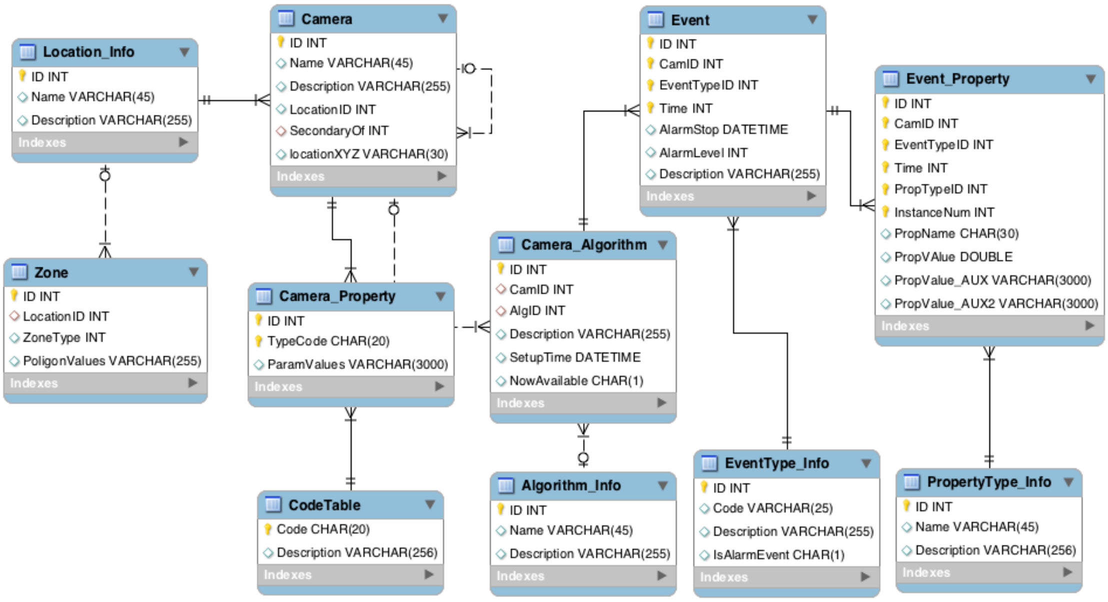

Figure 4.13: The Database Scheme of a Siemens' Internal Visual Surveillance Prototype System called VAP (Video Analytic Platform).

conceptual theory, although their focuses are in completely different directions. In rule engine terms, SQL is primarily a fact and relation(set) engine therefore does not support automatic pattern tracking that dynamically builds a rule tree at run time. However, giving up the *'automatic'* inference aspect, and by taking static view of all possible queries, SQL can do the other, to a limited extent. Therefore, to contrast the performance to the case of using traditional database, we manually expanded the chaining of the rules for each queries *'Q1-Q7'* in form of SQL. To see the influence of different relational data models, the SQL expansion is done for two different relational models. The first one is the flat-file style model as shown in Figure 4.13 explained in Section 4.5. We have also used a more complex data model that covers many other aspects of surveillance system. The model is shown in Figure 4.13 and is used in one of Siemens' internal prototype implementations of surveillance system called VAP(Video Analytic Platform). Table 4.4 shows some of SQL query examples expanded upon the two different relational data models.

We have also populated instances of the metadata vocabulary set used for case study II in Section 4.5. As shown in Table 4.1, originally, 40094 instances were extracted from 8

Query	Semantics / Relational Model (Scheme) / Corresponding SQL Query	
'Q1'	Find a Truck following an Emergency Vehicles	
	Flat-File DB scheme	VAP-DB scheme
	select distinct hl.CamID, hl.Time, v.time from hazardlight hl, vehicletype vt, vehicletrajectory v where hl.CamID = vt.CamID and vt.CamID = v.CamID and vt.VehicleID = v.VehicleID and vt.Type = 1 and v.Time > hl.Time and hl.Time + 80 > v.Time (4.15)	select distinct ep1.camid, ep1.time, ep2.time from event_property ep1, event_property ep2 where ep1.eventtypeid = 2 and ep1.proptypeid = 7 and ep2.eventtypeid = 13 and ep2.proptypeid = 9 and ep2.propvalue = 1 and ep1.camid = ep2.camid and ep2.time > ep1.time and ep1.time + 80 > ep2.time (4.16)
'Q6'	Find a Truck following another Truck	
	Flat-File DB scheme	VAP-DB scheme
	select distinct v.CamID, v.Time, d.time from distance d, vehicletrajectory v, vehicletype vt, vehicletype vt2 where vt.Type = 1 and vt.camID != 0 and v.CamID = vt.CamID and v.VehicleID = vt.VehicleID (4.17) and d.CamID = v.CamID and v.VehicleID = d.VehicleID and d.Distance < 25 and vt2.Type = 1 and vt2.CamID = vt.CamID and d.AssocVehicleID = vt2.VehicleID	select distinct ep2.camid, ep2.time, ep1.time from event_property ep1, event_property ep2, event_property ep3, .event_property ep4, event_property ep5, event_property ep6, event_property ep7 where ep2.eventtypeid = 13 and ep2.camid != 0 and ep2.proptypeid = 9 and ep2.propvalue = 1 and ep1.eventtypeid = 1 and ep1.camid = ep2.camid and ep3.id = ep1.id and ep3.eventtypeid = 1 and ep3.proptypeid = 1 and ep4.id = ep2.id and ep4.eventtypeid = 13 and ep4.proptypeid = 1 and ep3.propvalue = ep4.propvalue and ep1.proptypeid = 6 and ep1.propvalue < 25 and ep5.eventtypeid = 13 and ep5.proptypeid = 9 and ep5.propvalue = 1 and ep2.camid = ep5.camid and ep6.id = ep5.id and ep6.eventtypeid = 13 and ep6.proptypeid = 1 and ep7.id = ep1.id and ep7.eventtypeid = 1 and ep7.proptypeid = 2 and ep6.propvalue = ep7.propvalue (4.18)

Table 4.4: Some Examples of SQL Queries for *'Q1-Q7'*.

Data Set	# instances	time of each video sources	total time of 8 video sources
Original Case Study 2	# 40094	≒ 40 min	≒ 5 hours 30 min
CS2-0.5	27290	≒ 30 min	≒ 4 hours
CS2-01	42444	≒ 1 hours	≒ 8 hours
CS2-02	84888	≒ 2 hours	≒ 16 hours
CS2-03	127332	≒ 3 hours	≒ 24 hours
CS2-04	169776	≒ 4 hours	≒ 32 hours
CS2-05	212220	≒ 5 hours	≒ 40 hours
CS2-06	254664	≒ 6 hours	≒ 48 hours
CS2-07	297108	≒ 7 hours	≒ 56 hours
CS2-08	339552	≒ 8 hours	≒ 64 hours
CS2-24	1018195	≒ 24 hours	≒192 hours
CS2-48	2036390	≒ 48 hours	≒ 384 hours

Table 4.5: Statistics of Dataset prepared for the Scalability Evaluation of Rule-engines and Relational Database.

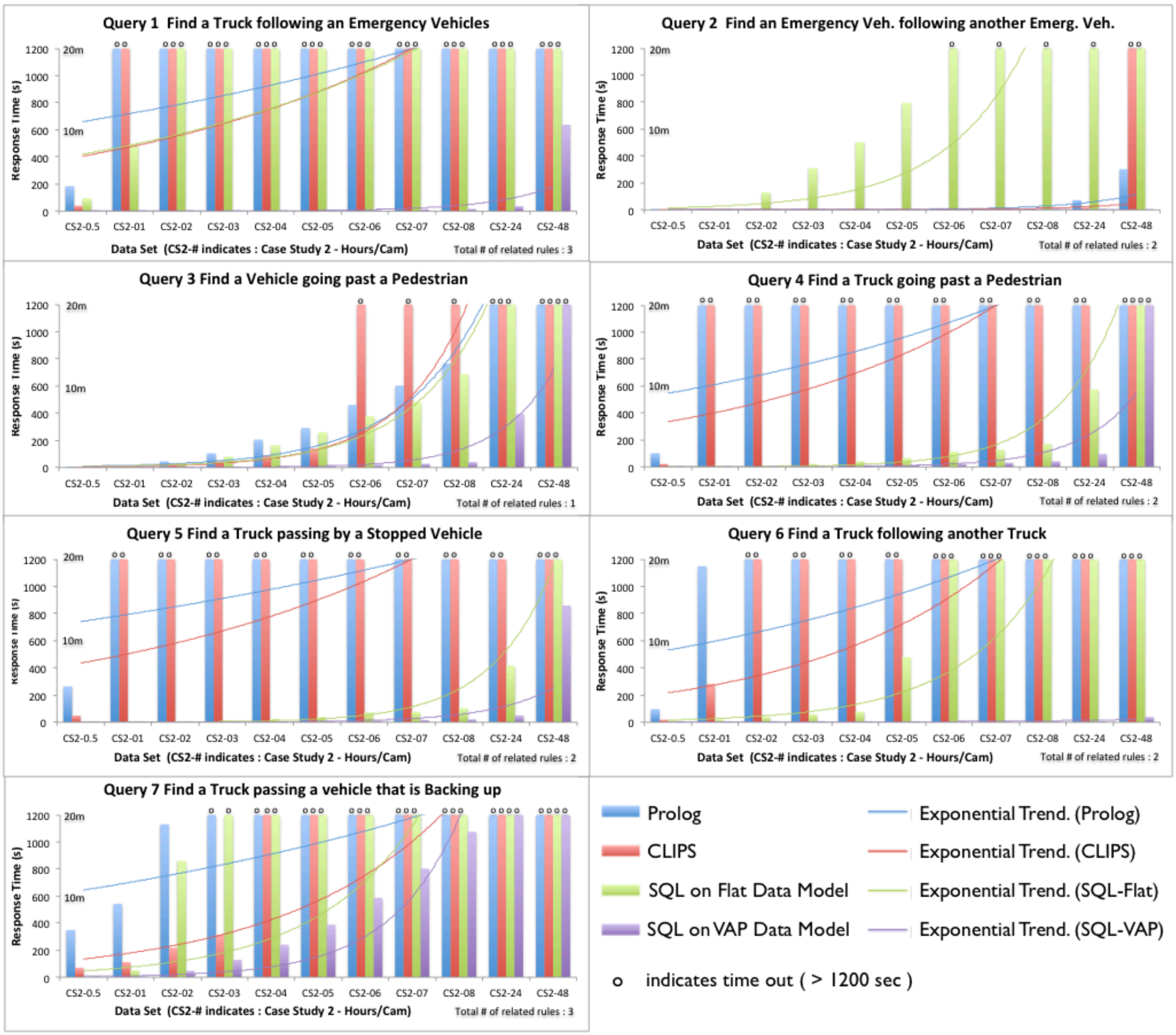

Figure 4.14: Performance Evaluation Result of Different Rule-engines and Relational Database Models. The Figures show the Average Response Time on Different Datasets shown in Table 4.5. The '∘' indicates Time-out (> 1200 sec).

different video sources. The total volume of the data is about 5 hours and 30 minutes, that is about 40 minutes per each source. Based on the original data set, we have populated dummy instances to prepared data sets as shown in Table 4.5. The data sets are designed to contain number of dummy instances correspond to varying hours. For example, the data set CS2-48 contains, 2036390 instances that corresponds to 384 hours (48hours / source × 8 source).

For the evaluation, we set SWI-Prolog 5.6.51, CLIPS 6.3 and MySQL 5.0 on a core2duo T7200 1.8 Ghz, 2Gb Ram Windows XP machine. Figure 4.14 shows the evaluation results. Overall, SQL performs better than rule engines as expected. However, this was not always the case that SQL performs better than rule engines. For example, in the case of *'Q2'*, the VAP scheme showed worst result. Indeed, the cases *'Q1'*, *'Q3'* and *'Q7'* show similar results between CLIPS and VAP Scheme. This seems because the relational models (schemes) were not optimized as it is usually done in the reality of application development. Therefore, it seems that there would be some optimization issues such as setting proper indices and separating tables according to correlated data density, etc. Overall, CLIPS performed better than Prolog. The rules can also be optimized considering the data distributions so that it can traverse less amount of instances first in the case of conjunctive rules. Therefore, speaking overall assessment on the evaluation, it seems that rule engines could also cover about 150,000 to 200,000 instances of data (in total about 32 to 40 hours 8 camera video sources in our setting) within 5 minutes response time, when the rule optimization is also considered. Nevertheless, traditional relational database seems still promising in terms of response time although they can not support automated inference features such as backtracking/ forwardchaining, unification, lists, or adhoc nested structures. Therefore, as a hybrid approach, one could consider the automated rules to single SQL query mapping mechanism, that can be triggered once rule engine binds rule search tree. To this extent, it appears many literatures on this issue. Some of them are to simply interface rule engine with relational database [39, 106, 112], to translate and to compile rules in prolog to SQL [55] and to optimize rules relational query system [89].

4.7 Chapter Summary

In summary of lessons learned in this chapter, while the attempt of using ontological metadata representation and logic programming showed advantage on flexibility of representing and resolving epistemic contextual semantics, the scalability is not sufficient *enough and need to be addressed in the future research.*

Intelligent analysis of high-level semantics is an important capability in visual surveillance systems with high potential usage. Reminding the requirements discussed in Section 1.2.4, such efficient semantic analysis of visual surveillance data requires a system oriented approach that optimally combines the individual legacy vision analytic power and its optimal use. In this chapter, we discussed the software components, data processing pipeline and system architecture design. Among others, the main design philosophy of our approach was to maximize utilization of available analytic metadata together with context information. In this view, the proper vision analytic metadata representation and the contextual knowledge expressive power of the reasoning framework are critical issues to accomplish the goal. For the former requirement, we introduced ontological metadata representation and for the latter requirement, we adopted logic programming. The main advantage of the presented approach is in the *'flexibility'* of representing and resolving epistemic contextual semantics by leveraging logic programming and a description logic based ontological data representation model (this is due to the *'modularity'* as explained in Section 2.3.3).

While the reasoning capability can be applied both the real-time analysis and the forensic post analysis, the two case studies show the potential and the feasibility of the proposed approach, especially in forensic sense of retrieval. To be fully applicable for practical real applications, the reasoning power should be *'scalable'*. Section 4.6 shows performance benchmark of triple stores and rule-engines against different scale of metadata (i.e., fact base in view of rule-engines). Unfortunately, it seems that logical formalism based approaches and the triple stores of ontological metadata are by themselves not sufficiently scalable. The *'scalability'* problem is common issue in the realm of ontology related research field and also expert system related researches. One good news, however, is that there have been undergoing active research focus on the *'scalability'* issue. We have briefly introduced some possible remedies shown in literatures. In addition, the *'scalability'* of low-level feature matching is also important and is an active research topic in the realm of high dimensional vector indexing field. Therefore, we believe and hope that we could benefit from those researches to resolve the *'scalability'* issue in the near future.

Another critical and rather essential issue is the proper handling of *'uncertain'* nature of vision analytic metadata and knowledge models. In the rest of this dissertation we will further discuss this aspect in detail.

5 Contradictory Information Handling

Thus far, we have discussed architectural aspect of the proposed approach to high-level semantic analysis of visual surveillance data. The main advantage of the proposed approach is on the high degree of *'modularity'* that makes more *'flexible'* contextual knowledge modeling possible. Besides the ability of flexible context modeling, *'uncertainty'* handling mechanism is also an important issue. While we have briefly explained the extension of logic programming with subjective logic (see Section 4.3.2), the detailed aspect to support *'epistemic reasoning'* under *'uncertainty'* is not covered in the previous Section. In this chapter, we further deal with the *'uncertainty'* aspect starting with the discussion on the *'default reasoning'* that supports *'non monotonicity'* explained in the preliminaries Section 5.5.

5.1 Introduction

In forensic analysis of visual surveillance data, ***'default reasoning'*** can play an important role for deriving plausible semantic conclusions under *'imprecise'*, *'incomplete'*, *'conflict'* and *'contradictory'* information about scenes. In default reasoning, not only facts (analytic metadata) but also rules (contextual knowledge) are the sources of *'imprecise'* and *'contradictory'* information. To be epistemically *'non-monotonic'*, in such reasoning, not only the proper representation of *'epistemic belief strength'* about given piece of information but also the proper *'principled fusion'* of the information is required. A discrete species of ***Bilattice*** for multivalued default logic is one that demonstrated default reasoning in visual surveillance. In this chapter, we present an approach to default reasoning using subjective logic that acts in a continuous space. As an uncertainty representation and handling formalism, subjective logic bridges Dempster Shafer belief theory and second order Bayesian, thereby making it attractive tool for artificial reasoning. For the verification of the proposed approach, we further extend the inference scheme on the bilattice for multivalued default logic to L-fuzzy set based logics that can be modeled with continuous species of bilattice structures. We present some illustrative case studies in visual surveillance scenarios to contrast the proposed approach with such L-fuzzy set based approaches.

5.2 Background and Motivation

The main objectives of this chapter is as follows: 1) to bestow *'default reasoning'* capability upon our subjective logic extended logic programming framework, 2) to compare the proposed framework with other uncertainty formalisms that could alternatively model *'default reasoning'* to better position the subjective logic based approach.

The proposed architecture and reasoning framework shown in Chapter 4, can be regarded as an *'extensional approach'* in the view of artificial intelligence as explained in Section 2.3. Extensional approaches treat knowledge as conditional rules that are labeled with uncertainty [130]. For the labeling uncertainty, in this dissertation, we proposed the use of subjective logic [93]. However, when it comes to uncertainty representation formalisms, there are number of other formalisms such as Bilattice [69], fuzzy set based fuzzy logic [176, 177], Dempster Shafer belief theory [149] and traditional probability based Bayesian approaches [33], etc. Therefore, for the proper choice of uncertainty formalism, it is important to know their characteristics and behind philosophy on representing and handling uncertainty.

As explained in Section 3.1, subjective logic [92, 93] is also one such uncertainty representation and handling formalism that can be seen as extended theory derived from both the Dempster Shafer belief theory and the second order Bayesian. From Dempster Shafer belief theory, subjective logic inherits the philosophy of explicit representation of ignorance about knowledge in a model called subjective opinion triangle that can be also mapped into beta distribution. The operators of subjective logic are also derived in the sense of Bayesian. Unlike traditional Dempster Shafer evidence fusion method, that is known to yield counter intuitive result when it is operated with highly contradictory evidences and also known to be inconsistent with Bayes' rule, subjective logic comes with similar opinion fusion operators that are robust even with such highly contradictory evidences [92]. Compared with bilattice that mainly consists of two lattices, one representing degree of truth and the other representing degree of information respectively, the degree of information concept is similar to the degree of ignorance in subjective opinion. The main difference between bilattice and subjective logic is the operators. While bilattice comes with four operators that are compositionally defined based on two lattice operators *meet* and *join* from the perspective of set theory, subjective logic comes with 12 operators defined rather in Bayesian sense. Another formidable uncertainty handling formalism, fuzzy logic is based on fuzzy set theory that relies on degree of membership concept for a knowledge segment and again this is similar to the concept of partial ignorance in subjective logic. Interestingly, it is known that some extensions of fuzzy logics can be modeled with (bi-)lattice structures. One thing worth to note concerning fuzzy logic is that, even though there are Zadeh's original logical operators, there are yet another ways of defining logical operators as well. However, due to this aspect, there is inconsistent between fuzzy logic operators and classical probability calculus, thereby often criticized by statisticians who prefer Bayesian [179]. Thus, we advocate that above aspects make the use of subjective logic attractive as a means of representing and handling uncertainty for artificial reasoning.

In addition to uncertainty representation aspect, what is also important is the uncertainty handling in a way supporting non-monotonic property. In reality, the truthness of a partial knowledge segment is often easy to be fragile, because there can be potentially possible contradictions or counter examples about the given knowledge segment. Due to this aspect, the property of retracting and updating existing beliefs upon acquisition of new information (aka. belief revision) is essential. Default reasoning introduced by Reiter

[143] is one such non-monotonic reasoning method especially under contradictory knowledge segments. Default reasoning allows expressing a segment of knowledge as being *'true by default'* or *'generally true'*, but could be proven false upon arrival of new information.

A discrete species of bilattice structure that represents multivalued default logic is one that is used to model default reasoning and demonstrated the usefulness on performing human identity maintenance and contextual reasoning of event in visual surveillance domain [153, 154]. As noted above, the degree of truth and the degree of information concepts in bilattice are similar to the ones in subjective logic. Focusing on the similarity, we examine subjective logic operators that have corresponding semantic behavior to the operators defined on bilattice framework. As mentioned above, what is also interesting is that some continuous species of bilattice structures are often used to represent two different species of fuzzy logic. Namely, intuitionistic (or interval-valued) fuzzy logic that can be modeled with so-called *'triangle'* bilattice and fuzzy Belnap logic (i.e., fuzzified four-valued logic, $\mathcal{FOUR}$) that can be also modeled with so-called *'square'* bilattice [18]. The relationship between these two fuzzy species of bilattice structures is studied in the work of Cornelis et al. [42, 43] and Arieli et al. [18, 19]. Interestingly, the uncertainty representation in intuitionistic fuzzy logic (*'triangle'*) is very similar to that of the opinion triangle. Therefore, to verify the proposed subjective logic based default reasoning approach and to study its similarity and dissimilarity with fuzzy logics, we further extend the inference scheme defined on the discrete bilattice structure for the multivalued default logic onto the two continuous species of bilattice structures. To better verify and contrast the characteristics of the proposed approach, we present some illustrative case study examples in typical visual surveillance scenarios. We then compare the default reasoning results yielded from the proposed subjective logic based approach, bialttice for multivalued default logic, the intuitionistic fuzzy logic (*'triangle'*) and the fuzzy Belnap logic (*'square'*).

We believe this way of comparison better position the subjective logic as a tool for artificial reasoning and also give us better insights on the correlations among different uncertainty formalisms. Namely, by the inherent nature of Subjective logic, it gives a bridge between Dempster Shafer belief theory and Bayesian. Then by the comparison in this work on modeling default reasoning, it shows the bridge among subjective logic, fuzzy logics and bilattices as well.

5.3 Related Work

As higher-level semantic analysis of visual surveillance data is gaining growing attention, the flexibility on knowledge representation and proper uncertainty handling mechanism is becoming more important. To address this aspect, there has been some work on the use of logic programming languages due to the expressive power and on the use of different uncertainty handling formalisms. In general, such approaches can be referred to as *'extensional approach'*. Extensional approaches (also known as rule-based systems) treat uncertainty as a generalized truth value attached to formulas and compute the uncertainty of any formula as a function of the uncertainties of its sub formulas [130]. Akdemir et al. [13] used an ontology structure for activity analysis, but with no uncertainty handling mechanism. Shet et al. [152] introduced a system that adopts Prolog based logic programming for higher-level situation reasoning in visual surveillance. The same authors adopted bilattice based multivalued default reasoning for identity maintenance of human detection results and context reasoning [153, 154]. Jianbing et al. [110] adopted Dempster Shafer belief theory with the use of rule-based system for bus surveillance scenario. Anderson et al. [17] adopted fuzzy logic to model and analyze human activity for video based eldercare scenario. While different uncertainty handling formalisms are introduced with logic programming based knowledge modeling, principled handling of default reasoning has been only demonstrated by the bilattice based approach [153, 154] (refer to Section 2.4.3 for more detailed comparision).

When it comes to bilattice framework itself, it is known that some continuous species of bilattice structures that are called *'triangle'* and *'square'* correspond to intuitionistic fuzzy logic and fuzzy Belnap logic, respectively [18]. Naturally, there has been comparative study on the characteristics between intuitionistic fuzzy logic and fuzzy Belnap logic [42, 43]. Atanassov [22] introduced a transformation between these two fuzzy logics and proved that the transformation is bijective. The use of square bilattice is demonstrated to improve human detection results by the use of rule-based reasoning given high false alarm rate and partial occlusion based output of different body parts based detectors [155] with the similar inference scheme shown in their previous work [153, 154]. In this chapter, we show that the default reasoning behavior on multivalued default and square bilattice can be also modeled using subjective logic. Relying on the study of Atanassov [22] and Cornelis et al. [42, 43], we also show the correspondence among subjective logic, intuitionistic fuzzy logic (*'triangle'*) and fuzzy Belnap logic (*'square'*).

5.4 Preliminaries

This section gives an overview of the fundamental background about uncertainty representation and handling formalisms that will be discussed in this chapter in the view of default reasoning. The preliminaries will cover bilattice theory, and two extensions of fuzzy logics, namely, intuitionistic fuzzy logic and fuzzy Belnap logic. Refer to Chapter 3 for the theoretical overview of subjective logic theory and logic programming.

5.4.1 Bilattice Theory

Bilattices are algebraic structures which are mainly built on top of the concept *poset* (i.e., partially ordered set, which is a generalization of ordering elements in a set, in terms of a property of our interest) introduced by Ginsberg [69] and elaborated by Fitting [60]. Ginsberg's formal definition of bilattice is as follows [69].

Definition 18. ***(Partial Order) .*** *A partial order is a binary relation $\leq$ over a set S which is reflexive, antisymmetric and transitive, i.e., for all a,b and c in S, satisfies : a) $a \leq a$ (reflexive) b) if $a \leq b$ and $b \leq a$ then $a = b$ (antisymmetric) c) if $a \leq b$ and $b \leq c$ then $a \leq c$ (transitive).*

Definition 19. ***(Poset)* .** *A set S with a partial order $(S, \leq)$ is called partially ordered set (or poset).*

Definition 20. ***(Lattice)* .** *A poset L with a partial order is a lattice $(L, \leq, \wedge, \vee)$ if it satisfies the following two axioms:*

a) for any two elements a and b of L, the set $\{a, b\}$ has a least upper bound $\vee$ (join).

b) for any two elements a and b of L, the set $\{a, b\}$ has a greatest lower bound $\wedge$ (meet).

Definition 21. ***(Pre-bilattice)* .** *A pre-bilattice is a structure $\mathcal{B} = (B, \leq_t, \leq_k)$, where B is a nonempty set containing at least two elements, and $(B, \leq_t)$, $(B, \leq_k)$ are complete lattices (for which all subsets are also lattices).*

Definition 22. ***(Bilattice)* .** *A bilattice is a structure $\mathcal{B} = (B, \leq_t, \leq_k, \neg)$, such that $(B, \leq_t, \leq_k)$ is a pre-bilattice and $\neg$ is a unary operation on B that has the following properties: for every x, y in B,*

a) if $x \leq_t y$ then $\neg x \geq_t y$, b) if $x \leq_k y$ then $\neg x \leq_k \neg y$, c) $\neg\neg x = x$.

The name *'bi'*- lattice indicates that it is a structure consists of two lattices. Lattices are any *poset* that are possible to define *meet* $\wedge$ (aka. greatest lower bound) and *join* $\vee$ (aka. least upper bound) for any two elements in it. A partial order is a generalization of ordering, i.e., a binary relation $\leq$ over a set S which is reflexive, antisymmetric and transitive. Lattices are often expressed as a graph whose edges represent the binary relation of *'partial order'* between two elements that can be directly compared. There can be elements a and b in the lattice L for which an order between them cannot be determined. However, greatest lower bound *meet* $(a \wedge b)$ and the lowest upper bound *join* $(a \vee b)$ can always be determined for any of two elements a and b in the lattice L. Namely, by considering two sets that contain elements that are greater than a and greater than b respectively, the lowest element which can be found in both of the sets is the meet (and join can be similarly defined). Figure 5.1 (a) shows a lattice that some elements in it, for example, a and b are incomparable but still has $a \wedge c = f$ and $a \vee c = t$. Conventionally in lattice theory, '1' represents the greatest elements and '0' is the lowest elements and

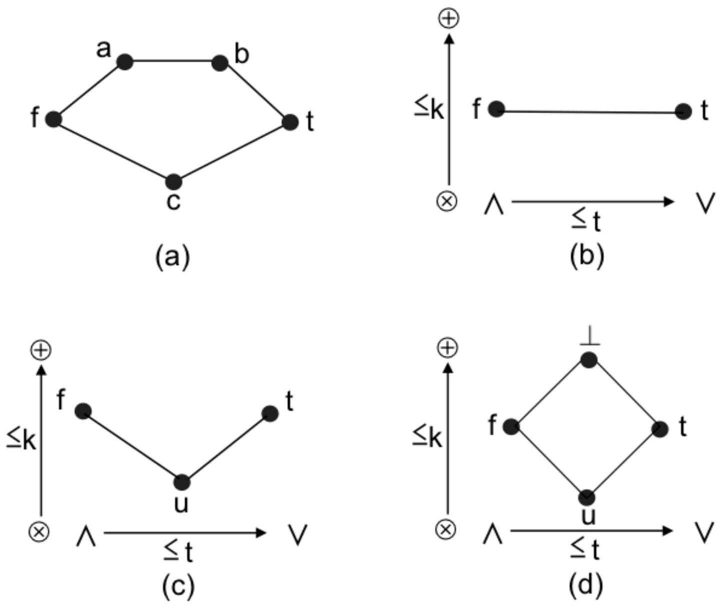

Figure 5.1: Examples of Lattice and Bilattices (a) a Lattice (b) Bilattice corresponding to Traditional Binary Logic (c) Bilattice corresponding to Three-valued Logic (d) Bilattice corresponding to Belnaps Four-Valued Logic, $\mathcal{FOUR}$. (Refer to [69] for more detail).

therefore for any a in lattice L, $a \wedge 1 = a$, $a \vee 1 = 1$, $a \wedge 0 = 0$ and $a \vee 0 = a$. Bilattices provide semantics for reasoning by considering one lattice with partial order in terms of degree of truth $\leq_t$ and the other lattice with partial order in terms of degree of information $\leq_k$ (note that, the semantics of degree of information often can be seen as degree of specificity of the information as well).

To avoid the confusion that would arise from using the same symbols *meet* $\wedge$ and *join* $\vee$ for both lattices, following Fitting, we use the symbols *meet* $\otimes$ and *join* $\oplus$ for the lattice with partial order $\leq_k$ [60]. While the meaning of $\wedge$ and $\vee$ corresponds to the standard logical role of conjunction and disjunction, the meaning of $\otimes$ and $\oplus$ are less intuitive. Fitting named $\otimes$ as *consensus* operator in the sense that it derives the most degree of information agreed upon two operands [60, 61]. Likewise $\oplus$ is named as *gullibility* operator in the sense that it accepts any degree of information upon two operands. In bilattices, therefore, when the *gullibility* operator $\oplus$ is used, getting more information pushes the overall belief towards true or false with more degree of information except in case of contradiction. Figure 5.1 (b) (c) (d) shows different bilattice structures that can model different logics.

5.4.2 L-Fuzzy Set Based Logics (Interval-valued Fuzzy Logic, Intuitionistic Fuzzy Logic and Fuzzy Belnap Logic)

Since the introduction of Fuzzy set theory and Fuzzy logic by Zadeh [176, 177], it became popular as a formalism for representing imprecise or vague linguistic concepts (e.g. such as *'hot'*, *'cold'*, *'fast'*, etc.). The basic idea is to introduce a fuzzy membership function (conventionally denoted as μ) as a measure of vagueness to elements in a set and it is called fuzzy set. The membership functions are defined to map an element u to a value within the interval $[0, 1]$ (i.e. $\mu(u) \rightarrow [0, 1]$), thereby assigning exact value makes all elements in the fuzzy set to be ordered and comparable. Due to this aspect, there has been arguing that this makes them inadequate for dealing with incomparable uncertain information [42]. There have been some remedies for this aspect. Noting on the footnote comment of Zadeh saying *'in a more general setting, the range of the membership function can be taken to be a suitable partially ordered set P.'* (p. 359 of [176]), Goguen introduced L-fuzzy set that uses a membership function that maps an element u to a value in a lattice (i.e. $\mu(u) \rightarrow L$) [70]. Interval based representation of degree of membership is also introduced with the name of interval-valued fuzzy sets (IVFSs, for short) by Gehrke et al. [68]. In IVFSs, an element u is mapped into a subinterval within (i.e. $\mu(u) \rightarrow (v_l, v_u) \in [0, 1]^2, v_l \leq v_u$). Intuitionistic fuzzy set theory (IFSs, for short) introduced by Atanassov [21] additionally adopts a non membership function ν, with a weaker constraint $\nu \leq 1 - \mu$ (note that, in the sense of Zadeh's original fuzzy set the ν is implicitly assumed to be $\nu = 1 - \mu$). Naturally, the socalled amount of *'indeterminacy'* or *'missing information'* can be defined by the value $\pi = 1 - \nu - \mu$. By dropping the constraint $\mu + \nu \leq 1$ and introducing even weaker constraint $\mu + \nu \leq 2$, we get fuzzy Belnap set. Unlike IVFSs that requires one to address values to be $v_l \leq v_u$ as lower bound and upper bound of imprecision, IFSs allows one to

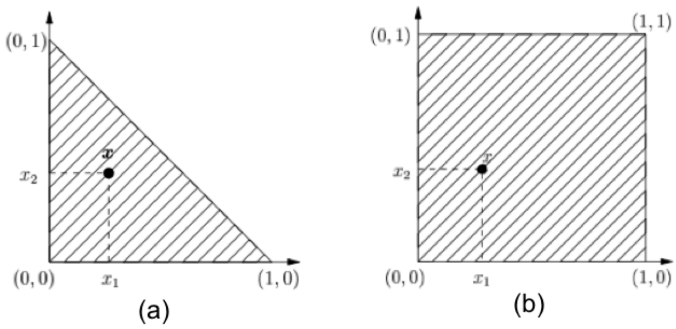

Figure 5.2: Lattices (a) $\mathcal{L}^*$ Triangle corresponding to Intuitionistic Fuzzy Logic (and also can be seen as $\mathcal{L}^I$ Triangle for Interval-Valued Fuzzy Logic), and (b) $\mathcal{L}_\Box$ Square corresponding to Fuzzy Belnap Logic.

address the positive and the negative side of an imprecise concept separately. Cornelis et al. [44], however, showed that IVFSs can also be represented in the form of IFSs, in other words the two are isomorphic and the truth values in IFSs can be represented as intervals (e.g. by the mapping $f(x_l, x_u) = (x_l, 1 - x_u)$). As traditional fuzzy set is used for fuzzy logic as a measure of uncertainty on a proposition, IVFSs, IFSs and fuzzy Belnap set are adopted for interval-valued fuzzy logic (IVFL, for short), intuitionistic fuzzy logic (IFL, for short) and the fuzzy Belnap logic (aka. Fuzzified four-valued logic, $\mathcal{FOUR}$). Following Goguen [70], IVFL and IFL can be defined on *'triangle* species of lattices denoted $\mathcal{L}^{\mathcal{I}}$ and $\mathcal{L}^*$ respectively. Fuzzy Belnap logic can be defined on the *'square'* lattice denoted $\mathcal{L}_\Box$. Therefore, IVFSs, IFSs and Fuzzy Belnap logics are kind of L-fuzzy logic. The formal definition of *'square'* and *'triangle'* are as follows [19, 43, 60].

Definition 23. ***($\mathcal{L}_\Box$ Square lattice for fuzzy Belnap logic) .*** *$\mathcal{L}_\Box = (L_\Box, \leq_\Box)$, where $L_\Box = [0,1]^2$ and $(x_1, x_2) \leq_\Box (y_1, y_2)$ iff $x_1 \leq y_1$ and $x_2 \geq y_2$.*

Definition 24. ***($\mathcal{L}^*$ Triangle lattice for IFSs) .*** *$\mathcal{L}^* = (L^*, \leq_{L^*})$, where $L^* = \{(x_1, x_2) \in [0,1]^2 | x_1 + x_2 \leq 1\}$ and $(x_1, x_2) \leq_{L^*} (y_1, y_2)$ iff $x_1 \leq y_1$ and $x_2 \geq y_2$.*

Definition 25. ***($\mathcal{L}^I$ Triangle lattice for IVFSs) .*** *$\mathcal{L}^I = (L^I, \leq_{L^I})$, where $L^I = \{(x_1, x_2) \in [0,1]^2 | x_1 \leq x_2\}$ and $[x_1, x_2] \leq_{L^I} [y_1, y_2]$ iff $x_1 \leq y_1$ and $x_2 \leq y_2$.*

Figure 5.2 shows the corresponding graphical interpretation of IFL and fuzzy Belnap logic. As it was shown in Figure 5.1 (c) and (d), they correspond to continuous extension of three valued logic and Belnap logic (Figure 5.2 (a) and (b), respectively). In epistemic sense, the values $(0,0)$ corresponds to *'unknown'*, $(1,0)$ corresponds to *'true'*, $(0,1)$ corresponds to *'false'*, (x,x) corresponds to *'undecidable'* and $(1,1)$ corresponds to *'contradiction'*. In Belnap, the *'contradiction'* point is a special case of *'undecidable'* points considered to have even more information than *'definite truth'* or *'definite false'*. In IFL and IVFL, however, this way of *'contradiction'* state is not allowed because IFL and IVFL do not allow epistemic points that is considered to have even more information than *'definite truth'* or *'definite false'*.

5.5 Default Reasoning

Defaults (default assumptions) are statements that can be interpreted as *'normally, typically, generally true or false'* as a rule. Contrary to defaults, statements that express explicit truth or falsity are called definite rules. In practice, the need to make default assumptions often occurs in cases where the information at hand is uncertain, incomplete and potentially contradictory. Default reasoning attempts to draw plausible conclusions based on known defaults and definite rules. Therefore, in default reasoning, conclusions can be changed upon acquisition of new information (i.e., *'nonmonotonicity'*). In logic, Reiter formalized such reasoning aspects as default logic theory using default rules [143]. In the following we give a brief overview on how rules are expressed in logic programming and a brief introduction to default logic.

5.5.1 Reiter's Default Logic

This section describes Reiter's formalization of default logic and an example of default reasoning in visual surveillance.

Definition 26. ***(Default Theory)*** [143] **.** *Let $\Delta = (D, W)$ be a default theory, where W is a set of logical formulae (rules and facts) also known as the definite rules and D is a set*

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Figure 5.3: Illustrative Example of Default Reasoning Scenario in Visual Surveillance.

of default rules of the form $\frac{\alpha:\beta}{\gamma}$*, where* α *is known as the precondition,* β *is known as the justification and* γ *is known as the conclusion.*

Any default rule $dr \in D$ can be also written as '$\gamma \leftarrow \alpha, not(\neg\beta)$', where *not* means the negation by failure to prove. The interpretation of such rule is that, if the precondition α is known to be true, and if there were no explicit violations of the justification (facts and rules that entails $\neg\beta$) then it is possible to derive the conclusion γ.

Example 2. ***(A Default Reasoning Scenario in Visual Surveillance) .*** Assume a scene as depicted in Figure 5.3 with two cameras observing the upper and lower parts of an escalator respectively. The scene also shows stairs next to the escalator. Consider the following set of rules :

$$\begin{aligned} \neg escalator\_working(T) &\leftarrow people\_take\_stairs(T), \\ &\quad\; not(escalator\_blocked(T)) \\ escalator\_blocked(T) &\leftarrow crowd\_at\_entrance(T) \end{aligned}$$
$$\text{where, } \neg escalator\_working(T) \in D \text{ and } escalator\_blocked(T) \in W.$$

Assume that Cam1 continuously observes that people appear to be using the stairs and generates a set of facts as $\{people\_take\_stairs(T_1)\}_{Cam1}$. Based on the current set of facts and rules, by default, the rule $\neg escalator\_working(T_1)$ is satisfied because we can not explicitly prove $escalator\_blocked(T_1)$. However, at a later time we observe a crowd in front of the escalator and as soon as of Cam2 generates a set of facts $\{crowd\_at\_entrance(T_2)\}_{Cam2}$ from its observations, then, the proposition $\neg escalator\_working(T_2)$ is no longer supported and is withdrawn.

5.5.2 Bilattice based Multivalued Default Logic

Ginsberg showed the use of bilattice structure to model default reasoning aspect and extended the structure to generalized default reasoning framework called multivalued default logic (aka. prioritized default logic) for artificial reasoning [69]. Ginsberg's bilattice structures also inherits the behind philosophy of Belnap logic in the sense that they also adopt the epistemic states *'unknown'* and *'contradictory'*. To distinguish definite truth and default truth value, default truth values assumed to have different amount of truth and different amount of information are also introduced. Figure 5.4 shows Belnap logic that has no default truth values (i.e., traditional four-valued logic), default logic and multivalued default logic respectively. Based on this truth value setup and the four bilattice operators, each bilattice operator can be seen as a truth functional binary operator on those values. Table 5.1 shows (a) the truth table of Belnap logic that has no defaults and (b) default logic that has default true and default false as epistemic states. Based on the truth functional binary operators, inference on the bilattice framework is defined in terms of truth assignment and closure as follows [69].

Definition 27. ***(Truth Assignment) .*** *Given a declarative language L and a Bilattice $\mathcal{B}$, a truth assignment is a function $\phi : L \to \mathcal{B}$.*

Definition 28. ***(Closure) .*** *Given a knowledge base $\mathcal{K}$ in form of declarative language and a truth assignment labeling each sentence $k \in \mathcal{K}$ with a truth value and a Bilattice $\mathcal{B}$, then the closure ϕ of a given query sentence q denoted $cl(\phi)(q)$, is the truth assignment function such that: $cl(\phi)(q) : \{p, p' | \forall S, S' \subseteq \mathcal{K}, S \models q, S' \models \neg q, p \in S, p' \in S'\} \to \mathcal{B}$.*

In other words, the implication of $cl(\phi)(q)$ is a functional mapping from the *'enumeration of all sentences'* that can entail (denoted by the symbol '$\models$') q and its contradictory information $\neg q$ to a *'truth value in bilattice $\mathcal{B}$'*. For example, if ϕ labels sentences $\{p, q \leftarrow p\} \in \mathcal{K}$ as true; i.e., $\phi(p) = T$ and $\phi(q \leftarrow p) = T$, then $cl(\phi)$ should also label q as true as it is information entailed by $\mathcal{K}$.

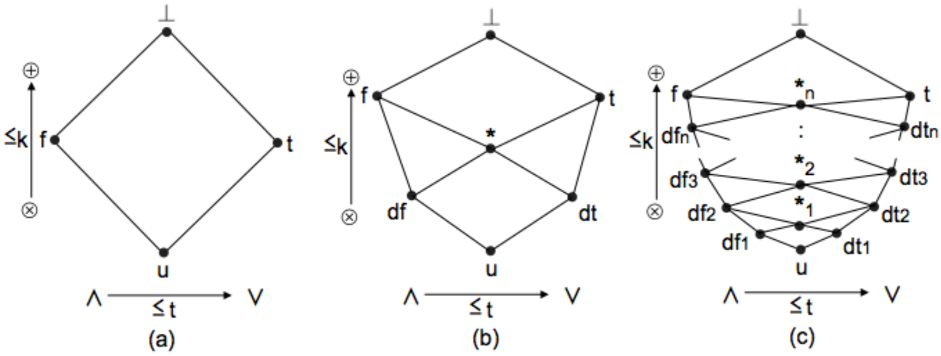

Figure 5.4: (a) Belnap Logic, $\mathcal{FOUR}$ (b) Default Logic (c) Multivalued (Prioritized) Default Logic.

(a)

$\wedge$	$\perp$	T	F	U
$\perp$	$\perp$	$\perp$	F	F
T	$\perp$	T	F	U
F	F	F	F	F
U	F	U	F	U

$\vee$	$\perp$	T	F	U
$\perp$	$\perp$	T	$\perp$	T
T	T	T	T	T
F	$\perp$	T	F	U
U	T	T	U	U

$\otimes$	$\perp$	T	F	U
$\perp$	$\perp$	T	F	U
T	T	T	U	U
F	F	U	F	U
U	U	U	U	U

$\oplus$	$\perp$	T	F	U
$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
T	$\perp$	T	$\perp$	T
F	$\perp$	$\perp$	F	F
U	$\perp$	T	F	U

(b)

$\wedge$	$\perp$	T	F	$\ast$	DT	DF	U
$\perp$	$\perp$	$\perp$	F	F	F	F	F
T	$\perp$	T	F	$\ast$	DT	DF	U
F	F	F	F	F	F	F	F
$\ast$	F	$\ast$	F	$\ast$	$\ast$	DF	DF
DT	F	DT	F	$\ast$	DT	DF	U
DF	F	DF	F	DF	DF	DF	DF
U	F	U	F	DF	U	DF	U

$\vee$	$\perp$	T	F	$\ast$	DT	DF	U
$\perp$	$\perp$	T	$\perp$	T	T	T	T
T	T	T	T	T	T	T	T
F	$\perp$	T	F	$\ast$	DT	DF	U
$\ast$	T	T	$\ast$	$\ast$	DT	$\ast$	DT
DT	T	T	DT	DT	DT	DT	DT
DF	T	T	DF	$\ast$	DT	DF	U
U	T	T	U	DT	DT	U	U

$\otimes$	$\perp$	T	F	$\ast$	DT	DF	U
$\perp$	$\perp$	T	F	$\ast$	DT	DF	U
T	T	T	$\ast$	$\ast$	DT	DF	U
F	F	$\ast$	F	$\ast$	DT	DF	U
$\ast$	$\ast$	$\ast$	$\ast$	$\ast$	DT	DF	U
DT	DT	DT	DT	DT	DT	U	U
DF	DF	DF	DF	DF	U	DF	U
U	U	U	U	U	U	U	U

$\oplus$	$\perp$	T	F	$\ast$	DT	DF	U
$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
T	$\perp$	T	$\perp$	T	T	T	T
F	$\perp$	$\perp$	F	F	F	F	F
$\ast$	$\perp$	T	F	$\ast$	$\ast$	$\ast$	$\ast$
DT	$\perp$	T	F	$\ast$	DT	$\ast$	DT
DF	$\perp$	T	F	$\ast$	$\ast$	DF	DF
U	$\perp$	T	F	$\ast$	DT	DF	U

Table 5.1: Truth Table of Bilattice Operators on (a) Belnap (b) Default Logic (The unshaded Part is exactly compatible with Belnap).

Definition 29. ***(Default Inference) .*** *Given a query sentence q and given S and S' that are sets of sentences such that $S \models q$ and $S' \models \neg q$, then the default inference is the truth value assignment closure $cl_{di}(\phi)(q)$ given by :*

$$cl_{di}(\phi)(q) = \bigoplus_{S \models q} u \vee [\bigwedge_{p \in S} cl(\phi)(p)] \oplus \neg \bigoplus_{S' \models \neg q} u \vee [\bigwedge_{p \in S'} cl(\phi)(p)]. \tag{5.1}$$

Informally, Equation 5.1 states that for n sets of sentences S that entails q, we first collect the lowest upper bound ('$\wedge$') that every sentence in S_i can agree on, then take it if the result contains more truth than u (unknown) along the partial order $\leq_t$. For each of these truth values of S_i, we evaluate the amount of information and choose the most informative (certain or specific) one among them using greatest lower bound ('$\oplus$') along the partial order $\leq_k$. We do the same process for all S'_i and by understanding the result from S' as contradictory hypothesis, we again collect the most informative one and apply the negation operation. Finally, both resulting intermediate values for S_i and S'_i are joined along the partial order $\leq_k$ again using greatest lower bound ('$\oplus$').

5.6 Default Reasoning using Subjective Logic

This section describes an inference mechanism for default reasoning using subjective logic. We will discuss how multiple logical values (i.e., default and definite truth values) can be modeled in subjective logic's opinion space. Thereafter, we propose an approach to modeling default reasoning based on subjective logic operators by analyzing the default reasoning mechanism on bilattice and identifying corresponding and analogous behaviour.

5.6.1 Fusing Contradicting Dogmatic Opinions in Subjective Logic

The core idea of default theory in Definition 26 is on the ability to discard a weaker belief by updating current belief based on a more specific or stronger belief. Among other approaches, the main strategy of the billatice based default inference shown in Definition 29 was to prioritize possible states of beliefs in an ordered set and to update lower ordered belief with higher ordered belief. Especially, the *'join operator'* $\oplus$ played an important role for combining competing truth values, that for example, draws an *'undecidible'* point when it fuses two conflicting beliefs at the same level of information (i.e., df_n and dt_n in Figure 5.4 (c)).

As explained in Section 3.1, subjective logic uses theoretical elements from the Dempster-Shafer belief theory that the Dempster's rule (see Definition 7) plays the central role for fusing different belief mass assignments. Benferhat et al. [26] introduced the use

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of Dempster Shafer belief theory for default reasoning. However, Dempster's rule has been criticized mainly because highly contradicting beliefs tend to produce counterintuitive results [98]. Among others, the critique is further formulated and discussed in the form of examples by Zadeh [178]. Audun Jøsang et al. [98, 94] also introduced such an information fusion method called *'consensus'* operator and showed that it is robust even in the highly contradicting cases.

In this section, therefore, we will briefly review how conflicting or contradictory information can be fused using subjective logic's *'consensus'* operator and contrast to Dempster's rule (see Definition 7).

The consensus operator is designed to *'fairly'* reflect two opinions in a single opinion. Informally, this means that, each opinion respect the other as much of their ignorance. The formal definition of *'consensus'* operator is as follows.

Definition 30. ***(Consensus** $\oplus_{sl}$**)*** [94] **.** *Let $w_x^A = (b_x^A, d_x^A, i_x^A, a_x^A)$ and $w_x^B = (b_x^B, d_x^B, i_x^B, a_x^B)$ be opinions respectively held by agents A and B about the same state x, and let $k = i_x^A + i_x^B - i_x^A i_x^B$. When $i_x^A, i_x^B \to 0$, the relative dogmatism between w_x^A and w_x^B is defined by γ so that $\gamma = i_x^B / i_x^A$ and $\gamma = 1$, when both i^A and i^B are exactly 0 (i.e., when A and B forms dogmatic opinions). Let $w_x^{A,B} = (b_x^{A,B}, d_x^{A,B}, i_x^{A,B}, a_x^{A,B})$ be the opinion such that:*

$$k \neq 0 : \begin{cases} b_x^{A,B} = (b_x^A i_x^B + b_x^B i_x^A)/k \\ d_x^{A,B} = (d_x^A i_x^B + d_x^B i_x^A)/k \\ i_x^{A,B} = (i_x^A i_x^B)/k \\ a_x^{A,B} = \frac{a_x^A i_x^A + a_x^B i_x^A - (a_x^A + a_x^B) i_x^A i_x^B}{i_x^A + i_x^B - 2 i_x^A i_x^B} \end{cases} \qquad k = 0 : \begin{cases} b_x^{A,B} = \frac{\gamma b_x^A + b_x^B}{\gamma + 1} \\ d_x^{A,B} = \frac{\gamma d_x^A + d_x^B}{\gamma + 1} \\ i_x^{A,B} = 0 \\ a_x^{A,B} = \frac{\gamma a_x^A + a_x^B}{\gamma + 1} \end{cases}$$

Then $w_x^{A,B}$ is called the consensus opinion between w_x^A and w_x^B, representing an imaginary agent $[A, B]$'s opinion about x, as if that agent represented both A and B. By using the symbol $\oplus$ to designate this operator, we define $w_x^{A,B} = w_x^A \oplus w_x^B$.

The behind philosophy of consensus operator is similar to the one of Bayesian. Namely, assuming two independent observations (opinions) A and B about a proposition x, it reflects both observations (opinions) in a *'fair and equal way'*. In Bayesian, the observation can be represented with the n number of sampling containing α number of positives and β number of negatives. Especially, using beta distribution, this can be represented as $Beta(p(x)|\alpha, \beta)$. Consider two independent observations observations (α^A, β^A) and (α^B, β^B). To be *'fair'*, each single observation should be reflected with the same weight, regardless of the total number of observations. Therefore, the fused observation should be the observation representing $(\alpha^A + \alpha^B, \beta^A + \beta^B)$ that represents again a beta distribution $Beta(p(x)|\alpha^A + \alpha^B, \beta^A + \beta^B)$. According to the opinion to beta distribution mapping scheme shown in Equation 3.9, this can be formulated as follows.

$$\begin{cases} Beta(p(x)|\alpha^A, \beta^A) \\ \Downarrow \\ b^A = \frac{\alpha^A}{\alpha^A+\beta^A+2} \\ d^A = \frac{\beta^A}{\alpha^A+\beta^A+2} \\ i^A = \frac{2}{\alpha^A+\beta^A+2} \end{cases} \oplus \begin{cases} Beta(p(x)|\alpha^B, \beta^B) \\ \Downarrow \\ b^B = \frac{\alpha^B}{\alpha^B+\beta^B+2} \\ d^B = \frac{\beta^B}{\alpha^B+\beta^B+2} \\ i^B = \frac{2}{\alpha^B+\beta^B+2} \end{cases} = \begin{cases} Beta(p(x)|\alpha^A + \alpha^B, \beta^A + \beta^B) \\ \Downarrow \\ b^{A,B} = \frac{\alpha^A+\alpha^B}{\alpha^A+\alpha^B+\beta^A+\beta^B+2} \\ d^{A,B} = \frac{\beta^A+\beta^B}{\alpha^A+\alpha^B+\beta^A+\beta^B+2} \\ i^{A,B} = \frac{2}{\alpha^A+\alpha^B+\beta^A+\beta^B+2} \end{cases} \tag{5.2}$$

Above (5.2), should be the same as the result derived by the Definition 30. For example, the consensus calculation of $b^{A,B}$ in opinion space is as follows.

$b^{A,B} = \frac{b^A i^B + b^B i^A}{i^A + i^B - i^A i^B}$,

by replacing $b^A, i^B, b^B, i^A$and $i^B$with the corresponding elements shown in (5.2).

$$\begin{cases} b^A i^B + b^B i^A = \frac{\alpha^A}{\alpha^A+\alpha^A+2}\frac{2}{\alpha^B+\alpha^B+2} + \frac{\alpha^B}{\alpha^B+\alpha^B+2}\frac{2}{\alpha^A+\alpha^A+2} = \frac{2(\alpha^A+\alpha^B)}{(\alpha^A+\beta^A+2)(\alpha^B+\beta^B+2)} \\ i^A + i^B - i^A i^B = \frac{2}{\alpha^A+\alpha^A+2} + \frac{2}{\alpha^B+\alpha^B+2} - \frac{4}{(\alpha^A+\beta^A+2)(\alpha^B+\beta^B+2)} = \frac{2(\alpha^A+\alpha^B+\beta^A+\beta^B+2)}{(\alpha^A+\beta^A+2)(\alpha^B+\beta^B+2)} \\ \rightarrow b^{A,B} = \frac{b^A i^B + b^B i^A}{i^A + i^B - i^A i^B} = \frac{\frac{\cancel{2}(\alpha^A+\alpha^B)}{\cancel{(\alpha^A+\beta^A+2)(\alpha^B+\beta^B+2)}}}{\frac{\cancel{2}(\alpha^A+\alpha^B+\beta^A+\beta^B+2)}{\cancel{(\alpha^A+\beta^A+2)(\alpha^B+\beta^B+2)}}} = \frac{\alpha^A+\alpha^B}{\alpha^A+\alpha^B+\beta^A+\beta^B+2}. \end{cases} \tag{5.3}$$

$d^{A,B}$ and $i^{A,B}$ can be proven similarly as above. Reminding the discussions in Section 3.1.3, in fact, the *'consensus'* operator is applying the bayesian theorem itself. The essence of bayesian theorem is *'posterior* $\propto$ *prior* $\times$ *likelihood'*. As the beta distributions are *'conjugated family'* of distributions, we know that the multiplication can be simplified by just adding the corresponding index values as follows.

$$Beta(p(x)|\alpha^A, \beta^A) \times Beta(p(x)|\alpha^B, \beta^B) = Beta(p(x)|\alpha^A + \alpha^B, \beta^A + \beta^B) \tag{5.4}$$

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$x \in 2^{\Theta}$	m^A_{Θ}	m^B_{Θ}	Dempster's rule	Non-normalised Dempster's rule	Consensus operator
Peter	0.98	0.00	0.490	0.0098	$w^{A,B}_{peter} = (0.492, 0.503, 0.005, 1/3)$, E=0.494
Paul	0.01	0.01	0.015	0.0003	$w^{A,B}_{Paul} = (0.010, 0.985, 0.005, 1/3)$, E=0.012
Mary	0.00	0.98	0.490	0.0098	$w^{A,B}_{Mary} = (0.492, 0.503, 0.005, 1/3)$, E=0.494
Θ	0.01	0.01	0.005	0.0001	-
$\emptyset$	0.00	0.00	0.000	0.9800	-

Table 5.2: An Example of Applying Dempster's Rule and Consensus Operator for Uncertain Belief Fusion [94].

$x \in 2^{\Theta}$	m^A_{Θ}	m^B_{Θ}	Dempster's rule	Non-normalised Dempster's rule	Consensus operator
Peter	0.99	0.00	0.000	0.0000	$w^{A,B}_{peter} = (0.495, 0.505, 0.000, 1/3)$, E=0.495
Paul	0.01	0.01	1.000	0.0001	$w^{A,B}_{Paul} = (0.010, 0.990, 0.000, 1/3)$, E=0.010
Mary	0.00	0.99	0.000	0.0000	$w^{A,B}_{Mary} = (0.495, 0.505, 0.000, 1/3)$, E=0.495
Θ	0.00	0.00	0.000	0.0000	-
$\emptyset$	0.00	0.00	0.000	0.9999	-

Table 5.3: An Example of Applying Dempster's Rule and Consensus Operator for Dogmatic Belief Fusion - Dempster's Rule Deriving Counter Intuitive Result [94].

Even though the *'numerical multiplication'* '$\times$' was used on beta distribution level of calculation, it is important to note that, the *'logical multiplication (conjunction)'* '$\wedge$' in subjective logic is different to the consensus operator. While *'consensus'* concerns about different opinions about the *'same'* proposition, *'logical multiplication (conjunction)'* deals with different opinions about *'dfferent'* propositions, and concerns about the chances that both may correct. For example, given two different binary frame of concern $X = \{x, \neg x\}$ and $Y = \{y, \neg y\}$, $w^A_x \wedge w^B_y$ concerns about the chances being $\{xy\}$ among $X \times Y = \{xy, x\neg y, \neg xy, \neg x\neg y\}$ (see Definition 31 for detail).

Now, let us again consider the Example 1 in Section 3.1.1, that Zadeh [178] used to criticize Dempster's rule. The example deals with a murder case with three suspects; Peter, Paul and Mary. Assume two conflicting testimonies. In Section 3.1.1, we have reviewed the result (see Table 3.1) of dempster's rule when the testimonies have some amount of uncertainty in the testimony. Similarly, Table 5.2 shows the result of applying consensus operator to the same example. As shown in Table 5.2, both Dempster's rule and consensus operator derived similar results. However, as shown in Table 5.3, if we fuse more highly contradicting beliefs having no uncertainty, Dempster's rule derives counter intuitive result saying that Paul is the suspect. Unlike Dempster's rule, consensus operator derived reasonable result similar to the case of Table 5.2.

5.6.2 On the Interpretation of Contradictory Point in Subjective Logic

As discussed, in subjective logic, fusing two contradictory information derives an opinion that reflects both opinions fairly. In the case of *'absolutely conflicting dogmatic opinions'*, namely definite true $w^A = (1, 0, 0)$ and definite false $w^B = (0, 1, 0)$, the consensus operator derives $w^{A,B} = (0.5, 0.5, 0)$ which is also a dogmatic opinion. *Cognitively speaking*, this opinion can be interpreted in two different views.

In bayesian view, such *'conflict'* or *'contradiction'* about a proposition x means that (no matter how we have observed) we have infinite positive observations and also infinite negative observations at the same time (i.e., $Beta(\alpha \to \infty, \beta \to \infty) = w(0.5, 0.5, 0)$). Namely, in this view, full contradiction would be linguistically said as *'a situation that we know it could occur or not occur at the same absolutely definite rate* $1/2$ *with no doubt'*. Given this, assume that a person should finally determine whether the proposition x would occur or not. The person may say it seems *'could happen with probability 0.5 but also could not happen with probability 0.5'* or more simply *'half and half'*.

While in logical view, the *'conflict'* or *'contradiction'* simply indicates the logically inconsistent state itself, where definite true and definite false can arise for the same proposition. In this sense, it most likely indicates logical error or wrong set up in the considered frame of discernment. Namely, in this view, the focus is on the fact that the resulting opinion had been derived through both the definite true and the definite false. In some multi-valued logic formalisms such as Belnap, this state is labeled with a symbol such as '$\perp$' called *'(full) contradiction'*. Especially, in bilattice based logic frameworks, the *'contradictory point'* is defined to have even more information than *'definite truth'* and *'definite false'* as shown in Table 5.4 and Figure 5.5. This is due to the restriction that it is based on a set theory (i.e., poset), in which every element should be ordered in a way it has both the *'meet'* and the *'join'* operators. (Note that, the *'contradictory'* point is *'the only point'* that is defined to have even more information than definite true and false as shown in Section 5.4.1).

In summary, while the former focusses on the interpretation of a given opinion itself, the latter rather focusses on the process how an opinion had been derived. To address this discussion, Audun Jøsang [94] proposed an extra parameter called *'degree of conflict'*. Following is the statement of Audun Jøsang in page 13 of [94].

'An argument that could be used against our consensus operator, is that it does not give any indication of possible belief conflict. Indeed, by looking at the result only, it does not tell whether the original beliefs were in harmony or in conflict, and it would have been nice if it did. ***A possible way to incorporate the degree of conflict is to add an extra conflict parameter****. This could for example be the belief mass assigned to $\emptyset$ in Non-normalised Dempster's rule, which in the opinion notation can be defined as $c_x^{A,B} = b_x^A d_x^B + b_x^B d_x^A$ where $c_x^{A,B} \in [0,1]$. The consensus opinion with conflict parameter would then be expressed as $w_x^{A,B} = (b_x^{A,B}, d_x^{A,B}, i_x^{A,B}, a_x^{A,B}, c_x^{A,B})$. The conflict parameter would only be relevant for combined belief, and not for original beliefs. A default value $c = -1$ could for example indicate original belief, because a default value $c = 0$ could be misunderstood as indicating that a belief comes from combined harmonious beliefs, even though it is an original belief.'*

The *'conflict'* parameter above can be used as an indicator to inform that the opinion is derived by consensus. Following the setup, fusing definite true $w_x^A = (1, 0, 0, 1/2, -1)$ and definite false $w_x^B = (0, 1, 0, 1/2, -1)$ would introduce the *'conflict'* parameter $c^{A,B} = 1 \cdot 1 + 0 \cdot 0 = 1$. Therefore, we get, $w_x^{A,B} = (0.5, 0.5, 0, 1/2, 1)$. This gives us the clue how the opinion was derived. Comparing $w_x^Z = (0.5, 0.5, 0, 1/2, -1)$ and $w_x^{A,B} = (0.5, 0.5, 0, 1/2, 1)$, while the former indicates one independent observation based opinion and the latter indicates that it was fused by two highly contradicting information, the actual distribution of them are the same. Namely, once an opinion is derived, one practical interpretation could be to indicate to the application that there is a conflict situation. However, any subsequent numerical calculation upon the resulting opinion will be the same, no matter how it had been derived. Therefore, in this dissertation, we will not explicitly use the *'conflict'* parameter but rather simply accept the Bayesian view for dealing with *'(full) contradictory'* state. Then, we will further discuss this aspect again in Discussion 1 in Section 5.9.1.

5.6.3 Mapping Multi-Logic-Values into Opinion Space

In this section we discuss the possible mapping of multiple logic values into opinion space. We start with noting that the implications of $\leq_t$ and $\leq_k$ in bilattice are similar to the concept of truth and ignorance in subjective opinion space when visualized in the opinion triangle. As shown in Figure 5.4 (b) and (c), d_{tn} and d_{fn} indicate different levels of incomplete truth or falsity. The more certain and specific knowledge is obtained, the higher level of default values result. The degree of certainty or specificity can be considered as degree of information, therefore, the levels of default values can be ordered along the partial order $\leq_k$. Along the information order $\leq_k$, for each pair of d_{tn} and d_{fn} there exist corresponding undecidible states $*_n$. As shown in Figure 5.4 (b) and (c), $*_n$ are assumed to have more information than their corresponding d_{tn} and d_{fn}. Unknown state U is one of the undecidible states with zero degree of information. Similarly, the full contradictory state $\perp$, that can be reached via definite true (full belief) and definite false (full disbelief), is also one of the undecidible states with maximum degree of information.

In the sense of degree of information, however, assigning even higher degree of information than definite true or false to the full contradiction point $\perp$ is an interesting aspect to discuss. In bilattice, the full contradictory state $\perp$ is considered to have even more information than definite true or false. While in subjective logic, it is again a dogmatic opinion having full degree of information (no ignorance). As discussed in the previous Section 5.6.2, however, this difference can be compromised by adding another dimension of information, (i.e., the *conflict parameter* $c = 1$) to explicitly indicate that a fused opinion had been derived via the definite true and the definite false. Therefore, strictly speaking, the bilat-

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tice way of representing *'full contradiction'* in subjective logic is $(0.5, 0.5, 0, a, c = 1)$. To distinguish this slight difference, we will denote the maximum undecidible point (namely, full contradictory point) in bilattice as $\perp_{bl}$ and $\perp_{sl} = (0.5, 0.5, 0, a, 1)$ for subjective logic (henceforce, the subscripts $_{bl}$ and $_{sl}$ denote bilattice and subjective logic respectively).

Except the full contradictory point, the rest of undecidible states $*_n$ can be defined in the opinion triangle as of bilattice. Additionally, such $*_n$ should be able to reach via an operation as of *join* operator $\oplus_{bl}$ in bilattice on the partial order $\leq_k$. This aspect can be modeled with subjective logic *consensus* operator $\oplus_{sl}$ (see Definition 30). By definition, when d_{tn} and d_{fn} (having the same level of ignorance) are fused with the *consensus* operator $\oplus_{sl}$, it always yields an opinion in the middle of opinion triangle with less ignorance. The only exception to this is the case of fusing definite true and definite false that yields an opinion in the middle between definite true and definite false again with no ignorance. For example, if we consider a tiny amount of ignorance[1] ε and take $(1 - \varepsilon, 0, \varepsilon) = t'$ and $(0, 1 - \varepsilon, \varepsilon) = f'$ as any default true or default false, then fusing t' and f' in terms of degree of information in subjective logic $t' \oplus_{sl} f' = *'$ will always draw the values with less ignorance $\varepsilon' < \varepsilon$ (see definition of i in the case of $k \neq 0$ in Definition 30). Following is the formal proof of this aspect.

$$
\begin{aligned}
&\textbf{Premise :}\\
&\text{Given two opinions } w_x^A = (1-\varepsilon, 0, \varepsilon) \text{ and } w_x^B = (0, 1-\varepsilon, \varepsilon)\ \text{, where } \varepsilon \in (0,1)\\
&w_x^A \oplus w_x^B = \left(\frac{2(1-\varepsilon)}{2-\varepsilon}, \frac{2(1-\varepsilon)}{2-\varepsilon}, \frac{\varepsilon}{2-\varepsilon}\right) \text{ satisfies } \varepsilon > \frac{\varepsilon}{2-\varepsilon}.\\
&\textbf{Proof :}\\
&\varepsilon > \frac{\varepsilon}{2-\varepsilon} \Rightarrow \cancel{\varepsilon}(2-\varepsilon) > \cancel{\varepsilon} \Rightarrow 2-\varepsilon > 1,\ \text{by the fact that } \varepsilon \in (0,1) \Rightarrow (1,2) > 1\\
&\therefore \varepsilon > \frac{\varepsilon}{2-\varepsilon}\ \square
\end{aligned}
\tag{5.5}
$$

This behavior is exactly the same as what bilattice based default structures capture, thus, in the sense of ordering along $\leq_k$, $t' \leq_k *'$ and $f' \leq_k *'$. The only exception to this is when definite true and false are fused. Namely, $t \oplus_{sl} f = \perp_{sl}$ (see Definition 30). This means that only for this single point there is no exact correspondence but rather an approximate correspondence. Consequently this point in the opinion triangle is denoted

[1] Benferhat et al. [26] used similar idea of introducing ε for Dempster Shafer belief theory based default reasoning.

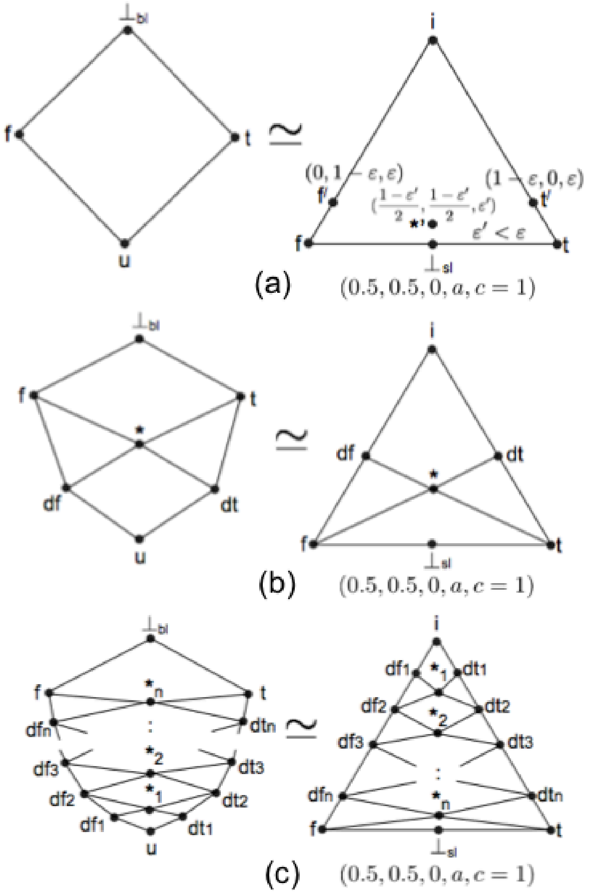

Figure 5.5: (a)The Bilattice and Opinion Triangle Space for Belnap $\mathcal{FOUR}$, (b) The Bilattice and Opinion Triangle Space for Default Logic, (c) The Bilattice and Opinion Triangle Space for Multivalued Default Logic.

as $\perp_{sl}$ as depicted in Figure 5.5 (a). Figure 5.5 (a) depicts that the correspondence of the bilattice used for Belnap logic, $\mathcal{FOUR}$ and the opinion triangle for example, by mapping $t_{bl} \simeq t_{sl} = (1, 0, 0, a)$, $f_{bl} \simeq f_{sl} = (0, 1, 0, a)$, $u_{bl} \simeq i_{sl} = (0, 0, 1, a)$ and $\perp_{bl} \simeq \perp_{sl} = (0.5, 0.5, 0, a, c = 1)$. As mentioned in Section 5.6.2, however, considering that the *conflict parameter* c does not affect to its subsequent calculation, we will not explicitly use the *conflict parameter* in this dissertation. For the default values, following the discussion above and nontheless values in bilattice are elements of a finite set and the opinion triangle is of continuous domain, we could pick points along the side edges of the opinion triangle. However, it is important to note that picking the logical values should be done in the sense of selecting opinion points to put in a set that satisfies ordering like bilattice. Therefore, there can be many ways of such mapping as long as we pick values such that:

$$\begin{aligned}&1.\ df_n \text{ and } dt_n \text{ have the same amount of ignorance.}\\&2.\ ignorance(dt_{n-1} \oplus df_{n-1} = \star_{n-1}) > ignorance(dt_n \oplus df_n = \star_n).\end{aligned} \tag{5.6}$$

Figure 5.5 (b) shows an example of such mapping of default true and default false to the opinion triangle. In the same manner, we can extend such mapping to the generalized multivalued default logic in which each of the defaults can be considered with different priority levels as shown in Figure 5.5 (c).

5.6.4 Default Inference using Subjective Logic Operators

Now, bearing in mind the default inference mechanism defined on bilattice (see Equation 5.1), we examined subjective logic operators corresponding to $\wedge_{bl}$, $\vee_{bl}$, $\otimes_{bl}$ and $\oplus_{bl}$. Concerning the semantic interpretation of $\wedge_{bl}$ and $\vee_{bl}$ representing the standard logical role of *conjunction* and *disjunction*, subjective logic *conjunction* ($\cdot_{sl}$) and *disjunction* ($\sqcup_{sl}$) operators are examined. For the $\otimes_{bl}$ and $\oplus_{bl}$ operators representing *consensus* and *gullibility* , subjective logic *consensus* $\oplus_{sl}$ and *addition* $+_{sl}$ operators are examined. Table 5.4 shows the examination results about the correspondence between each of semantically corresponding operator pairs. Interestingly, the interpretation of *consensus* operator $\otimes_{bl}$ does not match to the *consensus* $\oplus_{sl}$ in subjective logic. Rather, *consensus* $\oplus_{sl}$ exhibits characteristics corresponding to the *gullibility* operator ($\oplus_{bl}$) in bilattice. The subjective logic *addition* operator $+_{sl}$ showed completely different truth table compared to both the $\oplus_{bl}$ and the $\otimes_{bl}$. This is because the operator $+_{sl}$ is about adding any beliefs, subtracting any disbelief and averaging ignorance from both operands. Thereby, it tends to force rapid change of belief towards truth direction. The operator $\otimes_{bl}$ seems not to have any corresponding operator in subjective logic. In Table 5.4, the black points represent that those values are identical to the ones in bilattice space. Contrary, the red points indicate that the values are slightly different from the ones in bilattice space. This is due to the difference that the bilattice operators are defined on a discrete set and the operators in subjective logic are

$\wedge_{bl}$	$\perp$	T	F	$*$	DT	DF	U
$\perp$	$\perp$	$\perp$	F	F	F	F	F
T	$\perp$	T	F	$*$	DT	DF	U
F	F	F	F	F	F	F	F
$*$	F	$*$	F	$*$	$*$	DF	DF
DT	F	DT	F	$*$	DT	DF	U
DF	F	DF	F	DF	DF	DF	DF
U	F	U	F	DF	U	DF	U

$\vee_{bl}$	$\perp$	T	F	$*$	DT	DF	U
$\perp$	$\perp$	T	$\perp$	T	T	T	T
T	T	T	T	T	T	T	T
F	$\perp$	T	F	$*$	DT	DF	U
$*$	T	T	$*$	$*$	DT	$*$	DT
DT	T	T	DT	DT	DT	DT	DT
DF	T	T	DF	$*$	DT	DF	U
U	T	T	U	DT	DT	U	U

$\oplus_{bl}$	$\perp$	T	F	$*$	DT	DF	U
$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
T	$\perp$	T	$\perp$	T	T	T	T
F	$\perp$	$\perp$	F	F	F	F	F
$*$	$\perp$	T	F	$*$	$*$	$*$	$*$
DT	$\perp$	T	F	$*$	DT	$*$	DT
DF	$\perp$	T	F	$*$	$*$	DF	DF
U	$\perp$	T	F	$*$	DT	DF	U

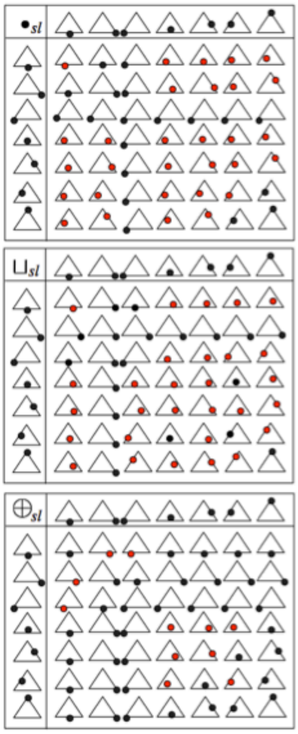

Table 5.4: Comparison of Operators used in $cl(\phi)$ shown in Equation (5.1) (Black - exact match and Red - slight different match).

defined on continuous space. However, the semantics of the corresponding operators are the same as it is explained below.

Definition 31. ***(Conjunction*** $\cdot_{sl}$***)*** [97] **.** *Let Θ_X and Θ_Y be two frames and let x and y be propositions about state in Θ_X and Θ_Y respectively. Let $w_x = (b_x, d_x, i_x, a_x)$ and $w_y = (b_y, d_y, i_y, a_y)$ be an agent's opinions about x and y, then conjunctive opinion denoted as $w_x \cdot w_y$ is $w_{x\wedge y} = (b_{x\wedge y}, d_{x\wedge y}, i_{x\wedge y}, a_{x\wedge y})$ such that :*

$$\begin{cases} b_{x\wedge y} = b_x b_y + \frac{(1-a_x)a_y b_x i_y + a_x(1-a_y)i_x b_y}{1-a_x a_y} \\ d_{x\wedge y} = d_x + d_y - d_x d_y \\ i_{x\wedge y} = i_x i_y + \frac{(1-a_y)b_x i_y + (1-a_x)i_x b_y}{1-a_x a_y} \\ a_{x\wedge y} = a_x a_y \ . \end{cases}$$

Definition 32. ***(Disjunction*** $\sqcup_{sl}$***)*** [97] **.** *Let Θ_X and Θ_Y be two frames and let x and y be propositions about state in Θ_X and Θ_Y respectively. Let $w_x = (b_x, d_x, i_x, a_x)$ and*

$w_y = (b_y, d_y, i_y, a_y)$ be an agent's opinions about x and y, then disjunctive opinion denoted as $w_x \sqcup w_y$ is $w_{x\vee y} = (b_{x\vee y}, d_{x\vee y}, i_{x\vee y}, a_{x\vee y})$ such that :

$$\begin{cases} b_{x\vee y} = b_x + b_y - b_x b_y \\ d_{x\vee y} = d_x d_y + \frac{a_x(1-a_y)d_x i_y + (1-a_x)a_y i_x d_y}{a_x + a_y - a_x a_y} \\ i_{x\vee y} = i_x i_y + \frac{a_y d_x i_y + a_x i_x d_y}{a_x + a_y - a_x a_y} \\ a_{x\vee y} = a_x + a_y - a_x a_y \ . \end{cases}$$

The truth functional table of logical conjunction (disjunction) in discrete space should be a function that is closed to its discrete set of truth values. Therefore, considering the interpretation of conjunction (disjunction), the binary operator $\wedge_{bl}$ ($\vee_{bl}$) should pick the greatest upper bound (lowest upper bound) element of given two operands. This forces, for example, $T \wedge_{bl} DT$ ($F \vee_{bl} DF$) to be DT (DF). However, if we were not restricted by discrete set of values, as is the case in subjective logic, we would expect values in between T and DT (F and DF) in a sense that the conjunction (disjunction) operation are interpreted as intersection (union) of both belief values. This aspect is mainly captured by the definition of $b_{x\wedge y}$ ($b_{x\vee y}$). The amount of ignorance is also captured by $i_{x\wedge y}$ ($i_{x\vee y}$) so that it can consider both ignorance values of given two belief values. This aspect is the main source where the differences of the truth table comes from in Table 5.4. Thus, considering the semantics of conjunction and disjunction, subjective logic's conjunction and disjunction operators model the meaning of the operators under partial ignorance correctly, but with an additional aspect that is only meaningful in a continuous space.

Similarly to conjunction and disjunction, the operator $\oplus_{bl}$ is also defined to pick a value among the given discrete set of values. The selection is done in the sense that it chooses any information that can be accepted upon both operands. In subjective logic, the consensus operator $\oplus_{sl}$ sees each of operands as one that have observed continuous amount of positive and negative evidence, thereby, summing up both observations into one opinion. As discussed in the previous Section 5.6.3, this is similar to interpreting the semantics of consensus from a bayesian perspective and it will increase the amount of information but cannot be restricted to be a discrete value. This aspect is captured by the definition of $i_x^{A,B}$

in Definition 30, except in the case of dogmatic opinions having no ignorance. Therefore, the meaning of $\oplus_{bl}$ is modeled also in the sense of partial order $\leq_k$ in bilattice, i.e., that the derived value of given two operands should have more information (less ignorance in subjective logic). Thus, $\oplus_{sl}$ operator in subjective logic models fusing uncertain beliefs in a way that it increases the degree of information.

Based on the consideration about the semantics of operators shown above, we now defined the truth assignment and closure operation for default reasoning using subjective logic.

Definition 33. ***(Truth Assignment$_{sl}$) .*** *Given a declarative language L and Subjective Opinion Space O, a truth assignment is a function $\phi_{sl} : L \to O$.*

Definition 34. ***(Closure$_{sl}$) .*** *Given a knowledge base $\mathcal{K}$ in form of declarative language and a truth assignment labeling each sentence $k \in \mathcal{K}$ with a truth value and Subjective Opinion Space O, then $cl_{sl}(\phi)(q)$, the closure ϕ of a given query sentence q, is the truth assignment function such that :*
$cl_{sl}(\phi)(q) : \{p, p' | \forall S, S' \subseteq \mathcal{K}, S \models q, S' \models \neg q, p \in S, p' \in S'\} \to O$.

Definition 35. ***(Default Inference$_{sl}$) .*** *Given a query sentence q and given S and S' that are sets of sentences such that $S \models q$ and $S' \models \neg q$, then the default inference is the truth value assignment closure $cl_{sl_{di}}(\phi)(q)$ given by :*

$$cl_{sl_{di}}(\phi)(q) = \bigoplus_{S \models q} u \sqcup [\prod_{p \in S} cl_{sl}(\phi)(p)] \oplus \neg \bigoplus_{S' \models \neg q} u \sqcup [\prod_{p \in S'} cl_{sl}(\phi)(p)] \,. \tag{5.7}$$

In fact, reminding the *'Opinion Assignment'* of Definition 16 described in Section 4.3.2, Definition 33 corresponds to the Definition 16 - 1 and 2. Similarly, Definition 34 and Definition 35 corresponds to the Definition 16 - 4. In this dissertation, we will use the term *'Opinion Assignment'* to emphasize the use of subjective opinion. In this chapter, however, we will use the term *'Truth Assignment'* instead of the term *'Opinion Assignment'* also to consider the use of other *'uncertainty representation formalisms'* (such as bilattice and L-fuzzy sets) and to compare with them.

5.7 Multivalued Default Logic, Square Bilattice and Default Opinion

In this section, as it was done for subjective logic, we extend the discrete bilattice based multivalued default logic to L-fuzzy logics in continuous space. We then describe some properties of L-fuzzy logic representations on bilattice structure and review the possibility on enabling default reasoning. In the preliminaries section, we have considered IVFL, IFL and fuzzy Belnap logic in terms of truth value order ($\leq_t$). However, as we have examined in the previous section, the operators (especially, $\oplus_{bl}$) on degree of information ($\leq_k$) play an important role to model default reasoning aspect as to the case of discrete species bilattice of multivalued default logic. There has been some work on representing IVFSs, IFSs and Fuzzy Belnap using bilattice. Following is the definitions of *'triangle'* and *'square'* in the context of bilattice. Arieli et al. introduced the following definitions of *'square'* bilattice for fuzzy-belnap logic and *'triangle'* bilattice for IVFL [19].

Definition 36. ***($\mathcal{L}^2$ Square bilattice) .*** *Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice. A (bilattice-based) square is a structure $\mathcal{L}^2 = (L \times L, \leq_t, \leq_k, \neg)$, where $\neg(x_1, x_2) = (x_2, x_1)$, and*

a) $(x_1, x_2) \leq_t (y_1, y_2) \Leftrightarrow x_1 \leq_L y_1$ and $x_2 \geq_L y_2$,

b) $(x_1, x_2) \leq_k (y_1, y_2) \Leftrightarrow x_1 \leq_L y_1 and x_2 \leq_L y_2$.

When we set $L = [0, 1]$, this captures the Atanassov's idea on intuitionistic fuzzy sets in the sense that it distinguishes between membership function μ and a non-membership ν, but without imposing the restriction $\mu + \nu \leq 1$. Denoting the join and meet operations of the complete lattice $\mathcal{L}$ by $\wedge_L$ and $\vee_L$, the following four operators are defined for (x_1, x_2), (y_1, y_2) in L^2,

$$\begin{aligned} &(x_1, x_2) \wedge (y_1, y_2) = (x_1 \wedge_L y_1, x_2 \vee_L y_2) \quad &(x_1, x_2) \vee (y_1, y_2) = (x_1 \vee_L y_1, x_2 \wedge_L y_2) \\ &(x_1, x_2) \otimes (y_1, y_2) = (x_1 \wedge_L y_1, x_2 \wedge_L y_2) \quad &(x_1, x_2) \oplus (y_1, y_2) = (x_1 \vee_L y_1, x_2 \vee_L y_2)\ . \end{aligned} \tag{5.8}$$

	t-norms for $\wedge_L$	t-conorms for $\vee_L$
Min/Max	$\mathcal{T}_{\min}(a,b) = \min\{a,b\}$,	$\mathcal{S}_{\max}(a,b) = \max\{a,b\}$
Product/Sum	$\mathcal{T}_{\text{prod}}(a,b) = a \cdot b$	$\mathcal{S}_{\text{sum}}(a,b) = a + b - a \cdot b$
Łukasewicz	$\mathcal{T}_{\text{Luk}}(a,b) = \max\{0, a+b-1\}$.	$\mathcal{S}_{\text{Luk}}(a,b) = \min\{a+b, 1\}$
Drastic	$\mathcal{T}_{\text{D}}(a,b) = \begin{cases} b & \text{if } a = 1 \\ a & \text{if } b = 1 \\ 0 & \text{otherwise.} \end{cases}$	$\mathcal{S}_{\text{D}}(a,b) = \begin{cases} b & \text{if } a = 0 \\ a & \text{if } b = 0 \\ 1 & \text{otherwise,} \end{cases}$
Nilpotent min/max	$\mathcal{T}_{\text{nM}}(a,b) = \begin{cases} \min(a,b) & \text{if } a+b > 1 \\ 0 & \text{otherwise} \end{cases}$	$\mathcal{S}_{\text{nM}}(a,b) = \begin{cases} \max(a,b) & \text{if } a+b < 1 \\ 1 & \text{otherwise.} \end{cases}$
Hamacher Prod/Einstein	$\mathcal{T}_{\text{H}_0}(a,b) = \begin{cases} 0 & \text{if } a = b = 0 \\ \frac{ab}{a+b-ab} & \text{otherwise} \end{cases}$	$\mathcal{S}_{\text{H}_2}(a,b) = \dfrac{a+b}{1+ab}$

Table 5.5: Some of *t-norms* and *t-conorms*.

The $\wedge_L$ and $\vee_L$ can be defined by a *t-norm* and a *t-conorm* in the sense that they generalize intersection and union in lattice space that can be seen as a metric space that satisfies triangle inequity. (note that, a *t-norm* is a function $\mathcal{T} : [0,1] \times [0,1] \rightarrow [0,1]$ that satisfies commutative, monotonicity, associative and one act as identity element. And the same is for a *t-conorm* $\mathcal{S} : [0,1] \times [0,1] \rightarrow [0,1]$ by replacing the last constraint with zero identity constraint). Table 5.5 shows some of well known *t-norms* and *t-conorms*. Choosing a pair of them to use for $\wedge_L$ and $\vee_L$ on the lattice L , we can define *meet* $\wedge$ and *join* $\vee$ for partial order $\leq_t$, and *meet* $\otimes$ and *join* $\oplus$ for partial order $\leq_k$ on the bilattice $\mathcal{L}^2$ according to Equation 5.8 shown above. Therefore, considering the semantics of the Equation 5.1 (see Definition 29), we can directly apply the same inference scheme to bilattice based fuzzy Belnap logic. Similarly to square bilattice for fuzzy Belnap logic, triangle for IVFSs can be defined as follows.

Definition 37. *($\mathcal{I}(\mathcal{L})$ **Triangle bilattice for IVFSs**) . Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice. and $I(L) = \{[x_1, x_2] | (x_1, x_2) \in L^2, x_1 \leq_L x_2\}$. A (bilattice-based) triangle is a structure $\mathcal{I}(\mathcal{L}) = (I(L), \leq_t, \leq_k)$, where*

a) $[x_1, x_2] \leq_t [y_1, y_2] \Leftrightarrow x_1 \leq_L y_1$ and $x_2 \leq_L y_2$, *b)* $[x_1, x_2] \leq_k [y_1, y_2] \Leftrightarrow x_1 \leq_L y_1$ *and* $x_2 \geq_L y_2$ *.*

Though the definitions of triangle bilattice for IFSs are not explicitly defined in their work, we c-storean easily introduce the triangle structure for IFSs following the Definition 37.

Definition 38. ***($\mathcal{I}^*(\mathcal{L})$ Triangle bilattice for IFSs) .*** *Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice. and $I(L) = \{[x_1, x_2]|(x_1, x_2) \in L^2, x_1 + x_2 \leq_L 1_L\}$. A (bilattice-based) triangle is a structure $\mathcal{I}^*(\mathcal{L}) = (I^*(L), \leq_t, \leq_k, \neg)$, where $\neg(x_1, x_2) = (x_2, x_1)$, and*

a) $[x_1, x_2] \leq_t [y_1, y_2] \Leftrightarrow x_1 \leq_L y_1$ *and* $x_2 \geq_L y_2$*,*

b) $[x_1, x_2] \leq_k [y_1, y_2] \Leftrightarrow x_1 \leq_L y_1$ *and* $x_2 \leq_L y_2$ *.*

As in the case of square bilattice, by setting $L = [0, 1]$, the structure correspond to $\mathcal{L}^{\mathcal{I}}$ and $\mathcal{L}^*$. However, Arieli et al. also showed that $\mathcal{I}(\mathcal{L})$ is in fact not a (pre-) bilattice, since the substructure is not a lattice because the *lub (least upper bound, join $\vee_k$)* of any two elements does not always exist [19]. This corresponds to the interesting aspect that, the triangles do not allow explicit representation of the epistemic state *'contradictory'* in terms of degree of information (note that opinion triangle of subjective logic has the same aspect). Therefore, for example, the full truth and full falsity in triangle are not comparable in terms of degree of information. But still $(I(L), \leq_k)$ is a partially ordered set, therefore the triangle is very much in the same spirit as bilattices. This property is also same in the case of $\mathcal{I}^*(\mathcal{L})$. They have also proved that *t-norms* and *t-conorms* for the $\leq_k$-order can't be properly defined by introducing some theorems such as *t-representability* theorem, etc. However, they showed that any *t-norms* or *t-conorms* definable in classical fuzzy set theory have extensions to IVFSs along the partial order $\leq_t$ in a compositional manner. Due to this aspect, it seems that bilattices are not always the key to model the adequate properties of IVFL and IFL but is quite much adequate for modeling fuzzy Belnap logic. Therefore, in the context of bilattice, the default inference scheme, Equation 5.8 can not be set up on IVFL or IFL. While the *meet* $\otimes$ and *join* $\oplus$ operators can not be defined on $\mathcal{I}(\mathcal{L})$ and $\mathcal{I}^*(\mathcal{L})$, however, there is an useful mapping between *square* $\mathcal{L}_\square$ and *triangle*

$\mathcal{L}^*$. In the work [22], Atanasov, the founder of intuitionistic fuzzy logic, further studied on the relationship between the *'triangle'* and the *'square'*, and defined following two bijective transformations F and G from $\mathcal{L}_\square$ to $\mathcal{L}^*$, defined for $(x_1, x_2) \in [0,1]^2$ such that,

$$F(x_1,x_2) = \begin{cases} (0,0) & \text{if } x_1 = x_2 = 0 \\ \left(\frac{x_1^2}{x_1+x_2}, \frac{x_1 x_2}{x_1+x_2}\right) & \text{if } x_1 \geq x_2 \\ \left(\frac{x_1 x_2}{x_1+x_2}, \frac{x_2^2}{x_1+x_2}\right) & \text{if } x_1 < x_2 \,. \end{cases} \quad (5.9)$$

$$G(x_1,x_2) = \begin{cases} \left(x_1 - \frac{x_2}{2}, \frac{x_2}{2}\right) & \text{if } x_1 \geq x_2 \\ \left(\frac{x_1}{2}, x_2 - \frac{x_1}{2}\right) & \text{if } x_1 < x_2 \,. \end{cases} \quad (5.10)$$

Then later, Cornelis et al. showed that the bijective mapping does not preserve the order, therefore not lattice isomorphism [44]. However, as for the *triangle* perspective interpretation of values in *square*, it is still useful. Therefore, rather than directly model default reasoning scheme for IFSs and IFSs, we do reasoning on *square* bilattice for fuzzy Belnap logic, then transform the derived result using above Equations (4) (5). Definition 27, Definition 28 and Definition 29 hold on *square* $\mathcal{L}^2$, and we will again distinguish the inference on *square*, by denoting subscript $_{\mathcal{L}^2}$ to the closure operation i.e. $cl_{\mathcal{L}^2_{di}(\phi)(q)}$. For $\mathcal{I}(\mathcal{L})$ and $\mathcal{I}^*(\mathcal{L})$, we define following projection function relying on the above two possible mappings.

Definition 39. *($cl^F_{\mathcal{I}^*(\mathcal{L})}$ **F-Interpretation**) . Given a reasoning result $cl_{\mathcal{L}^2_{di}(\phi)(q)}$ on square $\mathcal{L}^2$, the F-Interpretation is the function such that $cl^F_{\mathcal{I}^*(\mathcal{L})} = F(cl_{\mathcal{L}^2_{di}}(\phi)(q))$, where the function F corresponds to Equation (5.9).*

Definition 40. *($cl^G_{\mathcal{I}^*(\mathcal{L})}$ **G-Interpretation**) . Given a reasoning result $cl_{\mathcal{L}^2_{di}}(\phi)(q)$ on square $\mathcal{L}^2$, the G-Interpretation is the function such that $cl^G_{\mathcal{I}^*(\mathcal{L})} = G(cl_{\mathcal{L}^2_{di}}(\phi)(q))$, where the function G corresponds to Equation (5.10).*

Reminding that the IVFSs and IFSs are isomorphic [44], in this chapter, we will show default reasoning on $\mathcal{L}^2$ and its F and G interpretations to IFSs. The interpretations can give us a shedding insight on comparing the reasoning result of the presented subjective logic based approach with IFL and fuzzy Belnap logic, because the uncertainty representation using μ and ν in IFL is pretty much similar to the one of subjective logic.

5.8 Case Study

This section deals with illustrative default reasoning examples for visual surveillance scenarios. To verify our approach and also to contrast with L-fuzzy logic based approaches, we will reuse two examples demonstrated by Shet et al. [153, 154] and one scenario in typical airport scene that is also inspired by Shet et al. [152]. Then we compare the proposed default inference approach Equation 5.7 to the one of bilattice based default reasoning and its extension to $\mathcal{L}^2$. The reasoning on $\mathcal{L}^2$ will be also interpreted in the view of $\mathcal{L}^*$ (i.e. $\mathcal{I}^*(\mathcal{L})$ psuedo-bilattice of $\mathcal{L}^*$) with the interpretations $cl^F_{\mathcal{I}^*(\mathcal{L})}$ and $cl^G_{\mathcal{I}^*(\mathcal{L})}$. In this section, we will not directly concern about IVFL due to the isomorphism between IVFSs and IFSs. We set truth values as follows :

$$
\begin{array}{ll}
T \simeq (1,0,0)_{sl} \simeq (1,0)_{\mathcal{L}^*} \simeq (1,0)_{\mathcal{L}^2} & F \simeq (0,1,0)_{sl} \simeq (0,1)_{\mathcal{L}^*} \simeq (0,1)_{\mathcal{L}^2} \\
DT \simeq (0.5,0,0.5)_{sl} \simeq (0.5,0)_{\mathcal{L}^*} \simeq (0.5,0)_{\mathcal{L}^2} & DF \simeq (0,0,5,0.5)_{sl} \simeq (0,0.5)_{\mathcal{L}^*} \simeq (0,0.5)_{\mathcal{L}^2} \\
U \simeq (0,0,1)_{sl} \simeq (0,0)_{\mathcal{L}^*} \simeq (0,0)_{\mathcal{L}^2} & * \simeq (x,x,1-2x)_{sl} \simeq (x,x)_{\mathcal{L}^*} \simeq (x,x)_{\mathcal{L}^2}
\end{array}
$$

i.e. undecided or contradiction with some amount of uncertainty (degree of information) on opinion triangle (on L-fuzzy sets), and $\perp \simeq (0.5,0.5,0)_{sl} \simeq (0.5,0.5)_{\mathcal{L}^*} \simeq (1,1)_{\mathcal{L}^2}$ *(i.e. the full contradiction).*

These mappings are reasonable in the sense that the uncertainty representation of opinion triangle and IFL $\mathcal{I}^*(\mathcal{L})$ are similar except the atomicity value of opinion triangle. For the simplicity we assume that all the propositional knowledge we consider are balanced, therefore we set the atomicity of opinion triangle as default $a = 0.5$, and we will not explicitly denote a. For the rest of truth values we will use opinion triple representation (b, d, i). For values of opinion triangle, $\mathcal{I}^*(\mathcal{L})$ and $\mathcal{L}^2$, that are slightly different to above symbols but still can be interpreted as one of them, we will denote it with superscript $'$. (e.g. $* \simeq *', DT \simeq DT', etc.$).

Example 3. ***(Identity Inference)*** [153, 154] **.** Assume the following truth assignment and set of rules about determining whether two individuals observed in an image should be considered as being one and the same.

$$\begin{aligned}
\phi[\neg equal(P_1, P_2) \leftarrow distinct(P_1, P_2)] &= DT \\
\phi[equal(P_1, P_2) \leftarrow appear_similar(P_1, P_2)] &= DT \\
\phi[appear_similar(a, b)] &= T \\
\phi[distinct(a, b)] &= T
\end{aligned}$$

Given two default true rules and facts that can be seen as definite true, the inference for default logic shown in [153] with bilattice and the default inference with subjective logic are as follows.

$$\begin{aligned}
&cl_{bl_{di}}(\phi)(equal(a, b)) \\
&\quad = [U \vee (T \wedge DT)] \oplus \neg[U \vee (T \wedge DT)] = [U \vee DT] \oplus \neg[U \vee DT] = DT \oplus DF = * \\
&cl_{sl_{di}}(\phi)(equal(a, b)) \\
&\quad = [U \sqcup (T \bullet DT)] \oplus \neg[U \sqcup (T \bullet DT)] = [U \sqcup (0.67, 0, 0.33)] \oplus \neg[U \sqcup (0.67, 0, 0.33)] \\
&\quad = (0.67, 0, 0.33) \oplus \neg(0.67, 0, 0.33) = (0.67, 0, 0.33) \oplus (0, 0.67, 0.33) = (0.4, 0.4, 0.2) = *'
\end{aligned}$$

And as shown in Table 5.5, choosing one of *t-norm* and *t-conorm* pair, and applying Equation 5.8 in Definition 36, we get following inference result derived on $\mathcal{L}^2$, and its interpretations $cl^F_{\mathcal{I}^*(\mathcal{L})}$ and $cl^G_{\mathcal{I}^*(\mathcal{L})}$ on $\mathcal{I}^*(\mathcal{L})$. The reasoning results are as follows.

$$\begin{aligned}
&cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(equal(a, b)) \\
&\quad = [U \vee (T \wedge DT)] \oplus \neg[U \vee (T \wedge DT)] = [(0,0) \vee \{(1,0) \wedge (0.5,0)\}] \oplus \neg[(0,0) \vee \{(1,0) \wedge (0.5,0)\}] \\
&\quad = [(0,0) \vee (\min(1, 0.5), \max(0,0))] \oplus \neg[(0,0) \vee (\min(1, 0.5), \max(0,0))] \\
&\quad = [(0,0) \vee (0.5,0)] \oplus \neg[(0,0) \vee (0.5,0)] = [\max(0, 0.5), \min(0,0)] \oplus \neg[\max(0, 0.5), \min(0,0)] \\
&\quad = (0.5, 0) \oplus \neg(0.5, 0) = (0.5, 0) \oplus (0, 0.5) = (\max(0.5, 0), \max(0, 0.5)) = (0.5, 0.5) = *' \\
&cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(equal(a, b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5, 0.5)) = (\tfrac{0.5^2}{0.5+0.5}, \tfrac{0.5 \cdot 0.5}{0.5+0.5}) = (0.25, 0.25) = *' \\
&cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(equal(a, b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5, 0.5)) = (0.5 - \tfrac{0.5}{2}, \tfrac{0.5}{2}) = (0.25, 0.25) = *
\end{aligned}$$

$$\begin{aligned}
&cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(equal(a, b)) \\
&\quad = [U \vee (T \wedge DT)] \oplus \neg[U \vee (T \wedge DT)] = [(0,0) \vee \{(1,0) \wedge (0.5,0)\}] \oplus \neg[(0,0) \vee \{(1,0) \wedge (0.5,0)\}]
\end{aligned}$$

$$= [(0,0) \vee (1 \cdot 0.5, 0+0-0 \cdot 0)] \oplus \neg[(1 \cdot 0.5, 0+0-0 \cdot 0)] = [(0,0) \vee (0.5,0)] \oplus \neg[(0,0) \vee (0.5,0)]$$

$$= [0+0.5-0 \cdot 0.5, 0 \cdot 0] \oplus \neg[0+0.5-0 \cdot 0.5, 0 \cdot 0] = (0.5,0) \oplus \neg(0.5,0) = (0.5,0) \oplus (0,0.5)$$

$$= (0.5+0-0.5 \cdot 0, 0.5+0-0.5 \cdot 0) = (0.5,0.5) = *'$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0.5)) = (0.25,0.25) = *'$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0.5)) = (0.25,0.25) = *'$$

$$cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))$$

$$= [U \vee (T \wedge DT)] \oplus \neg[U \vee (T \wedge DT)] = [(0,0) \vee \{(1,0) \wedge (0.5,0)\}] \oplus \neg[(0,0) \vee \{(1,0) \wedge (0.5,0)\}]$$

$$= [(0,0) \vee (\max(0, 1+0.5-1), \min(0+0,1))] \oplus \neg[(0,0) \vee (\max(0,1+0.5-1), \min(0+0,1))]$$

$$= [(0,0) \vee (0.5,0)] \oplus \neg[(0,0) \vee (0.5,0)] = [\min(0+0.5,1), \max(0,0+0-1)]$$

$$\oplus \neg[\min(0+0.5,1), \max(0,0+0-1)] = (0.5,0) \oplus \neg(0.5,0) = (0.5,0) \oplus (0,0.5)$$

$$= (\min(0.5+0,1), \min(0+0.5,1)) = (0.5,0.5) = *'$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0.5)) = (0.25,0.25) = *'$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0.5)) = (0.25,0.25) = *'$$

$$cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))$$

$$= ([U \vee (T \wedge DT)] \oplus \neg[U \vee (T \wedge DT)] = [(0,0) \vee \{(1,0) \wedge (0.5,0)\}] \oplus \neg[(0,0) \vee \{(1,0) \wedge (0.5,0)\}]$$

$$= [(0,0) \vee (\mathcal{T}_D(1,0.5), \mathcal{S}_D(0,0))] \oplus \neg[(0,0) \vee (\mathcal{T}_D(1,0.5), \mathcal{S}_D(0,0))] = [(0,0) \vee (0.5,0)]$$

$$\oplus \neg[(0,0) \vee (0.5,0)] = [\mathcal{S}_D(0,0.5), \mathcal{T}_D(0,0)] \oplus \neg[\mathcal{S}_D(0,0.5), \mathcal{T}_D(0,0)]$$

$$= (0.5,0) \oplus \neg(0.5,0) = (0.5,0) \oplus (0,0.5) = (\mathcal{S}_D(0.5,0), \mathcal{S}_D(0,0.5)) = (0.5,0.5) = *'$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0.5)) = (0.25,0.25) = *'$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0.5)) = (0.25,0.25) = *'$$

$$cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))$$

$$= [U \vee (T \wedge DT)] \oplus \neg[U \vee (T \wedge DT)] = [(0,0) \vee \{(1,0) \wedge (0.5,0)\}] \oplus \neg[(0,0) \vee \{(1,0) \wedge (0.5,0)\}]$$

$$= [(0,0) \vee (\mathcal{T}_{nM}(1,0.5), \mathcal{S}_{nM}(0,0))] \oplus \neg[(0,0) \vee (\mathcal{T}_{nM}(1,0.5), \mathcal{S}_{nM}(0,0))]$$

$$= [(0,0) \vee (0.5,0)] \oplus \neg[(0,0) \vee (0.5,0)] = [\mathcal{S}_{nM}(0,0.5), \mathcal{T}_{nM}(0,0)] \oplus \neg[\mathcal{S}_{nM}(0,0.5), \mathcal{T}_{nM}(0,0)]$$

$$= (0.5,0) \oplus \neg(0.5,0) = (0.5,0) \oplus (0,0.5) = (\mathcal{S}_{nM}(0.5,0), \mathcal{S}_{nM}(0,0.5)) = (0.5,0.5) = *'$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0.5)) = (0.25,0.25) = *'$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0.5)) = (0.25,0.25) = *'$$

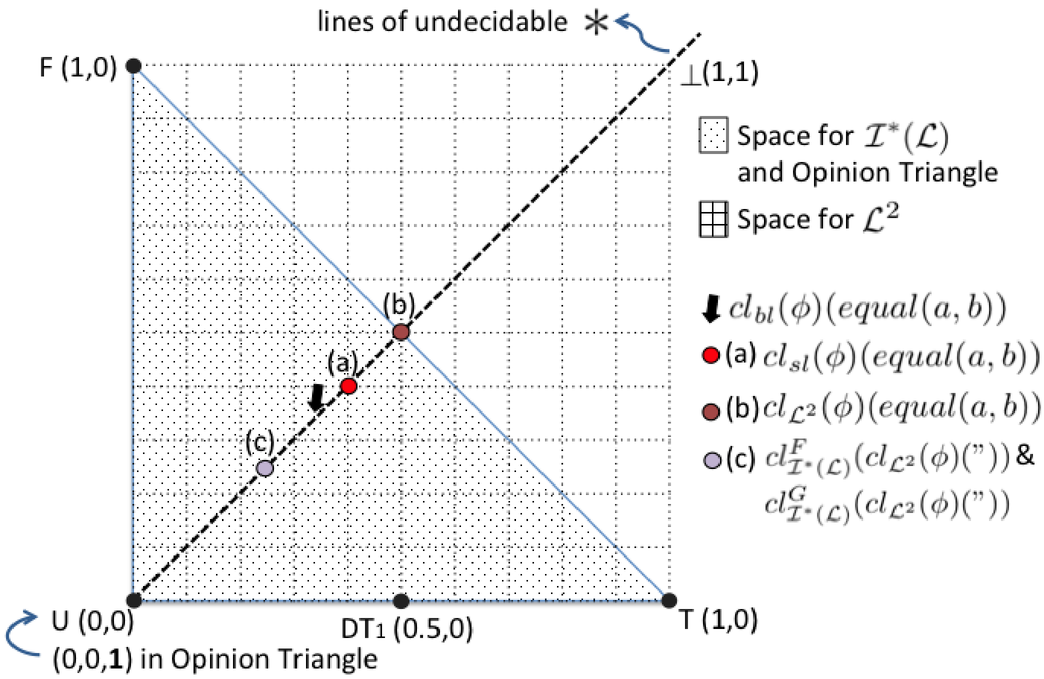

Figure 5.6: Reasoning Results of Example 3 in Opinion Space and $\mathcal{L}^2$ & $\mathcal{I}^*(\mathcal{L})$.

$$
\begin{aligned}
&cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b)) \\
&\quad = [U \vee (T \wedge DT)] \oplus \neg[U \vee (T \wedge DT)] = [(0,0) \vee \{(1,0) \wedge (0.5,0)\}] \oplus \neg[(0,0) \vee \{(1,0) \wedge (0.5,0)\}] \\
&\quad = [(0,0) \vee (\mathcal{T}_{H0}(1,0.5), \mathcal{S}_{H2}(0,0))] \oplus \neg[(0,0) \vee (\mathcal{T}_{H0}(1,0.5), \mathcal{S}_{H2}(0,0))] \\
&\quad = [(0,0) \vee (0.5,0)] \oplus \neg[(0,0) \vee (0.5,0)] = [\mathcal{S}_{H2}(0,0.5), \mathcal{T}_{H0}(0,0)] \oplus \neg[\mathcal{S}_{H2}(0,0.5), \mathcal{T}_{H0}(0,0)] \\
&\quad = (0.5,0) \oplus \neg(0.5,0) = (0.5,0) \oplus (0,0.5) = (\mathcal{S}_{H2}(0.5,0), \mathcal{S}_{H2}(0,0.5)) = (0.5,0.5) = *' \\
&cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0.5)) = (0.25,0.25) = *' \\
&cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0.5)) = (0.25,0.25) = *'
\end{aligned}
$$

Figure 5.6 shows the graphical representation of above reasoning results. The resulting opinion $(0.4, 0.4, 0.2)$ represents same amount of degree of truth and false with uncertainty. This can also be represented as undecided state $*'$. All reasoning results on $\mathcal{L}^2$ also yielded similar result $(0.5, 0.5)$ and the same F/G interpretations $(0.25, 0.25)$. Thus, the semantics of results from the discrete bilattice for multivalued default logic, bilattice based L-fuzzy logics and subjective logic are the same. While the uncertainty representation semantics of subjective logic is similar to IFL, when the reasoning result on fuzzy Belnap logic is interpreted, the distance between subjective opinion and the value of IFL was bigger than the one of fuzzy Belnap.

Example 4. ***(Identity Inference with contextual cues)*** [153, 154] **.** Assume that a person enters an office room that we believe to be empty and closed (no other exit). Suppose that after a while, another person appears from the room who seems dissimilar from the first person. In this case, inferring equality based on appearance matching is a weaker default than inferring equality based on the fact that person entered and exited an empty closed world. This aspect can be represented as following truth assignment and set of rules.

$$\begin{aligned}
\phi[\neg equal(P_1, P_2) \leftarrow \neg appear_similar(P_1, P_2)] &= DT_1 \\
\phi[equal(P_1, P_2) \leftarrow enter_closed_world(P_1, X, T_1), & \\
exit_closed_world(P_2, X, T_2), T_2 > T_1, & \\
empty_before(X, T_1), empty_after(X, T_2), & \\
not(enter_or_exit_between(P_3, T_1, T_2)).] &= DT_2 \\
\phi[\neg appear_similar(a, b)] &= T \\
\phi[enter_closed_world(a, office, 400)] &= T \\
\phi[exit_closed_world(b, office, 523)] &= T \\
\phi[empty_before(office, 400)] &= T \\
\phi[empty_after(office, 523)] &= T \\
\phi[\neg(enter_or_exit_between(p_3, 400, 523)] &= T
\end{aligned}$$

The Inference in this setup is multivalued default reasoning and the bilattice based inference result shown in Shet et al. [153] and the result of subjective logic based inference are as follows.

$$\begin{aligned}
&cl_{bl_{di}}(\phi)(equal(a, b)) \\
&\quad = [U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)] \oplus \neg [U \vee (T \wedge DT_1)] = [U \wedge DT_2] \oplus \neg [U \wedge DT_1] = DT_2 \oplus DF_1 = DT_2 \\
&cl_{sl_{di}}(\phi)(equal(a, b))
\end{aligned}$$

$$= [U \sqcup (T \bullet T \bullet T \bullet T \bullet T \bullet DT_2)] \oplus \neg[U \sqcup (T \bullet DT_1)] = [U \sqcup (0.9, 0, 0.1)] \oplus \neg[U \sqcup (0.67, 0, 0.33)]$$
$$= (0.9, 0, 0.1) \oplus \neg(0.67, 0, 0.33) = (0.9, 0, 0.1) \oplus (0, 0.67, 0.33) = (0.75, 0.17, 0.08) = DT_2'$$

Choosing a pair of *t-norm* and *t-conorm*, inference result derived on $\mathcal{L}^2$, and its interpretations $cl^F_{\mathcal{I}^*(\mathcal{L})}$ and $cl^G_{\mathcal{I}^*(\mathcal{L})}$ on $\mathcal{I}^*(\mathcal{L})$ are as follows.

$$cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(equal(a, b))$$
$$= [U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)] \oplus \neg[U \vee (T \wedge DT_1)] = [U \vee (T \wedge DT_2)] \oplus \neg[U \vee (T \wedge DT_1)]$$
$$= [(0, 0) \vee ((1, 0) \wedge (0.8, 0))] \oplus \neg[(0, 0) \vee ((1, 0) \wedge (0.5, 0))] = [(0, 0) \vee (\min(1, 0.8), \max(0, 0))]$$
$$\oplus \neg[(0, 0) \vee (\min(1, 0.5), \max(0, 0))] = [(0, 0) \vee (0.8, 0)] \oplus \neg[(0, 0) \vee (0.5, 0)]$$
$$= (\max(0, 0.8), \min(0, 0)) \oplus \neg(\max(0, 0.5), \min(0, 0)) = (0.8, 0) \oplus \neg(0.5, 0) = (0.8, 0) \oplus (0, 0.5)$$
$$= (\max(0.8, 0), \max(0, 0.5)) = (0.8, 0.5) = DT_2'$$
$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(equal(a, b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.8, 0.5)) = (\tfrac{0.8^2}{0.8+0.5}, \tfrac{0.8 \cdot 0.5}{0.8+0.5}) = (0.49, 0.3) = DT_2'$$
$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(equal(a, b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.8, 0.5)) = (0.8 - \tfrac{0.5}{2}, \tfrac{0.5}{2}) = (0.55, 0.25) = DT_2'$$

$$cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(equal(a, b))$$
$$= [U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)] \oplus \neg[U \vee (T \wedge DT_1)] = [U \vee (T \wedge DT_2)] \oplus \neg[U \vee (T \wedge DT_1)]$$
$$= [(0, 0) \vee ((1, 0) \wedge (0.8, 0))] \oplus \neg[(0, 0) \vee ((1, 0) \wedge (0.5, 0))] = [(0, 0) \vee (1 \cdot 0.8, 0 + 0 - 0 \cdot 0)]$$
$$\oplus \neg[(0, 0) \vee (1 \cdot 0.5, 0 + 0 - 0 \cdot 0)] = [(0, 0) \vee (0.8, 0)] \oplus \neg[(0, 0) \vee (0.5, 0)]$$
$$= (0 + 0.8 - 0 \cdot 0.8, 0 \cdot 0)) \oplus \neg(0 + 0.5 - 0 \cdot 0.5, 0 \cdot 0) = (0.8, 0) \oplus \neg(0.5, 0) = (0.8, 0) \oplus (0, 0.5)$$
$$= (0.8 + 0 - 0.8 \cdot 0, 0 + 0.5 - 0 \cdot 0.5) = (0.8, 0.5) = DT_2'$$
$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(equal(a, b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.8, 0.5)) = (0.49, 0.3) = DT_2'$$
$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(equal(a, b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.8, 0.5)) = (0.55, 0.25) = DT_2'$$

$$cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(equal(a, b))$$
$$= [U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)] \oplus \neg[U \vee (T \wedge DT_1)] = [U \vee (T \wedge DT_2)] \oplus \neg[U \vee (T \wedge DT_1)]$$
$$= [(0, 0) \vee ((1, 0) \wedge (0.8, 0))] \oplus \neg[(0, 0) \vee ((1, 0) \wedge (0.5, 0))]$$
$$= [(0, 0) \vee (\max(0, 1 + 0.8 - 1), \min(0 + 0, 1))] \oplus \neg[(0, 0) \vee (\max(0, 1 + 0.5 - 1), \min(0 + 0, 1))]$$
$$= [(0, 0) \vee (0.8, 0)] \oplus \neg[(0, 0) \vee (0.5, 0)] = (\min(0 + 0.8, 1), \max(0, 0 + 0))$$
$$\oplus \neg(\min(0 + 0.5, 1), \max(0, 0 + 0)) = (0.8, 0) \oplus \neg(0.5, 0) = (0.8, 0) \oplus (0, 0.5)$$

$$= (\min(0.8+0,1), \min(0+0.5,1)) = (0.8, 0.5) = DT_2'$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.8,0.5)) = (0.49, 0.3) = DT_2'$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.8,0.5)) = (0.55, 0.25) = DT_2'$$

$$cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))$$

$$= [U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)] \oplus \neg[U \vee (T \wedge DT_1)] = [U \vee (T \wedge DT_2)] \oplus \neg[U \vee (T \wedge DT_1)]$$

$$= [(0,0) \vee ((1,0) \wedge (0.8,0))] \oplus \neg[(0,0) \vee ((1,0) \wedge (0.5,0))] = [(0,0) \vee (\mathcal{T}_D(1,0.8), \mathcal{S}_D(0,0))]$$

$$\oplus \neg[(0,0) \vee (\mathcal{T}_D(1,0.5), \mathcal{S}_D(0,0))] = [(0,0) \vee (0.8,0)] \oplus \neg[(0,0) \vee (0.5,0)]$$

$$= (\mathcal{S}_D(0,0.8), \mathcal{T}_D(0,0)) \oplus \neg(\mathcal{S}_D(0,0.5), \mathcal{T}_D(0,0))(0.8,0) \oplus \neg(0.5,0) = (0.8,0) \oplus (0,0.5)$$

$$= (\mathcal{S}_D(0.8,0), \mathcal{S}_D(0,0.5)) = (0.8,0.5) = DT_2'$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.8,0.5)) = (0.49, 0.3) = DT_2'$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.8,0.5)) = (0.55, 0.25) = DT_2'$$

$$cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))$$

$$= [U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)] \oplus \neg[U \vee (T \wedge DT_1)] = [U \vee (T \wedge DT_2)] \oplus \neg[U \vee (T \wedge DT_1)]$$

$$= [(0,0) \vee ((1,0) \wedge (0.8,0))] \oplus \neg[(0,0) \vee ((1,0) \wedge (0.5,0))] = [(0,0) \vee (\mathcal{T}_{nM}(1,0.8), \mathcal{S}_{nM}(0,0))]$$

$$\oplus \neg[(0,0) \vee (\mathcal{T}_{nM}(1,0.5), \mathcal{S}_{nM}(0,0))] = [(0,0) \vee (0.8,0)] \oplus \neg[(0,0) \vee (0.5,0)]$$

$$= (\mathcal{S}_{nM}(0,0.8), \mathcal{T}_{nM}(0,0)) \oplus \neg(\mathcal{S}_{nM}(0,0.5), \mathcal{T}_{nM}(0,0)) = (0.8,0) \oplus \neg(0.5,0) = (0.8,0) \oplus (0,0.5)$$

$$= (\mathcal{S}_{nM}(0.8,0), \mathcal{S}_{nM}(0,0.5)) = (0.8,0.5) = DT_2'$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.8,0.5)) = (0.49, 0.3) = DT_2'$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.8,0.5)) = (0.55, 0.25) = DT_2'$$

$$cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))$$

$$= [U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)] \oplus \neg[U \vee (T \wedge DT_1)] = [U \vee (T \wedge DT_2)] \oplus \neg[U \vee (T \wedge DT_1)]$$

$$= [(0,0) \vee ((1,0) \wedge (0.8,0))] \oplus \neg[(0,0) \vee ((1,0) \wedge (0.5,0))] = [(0,0) \vee (\mathcal{T}_{H0}(1,0.8), \mathcal{S}_{H2}(0,0))]$$

$$\oplus \neg[(0,0) \vee (\mathcal{T}_{H0}(1,0.5), \mathcal{S}_{H2}(0,0))] = [(0,0) \vee (0.8,0)] \oplus \neg[(0,0) \vee (0.5,0)]$$

$$= (\mathcal{S}_{H2}(0,0.8), \mathcal{T}_{H0}(0,0)) \oplus \neg(\mathcal{S}_{H2}(0,0.5), \mathcal{T}_{H0}(0,0)) = (0.8,0) \oplus \neg(0.5,0)$$

$$= (0.8,0) \oplus (0,0.5) = (\mathcal{S}_{H2}(0.8,0), \mathcal{S}_{nM}(0,0.5)) = (0.8,0.5) = DT_2'$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.8,0.5)) = (0.49, 0.3) = DT_2'$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.8,0.5)) = (0.55, 0.25) = DT_2'$$

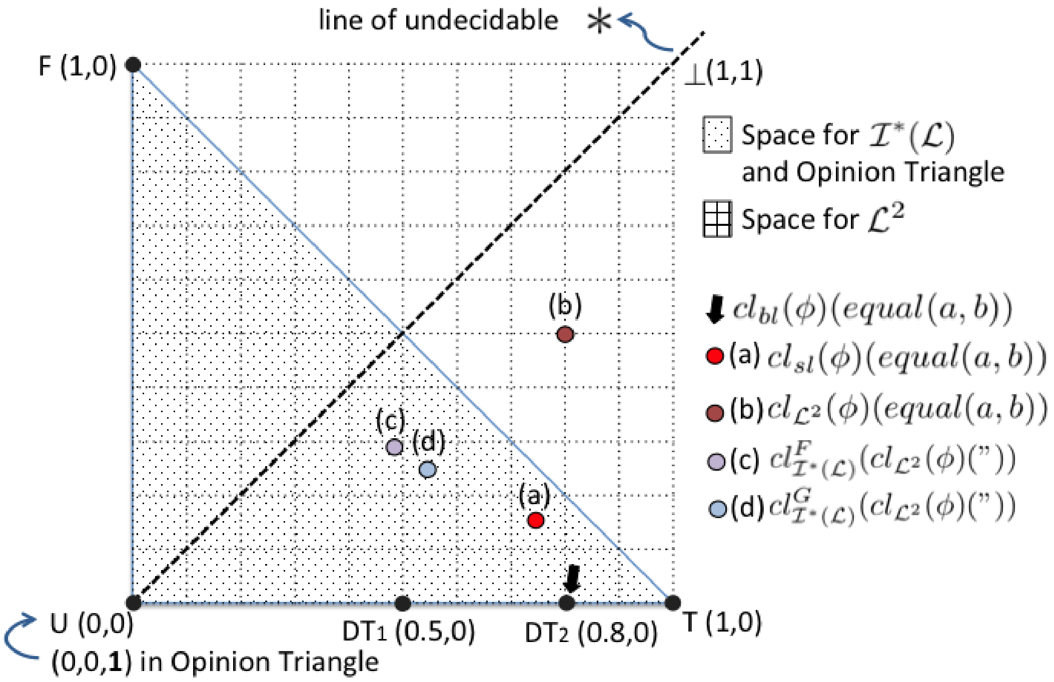

Figure 5.7: Reasoning Results of Example 4 in Opinion Space and $\mathcal{L}^2$ & $\mathcal{I}^*(\mathcal{L})$.

Figure 5.7 shows above reasoning results. The opinion labeled (a) in opinion triangle is the most closest one to DT_2 compared with L-fuzzy logic based results but a bit biased to center than original DT_2. In the sense of truth value, this is the same to the cases of yielded values on $\mathcal{L}^2$. However, in these cases, the amount of falsity is also relatively high therefore the point is located in the area of overflowed information ($\mu+\nu > 1$). The F and G interpretations are also a bit different but very close to each other. Nonetheless, in the sense that all reasoning results are pointed on the right-hand side of the line of undecidable *, semantically, this can be interpreted as the meaning of week truth like DT_2. Thus, the semantics of results from the discrete bilattice for multivalued default logic, the bilattice based L-fuzzy logics and subjective logic are the same.

Example 5. ***(Theft Inference with contextual cues)*** *(The scenario and rules for 'theft' have been inspired by Shet et al.* [152]) **.** Assume a typical airport surveillance as depicted in Figure 5.8 with two cameras. Suppose that a human P_1 carrying an object (Baggage) B is observed and stayed around telephone booth in $Cam1$. After a while he disappears from the view of $Cam1$ without taking his baggage B. Subsequently, P_2 enters the scene, picks up the baggage and leaves. In parallel, according to $Cam2$, it seems that P_1 and P_2 belong to a same group of people so the two people are considered as friends. In this scenario, based on the possession relation between an object and person, we could build a default rule to infer whether a person is a thief or not. Similarly, based on the friend relation we can also build a bit stronger default rule saying possessing object of friend is not thief. This aspect is depicted as following truth assignment and set of rules.

$$
\begin{aligned}
\phi[theft(P,B,T) \leftarrow human(P), package(B),& \\
possess(P,B,T), \neg(belongs(B,P,T))] &= DT_1 \\
\phi[\neg theft(P,B,T) \leftarrow human(P), package(B),& \\
possess(P,B,T), belongs(B,P,T)] &= DT_1 \\
\phi[\neg theft(P,B,T) \leftarrow human(P), package(B),& \\
possess(P,B,T), \neg(belongs(B,P,T)),& \\
friend(P,P'), belongs(B,P',T)] &= DT_2 \\
\phi[human(P)] &= T \\
\phi[package(B)] &= T \\
\phi[possess(P,B,T)] &= T \\
\phi[\neg(belongs(B,P,T))] &= T \\
\phi[friend(P,P')]_{cam2} &= T \\
\phi[belongs(B,P',T)]_{cam2} &= T \\
where,\ DT_1 \simeq (0.5,0,0.5)\ and\ DT_2 \simeq (0.8,0,0.2)&
\end{aligned}
$$

Given above rules and facts (gathered till P_2 is picking up the baggage B), inferring whether the person is a thief or not is shown as follows.

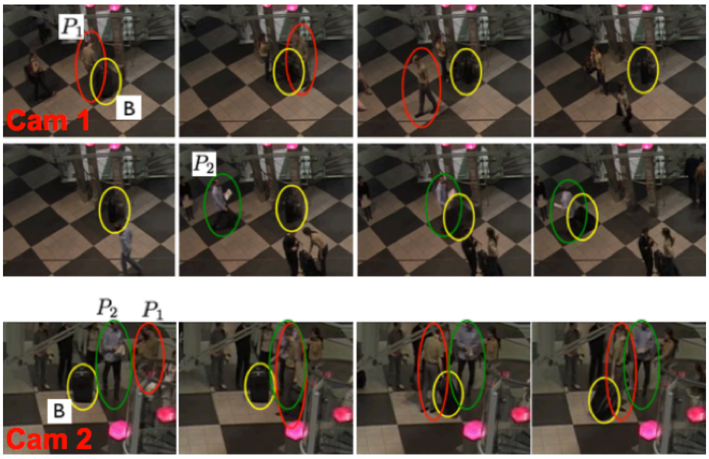

Figure 5.8: Illustrative Scenario Setup of Example 5.

Case1. (Inference relying only on Cam1).

$$cl_{bl_{di}}(\phi)(theft(P,B,T)) = [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] = [U \vee DT_1] = DT_1$$

$$cl_{sl_{di}}(\phi)(theft(P,B,T)) = [U \sqcup (T \bullet T \bullet T \bullet T \bullet DT_1)] = [U \sqcup (0.74, 0, 0.26)] = (0.74, 0, 0.26) = DT_2'$$

Choosing a pair of *t-norm* and *t-conorm*, inference result derived on $\mathcal{L}^2$, and its interpretations $cl^F_{\mathcal{I}^*(\mathcal{L})}$ and $cl^G_{\mathcal{I}^*(\mathcal{L})}$ on $\mathcal{I}^*(\mathcal{L})$ are as follows.

$$\begin{aligned} & cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T)) \\ &\quad = [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] = [U \vee (T \wedge DT_1)] = [(0,0) \vee ((1,0) \wedge (0.5,0))] \\ &\quad = [(0,0) \vee (\min(1,0.5), \max(0,0))] = [(0,0) \vee (0.5,0)] = (\max(0,0.5), \min(0,0)) \\ &\quad = (0.5,0) = DT_1 \end{aligned}$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (\tfrac{0.5^2}{0.5}, \tfrac{0.5 \cdot 0}{0.5+0}) = (0.5,0) = DT_1$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (0.5 - \tfrac{0}{2}, \tfrac{0}{2}) = (0.5,0) = DT_1$$

$$\begin{aligned} & cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T)) \\ &\quad = [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] = [U \vee (T \wedge DT_1)] = [(0,0) \vee ((1,0) \wedge (0.5,0))] \\ &\quad = [(0,0) \vee (1 \cdot 0.5, 0 + 0 - 0 \cdot 0)] = [(0,0) \vee (0.5,0)] = (0 + 0.5 - 0 \cdot 0.5), 0 \cdot 0) = (0.5,0) = DT_1 \end{aligned}$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (0.5,0) = DT_1$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (0.5,0) = DT_1$$

$$cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))$$

$= [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] = [U \vee (T \wedge DT_1)] = [(0,0) \vee ((1,0) \wedge (0.5,0))]$

$= [(0,0) \vee (\max(0, 1+0.5-1), \min(0+0,1))] = [(0,0) \vee (0.5,0)]$

$= (\min(0+0.5,1), \max(0, 0+0-1)) = (0.5,0) = DT_1$

$cl^{F}_{\mathcal{I}^*(\mathcal{L})}(cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^{F}_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (0.5,0) = DT_1$

$cl^{G}_{\mathcal{I}^*(\mathcal{L})}(cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^{G}_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (0.5,0) = DT_1$

$cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))$

$= [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] = [U \vee (T \wedge DT_1)] = [(0,0) \vee ((1,0) \wedge (0.5,0))]$

$= [(0,0) \vee (\mathcal{T}_D(1,0.5), \mathcal{S}_D(0.5,0))] = [(0,0) \vee (0.5,0)] = (\mathcal{S}_D(0,0.5), \mathcal{T}_D(0,0)) = (0.5,0) = DT_1$

$cl^{F}_{\mathcal{I}^*(\mathcal{L})}(cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^{F}_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (0.5,0) = DT_1$

$cl^{G}_{\mathcal{I}^*(\mathcal{L})}(cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^{G}_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (0.5,0) = DT_1$

$cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))$

$= [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] = [U \vee (T \wedge DT_1)] = [(0,0) \vee ((1,0) \wedge (0.5,0))]$

$= [(0,0) \vee (\mathcal{T}_{nM}(1,0.5), \mathcal{S}_{nM}(0.5,0))] = [(0,0) \vee (0.5,0)] = (\mathcal{S}_{nM}(0,0.5), \mathcal{T}_{nM}(0,0))$

$= (0.5,0) = DT_1$

$cl^{F}_{\mathcal{I}^*(\mathcal{L})}(cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^{F}_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (0.5,0) = DT_1$

$cl^{G}_{\mathcal{I}^*(\mathcal{L})}(cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^{G}_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (0.5,0) = DT_1$

$cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))$

$= [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] = [U \vee (T \wedge DT_1)] = [(0,0) \vee ((1,0) \wedge (0.5,0))]$

$= [(0,0) \vee (\mathcal{T}_{H0}(1,0.5), \mathcal{S}_{H2}(0.5,0))] = [(0,0) \vee (0.5,0)] = (\mathcal{S}_{H2}(0,0.5), \mathcal{T}_{H0}(0,0))$

$= (0.5,0) = DT_1$

$cl^{F}_{\mathcal{I}^*(\mathcal{L})}(cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^{F}_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (0.5,0) = DT_1$

$cl^{G}_{\mathcal{I}^*(\mathcal{L})}(cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^{G}_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (0.5,0) = DT_1$

Relying only the facts generated by $Cam1$ (those are not explicitly subscripted with camera id), all of above inference concluded $cl(\phi)(theft(P_2,B,T)) = DT_1$ except subjective logic based approach that derived DT_2'. However, the semantic interpretation of the results are all the same. Namely. theft has taken place with low confidence.

Case2. (Inference relying on Cam1 and Cam2)

$cl_{bl_{di}}(\phi)(theft(P,B,T))$

$= [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] \oplus \neg[U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)] = [U \vee DT_1] \oplus \neg[U \vee DT_2]$

$= DT_1 \oplus \neg DT_2 = DT_1 \oplus DF_2 = DF_2$

$cl_{sl_{di}}(\phi)(theft(P,B,T))$

$= [U \sqcup (T \bullet T \bullet T \bullet T \bullet DT_1)] \oplus \neg[U \sqcup (T \bullet T \bullet T \bullet T \bullet T \bullet T \bullet DT_2)]$

$= [U \sqcup (0.74, 0, 0.26)] \oplus \neg[U \sqcup (0.9, 0, 0.1)] = (0.74, 0, 0.26) \oplus \neg(0.9, 0, 0.1)$

$= (0.74, 0, 0.26) \oplus (0, 0.9, 0.1) = (0.22, 0.7, 0.08) = DF'_2$

Choosing a pair of *t-norm* and *t-conorm*, inference result derived on $\mathcal{L}^2$, and its interpretations $cl^F_{\mathcal{I}^*(\mathcal{L})}$ and $cl^G_{\mathcal{I}^*(\mathcal{L})}$ on $\mathcal{I}^*(\mathcal{L})$ are as follows.

$cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))$

$= U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] \oplus \neg[U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)]$

$= [U \vee (T \wedge DT_1)] \oplus \neg[U \vee (T \wedge DT_2)] = [(0,0) \vee ((1,0) \wedge (0.5,0))] \oplus \neg[(0,0) \vee ((1,0) \wedge (0.8,0))]$

$= [(0,0) \vee (\min(1,0.5), \max(0,0))] \oplus \neg[(0,0) \vee (\min(1,0.8), \max(0,0))]$

$= [(0,0) \vee (0.5,0)] \oplus \neg[(0,0) \vee (0.8,0)] = (\max(0,0.5), \min(0,0)) \oplus \neg(\max(0,0.8), \min(0,0))$

$= (0.5,0) \oplus \neg(0.8,0) = (0.5,0) \oplus (0,0.8) = (\max(0.5,0), \max(0,0.8)) = (0.5,0.8) = DF'_2$

$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0.8)) = (\frac{0.5\cdot 0.8}{0.5+0.8}, \frac{0.8^2}{0.5+0.8}) = (0.3, 0.49) = DF'_1$

$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0.8)) = (\frac{0.5}{2}, 0.8 - \frac{0.5}{2}) = (0.25, 0.55) = DF'_1$

$cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))$

$= [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] \oplus \neg[U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)]$

$= [U \vee (T \wedge DT_1)] \oplus \neg[U \vee (T \wedge DT_2)] = [(0,0) \vee ((1,0) \wedge (0.5,0))] \oplus \neg[(0,0) \vee ((1,0) \wedge (0.8,0))]$

$= [(0,0) \vee (1 \cdot 0.5, 0+0-0\cdot 0)] \oplus \neg[(0,0) \vee (1 \cdot 0.8, 0+0-0\cdot 0)] = [(0,0) \vee (0.5,0)] \oplus \neg[(0,0) \vee (0.8,0)]$

$= (0 + 0.5 - 0 \cdot 0.5, 0 \cdot 0) \oplus \neg(0 + 0.8 - 0 \cdot 0.8, 0 \cdot 0) = (0.5,0) \oplus \neg(0.8,0) = (0.5,0) \oplus (0,0.8)$

$= (0.5 + 0 - 0.5 \cdot 0, 0 + 0.8 - 0 \cdot 0.8) = (0.5, 0.8) = DF'_2$

$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0.8)) = (0.3, 0.49) = DT'_2$

$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0.8)) = (0.25, 0.55) = DT'_2$

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$$cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))$$
$$= [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] \oplus \neg[U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)]$$
$$= [U \vee (T \wedge DT_1)] \oplus \neg[U \vee (T \wedge DT_2)] = [(0,0) \vee ((1,0) \wedge (0.5,0))] \oplus \neg[(0,0) \vee ((1,0) \wedge (0.8,0))]$$
$$= [(0,0) \vee (\max(0, 1+0.5-1), \min(0+0,1)] \oplus \neg[(0,0) \vee (\max(0,1+0.8-1), \min(0+0,1)]$$
$$= [(0,0) \vee (0.5,0)] \oplus \neg[(0,0) \vee (0.8,0)]$$
$$= (\min(0+0.5,1), \max(0,0+0-1)) \oplus \neg(\min(0+0.8,1), \max(0,0+0-1))$$
$$= (0.5,0) \oplus \neg(0.8,0) = (0.5,0) \oplus (0,0.8) = (\min(0+0.5,1), \min(0+0.8,1)) = (0.5,0.8) = DF_2'$$
$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0.8)) = (0.3,0.49) = DT_2'$$
$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0.8)) = (0.25,0.55) = DT_2'$$

$$cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))$$
$$= [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] \oplus \neg[U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)]$$
$$= [U \vee (T \wedge DT_1)] \oplus \neg[U \vee (T \wedge DT_2)] = [(0,0) \vee ((1,0) \wedge (0.5,0))] \oplus \neg[(0,0) \vee ((1,0) \wedge (0.8,0))]$$
$$= [(0,0) \vee (\mathcal{T}_D(1,0.5), \mathcal{S}_D(0,0)] \oplus \neg[(0,0) \vee (\mathcal{T}_D(1,0.8), \mathcal{S}_D(0,0)]$$
$$= [(0,0) \vee (0.5,0)] \oplus \neg[(0,0) \vee (0.8,0)] = (\mathcal{S}_D(0,0.5), \mathcal{T}_D(0,0)) \oplus \neg(\mathcal{S}_D(0,0.8), \mathcal{T}_D(0,0))$$
$$= (0.5,0) \oplus \neg(0.8,0) = (0.5,0) \oplus (0,0.8) = (\mathcal{S}_D(0,0.5), \mathcal{S}_D(0,0.8)) = (0.5,0.8) = DF_2'$$
$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0.8)) = (0.3,0.49) = DT_2'$$
$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0.8)) = (0.25,0.55) = DT_2'$$

$$cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))$$
$$= [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] \oplus \neg[U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)]$$
$$= [U \vee (T \wedge DT_1)] \oplus \neg[U \vee (T \wedge DT_2)] = [(0,0) \vee ((1,0) \wedge (0.5,0))] \oplus \neg[(0,0) \vee ((1,0) \wedge (0.8,0))]$$
$$= [(0,0) \vee (\mathcal{T}_{nM}(1,0.5), \mathcal{S}_{nM}(0,0)] \oplus \neg[(0,0) \vee (\mathcal{T}_{nM}(1,0.8), \mathcal{S}_{nM}(0,0)]$$
$$= [(0,0) \vee (0.5,0)] \oplus \neg[(0,0) \vee (0.8,0)] = (\mathcal{S}_{nM}(0,0.5), \mathcal{T}_{nM}(0,0)) \oplus \neg(\mathcal{S}_{nM}(0,0.8), \mathcal{T}_{nM}(0,0))$$
$$= (0.5,0) \oplus \neg(0.8,0) = (0.5,0) \oplus (0,0.8) = (\mathcal{S}_{nM}(0,0.5), \mathcal{S}_{nM}(0,0.8)) = (0.5,0.8) = DF_2'$$
$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0.8)) = (0.3,0.49) = DT_2'$$
$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0.8)) = (0.25,0.55) = DT_2'$$

$$cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))$$
$$= [U \vee (T \wedge T \wedge T \wedge T \wedge DT_1)] \oplus \neg[U \vee (T \wedge T \wedge T \wedge T \wedge T \wedge T \wedge DT_2)]$$

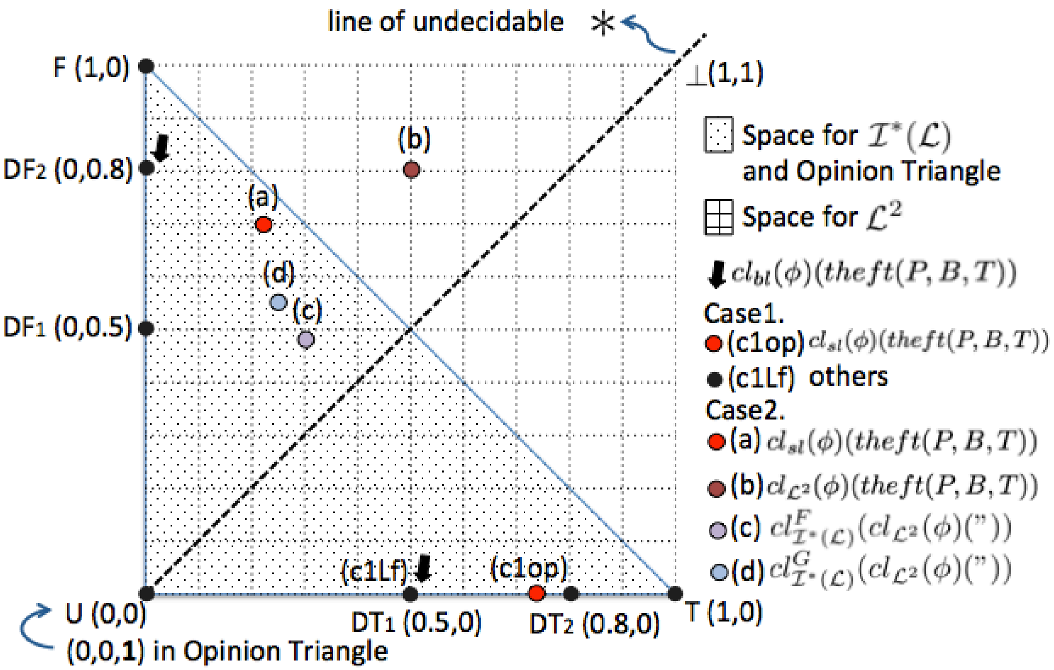

Figure 5.9: Reasoning Results of Example 5 in Opinion Space and $\mathcal{L}^2$ & $\mathcal{I}^*(\mathcal{L})$.

$$= [U \vee (T \wedge DT_1)] \oplus \neg[U \vee (T \wedge DT_2)] = [(0,0) \vee ((1,0) \wedge (0.5,0))] \oplus \neg[(0,0) \vee ((1,0) \wedge (0.8,0))]$$

$$= [(0,0) \vee (\mathcal{T}_{H0}(1,0.5), \mathcal{S}_{H2}(0,0)] \oplus \neg[(0,0) \vee (\mathcal{T}_{H0}(1,0.8), \mathcal{S}_{H2}(0,0)] = [(0,0) \vee (0.5,0)]$$

$$\oplus \neg[(0,0) \vee (0.8,0)] = (\mathcal{S}_{H2}(0,0.5), \mathcal{T}_{H0}(0,0)) \oplus \neg(\mathcal{S}_{H2}(0,0.8), \mathcal{T}_{H0}(0,0)) = (0.5,0) \oplus \neg(0.8,0)$$

$$= (0.5,0) \oplus (0,0.8) = (\mathcal{S}_{H2}(0,0.5), \mathcal{S}_{H2}(0,0.8)) = (0.5,0.8) = DF_2'$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5,0.8)) = (0.3,0.49) = DT_2'$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(theft(P,B,T))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5,0.8)) = (0.25,0.55) = DT_2'$$

Figure 5.9 shows above reasoning results. In case of Case1, subjective logic yielded a rather strong opinion $(0.74, 0, 0.26)$ that is rather close to DT_2 than DT_1 that other approaches yielded. Contrary to Case 1, when we take more information also from $Cam2$, all of above inference concluded $cl(\phi)(theft(P_2, B, T)) = DF_2$ and DF_2'. Namely, no theft has taken place with rather high confidence. The opinion DF_2' $(0.22, 0.7, 0.08)$ in opinion triangle is the most closest one to DF_2 compared to L-fuzzy logic based results. In the sense of truth value, this is the same to the cases of yielded values on $\mathcal{L}^2$. However, in this case, the amount of falsity is also relatively high therefore the point is located in the area of overflowed information ($\mu + \nu > 1$). The F and G interpretations are both close to each other however the interpretation is rather closer to DF_1. Nonetheless, in the sense that all reasoning results are pointed on the left-hand side of the line of undecidable *, semantically, this can be interpreted as the meaning of week false like DF_2. Thus, the semantics of results from the discrete bilattice for multivalued default logic, the bilattice based L-fuzzy logics and subjective logic are the same.

As shown with above illustrative visual surveillance inference scenarios, the proposed default reasoning mechanism Equation 5.7 semantically well models default reasoning and that is so in the case of L-fuzzy logics.

5.9 Discussion

As shown in the previous section with examples of default reasoning in visual surveillance, both subjective logic and L-fuzzy logics (especially, IFL and fuzzy Belnap logic) seem relevant for the use of approximate default reasoning. What makes IFSs attractive compared to other fuzzy set extensions is that it makes geometrical interpretations possible, thereby, the combination of membership and non-membership functions can be calculated in the Euclidean plane with a Cartesian coordinate system. [23]. This aspect is also the same in Subjective logic because subjective logic also makes the geometrical interpretations of Beta probability distributions possible in the Euclidean plane. However, there are still some properties worth to discuss to contrast these approaches. In this section, we give a comparison on the property of both approaches in the view of logical soundness and feasibility in visual surveillance.

5.9.1 Which way to go? Subjective Logic vs. L-fuzzy Logics

As we noted in the previous default reasoning section, the initial idea of discrete bilattice assumed an epistemic state called *'unknown'* and *'contradiction' (defined to have even more information than definite true or false)* following Belnaps four-valued logic. In the sense of discrete valued logic such as multivalued default logic, the only way of reaching to the overflowed information state is through the handling of definite true and definite false, and *'contradiction'* $\perp$ is the only epistemic state that is defined in the overflowed information area. However, as shown in the previous examples, this is not true on $\mathcal{L}^2$, because we can encounter values in the area of $(\mu + \nu > 1)$. Similarly to the discussion we have reviewed in Section 5.6.2, regarding the meaning of the area, and more specifically the epistemic state *'contradiction'*, there has been many discussions on the significance of the epistemic state in the view of logic. A critical review can be found in a work of Urquhart [166]. There has been also some report on the possible problems that could be introduced for the formal specification of software systems such as non-termination error in a work of Hähnle [77]. Dubois, D. [56] formally showed the problems that can arise in Belnap logic, in the sense of logical soundness. In the following, we briefly review the discussions introduced by [56].

Discussion 1. ***(Paradoxes in the truth table of multivalued logics) .*** Table 5.1 (a) and (b) show the truth table of the Belnap's four-valued logic and the default logic. In both logics, the conjunction and disjunction are problematic when applied to the two extreme epistemic states *'Unknown'* U and *'Contradiction'* $\perp$. For instance in Belnap logic, we have $U \wedge U = U$ and $U \vee U = U$. Assume that we attached U to a proposition p and consider $p \wedge \neg p$ and $p \vee \neg p$. In Belnap logic, the former and latter both are U because $U \wedge \neg U = U \wedge U = U$ and $U \vee \neg U = U \vee U = U$. However, in classical logic sense, the former should be false and the latter should be true. The same anomaly is also introduced in the case of $\perp$. According to the truth table, it claims $\perp \wedge \perp = \perp$ and $\perp \vee \perp = \perp$. For $p \wedge \neg q$ and $p \vee \neg q$, again, we get $\perp$ for both. It breaks the tautology in classical logic sense [56]. Similar anomaly can be found in the case of $U \wedge \perp = F$ and $U \vee \perp = T$.

From a common sense, the results are counterintuitive and this was even to Belnap because he stated that this is an unavoidable consequence of his formal setting [56]. This aspect can be problematic in the following scenario. Consider a proposition p and q, and agent A_1 and A_2 saying the p is T and F respectively and this is why p is $\perp$. Because A_1 and A_2 say nothing about q we assume q is U. Now, $p \wedge q = \perp \wedge U = F$ that is again

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counter intuitive [56]. This aspect also leads to debatable epistemic value assignments. Suppose two atomic propositions p and q with epistemic state assignment $\phi(p) = \perp$ and $\phi(q) = U$. Then $\phi(p \wedge q) = F$ as noted above. But since Belnap negation is such that $\phi(\neg p) = \perp$ and $\phi(\neg q) = U$, we also get $\phi(\neg p \wedge q) = \phi(p \wedge \neg q) = \phi(\neg p \wedge \neg q) = F$. Hence, $\phi((p \wedge q) \wedge (\neg p \wedge q) \wedge (p \wedge \neg q) \wedge (\neg p \wedge \neg q)) = \phi(p \wedge q) \wedge \phi(\neg p \wedge q) \wedge \phi(p \wedge \neg q) \wedge \phi(\neg p \wedge \neg q) = F$, however, according to the truth table, $(p \wedge q) \wedge (\neg p \wedge q) \wedge (p \wedge \neg q) \wedge (\neg p \wedge \neg q) = F \wedge F \wedge T \wedge T = \perp$. This means $\phi(\perp) = F$, therefore hardly acceptable again [56].

This aspect shows that, for any logical connectives $*$, $\phi(p) * \phi(q) \neq \phi(p * q)$ in Belnap logic, namely, Belnap logic is *'non-commutative'*. In other words, an epistemic value on each proposition can not characterize a single epistemic value for the combination of the propositions. This aspect also hold in the fuzzy Belnap logic as well, because regardless what *t-norms* and *t-conorms* we choose, the truth table values corresponding to definite true, definite false, unknown and contradictory values will have the same truth functional values as of discrete Belnap logic.

Unlike fuzzy Belnap logic, in subjective logic we can avoid this problem by the use of atomicity value a, therefore subjective logic better captures the spirit of classical logic[2]. Consider the same case of $U \wedge U$ and $U \vee U$. As shown in Figure 5.10 (a), for subjective logic opinion $w = (0, 0, 1, 0.5)$ which corresponds to U, subjective logic conjunction also draws full ignorance but with different atomicity, namely $(0, 0, 1, 0.5) \wedge \neg(0, 0, 1, 0.5) = (0, 0, 1, 0.25)$. The semantics is clear. Namely, for a proposition that is known to be binary event that an agent has a full ignorance on the truth of it, the conjunction also draws full uncertainty but, following the spirit of probabilistic conjunction, it comes with the atomicity that is the product of both atomicity (i.e. $0.5 \cdot 0.5$ in this case). Therefore, even if we get full ignorance, when it is interpreted in terms of Beta distribution, the overall expectation should be biased to falsity as traditional logic yields F. This is the same in the case of $U \wedge U$ that yields a full ignorance opinion but its atomicity is biased to T. The

[2]Strictly speaking, subjective logic does not allow a *'truly full uncertain' state*, in the sense that subjective logic adds additional dimension of information called atomicity (prior) value. This is the main cause of the difference shown in Figure 5.10 (a).

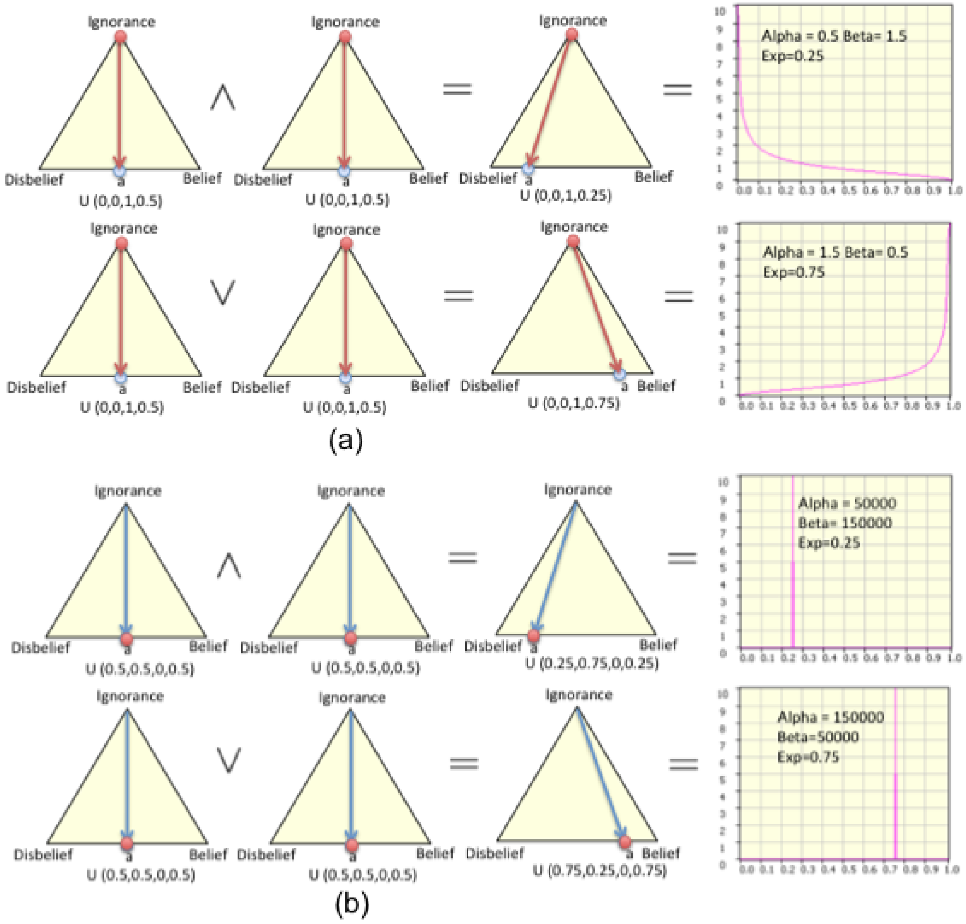

Figure 5.10: (a) $U \wedge U$ and $U \vee U$ (b) $\perp \wedge \perp$ and $\perp \vee \perp$ in Subjective Logic.

similar aspect holds in the case of $\perp \wedge \perp = \perp$ and $\perp \vee \perp = \perp$ [3]. For $p \wedge q$ and $p \vee \neg q$, classical logic should draw T and F. As shown in Figure 5.10 (b), considering the epistemic state of contradiction as $w = (0.5, 0.5, 0, 0.5)$, we get $(0.5, 0.5, 0, 0.5) \wedge \neg(0.5, 0.5, 0, 0.5) = (0.25, 0.75, 0, 0.25)$ that is biased to disbelief. Note that, both in Figure 5.10 (a) and (b) we have the same probability expectation values. However, when represented as Beta

[3] This is due to the main difference that subjective logic offers the Bayesian view on interpreting contradiction as discussed in Section 5.6.2. Strictly speaking, Belnap like logical view of contradiction can be also explicitly distinguished using the *'conflict parameter'* in subjective logic (see Section 5.6.2), even though the use of the parameter does not affect to subsequent calculation and therefore we did not explicitly use the parameter in this dissertation. However, for some applications in which it is very important to know and to distinguish the occurrence of the logical view of conflict, so to regard it as unhealthy system status and therefore, for example to halt the system, we could explicitly use the *'conflict'* parameter.

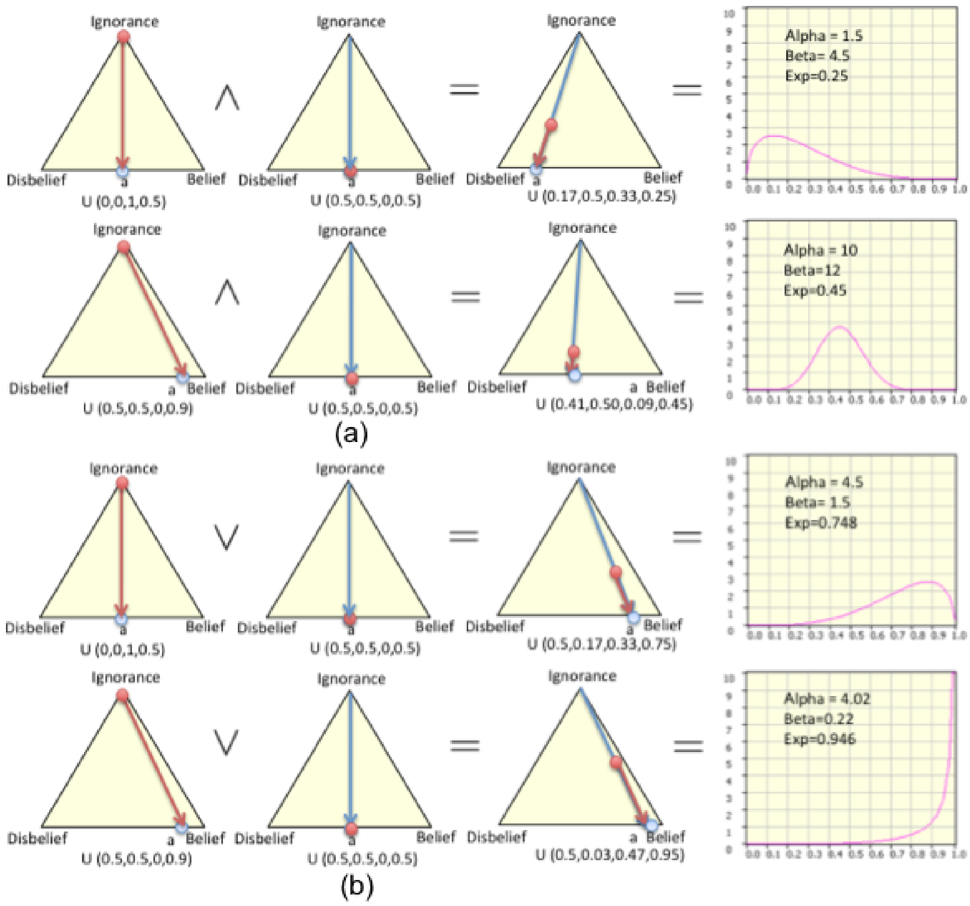

Figure 5.11: (a) Two Examples of $U \wedge \bot$ (b) Two Examples of $U \vee \bot$ in Subjective Logic.

distribution, while (b) is almost certain because we have rather pick distribution (b) is almost uncertain. This aspect is directly captured in the opinion triangle by the value of ignorance. Now, for the counter intuitive cases of $U \wedge \bot = F$ and $U \vee \bot = T$ in Belnap logic, subjective logic draws a bit different epistemic states. Figure 5.11 (a) depicts two cases of $U \wedge \bot$ one more biased to F and the other more biased to $\bot$. The basic idea is that we take more atomicity in the case of unknown opinion. The same aspect is captured in the case of $U \vee \bot$ as shown in Figure 5.11 (b). Figure 5.11 (a) more intuitively explains the above mentioned agent scenario with two propositions p and q. Again, consider the truth value on proposition p to be $\bot$ because agent A_1 and A_2 saying the p is T and F. For the proposition q, we assign U because agent A_1 and A_2 say nothing about q. Now,

$p \wedge q$ can be determined differently as shown in Figure 5.11 (a).

Finally, the last aspect on whether commutative computation is possible or not in Belnap logic, has no problem in subjective logic [98, 97, 94]. Suppose two atomic propositions p and q with epistemic state assignment $\phi(p) = \perp = (0.5, 0.5, 0, 0.5)$ and $\phi(q) = U = (0, 0, 1, 0.5)$. Then, $\phi(p \wedge q) = (0.17, 0.5, 0.33, 0.25)$. The negation in this case is $\phi(\neg p) = \perp = (0.5, 0.5, 0, 0.5)$ and $\phi(\neg q) = U = (0, 0, 1, 0.5)$, we also get $\phi(\neg p \wedge q) = \phi(p \wedge \neg q) = \phi(\neg \wedge \neg q) = (0.17, 0.5, 0.33, 0.25)$. Hence,

$$
\begin{aligned}
&\phi((p \wedge q) \wedge (\neg p \wedge q) \wedge (p \wedge \neg q) \wedge (\neg p \wedge \neg q)) \\
&\quad = \phi(p \wedge q) \wedge \phi(\neg p \wedge q) \wedge \phi(p \wedge \neg q) \wedge \phi(\neg p \wedge \neg q) \\
&\quad = (0.17, 0.5, 0.33, 0.25) \wedge (0.17, 0.5, 0.33, 0.25) \wedge (0.17, 0.5, 0.33, 0.25) \wedge (0.17, 0.5, 0.33, 0.25) \\
&\quad = (0.07, 0.75, 0.18, 0.07) \wedge (0.17, 0.5, 0.33, 0.25) \wedge (0.17, 0.5, 0.33, 0.25) \\
&\quad = (0.03, 0.88, 0.1, 0.02) \wedge (0.17, 0.5, 0.33, 0.25) = (0.01, 0.94, 0.05, 0.01)
\end{aligned}
$$

In subjective logic, regardless of the order how we calculate opinions, we get the same result as follows.

$$
\begin{aligned}
&\phi((p \wedge q) \wedge (\neg p \wedge q) \wedge (p \wedge \neg q) \wedge (\neg p \wedge \neg q)) \\
&\quad = \phi((0.17, 0.5, 0.33, 0.25) \wedge (0.5, 0.5, 0, 0.5) \wedge q \wedge p \wedge \neg q \wedge \neg p \wedge \neg q) \\
&\quad = \phi((0.11, 0.75, 0.14, 0.13) \wedge (0, 0, 1, 0.5) \wedge p \wedge \neg q \wedge \neg p \wedge \neg q) \\
&\quad = \phi((0.05, 0.75, 0.2, 0.07) \wedge (0.5, 0.5, 0, 0.5) \wedge \neg q \wedge \neg p \wedge \neg q) \\
&\quad = \phi((0.03, 0.88, 0.10, 0.04) \wedge (0, 0, 1, 0.5) \wedge \neg p \wedge \neg q) \\
&\quad = \phi((0.01, 0.88, 0.11, 0.02) \wedge (0.5, 0.5, 0, 0.5) \wedge \neg q) \\
&\quad = \phi((0.01, 0.94, 0.05, 0.01) \wedge (0, 0, 1, 0.5)) = \phi(0, 01, 0.94, 0.05, 0.01) = (0, 01, 0.94, 0.05, 0.01)
\end{aligned}
$$

We believe, above aspect makes subjective logic more solid and sound logic formalism under uncertainty. Especially, compared to fuzzy Belnap logic, the operational order does not affect on the final result. This is an important aspect, because in fuzzy-Belnap logic, once we reach at the contradictory point, there is no easy way to escape from the state unless we use the meet operator $\otimes$ along the partial order $\leq_k$. Namely, Belnap logic is non-associative. Therefore, in fuzzy Belnap logic, the sequence of information arrival is important, however, so is not in subjective logic because subjective logic supports associativity [98, 97, 94]).

Discussion 2. ***(Feasibility in visual surveillance) .*** In the previous section for the examples, we assigned definite true T and definite false F for the logical facts. In practice, however, such symbolic logical facts are generated from the vision analytics. Because the vision analytics tend to rely on machine learning and pattern recognition techniques, in general, the values will be also noisy. Indeed, in practice, it would be more realistic to attach arbitrary amount of beliefs even to the logical rules rather than values such as DT, DF, DT_1, DF_1, etc. In the previous examples, the L-fuzzy logic based approaches generated the same result regardless how we choose *t-norms* and *t-conorms*. Therefore, in this discussion, we will examine how the uncertainty introduced on facts and rules, and how the choices on *t-norms* and *t-conorms* could affect the reasoning result. Consider Example 3 in the previous section with slightly different settings as follows.

$$\begin{aligned}
\phi[\neg equal(P_1, P_2) \leftarrow distinct(P_1, P_2)] &= (0.5, 0.1)_{\mathcal{L}^2} = (0.5, 0.1, 0.4)_{sl} = r_1 \\
\phi[equal(P_1, P_2) \leftarrow appear_similar(P_1, P_2)] &= (0.5, 0.1)_{\mathcal{L}^2} = (0.5, 0.1, 0.4)_{sl} = r_2 \\
\phi[appear_similar(a, b)] &= (0.6, 0.3)_{\mathcal{L}^2} = (0.6, 0.3, 0.1)_{sl} = f_1 \\
\phi[distinct(a, b)] &= (0.3, 0.4)_{\mathcal{L}^2} = (0.3, 0.4, 0.3)_{sl} = f_2
\end{aligned}$$

In above setting, given two rules that are considered with the same amount of significance, we attach more strong belief to f_1. Therefore, the expected result is that the two persons maybe the same one but not quite certainly. Applying the same inference mechanism Equation 5.1 for L-fuzzy logics and (2) for subjective logic, the inference results are as follows. (note that, above setting is not applicable to the case of discrete bilattice species for multivalued default logic.).

$$\begin{aligned}
&cl_{sl_{di}}(\phi)(equal(a, b)) \\
&\quad = [U \sqcup (f_1 \bullet r_2)] \oplus \neg[U \sqcup (f_2 \bullet r_1)] \\
&\quad = [U \sqcup (0.6, 0.3, 0.1) \bullet (0.5, 0.1, 0.4)] \oplus \neg[U \sqcup (0.3, 0.4, 0.3) \bullet (0.5, 0.1, 0.4)]
\end{aligned}$$

$$= [(0,0,1) \sqcup (0.4, 0.37, 0.23)] \oplus \neg[(0,0,1) \sqcup (0.24, 0.46, 0.3)]$$

$$= (0.4, 0.07, 0.53) \oplus \neg(0.24, 0.09, 0.67) = (0.4, 0.07, 0.53) \oplus (0.09, 0.24, 0.67) = (0.37, 0.21, 0.42)$$

As shown in Table 5.5, choosing one of *t-norm* and *t-conorm* pair and applying Equation 5.8 in Definition 36, we get following inference results derived on $\mathcal{L}^2$, and its interpretations $cl^F_{\mathcal{I}^*(\mathcal{L})}$ and $cl^G_{\mathcal{I}^*(\mathcal{L})}$ on $\mathcal{I}^*(\mathcal{L})$.

$$cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))$$

$$= [U \vee (f_1 \wedge r_2)] \oplus \neg[U \vee (f_2 \wedge r_1)] = [U \vee (0.6, 0.3) \wedge (0.5, 0.1)] \oplus \neg[U \vee (0.3, 0.4) \wedge (0.5, 0.1)]$$

$$= [U \vee (\min(0.6, 0.5), \max(0.3, 0.1))] \oplus \neg[U \vee (\min(0.3, 0.5), \max(0.4, 0.1))]$$

$$= [(0,0) \vee (0.5, 0.3)] \oplus \neg[(0,0) \vee (0.3, 0.4)] = (\max(0, 0.5), \min(0, 0.3)) \oplus \neg(\max(0, 0.3), \min(0, 0.4))$$

$$= (0.5, 0) \oplus \neg(0.3, 0) = (0.5, 0) \oplus (0, 0.3) = (\max(0.5, 0), \max(0, 0.3)) = (0.5, 0.3)$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.5, 0.3)) = (\tfrac{0.5^2}{0.5+0.3}, \tfrac{0.5 \cdot 0.3}{0.5+0.3}) = (0.31, 0.19)$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{\min/\max}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.5, 0.3)) = (0.5 - \tfrac{0.3}{2}, \tfrac{0.3}{2}) = (0.35, 0.15)$$

$$cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))$$

$$= [U \vee (f_1 \wedge r_2)] \oplus \neg[U \vee (f_2 \wedge r_1)] = [U \vee (0.6, 0.3) \wedge (0.5, 0.1)] \oplus \neg[U \vee (0.3, 0.4) \wedge (0.5, 0.1)]$$

$$= [U \vee (0.6 \cdot 0.5, 0.3 + 0.1 - 0.3 \cdot 0.1)] \oplus \neg[U \vee (0.3 \cdot 0.5, 0.4 + 0.1 - 0.4 \cdot 0.1)]$$

$$= [(0,0) \vee (0.3, 0.37)] \oplus \neg[(0,0) \vee (0.15, 0.46)] = (0 + 0.3 - 0 \cdot 0.3, 0 \cdot 0.37) \oplus \neg(0 + 0.15 - 0 \cdot 0.15, 0 \cdot 0.15)$$

$$= (0.3, 0) \oplus \neg(0.15, 0) = (0.3, 0) \oplus (0, 0.15) = (0.3 + 0 - 0.3 \cdot 0, 0 + 0.15 - 0 \cdot 0.15) = (0.3, 0.15)$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.3, 0.15)) = (\tfrac{0.3^2}{0.3+0.15}, \tfrac{0.3 \cdot 0.15}{0.3+0.15}) = (0.2, 0.1)$$

$$cl^G_{\mathcal{I}^*(\mathcal{L})}(cl^{prod/sum}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^G_{\mathcal{I}^*(\mathcal{L})}((0.3, 0.15)) = (0.3 - \tfrac{0.15}{2}, \tfrac{0.15}{2}) = (0.225, 0.075)$$

$$cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))$$

$$= [U \vee (f_1 \wedge r_2)] \oplus \neg[U \vee (f_2 \wedge r_1)] = [U \vee (0.6, 0.3) \wedge (0.5, 0.1)] \oplus \neg[U \vee (0.3, 0.4) \wedge (0.5, 0.1)]$$

$$= [U \vee (\max(0, 0.6 + 0.5 - 1), \min(0.3 + 0.1, 1)] \oplus \neg[U \vee (\max(0, 0.3 + 0.5 - 1), \min(0.4 + 0.1, 1)]$$

$$= [(0,0) \vee (0.1, 0.4)] \oplus \neg[(0,0) \vee (0, 0.5)] = (\min(0 + 0.1, 1), \max(0, 0 + 0.4 - 1))$$

$$\oplus \neg(\min(0 + 0, 1), \max(0, 0 + 0.5 - 1)) = (0.1, 0) \oplus \neg(0, 0) = (0.3, 0) \oplus (0.0)$$

$$= (\min(0.3 + 0, 1), \min(0 + 0, 1)) = (0.3, 0)$$

$$cl^F_{\mathcal{I}^*(\mathcal{L})}(cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^F_{\mathcal{I}^*(\mathcal{L})}((0.3, 0)) = (\tfrac{0.3^2}{0.3+0}, \tfrac{0.3 \cdot 0}{0.3+0}) = (0.3, 0)$$

$cl^{G}_{\mathcal{I}^*(\mathcal{L})}(cl^{Luk}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^{G}_{\mathcal{I}^*(\mathcal{L})}((0.3,0)) = (0.3 - \frac{0}{2}, \frac{0}{2}) = (0.3,0)$

$cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))$

$$= [U \vee (f_1 \wedge r_2)] \oplus \neg[U \vee (f_2 \wedge r_1)] = [U \vee (0.6,0.3) \wedge (0.5,0.1)] \oplus \neg[U \vee (0.3,0.4) \wedge (0.5,0.1)]$$
$$= [U \vee (\mathcal{T}_D(0.6,0.5), \mathcal{S}_D(0.3,0.1)] \oplus \neg[U \vee (\mathcal{T}_D(0.3,0.5), \mathcal{S}_D(0.4,0.1)]$$
$$= [(0,0) \vee (0,1)] \oplus \neg[(0,0) \vee (0,1)] = (\mathcal{S}_D(0,0), \mathcal{T}_D(0,1)) \oplus \neg(\mathcal{S}_D(0,0), \mathcal{T}_D(0,1))$$
$$= (0,0) \oplus \neg(0,0) = (0,0) \oplus (0,0) = (\mathcal{S}_D(0,0), \mathcal{S}_D(0,0)) = (0,0)$$

$cl^{F}_{\mathcal{I}^*(\mathcal{L})}(cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^{F}_{\mathcal{I}^*(\mathcal{L})}((0,0)) = (0,0)$

$cl^{G}_{\mathcal{I}^*(\mathcal{L})}(cl^{drastic}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^{G}_{\mathcal{I}^*(\mathcal{L})}((0.3,0)) = (0,0)$

$cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))$

$$= [U \vee (f_1 \wedge r_2)] \oplus \neg[U \vee (f_2 \wedge r_1)] = [U \vee (0.6,0.3) \wedge (0.5,0.1)] \oplus \neg[U \vee (0.3,0.4) \wedge (0.5,0.1)]$$
$$= [U \vee (\mathcal{T}_{nM}(0.6,0.5), \mathcal{S}_{nM}(0.3,0.1)] \oplus \neg[U \vee (\mathcal{T}_{nM}(0.3,0.5), \mathcal{S}_{nM}(0.4,0.1)]$$
$$= [(0,0) \vee (0.5,0.3)] \oplus \neg[(0,0) \vee (0,0.4)] = (\mathcal{S}_{nM}(0,0.5), \mathcal{T}_{nM}(0,0.3)) \oplus \neg(\mathcal{S}_{nM}(0,0), \mathcal{T}_{nM}(0,0.4))$$
$$= (0.5,0) \oplus \neg(0,0) = (0.5,0) \oplus (0,0) = (\mathcal{S}_{nM}(0.5,0), \mathcal{S}_{nM}(0,0)) = (0.5,0)$$

$cl^{F}_{\mathcal{I}^*(\mathcal{L})}(cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^{F}_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (\frac{0.5^2}{0.5+0}, \frac{0.5 \cdot 0}{0.5+0}) = (0.5,0)$

$cl^{G}_{\mathcal{I}^*(\mathcal{L})}(cl^{Nilpotent}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^{G}_{\mathcal{I}^*(\mathcal{L})}((0.5,0)) = (0.5 - \frac{0}{2}, \frac{0}{2}) = (0.5,0)$

$cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))$

$$= [U \vee (f_1 \wedge r_2)] \oplus \neg[U \vee (f_2 \wedge r_1)] = [U \vee (0.6,0.3) \wedge (0.5,0.1)] \oplus \neg[U \vee (0.3,0.4) \wedge (0.5,0.1)]$$
$$= [U \vee (\mathcal{T}_{H0}(0.6,0.5), \mathcal{S}_{H2}(0.3,0.1)] \oplus \neg[U \vee (\mathcal{T}_{H0}(0.3,0.5), \mathcal{S}_{H2}(0.4,0.1)]$$
$$= [(0,0) \vee (0.375,0.39)] \oplus \neg[(0,0) \vee (0.23,0.48)] = (\mathcal{S}_{H2}(0,0.375), \mathcal{T}_{H0}(0,0.39))$$
$$\oplus \neg(\mathcal{S}_{H2}(0,0.23), \mathcal{T}_{H0}(0,0.48)) = (0.375,0) \oplus \neg(0.23,0) = (0.375,0) \oplus (0,0.23)$$
$$= (\mathcal{S}_{H2}(0.375,0), \mathcal{S}_{H2}(0,0.23)) = (0.375,0.23)$$

$cl^{F}_{\mathcal{I}^*(\mathcal{L})}(cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^{F}_{\mathcal{I}^*(\mathcal{L})}((0.375,0.23)) = (\frac{0.375^2}{0.375+0.23}, \frac{0.375 \cdot 0.23}{0.375+0.23}) = (0.23,0.14)$

$cl^{G}_{\mathcal{I}^*(\mathcal{L})}(cl^{Hamacher}_{\mathcal{L}^2_{di}}(\phi)(equal(a,b))) = cl^{G}_{\mathcal{I}^*(\mathcal{L})}((0.375,0.23)) = (0.375 - \frac{0.23}{2}, \frac{0.23}{2}) = (0.26,0.12)$

Figure 5.12 shows above results in opinion space, $\mathcal{I}^*(\mathcal{L})$ and $\mathcal{L}^2$. Unlike the case of using values lying on the boundary of the spaces such as T, F, DT_1, DF_1, when internal values in spaces are used, the reasoning results are quite dependent on the choice of *t-norms*

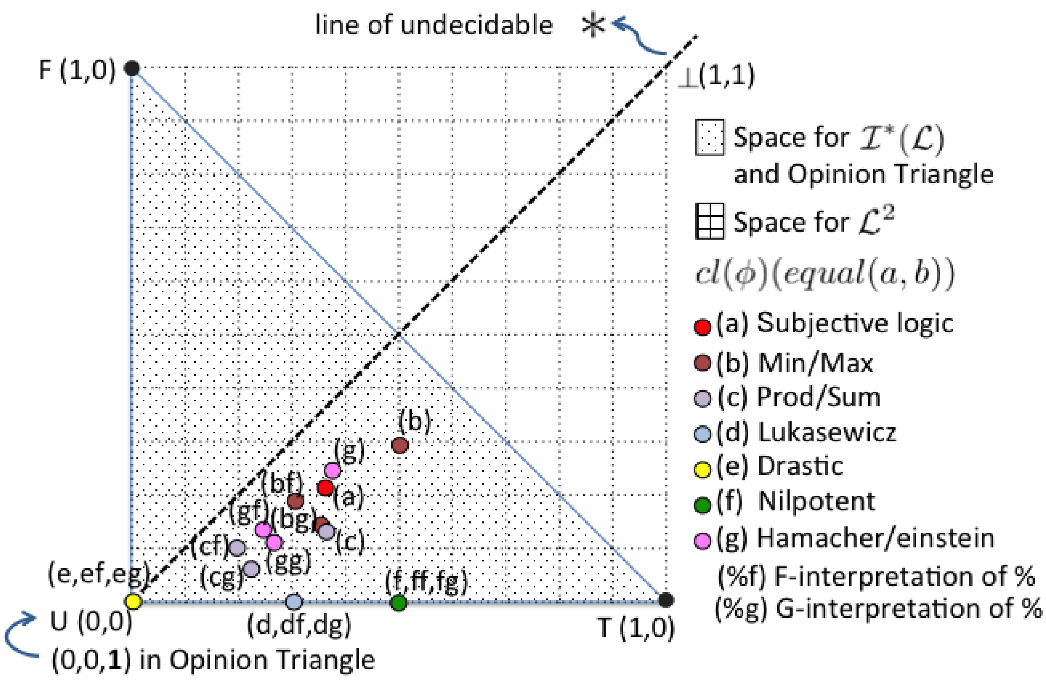

Figure 5.12: Reasoning Results of Modified Example 3 in Opinion Space and $\mathcal{L}^2$ & $\mathcal{I}^*(\mathcal{L})$.

and *t-conorms*. However, what pair of *t-norms* and *t-conorms* to use is not easy to answer. This is a problem common to all fuzzy set based applications. Typically, connectives are categorized by the properties they satisfy. Lukasewicz connectives are in some sense the most interesting one because they satisfy the most properties of binary connectives, but it does not mean that they are best suited for each application. This aspect is sometimes also attacked by statisticians who prefer Bayesian theory. However, Product / Sum connectives are interesting in Bayesian sense, because Product *t-norm* and Sum *t-conorm* resemble probabilistic conjunction and disjunction. For instance, following Equation 5.8, fuzzy Belnap connectives on $\mathcal{L}^2$ that are compositionally defined upon Product *t-norm* and Sum *t-conorm* pair are as follows :

$$
\begin{aligned}
(b_x, d_x) \wedge_{bl} (b_y, d_y) &= (b_x b_y, d_x + d_y - d_x d_y) \\
(b_x, d_x) \vee_{bl} (b_y, d_y) &= (b_x + b_y - b_x b_y, d_x d_y) \\
(b_x, d_x) \otimes_{bl} (b_y, d_y) &= (b_x b_y, d_x d_y) \\
(b_x, d_x) \oplus_{bl} (b_y, d_y) &= (b_x + b_y - b_x b_y, d_x + d_y - d_x d_y)
\end{aligned}
$$

Chapter 5

As mentioned throughout this chapter, subjective logic has solid mathematical basis in Bayesian perspective on dealing binary (crisp) event (see preliminaries and definitions operators). Therefore, it is worth to compare above Product / Sum connectives with the ones in subjective logic in the definition level. For example, given two opinions $w_x = (b_x, d_x, i_x, a_x)$ and $w_y = (b_y, d_y, i_y, a_y)$, the conjunction operator of subjective logic generates following elements (see Definition 31) :

$$\begin{aligned}
&b_{x \wedge_{sl} y} = b_x b_y + \tfrac{(1-a_x)a_y b_x i_y + a_x(1-a_y) i_x b_y}{1-a_x a_y} \\
&d_{x \wedge_{sl} y} = d_x + d_y - d_x d_y \\
&i_{x \wedge_{sl} y} = i_x i_y + \tfrac{(1-a_y) b_x i_y + (1-a_x) i_x b_y}{1-a_x a_y} \\
&a_{x \wedge_{sl} y} = a_x a_y
\end{aligned}$$

and the disjunction of the two opinions are defined as follows (see Definition 32):

$$\begin{aligned}
&b_{x \vee_{sl} y} = b_x + b_y - b_x b_y \\
&d_{x \vee_{sl} y} = d_x d_y + \tfrac{a_x(1-a_y) d_x i_y + (1-a_x) a_y i_x d_y}{a_x + a_y - a_x a_y} \\
&i_{x \vee_{sl} y} = i_x i_y + \tfrac{a_y d_x i_y + a_x i_x d_y}{a_x + a_y - a_x a_y} \\
&a_{x \vee_{sl} y} = a_x + a_y - a_x a_y
\end{aligned}$$

the consensus $\otimes_{sl}$ of the two opinions are defined as follows (see Definition 30):

$$\begin{aligned}
&b_x^{A,B} = (b_x^A i_x^B + b_x^B i_x^A)/k \\
&d_x^{A,B} = (d_x^A i_x^B + d_x^B i_x^A)/k \\
&i_x^{A,B} = (i_x^A i_x^B)/k \\
&a_x^{A,B} = \tfrac{a_x^A i_x^A + a_x^B i_x^A - (a_x^A + a_x^B) i_x^A i_x^B}{i_x^A + i_x^B - 2 i_x^A i_x^B} \\
&where,\ k = i_x^A + i_x^B - i_x^A i_x^B .
\end{aligned}$$

Although conjunctions (disjunctions) of fuzzy Belnap on Product / Sum and of subjective logic look similar, they are not exactly the same. The reason is because the definition in subjective logic is defined so that it can model a beta distribution that approximates the resulting function by multiplying (comultiplying) the two of corresponding beta distributions of the given two opinions w_x and w_y [97] (note that, the result of multiplication and comultiplication of two beta functions are not always beta function, [97]).

Similarly, while the *join* operator $\oplus_{bl}$ on Product / Sum just sum both the belief and the disbelief, subjective logic calculation is designed so that it can model the beta distribution derived by merging each pair of parameters of the beta distributions correspond to the given two opinions w_x and w_y [94]. Indeed, through (5.2), (5.3) and (5.4) in Section 5.6.3, we have shown that the *'consensus'* operator itself is the calculation of *'Bayes Theorem'* itself. Due to this aspect, even compared with Product / Sum fuzzy Belnap connectives, subjective logic stays closer to the Bayesian aspect.

When it comes to visual surveillance, number of vision analytics are based on the pattern recognition and machine learning techniques that are also (in many cases) based on Bayesian statistics rather than fuzzy theory. Noting this aspect, we advocate subjective logic could be better suited for visual surveillance applications especially when we want to stay closer to the way that usual vision analytics generate uncertain symbolic facts.

5.10 Chapter Summary

In summary of lessons learned in this chapter, the attempt to modeling default reasoning using subjective logic has given several insights. 1) It shows the feasibility of proposed reasoning scheme on handling contradictory information. 2) While L-fuzzy logics and subjective logic have commonality in representing epistemic status, subjective logic is more close to Bayesian on operation of such status. 3) The logical soundness of the proposed approach makes it attractive for visual surveillance scenarios compared with L-fuzzy logic based default reasoning approaches.

In this chapter, we proposed subjective logic based inference framework for default reasoning, and demonstrated its use for high level semantic analysis of visual surveillance scenes. Default reasoning is an important aspect of human like non-monotonic reasoning under incomplete and imprecise knowledge, that can play an important role for deriving plausible conclusions for many applications. Especially, in the forensic sense of visual surveillance that needs to reason about a propositional hypothesis to be investigated after an incident or a report, it is natural to examine all positive and negative contextual evidences that are related to the given hypothesis and fuse them to derive plausible conclusion based on default reasoning. The keys to enable default reasoning are 1) representing incompleteness of knowledge and 2) providing appropriate inference mechanism to draw plausible semantic conclusions by aggregating that knowledge. We adopted subjective logic due to its property of representing belief with ignorance and its rich set of operators for handling uncertain beliefs. To contrast the properties and advantage of the proposed approach, we also applied the inference scheme on L-fuzzy set based logics. The case study results show that the proposed approach and L-fuzzy set based approaches can be an alternative tool to model default reasoning. Among the L-fuzzy logics, intuitionistic fuzzy logic is very similar to the uncertainty representation scheme of subjective logic. While the generalized intuitionistic fuzzy logic, that is fuzzy Belnap logic, can be defined on a bilattice structure with operators regarding degree of information, intuitionistic fuzzy logic could not be fully defined on a bilattice structure because the *join* operator along the axis of degree of information can not be defined. Contrary to intuitionistic fuzzy logic, even though it also has triangle structure, subjective logic provides an operator called *consensus* that has very similar behaviour as the *join* operator on degree of information in bilattice. This is because when two opinions are fused by the *consensus* operator, it always decreases ignorance in the derived opinion except in the case of fusing definite true (full belief) and definite false (full disbelief). Due to this aspect, the comparison of subjective logic based default reasoning with intuitionistic fuzzy logic was done via a mapping between the fuzzy Belnap logic and the intuitionistic fuzzy logic. The reasoning result of both the subjective logic and the fuzzy Belnap logic seem reasonable. However, as noted in the discussion section, fuzzy Belnap logic has some problems. 1) the truth table has some problematic aspects, thereby logically not sound. 2) due to 1) the sequence of getting information is critical. 3) due to 1) once the epistemic state is reached to the *contradictory* state, it is not easy to escape that state. 4) the basic four logical operators in L-fuzzy logics can be determined in many ways, therefore, the semantics of the operators are not sound and clear in Bayesian sense. Due to these aspects, we advocate subjective logic has advantages as a tool for artificial reasoning in visual surveillance. Because, in visual surveillance, due to the flexibility, and instability of the vision analytics, we can not guarantee the sequence of getting information, therefore, the reasoning system should be robust against the information acquisition sequence. Indeed, most of the vision analytics are based on probabilistic theory, therefore, the values from those analytic modules could be well interpreted in subjective logic. Beside these aspects, there is yet another advantage of the proposed approach, that is the ability of default reasoning can be fulfilled within a single subjective logic based reasoning framework that can also offer additional potential usage such as bidirectional conditional modeling [81], reputation based belief decaying, etc. [80]. Therefore, enabling default reasoning to subjective logic could offer better expressive power for modeling and reflecting real world situation.

There are, however, still open issues such as comparing the introduced inference scheme to more complicated situational reasoning. Therefore, our future research will cover such comparisons and applying the shown approach to more complicated scenarios using automatically generated large scale data.

6 Vague Proposition Modeling and Handling

In the previous chapter, we have explained that subjective logic can be a good means to model *'default reasoning'*. The main idea was to label truth of facts and rules with subjective opinion (see Definition 33) and to fuse both positive and negative sources of information together (see Definition 35).

In the big picture of our *'Opinion Assignment'* concept described in Definition 16 of Section 4.3.2, the former corresponds to 1 - 2 of Definition 16 and the latter corresponds to Definition 16 - 4. However, there is yet another type of opinion assignment depicted in Definition 16 - 3. While 1 - 2 of Definition 16 assign a fixed static opinion value to facts and rules, Definition 16 - 3 assigns an opinion dynamically, according to the subjective logic calculation scheme written in the rule body. In this chapter, we will further explore rule modeling in type of Definition 16 - 3.

6.1 Introduction

This chapter presents an approach to modeling vague contextual rules using subjective logic for forensic visual surveillance. Unlike traditional real-time visual surveillance, forensic analysis of visual surveillance data requires mating of high level contextual cues with observed evidential metadata where both the specification of the context and the metadata suffer from uncertainties. To address this aspect, in Chapter 4, we proposed the use of declarative logic programming to represent and reason about contextual knowledge, and the use of subjective logic for uncertainty handling. Upon this approach, in the previous Chapter 5, we have demonstrated that subjective logic can be a good means to model *'default reasoning'*. The main idea was to label generalized truth or priority of facts and rules with subjective opinion and to fuse both positive and negative sources of information together.

However, there are often cases that the truth value of rule itself is also uncertain thereby, uncertainty assignment of the rule itself should be rather functional. *'the more X then the more Y'* type of knowledge is one of the examples. To enable such type of rule modeling, in this chapter, we propose a reputational subjective opinion function upon logic programming, which is similar to fuzzy membership function but can also take uncertainty of membership value itself into account. Then we further adopt subjective logic's fusion operator to accumulate the acquired opinions over time. To verify our approach, we present a preliminary experimental case studies on reasoning likelihood of being a good witness that uses metadata extracted by a person tracker and evaluates the relationship between the tracked persons. The case study is further extended to demonstrate more complex forensic reasoning by considering additional contextual rules.

6.2 Background and Motivation

In the pipeline of our proposed approach (see Chapter 4), intermediate metadata comes from vision analytics and additional visual or non visual contextual cues are encoded as either symbolized facts or rules. Then uncertainty comes with vision analytics are represented as subjective opinions and attached to their symbolized facts. Similarly, uncertainty as general trustworthiness or priority among rules is also represented and attached to given contextual rules. Once such uncertainty attachment is done, principled inference, which is often nonmonotonic, is conducted. The examples of such principled inferences are default reasoning [143] to handle inconsistent information (see Chapter 5), abduction [50] to find most probable hypothesis of given observation and belief revision over time upon the change of observation (see Chapter 8), etc. Therefore, appropriate uncertainty assignment plays an important role for proper inference.

Unlike above mentioned opinion assignment to logical rules, however in reality, there are linguistically vague logical rules that hinders assigning a single opinion to a rule does not make sense but opinion assignment itself should be done rather functionally. In this chapter we will refer to this type of rule as *'Vague Rule'*. An example rule in this type is *'the more X then the more Y'*. Humans are very skillful in dealing such type of rules. To enable such type of rule handling in our framework, we first examine a subjective logic operator called *'reputation operator'*. We then propose a reputational subjective opinion function that is similar to fuzzy membership function but can also take uncertainty of membership value itself into consideration. To demonstrate reasoning under such vague rules, we present a preliminary experimental case study by intentionally restricting the type of available metadata to the results from human detection and tracking algorithms. Automatic human detection and tracking is one of the common analytics and becoming more widely employed in automated visual surveillance systems. The typical types of meta-information that most human detection analytic modules generate comprise, for instance, localization information such as coordinate, width, height, time and (optionally) additional low-level visual feature vectors. We intend to use further such information for evaluating the relationship between two persons and, more specifically, for estimating whether one person could serve as a witness of another person in a public area scene. Examples for (linguistic) domain knowledge applicable to this scenario include: 1) (At least) two distinct people are required for building a relationship. 2) The closer the distance between two people is, the higher is the chance that they may identify each other. 3) If two persons approach each other directly (face-to-face) then there is a higher chance that they can identify each other. Such linguistic knowledge can be modeled and encoded as rules by the proposed approach. The case study is further extended to demonstrate more complex forensic reasoning by considering additional contextual rules together with the shown vague rules.

6.3 Proposed Approach

The proposed vague rule modeling approach mainly relies on the logic programming extended with subjective logic (see Section 4.3.2). Firstly, for a given propositional knowledge, we assume a fuzzy-like membership function that grades degree of truth. Then we focus on that the interpretation of such membership function can be dogmatic, thereby, when the function is projected on the opinion space, it only lays on the bottom line of the opinion space. Indeed, in many cases, the exact shape of the function is hard to determine. To address this aspect, we introduce a reputational function that evaluate the trust worthiness of the fuzzy-like membership function. Then we introduce accumulation of the resulted opinions overtime.

6.3.1 Vague Propositional Rules

In logic programming, a conditional proposition '$y \leftarrow x$' is interpreted as *'IF x THEN y'*. However, there are often cases that we may want to interpret the meaning as *'the more x then the more y'* or *'the more x then the less y'*, etc. In this case, the opinion attached to the consequence of the rule should be rather functional in terms of the elements within the rule body. Therefore, the opinion assignment suit to this interpretation is Definition 16. In the sense of intrinsic linguistic uncertainty of the rule, it resembles fuzzy rules shown by Anderson et al. [17, 177]. In the work, quantitative low level features of human detection results such as 'centroid', 'eigen-based height' and 'ground plane normal similarity' are linguistically mapped into non-crisp premises (i.e. fuzzy sets) as '(H)igh', '(M)edium', '(L)ow' and '(V)ery Low'. Then fuzzy rules defines the conjunctive combination of those linguistic symbols to draw higher semantics such as 'Upright', 'In Between' and 'On the ground' (e.g. '$Upright(L) \leftarrow Centroid(H), EigenHeight(M), Similarity(H)$' [17]). Therefore, introducing appropriate fuzzy membership functions for each linguistic terms and proper handling of the membership functions is of important issue. In this view, Mizumoto et al. [119] showed comparison of sophisticated mathematical handling of ambiguous concepts such as 'more or less' having various shapes. One another thing worth to note concerning Fuzzy Logic is that, even if there are Zadeh's original logical operators, there are yet another ways of defining logical operators as well. For example, for given two quantitative variables x and y come with corresponding membership functions μ_a and μ_b, Zadeh's AND operator is defined as 'x AND $y = min(\mu_a(x), \mu_a(y))$'. In socalled *'t-norm fuzzy logic'*, any form of *t-norms* can be considered as AND operators (see Table 5.5). (note that, *t-norms* also played an important role to define lattice operators of L-fuzzy sets as shown in Definition 5.8 of Chapter 5). For example, in the case of using product *t-norm*, the AND operator can be defined as 'x AND $y = \mu_a(x) \cdot \mu_b(x)$' [73]. This aspect still remains controversial among most statisticians, who prefer Bayesian logic [179]. Contrary, as explained in the section Section 3.1, subjective logic can be interpreted in the sense of bayesian and also the final quantitative opinion space can also be interpreted in the sense of fuzziness (i.e. 'very certainly true', 'less certainly true', etc). This way, we believe that subjective logic can better bridges the interpretation of fuzzy intuitive concepts in bayesian sense. The basic idea of our approach is as follows :

- 1. For a given propositional rule 'the less (more) $y \leftarrow$ the more x' we could introduce a membership-like function $\mu_i : x \rightarrow y$.

- 2. It is clear that the function μ_i should be monotonically decreasing (increasing) but the shape is not quite clear.

- 3. Considering potentially possible multiple membership like functions μ_i, however the values of $\mu_i(x)$ at the two extreme point of ($min_x \leq x \leq max_x$) tend to converge but the values in between are diverge therefore, the values of later cases are more uncertain.

- 4. Considering the aspect of 3. we introduce socalled reputational opinion function on the function μ_i and combine it with raw opinion obtained from μ_i using subjective logic's reputation operator.

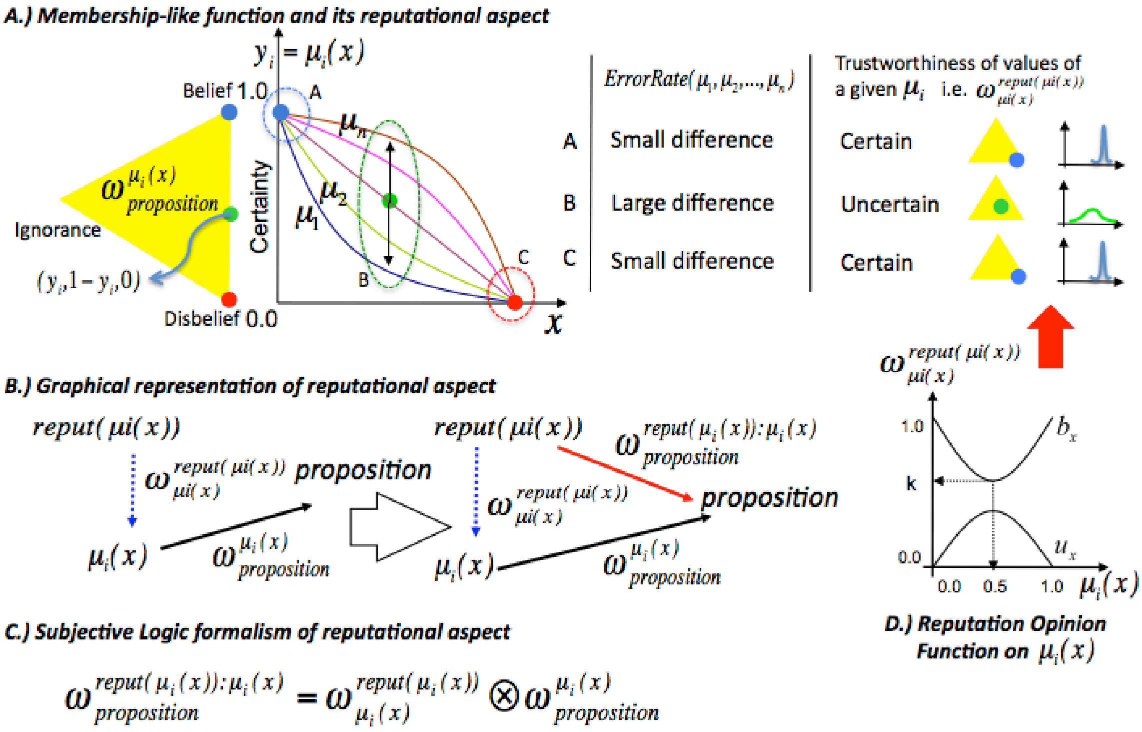

Figure 6.1: Vague Rule Modeling using Subjective Logic's Reputation Operator.

This idea is depicted in Figure 6.1, where the actual reputational operation is defined as follows :

Definition 41. ***(Reputation)*** *[99]* **.** *Let A and B be two agents where A's opinion about B's recommendations is expressed as $w^A_B = \{b^A_B, d^A_B, u^A_B, a^A_B\}$, and let x be a proposition where B's opinion about x is recommended to A with the opinion $w^B_x = \{b^B_x, d^B_x, u^B_x, a^B_x\}$. Let $w^{A:B}_x = \{b^{A:B}_x, d^{A:B}_x, u^{A:B}_x, a^{A:B}_x\}$ be the opinion such that:*

$$\begin{cases} b^{A:B}_x = b^A_B b^B_x & d^{A:B}_x = d^A_B d^B_x \\ u^{A:B}_x = d^A_B + u^A_B + b^A_B u^B_x & a^{A:B}_x = a^B_x \end{cases}$$

then $w^{A:B}_x$ is called the reputation opinion of A. By using the symbole $\otimes$ to designate this operation, we get $w^{A:B}_x = w^A_B \otimes w^B_x$.

For actual evaluation of a given function μ_i, an opinion assignment function on the given μ_i need to be defined. Although there could be also another ways of such function, in our approach, this is modeled as follows :

$$w^{reput^{\mu_i(x)}}_{\mu_i(x)} = \begin{cases} b_x = k + 4(1-k)(\mu_i(x) - \frac{1}{2})^2 \\ d_x = \frac{1-b_x}{Dratio} \\ u_x = 1 - b_x - d_x \end{cases} \quad (6.1)$$

where k, represents the minimum boundary of belief about the value from $\mu_i(x)$, and the $Dratio$ indicates the ratio for assigning the residue of the value μ_i to disbelief and uncertainty. This is depicted as Figure 6.1 - D.

6.4 Case Study I

6.4.1 Scenario Setting for Case Study I

At this stage we focused on evaluating the modeling approach itself rather than the reliability of the person detection algorithm. Therefore, we manually annotated a test video from one of i-LIDS [3] data sample with ground truth metadata for human detection comprising bounding boxes and timing information (shown in Figure 6.2). In total, 1 minute of test video was annotated in which there are 6 people. For our purposes, we intentionally marked one person as suspect. Then we encoded following linguistic contextual knowledge according to the proposed approach as explained in Section 4.3.2. 1) (At least) two distinct people are required for building a relationship 2) The closer the distance between two people is, the higher is the chance that they can identify each other. 3) If two persons approach each other directly (face-to-face) then there is a higher chance that they can identify each other. Then we calculate subjective opinions between the person marked as suspect and other human instances over time.

6.4.2 Uncertainty Modeling

6.4.2.1 Distance

The distance between a pair of people would be one of the typical pieces of clue for reasoning whether one person could serve as a witness of another person. This relates to the

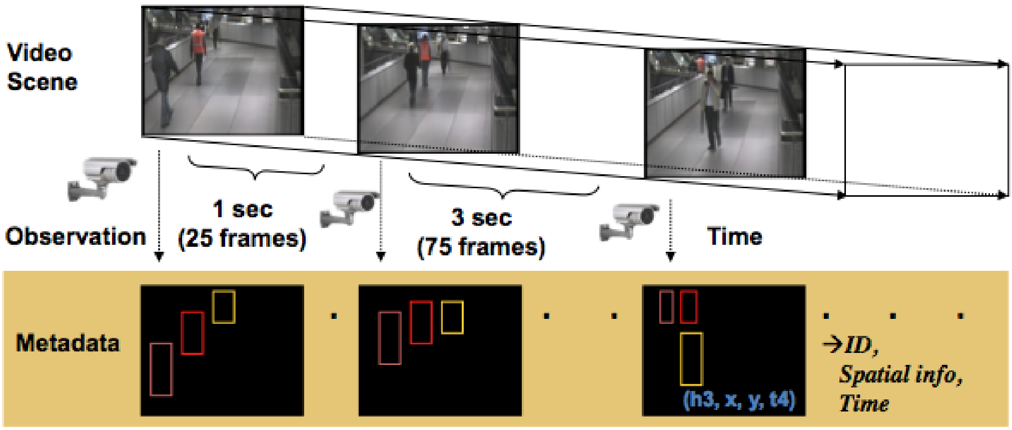

Figure 6.2: Scenario Setting for Case Study I.

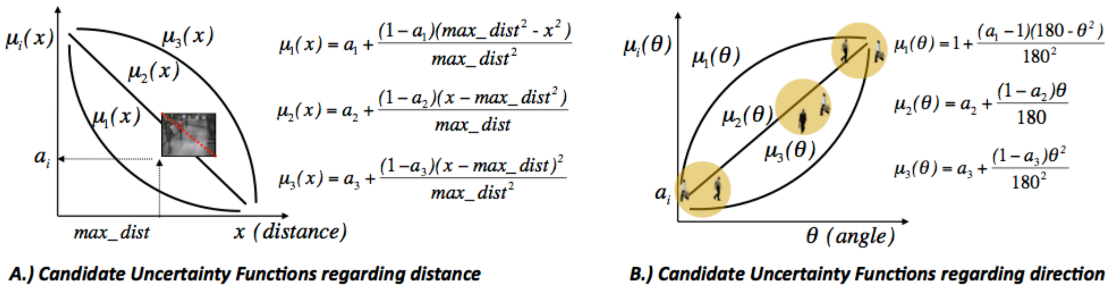

Figure 6.3: Candidate Uncertainty Functions regarding Distance and Direction.

general human knowledge that *'The closer two people are in distance, the more chances of perceiving the other are'.* Humans are very adapted to operating upon such type of uncertain and ambiguous knowledge. Exactly modeling such a relation is not trivial, but we can approximate it with a monotonic decreasing function about the possibility of perceiving each other. This aspect is depicted as three possible curves in the middle of Figure 6.3 - A.), where x represents the distance between the persons as calculated from the person detection metadata and μ_i represents the likelihood that two persons at this distance would perceive each other, $maxdist$ is the maximum possible (i.e diagonal) distance in a frame and a_i is the estimated probability that two humans could've recognized each other at the $maxdist$ distance. However, the value derived from such function is not fully reliable due to the variety of real world and uncertainty in the correctness of the function and uncertainty in the distance value itself. Considering the aspect of distance, it is clear that both extreme cases i.e. very close or very far are much more certain than in the middle of the range. Thus, to better model the real world situation, the reputational opinion function need to be applied to any chosen function μ_i. This is modeled as opinion on the reliability of $\mu_i(x)$ by applying Equation 6.1. In order to evaluate the impact of choosing different functions in Figure 6.3 - A.), three different types of μ_i functions (a concave, convex and linear) have been applied. The derived reputational opinions showed similar aspects having peaks of certain belief at each extreme cases as shown in Figure 6.4.

6.4.2.2 Direction

Similarly, we also used direction information between two persons. The linguistic knowledge to be modeled is *'if two persons approach each other directly (face-to-face) then there is a higher chances of perceiving each other'.* The corresponding direction-based relevance function is shown in Figure 6.3 - B.), where Θ represents the angle between the persons heading directions as calculated from the person detection metadata and μ_i represents the likelihood that two persons at the angle would perceive each other and a_i is the expected minimum probability that two humans could've recognized each other at any angle. However, again the trustworthiness of the values from such functions μ_i is uncertain, especially in the middle range of the Θ. To roughly model such aspect, for a chosen function $\mu_i(\Theta)$, the same reputational function from Equation 6.1 was used again. The impact of choosing different μ_i showed similar behavior as of direction based opinions as shown in Figure 6.4.

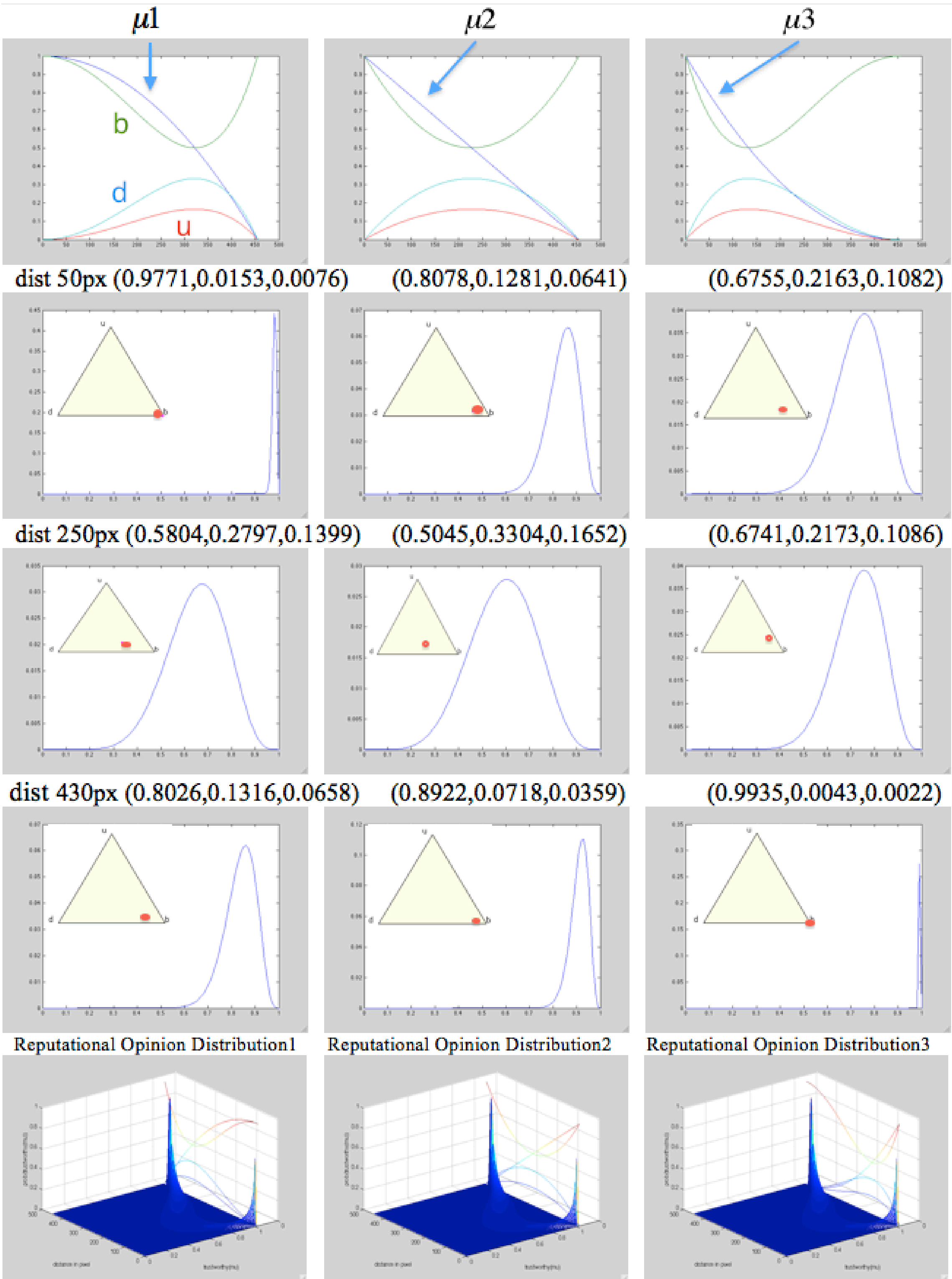

Figure 6.4: Samples of Reputational Opinion according to Distance and Equation 6.1.

6.4.3 Rule Encoding

In addition to the uncertainty modeling, logic programming is used to represent the given contextual rules as explained in Section 4.3.2. Encoded rules in form of Equation 4.2 are as follows :

$$w^{Rule1}_{witness(H_1,H_2,T_1)} \leftarrow \\ (w^{Human_Detector}_{human(H_1,T_1)} \wedge w^{Human_Detector}_{human(H_2,T_1)}) \otimes (w^{\mu_{dist}(d)}_{witness(H_1,H_2,T_1)} \otimes w^{reput^{\mu}(d)}_{\mu_{dist}(d)}) \tag{6.2}$$

$$w^{Rule2}_{witness(H_1,H_2,T_1)} \leftarrow \\ (w^{Human_Detector}_{human(H_1,T_1)} \wedge w^{Human_Detector}_{human(H_2,T_1)}) \otimes (w^{\mu_{dir}(d)}_{witness(H_1,H_2,T_1)} \otimes w^{reput^{\mu}(d)}_{\mu_{dir}(d)}) \tag{6.3}$$

$$w^{Rule3}_{witness(H_1,H_2,T_1)} \leftarrow (w^{Rule1}_{witness(H_1,H_2,T_1)} \wedge w^{Rule2}_{witness(H_1,H_2,T_1)}) \tag{6.4}$$

$$w^{Rule4}_{witness(H_1,H_2,T_n)} \leftarrow \oplus^{n}_{i=1} w^{Rule3}_{witness(H_1,H_2,T_i)} \tag{6.5}$$

The first Rule 6.2 starts considering the necessary condition, meaning that there should be a distinct pair of two people. Therefore the conjunction operation $\wedge$ (see Definition 31) on two opinions [97] is used that is very similar to the operation $P(A) \cdot P(B)$ except that in subjective logic the opinion can additionally represent ignorance. Then, for the resulting set of frames the reputational opinion about the distance opinions is calculated as described in Section 6.4.2. Each result is assigned to a new opinion with the predicate of the appropriate arity and is assigned the name of agent with the final belief values. In this case, the final opinion value represents that there is an opinion about two persons being potential witnesses of each other from an agent named $Rule1$. The second Rule 6.3 is almost same as Rule 6.2. The only different part of this rule is that the reputational opinion is about direction. The third Rule 6.4 combines the evidences coming from Rule 6.2 and (6.3). The conjunction operator $\wedge$ is used to reflect that for reliable positive resulting opinions both evidences should have appeared with a certain amount of belief. The last Rule 6.5 is about accumulating the belief over time using the consensus operator $\oplus$ [94] (see Definition 30). Figure 6.5 shows a graphical representation of the rules in a tree form.

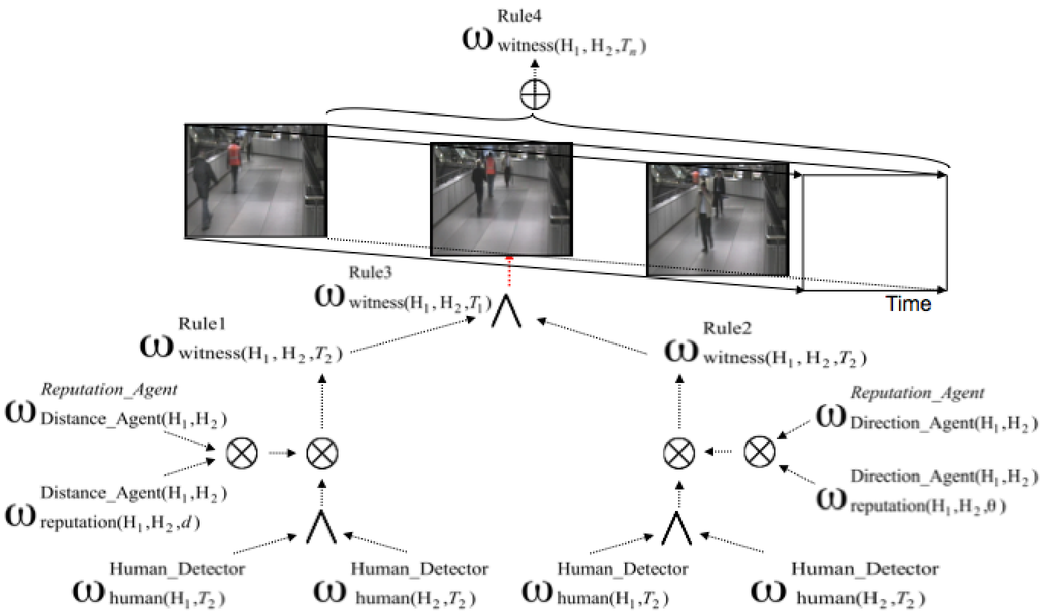

Figure 6.5: Tree Representation of used Rules.

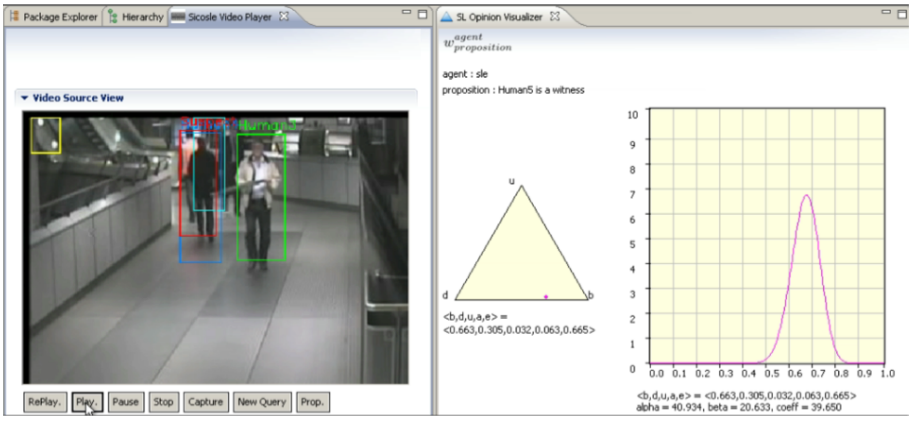

Figure 6.6: Visualization of the Experiment.

6.4.4 Experimental Result

Using the rules described in Section 6.4.3, we calculated subjective opinions between a person marked as suspect and other human instances over time. Figure 6.6 shows a snapshot of the visualization in the prototype comprising a video player and an opinion visualizer. While the video is being played the corresponding metadata is transformed into the corresponding opinion representation. The translated opinions are fed into the rule-engine

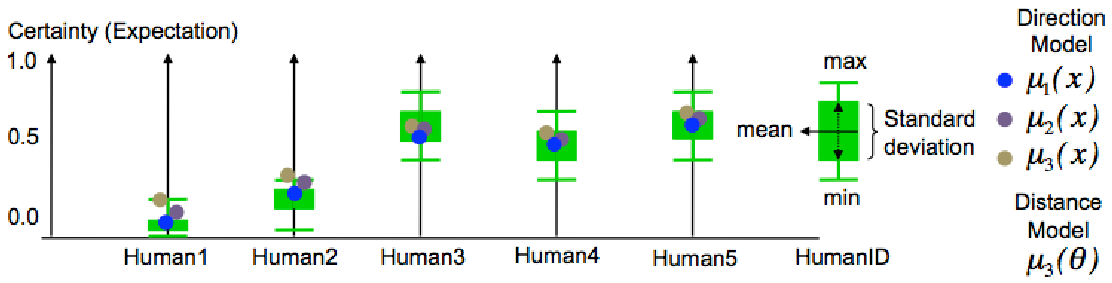

Figure 6.7: Experimental Result.

which automatically evaluates the rules. The right part of Figure 6.6 shows the opinion about the proposition *'human 5 is a witness for the suspects'* marked red and its corresponding mapping to beta distribution. For verification of these results, a questionnaire was prepared to collect scores about the witnessing chances for each of the 'pairs' in the scene (e.g. human1 and suspect, human2 and suspect , etc). 7 people from our lab took part in the questionnaire. Then changing the uncertainty functions on vague rules, we tested the behavior of the proposed approach to check whether it well models human intuition. Although there can be 9 possible combinations of uncertainty functions (i.e. 3 distance functions and 3 direction functions), to better contrast the impact of changing such uncertainty functions, we have fixed the direction function to the type of μ_3 defined in Figure 6.3 - B.) and tested with 3 different direction functions shown in Figure 6.3 - A.). Then the mean and standard deviation, *min* and *max* of the 'human opinions' were calculated and compared to the computed results. According to [93], the following criteria should be applied to the computed results.

1) The opinion with the greatest probability expectation is the greatest opinion.

2) The opinion with the least uncertainty is the greatest opinion.

3) The opinion with the least relative atomicity is the greatest opinion.

In the described experiment, due to the small size of possible pairs, only the first criterion was applied and the final expectation values of each opinion for candidate pairs were plotted jointly with the questionnaire based result as shown in Figure 6.7. The final result turns out to be following the tendency of questionnaire based human 'opinions'. The change of uncertainty function seems not introducing that critical differences. However, there were more differences between the expected values, when the final expectation values were low, for instance, though it was a slight differences, μ_3 tend to yield larger expectation value then μ_2 and μ_1. The differences ware smaller when the final expectation values were getting higher. However, in any cases, the order on the ranking of witnesses show the same results. Therefore, in the sense of human like reasoning, it seems that the proposed approach well models human intuition.

6.5 Case Study II

In this section, we further explorer the proposed case study scenario for more complex contextual forensic reasoning. Especially, we will consider the situation that is needed to be modeled in the sense of default reasoning [143] explained in Chapter 5.

6.5.1 Scenario Setting for Case Study II

(a) Witness additional clues (Talking on the Phone)

(b) Witness with additional clues (License Plate and Face Detection)

Figure 6.8: Scenario Setting for Case Study 2.

Let us consider a conceptual scenario that a security personnel wants to get suggestions of most probable witnesses of a selected suspect in a scene. Given an assumption that automatic vision analytics are running and extracting basic semantics, we will assume two virtual situations as shown in Figure 6.8, where, witnesses are reasoned according to the uncertain spatio-temporal rules as demonstrated in Section 6.4. In all situations we will assume that 'witness2' has higher opinion then 'witness1'. In addition to this, we will assume optional cases that additional evidential cues are detected. In Figure 6.8 - A.), 'witness2' is talking on the phone. In Figure 6.8 - B.), the optional case is the detection of a license plate of the car seems to belong to the 'witness1' and 'witness2' comes with face detection.

6.5.2 Reasoning Examples

Given the scenario with optional cases, we will also assume that 1). people usually don't recognize well when they are talking on the phone, 2). identifiable witness is a good witness. 3) License plate is better identifiable source than face detection because we can even fetch personal information of the owner easily. Therefore, under optional assumption, for example, in Figure 6.8 - A.), 'witness1' should be better witness, and in Figure 6.8 - B.), 'witness1' should be suggested as a better witness. This kind of non monotonic reasoning under inconsistent information can also be regarded as *'default reasoning'* [143]. In Chapter 5, we showed that this aspect can be modeled using subjective logic as well under the opinion assignment (see Definition 33) and inference mechanism (see Definition 35) shown in Chapter 5. Here, it is important to note that, unlike the case of vague rule modeling, the type of opinion assignment to prioritize belong to Definition 16 - 2. and the default inference scheme belongs to Definition 16 - 4. As shown in Section 5.8, we set $T \simeq (1, 0, 0)$ (full truth), $DT_1 \simeq (0.5, 0, 0.5)$ (weak default true), $DT_2 \simeq (0.8, 0, 0.2)$ (strong default true), $F \simeq (0, 1, 0)$ (full false), $DF_1 \simeq (0, 0.5, 0.5)$ (weak default false), $DF_2 \simeq (0, 0.8.0.2)$ (strong default false), $* \simeq (0.33, 0.33, 0, 34)$ (contradiction), $U \simeq (0, 0, 1)$ (full uncertainty) and $\perp \simeq (0.5, 0.5, 0)$ (full contradiction) [82, 83]. For the rest of truth values we will use opinion triple representation (b,d,i).

Chapter 6

Example 6. ***(Witness talking on the phone)*** **.** Assume the following set of rules about determining good witness including the uncertain spatio-temporal relation based witness reasoning rule described in Section 6.4.3. Then also assume the following opinion assignment that $witness2$ (denoted as wit_2) has higher opinion being the witness than $witness1$ (denoted as wit_1).

$$\begin{aligned}
\phi_{Rule}[w^{Rule4}_{witness(H_1)} \leftarrow \oplus^{n}_{i=1} w^{Rule3}_{witness(H_1,H_2,T_i)}] &= DT_1 \\
\phi_{Rule}[\neg w_{witness(H_1)} \leftarrow w_{talking_on_phone(H_1)}] &= DT_2 \\
\phi_{RuleEval}[w^{Rule4}_{witness(wit_1)}] &= (0.6, 0.15, 0.25) \\
\phi_{RuleEval}[w^{Rule4}_{witness(wit_2)}] &= (0.7, 0.10, 0.20)
\end{aligned}$$

Given two default true and default false rules and facts that can be seen as definite true, the inference for reasoning better witness using default logic with subjective logic is as follows.

$$\begin{aligned}
cl_{sl_{di}} \quad & (\phi)(w_{witness(wit_1)}) = [U \sqcup ((0.6, 0.15, 0.25) \cdot DT_1)] \\
& = [U \sqcup (0.44, 0.15, 0.41)] = (0.44, 0.15, 0.41) \sim (Expectation = 0.54) \\
cl_{sl_{di}} \quad & (\phi)(w_{witness(wit_2)}) = [U \sqcup ((0.7, 0.10, 0.20) \cdot DT_1)] \\
& = [U \sqcup (0.50, 0.10, 0.40)] = (0.50, 0.10, 0.40) \sim (Expectation = 0.60)
\end{aligned}$$

Above result shows that given the weak rules, ‘witness2’ is more probable witness candidate than ‘witness1’. Then, let us consider the weak opinion assignment to the additional contextual cue that witness2 is using the phone. This semantics can be interpreted as ‘the witness seems to using a phone but not quite sure’.

$$\phi_{fact}[w_{talking\_on\_phone(wit\_2)}] \quad = \quad (0.6, 0.15, 0.25)$$

Given the additional information, the inference on witness2 is being witness is as follows.

$$\begin{aligned}
cl_{sl_{di}} \quad & (\phi)(w_{witness(wit_2)}) \\
& = [U \sqcup ((0.7, 0.10, 0.20) \cdot DT_1)] \oplus \neg[U \sqcup ((0.6, 0.15, 0.25) \cdot DT_2)] \\
& = [U \sqcup (0.50, 0.10, 0.40)] \oplus \neg[U \sqcup (0.59, 0.15, 0.26)] \\
& = (0.50, 0.10, 0.40) \oplus \neg(0.59, 0.15, 0.26) \\
& = (0.50, 0.10, 0.40) \oplus (0.15, 0.59, 0.26) \\
& = (0.34, 0.47, 0.19) \sim (Expectation = 0.39)
\end{aligned}$$

The resulting opinion (0.34,0.47,0.19) on witness2’s being a good witness now weaker than (0.44,0.15,0.41) which is for the case of witness1’s being a good witness. The expectation values also captures this aspect. Thus, this result shows that the inference scheme well models human intuition.

Example 7. ***(Witness with Face Detection vs. License Plate Detection) .*** Consider the following set of rules about determining good witness and the following opinion assignment to capture the scenario described in Section 6.5.1 as depicted in Figure 6.8.

$$\begin{aligned}
\phi_{Rule}[w^{Rule4}_{witness(H_1)} \leftarrow \oplus^{n}_{i=1} w^{Rule3}_{witness(H_1,H_2,T_i)}] &= DT_1 \\
\phi_{Rule}[w_{witness(H_1)} \leftarrow w^{Rule4}_{witness(H_1)} \cdot w_{hasFaceDetectInfo(H_1)}] &= DT_1 \\
\phi_{Rule}[w_{witness(H_1)} \leftarrow w^{Rule4}_{witness(H_1)} \cdot w_{hasLicenseDetectInfo(H_1)}] &= DT_2 \\
\phi_{RuleEval}[w^{Rule4}_{witness(wit_1)}] &= (0.6, 0.15, 0.25) \\
\phi_{RuleEval}[w^{Rule4}_{witness(wit_2)}] &= (0.7, 0.10, 0.20) \\
\phi_{fact}[w_{hasLicenseDetectInfo(wit_1)}] &= (0.6, 0.15, 0.25) \\
\phi_{fact}[w_{hasFaceDetectInfo(wit_2)}] &= (0.6, 0.15, 0.25)
\end{aligned}$$

Given two default true and default false rules and facts that can be seen as definite true, the inference for reasoning better witness using default logic with subjective logic is as follows.

$$\begin{aligned}
cl_{sl_{di}}(\phi)(w_{witness(wit_1)}) &= [U \sqcup ((0.6, 0.15, 0.25) \cdot DT_1 \cdot (0.6, 0.15, 0.25) \cdot DT_2)] \\
&= [U \sqcup ((0.44, 0.15, 0.41) \cdot (0.59, 0.15, 0.26))] \\
&= (0.33, 0.28, 0.39) \sim (Expectation = 0.36) \\
cl_{sl_{di}}(\phi)(w_{witness(wit_2)}) &= [U \sqcup ((0.7, 0.10, 0.20) \cdot DT_1 \cdot (0.6, 0.15, 0.25) \cdot DT_1)] \\
&= [U \sqcup ((0.5, 0.1, 0.4) \cdot (0.44, 0.15, 0.41))] \\
&= (0.3, 0.24, 0.47) \sim (Expectation = 0.33)
\end{aligned}$$

Above result shows that given the evidences, 'witness2' is slightly more probable witness candidate than 'witness1' because license plate info is more informative thereby strongly considered than face related information by the opinion assignment. However, due to the opinion on the fact level is not certain, the values were not strongly forced the belief but rather increased the uncertainty in the final opinion. The expectation values also captures this aspect. Thus, this result show that the inference scheme well models human intuition.

6.6 Chapter Summary

In summary of lessons learned in this chapter, the proposed approach on dynamic assessment of a vague proposition offers more choices to model complex contextual human knowledge by enriching the expressive power of the framework. The proposed approach can be used with another principled reasoning scheme such as default reasoning. There are, however, still open issues on automatic assignment of proper priors and proper modeling of the reputation function, etc.

Intelligent forensic reasoning upon metadata acquired from automated vision analytic modules is an important aspect of surveillance systems with high usage potential. The knowledge expressive power of the reasoning framework and the ability of uncertainty handling are critical issues in such systems. In this chapter, based on our previous work on the use of logic programming with subjective logic, we extended the framework so that it can also handle vague propositional rules. The approach is mainly based on the fuzzy-like membership function and the reputational operation on it. The main advantage of the proposed approach is that it offers more choices to model complex contextual human knowledge by enriching the expressive power of the framework. The other advantage of the proposed approach is that the modeled vague rules can be used with another principled reasoning scheme. In this chapter, especially, we have demonstrated how the reasoning results from uncertain spatio-temporal rules could be used with default reasoning. Another interesting properties of the system is that, unlike traditional probability based conditional reasoning, this approach allows for representing lack of information about a proposition. We could also roughly assign our subjective priors with lack of information, and observations can also be represented with any degree of ignorance, therefore we believe this better reflects human intuition and real world situations. Another beneficial property is the flexibility of assigning opinions to formulae. Especially, rule can embed its own opinion calculation scheme thereby, allows for sophisticated propagation of opinions through the inference pipeline. There are, however, still several open issues such as how to better model the reputational function, how to automatically assign proper prior opinions to rules, etc. Although we still need to extend this concept to large scale data. We advocate that this work showed the potential of the proposed approach.

7 Hybrid Knowledge Modeling and Reasoning

Thus far, our discussions have been focused on the *'extensional'* aspect of knowledge representation and uncertainty handling. As reviewed in Section 2.3.3, while *'extensional'* approach has benefits on *'flexibility'* and *'expressive power'* due to the *'modularity'*, it also has some deficiencies such as *'improper handling of bidirectional inference'* that can be better handled in *'intensional'* approaches. Therefore, it would be worth if we could take benefits of both approaches in a single framework. In this chapter, we focus on the extension of our proposed reasoning framework to bestow *'intensional'* characteristics upon our proposed *'extensional'* system, especially in focus on enabling *'bidirectional inference'*.

7.1 Introduction

In forensic analysis of visual surveillance data, conditional knowledge representation and inference under uncertainty play an important role for deriving new contextual cues by fusing relevant evidential patterns. To address this aspect, both rule-based (aka. *extensional*) and state based (aka. *intensional*) approaches have been adopted for situation or visual event analysis. The former provides flexible expressive power and computational efficiency but typically allows only one directional inference. The latter is computationally expensive but allows bidirectional interpretation of conditionals by treating antecedent and consequent of conditionals as mutually relevant states (see Section 2.4 for details). In visual surveillance, considering the varying semantics and potentially ambiguous causality in conditionals, it would be useful to combine the expressive power of rule-based system with the ability of bidirectional interpretation. In this chapter, we propose a hybrid approach that, while relying mainly on a rule-based architecture, also provides an intensional way of on-demand conditional modeling using conditional operators in subjective logic. We first show how conditionals can be assessed via explicit representation of ignorance in subjective logic. We then describe the proposed hybrid conditional handling framework. Finally, we present several experimental case studies from a typical public domain visual surveillance scenes.

7.2 Background and Motivation

In recent years, there has been an increasing research focus on higher level semantic reasoning for visual surveillance data by augmenting low level computer vision modules with high level contextual cues (see Section 1.1 for detailed background on this). Considering the variety of possible semantics in surveillance scenes, arising from complex spatio-temporal events, intelligent high level semantic reasoning should provide a means of fusing evidence from multiple, *'uncertain'*, *'incomplete'* and potentially *'contradictory'* information sources. The key challenges for such high level reasoning approaches are the choice of an appropriate contextual knowledge representation and the proper handling of uncertainty.

As we have reviewed in Section 2.3 and Section 2.4, depending on how such approaches handle uncertainty, they can be roughly categorized into *'extensional'* and *'intensional'* approaches [130]. Extensional approaches also known as rule-based systems treat uncertainty as a generalized truth value attached to formulas and compute the uncertainty of any formula as a function of the uncertainties of its sub formulas. In intensional approaches, also known as state based approaches, antecedents and consequents in conditionals are treated as *'subsets of possible states'* and handle uncertainty taking into account relevance between the two states. Extensional approaches have advantages in the *'flexibility'* and *'expressive powe'* due to their ability to derive a new proposition based only on what is currently known (a) regardless of anything else in the knowledge base (*'locality'*) and (b) regardless of how the current knowledge was derived (*'detachment'*). '*locality* and *detachment*' are together referenced to as '*modularity*' [130].

Extensional approaches however, are considered semantically less clear compared to intensional approaches. This deficit of semantic clarity comes from how conditionals are interpreted. In rule-based frameworks, conditionals are interpreted by 'Modus Ponens' [1]. Therefore in such systems, typically, reverse interpretation (also known as abduction) is not possible. Although it is possible to explicitly add reverse conditionals, this would introduce cycles that can adversely impact the reasoning mechanism [130]. Another problem with traditional rule-based frameworks is the quantitative uncertainty assessment function of conditional formulas. Due to the inference direction and modularity, the uncertainty propagation is limited to one direction and, thus hinders considering relations between antecedent and consequent in conditional formulas. This is in contrast to traditional Bayesian reasoning that focusses on the probabilistic relation between antecedent and consequent. In such intensional approaches, the uncertainty of any formula is computed by combining possible worlds via set theory operations. Such approaches exhibit neither the property of locality nor detachment. In other words, they are unable to derive new information unless they have exhaustively accounted for all other information sources in the knowledge base, that could possibly influence the final proposition to be reasoned about. While this process provides better semantic clarity, it comes with an exponential computational cost. Another property intensional approaches buy us is the ability to perform *'bi-directional inference'*. Since intensional approaches possess complete knowledge of all information sources influencing all variables of interest, it is possible to ask arbitrary queries of the model without apriori committing to one direction of inference unlike in extensional appraoches.

In visual surveillance, there are cases that antecedent and consequent appear to be related but the causality direction is ambiguous due to the complexity and variety of semantics. Thus, it would be desirable to combine the two approaches to achieve more context modeling power and computational efficiency. In this chapter, we present a hybrid approach to conditional evidence fusion that leverages the avantages of both extensional

[1] The rule of deduction (given 'If P then Q' infer 'P therefore Q').

and intensional approaches using subjective logic. Subjective logic theory provides operators for both abduction as well as deduction [100, 95, 140] thereby, making it possible to interpret conditionals bidirectionally, in a Bayesian sense.

7.3 Conditional Operators in Subjective Logic

In this section, we summarize the derivation of conditional operators in subjective logic, as presented by Jøsang et al, in [100, 95, 140]. Given a conditional of the form '$y \leftarrow x$' we interpret the meaning as *'IF x THEN y'*. In traditional binary logic, this is seen in a truth-functional manner following the truth table called *'material implication'* such that any antecedent x being true forces y to be evaluated as true as well. However, in practice there are examples of false conditionals with false antecedent and true consequent. Therefore, the more natural interpretation is that *'The truth value of x is relevant to the truth value of y'*. In the sense of relevance connection between x and y, the interpretation should also consider the case '$y \leftarrow \neg x$' for completeness, so we can properly handle the case that the antecedent x is false. Following classical probability theory [159], a conditional of the form '$y \leftarrow x$' can be interpreted in terms of probability calculus and can be expressed via binomial conditional deduction as:

$$
\begin{aligned}
&p(y||x) = p(x)p(y|x) + p(\overline{x})p(y|\overline{x}) \\
&\textit{where the terms are defined as follows:} \\
&p(y|x) : \textit{the conditional probability of } y \text{ given } x \textit{ is TRUE} \\
&p(y|\overline{x}) : \textit{the conditional probability of } y \textit{given } x \textit{ is FALSE} \\
&p(x) : \textit{the probability of the antecedent } x \\
&p(\overline{x}) : \textit{the probability of the antecedent's complement} \\
&p(y||x) : \textit{the deduced probability of the consequent } y
\end{aligned}
\tag{7.1}
$$

Note that, the notation $p(y||x)$ is meant to indicate that the truth value of y is evaluated with both the positive and the negative conditionals. This notation is only meaningful in a probabilistic sense. In practice, however, $p(y|x)$ or $p(y|\overline{x})$ often cannot be determined directly. In such cases, the required conditionals can be correctly derived by inverting the available conditionals if we know the base rate $p(y)$ (that is denoted as $a(y)$ in subjective logic), using Bayes' theorem as follows:

$$p(y|x) = \frac{a(y)p(x|y)}{a(y)p(x|y) + a(\overline{y})p(x|\overline{y})} \tag{7.2}$$

In subjective logic, $a(y)$ is interpreted as a subjective opinion $w_y(0, 0, 1, a_y)$. Such an opinion having full ignorance is called *'vacuous'* opinion. It represents a measure of prior knowledge. Similarly, $p(y|\overline{x})$ can be derived as follows:

$$p(y|\overline{x}) \;=\; \frac{a(y)p(\overline{x}|y)}{a(y)p(\overline{x}|y) + a(\overline{y})p(\overline{x}|\overline{y})} = \frac{a(y)(1 - p(x|y))}{a(y)(1 - p(x|y)) + a(\overline{y})(1 - p(x|\overline{y}))} \tag{7.3}$$

Let the term *'child frame'* denote a traditional state space of mutually disjoint states and let the term *'parent frame'* denote the evidence that was obtained. Bearing the notation $p(\text{consequence}|\text{antecedent})$ in mind, if the *'parent frame'* is about the antecedent and the *'child frame'* is the consequent, then such inference is called *'deduction'*. Abductive reasoning is the case when the *'parent frame'* is the consequent and *'child frame'* is the antecedent as described in [95]. In subjective logic, conditionals can be generalized to multinomial frames. However, in this section, we will focus on the binomial case only. Deduction on binomial frames is as follows.

Definition 42. ***(Deduction)*** [100] **.** *Let $\Theta_X = \{x, \overline{x}\}$ and $\Theta_Y = \{y, \overline{y}\}$ be two frames with arbitrary mutual dependency. Let $w_x = (b_x, d_x, i_x, a_x)$, $w_{y|x} = (b_{y|x}, d_{y|x}, i_{y|x}, a_{y|x})$ and $w_{y|\overline{x}} = (b_{y|\overline{x}}, d_{y|\overline{x}}, i_{y|\overline{x}}, a_{y|\overline{x}})$ be an agent's respective opinions about x being true, about y being true given x is true and about y being true given x is false, then deductive conditional opinion $w_{y||x} = (b_{y||x}, d_{y||x}, i_{y||x}, a_{y||x})$ is expressed as :*

$$w_{y||x} = w_x \circledcirc (w_{y|x}, w_{y|\overline{x}}) \tag{7.4}$$

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where $\circledcirc$ *denotes the general conditional deduction operator for subjective opinions and,* $w_{y||x}$ *is defined by*

$$\begin{cases} b_{y||x} = b^I_y - a_y K \\ d_{y||x} = d^I_y - (1-a_y)K \\ i_{y||x} = i^I_y + K \\ a_{y||x} = a_y \end{cases} \quad where, \begin{cases} b^I_y = b_x b_{y|x} + d_x b_{y|\overline{x}} + i_x(b_{y|x}a_x + b_{y|\overline{x}}(1-a_x)) \\ d^I_y = b_x d_{y|x} + d_x d_{y|\overline{x}} + i_x(d_{y|x}a_x + d_{y|\overline{x}}(1-a_x)) \\ i^I_y = b_x i_{y|x} + d_x i_{y|\overline{x}} + i_x(i_{y|x}a_x + i_{y|\overline{x}}(1-a_x)) \end{cases}$$

and K *can be determined according to 3 different selection criteria detailed in* [100].

Abduction on binomial frames is as follows.

Definition 43. ***(Abduction)*** [140] **.** *Let* $\Theta_X = \{x, \overline{x}\}$ *and* $\Theta_Y = \{y, \overline{y}\}$ *be two frames with arbitrary mutual dependency. Let* $w_y = (b_y, d_y, i_y, a_y)$, $w^{vac}_x = (0, 0, 1, a_x)$, $w_{y|x} = (b_{y|x}, d_{y|x}, i_{y|x}, a_{y|x})$ *and* $w_{y|\overline{x}} = (b_{y|\overline{x}}, d_{y|\overline{x}}, i_{y|\overline{x}}, a_{y|\overline{x}})$ *be an agent's respective opinions about observed consequent* y *being true, vacous subjective opinion about the base rate of the hypothesis* x *,about* y *being true given* x *is true, and about* y *being true given* x *is false, then abductive conditional opinion* $w_{x\overline{||}y} = (b_{x\overline{||}y}, d_{x\overline{||}y}, i_{x\overline{||}y}, a_{x\overline{||}y})$ *about* x *being cause of observed consequent* y *is expressed as :*

$$w_{x\overline{||}y} = w_y \overline{\circledcirc}(w_{y|x}, w_{y|\overline{x}}, w^{vac}_x) = w_y \circledcirc (w_{x|y}, w_{x|\overline{y}}) \tag{7.5}$$

where $\overline{\circledcirc}$ *denotes the general conditional abduction operator for subjective opinions, then the inverted conditionals* $w_{x|y}$,$w_{x|\overline{y}}$ *in the right hand side of the equation can be derived using the following formula,*

$$\begin{aligned} w_{x|y} &= \frac{w^{vac}_x \cdot w_{y|x}}{w^{vac}_x \circledcirc (w_{y|x}, w_{y|\overline{x}})} \\ w_{x|\overline{y}} &= \frac{w^{vac}_x \cdot \neg w_{y|x}}{w^{vac}_x \circledcirc (\neg w_{y|x}, \neg w_{y|\overline{x}})} \end{aligned} \tag{7.6}$$

thereby, we can calculate the $\overline{\circledcirc}$ *operation with the* $\circledcirc$ *operator and inverted conditionals.*

Note that Equation 7.6 involves multiplication and division operators as well as deduction operator $\circledcirc$ (see Equation 7.4 and Table 3.2 in Definition 43). The multiplication in subjective logic is also called *'conjunction'* and the formal definition is shown in Definition 31. The inverse operation to multiplication is division. The quotient of opinions about propositions x and y represents the opinion about a proposition z which is independent of y such that $w_x = w_{y \wedge z}$. The formal definition of division operator is as follows.

Definition 44. ***(Division /)*** [97] **.** *Let Θ_X and Θ_Y be two frames and let x and y be propositions about state in Θ_X and Θ_Y respectively. Let $w_x = (b_x, d_x, i_x, a_x)$ and $w_y = (b_y, d_y, i_y, a_y)$ be an agent's opinions about x and y, then division opinion denoted as w_x/w_y is $w_{x \overline{\wedge} y} = (b_{x \overline{\wedge} y}, d_{x \overline{\wedge} y}, i_{x \overline{\wedge} y}, a_{x \overline{\wedge} y})$ such that :*

Defined only when $a_x \leq a_y,\ d_x \geq d_y, b_x \geq \frac{a_x(1-a_y)(1-d_x)b_y}{(1-a_x)a_y(1-dy)}$ *and* $i_x \geq \frac{(1-a_y)(1-d_x)i_y}{(1-a_x)(1-dy)}$

$$\begin{cases} \text{if } a_x < a_y, \\ b_{x \overline{\wedge} y} = \dfrac{a_y(b_x + a_x i_x)}{(a_y - a_x)(b_y + a_y i_y)} - \dfrac{a_x(1-d_x)}{(a_y - a_x)(1-d_y)} \\ d_{x \overline{\wedge} y} = \dfrac{d_x - d_y}{1 - d_y} \\ i_{x \overline{\wedge} y} = \dfrac{a_y(1-d_x)}{(a_y - a_x)(1-d_y)} - \dfrac{a_y(b_x + a_x i_x)}{(a_y - a_x)(b_y + a_y i_y)} \\ a_{x \overline{\wedge} y} = a_x / a_y \end{cases} \qquad \begin{cases} \text{if } 0 < a_x = a_y, \\ b_{x \overline{\wedge} y} = \dfrac{\gamma(1-d_x)}{1-d_y} \\ d_{x \overline{\wedge} y} = \dfrac{d_x - d_y}{1 - d_y} \\ i_{x \overline{\wedge} y} = \dfrac{(1-\gamma)(1-d_x)}{1-d_y} \\ a_{x \overline{\wedge} y} = a_x / a_y \end{cases}$$

$$\textit{where}, \gamma = \frac{a_y(1-a_y)}{(a_y - a_x)(b_y + a_y i_y)} \left(\frac{(1-d_y)b_x}{1-d_x} - b_y \right) + \frac{b_y}{b_y + a_y i_y}.$$

Refer to [97] for the detailed explanation on division operators.

7.4 Proposed Approach

The proposed hybrid conditional handling framework mainly relies on rule-based system that enables logic programming proposed in Section 4.3.2. The rule-based system is extended to allow representation of rules using subjective opinions and operators. The conditional knowledge directly encoded as such rules are handled in extensional manner. To handle conditionals in intensional manner, special types of predefined rules are introduced. Such predefined rules drive the abduction and deduction operation in the subjective logic framework in the presence of required prior opinions for the conditionals. We will first give a brief overview how rules are expressed in logic programming. Thereafter, comes with further details of the framework.

7.4.1 Extensional Layer

In Section 4.3.2, we proposed a logic programming framework extended with subjective logic. In this section, we will briefly explain the framework again. In the proposed framework, the CLIPS [1] rule engine was used as a basis to provide flexibility for defining complex data structure as well as for providing a rule resolving mechanism. To extend this system, a data structure '$opinion(agent, proposition, b, d, i, a)$' was defined that can be interpreted as a fact of arity 6 with the following terms, *agent (opinion owner)*, *proposition*, *belief*, *disbelief*, *ignorance*, and *atomicity*. To represent propositions, we extended the structure so that it can take arity n properties as well. Therefore, given a predicate p the proposition can be described as '$p(a_1, a_2, ..., a_n)$'. In our system, therefore, each fact is represented as the form of '$w^{agent}_{p(a_1,a_2,...,a_n)}$'. Namely, rules are defined with the opinion and proposition structure. Additionally, functions of subjective logic operators taking opinions as parameters were defined. Since in rule-based systems, actions can be executed in the head part of the rule, the uncertainty assessment '$\phi : rules \rightarrow opinion$' operation, defined in Definition 16, can be defined in the head part of rules using subjective logic operators and opinions shown in rule body. This aspect is depicted in Section 4.3.2 as follows :

$$
\begin{aligned}
&\textit{Rule Head (ACTION) :}\\
&\qquad\qquad \textit{Assert new Opinion } w^{a_c}_{p_c(a_{c1},..,a_{cn})},\\
&\qquad\qquad \text{where } w^{a_c}_{p_c(a_{c1},..,a_{cn})} = w^{a1}_{p1(a_{11},..,a_{1n})} \circledast .. \circledast w^{ai}_{pn(a_{i1},..,a_{in})}\\
&\leftarrow\\
&\textit{Rule Body :}\\
&\qquad\qquad w^{a1}_{p1(a_{11},..,a_{1n})}, .., w^{ai}_{pn(a_{i1},..,a_{in})}
\end{aligned}
\tag{4.1}
$$

Due to the redundancy that arises when describing rules at the opinion structure level, we will use abbreviated rule formulae as follows :

$$w^{a_c}_{p_c(a_{c1},..,a_{cn})} \leftarrow w^{a1}_{p_1(a_{11},..,a_{1n})} \circledast ... \circledast w^{ai}_{p_n(a_{i1},..,a_{in})} \tag{4.2}$$

where $\circledast$ indicates one of subjective logic's operators. More concretely, this implies checking for the existence of each opinion that matches property constraints among the propositions of opinions. In general, rules in the form of Equation 4.2 can also be seen as conditionals. Thus, this way of representing rules and handling uncertainty can be considered as extensional approach and in our framework such rules represent the extensional layer.

7.4.2 Intensional Layer

In the intensional layer, a conditional proposition '$y \leftarrow x$' is viewed in the sense of relevance connection between x and y. To generalize this view, we will consider x and y as two facts with arbitrary mutual dependency without restricting the influence direction. Then we can set two conditionals '$y \leftarrow x$' and '$x \leftarrow y$'. This results in 4 possible conditional operations as shown in Table 7.1, namely, deduction and abduction for each conditionals. In the sense of querying for the cases in Table 7.1, Case 1 can be interpreted as *'given an observational opinion x give me an opinion that this would make y happen'*. Case 2 means *'given an observational opinion y give me an opinion that it was caused by x'*. Case 3 is *'given an observational opinion y give me an opinion that this would make x happen'*. Finally, Case 4 says *'given an observational opinion x give me an opinion that it was caused by y'*. As shown in Section 7.3, the calculation of those operations requires incidental opinions (i.e. priors) also. According to Equation 7.4 and Equation 7.5 $w_{x\overline{||}y} = w_y \overline{\circledcirc}(w_{y|x}, w_{y|\overline{x}}, w_x^{vac}) = w_y \circledcirc (w_{x|y}, w_{x|\overline{y}}) = w_{x||y}$ by getting inverse opinions using Equation 7.6. Thus, Case 2 and Case 3 are equal. Similarly, Case 1 and Case 4 are also equal. This implies that an abduction on a proposition '$y \leftarrow x$' can be interpreted as a deduction on a proposition '$x \leftarrow y$'. Thus, given a single query shown in Table 7.1 we have two choices of conducting this conditional depending on available priors. Therefore, a decision making process, examining the availability of required priors in the rule and fact

		Query	Observation		Operator	Priors					
			w_x	w_y		$w_{x\|y}$	$w_{x\|\overline{y}}$	$w_{y\|x}$	$w_{y\|\overline{x}}$	w_x^{vac}	w_y^{vac}
$y \leftarrow x$	Case 1	$w_{y\|\|x}$	$\surd$		$\circledcirc$			$\surd$	$\surd$		
	Case 2	$w_{x\overline{\|\|}y}$		$\surd$	$\overline{\circledcirc}$			$\surd$	$\surd$	$\surd$	
$x \leftarrow y$	Case 3	$w_{x\|\|y}$		$\surd$	$\circledcirc$	$\surd$	$\surd$				
	Case 4	$w_{y\overline{\|\|}x}$	$\surd$		$\overline{\circledcirc}$	$\surd$	$\surd$				$\surd$

Table 7.1: Possible Subjective Conditional Operations given x and y with Arbitrary Mutual Dependency.

base, can be derived in form of a deterministic conjunctive logic program. Following this scheme, for example, Case 1 corresponds to the rule

$$Calculate\ w_{y||x} \leftarrow w_x, w_{y|x}, w_{y|\overline{x}} \tag{7.7}$$

saying if there exist three facts w_x, $w_{y|x}$ and $w_{y|\overline{x}}$ calculate $w_{y||x}$. Following Equation 4.2, the rule for Case 1 (i.e., Equation 7.7) will just be denoted as follows.

$$w_{y||x} \leftarrow w_x \circledcirc (w_{y|x}, w_{y|\overline{x}}) \tag{7.8}$$

Other rules corresponding to the rest of the Cases 2 - 4 can be derived similarly. Note that such rules are actually not about the context itself but about a mechanism how the conditionals need to be handled and, therefore, those rules should be considered as meta-rules. We consider those rules being part of the *intensional layer*. The CLIPS rule engine provides prioritization of rules. By adjusting priorities of the meta-level rules and other extensional rules, we can decide when the meta-level rules should be triggered. For example, we could first apply the top priority group of extensional rules to infer occurrence of some evidences that may be used for intensional layer. Then, we could intensionally set meta-level rules to be the second priority group. Setting lowest priority group of extensional rules, the inference result from intensional layer could be also used for extensional way of inference. This way, the proposed approach can provide a combination of *'extensional'* and *'intensional'* approaches.

7.5 Case Study I

7.5.1 Scenario Setting for Case Study I

Figure 7.1: Airport Surveillance Scenes for Case Study.

This section describes a case study for the application of forensic conditional reasoning using the proposed hybrid framework on metadata acquired using low level computer vision analytics. Low level video analytic modules generate a rich set of metadata upon which the proposed hybrid reasoning can be performed to detect more complex events. This metadata captures the output of modules that detect 'humans' at a particular location and the time, modules that detect humans passing, a pre-defined region etc. A detailed description of these modules is outside the scope of this chapter. Figure 7.1 shows frames from a video sequence captured from a visual surveillance system observing a typical airport scene. We also have access to information such as pre-annotated zones of stairs, escalator, ATM machines, etc. Now assume we need to query this video sequence to detect all time instances when the escalator was non-functional. Please note, we do not have any low-level computer vision module that can directly detect such an event. We wish to formulate rules to detect this event based on circumstantial evidence of how the behavior of the humans in the scene changes in response to the escalator not working.

In this case study we will show how we encode rules in the proposed hybrid reasoning framework to capture our knowledge of how we expect people behavior to change in response to this event, and further show how we can use this knowledge to reason about instances when the escalator is not-functional, given certain observations. Consider the following rules :

Conditional Knowledge Model - Case1

- Rule 1 - *"if escalator is working, people usually use escalator"* or
- Rule 2 - *"if escalator is not working, people usually use stairs"*.

Above knowledge can be seen as rules such that

'*people use escalator* $\leftarrow$ *escalator working*' or

'*people use stairs* $\leftarrow \neg$ *escalator working*'.

At the same time we could also think about rather deductive knowledge as follows:

Conditional Knowledge Model - Case2

- Rule 3 - *"if people use escalator, usually escalator is working"* or
- Rule 4 - *"if people use stairs, usually escalator is not working"*

Above knowledge can be seen as rules such that

'*escalator working* $\leftarrow$ *people use escalator*' or

'$\neg$*escalator working* $\leftarrow$ *people use stairs*'

In this scenario, knowledge about when people use the escalator or the stairs is essential. Therefore, based on the output of low level vision modules, we need to derive rules to reason about the tendency of *'people use escalator'* or *'people use stairs'*. Consider the following rules:

Observational Knowledge Model

- Rule 5 - *"if a person is on escalator, he/she is using escalator "*
- Rule 6 - *"if a person is on stairs, he/she is using stairs "*

Above knowledge can be seen as rules such that

'*he/she uses escalator* $\leftarrow$ *person on escalator zone*' or

'*he/she uses stairs* $\leftarrow$ *person on stairs zone*'

These rules capture our assumption that humans tend to always be in a state of 'action' when interacting with certain structures within a building. Therefore, when a low level vision module detects a human, at a particular time instant, on a structure such as stairs or escalator, we assume that the human is in the process of 'using' it to go somewhere. Once such an evidence is built, we can assess the plausibility (opinion in Subjective Logic) about escalator's working or not using either Case 1 (abductive rules) or Case 2 (deductive rules) type of rules.

7.5.2 Rule Modeling

This section deals with rule modeling for the scenario, based on the system setting described in Section 7.5.1. In both conditional knowledge models shown in Section 7.5.1, the observation about people use escalator or stairs serves as basis for the assessment. Therefore, we first start with observational knowledge model described in Section 7.5.1. Assuming opinions about observational metadata from human detection and annotation, we will use opinions such as $w^{HumanDetector}_{human(H_i,Loc_{H_i},T_j)}$, $w^{Annotation}_{escalator(E_1,Loc_{E_1},T_j)}$, $w^{Annotation}_{stairs(S_1,Loc_{S_1},T_j)}$, etc (where for example, the first notation says, there is an opinion from $HumanDetector$ about a proposition *human* (i.e., a human exists, more concretely) with id H_i and localization information Loc_{H_i} at time T_j). Depending on whether or not a human is 'within' a pre-annotated zone $Zone_k$ in the scene, low level modules also generate an opinion of the form $w^{GeometryAgent}_{within(H_i,Zone_k,T_j)}$. We combine these opinions with the conjunction operator $\wedge$ [93, 97] of subjective logic as follows.

$$w_{a\_human\_uses\_escalator(H_i,T_j)} \leftarrow \\ (w^{HumanDetector}_{human(H_i,Loc_{H_i},T_j)} \wedge w^{Annotation}_{escalator(E_1,Loc_{E_1},T_j)} \wedge w^{GeometryAgent}_{within(H_i,E_1,T_j)}) \tag{7.9}$$

$$w_{a\_human\_uses\_stairs(H_i,T_j)} \leftarrow \\ (w^{HumanDetector}_{human(H_i,Loc_{H_i},T_j)} \wedge w^{Annotation}_{stairs(S_1,Loc_{S_1},T_j)} \wedge w^{GeometryRule}_{within(H_i,S_1,T_j)}) \tag{7.10}$$

While above Rule 7.9 and Rule 7.10 will fire for every single human detection, we are interested in accumulating all the instances of evidences from these rules, to get a single opinion about *'people_use_escalator'*. An opinion from Rule 7.9 or Rule 7.10 is too weak evidence to judge the proposition *'people_use_escalator'*. Therefore, we first apply the *'reputation (discount)'* operator $\otimes$ [99] (see Definition 41) to every single detections as follows.

$$\begin{aligned}
&w^{byH_i}_{people_use_escalator(T_j)} \leftarrow \\
&\qquad (w_{a_human_uses_escalator(H_i,T_j)} \otimes w^{a_human_uses_escalator}_{people_use_escalator}) \\
&w^{byH_i}_{people_use_stairs(T_j)} \leftarrow \\
&\qquad (w_{a_human_uses_stairs(H_i,T_j)} \otimes w^{a_human_uses_stairs}_{people_use_stairs})
\end{aligned} \tag{7.11}$$

Then, we need to fuse each of decisions made from above Rule 7.11. Using consensus operator $\oplus$ [93, 94] in subjective logic (see Definition 30 Section 5.6.3), we will combine both cases of people use escalator or stairs into one observational opinion. The encoded rule is as follows.

$$\begin{aligned}
&w_{people_use_escalator(T_j)} \leftarrow \\
&\qquad (\oplus^{n}_{i=1} w^{byH_i}_{people_use_escalator(T_j)}) \bigoplus (\oplus^{n}_{i=1} \neg w^{byH_i}_{a_people_use_stairs(T_j)})
\end{aligned} \tag{7.12}$$

Rule 7.12 cumulates every single human detection's contribution to the opinion about *'a human uses escalator'*. In the similar way, it also cumulates every single human detection's contribution to the opinion on *'human uses stairs'*. Both cumulated opinions are again fused by consensus operator. In the context that we fuse both positive and negative evidences, it resembles the subjective logic based *'default inference'* (see Definition 35). Furthermore, this is also similar to the binomial observation that deals with number of positive and negative evidences (see Section 3.1.3), but with uncertainty consideration. For the simplicity of illustrative explanation, assume that we had perfect human detector and perfect geometry agent, so every results from Rule 7.9 and Rule 7.10 were definite true.

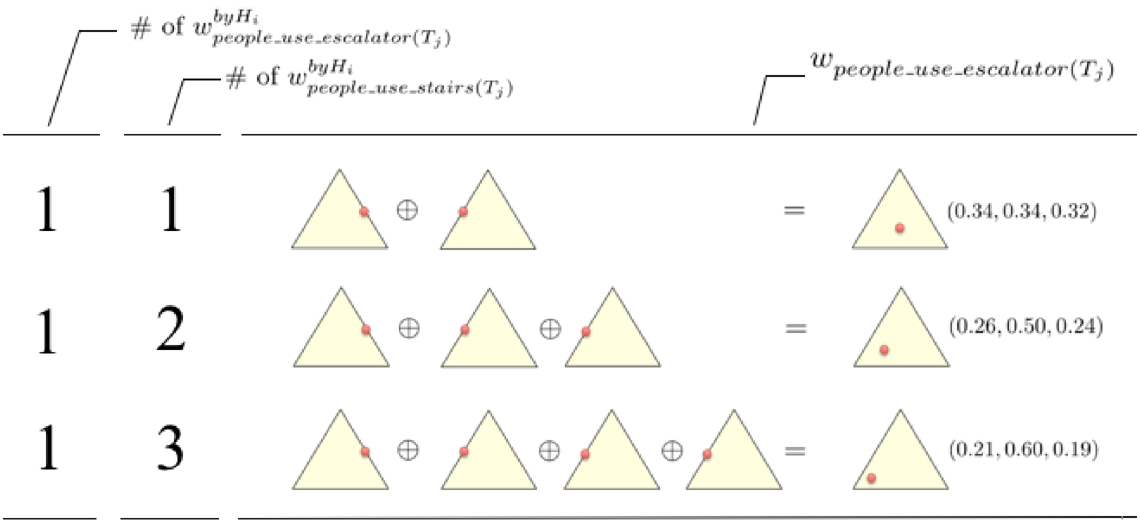

Figure 7.2: Examples of Applying Rule 7.12 based on Different # of Observations.

Setting both the discount opinions as $w^{a\_human\_uses\_escalator}_{people\_use\_escalator} = w^{a\_human\_uses\_stairs}_{people\_use\_stairs} = (0.5, 0, 0.5)$, every single opinion of $w^{byH_i}_{people\_use\_escalator(T_j)}$ and $w^{byH_i}_{people\_use\_stairs(T_j)}$ also becomes $(0.5, 0, 0.5)$. Then, based on this assumption, Figure 7.2 shows examples of calculating opinions about *'people_ use_ escalator'* given different number of observations at a given time T_j.

As explained in Section 7.4.1 these rules belong to *'extensional layer'*. Now, to model the conditional knowledge models from Section 7.5.1 that work at the *'intensional layer'*, we need to collect certain prior opinions. In our case, by setting x as *escalator_working* and y as *people_use_escalator*, we could assign opinions for the items shown below.

- $w_{escalator\_working|people\_use\_escalator}$
- $w_{escalator\_working|\neg people\_use\_escalator}$
- $w_{people\_use\_escalator|escalator\_working}$
- $w_{people\_use\_escalator|\neg escalator\_working}$
- $w^{vac}_{escalator\_working}$
- $w^{vac}_{people\_use\_escalator}$

Once Rule 7.9 $\sim$ Rule 7.12 are triggered, based on available information among above priors and according to Table 7.1, appropriate conditional reasoning rules are dynamically triggered. Although, there exist formal ways of extracting priors from data, we will for this case study, use pre-defined subjective prior opinions for the convenience of our discussion and considering the scope of this chapter (note that, however, in fact the use of such a

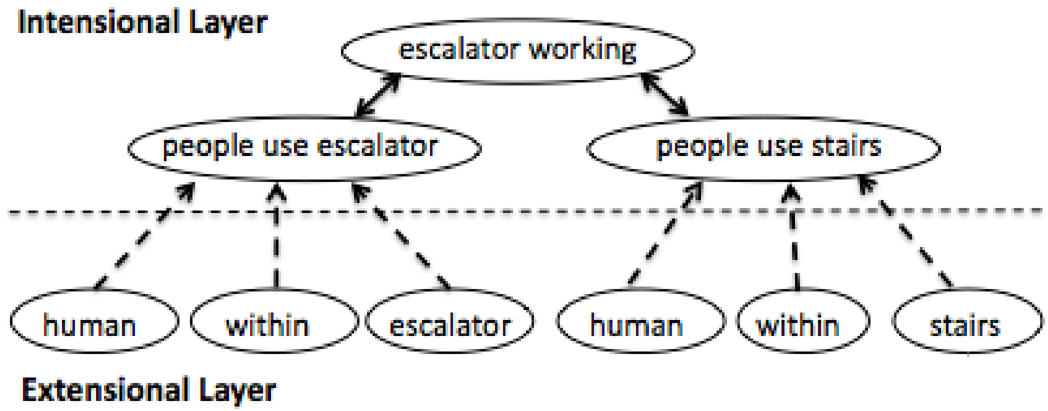

Figure 7.3: Graphical Model of used Conditional Knowledge.

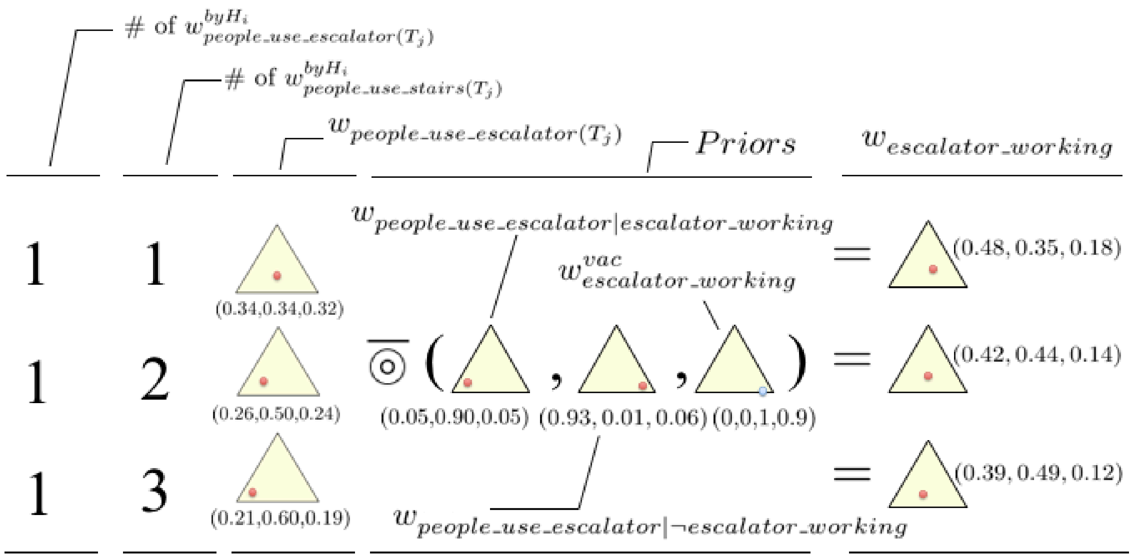

Figure 7.4: Illustration of Abductive Conditional Reasoning for Case Study I (see http://persons.unik.no/josang/sl/Op.html).

subjective opinion also makes sense in the view of subjectivism in Bayesian, that uses subjective priors as discussed in Section 3.1.3, refer to chapters on *'Comparing Bayesian and Frequentist Inferences'* of [33] for further explanation).

Example 8. ***(Is the Escalator Working?)*** **.** In the case of defining usual tendency (base rate) of escalator's working to 90%, opinion about people would use escalator when an escalator is working to (0.93,0.01,0.06) and opinion about people would still use escalator even if an escalator is not working to (0.05,0.9,0.05), following rule will be triggered.

$$\begin{aligned} &w_{escalator\_working(E_1,T_j)\overline{\|}people\_use\_escalator(T_j)} \leftarrow \\ &\quad w_{people\_use\_escalator(T_j)} \overline{\circledcirc} (w_{people\_use\_escalator(T_j)|escalator\_working(E_1,T_j)}, \\ &\quad\quad w_{people\_use\_escalator(T_j)|\neg escalator\_working(E_1,T_j)}, w^{vac}_{escalator\_working(E_1,T_j)}) \end{aligned} \tag{7.13}$$

Figure 7.3 depicts graphical model of introduced conditionals for the case study and Figure 7.4 shows an illustrative example of Rule 7.13 in the case of observational opinions from Rule 7.9 ∼ Rule 7.12 were as depicted in Figure 7.2. In the case we observed 1 person using escalator and 1 person using stairs, by the base rate and prior, we get more belief on escalator working but with high uncertainty. As we observe more people using stairs, we get stronger disbelief on the proposition *'escalator_working'*.

7.5.3 Experimental Proof of Concept

As proof of concept, a demonstrator has been built. CLIPS rule-engine was used to implement subjective logic extended logic programming for the proposed reasoning framework. Eclipse platform was used to integrate other modules such as opinion visualization, video parser, metadata parser and manipulation UI, etc. At this stage of case study, we used 300 seconds of airport video surveillance data. Then we manually annotated human detection metadata such as bounding box and time information per seconds as shown in Figure 7.1 with a background annotation (e.g. escalator, stairs, etc). We first set the necessary priors as shown in Figure 7.4. To consider each of single human detection based judgement as a reasonably weak and uncertain evidence, we used strong discount opinion factor as $w^{a\_human\_uses\_escalator}_{people\_use\_escalator} = w^{a\_human\_uses\_stairs}_{people\_use\_stairs} = (0.3, 0.05, 0.65)$. For more intuitive case study, we first considered a real-time analysis setup. Meaning that we loaded Rule 7.9 - 7.13 and injected low level metadata along the video playing in synchronized manner. To see how belief changes over time, we additionally introduced a rule as follows.

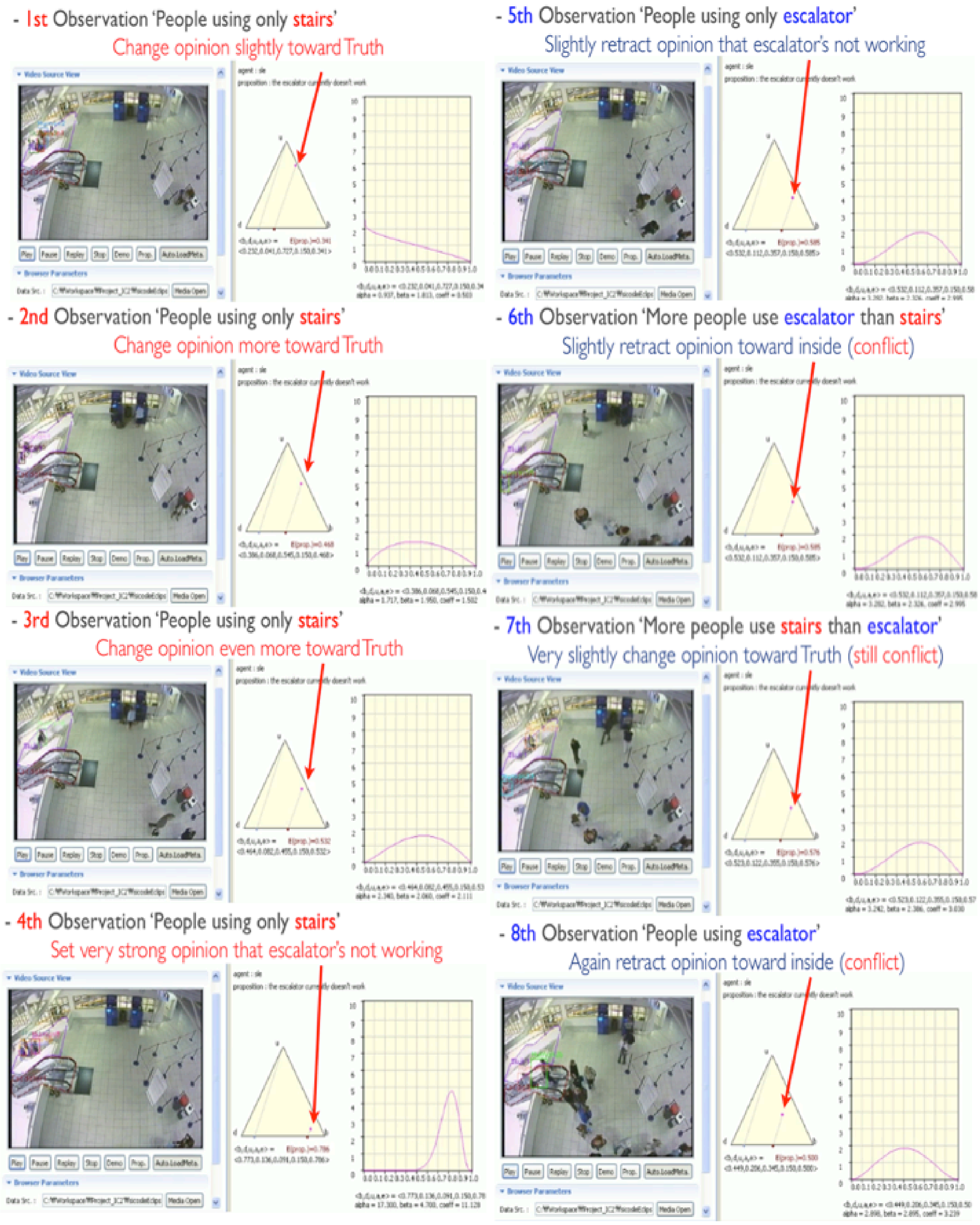

Figure 7.5: Screen Captures demonstrating Belief Revision on the Proposition *'the escalator is not working'*.

$$w^{Accumulated\_Over(T_1,T_n)}_{people\_use\_escalator(T_j)} \leftarrow w^{Accumulated\_Over(T_1,T_{n-1})}_{people\_use\_escalator(T_{n-1})} \bigoplus (w_{a\_people\_use\_stairs(T_n)}) \tag{7.14}$$

Then we kept tracking the change of the derived opinion about the proposition *'escalator is not working'*. When the system gets opinions about human detection, rules were automatically triggered. Figure 7.5 shows the change of opinion on *'escalator is not working'* along a timely sequential multiple observations. For example, the 4th observation in Figure 7.5 shows a scene that many people were using stairs while nobody was using the escalator. In this case, the system correctly computed a strong opinion that the escalator is not working. While, 5th observation shows an example of retracting current opinion upon observation of *'people use escalator'*. This demonstrates the *'non-monotonicity'* in terms of *'belief revision'* over time. The demo on this case study shows that it well simulate human intuition.

7.6 Case Study II

In this section, we further explorer the proposed approach with a virtual scenario.

7.6.1 Scenario Setting for Case Study II

Let us consider a conceptual scenario that a security personnel wants to find a scene of parking a vehicle, that seems to be done by a *'novice driver'*. Let us also assume a virtual system setting such that: parking slots are pre-annotated with a polygon, vision analytics can detect moving and stopped vehicles with motion vector and geometric localization information (i.e., coordinate, width and height). Given the assumption, Figure 7.6 shows some of sequential video footages [2]. In Figure 7.6, parking slots are annotated with blue polygons and vehicles with no motion vectors are labelled with pink ovals. Vehicles with motion vectors are marked with red boxes and red arrows with its vehicle *'id'*. Humans

[2]The video footages are inspired and intensionally synthesized based on the original YouTube video on http://www.youtube.com/watch?v=_rHY1qKLLws&feature=related.

Chapter 7

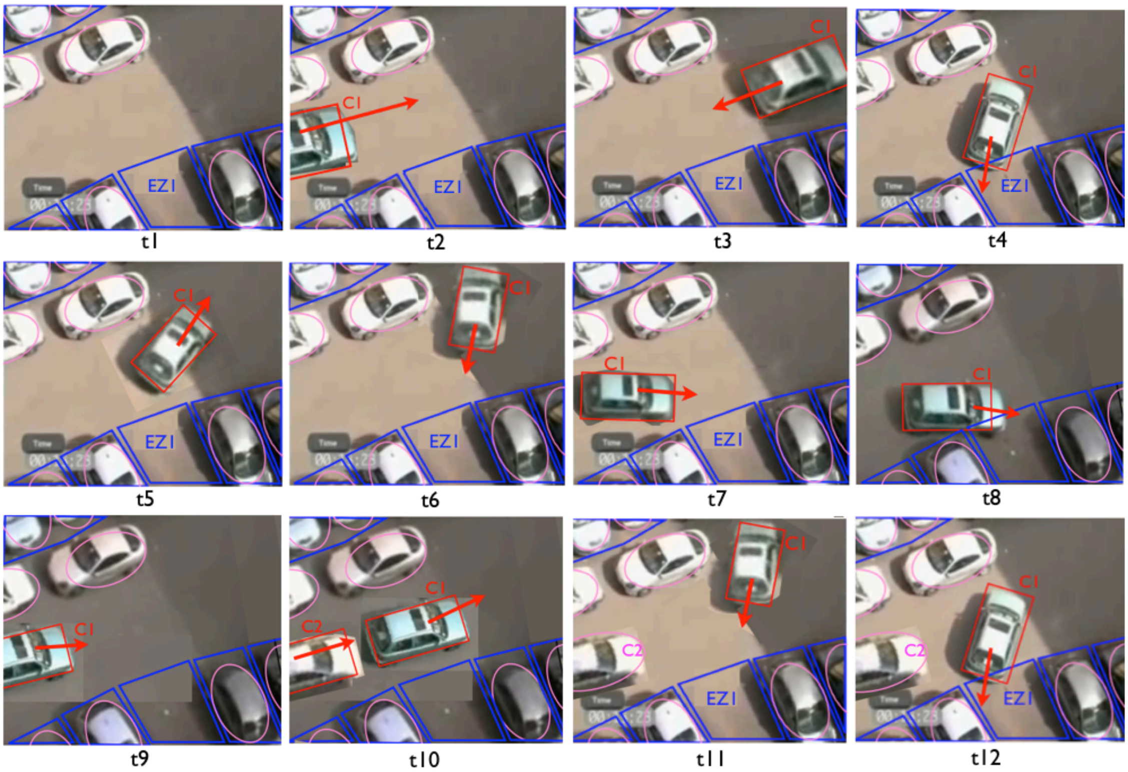

Figure 7.6: Scenario Setting for Case Study 2.
- Two Cars, $C1$ and $C2$. An Empty Parking Slot $EPS1$.-

can easily infer lots of implied high level semantics of Figure 7.6 as follows.

'It seems that the car $C1$ was trying to park to an empty parking slot $EPS1$ several times. In the meanwhile the car was trying to park, another car $C2$ appeared. The newly appeared car $C2$ seems to be blocked due to the car $C1$, that is still trying to park (see $t9 \sim t12$ in Figure 7.6). Because the car $C1$ was very sluggish at parking, it seems that the driver of the car $C1$ was a novice driver. If the driver of $C1$ was a novice driver, there is a possibility that the car $C1$ could have scratched other cars. If such case was really occurred, the car $C2$ would be a good witness to get more information on the scene and on the driver of $C1$, etc.'

In this case study, we will show how we encode some of above knowledge as rules in the proposed hybrid reasoning framework[3]. Consider the following rules :

Conditional Knowledge Model - Case1

- Rule 1 - *"if a novice driver is driving, he/she is not good at parking"* or
- Rule 2 - *"if a normal driver is driving, he/she is reasonably good at parking"*.

Above knowledge can be seen as rules such that

'$\neg$ *good at parking* $\leftarrow$ *novice driver*' or

'*good at parking* $\leftarrow$ $\neg$ *novice driver*'.

At the same time, we could also think about rather deductive knowledge as follows:

Conditional Knowledge Model - Case2

- Rule 3 - *"if he/she is not good at parking, usually a novice driver is driving"* or
- Rule 4 - *"if he/she is reasonably good at parking, usually not a novice driver"*

Above knowledge can be formulated as rules such that

'*novice driver* $\leftarrow$ $\neg$ *good at parking*' or

'$\neg$ *novice driver* $\leftarrow$ *good at parking*'

In this scenario, the knowledge about *'whether a parking was good or not'* is essential. Therefore, based on the output of low level vision modules, we need to derive rules to get opinions on *'good at parking'* or *'not good at parking'*. Consider the following rules:

Observational Knowledge Model

- Rule 5 - *"if a car comes in to, and goes out of an empty parking slot multiple times, it probably is not a good parking"*
- Rule 6 - *"if a car comes in to, and goes out of an empty parking slot few times, it probably is a good parking"*

Above knowledge can be seen as rules such that

'$\neg$ *good parking* $\leftarrow$ *areas of car and empty slot are overlapped multiple times*' or

'*good parking* $\leftarrow$ *areas of car and empty slot are overlapped only few times*'

[3] Note that, in this scenario, by assumption, we do not have any low-level computer vision module that can directly detect the high level semantics such as *'novice driver'*, etc.

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The implication of Rule 5 and 6 is similar to the one of binomial observations in the context that the Rule 5 and 6 also observe multiple times of positive evidences (see Section 3.1.3 for more details on binomial observation). However, in our context, Rule 5 and 6 only matter positive evidences (i.e., occurrence of *'geometric overlap between the car and the empty parking slot of interest'*). It can be also stated as *'the more times the car tries to park, the higher is the likelihood that it was driven by a novice driver'* which is linguistically similar to the *'vague rules'* discussed in the previous Chapter 6. However, while Rule 5 and 6 deal with the number of occurrence of the event (defined in rule head) itself, the vague rules discussed in Chapter 6 deal with an attribute implied in the condition (rule body) to occur an event (defined in rule head) [4] (namely, vague rules in Chapter 6 concern about intrinsic parameters within a *'predicate'* based proposition[5]). Therefore, the Rule 5 and Rule 6 should be designed as an increasing function of the number of *'parking trial'*. Namely, the more observational instances of evidence on *'parking trial'* should entail the stronger opinion on *'not good at parking'*.

7.6.2 Rule Modeling

Based on the discussion of previous Section 7.6.1, this section deals with actual rule modeling for the given scenario. In both conditional knowledge models shown in Section 7.6.1, the observation about the goodness of parking serves as basis for the assessment. Therefore, we first start with observational knowledge model described in Section 7.6.1. Assuming opinions about observational metadata from vehicle detection and annotation, we will use opinions such as $w^{VehicleDetector}_{vehicle(V_i,Loc_{V_i},MV_{V_1},T_j)}$, $w^{Annotation}_{parking\_slot(PS_k,Loc_{PS_k},T_j)}$, etc (where for example, the first notation says, there is an opinion from *'VehicleDetector'* about a proposition *'vehicle'* (i.e., a vehicle exists, more concretely) with id V_1, localization information Loc_{V_1} and motion vector information MV_{V_1} at time T_j).

[4] In Chapter 6, *'direction'* and *'distance'* are the concerned attributes.

[5] See Section 3.2.2 for detailed explanation about the difference between normal propositional logic and first-order predicate logic. Also note that, the rule engine used in our framework is based on the first-order predicate logic (see Section 4.3.2).

To infer whether a parking slot is occupied or not, we check if a stopped vehicle is within the parking slot, examining whether the motion vector of the car is Null. Therefore, we get a rule as follows.

$$\begin{aligned}
&w^{byV_i}_{occupied_parking_slot(V_i,PS_k,Loc_{PS_k},T_j)} \leftarrow \\
&\quad (w^{VehicleDetector}_{vehicle(V_i,Loc_{V_i},NULL,T_j)} \wedge w^{Annotation}_{parking_slot(PS_k,Loc_{PS_k},T_j)} \wedge w^{GeometryAgent}_{within(V_i,PS_k,T_j)}) \\
&w^{byV_i}_{empty_parking_slot(PS_k,Loc_{PS_k},T_j)} \leftarrow \neg w^{byV_i}_{occupied_parking_slot(_,PS_k,Loc_{PS_k},T_j)}
\end{aligned} \tag{7.15}$$

As indicated by the agent markup *'byV$_i$'*, Rule 7.15 derives an opinion about the proposition *'empty_parking_slot*(PS_k, Loc_{PS_k}, T_j)*'* only when some amount of overlap occurred by a vehicle *'V$_i$'* and a parking slot *'PS$_k$'*. By default, however, every parking slots have chances of being empty, even if there are no clues about occupancy. Consider following truth assignment where, DT indicates *'default true'*, where any value reasonably close to *'true'* but rather *'uncertain'* can be a candidate of DT as long as we fix the value throughout reasoning pipeline (see Definition 16 and Definition 35). In this case, following examples in Chapter 5, we assign $DT = (0.5, 0, 0.5)$ (see Section 5.8).

$$\phi_{Rule}[w_{empty\_parking\_slot(SP_k,Loc_{PS_k},T_j)} \leftarrow w_{parking\_slot(PS_k,T_j)}] \quad = \quad DT \tag{7.16}$$

In this setup, following the *'default reasoning'* scheme introduced in Chapter 5 (see Definition 35 for detail), we can define a rule as follows.

$$\begin{aligned}
&w^{byDefaultInference}_{empty_parking_slot(PS_k,Loc_{PS_k},T_j)} \leftarrow \\
&\quad (w_{parking_slot(PS_k,T_j)} \wedge DT) \oplus (\wedge^n_i w^{byV_i}_{empty_parking_slot(PS_k,Loc_{PS_k},T_j)})
\end{aligned} \tag{7.17}$$

Now, we introduce a rule to know whether a car is trying to park. Consider a rule as follows.

$$\begin{aligned}
&w^{byV_{i,j}}_{trial_of_parking(V_i,PS_k,T_j)} \leftarrow \\
&\quad (w^{VehicleDetector}_{vehicle(V_i,Loc_{V_i},NULL,T_j)} \wedge w^{byDefaultInference}_{empty_parking_slot(PS_k,Loc_{PS_k},T_j)} \wedge w^{GeometryAgent}_{overlap(V_i,PS_k,T_j)})
\end{aligned} \tag{7.18}$$

While above Rule 7.18 will fire for every single parking trial, we are interested in accumulating all the instances of Rule 7.18, to get a single opinion about *'good_ at_ parking'*. Therefore, the accumulation should form a decreasing opinion function[6] in terms of number of observations. To achieve this, we first apply the *'reputation (discount)'* operator $\otimes$ [99] (see Definition 41) to every single parking trial detection as follows, because a single instance of *'trial_ of_ parking'* is a weak evidence to judge the proposition *'good_ at_ parking'*.

$$\begin{aligned} &w^{byV_{i,j}}_{good\_at\_parking(V_i,PS_k,T_j)} \leftarrow \\ &\qquad (w^{byV_{i,j}}_{trial\_of\_parking(V_i,PS_k,T_j)} \otimes w^{trial\_of\_parking}_{good\_at\_parking}) \end{aligned} \tag{7.19}$$

Then, we need to fuse each of decisions made from above Rule 7.19. By default, however, it would be expected to be a normal parking, and the number of parking trials would form a negative evidence against good parking. Therefore, using consensus operator $\oplus$ [93, 94] in subjective logic (see Definition 30 Section 5.6.3), the encoded rule becomes as follows.

$$w_{good\_at\_parking(V_i,PS_k,T_j)} \quad \leftarrow \quad DT \bigoplus \neg(\oplus^{n}_{j=1} w^{byV_{i,j}}_{good\_at\_parking(V_i,PS_k,T_j)}) \tag{7.20}$$

Rule 7.20 cumulates amount of contribution of every single instance of *'trial_ of_ parking'*, to the opinion on *'good at parking'*. By the nature of consensus operator $\oplus$ and $\neg$, the more instances from Rule 7.18 and Rule 7.19 (i.e., *'trial_ of_ parking'*) we detect, the weaker belief on *'good_ at_ parking'* we get. For the illustrative explanation of this aspect, assume that we had perfect vehicle detector and perfect geometry agent, so every results from Rule 7.19 were definite true. Setting the discount opinion as $w^{trial\_of\_parking}_{good\_at\_parking} = (0.33, 0, 0.67)$[7], every single opinion of $w^{byV_{i,j}}_{good\_at\_parking(V_i,PS_k,T_j)}$ also becomes $(0.33, 0, 0.67)$. Then, based on this assumption, Figure 7.7 shows examples of calculating opinions $w_{good\_at\_parking}$ by the by Rule 7.15 $\sim$ Rule 7.20, given different number of parking trials of V_i. In Figure 7.7, as more instances of parking trials are considered, Rule 7.20 set stronger disbelief on

[6]It becomes increasing opinion function in case we consider *'NOT good at parking'*.

[7]The implication of this assignment is to consider at least three times of parking trial to decide possibility of being a bad parking. Therefore, by intuition, we assigned 1/3 belief of full belief.

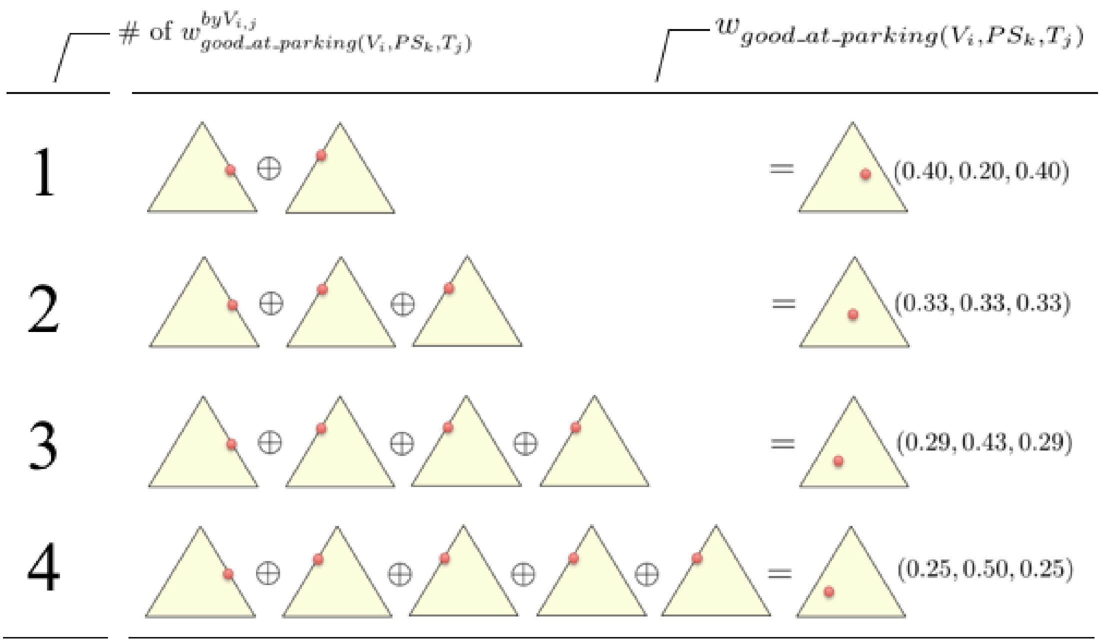

Figure 7.7: Examples of Applying Rule 7.20 based on Different # of Observations on *'good_at_parking'* proposition by the Vehicle *'V_i'*.

'good_at_parking' proposition.

Thus far, we have reviewed rules belong to *'extensional layer'*. Now to address the conditional knowledge models from Section 7.6.1 that work at the *'intensional layer'*, we need to collect certain prior opinions. In our case, by setting x as *novice_driver* and y as *good_at_parking*, we could assign opinions for the items shown below.

- $w_{novice\_driver|good\_at\_parking}$
- $w_{novice\_driver|\neg good\_at\_parking}$
- $w_{good\_at\_parking|novice\_driver}$
- $w_{good\_at\_parking|\neg novice\_driver}$
- $w^{vac}_{novice\_driver}$
- $w^{vac}_{good\_at\_parking}$

Once Rule 7.15 ~ Rule 7.20 are triggered, given available priors, one of following meta-level rules (7.8) in *'intensional layer'* are triggered.

$$
\begin{aligned}
& w_{novice\_driver(V_i)\overline{|}|good\_at\_parking(V_i)} \leftarrow \\
& \quad w_{good\_at\_parking(V_i)} \overline{\circledcirc} (w_{good\_at\_parking(V_i)|novice\_driver(V_i)}, \\
& \qquad\qquad w_{good\_at\_parking(V_i)|\neg novice\_driver(V_i)}, w^{vac}_{novice\_driver(V_i)})
\end{aligned}
\tag{7.21}
$$

$$
\begin{aligned}
&w_{good\_at\_parking(V_i)||novice\_driver(V_i)} \leftarrow \\
&\quad w_{good\_at\_parking(V_i)} \circledcirc (w_{novice\_driver(V_i)|good\_at\_parking(V_i)}, \\
&\qquad w_{novice\_driver(V_i)|\neg good\_at\_parking(V_i)})
\end{aligned} \tag{7.22}
$$

Note that, Rule 7.21 and Rule 7.22 use different priors, but returns an opinion on the same proposition that is *'novice_ driver'*. Namely, '$w_{novice\_driver(V_i)}$'.

7.6.3 Illustrative Examples as Proof of Concept

In this section, we show conceptual reasoning example given the scenario, rules and assumptions described thus far.

Example 9. ***(Novice Driver) .*** Assume that we have detected a vehicle with id $C1$ and a parking slot $EPS1$ as depicted in Figure 7.6. Let us also assume that we have observational opinions on *'good_ at_ parking'* of the $C1$ to the slot $EPS1$ as shown in Figure 7.7. Assume that 10% of drivers are novice driver in general. Consider the case we have intuition based subjective opinions on the proposition *'good_at_parking|novice_driver'* and also on the proposition *'good_at_parking|¬novice_driver'* as follows [8].

- $\bullet w_{good\_at\_parking|novice\_driver} = (0.05, 0.85, 0.1, 0.5)$
- $\bullet w_{good\_at\_parking|\neg novice\_driver} = (0.9, 0.05, 0.05, 0.5)$
- $\bullet w^{vac}_{novice\_driver} = (0.0, 0.0, 1.0, 0.10) \quad (i.e., 10\%)$

then, according to Table 7.1, Rule 7.21 will be triggered. Given the assumptions and the priors above, Figure 7.8 shows an illustrative example of applying Rule 7.21.

In the case there was only one observational instance on *'good_ at_ parking'*, the computational result yields more *'disbelief'* than *'belief'* about the proposition *'novice_ driver'*. However, as we get multiple parking trial based multiple instance of *'good_ at_ parking'* opinions, it smoothly changes its opinion toward stronger belief about the proposition *'novice_ driver'*. This shows that the reasoning results coincide with human intuition.

[8] Note that, in fact, such subjective opinions (based on human intuition) can be regarded as subjective Bayesian priors as discussed in Section 3.1.3. Refer to [33] for details of Bayesian statistics and its comparison with traditional frequentist methods.

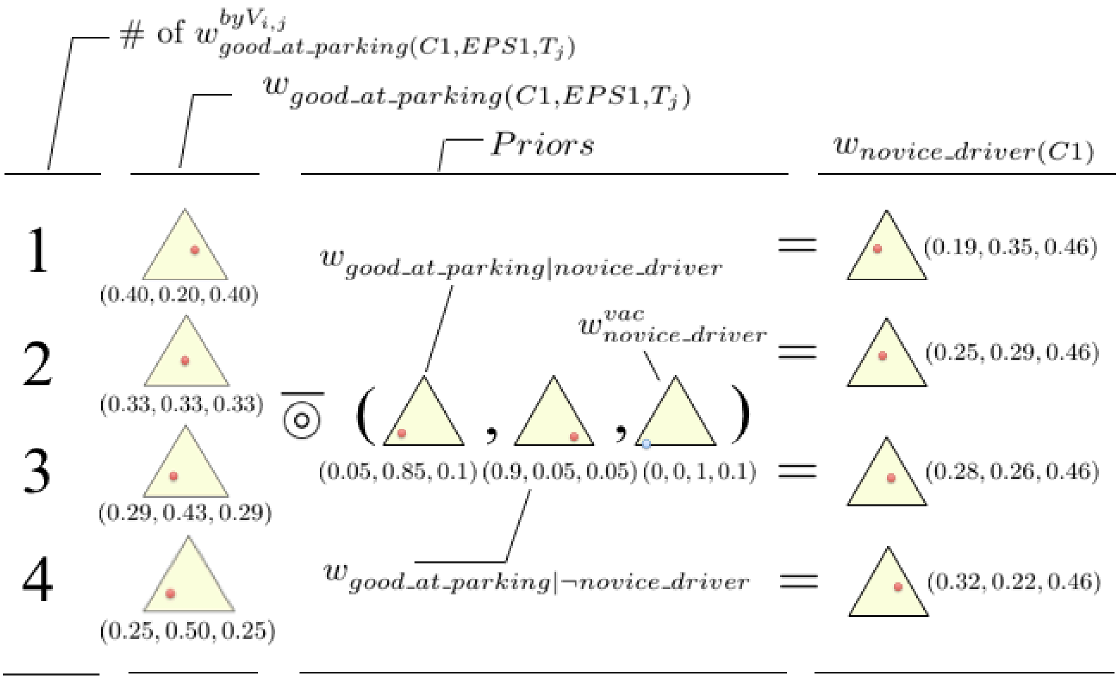

Figure 7.8: Illustration of Abductive Conditional Reasoning for Case Study 2 (see http://persons.unik.no/josang/sl/Op.html).

Example 10. ***(Illegal Parking)*** **.** Consider the same situation as the one of Example 9. Assume that we have detected a new vehicle with id $C2$. $C2$ slowly entered in the scene and suddenly stopped after a while, as depicted in the footages whthin the period ($t9 \sim t12$) in Figure 7.6. Assume a set of rules about determining *'illegal parking'* and the opinion assignments to the rules as follows.

$$\begin{aligned}
\phi_{Rule}[w_{illegal_parking(V_i,T_j)} \leftarrow w^{VehicleDetector}_{vehicle(V_i,Loc_{V_i},NULL,T_j)}] &= (0.2, 0, 0.8) \\
\phi_{Rule}[\neg w_{illegal_parking(V_i,T_j)} \leftarrow w^{byV_i}_{occupied_parking_slot(V_i,PS_k,Loc_{PS_k},T_j)}] &= (0.9, 0, 0.1) \\
\phi_{Rule}[\neg w_{\neg illegal_parking(V_i,T_j)} \leftarrow w^{byV_i}_{blocked_vehicle(V_i,T_j)}] &= (0.9, 0, 0.1)
\end{aligned}$$

$$w_{blocked\_vehicle(V_i,T_j)} \leftarrow w_{novice\_driver(V_i)} \wedge w^{GeometryAgent}_{motion\_vector\_before\_stop\_heading(V_i,Vj)} \tag{7.23}$$

where, $w^{byV_i}_{occupied\_parking\_slot(V_i,PS_k,Loc_{PS_k},T_j)}$ is the opinion determined by the Rule 7.15 and $w_{novice\_driver(V_i)}$ is the opinion after Rule 7.21 and Rule 7.22 are triggered in the *'intensional layer'*.

Consider opinion assignments for the case that $C1$ has tried to park 4 times. For this example, we will take the corresponding opinion w_{novice_driver} from the Figure 7.8 of Example 9. Also assume that our geometry agent is robust and reliable.

$$\begin{aligned}
\phi_{RuleEval}[w^{\#ofinst.=4}_{novice_driver(C1)}] &= (0.32, 0.22, 0.46) \\
\phi_{fact}[w^{GeometryAgent}_{motion_vector_before_stop_heading(C2,C1)}] &= T = (1, 0, 0) \\
\phi_{fact}[w^{VehicleDetector}_{vehicle(C2,Loc_{C2},NULL,T_j)}] &= T = (1, 0, 0)
\end{aligned}$$

Given this setting, the inference on *'C2 parked illegally'* can be conducted as follows.

$$\begin{aligned}
cl_{sl_{di}} \quad & (\phi)(w_{illegal_parking(C2)}) \\
& = [U \sqcup ((1, 0, 0) \cdot DT_1)] \oplus \neg[U \sqcup (((0.32, 0.22, 0.46) \cdot (1, 0, 0)) \cdot DT_2)] \\
& = [U \sqcup (1, 0, 0) \cdot (0.2, 0, 0.8)] \oplus \neg[U \sqcup ((0.47, 0.22, 0.31) \cdot (0.95, 0, 0.05))] \\
& = [U \sqcup (0.47, 0, 0.53)] \oplus [U \sqcup \neg(0.55, 0.22, 0.23)] \\
& = (0.47, 0, 0.53) \oplus (0.22, 0.55, 0.23) \\
& = (0.35, 0.46, 0.19)
\end{aligned}$$

The entailed opinion (0.35,0.46,0.19) can be linguistically interpreted as *'it may not be an illegal parking, but with some amount of uncertainty'*. In the same way, the other stopped vehicles labelled with pink ovals can be regarded as *'illegal parking'* when they are not parked within a predefined parking slot (see Figure 7.6). This example shows that the inference result from the *'intensional layer'* can be again used for another semantic reasoning in the *'extensional layer'*.

7.7 Chapter Summary

In summary of lessons learned in this chapter, we have demonstrated how conditional premises can be factorized and handled in a Bayesian sense by using subjective logic's deduction and abduction operators. The main advantage of the proposed hybrid approach is that it offers more choices to handle the trade-offs between expressive power, semantic clarity and computational efficiency. However, there are still several open issues such as *how to extend this concept to multinomial frame based event, how to automatically get the prior opinions, etc.*

In this chapter, we proposed a hybrid approach to conditional evidence fusion that mates extensional and intensional approaches of conditional knowledge representation and inference under uncertainty, for high level semantic analysis of visual surveillance data. For uncertainty handling, subjective logic was adopted on top of rule-based framework. The main advantage of the proposed hybrid approach is that it offers more choices to handle the trade-offs between expressive power, semantic clarity and computational efficiency. For highly varying and complex compositional events in visual surveillance data, we could benefit from the expressive power and modularity of the extensional approach, while leveraging the intensional interpretation of conditionals to facilitate bi-directional inference.

There are, however, still several open issues such as how to extend this concept to multinomial frame based event, how to automatically get the prior opinions, etc. Although we still need to extend this concept to large scale data. We advocate that this work showed the potential of the proposed approach. One of interesting properties of the system is that, unlike traditional probability based conditional reasoning, this approach allows for representing lack of information about a proposition. We could assign priors with lack of information, and observations can also be represented with any degree of ignorance, therefore we believe this better reflects human intuition and real world situations. Another beneficial property is the flexibility of defining rules by taking a layered approach to leverage advantages of both *extensional* and the *intensional* approaches.

8 Logical Abduction under Uncertainty

Thus far, our discussions and reasoning examples have dealt reasoning scenarios that can be triggered by a *'specific'* question to be inferred. In reality, however, humans tend to reason about a situation rather in a diagnostic manner, without setting a specific proposition to be reasoned about but just by considering set of observations of their interest. Such a reasoning is called logical abduction and this chapter represents our approach to achieve the abductive logical reasoning.

8.1 Introduction

This chapter proposes an approach to enable automatic generation of probable semantic hypotheses for a given set of collected observations for forensic visual surveillance. As video analytic power exploited in visual surveillance is getting matured, the more automatically generated intermediate semantic metadata became available. In the sense of forensic reuse of such data, the majority of approaches have been focused on specific semantic query based scene analysis. However, in reality, there are often cases in which it is more natural to reason about the most probable semantic explanation of a scene given a collection of specific semantic evidences. In general, this type of diagnostic reasoning is known as abduction[1]. To enable such a semantic reasoning, in this chapter, we propose a layered reasoning pipeline that combines abductive logic programming together with backward and forward chaining based deductive logic programming. To rate derived hypotheses, we

[1] note that, while the subjective logic *'abduction'* operator discussed in Chapter 7 deals the *'bidirectional interpretation'* of a conditional rule, *'logical abduction'* deals a diagnostic mechanism to find most probable hypothesis given set of observations

apply subjective logic. We present a conceptual case study in a distributed camera based scenario. The case study shows the potential and feasibility of the proposed approach for forensic analysis of visual surveillance data.

8.2 Background and Motivation

As discussed in Section 1.1, recent advances in computer vision technology have made it possible to analyze specific patterns of abnormal human or object behavior in visual surveillance. When it comes to forensic reuse of such analytics, however, it is still beyond the sufficient intelligence due to the variety of possible semantics and complex plots implied in surveillance scenes. Such forensic semantic analysis of visual surveillance, therefore, requires intelligent reuse of low level vision analytic results in a context sensitive manner.

To address this aspect, there has been some work on the use of declarative logic formalism to represent and reason about high-level contextual semantic knowledge which is referenced to as *'extensional'* approach. In Section 2.4.3, we have reviewed such approaches appear in literature. After the review and comparison, in Chapter 4, we have also proposed an *'extensional'* approach that uses logic programming with subjective logic theory.

In such approaches, reasoning is triggered with a specific high level semantic query to decide the existence of metadata patterns that semantically satisfy the given query. However, in reality, there are often cases require to ask about the most probable semantic explanation of a scene based on specific evidential semantic observations. That is, when the former approach is applied to semantic retrieval, the security personnel is required to know what semantics to ask. While the later would, ideally, let users collect evidences of their interest and do simplify queries into a single *'why?'* or *'what?'* type of query. Enabling such type of reasoning would be advantageous especially in the situation that the semantic ambiguity of a scene is highly implied or multiple semantic interpretations of a scene are possible. In general, this type of diagnostic reasoning is known as abduction [131]. In this chapter, we present a layered pipeline of reasoning flow that adopts abductive logic programming together with backward and forward chaining based deductive logic programming and combine subjective logic [92, 93] for the rating of derived diagnostic hypotheses. We then present a conceptual case study from a distributed camera based scenario to show the potential and feasibility of the proposed approach in the sense of diagnostic forensic.

8.3 Related Work

To achieve better expressive power and flexibility on context modelling and reasoning of visual surveillance data, there has been some work on the use of declarative logic formalisms and on the use of different uncertainty handling formalisms. As we have already reviewed in Section 2.4.3, Akdemir et al. [13] used an ontology for human activity recognition, but without uncertainty handling. Shet et al. [152, 153] proposed a system that adopts Prolog based logic programming extended with the bilattice framework [69] for default logic [143] based situation reasoning and human identity maintanance. Jianbing et al. [110] used rule-based reasoning with Dempster Shafer's belief theory [149] for a bus surveillance scenario. Anderson et al. [17] used fuzzy logic [177] to model human activity for video based eldercare. Based on our previous work, [78, 81, 80, 82, 83, 79], in this dissertation, we proposed the use of logic programming and subjective logic [92, 93] to encode contextual knowledge with uncertainty handling, then demonstrated bidirectional conditional inference and default reasoning for visual surveillance scenarios. In the sense of reasoning, such logic framework based approaches require a specific query to reason about, and yield an output on the truth of the given query.

However, in the sense of forensic use of visual surveillance data, rather diagnostic reasoning so called *'logical abduction'* is also required. In such diagnostic reasoning, the initial input to the reasoning system would be desired to be a selected set of interested event, then the query can be converged into a single *'why?'* or *'what?'* type of query to get explanations or hypotheses to the best of given knowledge base. There has been also some work on the use of abduction. Espinosa et al. [132] showed description logic [2] based abduction example getting explanations about an image given ontology based image annotation. Ferrin et al. [59] also used description logic for visual surveillance to show that abduction can be used for realtime *'car theft'* event detection. While both work showed the potential of abduction in interpreting media contents, the use of description logic makes it hard to consider uncertainty handling or extending the reasoning framework so that it can also conduct other types of reasoning such as default reasoning. In contrast to the previous work, we take more pragmatic approach based on logic programming including abductive logic programming and rate results with subjective logic with more focus on situation analysis rather than atomic event analysis.

8.4 Abductive Reasoning

Abduction means a method of logical inference finding plausible explanations (hypotheses) for a given set of observations. Peirce [131] showed that it can be seen as inverse modus ponens and formulated it as $\Sigma \cup \Delta \models \Gamma$, where Σ represents the background knowledge, Γ represents observations, Δ represents the explanations to be computed and the symbol '$\models$' represents logical entailment. During the 90s this aspect has been solidly studied in the field of AI in the context of logic programming and came up with abductive logic programming [50]. In this section we provide a brief introduction to abductive logic programming.

8.4.1 Abductive Logic Programming

An abductive logic programming theory is defined as a triple (P, A, IC) consisting of a logic program P, a set of ground abducible atoms A and a set of classical logic formulas IC, called the integrity constraints, such that no $p \in A$ occurs in the head of a rule of P. The logic program P also consists of rules R and literals L, namely, $P = (R, L)$. Then for a given query observation Q *(note that, in abductive logic programming, Q is not a set of observations but rather a single observational query)*, an abductive explanation for the observation is a set $\Delta \subseteq A$ such that : 1) $P \cup \Delta \models Q$. 2) $P \cup \Delta \models IC$. 3) $P \cup \Delta$ *is consistent*. This mean that the derived explanation Δ and P should be able to entail Q. At the same time, should not violate the constraints IC. In the sense of Peirce's definition of abduction, (P, A, IC) corresponds to Σ and Q is the Γ and Δ is the output. Following shows a simple example of abductive logic programming.

Example 11. ***(Abduction)* .** Assume an abductive logic program (P, A, IC), where $P = (R, L)$ such that :

$$
\begin{aligned}
R &= \{ \; grass\_is\_wet \leftarrow it\_rained \\
&\qquad grass\_is\_wet \leftarrow the\_sprinkler\_was\_on \; \} \\
L &= \{ \; the\_sun\_was\_shining \; \} \\
A &= \{ \; it\_rained \; , \; the\_sprinkler\_was\_on \; \} \\
IC &= \{ \; false \leftarrow it\_rained, \; the\_sun\_was\_shining \; \}
\end{aligned}
$$

then for a given observational query $Q = grass\_is\_wet$, the proper explanation $\Delta = \{the\_sprinkler\_was\_on\}$ should be derived because it is the consistent explanation given the constraint IC and the fact $the\_sun\_was\_shining$ in P. (note that, in normal logic programming the output is truth value about the query). There has been some work on enabling the shown concept of abductive logic programming.

8.4.2 Abductive Logic Programming Frameworks

Most of the implementations of Abductive Logic Programming extend the SLD-resolution [66] based computational model of logic programming. (note., resolution is a procedure for showing whether or not a given set of facts are valid according to currently known rule sets. The SLD-resolution is known to be a computationally efficient resolution algorithm). SLDNFA [51] is one of well known algorithms to fulfill abduction itself. ACLP [101], CIFF, SCIFF [6] and Asystem [125] are some of known implementations of abductive logic framework. Besides, ABDUAL and ProLogICA., etc., appear in literature but the project sites are no longer appear. Indeed, it seems that no prominent benchmark or comparison of those systems appear. Although some are based on Prolog, some of them are also based on other languages such as ECliPSe language [57]. In this dissertation, considering availability of executable binary, simplicity on system integration issues, we have used ACLP [101].

8.5 Proposed Approach

The objective of the presented framework is to enable abductive semantic reasoning and its quantitative evaluation given multiple observational evidence input. For the core part of the logical abduction, abductive logic programming plays an important role. As shown in Section 8.4, however, abductive logic programming also requires a specific single query to be explained and this is far from our intention. Although we could collect a set of interested observations and put them as part of facts (literals L) in P of (P, A, IC), still the reasoning should be triggered by a single query. To address this aspect, we take layered pipeline concept throughout multiple observational input processing, abduction and evaluation of derived hypotheses. To process the initial multiple observational input, we adopt a forward chaining based rule engine to derive a possible set of semantics to be asked. Unlike backward chaining based rule engine such as prolog that only tracks back patterns that satisfy the given single query, a forward chaining based rule engine derives all deductive logical conclusions given input literals and rule set at one execution. For this part, CLIPS [1] rule engine was used. Once we get the list of possible semantics to be asked and explained, for each of them we conduct the abduction and get a list of explanations. For the abductive reasoning part, we adopt an implementation of abductive logic programming framework. For the prototyping purpose, ACLP [101] implementation of abductive logic programming was used due to its relatively simple structure among other frameworks such as ASystem [125]. For evaluation of the derived hypotheses we adopt subjective logic extended deductive rule based framework. Namely, for each of the potential explanations, we conduct backward chaining based reasoning with subjective logic based uncertainty evaluation. This layered pipeline is depicted in Figure 8.1. For input video sources with analytic results, we first select a set of interested evidences to form Γ,

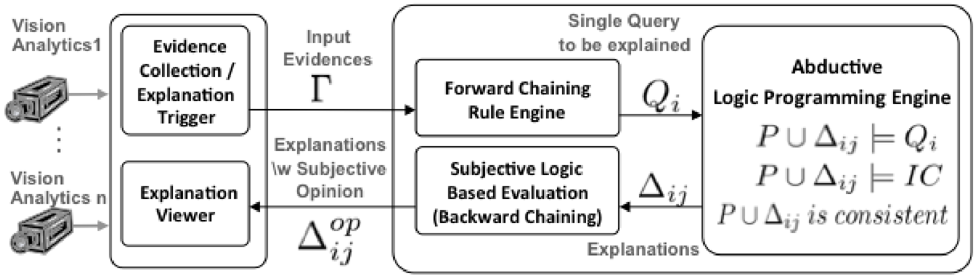

Figure 8.1: The Abductive Reasoning Framework.

the set of observations which is then sent to the forward chaining based rule engine to reason about possible semantics to be asked. For each of the semantics to be asked, we form a single query Q which is sent to the abductive logic programming engine. Once we get explanation set Δ, this is evaluated with the subjective logic based backward chaining engine. Finally, the result is presented on explanation viewer.

8.6 Case Study

This section deals with an illustrative case study on the application of forensic abductive reasoning in a typical distributed camera based scenario. We will assume metadata acquired from low level computer vision analytics such as human detection, vehicle detection and object detection, etc. As explained in Section 8.4 and Section 8.5, in our setup, such metadata will be considered as logical facts (literals L, in the sense we care both positive and negative facts) and will be put into logic program P together with context rules R (note that, $P = (R, L)$). Assuming a security personnel examining a video scene for certain forensic reason, the query for the abductive reasoning will also be assumed to be triggered by Γ, the set of collected evidential metadata on his interest. Given the Γ, we first use forward chaining based deductive logic programming to derive possible semantic queries Q_i to be explained and will be sent to the core abductive logic programming part to get the set of explanations Δ_{ij}. Then finally, we will evaluate the resulting Δ_{ij} with subjective logic to get opinion attached set of explanations Δ_{ij}^{op}.

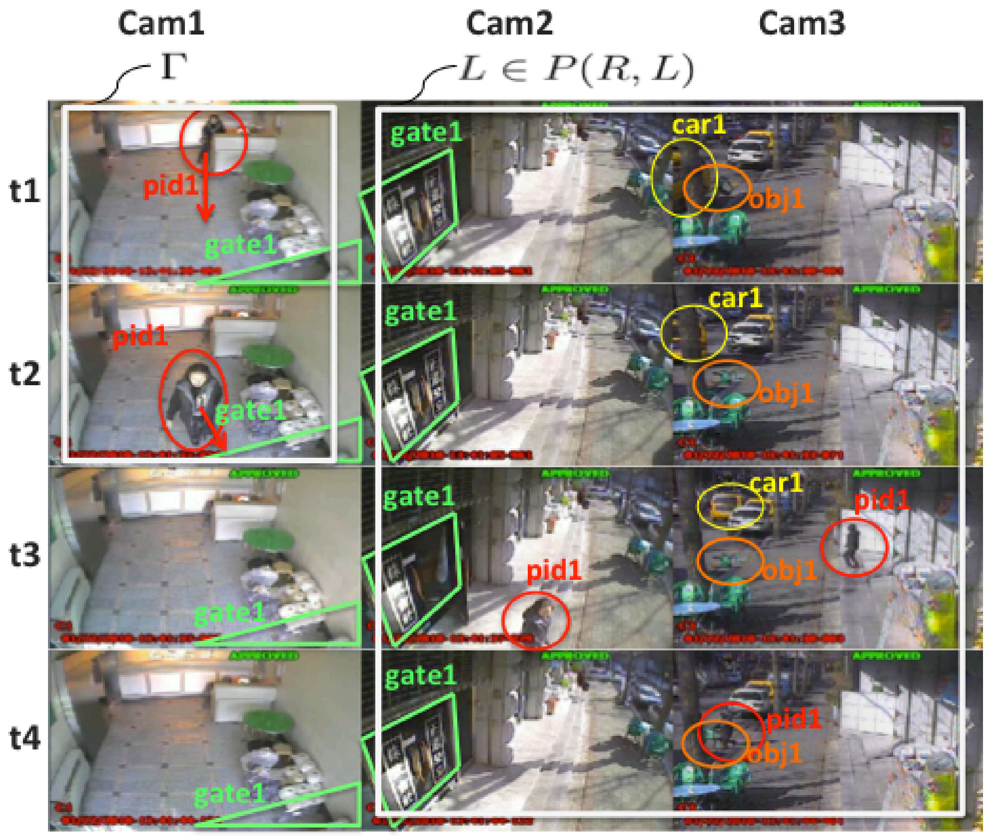

Figure 8.2: The Setup for Case Study Scenario with 3 Cameras Deployed.

Example 12. ***Scenario (Why the person suddenly went outside ?) .*** Assume a typical distributed visual surveillance with three cameras deployed as shown in Figure 8.2. Cam1 monitors the lobby of the building, Cam2 and Cam3 monitor front of the gate in different outside views. Suppose a security personnel was watching a scene of Cam1 and got interested in why the person who was sitting at the front desk suddenly stood up and went outside the building. From the scene of Cam1, he collected metadata such as the person's standing up and heading to the predefined gate zone as depicted with the box labelled with Γ in Figure 8.2.

$$\Gamma_{(t1..t2)} = \{ \quad human(pid1), gate(gate1), standing\_up(pid1), heading(pid1, gate1) \quad \}$$

Given the Γ, suppose the system had the following deductive logic program segment.

$go\_outside(P) \leftarrow human(P), gate(G), heading(P, G)$

The use of the forward chaining rule engine with the given Γ will then assert new semantic derivations Q_i as follows.

$$Q_1 = \quad go\_outside(pid1)$$

Now, each Q_i will be considered as the query to be explained by the abductive reasoning framework. Suppose also that, at the same time of the event in Cam1, there were a vehicle parked in front of the gate and also a chair beside the vehicle. When the vehicle leaves, it hit the chair and the chair fell down on the road. These are captured in Cam3 in Figure 8.2 and the analytic metadata of the scene is stored in form of literals L. Consider an abductive logic program (P, A, IC) containing context rules R, integrity constraint IC and abducibles A as follows (note that, $P = (R, L)$).

$$L = \{ \quad vehicle(car1), object(obj1), gate(gate1) \ \}$$

$$\begin{aligned}
R = \{ \qquad go_outside(X) &\leftarrow to_do_smt(X, Y) \\
go_outside(X) &\leftarrow saw_smt(X, Y) \\
to_do_smt(X, Y) &\leftarrow take_a_veh(X), vehicle(Y) \\
to_do_smt(X, Y) &\leftarrow buy_smt(X), mart(Y) \\
saw_smt(X, Y) &\leftarrow veh_obj_acc(Y), mng_veh_obj_acc(X) \\
saw_smt(X, Y) &\leftarrow veh_prs_acc(Y), mng_veh_prs_acc(X) \\
veh_obj_acc(Y) &\leftarrow vehicle(Y), object(Z). \\
veh_prs_acc(Y) &\leftarrow vehicle(Y), person(Z) \ \}
\end{aligned}$$

$$\begin{aligned}
IC = \{ \quad false &\leftarrow buy_smt(X), take_a_veh(X) \\
false &\leftarrow buy_smt(X), mng_veh_obj_acc(X) \\
false &\leftarrow buy_smt(X), mng_veh_prs_acc(X) \\
false &\leftarrow take_a_veh(X), mng_veh_obj_acc(X) \\
false &\leftarrow take_a_veh(X), mng_veh_prs_acc(X) \ \}
\end{aligned}$$

$$A = \{ \quad buy\_smt, take\_a\_veh, mng\_veh\_obj\_acc, mng\_veh\_prs\_acc \quad \}$$

The rule set R means that, a person may go outside to do something outside or because the person saw some event. Taking a vehicle or buying something can be seen as doing something in case that there was a vehicle to take or a mart to go to buy something. Similarly, if there was an accident in which a vehicle and an object are involved and the person is trying to manage the accident somehow, it implies that the person noticed the event. This is also same in the case of an accident in which a vehicle and a person were involved. In these cases, both a vehicle and an object or both a vehicle and a person should be existing at the same time. As a constraint, the integrity constraint rule IC means that some of those event should happen exclusively. Given the knowledge, the candidate semantic predicates we may want to derive by the execution of abduction are designated in the set of abducibles A. Now, with the given above abductive logic program and an input query $Q_1 = go\_outside(pid1)$, the abductive logic procedure will derive explanations as follows.

$$\begin{aligned}\Delta_{11} = \{ \quad & not(mng\_veh\_prs\_acc(pid1)), not(mng\_veh\_obj\_acc(pid1)), \\ & not(buy\_smt(pid1)), take\_a\_veh(pid1) \ \}\end{aligned}$$

$$\Delta_{12} = \{ \quad not(take\_a\_veh(pid1)), not(buy\_smt(pid1), mng\_veh\_obj\_acc(pid1) \ \}$$

In above scenario, we get two hypotheses sets for a given query Q_1. In the next step, we examine each of the elements in Δ_{11} and Δ_{12} by the use of the subjective logic extended logic programming as explained in Section 4.3.2. Consider following logical rule segments.

$$\begin{aligned} & w_{take\_a\_veh(P_1,V_1)} \leftarrow \\ & \quad w^{Human\_Detector}_{human(H_1,T_1,X_1,Y_1)} \wedge w^{Vehicle\_Detector}_{vehicle(V_1,T_1,X_2,Y_2)} \wedge \\ & \quad w^{Geometry\_Manager}_{overwlap(X_1,Y_1,X_2,Y_2)} \end{aligned} \tag{8.1}$$

$$\begin{aligned} & w_{mng\_veh\_obj\_acc(P_1,V_1)} \leftarrow \\ & \quad w^{Human\_Detector}_{human(H_1,T_1,X_1,Y_1)} \wedge w^{Vehicle\_Detector}_{vehicle(V_1,T_1,X_2,Y_2)} \wedge \\ & \quad w^{Object\_Detector}_{object(O_1,T_1,X_3,Y_3)} \wedge w^{Geometry\_Manager}_{overwrap(X_2,Y_2,X_3,Y_3)} \wedge \\ & \quad w^{Geometry\_Manager}_{near(X_1,Y_1,X_3,Y_3)} \end{aligned} \tag{8.2}$$

Rule 8.1 corresponds to the abducible $take\_a\_veh(pid1)$ in Δ_{11} and Rule 8.2 corresponds to the abducible $mng\_veh\_obj\_acc(pid1)$ in Δ_{12} respectively. In our scenario, given the rules and metadata in form of literals L, both rules can be satisfied. Another aspect to consider is the negative conditions required not to satisfy in Δ. In above cases, Δ_{11} requires 3 exclusive abducibles not to be satisfied and Δ_{12} requires 2 exclusive abducibles not to be satisfied. Indeed, due to the uncertainty implied in metadata, rest of the abducibles with negation could also come up with certain amount of uncertainty that are also represented as subjective opinions. In this case, we will take a subjective logic's complement operation on the derived opinions. In case we can't even prove an abducible predicate in Δ given logic program P, we will simply assign full ignorance opinion. Considering the positive abducibles and negative abducibles to be satisfied, it can be seen as default reasoning [143] and, therefore, we will conduct default reasoning using subjective logic [82, 83] introduced in Chapter 5. In this work, we slightly modify the reasoning scheme shown in Definition 35 to make it fit in the case of reasoning on Δ. The variant definition is as follows.

Definition 45. ***(Default Inference on Δ) .*** *Given a set Δ, containing opinion assigned elements, the default inference is the truth value assignment closure $cl(\phi)(\Delta)$ given by :*

$$cl(\phi)(\Delta) = [\bigwedge_{p \in \Delta} w_{(p)}] \bigoplus \neg [\bigwedge_{\neg p \in \Delta} w_{(p)}] \tag{8.3}$$

,where $\wedge$, $\oplus$ and $\neg$ represent subjective logic's Conjunction, Consensus and Complement operators respectively.

Now, consider the case that uncertainty assignment is as follows, that is, having the same amount of opinion on each positive evidences in Δ_{11} and Δ_{12}. Namely,

$w_{take\_a\_veh(pid1)} = w_{mng\_veh\_obj\_acc(pid1)} = (0.6, 0.3, 0.1)$.

Assume that, the system failed to prove $mng\_veh\_prs\_acc$ $(pid1)$ and come up with $buy\_smt(pid1)$ with relatively high confidence that it was not happened, as opinion assignment follows:

$w_{mng\_veh\_prs\_acc(pid1)} = (0, 0, 1)$.

$w_{buy\_smt(pid1)} = (0.1, 0.8, 0.1)$.

Given this set up, applying Definition 45, the final evaluation results of each Δ_i is as follows:

$$
\begin{aligned}
\Delta_{11}^{op} = cl(\phi)(\Delta_{11}) = \quad & (0.6, 0.3, 0.1) \oplus \\
& \neg[(0, 0, 1) \wedge (0.6, 0.3, 0.1) \wedge (0.1, 0.8, 0.1)] \\
& = (0.6, 0.3, 0.1) \oplus \neg(0.04, 0.86, 0.1) \\
& = (0.6, 0.3, 0.1) \oplus (0.86, 0.04, 0.1) \\
& = (0.77, 0.18, 0.05), Exp = 0.80 \\
\Delta_{12}^{op} = cl(\phi)(\Delta_{12}) = \quad & (0.6, 0.3, 0.1) \oplus \\
& \neg[(0.6, 0.3, 0.1) \wedge (0.1, 0.8, 0.1)] \\
& = (0.6, 0.3, 0.1) \oplus \neg(0.08, 0.86, 0.06) \\
& = (0.6, 0.3, 0.1) \oplus (0.86, 0.08, 0.06) \\
& = (0.79, 0.17, 0.04), Exp = 0.82
\end{aligned}
$$

In above example, the final result rated that the hypothesis Δ_{12} is a bit more plausible than Δ_{11} ($\because$ Δ_{12} has more belief and less uncertainty than Δ_{11}, the expectation value Exp of Δ_{12} is therefore, greater also). However, the reason the opinion gap between these two Δ_1's are not bigger is due to the limited set of rules and metadata we assumed. Therefore, thorough domain knowledge engineering and rule derivation would be critical on applying the proposed approach.

8.7 Chapter Summary

In summary of lessons learned in this chapter, we proposed a system enabling diagnostic abduction based reasoning. It mainly consists of tow layers. 1) a layer that reason about queries to be asked and 2) a layer that uses abductive logic programming to draw potential hypotheses given set of observations. Our demonstration showed the feasibility of using the proposed approach on diagnostic queries. For practical use of this approach, however, thorough domain engineering and rule derivation should be further tackled.

In this chapter, we proposed an abductive reasoning framework for forensic visual surveillance. For logical abduction, abductive logic programming framework was adopted together with forward chaining based rule engine. The resulting hypotheses are evaluated by a rule based system that is extended with subjective logic operators. The main advantage of the proposed approach is that it doesn't require the user to ask a specific query but let them concentrate on collecting observations of their interest. For a collected set of observations, the system fulfills an abduction to suggest probable hypotheses and rate the uncertainty of each hypothesis. We believe this way of using forensic visual surveillance will offer the user more choices on semantic analysis.

There are, however, still several open issues such as how to accurately define context rules so that the derived hypotheses make sense, how to scale up the proposed approach to large scale data, etc. Among them, as shown in the case study, we believe thorough domain engineering and rule derivation is very critical because the final result would be affected by this aspect. Nevertheless, we believe the proposed system has benefit and unique property in the way of composing query especially in the forensic situation that the semantic ambiguity of a scene is high or multiple semantic interpretations of a scene are possible.

9 Conclusion

In this chapter, we provide an architectural big picture of this dissertation, summarize main contributions of this dissertation, highlight open issues and discuss extensions to this work.

9.1 The Big Picture - Architectural View of the Work

This dissertation focussed on issues related to intelligent high-level semantic analysis of visual surveillance data.

In Chapter 4, we discussed the software components, data processing pipeline and system architecture design as shown in Figure 9.1. Especially, we focused on the reuse of intermediate metadata acquired from vision analytics with a discussion of experimental evaluation on query performance in different software component settings (i.e., ontology, logic programming and traditional databased techniques). Figure 9.1 - a) shows the key topics discussed in the Chapter 4.

Based on the proposed architecture, it has been extended to enable advanced reasoning aspects such as *'default reasoning'* (in Chapter 5), *'vague rule modeling'* (in Chapter 6), *'bidirectional interpretation of conditionals'* (in Chapter 7) and *'diagnostic abduction'* (in Chapter 8). Because these topics were dealt mainly in logical reasoning sense, these chapters correspond to the shaded area shown in Figure 9.1 - b). Especially, for the diagnostic abduction discussed in Chapter 8, we have introduced an abductive logic programming part as shown in Figure 9.1 - c).

Throughout Chapter 4 - Chapter 8, rule sets played an important role in visual

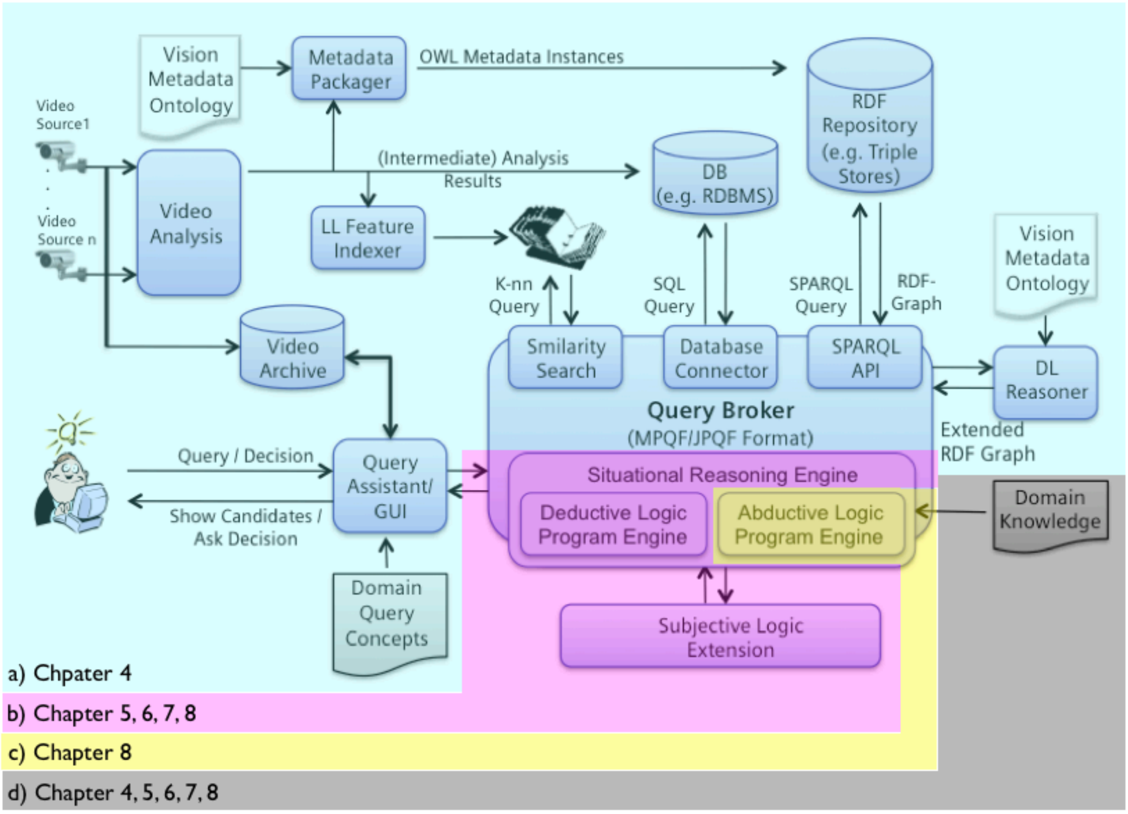

Figure 9.1: The Architectural Bic Picture and Topic Roadmap in terms of Chapters of this Dissertation.

surveillance scenarios. Such rule sets can be regarded as domain knowledge. Therefore, the shaded part of Figure 9.1 - d) corresponds to these chapters.

9.2 Benefits and Drawbacks

The main advantage of the presented approach is in the *'flexibility'* of representing and resolving epistemic contextual semantics by leveraging logic programming based data representation model (this is due to the *'modularity'* as explained in Section 2.3.3). Based on the *'flexible'* knowledge *'expressive'* power, we have bestowed advanced reasoning features.

The supported reasoning features by approaches appear in literature and covered by our proposed approach is summarized in Table 9.1 (see Section 2.4.3 for details). The table shows that the coverage of our subjective logic based approach is most broad. While

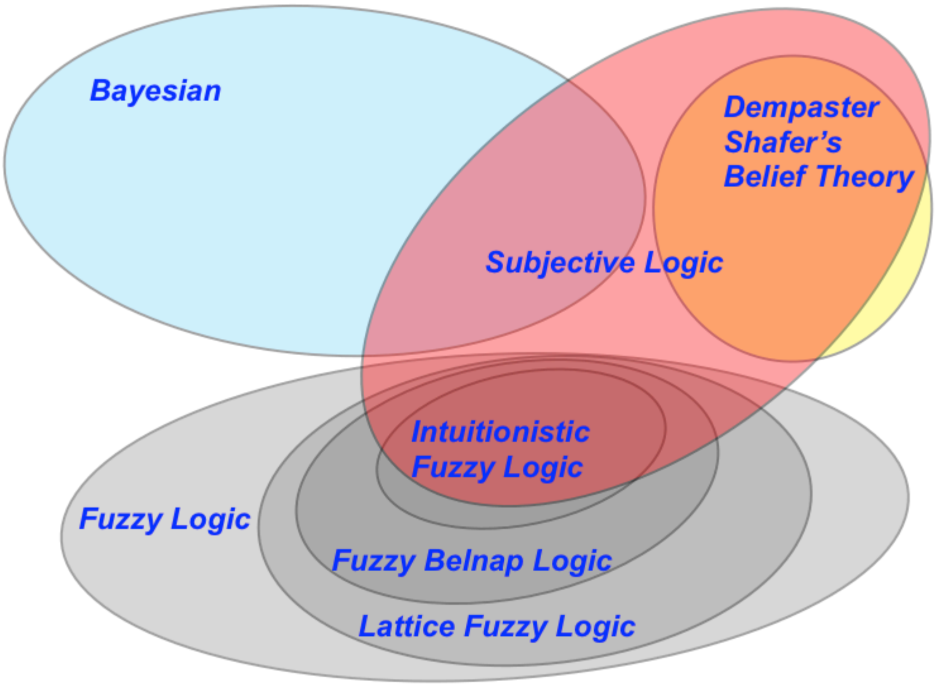

Figure 9.2: Positioning Subjective Logic among Other Uncertainty Representation Formalisms.

some of previous work support advanced features such as *'default reasoning'*, *'vague rule modeling'* and *'belief revision'*, some features such as *'bidirectional inference'* and *'diagnostic abduction'* are only supported by our approach. To cope with such *'non-monotonic'* behavior, in this dissertation, we adopted subjective logic for uncertainty representation formalism. However, subjective logic itself is remain within the expressive power of propositional logic as of may *'intensional'* approaches. In this dissertation to cope with complex semantics, we adopted *'predicate'* logic that is based on first-order logic. For the flexible manipulation of facts and predicates on the need of arithmetic calculation, etc., we also benefited from procedural handling of them.

The discussion in Section 5.9.1 shows that our approach using subjective logic is *'logically'* robust. Especially, compared with bilattice based approach that can be considered as Fuzzy-Belnap, Discussion 1 shows that our approach is free from the *'paradoxes in the truth table of multivalued logics'*. Discussion 2 shows that the *'logical soundness'* is impor-

Approach	Ogale / Yuri et al. [126, 88]	Akdemir et al. [13]	Jianbing et al. [110]	Shet et. al [152, 155]	Anderson / Dorado et al. [17, 54]	Our approach [81, 80, 82] [83, 79]
Knowledge Modeling	Rule Based (Prop. L)	Ontology (DL)	Rule Based (FOL)	Rule Based (FOL)	Rule Based (Prop. L)	Rule Based (FOL)
Uncertainty Formalism	Stochastic Grammar	-	Dempster Shafer	Bilattice	Fuzzy Logic	Subjective Logic
Default Reasoning	-	-	-	√	-	√ Chapter 5
Vague Rule Modeling	-	-	-	-	√	√ Chapter 6
Bidirectional Inference	-	-	-	-	-	√ Chapter 7
Diagnostic Abduction	-	-	-	-	-	√ Chapter 8
Belief Revision	-	-	-	√	-	√ Chapter 5-8

Table 9.1: The Coverage of the Proposed Approach in terms of Reasoning Power.

tant in visual surveillance system. It makes the use of subjective logic attractive. In the sense that the Fuzzy-Belnap has similar uncertainty representation tuple, it resembles subjective logic. In addition to this, in Section 3.1.2, we have shown that the subjective logic is derived from Dempster Shafer belief theory. Similarly, In Section 3.1.3, we have reviewed that subjective logic is a special case of Bayesian, that uses Beta family of conjugated distribution among other distributions. Figure 9.2 shows our view of positioning subjective logic among other uncertainty formalism. The reason for the complementary area between subjective logic and Bayesian is because subjective logic can not handle other prior distributions that can not be approximated to beta distribution. Naturally, subjective logic can not handle multi-modal distributions or non-conjugated form of distributions. Moreover, some of advanced techniques in Bayesian approach such as MCMC (Markov Chain Monte Carlo) [33] is not captured in subjective logic theory. However, the fact that it bridges multiple uncertainty formalism gives us a good intuition for interpreting and operating uncertainty from multiple perspectives.

For contradictory and imprecise knowledge based reasoning, we could benefit from our subjective logic based *'default reasoning'*. Especially, in the forensic sense of visual surveillance that needs to reason about a propositional hypothesis to be investigated after an incident or a report, it is natural to examine all positive and negative contextual evidences that are related to the given hypothesis and fuse them to derive plausible conclusion based on default reasoning.

The support of *'vague rules'* allows for representing lack of information about a proposition. Therefore, we could roughly assign our subjective priors with lack of information, and observations can also be represented with any degree of ignorance, therefore we believe this better reflects human intuition and real world situations. Especially, rule can embed its own opinion calculation scheme thereby, allows for sophisticated propagation of opinions through the inference pipeline. In Chapter 6, especially, we have demonstrated how the reasoning results from uncertain spatio-temporal rules could be used with default reasoning.

Another important advantage is the support of *'bidirectional'* interpretation of conditionals. The advantage is that it offers more choices to handle the trade-offs between expressive power, semantic clarity and computational efficiency. For highly varying and complex compositional events in visual surveillance data, we could benefit from the expressive power and modularity of the extensional approach, while leveraging the intensional interpretation of conditionals to facilitate bi-directional inference.

Finally, the main advantage of the *'diagnostic logical abduction'* is that it doesn't require the user to ask a specific query but let them concentrate on collecting observations of their interest. For a collected set of observations, the system fulfills an abduction to suggest probable hypotheses and rate the uncertainty of each hypothesis. We believe this way of using forensic visual surveillance will offer the user more choices on semantic analysis.

There are, however, drawbacks as well. One of the biggest drawbacks is the *'scalability'* against large scale data. To be fully applicable for practical real applications, the reasoning power should be *'scalable'*. Section 4.6 shows performance benchmark of triple stores and rule-engines against different scale of metadata (i.e., fact base in view of rule-engines). Unfortunately, it seems that logical formalism based approaches and the triple stores of ontological metadata are by themselves not sufficiently scalable. The *'scalability'* problem is common issue in the realm of ontology related research field and also expert system related researches. One good news, however, is that there have been undergoing active research focus on the *'scalability'* issue. We have briefly introduced some possible remedies shown in literatures. In addition, the *'scalability'* of low-level feature matching is also important and is an active research topic in the realm of high dimensional vector indexing field. Therefore, we believe and hope that we could benefit from those researches to resolve the *'scalability'* issue in the near future.

9.3 Open Issues and Future Work

Besides the benefits and drawbacks discussed in the previous section, there are still open issues around the proposed approach.

While the proposed approach offers *'flexibility'* and *'expressive power'* for knowledge modeling, the actual *'knowledge acquisition'* and *'expression'* should be done by *'domain experts'*. Therefore, how to accurately define context rules so that the derived reasoning results make sense is an important issue. As shown especially in the case study of Section 8.7, we believe thorough domain engineering and rule derivation is very critical because the final result would be affected by this aspect.

This is also related to an important issue on the verification of the proposed approach applying to more complicated situational reasoning. Therefore, it is important to generate and share large scale video data set that contain rich *'epistemic'* semantics. For the verification and comparison, the video data set should also come with ground truth. However, unlike data sets for traditional vision analytics such as human or object detection and tracking, such *'epistemic'* semantic data set is hard to generate in that : 1) it would naturally contain not only multiple objects but also multiple human instances who should regally agree on the use of the data set. 2) it would also naturally deal distributed multiple camera that each instance should be labeled properly across the camera sources. 3) the scenarios should be throughly considered before we build up such data set. Therefore, we believe proper data set should be generated in a collaborative manner by many researchers as it was done for TRECVID [123].

There are several open issues also related to subjective logic formalism such as how to better model the reputational function, how to automatically assign proper prior opinions to rules, how to extend this concept to multinomial frame based event, etc. Especially, how to interpret uncertainty values

Therefore, our future research will cover above mentioned issues and extend the shown approach to more complicated scenarios using automatically generated large scale data.

9.4 Summary and Epilogue

We believe that there are several important contributions in this dissertation. First, this dissertation lays out a systematic approach and methodical support for a more intelligent semantic reasoning system, especially in the sense of *'epistemic'* forensic (-post) analysis of visual surveillance data. Traditionally, most of approaches to forensic sense of visual surveillance applications have remained in signal/low-level feature processing. Content-based retrieval is another remarkable approach that tries to search a scene that matches to given image queries. Such methods have been limited to handle only simple and deterministic queries due to their heavy dependency on signal level processing. Only a few researches have been attempted to handle complex semantic aspects such as using traditional database and ontology technics. However, such work lacks handling uncertainty and heavily rely on deterministic data scheme thereby, the data processing components are too much tied on the pre-defined scheme. Even a small change hits the consistency of the system thereby, the change forces entire system to be modified. Unlike legacy approaches, this work proposes the use of logic programming that is extended with subjective logic theory for uncertainty handling and epistemic reasoning. To the best of our knowledge, this is the first attempt that subjective logic theory is applied to visual surveillance application.

It is worth to note that one remarkable research has been attempted in a similar manner using bilattice framework implemented with prolog. The bilattice framework demonstrated reasoning power such as belief revision and default reasoning. However, it is important to note that the bilattice framework based approach lacks soundness in terms of logic theory. Indeed, the operations in bilattice framework are rather related to fuzzy theory than traditional probability based Bayesian. Most of the vision analytic modules today tend to based on Bayesian approach throughout its design and training phase. Therefore, for system engineering to reuse such analytic data, providing a framework that can be also reasoned in a Bayesian sense is preferred. In contrast, subjective logic not only provides Bayesian sense of interpreting reasoning data but also bridges many aspects of bilattice based approach, fuzzy based approach, dempster shaferian approach and Bayesian approach. Especially, we have demonstrated how conditional premises can be factorized and handled in a Bayesian sense by using subjective logic's deduction and abduction operators.

In this dissertation, we also demonstrate dynamic assessment of a vague proposition by introducing a reputation concept. We believe this way, we better capture human intuition on modeling and handling vague propositions. We further extend our approach to model default reasoning behaviour. To the best of our knowledge, this is also the first time on modeling default reasoning in a schematic way using subjective logic operators. A comprehensive comparison with L-fuzzy logics and billatice theory is conducted. This comparison shows many overlap between subjective logic theory and bilattice in the behavioral aspects of handling contradictory information. However, through the comparison we showed that subjective logic is more robust in handling logical premises. This is again due to the lack of soundness in bilattice that often derives counter intuitive reasoning result unlike subjective logic theory.

Based on the proposed approach, we further extend the system so that it can handle abductive reasoning. Semantically speaking, such abductive queries are more complex to handle due to its ambiguity. We tackled this by adding a layer that reason about queries to be asked and a layer that uses abductive logic programming to draw potential hypotheses given set of observations. Our demonstration showed the feasibility of using the proposed approach on diagnostic queries. To the best of our knowledge, this is also the first time using subjective logic for assessment of abductive reasoning result.

The attempt to semantic reasoning done in this dissertation has given several insights into the design of intelligent visual surveillance system. Development of better intelligent visual surveillance systems will require feedback. That is, deriving machine-processable reasoning categories to develop a semantic reasoning mechanism and the systems are often made by using the insights during the attempt to building such a system. In fact, this dissertation especially provides a comprehensive insights on the relationship between different uncertainty formalisms. Preliminary results indicate that this approach is feasible towards semantic reasoning for visual surveillance applications.

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