Improved Analytical Models of SNIR Degradation in Presence of Pulsed Signals and Impact of Code-Pulse Synchrony

F. Soualle, M. Cattenoz, C. Zecha, EADS-Astrium K. Giger, Technische Universität München

Email: francis.soualle@astrium.eads.net

BIOGRAPHY

Francis Soualle graduated in 1995 with a Diplom-Engineer in mechanics at the "Institut catholique d'arts et métiers", and in 1998 in Digital Communication Techniques at the "Ecole Supérieure d'Electricité", both in France. Since 2000 he has been working in the field of navigation satellites as system engineer at EADS Astrium. After being involved in several studies on radio frequency interference modeling and GNSS receiver performances, he now concentrates especially on GNSS compatibility and interoperability issues, signal design and candidate architectures for the future European satellite navigation system, Galileo.

Mathieu Cattenoz received the M.S. degrees in electrical engineering from both "Ecole Supérieure d'Electricité" (Supélec), France and Technische Universität München (TUM), Germany, in 2011. In his M.S. thesis, he has worked on the modeling of the impact of pseudolite signals onto the GNSS receiver performance at EADS Astrium.

Kaspar Giger received the M.S. degree in electrical engineering and information technology from the Swiss Federal Institute of Technology (ETH), Zurich, Switzerland, in 2006. He is currently pursuing the Ph.D. degree at the Institute for Communications and Navigation, Technische Universität München, Munich, Germany, working on new signal tracking algorithms.

Christian Zecha holds a PhD in physics. Since 2006 he works as a system engineer in the satellite navigation department at EADS Astrium. His main working areas are modelling and simulation of receiver front-ends, development of tracking algorithms and analysis of GNSS receiver performance.

ABSTRACT

An increasing number of studies investigate the impact of pulsed interferences in the frequency bands allocated to satellite navigation and show that such interferences can be considered as a significant threat for GNSS receiver performances. Analytical models which include on the one side the pulse parameters (pulse peak power, pulse duty cycle or pulse repetition frequency) and on the other side the receiver front-end parameters (ADC levels, filter bandwidth and pulse blanking threshold) already allow to closely evaluate the corresponding degradations. However, the impact of the spreading codes and especially their interaction with the interfering pulses is usually neglected.

It is indeed often assumed that the randomness of the spreading codes enables to ignore any repetitive chip pattern of the code that could correlate with periodical pulse sequences. Moreover, if the code segments corrupted by the high power pulses are blanked too, they are not used afterwards in the correlation process. The balancedness of the remaining codes should not lead to any artifact in the signal-to-noise plus interference ratio (SNIR) estimated with the correlator output. Hence, for pure random codes, the pulse positions should not influence the statistical properties of the correlator output. If however the codes contain series of identical chips, socalled runs, which are synchronized and similar to the pulse sequence and if these runs are not perfectly blanked, the effective SNIR will likely be modified. Although the design of navigation codes should guarantee that such periodical chip patterns do not exist, detailed investigations show that certain properties of non-strictly random codes could still give rise to some unexpected results for the SNIR.

It is consequently the intention of this paper to first set-up a more accurate analytical model for the impact of pulsed signals onto the receiver SNIR and secondly to show that in some specific conditions, non-ideal code properties might lead to anomalies of the SNIR degradation in a pulsed interference environment. Such conditions are 1) Non-ideal randomness properties of the spreading sequences, 2) Pulse interference sequences which are synchronized with sub-patterns of the codes and 3) Specific configurations of the receiver front-end which favor such SNIR degradations.

1 INTRODUCTION

Typically, a GNSS receiver can face high power pulsed signals in two situations. In the first one, the pulses constitute interfering signals, transmitted by external systems, and having no commonalities with the GNSS signals. This is the case, for example, of the DME/TACAN pulsed signals transmitted in the frequency bands shared with the Galileo E5 signals. In the second one, the pulses are transmitted by terrestrial emitters, aiming at improving the receiver's navigation performance (accuracy, availability) in a specific area, like an airport. Those sources, called pseudolites, share the frequency bands with the GNSS signals as well as many features of the signal structure.

In both cases it is mandatory to estimate how the corresponding pulsed signals will affect the receiver performance. This is usually quantified with the signal-to-noise plus interference ratio, SNIR, at the correlator output and analytical models help determining the pulse effects onto this figure of merit.

The objective of this paper is then to show the sensitivity of the SNIR with parameters related either to the receiver front-end, to the interfering pulses or to the spreading codes. For this purpose, the following structure has been adopted.

In the first part, the main functional blocks of the receiver front-end (bandlimiting filter, AGC, ADC and blanker) will be described, as well as their behaviour in presence of pulsed signals. Then, a literature survey of existing models used to evaluate the impact of pulsed signals onto the SNIR will follow. It will be shown that each of the models focuses only on one specific type of interfering scenario. It will also be shown that some characteristics like the AGC dynamic or the interaction between the pulse amplitude and the blanking threshold needs to be described more precisely to refine the existing models. As a consequence enhanced analytical models will be developed to evaluate the impact of pulsed interferences onto the SNIR. Monte Carlo simulations will enable to validate the corresponding analytical models.

In a second part, specific examples of receiver front-end configurations and pulse characteristics will be proposed to demonstrate the influence of the non-ideal randomness properties of the spreading sequences used for navigation. Here, the three main Golomb postulates of code randomness will be recalled, and particular attention will be paid to the balance of positive and negative code chips as well as the identification of segments of code, called runs, which are composed of identical chip values. Such runs can effectively interact with the pulse entering the correlator, leading to unexpected SNIR variations. Again, simulations will be used to verify that for some specific cases the "synchrony" between the pulses and the code runs could effectively lead to deviations of some dBs, w.r.t. standard analytical models for the SNIR.

In the final part, the consequences of the former results will be developed for specific applications. This concerns especially scenarios with a zero Doppler between navigation and pulsed signals which will magnify the non-randomness of the spreading codes. Pseudolite applications could belong to this category and an adequate spreading codes selection should therefore be made.

2 FRONT-END RECEIVER MODEL

2.1 FUNCTIONAL BLOCK DESCRIPTION

Considering a typical GNSS receiver front-end, Figure 1 represents the main functional blocks which will influence the SNIR evaluated with the correlator output.



Figure 1: Typical GNSS Receiver Front-End

Analogue filtering and down-conversion are merged in a single block. Here, the main focus is given to the interaction between the Automatic Gain Control (AGC), the Analog-to-Digital Converter (ADC) and the blanker that are now described in more details.

- The AGC will adapt the amplitude of the signal entering the ADC to minimize the quantization losses. The two main methods to achieve this objective consist in regulating either the percentiles of the ADC output distribution or the power of the ADC input over a short interval. This last solution has been retained.
- For the ADC, 2^N quantization levels are considered and all samples beyond the maximal levels $(\pm 2^{N-1})$ are clipped.
- The digital blanker sets to zero all samples whose absolute value is larger than the blanking threshold.

The following three figures show the signal variations after the low-pass filter, after the AGC and after the blanker. The corresponding curves have been generated with a software tool aiming at modeling the receiver front-end chain.



Figure 2: Analogue signal after low pass-filtering



Figure 3: Analogue signal after AGC



Figure 4: Digital signal after blanker

It can be observed from Figure 3 that the proposed AGC uses a gain based on a power estimated over a time window with duration comparable to the pulse duration.

During the pulse, the amplitude decay can be easily recognized and the pulse amplitude once compressed converges to a constant. Due to the relatively low AGC dynamic, the pulse is not compressed rapidly enough and the corresponding samples are blanked (Figure 4). In the remaining part of the compressed and un-blanked pulse, the amplitude of the noise (containing also the navigation signals at very low power) is much too low and is not adapted for later processing (for example large quantization losses have to be expected).

Once the pulse stops, the AGC gain increases again to adapt the amplitude of the noise samples to the ADC quantization grid. At the beginning of this recovery period, the amplitude of the noise and navigation signals is much too low and large quantization losses have to be expected again. Once the steady state is reached after pulse stops, it can be observed that the blanking threshold is set too low: thermal noise samples are also blanked, which leads to additional losses.

The former descriptions enabled to underline the role of each element of the receiver front-end and its incidence on the samples used to evaluate the SNIR in case of inappropriate setting. In the proposed example this corresponded to an AGC recovery time with same duration as the pulse, and a too low blanking threshold compared to the noise standard deviation. It is proposed now to verify quantitatively these observations.

2.2 MATHEMATICAL CONVENTIONS

In this section, the analytical expressions of the signal along the different blocks of the reception chain are described. The corresponding variables are indicated in Figure 1.

Here a single navigation satellite signal and a single pulsed signal are considered at reception. This simplified situation enables to better illustrate the receiver behavior and to derive the analytical models. The corresponding methodology can be extended for the situations with multiple satellite navigation signals (one of them being the desired one) and with multiple pulsed signals (interferers or pseudolite).

<u>Received Signal</u>

The baseband notation for the received signal r(t) can then be decomposed as follows:

 $r(t) = s^{s}(t) + \tilde{s}^{p}(t) + w(t)$ (1)

Where $s^{s}(t)$ represents the navigation signal, $\tilde{s}^{p}(t)$ the pulsed signal and w(t) the additive thermal noise. All three signals are considered in-phase, which represents a worst case situation for the impact of the pulsed signal.

- The navigation signal can itself be written as:

$$s^{s}(t) = \sqrt{P^{s}} \sum_{m=-\infty}^{m=+\infty} c_{m}^{s} p^{s}(t - mT_{c}^{s} - \tau^{s}) = \sqrt{P^{s}} c^{s}(t - \tau^{s})$$
Herein
(2)

 $c^{s}(t)$ is the spreading code sequence of length L and c^{s}_{m} is the mth chip value

 $p^{m}(t)$ is the chip waveform with a PSD, called $G_d(f)$,

normalized over the transmission bandwidth.

 T_c^s is the chip duration

 P^{s} is the power at receiver antenna output port

 τ is the code delay

- The pulsed signal, $\tilde{s}^{p}(t)$, is characterized by a Pulse Repetition Frequency (PRF), f_{p} , and a pulse duration (PD), T_{p} . The corresponding Pulse Duty Cycle (PDC)

is given by PDC = $T_p f_p$. The pulse has a peak power equal to P^p , a pulse shape $p^p(t)$, and a PSD, called $G_p(f)$, also normalized over the transmission bandwidth.

In the particular case of pulsed Pseudolite signals, only a portion of spreading codes will be transmitted during the pulse (see [4]).

- The additive thermal noise, w(t), is defined by a Gaussian distribution, p(w), with a Power Spectral Density (PSD) N_0 supposed flat over an infinite bandwidth.

<u>Signal After Filtering</u>

The equivalent low-pass filter h(t) is assumed to be a brickwall filter of one-sided bandwidth $\beta/2$ (β is the equivalent passband bandwidth). The filter output reads:

$$r_f(t) = h(t) * r(t)$$
(3)

The noise signal (n(t)=h(t)*w(t)) is white over the bandwidth β , and follows the corresponding distribution: $n(t) \sim N(0, \sigma^2)$ with $\sigma^2 = N_0 \cdot \beta$.

• Signal After AGC:

The AGC multiplies the filtered signal $r_f(t)$ with a gain G(t) to maintain a constant power at the input to the ADC. This gain is inversely proportional to the power estimated over the recovery time period (RT):

$$r_{g}(t) = G(t) \cdot r_{f}(t)$$

$$G^{2}(t) = \left(\frac{1}{\mathrm{RT}} \int_{t-RT}^{t} r_{f}^{2}(t) dt\right)^{-1}$$
(4)

This means that after multiplication with the AGC gain, the signal $r_g(t)$ has unit power over any time window of duration RT. If the AGC input would be a noise with variance σ^2 , the AGC output, $r_g(t)$, has variance one (unit power). This level is then used as reference for the quantization grid of the ADC.

• Signal After ADC:

The role of the ADC is two-fold:

- It samples the signal $r_g(t)$ at a sampling rate f_s equal to the Nyquist frequency $f_s=\beta$.
- It quantizes the signal $r_g(t)$ into a discrete valued signal that can take 2^N values if it belongs to the interval [- L_{clip} , L_{clip}]. L_{clip} is called the clipping voltage. If $r_g(t)$ exceeds this interval it is clipped as follows:

$$f_{ADC}(r_g(t)) = \begin{cases} -L_{clip} & \text{if } r_g(t) < -L_{clip} \\ L_{clip} & \text{if } r_g(t) > +L_{clip} \\ r_d(t) & \text{otherwise} \end{cases}$$
(5)

For typical receiver implementations the blanking threshold is smaller than the clipping voltage and the effects of the clipping can be ignored. Because one of the main objectives was to observe the effects of the blanking threshold, BTH, onto the SNIR, the clipping voltage is set much higher than its optimal value which minimizes the quantization losses (see [11]). This solution allows a larger range of variations for the BTH.

• Signal After Blanker:

The signal after the blanker is then given by:

$$x(t) = f_B(r_d(t)) = \begin{cases} r_d(t) & \text{if } |r_d(t)| < \text{BTH} \\ 0 & \text{otherwise} \end{cases}$$
(6)

It is important to note for the later mathematical derivations that $f_b(\cdot)$ is odd and linear over [-BTH, BTH].

• Correlation:

The final step of the signal processing chain consists in correlating the digital signal, x(t), with the local replica, i.e. the receiver internal code, $c^{s}(t)$. The correlation is performed over a code period of duration, T (also called coherent integration time).

$$C(i) = \frac{1}{T} \int_{(i-1)T}^{iT} x(t) \cdot c^{s}(t) dt$$
(7)

• **SNIR Estimator:**

The signal-to-noise plus interference ratio, SNIR is representative for the performance of the acquisition, the phase tracking loop and demodulation and in some extends to the code tracking loop (see [7]). The expression of the SNIR for the tracked satellite signal, s'(t), is given by (see also [9]).

$SNIR[C] = \frac{E^2[C]}{var[C]} $ (8)	3)
--	----

2.3 ASSUMPTIONS

For the derivations of the mathematical models for the SNIR, the following assumptions have been used:

- **[A.1]** The magnitude of the power of the satellite navigation signal is significantly smaller than the power of the thermal noise.
- **[A.2]** All signals are considered real. Derivations suppose that all energy is on the in-phase component.
- **[A.3]** The chip waveforms are considered two-valued (+1/-1). This corresponds to the most usual waveforms

(BPSK, BOC). Derivations could be extended to multi-level waveforms like the CBOC.

- [A.4] The amplitude of the pulse is constant.
- **[A.5]** The effects of band limitation on the navigation signals are ignored (β >>Gabor bandwidth of the received navigation signal). It means that the chip plateaus do not see any ripples and can be considered as constant. This is also true for the pulse.
- **[A.6]** The ADC samples at Nyquist rate $(f_s = \beta)$.
- **[A.7]** Quantization is assumed to be performed with high-enough resolution (large number of quantization levels) such that quantization losses can be neglected. Furthermore, no clipping effect will occur as long as BTH < $L_{\text{clip.}}$ As a consequence, $r_d(t) = r_g(t)$.
- **[A.8]** No transient effects of AGC are considered. As shown later in section 4, two AGC behaviors will be distinguished: a fast or a slow gain regulation will apply according to the value of the recovery time w.r.t. the pulsed duration. It will be seen that in each situation the AGC gain will take a constant value during the time intervals either with or without pulses.

The former assumptions will always be applicable in this paper. Now further assumptions will be used for the derivation of the closed form expressions for the SNIR. Note that for the analysis of the code/pulse synchrony effects, these assumptions will not apply (see section 7).

- [A.9] All spreading codes are random and independent.
- **[A.10]** Even if the pulses occur periodically, no time dependency between the pulses and the received spreading code exist (the PRF is not multiple of the inverse of the code period). In that way the pulse can arise in any portion of the code.
- **[A.11]** The portion of the spreading code during the pulse is considered random and balanced (as many "0" as "1" symbols), as well as the portion of the code out of the pulse.
- **[A.12]** The pulse takes positive and negative values with same probability.

3 EXISTING SNIR DEGRADATIONS MODELS

In literature several analytical models of the SNIR have been proposed. They usually give expressions for the SNIR of satellite and pseudolite signal tracking in presence of interference. The models differ according to the type of interference (continuous wave, narrow-band Gaussian noise, pseudolite signals, etc.), the interference power and the relative dynamic of the interference transmitter w.r.t. the ranging source. Here a review of the models considering pulsed interferences is proposed.

• Model for low power interfering pulses

In [3] the following expression for the SNIR in presence of a low power pulse is proposed:



• Model for multiple high and low power interfering pulses

The above model considers that no pulse is blanked. Therefore a different model was proposed in [3] to take into account concurrent reception of multiple pulses with low and high powers, which potentially may trigger the blanker:

$$SNIR = T_{int} \frac{S(1 - PDC_B)^2}{(1 - PDC_B).N_o + \sum_{i=1}^{N} PDC_i \int_{-\beta/2}^{\beta/2} G_d(f).\tilde{G}_{p,i}(f)df}$$
(10)

Herein

- PDC_B is the proportion of time when the signal level exceeds the blanking threshold.

- *N* represents the number of pulses with peak power lower than the blanking threshold. Here, each low power pulse with index *i* is characterized with a **non-normalized** PSD, $\tilde{G}_{n,i}(t)$, and a Pulse Duty Cycle, PDC_{*i*}.

• Model for multiple high power pseudolite pulses

In [5], the situation of K_p non-overlapping pseudolite pulses is analyzed. Their power is sufficiently high to saturate (clipped ADC) the front-end of a receiver which is not equipped with a blanker (note that this situation is not considered in the current paper since BTH < L_{clip}).

The SNIR for the tracked satellite signal is given by:

SNIR $- \frac{s.(1-K_p \cdot PDC)}{s.(1-K_p \cdot PDC)}$	(11)
$\operatorname{SIVIR}_{\operatorname{Sat}} = p \cdot K_p \cdot \operatorname{PDC} + (1 - K_p \cdot \operatorname{PDC})$	(11)

Herein

- $s = 10^{(S/I)_{typ}/10}$ represents the typical *post-correlation* signal-to-noise plus interference ratio in absence of pulsed pseudolite signals (without saturation). For the calculation of the interference power, I_{typ} , the average number of interfering signals and the average code cross-correlation function values are taken into account (in addition to the thermal noise). Furthermore, the powers are measured over the front-end filter bandwidth, β . [4] shows that in the particular case of the GPS CA signals, 10 satellites are visible and the average cross-correlation is -30 dB.

- $p = 10^{(P_{PL}/I)_{max}/10}$ represents the maximal *post-correlation* pulse-to-noise-plus-interference ratio (in saturation). In [4], it is supposed that the saturated power equals the thermal noise power into the front-end bandwidth, β (= N_0 , β). In comparison to *s*, this ratio is derived by considering the worst-case configuration of alignment between the spreading code of the tracked signal and one of the interfering pseudolites signals (in the case of GPS C/A codes this value is -21.6 dB [14])

When considering the tracking of one of the pseudolite signals, it is supposed again that the receiver operates in saturation. The SNIR becomes (from [5]):

SNIR –	$s_{\max} \cdot PDC$	(12)
SIVICPL =	$p \cdot (K_p - 1) \cdot \text{PDC} + (1 - (K_p - 1) \cdot \text{PDC})$	(12)

Herein $s_{\text{max}} = 10^{(S/I)_{\text{max}}/10}$ represents the maximal (saturated) signal-to-noise-plus-interference power ratio. In the case of the GPS CA code [5] shows that $(P/I)_{\text{max}}=(S/I)_{\text{max}}-21.6$ dB.

• Model considering the front-end receiver dynamic behavior

In [6] the SNIR model which considers the dynamic of the receiver front-end, and especially the Recovery Time (RT) of the AGC, is proposed for a single pulse supposed completely blanked.

$$SNIR = T_{int} \cdot (C / N_o) \cdot (1 - PRF \cdot (PDC + RT))$$
(13)

Hence, each SNIR expression is representative of a specific configuration of the receiver (application of a blanker, working in saturation mode, dynamics of the AGC), or of the interfering signals (high, low power, pulse overlapping). All of these expressions can be considered in some extends complementary. Now, it would be useful to have a more generalised expression which could cover these different situations Furthermore, simultaneously. it is proposed to additionally account for the following aspects:

- The blanking threshold amplitude and its effects on the thermal noise samples
- The contributions of the noise and the tracked signal during the pulse.
- For the calculation of the interference contribution, the properties of the partial cross-correlation should be considered during the pulse only (a section of epoch), instead of considering the cross-correlation properties over a complete code epoch. Moreover, the crosscorrelations of this model do not have to be evaluated only for code delays equal to an integer number of chips but also for code delays expressed in fraction of chip. Indeed, in that former case, the chip waveforms have an additional influence on the cross-correlation.
- Equations (11) and (12) considers the maximal crosscorrelation value which is a worst case appearing only a very small proportion of time, while most of crosscorrelation experiences between satellites and pseudolites will be much smaller.
- Finally the dynamic and the relative position between the interfering and desired signals have also to be covered.

It is therefore proposed to derive a set of analytical models which could include the previous existing models and take additionally into account some of the aforementioned aspects.

4 AGC BEHAVIOUR

4.1 PRESENTATION

Figure 3 represented the behavior of the output of the AGC whose recovery time was roughly equal to the pulse duration. This situation could be considered as non-optimal since the pulse was not totally blanked and a non-negligible part of the noise samples were useless for later correlation due to the transient time after pulse stopped. It

is now proposed to consider two extreme cases: a fast and a slow AGC. Here the analysis of corresponding signals at the output of the AGC will help supporting the analytical derivations for the SNIR model by highlighting the relevant contributions.

4.2 FAST AGC

Figure 5 represents the output of a fast AGC.



Figure 5: Signal after fast AGC

For a fast AGC, the gain is estimated on a time window much shorter than the duration of the pulse. Therefore:During the pulse, the AGC gain depends on both pulse and thermal noise powers:

$$G(t) = G_{on} = 1/\sqrt{\sigma^2 + P^p}$$

If the power of the pulsed signal is magnitudes stronger than the noise, the gain could even be simplified to

$$G_{on} \approx 1/\sqrt{P^{I}}$$

• Beyond the pulse, the power of the thermal noise, σ^2 , determines the AGC gain value, $G(t) = G_{\text{off}} = 1/\sigma$. As shown on Figure 5 the pulse is compressed and takes an amplitude equivalent to the normalized variance of the thermal noise present out of the pulse.

Hence it can be observed that for a fast AGC two different gain values G_{on}/G_{off} have to be considered.

It can be seen from Figure 5 that when the blanking threshold is configured such that the pulsed signal is suppressed, non-negligible parts of the noise and navigation signal would also be potentially blanked leading to undesirable SNIR degradations. Therefore the application of a blanker in combination with a fast AGC is questionable. But even without blanker, the fast AGC efficiently compresses the pulsed signal with a factor $1/\sqrt{P^{p}}$ and thus reduces its degrading contribution in the SNIR. If the navigation signals still exist during the pulse, they are so compressed that their contribution to the SNIR is marginal.

Finally, the quantization error for the noise samples outof-the pulse is identical to the one without pulse since the AGC gain takes the steady state and nominal value $1/\sigma$.

4.3 SLOW AGC

Figure 6 represents the output of a slow AGC.



For a slow AGC, the gain is now estimated on a time window whose duration encompasses several pulses. Therefore the AGC gain is identical during and aside the pulses and equals:

$$G(t) = G_{on} = G_{off} = 1/\sqrt{\sigma^2 + PDC \cdot P^p}$$

Hence, the pulse amplitude after AGC becomes:

$$G(t).\sqrt{P^{p}} = \sqrt{P_{p}/(\sigma^{2} + \text{PDC} \cdot P^{p})}$$
$$\approx \sqrt{1/\text{PDC}} \quad \text{if } \sigma^{2} << \text{PDC}.P^{p}$$

And the noise standard deviation becomes:

$$G(t).\sigma = \sqrt{\sigma^2 / (\sigma^2 + \text{PDC} \cdot P^p)}$$

$$\approx \sqrt{\sigma^2 / (\text{PDC} \cdot P^p)} \text{ if } \sigma^2 << PDC.P^p$$

Again, the $G_{\rm on}/G_{\rm off}$ notation can be used for later analytical derivations even though both gains are equal now. As a consequence, for a slow AGC, the general behavior of the signal before and after the pulse is identical, only the scale has changed as shown in Figure 6.

For a slow AGC, the application of the blanker is now worthwhile as soon as its amplitude is set between the standard deviation of compressed noise, and the pulse amplitude. For the interval without pulses and since the noise is now compressed, higher quantization losses have to be accounted since their amplitude is no more set optimally for the ADC quantization grid. However and because a large number of quantization bits (e.g. 8) has been considered as working assumption ([A.7]), even compressed the signals are quantized over enough levels which still enable to neglect the quantization losses.

For the interval with pulses, if the pulse is effectively suppressed due to an appropriate blanking threshold setting, this is also true for the noise and navigation signals (except for some marginal samples corresponding to the "negative" queues of the noise distribution, which draw the amplitude of the AGC output below the blanking threshold).

5 IMPROVED SNIR DEGRADATIONS MODELS

5.1 SEGMENTATION

Figure 5 and Figure 6 show that during each coherent integration interval, T, two contributions can be easily distinguished:

- For a fast AGC, the first interval contains only navigation signal and thermal noise samples and is not compressed. The second interval will contain the samples for the compressed pulse in addition to the noise and signal samples.
- For a slow AGC, the first interval contains also only navigation and thermal noise samples but now it is compressed with the gain: $1/\sqrt{\sigma^2 + \text{PDC} \cdot P^p}$. The second interval contains no samples if the blanker is appropriately set, or contains the pulsed signal, navigation signals and noise all compressed with the same former gain.

Because the spreading codes have been chosen as random, the signal during the pulse is independent from the signal outside the pulse ([A.10]). Therefore the mean (resp. variance) of the correlator output can be evaluated with the weighted average of the mean (resp. variance) of two partial correlator outputs. The first partial correlator output is obtained by correlating the segments without pulses, the second one is obtained by correlating the segments during the pulses. Note that for the variance, the noise and interference contributions are inversely proportional to their activity period (see [15]).

As a conclusion, the weighted mean and variance accounts for the PDC. This is shown on the following equations:

$E[C] = (1 - PDC) \cdot E[C_{off}] + PDC \cdot E[C_{on}]$	(14)
$\operatorname{var}[C] = (1 - \operatorname{PDC}) \cdot \operatorname{var}[C_{\operatorname{off}}] + \operatorname{PDC} \cdot \operatorname{var}[C_{\operatorname{on}}]$	(14)

Both mean and variances are then applied to the general SNIR expression in Eq. (8).

As a consequence the next sections will concentrate on the derivation of the statistical properties of the partial correlations between the local replica and:

- the thermal noise plus navigation signal, multiplied by the gain G_{off} , which is applicable aside the time intervals with pulses
- the pulsed signal (and thermal noise plus navigation signals) when multiplied by G_{on} .

Note that if the blanking threshold is set below the pulse amplitude, a large majority of samples are suppressed and the terms $E[C_{on}]$ and $var[C_{on}]$ are very small or even negligible in equation (14).

In the following sub-sections the analytical expressions for the contributions to the mean and variance will be derived for the following cases:

- Contribution of the noise and navigation signal in intervals without pulses (section 5.2)
- _ Contribution of the noise and navigation signal in intervals with pulses (section 5.3)
- Contribution of the pulsed signals (section 5.4) _

SIGNAL 5.2 TRACKED NOISE AND **CONTRIBUTIONS** TO THE PARTIAL-CORRELATION STATISTICS FOR **INTERVALS WITHOUT PULSES**

In this section, the analytical expressions for the mean and variance of the partial correlations evaluated in intervals without pulses are presented. In this situation the samples containing thermal noise and the navigation signals are multiplied with the gain $G_{\rm off}$ whose value differs according to the fast or slow AGC type. The exact mathematical derivations, which are based on Taylor expansion up to the order 1, are given in appendix A.

The expression for the mean of the correlator output is:

$$E[C_{\text{off}}] \approx G_{\text{off}} \sqrt{P'} \left[1 - 2Q_o \left(\frac{\text{BTH}}{G_{\text{off}} \sigma} \right) - \frac{\text{BTH}}{G_{\text{off}} \sigma} \sqrt{\frac{2}{\pi}} e^{-\frac{\text{BTH}^2}{2(G_{\text{off}} \sigma)^2}} \right]$$
(15)

The expression for the variance of the correlator output is:

1

 G_{a}^{2} βT

 $\operatorname{var}[C_{\operatorname{off}}] \approx$

$$\frac{1}{2} \frac{1}{2} \sigma^2 \left(1 - 2Q_2 \left(\frac{\text{BTH}}{G_{\text{off}} \sigma} \right) \right)$$
(16)

Note that in the former expressions the $Q_n(x)$ functions are defined by:

$$Q_n(x) = \int_{x}^{+\infty} \frac{x^n}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$
(17)

5.3 TRACKED NOISE SIGNAL AND **CONTRIBUTIONS** то THE PARTIAL-CORRELATION STATISTICS FOR **INTERVALS WIT PULSES**

In this section, the analytical expressions for the contribution of the noise to the mean and variance of the partial correlations during intervals containing pulses are presented. In this situation the samples containing the pulse, the thermal noise and the navigation signals are multiplied with the gain G_{on} . The mathematical derivations, again based on Taylor expansion up to the order 1, can be found in appendix B. It is recalled that an important assumption enabling the derivation of these quantities is that the pulse amplitude is constant ([A.4]).

The expression for the mean of the correlator output is:



The former expression supposes of course that the pulses are not blanked (see section 4 for more details).

5.4 CONTRIBUTION OF THE PULSES TO THE PARTIAL-CORRELATION STATISTICS

This section determines the contribution of the pulses when correlated with the navigation signals. According to the relative dynamic between the navigation signal source and the pulse emitter two different analytical models have to be considered.

• Spectral Separation Coefficient for dynamic configuration

In this situation it is supposed that the navigation signal source is moving w.r.t. the receiver and the terrestrial pulse emitter. This is mainly the case for a typical GNSS scenario using MEO satellites as ranging sources. Here, the relative delay between the local replica, locked to the satellite navigation signal, and the pulsed signal permanently drifts over time.

The most encountered and accepted analytical model to account for the effects of the interference onto the SNIR is based on the Spectral Separation Coefficient (SSC) model (e.g. [13], [12]). The SSC expression is given by:

$$SSC = \int_{-\beta/2}^{\beta/2} G_s(f) \cdot G_p(f) df$$
(20)

It is recalled that the PSDs of the desired, $G_s(f)$, and interfering, $G_p(f)$, signals are normalized over their respective transmission bandwidths.

When applying the SSC in the specific context of pulsed interference it is necessary to account for pulse duty cycle, as already proposed in equation (9). Note that equation (9) applies for pulsed with low peak power. For higher power it is necessary to account for the AGC gain which scales the corresponding power. Hence, the variance of the correlator output during the time interval containing the pulses becomes:

$$\{\operatorname{var}[C_{\mathrm{on}}]\}_{\mathrm{pulse}} = \frac{G_{\mathrm{on}}^2 \cdot P_p \,\mathrm{SSC}}{T}$$
(21)

Hence the AGC gain will effectively affect the impact of the pulsed signals by compressing their amplitude with $G_{\rm on}$.

By considering that the portion of the spreading codes during the pulse are well-balanced ([A.11]) the mean of the correlator output during such time intervals is null $(E[C_{on}] = 0)$.

• Waveform Convolution Coefficient for the static configuration

Here, the navigation signal source is fixed w.r.t. the terrestrial emitting source and the receiver is not moving. Two typical scenarios apply: For the first scenario, the navigation source is a pseudolite (usually emitting in pulses) while the pulsed source can either be an interfering source (radar) or another pseudolite located in its vicinity. The second scenario corresponds to a geostationary satellite emitting signals used for ranging, additionally to the MEO satellites. This could be the case of the SBAS (EGNOS, WAAS) systems using geostationary satellites to disseminate corrections data and possibly ranging sources.

In both situations the delay, $\tau_{s,p}$, between the replica and the pulses does not vary significantly. The use of the SSC is not exactly appropriate and the relative delay $\tau_{s,p}$ needs to be accounted. In appendix C the variance of the partial correlator with the pulse contribution is shown to become

$$\{\operatorname{var}[C_{\operatorname{on}}]\}_{\operatorname{pulse}} = G_{\operatorname{on}}^2 \cdot P_p \cdot \frac{T_c^s}{T} \cdot \operatorname{WCC}(\tau_{s,p})$$
(22)

Here WCC represents the Waveform Convolution Coefficient, function of $\tau_{s,p}$, and is defined as

$$WCC(\tau_{s,p}) = \frac{\left(p^{s}(\tau_{s,p}) * p^{p}(\tau_{s,p})\right)^{2}}{T_{c}^{s} \cdot T_{p}} * \sum_{m=-\infty}^{m=\infty} \delta(\tau_{s,p} - kT_{c})$$
(23)

The derivation of this equation supposes T_p multiple T_c .

In fact, from (21) and (22) it is shown that the SSC is proportional to the average of the WCC for $\tau_{s,p}$ varying between 0 and T_c .

$$SSC = T_c^{s} \left\langle WCC(\tau_{s,p}) \right\rangle_{0 \le \tau_{c,s} \le T_c}$$
(24)

• Consideration of the blanked samples

Because it is also necessary to consider the samples contained in the pulses which are blanked, the contribution of the pulse to the variance becomes:

$$\{\operatorname{var}[C_{on}]\}_{\operatorname{noise}} \approx G_{on}^{2} \cdot P^{p} \cdot \frac{T_{c}^{s}}{T} \cdot K_{CC} \cdot \left[\mathcal{Q}_{o} \left(\frac{-\operatorname{BTH} - G_{on} \sqrt{P_{p}}}{G_{on} \sigma} \right) - \mathcal{Q}_{o} \left(\frac{\operatorname{BTH} - G_{on} \sqrt{P_{p}}}{G_{on} \sigma} \right) \right]$$

$$(25)$$

The coefficient K_{cc} is either the WCC or the SSC according to the relative velocity between the interfering and navigation signal sources. For a dynamic situation K_{cc} will take the form of the Spectral Separation Coefficient and for a static situation K_{cc} will take the form of the Waveform Convolution Coefficient.

As a conclusion the variance of the partial correlator output during the pulse equals (considering the pulse and noise contributions):

 $\overline{\operatorname{var}[C_{\text{on}}]} = \left\{ \operatorname{var}[C_{\text{on}}] \right\}_{\text{noise}} + \left\{ \operatorname{var}[C_{\text{on}}] \right\}_{\text{pulse}}$ (26)

5.5 ANALYTICAL EXPRESSIONS FOR THE SNIR

In this section, it is proposed to provide the closed form expressions of the SNIR for specific settings of the blanking threshold corresponding to the following cases.

• Slow AGC with very high blanking threshold

Here it is supposed that the blanking threshold is set much larger than the pulse amplitude (no sample is blanked during the complete integration time). In that case, simplifications described in Appendix–D enable to deduce the general expressions of the mean and variance of the correlator output over a coherent integration:

$$E[C] = (1 - PDC) \cdot G_{off} \sqrt{P^s} + PDC \cdot G_{on} \cdot \sqrt{P^s}$$

$$var[C] = (1 - PDC) \frac{G_{off}^2 \sigma^2}{\beta T} + PDC \cdot \left(\frac{1}{\beta T} G_{on}^2 \sigma^2 + G_{on}^2 \cdot P^p \cdot \frac{T_c^s}{T} \cdot K_{cc}\right)$$
(27)

Based on these expressions, the closed form expression for the SNIR is (see Appendix–D):

$$SNIR = \frac{P^{s}}{\frac{\sigma^{2}}{\beta T} + PDC \cdot P^{p} \cdot \frac{T_{c}^{s}}{T} \cdot K_{cc}}$$
(28)

This expression implicitly means that the navigation signals samples can still contribute to the SNIR (no power loss of $(1-PDC)^2$ at numerator).

• Slow AGC with optimal blanking threshold

Here the blanking threshold is set such that the complete pulse is blanked but all noise samples during the periods without pulse are preserved (optimal situation). In this situation, all contributions to the mean or variance during the pulse are suppressed, and since $G_{off}=G_{on}$, for a slow AGC, the SNIR expression is:

SNIR –	$(1-PDC)P^{s}$	$T \cdot \frac{(1 - PDC)P^s}{s}$	(20)
SIVIN -	$\sigma^2/\beta T$	N _o	(29)

• Fast AGC without blanker

For a fast AGC, no blanker is applied since it was shown in section 4.2 that it does not provide any real benefit. Furthermore, it is supposed that the AGC gain compressed the thermal noise and navigation signals during the pulse in such a way that they do not influence either the mean or variance of the partial correlation. The closed-form expression for the SNIR becomes:

$$SNIR = \frac{(1 - PDC)^2 \cdot P^s}{(1 - PDC) \cdot \frac{\sigma^2}{\beta T} + PDC \cdot \sigma^2 \cdot \frac{T_c^s}{T} \cdot K_{cc}}$$
(30)

6 VALIDATION OF THE ANALYTICAL MODELS WITH MONTE-CARLO SIMULATIONS

6.1 SIMULATION PARAMETERS

For the validation of the former analytical models the parameters described in Table 1 have been considered.

Categ.	Parameter	Value	Comment
	Filter Bandwidth, β	40.92 MHz	
	AGC Recovery Time	1µs <rt<1s< th=""><th></th></rt<1s<>	
	ADC bits	8	
Receiver	Blanking Threshold	<i>∂</i> ²< BTH²< <i>∂</i> ²+10 dB	Reference is thermal noise power
	ADC sampling frequency, <i>f_s</i>	40.92 MHz	Nyquist
	Integration time	$T = 1 \mathrm{ms}$	
	Waveform	BPSK(5)	Galileo E6-like
Nov	Code Length	L = 5115	signals
Signal	Code Type	Random	Indiv. code per Monte-Carlo run
	Received Power	$P_1 = -158.5 \text{ dBW}$	Ant. output port.
Thermal Noise	PSD, N_0	-201.5 dBW/Hz	
	Pulse Duration	$PD = 27 \ \mu s$	
Pulsed Signal	Peak Power	$P^p = -110 \text{ dBW}$	Ant. output port.
	Pulse Repetition Frequency	1KHz <prf<4khz< th=""><th></th></prf<4khz<>	

Table 1: Parameters used for the validation

Based on the proposed parameters, the SNIR in absence of any pulsed interfering signal, without blanker and having infinite front-end bandwidth at the receiver equals

$$SNR = T \frac{P_l}{N_c} = 13 \, dB \tag{31}$$

6.2 RESULTS

• Validation for the periods without pulses

First the model is validated in absence of a pulse (thermal noise and navigation signal only). For this purpose, the analytical expressions (15) and (16) for the mean and variance of the correlator outputs have been directly included into equation (8). Figure 7 shows the variations of the SNR degradations as function of the blanking threshold. Here both analytical models and simulations results are shown (the intervals of confidence for the SNIR obtained with simulations are also represented)



Figure 7: SNIR Degradation without pulse with blanking threshold

As expected it can be verified that when the blanking threshold is set too low, the corresponding percentiles of the noise distribution are suppressed, which impacts strongly the SNR. It can also be stated that a very good match exists between results of the Monte-Carlo simulations and the analytical models.

• Validation of Relationship between SSC and WCC

In this section it is proposed to provide one simple example of calculation of WCC, and to verify the relationship between the SSC and the WCC (Eq. (24)). For this purpose the simple case of two BPSK signals with same waveform duration (1µs) is considered. This would be the case of a pulsed pseudolite signal using a BPSK chip modulation which would interfere with the reception of a navigation signal using a BPSK too. The following figure illustrates how equation (23) is applied in this specific scenario to generate the WCC as function of the delay $\tau_{s,p}$.

The last figure shows that the average of the WCC over the propagation time $\tau_{s,p}$ is 2/3. Application of equation (24) leads to $SSC_{BSPK,BPSK} = 10*\log 10(2/3 \cdot 1/T_c) =$ -61.8 dB.Hz⁻¹, which is a value that can be also be recovered by the application of equation (20).



Figure 8: WCC and SCC for BPSK-BPSK case As a conclusion Eq. (24) can be verified and therefore the SCC effectively represents the time average, w.r.t. the propagation times, of the WCC over one chip duration.

• Validation of SNIR model with pulses

In this section the SNIR model is validated when considering the presence of a pulse. Here the segmentation of the coherent integration time *T* is applied which leads to a combination of the mean and variance for both sub-intervals with and without pulses (see Eq. (14)). The PRF is set to 1 KHz (all other pulse parameters are listed in Table 1). Furthermore, two AGC types have been considered: the first one is a fast AGC using a recovery time of 1 μ s, much smaller than the pulse duration of 27 μ s. The second uses a recovery time of 1ms and can be considered as a slow AGC. Figure 9 shows the corresponding variations of the SNIR degradations w.r.t. a nominal SNIR of 13 dB.



Figure 9: SNIR Degradation with pulse and blanking for slow and fast AGC. PRF = 1KHz. $P_p=-110 \ dBW$

- For a fast AGC, the noise samples in the intervals without pulse have a nominal distribution that would be observed for a scenario considering only navigation signals $(G_{off}=1/\sigma^2)$ and the amplitude of the pulse, once compressed corresponds to the 1- σ percentile of the noise distribution (see Figure 5). Hence, the application of a blanker for pulse suppression can not be done without "scarifying" noise samples containing the navigation signals. This is the reason why the SNIR degradations are especially large for low blanking thresholds. When the threshold becomes larger (>5 dB) these degradations reduce, before converging to an asymptote representative of the effects of the non-blanked (but still compressed) pulse onto the SNIR, as given by Eq. (30):
- For a slow AGC the pulse and noise samples are compressed with the same gain $G_{\rm on}=G_{\rm off}$. Therefore the pulse still emerges from the thermal noise (see Figure 6) and it is possible to effectively suppress it when setting the blanking threshold appropriately. Now, if the blanking threshold is too high, the pulses are not blanked and will impact the SNIR. Eq. (28) now applies. When the blanking threshold decreases and reaches the pulse amplitude (~6dB on Figure 9) the pulse is suppressed. So the SNIR degradation is minimal and given by Eq. (29). Now, if the blanking threshold is further reduced, a similar effect as for the fast AGC occurs: the noise samples are blanked and the SNIR degradation increases again.

6.3 AGC DYNAMIC AND PULSE DURATION

The former simulations have been repeated for different values of the recovery time, varying between 1µs and 1s. Figure 10 shows the corresponding SNIR degradations.



Two typical behaviors can be recognized on this figure:

- For recovery times smaller than the pulse duration, the SNIR variations are very close (0.5 dB disparity) and can all be explained in a similar way to the curve in Figure 9. A deeper analysis shows that this behavior

occurs as long as RT<PD/2.

- Similarly, SNIR variations are very close for slow AGC which corresponds to RT > 4 PD. Explanations similar to those of Figure 9 can be proposed again.
- In between (PD/2<RT<4 PD) a transition zone shows an SNR degradation around 4 to 6 dB independent of the blanking threshold.

Hence, an AGC can be described as fast or slow only when comparing the pulse duration to the recovery time. It must be noted that the bounds PD/2 and 4 PD are only applicable for the proposed AGC implementation. Other implementations might provide other bounds, but this general trend should be preserved.

Finally, by choosing a recovery time having the same order of magnitude as the pulse duration, the averaged SNIR degradations are the largest when compared to the slow or fast AGC situations. The SNIR does not benefit from the slow or fast AGC advantages, but takes only the corresponding drawbacks. Figure 3 illustrated perfectly this property.

7 IMPACT OF SPREADING CODES AND CODE/PULSE SYNCHRONY

7.1 RANDOMNESS PROPERTIES

In [10], three main criteria, called *Golomb's postulates*, are proposed to measure the randomness of pseudo-random codes. These are

- *Balance* property: the number of 1's in every period of a sequence s differs from the number of 0's by at most one.
- *Two-levels auto-correlation property*: **s** is a sequence with a 2-level auto-correlation function (ACF):

ACF(k) = 1, if k=0, and ACF(k) = -1/N if $k\neq 0$

- *Run property*: a run of **s** is a subsequence consisting of only 0's or 1's which are neither preceded nor succeeded by the same symbol. The length of a run, called L_{run} , is the number of consecutive identical symbols (for example '10001' is a run of length 3). A run of successive 0's is called a *gap* while a run of successive 1's is called a *block*. For a code, the runs follow a specific distribution which depends on their length, L_{run} .

In [8] the application of the Golomb's postulates to spreading code families used for several navigation systems (Galileo, GPS, QZSS) enabled to identify some (minor) discrepancies. For example,

- One GPS-CA code (PRN 25) shows a run of 16 consecutive 1's between the 967th and 982th chips.
- One E5b-I code (PRN 45) shows a run of 25 consecutive 0's between the 6705th and 6729th chips.

7.2 IDENTIFICATION OF PULSE/CODE RUN SYNCHRONY

The former investigations have shown that relatively long sequences of consecutive and identical chips exist in the spreading codes. Because, 20 GPS CA code of 1ms duration are repeated within one data bit, it means that a small but non-negligible match exists between this repetitive code sequence and a stream of periodical pulses synchronized to the run locations. In case of the GPS CA code (PRN25) pulse sequences with PRF ~1 KHz and pulse duration ~16 μ s would be critical. Figure 13 shows such a code run / pulse synchrony for the GPS CA (PRN-25) during the first millisecond.



Figure 11: GPS-CA (PRN 25)-Pulse Alignment A similar synchrony can be created for the E5b-I (PRN45) codes (now over a duration of 2.5µs).



Figure 12: Galileo E5b-I (PRN 45)-Pulse Alignment

These two examples highlight a match between run and pulse sequences, when considering one pulse per primary code period of 1ms. It is now proposed to extend this investigation in a more systematical way by considering several pulses per code period. Indeed, it might also happen that K runs of same length L_{run} repeat periodically in one code duration. In terms of integrated energy during the correlation process, this would be equivalent to a single run of duration $K.L_{run}$. Furthermore, even if runs are not exactly of the same length but occur at periodical locations in the codes, again a good match with the periodic pulse sequence could appear.

Therefore, the systematic search consisted to characterize the pulsed sequences with a mask, $p_{PD,PS}(t)$, defined with two parameters: its pulse duration, PD, and its pulse spacing, PS, such that PRF = 1/(PD+PS).



Figure 13: Param. for Code /Pulse Synchrony Analysis

This mask takes the values +1/0 values and is correlated with the spreading code sequence, $c^{k}(t)$ using the +1/-1 voltage notation, over the code period. Note that if 1/PRF is smaller than the code period, $p_{\text{PD,PS}}(t)$ is repeated until reaching the corresponding code duration (leading to $\tilde{p}_{PD,PS}(t)$). This correlation function depends on PD, PS

and the delay between the pulse and the code, $\tau_{s,p}$, as shown in the following equation:

$$PR_CCF(PD, PS, \tau_{CP}) = \frac{1}{L} \sum_{p=1}^{L} c^{k}(t) \cdot \widetilde{p}_{PD, PS}(t - \tau_{s, p})$$
(32)

Note that it is supposed that the pulse duration is an integer number of chips (sufficient for the proposed pulse-run investigation).

The maximum of this correlation function over all delays, $\tau_{s,p,}$ while keeping PD and PS fixed, is retained to measure the degree of matching between the pulse sequences and the periodical runs.

The following figure shows the corresponding correlation maxima for the specific case of the GPS CA (PRN 25) sequences. Here the pulse duration varied between 1 chip (\sim 1µs) and 100 chips (\sim 100µs).



Figure 14: Pulse/Code Run correlation for the GPS-CA (PRN 25)

It can be observed that even if the corresponding correlation does not take large values (maximum at 5%), some specific PD/PS combinations lead to pulse sequences similar to the code ones. Furthermore, the better matches occur for small PS values. Now, because most of the pulsed interference affecting GNSS receivers have a PRF smaller than 5 KHz, it means that the effects of code run / pulse synchrony should be analysed primarily for pulse spacing larger than about 200 chips.

A "cut" of the corresponding mesh figure is proposed for a pulse duration of 100 μ s (~100 chips).



Figure 15: Pulse/Code Run correlation for the GPS-CA (PRN 25)- PD = 100 chips

It can be verified that some of the peaks take the form of a sinc² shape (see PS of 920 chips). This characteristic should be further analysed in more detail.

7.3 IMPACT OF CODE-RUN/PULSE SYNCHRONY ON SNIR

Because the pulse sequence is synchronized with the primary code, no variability in the correlation output has to be expected but rather an additional DC component. This DC component will add to the contribution of the received signal at the correlator and might therefore bias the SNIR estimator of equation (8).

It is now proposed to verify with Monte-Carlo simulations if the code run / pulse synchrony will effectively affect the SNIR as anticipated. For this purpose, it is proposed to align the pulse and the corresponding run to cover a worst case situation.

Here the special case of the GPS CA code (PRN 25) will be considered for illustration. The following table shows the corresponding characteristics.

	Chip Rate [MCps]	Run Length [Chip]	PRF [KHz]	PS [µs]	PD [µs]
GPS-CA (PRN 25)	1	16	1	~1007	~16
Table 2: Parameters used for pulse synchrony					

As far as the parameters for the receiver are concerned, a fast AGC with a recovery time of 1μ s (much smaller than the proposed pulse durations) has been applied. The peak power is kept to -110 dBW. Furthermore, no blanker is applied for such a fast AGC (see section 5.5).

Figure 16 shows the SNIR as function of the PRF when considering either random codes (black curve) of 1023 chip length, or the GPS CA PRN-25 when the code runs and the pulses are synchronized (blue curve).



Figure 16: SNIR for random code (L=1023) and the GPS CA (PRN 25) for Code Run / Synchrony

The former figure shows effectively that when the pulses are synchronised with the code runs an obvious increase of SNIR appears. This situation occurs for a PRF of 1 KHz. As soon as the PRF deviates from this value the corresponding abnormal behaviour can not be observed and the SNIR follows the one obtained with random codes (other peaks at 600Hz and 1400Hz are also visible which certainly correspond to other code run/ pulse synchronies).

The artefact of the SNIR is now justified. As explained, the code run / pulse synchrony leads to an additional DC component which is proportional to the pulse amplitude once compressed and to the pulse duty cycle. Therefore the general expression for the mean of the correlator output becomes (no navigation signal contribution):

$$E[C] = (1 - \text{PDC}) \cdot G_{off} \cdot \sqrt{P^s} + \text{PDC} \cdot G_{on} \sqrt{P^p}$$
(33)

For a fast AGC, the SNIR given by Eq. (30) becomes

$$SNIR = \frac{((1 - PDC) \cdot \sqrt{P^s} + PDC \cdot \sigma)^2}{(1 - PDC) \cdot \frac{\sigma^2}{\beta T} + PDC \cdot \sigma^2 \cdot \frac{T_c^s}{T} \cdot K_{cc}}$$
(34)

For the specific case of the GPS CA code PRN 25, which shows a run of length $L_{run} = 16$, PDC = $L_{run}/T = 1.56\%$, and $\sigma^2 = \beta \cdot N_0$. Application of the former equation leads to an SNIR of 17.2 dB or equivalently an improvement of 4.8 dB w.r.t the case of random codes. This value is very close to the one observed on Figure 16.

In order to consolidate the corresponding interpretation it is proposed to inverse the sign of the pulse, which is equivalent to an angle phasing of π , between the satellite and the pulse signals (eq. (1)). The red curve on Figure 16 shows the corresponding SNIR behaviour. Due to the sign inversion, the numerator of the SNIR (Eq. (34)) becomes $((1-PDC).\sqrt{P^s} - PDC.\sigma)^2$. In that case the numerical application leads to an SNIR of 0.8 dB or equivalently a reduction of 11.6 dB w.r.t the case of random codes. This value is close to the one observed on Figure 16 which consolidates the corresponding justification.

An analogy is now proposed between the spectral lines of the real navigation signal PSD and the code runs. Indeed both characteristics are the consequence of the non pure randomness of the codes. For pure random codes, the smooth PSD of navigation signal using a BPSK waveform should follows a { $f_c \cdot sinc^2(f/f_c)$ } function, where f_c is the chip rate. However, for pseudo-random codes the PSD will be composed of spectral lines spaced with the inverse of the code periodicity. In [14] it is shown that the deviations of these spectral line amplitudes w.r.t. the signal and code type. Figure 17 shows an example of the first four lines, spaced by 1 KHz in the special case of a GPS CA, as well as the envelop PSD, represented with a yellow line. Note that due to the data modulation each spectral line is replaced by a { $f_d \cdot sinc^2(f/f_d)$ } function, f_d being the data rate equal to 50 bps.



Figure 17: PSD and close-in for the GPS CA PRN-1 showing the spectral lines

If a narrowband or a CW interferer is tuned to the corresponding spectral line frequencies the degradations of the SNIR are much higher than for other carrier frequencies. This increase of the vulnerability of the navigation signal to narrow band interferences, caused by the non-randomness of the codes can be thus revealed by the analysis in the frequency domain. The current paper has just shown that a similar increase of the navigation signal sensitivity against pulsed signals could also be highlighted when analysing the code characteristics (code runs, balance) in the time domain. Hence both domains can be considered as complementary to highlight possible weaknesses of the navigation signals against interferers. Note that it would be interesting to verify if the frequencies corresponding to the periodical runs can be related to a specific excesses of spectral lines w.r.t. ideal smooth PSD. Indeed, in case of a spreading code including a periodic run sequence, the spectral line corresponding to the run periodicity is expected to exceed the smooth PSD of a truly random code.

It is important to note that the unexpected artefacts in SNIR are observed when both navigation and pulsed signals have the same phase. Now, the Doppler will certainly help reducing the corresponding effects by letting this phase vary over the different integrations. Similarly the pulse modulation (+1/-1) will also attenuate the average SNIR estimated over several coherent integrations. Nevertheless, some abnormal variations should remain in the SNIR for specific PRF.

7.4 SCENARIO OF CODE/PULSE SYNCHRONY AND RECOMMENDATIONS

Different scenarios can consequently lead to unexpected SNIR behaviors when code runs and pulses are synchronized. Again the condition is that the relative dynamic of the pulse transmitter and navigation signal source is low over a time period corresponding to several integration periods. This would be the case for:

- a navigation signal transmitted by a geostationary satellite, and pulsed interferer or a pseudolite.
- a navigation signal transmitted by pseudolite and again pulsed interferer or another pseudolite signal.

It is clear that other conditions related to the receiver front-end are also necessary to magnify the corresponding effects. This would be the case of a receiver equipped with a fast AGC (RT smaller than PD) or a receiver which is not equipped with a blanker and operates in saturation in presence of the pulsed interferer. Typical receivers not equipped with blanker could belong to the category of the mass market receivers, for example. Furthermore, the receiver should have a slow dynamic too.

8 CONCLUSION AND WAY FORWARD

The outcomes of this paper are manifolds:

- It proposed refined analytical models for the SNIR in presence of pulsed interferences, which better account for the blanking threshold value and the dynamic of the AGC regulation w.r.t. to the pulse duration. Here two main behaviors have been distinguished: a fast one (resp. slow) when the recovery time is significantly smaller (resp. larger) than the pulse duration.
- It showed that using a slow AGC usually leads to better performances because the pulsed signals still emerge from the thermal noise after AGC regulation which authorizes their suppression with the blanker. The use of a blanker for a fast AGC does not provide significant benefits. Now the AGC itself could serve as mitigation method by compressing the pulse. It is also recommended to avoid an intermediary zone when the pulse duration and recovery time are similar.
- It highlighted that the Spectral Separation Coefficient (SSC) could be used when the difference in propagation times between the navigation signal source and the receiver on a one side, and between the pulse emitter and the receiver varied sufficiently over several integration periods on the other side. For a fixed configuration, the Waveform Convolution Coefficient (WCC) should be used. Finally, it was demonstrated that the WCC and SCC were closely related.
- It showed that in the special cases when subsets of identical chips, called runs, and pulses were synchronized during several integrations, artifacts in the SNIR could appear. In that situation the impact of the pulses onto the SNIR can not be modeled by a random contribution but by a constant which impacts the SNIR. This is especially true for receivers using

fast AGC. The amplitude and sign of the corresponding variations depends on the pulse duration and the phasing between the navigation and pulse signals. Pulse modulation and non-zero Doppler will certainly reduce these artifacts without completely suppressing them. Such SNIR abnormal behaviors could be encountered when using either navigation signals transmitted by geostationary satellites or pseudolites. Hence, the runs increase the sensitivity of the navigation signal to periodical pulses, in the same way that the deviations of spectral lines w.r.t. the smooth PSD, applicable to pure random codes, increase the sensitivity to narrow band (CW) interfering signals.

As a way forward to this study, the following aspects could be further covered:

- The operational mode of a receiver working in saturation should be considered. This situation applies mainly for scenario involving pseudolite signals which usually lead the ADC in clipping. SNIR models similar to those of equations (11) and (10) could be developed when accounting additionally the AGC dynamic.
- The proposed refined models could be extended to pulses showing non-constant amplitude during the pulse duration (chirps signals for example). Nevertheless, the models proposed in this paper and applied for pulses with constant amplitude could already provide the worst case situations w.r.t. other pulses.
- The effects of quantization and filtering should also be accounted. Indeed, for the derivation of SNIR models it was always assumed that the quantization losses are negligible. However, for a slow AGC it can be shown that the compression of the navigation signals should increase the corresponding quantization losses. Here a similar approach to this applied in [11] could be followed.
- The refined SNIR models could be applied for dimensioning pseudolite systems. Here the results on the code/pulse synchrony could be taken into account for the selection of spreading codes for pseudolites which should show as less runs as possible.
- The investigations regarding the code run/pulse synchrony could be pursued, in order to verify if the SNIR artifacts really represents a marginal phenomenon or have to be taken into account more seriously (similarly to the receiver sensitivity against by CW interferers), at least for dedicated applications.

9 ACKNOWLEDGMENTS

The authors would like to thank her colleague Suzanne Schloetzer for her support during the preparatory work for this study.

10 REFERENCES

[1] Galileo SIS-ICD. Issue 1-0. May 23 2006. http://www.galileoju.com/page2.cfm [2] ICD-GPS-200 "Navstar GPS Space Segment/Navigation User Interfaces". 14 January 2003.

[3] "Suppression of Pulsed Interference Through Blanking". C. Hegarty, A. J. Van Dierendonck, D. Bobyn, M. Tran, T. Kim, and J. Grabowski, Proceedings of the IAIN World Congress, San Diego, p. 399, June 2000.

[4] "GPS Pseudolites: Theory, Design, and Applications. PhD Thesis". H.S. Cobb. Stanford University, 1997.

[5] "Self-Calibration of Pseudolite Arrays Using Self-Differencing Transceivers". E. A. Lemaster. Thesis. 2002.

[6] "Feasibility of sharing between radionavigationsatellite service receivers and the Earth explorationsatellite (active) and space research (active) services in the 1 251.5-1 300 MHz band". Modification to preliminary draft revision of Recommendation ITU-R RS.1347. Annex 9 to Document 7C/146-E. 25 September 2009.

[7] "Simplified Techniques for Analyzing the Effects of Non-white Interference on GPS Receivers". Chris. Hegarty. ION September 2002.

[8] "Correlation and Randomness Properties of the Spreading Coding Families for the Current and Future GNSSs". F.Soualle, Signal Workshop 2009 in DLR.

[9] "Effect of Narrowband Interference on GPS Code Tracking Accuracy". J.W. Betz. ION. NTM. 2000. Anaheim.

[10] "Shift Register Sequences", Solomon W. Golomb.

[11] "Presampling Filtering, Sampling and Quantization Effects on the Digital Matched Filter Performance", H. Chang, Proceedings of the International Telemetering Conference, San Diego, CA, 1982.

[12] "Effect of Narrowband Interference on GPS Tracking Accuracy". J Betz. ION NTM 2000, 26-28 January 2000, Anaheim, CA.

[13] "Interference Assessment Using Public Information". Jose Perello. Signal Workshop 2009. DLR Oberpfaffehofen.

[14] "*Global Positioning System: Theory and Application I*", volume 163 of Progress in Aeronautics and Astronautics. AIAA, 1996. B.W. Parkinson and J.J. Spilker, Jr editors.

[15] "GNSS Receiver Front-End Modelling for Satellite and Pseudolite Signals". Master Degree Thesis. Technische Universität München. M. Cattenoz. 2011.

APPENDIX A – DERIVATION OF SNIR IN PRESENCE OF THERMAL NOISE ONLY

The objective of this appendix is to derive the SNIR when the received signal is the combination of the thermal noise and the navigation signal only (no pulse is considered in eq. (1)). In that case, the signal power at AGC input is dominated by the thermal noise power which yields to a constant AGC gain, $G(t) = G_{\text{off}}$ The expressions for the mean and for the variance of the correlator output are now derived.

Expression for the Mean of the Correlator Output

The mean of the correlator outputs is given by:

$$\mathsf{E}[C_{off}] = \mathsf{E}\left[\frac{1}{T}\int_0^T x(t)c^s(t)dt\right] = \frac{1}{T}\int_0^T \mathsf{E}\left[x(t)c^s(t)\right]dt \quad (35)$$

Where x(t) is the received signal at ADC output and $c^{s}(t)$ is the replica. Here,

$$\mathsf{E}[x(t)c^{s}(t)] \approx \mathsf{E}[f_{B}(G_{off} \cdot (\sqrt{P^{s}}c^{s}(t) + n(t))) \cdot c^{s}(t)]$$
(36)

Based on the fact that $f_B(\bullet)$ is odd and n(t) has a symmetric distribution, the following equations hold :

$$\mathsf{E}[x(t)c^{s}(t)] = \mathsf{E}[x(t)c^{s}(t) | c^{s}(t) = 1]$$

$$= \mathsf{E}[f_{B}\left(G_{off}\left(\sqrt{P^{s}} + n(t)\right)\right)]$$

$$= \int_{-\infty}^{\infty} f_{B}\left(G_{off}\left(\sqrt{P^{s}} + n\right)\right) \cdot p(n)dn$$

$$(37)$$

Using the definition for the distribution of the noise n(t) yields:

$$\mathsf{E}[C_{off}] = G_{off} \underbrace{\int_{-\frac{\mathrm{BTH}}{G_{off}}}_{-\frac{\mathrm{BTH}}{G_{off}} - \sqrt{P^s}} \left(\sqrt{P^s} + n\right) \cdot \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{n^2}{2\sigma^2}} dn \quad (38)$$

Assuming that $G_{\text{off}} \sqrt{P^s} \ll \text{BTH}$, Taylor expansions can be used at first order to derive the following expression:

$$\mathsf{E}[x(t)c^{s}(t)] \approx G_{off} \sqrt{P^{s}} \left[1 - 2Q_{0} \left(\frac{\mathrm{BTH}}{G_{off} \sigma} \right) - \frac{\mathrm{BTH}}{G_{off} \sigma} \sqrt{\frac{2}{\pi}} e^{-\frac{\mathrm{BTH}^{2}}{2(G_{off} \sigma)^{2}}} \right]$$
(39)

Since $E[x(t)c^{s}(t)]$ is not dependent on time *t*, it can be finally deduced that:

$$\mathsf{E}[C_{off}] \approx G_{off} \sqrt{P^{s}} \left[1 - 2Q_{0} \left(\frac{\mathsf{BTH}}{G_{off} \sigma} \right) - \frac{\mathsf{BTH}}{G_{off} \sigma} \sqrt{\frac{2}{\pi}} e^{-\frac{\mathsf{BTH}^{2}}{2(G_{off} \sigma)^{2}}} \right]$$
(40)

Expression for the Variance of the Correlator Output

The variance of the correlator outputs is given by:

$$\mathsf{E}[C_{off}^{2}] = \mathsf{E}\left[\frac{1}{T^{2}}\int_{0}^{T}\int_{0}^{T}x(t)x(u)c^{s}(t)c^{s}(u)dtdu\right]$$
(41)

 $c^{s}(t)$ is independent of $c^{s}(u)$ when *t* and *u* do not belong to the same chip interval. Denoting by $I_{c}(t)$ the chip interval containing *t*: $\exists m_{1} \in \mathbb{Z}, t \in [m_{1}T_{c}, (m_{1}+1)T_{c}]$. In the following the integration interval is split into two complementary intervals to derive $E[C^{2}]$:

• interval where t and u are not on the same chip interval, i.e. t fixed and $u \notin I_c(t)$

• interval where t and u are on the same chip interval, i.e. t fixed and $u \in I_c(t)$

$$\mathsf{E}[C_{off}^{2}] = \underbrace{\frac{1}{T^{2}}}_{t \in [0,T]u \notin I_{c}(t)} \underbrace{\mathsf{x}(t) x(u) c^{s}(t) c^{s}(u) du dt}_{t \in [0,T]u \notin I_{c}(t)} + \underbrace{\frac{1}{T^{2}}}_{Term2} \underbrace{\mathsf{E}}_{t \in [0,T]u \notin I_{c}(t)} \underbrace{\mathsf{x}(t) x(u) c^{s}(t) c^{s}(u) du dt}_{Term2}$$

$$(42)$$

Each of the terms Term1 and Term2 is now derived.

1) Derivation of the Term1

Since t and u do not belong to the same chip interval, the double integral can be split into a product of integrals depending on t and u, and Term1 can be reduced to:

$$\operatorname{Term1} = \frac{1}{T^2} \int_{t \in [0,T]} \mathbb{E}[x(t)c^s(t)] dt \int_{u \notin I_c(t)} \mathbb{E}[x(u)c^s(u)] du$$

$$= \frac{T - T_c}{T} (\mathbb{E}[C])^2$$
(43)

2) Derivation of the Term2

Since c^l takes either 1 or -1 and $f_B(\bullet)$ is odd then:

$$\operatorname{Term2} = \frac{1}{T^2} \int_{t \in [0,T] u \in I_c(t)} \mathsf{E} \left[f_B \left(G_{off} \left(\sqrt{P^{t}} + n(t) \cdot c^{s}(t) \right) \right) \right] \cdot f_B \left(G_{off} \left(\sqrt{P^{t}} + n(u) \cdot c^{s}(u) \right) \right) dudt$$

$$(44)$$

The fact that the sampling frequency equals the Nyquist one ($\beta = f_s$) enables to consider the noise samples n(t) and n(u) as independent when taken at two different instants. Therefore the correlation function $E[n(t).n(u)] = \sigma^2 . \partial(t-u)$ After several steps, the following expression is established:

$$\operatorname{Term2} = \frac{1}{\beta T^{2}} \int_{0}^{T} \mathsf{E} \left[\left(f_{B} \left(G_{off} \left(\sqrt{P^{T}} + n(t) \right) \right)^{2} \right] dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} \right] dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{1}{\beta T} \right) (\mathsf{E} [C_{off}])^{2} dt + \left(\frac{T_{c}}{T} - \frac{T_{c}}{T} - \frac{T_{c$$

3) Final result for the variance

Finally, from the previous expressions, the variance of the correlator output can be deduced:

$$\operatorname{var}[C_{off}] = \mathsf{E}[C_{off}^{2}] - (\mathsf{E}[C_{off}])^{2}$$
$$= \frac{1}{\beta T} \left(\frac{1}{T} \int_{0}^{T} \mathsf{E}\left[\left(f_{B} \left(G_{off} \left(\sqrt{P^{s}} + n(t) \right) \right)^{2} \right] dt - \left(\mathsf{E}[C_{off}] \right)^{2} \right) \right]$$
(46)

Since $G_{\text{off}}\sqrt{P^s} \ll \text{BTH}$, it is possible to use again a Taylor expansion at first order. Therefore

$$\mathbf{E}\left[f_{B}^{2}\left(G_{1}\left(\sqrt{P^{T}}+n(t)\right)\right)\right] \approx G_{off}^{2}\sigma^{2}\left(1-2Q_{2}\left(\frac{\mathbf{B}\mathbf{T}\mathbf{H}}{G_{off}\sigma}\right)\right) + \left(G_{off}^{2}\cdot P^{s}\left[1-2Q_{0}\left(\frac{\mathbf{B}\mathbf{T}\mathbf{H}}{G_{off}\sigma}\right)-\left(\left(\frac{\mathbf{B}\mathbf{T}\mathbf{H}}{G_{off}\sigma}\right)^{2}+2\right)\cdot\frac{\mathbf{B}\mathbf{T}\mathbf{H}}{G_{off}\sigma}\sqrt{\frac{2}{\pi}}e^{-\frac{\mathbf{B}\mathbf{T}\mathbf{H}^{2}}{2(G_{off}\sigma)^{2}}}\right]\right)$$

$$(47)$$

Because this term does not depend on *t*:

$$\operatorname{var}[C_{off}] \approx \frac{1}{\beta T} \left(G_{off}^2 \sigma^2 \left(1 - 2Q_2 \left(\frac{\mathrm{BTH}}{G_{off} \sigma} \right) \right)$$
(48)

Expression for the SNIR of the Correlator Output

Injecting equations (40) and (48) into (8) yields



APPENDIX B – DERIVATION OF SNIR IN PRESENCE OF PULSED SIGNALS

The objective of this appendix is to derive the SNIR in a time interval when the received signal is dominated by the pulsed signal, with peak power P^p . In this situation the AGC gain G(t) is approximately constant: $G(t)\approx G_{on}$ during that time interval. Two effects will contribute to the variance of the correlator output. The first one is the variance caused by the noise samples suppressed by the blanker. The second one is the variance caused by the pulsed signals if it is not blanked. For the mean only the effect of the noise samples will be considered.

The following expression gives the expression of the blanker input (effect of ADC is neglected), showing the navigation and the predominant pulse signals.

$$x(t) = f_B \left(G_{on} \left(\sqrt{P^s} c^s(t) + \sqrt{P^p} . p^p(t) + n(t) \right) \right)$$
(50)

Expressions of the mean and variance due to the noise contribution (in presence of a pulsed signal)

Because the amplitude of the navigation signal is much lower than this of the pulse, the current situation can be compared to the previous one ("low-power signals only"). Now the interval in which the noise samples are not blanked becomes $[-BTH \pm G_{on}\sqrt{P^{p}}, BTH \pm G_{on}\sqrt{P^{p}}]$ instead of [-BTH, BTH].

The same methodology for the derivation of the mean and variance of the correlator output in presence of a pulse is therefore applied. Furthermore, the condition which enables to apply the Taylor expansion becomes:

$$\left| \mathsf{BTH} - G_{on} \sqrt{P^{p}} \right| >> G_{on} \sqrt{P^{s}}$$

In that case the contribution of the mean and average of the noise only are given by the two following equations:

$$\mathbf{E}[C_{on}] = G_{on}\sqrt{P^{s}} \left[Q_{0} \left(\frac{-\mathrm{BTH} - G_{on}\sqrt{P^{p}}}{G_{on}\sigma} \right) - Q_{0} \left(\frac{\mathrm{BTH} - G_{on}\sqrt{P^{p}}}{G_{on}\sigma} \right) - \frac{\mathrm{BTH}}{G_{on}\sigma} \frac{1}{\sqrt{2\pi}} \left(e^{-\frac{\left(\mathrm{BTH} - G_{on}\sqrt{P^{p}}\right)^{2}}{2(G_{on}\sigma)^{2}}} + e^{-\frac{\left(\mathrm{BTH} + G_{on}\sqrt{P^{p}}\right)^{2}}{2(G_{on}\sigma)^{2}}} \right) \right] \tag{51}$$



Expressions of the variance due to the pulsed signals

In addition, the pulsed signal will also impact the variance of the correlator output. For this purpose either the *Waveform Convolution Coefficient* (WCC) or *Spectreal Separation Coefficient* (SSC) are used, depending on the relative dynamic between the navigation signal and pulsed transmitters. This contribution to the variance equals:



See section 5.4 for the applicability of the coefficient K_{cc} which is either the WCC or the SSC.

Expression for the SNIR of the Correlator Output

Injecting equations (51), (52) and (53) into (8) yields

$$\operatorname{SNIR}[C_{on}] = \frac{(\mathsf{E}[C_{on}])^{2}}{\{\operatorname{var}[C_{on}]\}_{\operatorname{noise}} + \{\operatorname{var}[C_{on}]\}_{\operatorname{pulse}}}$$
(54)

APPENDIX C – EXPRESSION OF THE WAVEFORM CONVOLUTION COEFFICIENT

The objective of this appendix is to derive the expression of the variance caused by an interfering pulsed signal whose source can be considered as static w.r.t. to the navigation signal source, and supposing a minimal dynamic at receiver.

The corresponding variance can be expressed by:

$$\operatorname{var}\left[\frac{1}{T}\int_{0}^{T} (h_{\beta} * \tilde{s}^{p})(t) c^{s}(t) dt\right]$$
(55)

The following model for the pulsed signal will be considered:

$$\widetilde{s}^{p}(t) = \sqrt{P^{p}} \sum_{n=-\infty}^{n=+\infty} c_{n}^{p} p^{p}(t - nT^{p} - \tau_{s,p})$$
(56)

Herein T_p represents the pulse duration, $p^p(t)$ the pulse waveform and C_n^p the pulse value taking either +1 or -1 with equal probability when the pulse exists during the sub-intervals $[n \cdot T_p, (n+1) \cdot T_p]$ and 0 in the other cases. Furthermore, $\tau_{s,p}$ is the difference in propagation times between the navigation signal source and the receiver on a one side, and the pulsed signal source and the receiver on the other side. Here $\tau_{s,p}$ is considered as fixed.

Equation (55) is first derived in the situation when the chip duration equals the pulse duration $(T_c=T_p)$. Then the property $\mathsf{E}[c_n^p c_n^p c_m^s c_m^{s_i}] = \delta_{n,n'} \cdot \delta_{m,m'}$ which comes from

the randomness of the codes and of the pulse values ([A.11] and [A.10]), enables to demonstrate that:

With the fact that p^s is zero out of [0, T_c], the following expression applies:

$$\mathsf{E}\left[\left(\frac{1}{T}\int_{0}^{T}(h_{\beta}*\tilde{s})(t)c^{s}(t)dt\right)^{2}\right]$$
$$=\frac{P^{p}}{TT_{c}}\left(\int_{0}^{T_{c}}p^{p}(u)p^{s}(u-\tau_{s,p})du\right)^{2}*\sum_{n_{2}=-\infty}^{\infty}\delta(\tau_{s,p}-n_{2}T_{c})$$
(58)

For symmetrical waveforms such that $|p^{p}(t)| = |p^{p}(-t+T_{p})|$ and $|p^{s}(t)| = |p^{s}(-t+T_{c})|$, it is derived:

$$\mathsf{E}\left[\left(\frac{1}{T}\int_{0}^{T}(h_{\beta}*\tilde{s})(t)c^{s}(t)dt\right)^{2}\right]$$
$$=P^{p}\frac{T_{c}}{T}\left(\frac{1}{T_{c}}\left(p^{s}*p^{p}\right)(\tau_{s,p})\right)^{2}*\sum_{m=-\infty}^{\infty}\delta(\tau_{s,p}-mT_{c})$$
(59)

The Waveform Convolution Coefficient, $WCC(\tau_{s,p})$, function of $\tau_{s,p}$, is then defined by:

$$\operatorname{WCC}(\tau_{s,p}) = \left(\frac{1}{T_c} p^s(\tau^k) * p^p(\tau_{s,p})\right)^2 * \sum_{m=-\infty}^{\infty} \delta(\tau_{s,p} - kT_c)$$
(60)

In the general case when T_p is different from T_c , one has to extend the support of the waveforms for the pulse and the chip to the longest duration of both. Supposing T^c smaller than T^p , the support of the chip waveform should be extended over T^c by filling the corresponding waveform with zeros where it was not previously defined. Hence both waveforms will have the same time support. One is simply zero padded. Then a similar approach can be followed as previously. The WCC expression becomes now (here it is supposed that T_p is multiple of T_c):

$$WCC(\tau_{s,p}) = \frac{\left(p^{s}(\tau_{s,p}) * p^{p}(\tau_{s,p})\right)^{2}}{T_{c}^{s} \cdot T_{p}} * \sum_{m=-\infty}^{m=\infty} \delta(\tau_{s,p} - kT_{c})$$
(61)

Finally:

$$\operatorname{var}\left[\frac{1}{T}\int_{0}^{T}(h_{\beta}*\widetilde{s})(t)c^{s}(t)dt\right] = P^{p}\frac{T_{c}}{T}\operatorname{WCC}(\tau_{s,p})$$
(62)

APPENDIX D – CLOSED-FORM EXPRESSIONS OF THE SNIR FOR HIGH BLANKING THRESHOLDS

This appendix provides the closed-form expressions for the SNIR when considering that the blanking threshold is not used. Here no sample is blanked and the following simplifications can be used:



Simplified Mean and Variance expressions

Applying the former simplifications enables to deduce the different contributions to the mean and variance to the partial correlations.

- The contribution to the mean and variance caused by the thermal noise in absence of pulses (Eq. (15) and (16)) becomes

$E[C_{ m off}] pprox G_{ m off} \sqrt{P^s}$	(64)
$\operatorname{var}[C_{\operatorname{off}}] \approx rac{G_{\operatorname{off}}^2 \sigma^2}{eta T}$	(65)

- The contribution to the mean and variance caused by the thermal noise in presence of pulses (Eq. (18) and (19)) becomes

$E[C_{\rm on}] \approx G_{\rm on} \sqrt{P^s}$	(66)
$\{\operatorname{var}[C_{\operatorname{on}}]\}_{\operatorname{noise}} \approx \frac{G_{\operatorname{on}}^2 \sigma^2}{\beta T}$	(67)

- The contribution to the mean and variance caused by the interfering pulse signals (Eq. (25)) becomes

$$\{\operatorname{var}[C_{\operatorname{on}}]\}_{\operatorname{noise}} \approx G_{\operatorname{on}}^2 \cdot P^p \cdot \frac{T_c}{T} \cdot K_{CC}$$
(68)

Expression for the SNIR for a slow AGC

For a slow AGC ($G_{on} = G_{off} = 1/\sqrt{\sigma^2 + \text{PDC.}P^p} = G$) it is possible to introduce the former simplified expressions for the mean and variance into Eq. (27) which yields:

$E[C] = G \cdot \left((1 - \text{PDC}) \cdot \sqrt{P^s} + \text{PDC} \cdot \sqrt{P^s} \right)$	(69)
$\operatorname{var}[C] = G^{2}\left(\left(1 - \operatorname{PDC}\right)\frac{\sigma^{2}}{\beta T} + \operatorname{PDC} \cdot \left(\frac{1}{\beta T}\sigma^{2} + P^{p}\frac{T_{c}}{T}K_{CC}\right)\right)$	(70)

Including these expressions in Eq. (8) and neglecting the quantization losses during the pulse yields

$$SNIR = \frac{P^{s}}{\left(\left(1 - PDC\right)\frac{\sigma^{2}}{\beta T} + PDC \cdot \left(\frac{\sigma^{2}}{\beta T} + P^{p} \frac{T_{c}}{T} K_{cc}\right)\right)}$$

$$= \frac{P^{s}}{\frac{\sigma^{2}}{\beta T} + PDC \cdot P^{p} \cdot \frac{T_{c}}{T} \cdot K_{cc}}$$
(71)

Expression for the SNIR for a fast AGC

For a fast AGC ($G_{on} \approx 1/\sqrt{P^p}$ and $G_{off} = 1/\sigma$) it is possible to introduce again the former simplified expressions for the mean and variance into Eq. (27) which yields:

$$E[C] = (1 - PDC) \cdot \frac{1}{\sigma} \sqrt{P^s} + PDC \cdot \frac{\sqrt{P^s}}{\sqrt{P^p}}$$
(72)
$$var[C] = (1 - PDC) \frac{1}{\beta T} + PDC \cdot \left(\frac{1}{\beta T} \cdot \frac{1}{P^p} \cdot \sigma^2 + \frac{T_c}{T} \cdot K_{cc}\right)$$
(73)

Including these expressions in Eq. (8) yields

$$SNIR = \frac{\left((1 - PDC) \cdot \frac{1}{\sigma} \sqrt{P^s} + PDC \cdot \frac{\sqrt{P^s}}{\sqrt{P^p}}\right)^2}{(1 - PDC) \frac{1}{\beta T} + PDC \cdot \left(\frac{1}{\beta T} \cdot \frac{1}{P^p} \cdot \sigma^2 + \frac{T_c}{T} \cdot K_{cc}\right)}$$
(74)

Considering that the AGC gain compressed the navigation signals and thermal noise during the pulse in such a way that they do not influence neither the mean nor the variance of the partial correlation during pulse yields.

$$SNIR = \frac{(1 - PDC)^2 \cdot P^s}{(1 - PDC) \frac{\sigma^2}{\beta T} + PDC \cdot \sigma^2 \cdot \frac{T_c}{T} \cdot K_{cc}}$$
(75)