Differential integer ambiguity resolution with Gaussian a priori knowledge and Kalman filtering

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BIOGRAPHIES

Patrick Henkel studied electrical engineering and information technology at the Technische Universität München, Munich, Germany, and the École Polytechnique de Montréal, Canada. He then started his PhD on reliable carrier phase positioning, graduated with "summa cum laude" in 2010, and is now working towards his habilitation in the field of precise point positioning. He visited the Mathematical Geodesy and Positioning group at TU Delft in 2007, and the GPS Lab at Stanford University in 2008 and 2010. Patrick received the Pierre Contensou Gold Medal at the International Astronautical Congress in 2007, the first prize in Bavaria at the European Satellite Navigation Competition in 2010, and the Vodafone Award for his dissertation in 2011. He is one of the founders of Advanced Navigation Solutions - AMCONAV GmbH.

Patryk Jurkowski is a master student in electrical engineering and information technology at the Technische Universität München, Germany. He completed his bachelor of science with a thesis on "Baseline constrained ambiguity resolution with multiple frequencies" with distinction in 2010. He also received the first prize in Bavaria in the European Satellite Navigation competition in 2010 for a differential carrier phase positioning system. Patryk worked as a student trainee on EGNOS performance and integrity - level B for Galileo at EADS Astrium from 2006 to 2010. He founded Advanced Navigation Solutions - AMCONAV GmbH in 2011.

Christoph Günther studied theoretical physics at the Swiss Federal Institute of Technology in Zurich. He received his diploma in 1979 and completed his PhD in 1984. He worked on communication and information theory at Brown Boveri and Ascom Tech. From 1995, he led the development of mobile phones for GSM and later dual mode GSM/Satellite phones at Ascom. In 1999, he became head of the research department of Ericsson in Nuremberg. Since 2003, he is the director of the Institute of Communication and Navigation at the German Aerospace Center (DLR) and since December 2004, he additionally holds a Chair at the Technische Universität München (TUM). His research interests are in satellite navigation, communication and signal processing. He is also a founder of Advanced Navigation Solutions - AMCONAV GmbH.

ABSTRACT

In this paper, a new maximum a posteriori probability estimation of ambiguities and baselines is proposed for differential carrier phase positioning. It performs a recursive least-squares estimation with an extended Kalman filter, that uses double difference code and carrier phase measurements and Gaussian a priori knowledge about the baseline length, elevation/ pitch angle and azimuth/ heading. The maximum a posteriori probability estimator finds the optimum trade-off between a solution that minimizes the range residuals and one which is close to the priori knowledge.

It is shown that the Gaussian a priori knowledge enables a ten times faster convergence of the float solution, and it substantially suppresses multipath and, thereby, prevents divergence of float ambiguities and baselines. Moreover, the Gaussian a priori knowledge allows some errors in the a priori information, i.e. it is more robust than deterministic a priori knowledge.

INTRODUCTION

In the past two years, there has been a large number of accidents on German inland waterways: There were collisions of vessels and cargo ships, others run aground or against shore or watergates. The damages add up to six-digit Euro values per accident in average, and were often caused by navigation errors.

In this paper, a new method for reliable and accurate differential carrier phase positioning is described for maritime navigation. It is an extension of our recent paper [1], in which we proposed the maximum a posteriori probability estimation of ambiguities and baselines. The estimator uses Gaussian a priori knowledge on the baseline length, pitch angle and heading, and multi-frequency code carrier linear combinations to improve the reliability of ambiguity resolution. The a priori knowledge reduces the size of the search space, and the linear combinations increase the wavelength to noise ratio of the combinations, which enables a substantial reduction of the probability of wrong fixing. The Gaussian a priori knowledge was used for the search and fixed baseline solution, but not yet for the computation of the float solution.

The objectives of this paper are threefold: First, Gaussian a priori knowledge about the baseline length, pitch angle and heading is included in the computation of the float solution. Secondly, a state space model is used by an extended Kalman filter to further improve the accuracy of the float solution. Thirdly, the benefit of the Gaussian a priori knowledge is analyzed for environments with and without multipath. Note that the integer search is not discussed in this paper as it is described in details in [1]-[4].

Fig. 1 shows a coupled cargo ship and a baseline between two GNSS receivers, for which a new differential carrier phase positioning method is developed in this paper: The baseline length and elevation/ pitch angle are known to a certain extent and, thus, are introduced as a priori knowledge in the heading determination.

Fig. 1 Reliable navigation at German river Mosel: The heading shall be determined precisely by recursive least squares estimation with some Gaussian a priori knowledge about the baseline length and pitch angle.

Fig. 2 visualizes the benefit of baseline a priori knowledge on integer ambiguity resolution. The wavefronts from three satellites are depicted, which intersect in numerous points and, thus, leave an ambiguity. The search space can be constrained to a certain amount by the code solution (black circle), but the latter one is not yet accurate enough to find the correct ambiguity. The introduction of Gaussian a priori knowledge on the length and pitch angle of the baseline substantially reduces the size of the search space, but still allows some uncertainty in the a priori knowledge.

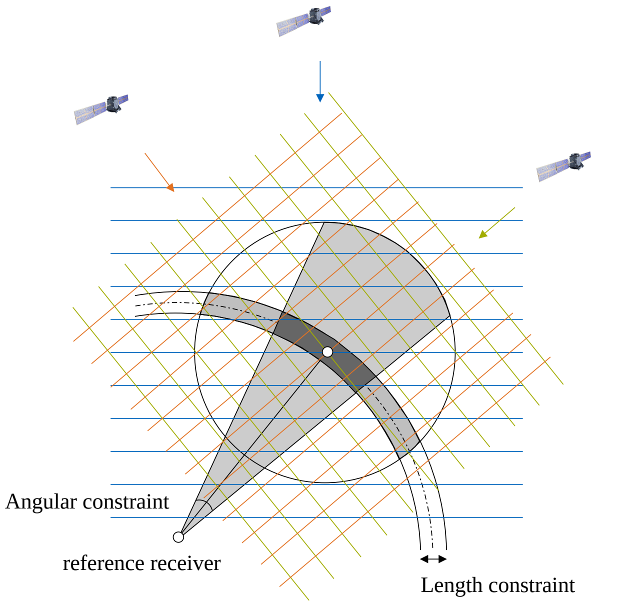

Fig. 2 Integer ambiguity search space: Its size is substantially reduced by some a priori knowledge about the baseline length and pitch angle.

ESTIMATION OF AMBIGUITIES AND BASELINE WITH GAUSSIAN A PRIORI KNOWLEDGE

Measurement model

The double difference code and carrier phases $\Delta\rho_n^k$, $\Delta\varphi_n^k$ from satellite $k \in \{1,\ldots,K\}$ at time n and the a priori knowledge about the baseline length l_{ap} and elevation/ pitch angle $\nu_{1,\mathrm{ap}}$ shall be considered in a single measurement vector, which is modeled as

$$\underbrace{\begin{bmatrix} \Delta\varphi_n^1 \\ \vdots \\ \Delta\varphi_n^K \\ \hline \Delta\rho_n^1 \\ \vdots \\ \Delta\rho_n^K \\ \hline l_{\mathrm{ap}} \\ \nu_{1,\mathrm{ap}} \end{bmatrix}}_{\boldsymbol{z}_n} = \left[\begin{array}{c|c|c} \Delta(\boldsymbol{e}_n^1)^T & 1 & \\ \vdots & \ddots & \\ \Delta(\boldsymbol{e}_n^K)^T & \quad 1 & \\ \hline \Delta(\boldsymbol{e}_n^1)^T & & \\ \vdots & & \\ \Delta(\boldsymbol{e}_n^K)^T & & \\ \hline & & 1 \\ & & \quad 1 \end{array}\right] \begin{bmatrix} \boldsymbol{\xi}_n \\ \boldsymbol{N} \\ \hline l_n \\ \nu_{1,n} \end{bmatrix} + \underbrace{\begin{bmatrix} \varepsilon_{\Delta\varphi_n^1} \\ \vdots \\ \varepsilon_{\Delta\varphi_n^K} \\ \hline \varepsilon_{\Delta\rho_n^1} \\ \vdots \\ \varepsilon_{\Delta\rho_n^K} \\ \hline \varepsilon_{l_{\mathrm{ap},n}} \\ \varepsilon_{\nu_{1,\mathrm{ap},n}} \end{bmatrix}}_{\boldsymbol{v}_n}, \qquad (1)$$

where Δe_n^k denotes the single difference of unit vectors pointing from satellite k and a reference satellite to the receiver, $\boldsymbol{\xi}_n$ is the baseline, $\boldsymbol{N}$ are the integer ambiguities, l_n is the true length and $\nu_{1,n}$ is the true pitch/ elevation angle of the baseline. It shall be parameterized in spherical coordinates in a local reference frame, i.e.

$$\boldsymbol{\xi}_n = \begin{bmatrix} l_n \cos(\nu_{1,n}) \cos(\nu_{2,n}) \\ l_n \cos(\nu_{1,n}) \sin(\nu_{2,n}) \\ l_n \sin(\nu_{1,n}) \end{bmatrix}. \tag{2}$$

Thus, there are only $K+3$ unknowns, which are in a non-linear relationship to the measurement vector $\boldsymbol{z}_n$. Note that the a priori knowledge is modeled as a stochastic quantity in (1), i.e. a Gaussian distribution with mean values l_n and $\nu_{1,n}$ and some errors $\varepsilon_{l_{\text{ap},n}}$ and $\varepsilon_{\nu_{1,\text{ap},n}}$. These errors were introduced as the true baseline parameters are not perfectly known in some applications, e.g. the length of a pusher train of coupled cargo ships is not stationary and also the pitch angle between the front and back of cargo ships slightly varies with the motion of the water. Consequently, a weighted least-squares baseline estimation based on the measurement model of (1) ensures a solution, which represents the optimum trade-off between low range residuals and a baseline close to the a priori knowledge.

State space model

The accuracy of the joint baseline and ambiguity estimation can be improved if a state space model is introduced. In maritime navigation, the length and pitch angle show only small variations over time, and the heading/ yaw angle is the primary parameter of interest. Therefore, the pitch angle, the heading and its rate, and the baseline length are included in the state vector and are modeled as Gauss-Markov processes, i.e.

$$\underbrace{\begin{bmatrix} \nu_{1,n} \\ \nu_{2,n} \\ \dot{\nu}_{2,n} \\ l_n \\ \boldsymbol{N} \end{bmatrix}}_{\boldsymbol{x}_n} = \underbrace{\left[\begin{array}{c|cc|cc} 1 & & & & \\ \hline & 1 & \delta t & & \\ & & 1 & & \\ \hline & & & 1 & \\ & & & & 1 \end{array}\right]}_{\boldsymbol{\Phi}} \underbrace{\begin{bmatrix} \nu_{1,n-1} \\ \nu_{2,n-1} \\ \dot{\nu}_{2,n-1} \\ l_{n-1} \\ \boldsymbol{N} \end{bmatrix}}_{\boldsymbol{x}_{n-1}} + \underbrace{\begin{bmatrix} w_{\nu_{1,n}} \\ w_{\nu_{2,n}} \\ w_{\dot{\nu}_{2,n}} \\ w_{l_n} \\ w_{\boldsymbol{N}} \end{bmatrix}}_{\boldsymbol{w}_n}, \tag{3}$$

where δt denotes the time interval between two measurements. Obviously, there is a non-linear relationship between the state vector $\boldsymbol{x}_n$ of (3) and the measurements $\boldsymbol{z}_n$ of (1), which are rewritten as

$$\boldsymbol{z}_n = \boldsymbol{h}_n(\boldsymbol{x}_n) + \boldsymbol{v}_n, \tag{4}$$

where $\boldsymbol{h}_n(\boldsymbol{x}_n)$ is implicitly defined by (1) and (3).

Recursive least-squares estimation with extended Kalman filter

In this paper, the baseline and ambiguity estimation shall be performed with a Kalman filter based on the measurement and state space models of (1) and (3). The Kalman filter includes a linear prediction, i.e.

$$\begin{aligned} \hat{\boldsymbol{x}}_{n+1}^- &= \boldsymbol{\Phi}_n \hat{\boldsymbol{x}}_n^+ \\ \boldsymbol{P}_{\hat{\boldsymbol{x}}_{n+1}^-} &= \boldsymbol{\Phi}_n \boldsymbol{P}_{\hat{\boldsymbol{x}}_n^+} \boldsymbol{\Phi}_n^T + \boldsymbol{Q}_{n+1}, \end{aligned} \tag{5}$$

where the second row is obtained from error propagation with $\boldsymbol{Q}_{n+1}$ representing the process noise covariance matrix. Once the new measurement is available, the state estimate is updated, i.e.

$$\hat{\boldsymbol{x}}_n^+ = \hat{\boldsymbol{x}}_n^- + \boldsymbol{K}_n \left(\boldsymbol{z}_n - \boldsymbol{h}_n(\hat{\boldsymbol{x}}_n^-) \right), \tag{6}$$

where $\boldsymbol{K}_n$ is the Kalman gain, which is computed such that the variance of the norm of the a posteriori state estimate is minimized, i.e.

$$\boldsymbol{K}_n = \arg\min_{K_n} \mathrm{E}\{\|\hat{\boldsymbol{x}}_n^+ - \mathrm{E}\{\hat{\boldsymbol{x}}_n^+\}\|^2\}. \tag{7}$$

This minimization of a variance of a highly non-linear function of Gaussian random variables can not be performed in closed form. Therefore, the function $\boldsymbol{h}_n(\boldsymbol{x}_n)$ shall be linearized around an initial state $\boldsymbol{x}_0$, i.e.

$$\begin{aligned} \boldsymbol{h}_n(\boldsymbol{x}_n) \;=\; & \boldsymbol{h}_n(\boldsymbol{x}_0) + \left.\frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial \nu_1}\right|_{\boldsymbol{x}=\boldsymbol{x}_0} \cdot (\nu_{1,n} - \nu_{1,0}) \\ & + \left.\frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial \nu_2}\right|_{\boldsymbol{x}=\boldsymbol{x}_0} \cdot (\nu_{2,n} - \nu_{2,0}) \\ & + \left.\frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial l}\right|_{\boldsymbol{x}=\boldsymbol{x}_0} \cdot (l_n - l_0) \\ & + \sum_{k=1}^{K} \left(\left.\frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial \boldsymbol{N}^k}\right|_{\boldsymbol{x}=\boldsymbol{x}_0} \cdot (\boldsymbol{N}_n^k - \boldsymbol{N}_0^k) \right), \end{aligned} \tag{8}$$

where all partial derivatives can be computed in closed form, and are stacked into a single matrix:

$$\boldsymbol{H}_n = \begin{bmatrix} \frac{\partial \boldsymbol{h}}{\partial \nu_{1,n}} & \frac{\partial \boldsymbol{h}}{\partial \nu_{2,n}} & \frac{\partial \boldsymbol{h}}{\partial \dot{\nu}_{2,n}} & \frac{\partial \boldsymbol{h}}{\partial l} & \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{N}^1} \cdots \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{N}^K} \end{bmatrix}. \tag{9}$$

This linearization of $\boldsymbol{h}_n(\boldsymbol{x}_n)$ enables the solution of (7) in closed form:

$$\boldsymbol{K}_n = \boldsymbol{P}_{\hat{\boldsymbol{x}}_n^-} \boldsymbol{H}_n^T \left(\boldsymbol{H}_n \boldsymbol{P}_{\hat{\boldsymbol{x}}_n^-} \boldsymbol{H}_n^T + \boldsymbol{\Sigma}_n \right)^{-1}, \tag{10}$$

where $\boldsymbol{\Sigma}_n$ denotes the covariance matrix of the measurement noise $\boldsymbol{v}_n$.

MAXIMUM A POSTERIORI PROBABILITY ESTIMATION OF AMBIGUITIES AND BASELINE

In this section, we derive the maximum a posteriori (MAP) probability estimator of the float solution, and show that it is equivalent to a least-squares estimator based on the measurement model of (1). Let us use the following model for the vector of double difference phase and code measurements of a single epoch without a priori knowledge:

$$\boldsymbol{\Psi} = \boldsymbol{H}\boldsymbol{\xi} + \boldsymbol{A}\boldsymbol{N} + \boldsymbol{b} + \boldsymbol{\varepsilon}, \quad (11)$$

with the double difference geometry matrix $\boldsymbol{H}$, the baseline vector $\boldsymbol{\xi} \in \mathcal{R}^{3\times 1}$, the ambiguity design matrix $\boldsymbol{A}$, the vector of integer ambiguities $\boldsymbol{N} \in \mathcal{Z}^{K\times 1}$, the vector of biases due to multipath $\boldsymbol{b}$, and measurement noise $\boldsymbol{\varepsilon}$. Note that $\boldsymbol{\Psi}$ can refer to both uncombined and combined measurements [5]-[8]. A maximum likelihood estimator determines the baseline and ambiguities such that their probability is maximized for a given set of measurements $\boldsymbol{\Psi}$, i.e.

$$\max_{\nu_1,\nu_2,l,\boldsymbol{N}} p\left(\nu_1,\nu_2,l,\boldsymbol{N}|\boldsymbol{\Psi}\right) = \max_{\nu_1,\nu_2,l,\boldsymbol{N}} p\left(\boldsymbol{\Psi}|\nu_1,\nu_2,l,\boldsymbol{N}\right)\cdot\frac{p(\nu_1,\nu_2,l,\boldsymbol{N})}{p(\boldsymbol{\Psi})}, \quad (12)$$

where the rule of Bayes was used. The first factor describes the measurement noise, which is typically modeled by a Gaussian distribution, i.e.

$$p\left(\boldsymbol{\Psi}|\nu_1,\nu_2,l,\boldsymbol{N}\right) = c_0\cdot e^{-\frac{1}{2}\|\boldsymbol{\Psi}-\boldsymbol{H}\boldsymbol{\xi}(\nu_1,\nu_2,l)-\boldsymbol{A}\boldsymbol{N}-\boldsymbol{b}\|^2_{\boldsymbol{\Sigma}^{-1}}}. \quad (13)$$

The second factor in (12) includes a joint probability density, which is interpreted as a priori knowledge. This a priori knowledge shall be modeled as statistically independent Gaussian distributions, i.e.

$$\begin{aligned} p(\nu_x) &= \frac{1}{\sqrt{2\pi\sigma^2_{\nu_{x,\mathrm{ap}}}}} e^{-\frac{(\nu_x-\nu_{x,\mathrm{ap}})^2}{2\sigma^2_{\nu_{x,\mathrm{ap}}}}}, \quad x\in\{1,2\} \\ p(l) &= \frac{1}{\sqrt{2\pi\sigma^2_{l_{\mathrm{ap}}}}} e^{-\frac{(l-l_{\mathrm{ap}})^2}{2\sigma^2_{l_{\mathrm{ap}}}}} \\ p(\boldsymbol{N}) &= \frac{1}{\sqrt{2\pi|\boldsymbol{\Sigma}_{\boldsymbol{N}}|}} e^{-\frac{1}{2}\|\boldsymbol{N}-\boldsymbol{N}_{\mathrm{ap}}\|^2_{\boldsymbol{\Sigma}^{-1}_{\boldsymbol{N}_{\mathrm{ap}}}}}. \end{aligned} \quad (14)$$

The distribution $p(\boldsymbol{\Psi})$ in (12) can be considered as a normalization:

$$p(\boldsymbol{\Psi}) = \int p\left(\boldsymbol{\Psi}|\nu_1,\nu_2,l,\boldsymbol{N}\right) p\left(\nu_1,\nu_2,l,\boldsymbol{N}\right) d\nu_1 d\nu_2 dl d\boldsymbol{N}, \quad (15)$$

which does not explicitly depend on the baseline parameters and ambiguities and, thus, does not affect the maximization of (12). Replacing the densities in (12) by (13) and (14), taking the logarithm, dividing by $-1/2$ and omitting irrelevant pre-factors turns the maximization into the following minimization:

$$\min_{\nu_1,\nu_2,l\boldsymbol{N}} \Big(\|\boldsymbol{\Psi}-\boldsymbol{H}\boldsymbol{\xi}(\nu_1,\nu_2,l)-\boldsymbol{A}\boldsymbol{N}\|^2_{\boldsymbol{\Sigma}^{-1}} + \sum_{x=1}^{2}\frac{(\nu_x-\nu_{x,\mathrm{ap}})^2}{\sigma^2_{\nu_{x,\mathrm{ap}}}} + \frac{(l-l_{\mathrm{ap}})^2}{\sigma^2_{l_{\mathrm{ap}}}} + \|\boldsymbol{N}-\boldsymbol{N}_{\mathrm{ap}}\|^2_{\boldsymbol{\Sigma}^{-1}_{N_{\mathrm{ap}}}}\Big) \quad (16)$$

where the first term describes the measurement residuals and the second one includes weighted offsets from the a priori known mean values. If an a priori knowledge about one or more baseline / ambiguity parameters is not available, the respective standard deviations are set to 0, which removes their contribution from the cost function. The weighted sum of 5 terms in (16) can also be written as a single weighted sum by augmenting the measurement vector by $\nu_{1,\mathrm{ap}}$, $\nu_{2,\mathrm{ap}}$, l_{ap}, and $\boldsymbol{N}_{\mathrm{ap}}$, which then results in the same measurement model as in (1).

BENEFIT OF GAUSSIAN A PRIORI KNOWLEDGE

In this section, the benefit of Gaussian a priori knowledge on the recursive least-squares estimation of ambiguities and baselines is analyzed. The measurement model of the previous section is slightly generalized to a block-wise processing, i.e. measurements were grouped together in blocks of 5 epochs to make the rate of the heading/ yaw angle observable. The single difference unit vectors $\Delta\boldsymbol{e}_n^k$ were computed from a snapshot of 8 visible satellites as observable at our institute in Munich once the full Galileo 27/3/1 Walker constellation becomes operational. All following simulation results refer to 5 Hz single frequency code and carrier phase measurements. The simulation scenarios of the baseline, measurement and process noises, and of the a priori knowledge are summarized in Tab. 1, and are considered typical for maritime navigation.

Tab. 1 Simulation scenario

Initial baseline: $l_0 = 100\,\mathrm{m}$, $\nu_{1_0} = 2^\circ$, $\nu_{2_0} = 45^\circ$, $\dot{\nu_{2_0}} = 1^\circ/\mathrm{s}$
Measurement noise: $\sigma_\varphi = 1\mathrm{cm}$, $\sigma_\rho = 1\mathrm{m}$
Process noise: $\sigma_{\nu_1} = 0.001^\circ$, $\sigma_{\dot{\nu}_2} = 0.03^\circ/\delta t$, $\sigma_l = 0$, $\delta t = 0.2\mathrm{s}$
A priori baseline length knowledge: $\sigma_{l_{\mathrm{ap}}} = 5\mathrm{cm}$
Setting of pitch a priori knowledge and multipath: Fig. 3-4: $\sigma_{\nu_{1,\mathrm{ap}}} = \infty$, no MP Fig. 5-6: $\sigma_{\nu_{1,\mathrm{ap}}} = 0.1^\circ$, no MP Fig. 8-9: $\sigma_{\nu_{1,\mathrm{ap}}} = \infty$, MP Fig. 10-11: $\sigma_{\nu_{1,\mathrm{ap}}} = 0.1^\circ$, MP

Fig. 3 shows the convergence of the constrained a posteriori float ambiguity estimates for $\sigma_{l_{\mathrm{ap}}} = 5$ cm but no a priori knowledge about the pitch angle. In this case, it takes 140 s until all float ambiguity estimates are at most 0.1 cycles apart from an integer number. The reduction of the float ambiguity errors within this time span clearly indicates the benefit of the state space model and a Kalman filter.

Fig. 4 shows the convergence process of the constrained a posteriori baseline parameters for the same simulation scenario. The estimate of ν_1 is much more noisy than the

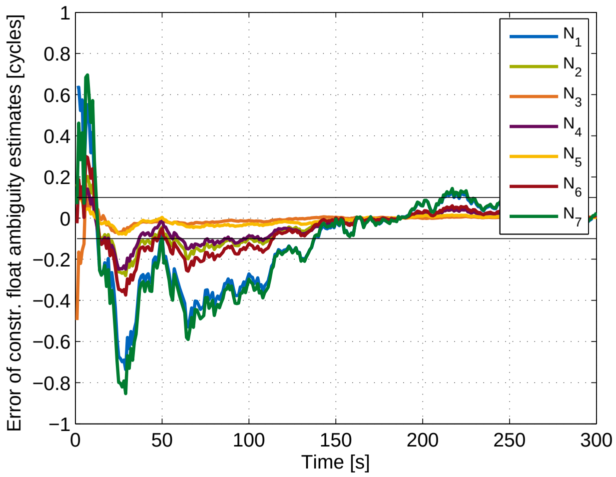

Fig. 3 Convergence of constrained float ambiguity estimates for Gaussian a priori knowledge with $\sigma_{l_{\text{ap}}} = 5\text{cm}$, $\sigma_{\nu_{1,\text{ap}}} = \infty$ and no multipath.

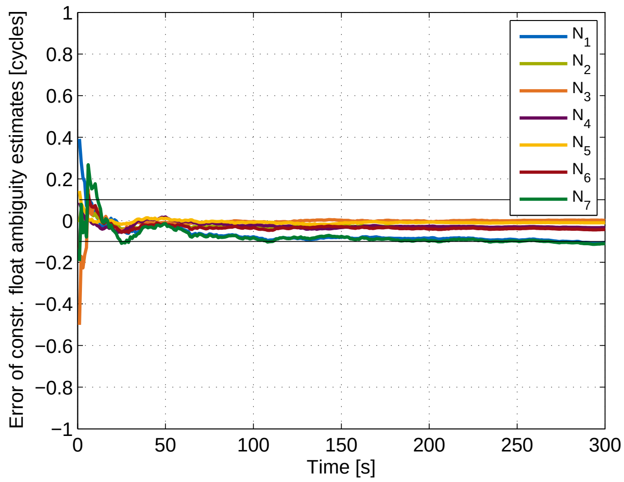

Fig. 5 Convergence of constrained float ambiguity estimates for Gaussian a priori knowledge with $\sigma_{l_{\text{ap}}} = 5\text{cm}$, $\sigma_{\nu_{1,\text{ap}}} = 0.1°$ and no multipath.

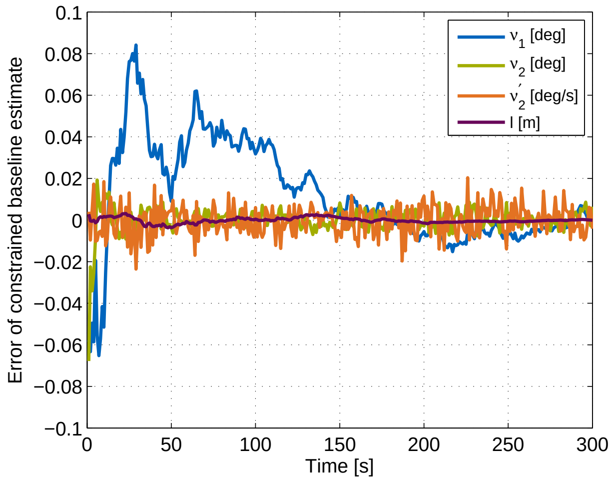

Fig. 4 Convergence of constrained baseline estimates for Gaussian a priori knowledge with $\sigma_{l_{\text{ap}}} = 5\text{cm}$, $\sigma_{\nu_{1,\text{ap}}} = \infty$ and no multipath.

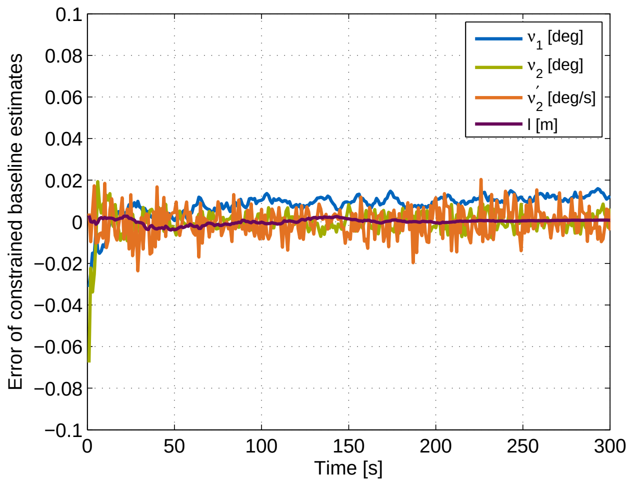

Fig. 6 Convergence of constrained baseline estimates for Gaussian a priori knowledge with $\sigma_{l_{\text{ap}}} = 5\text{cm}$, $\sigma_{\nu_{1,\text{ap}}} = 0.1°$ and no multipath.

one of ν_2, which is a consequence of their different absolute values: The sensitivity of the cost function of (16) w.r.t. ν_1 at $\nu_{1_0} = 2°$ is much smaller than the sensitivity w.r.t. ν_2 at $\nu_{2_0} = 45°$, which results in a noisier estimate. One can also observe the process noise after the convergence is reached.

Fig. 5 and 6 show the benefit of an improved a priori knowledge of the pitch angle: The reduction of the uncertainty to $\sigma_{\nu_{1,\text{ap}}} = 0.1°$ (which corresponds to a length uncertainty of 17.4 cm for $l = 100\text{ m}$) substantially shortens the convergence process, and enables reliable float ambiguities and baseline estimates within less than 10 epochs.

BENEFIT OF GAUSSIAN A PRIORI KNOWLEDGE IN THE PRESENCE OF MULTIPATH

In this section, the benefit of Gaussian a priori knowledge on the pitch angle shall be analyzed in the presence of code multipath. The latter one shall be modeled as independent random-walk processes for each double difference and is shown in Fig. 7. Note that the code multipath errors vary within 20 cm within the first 20 epochs, which enables an unbiased convergence of float ambiguities. It shall be noted that this is an optimistic assumption but that there is a need for an initialization in an environment without too severe multipath.

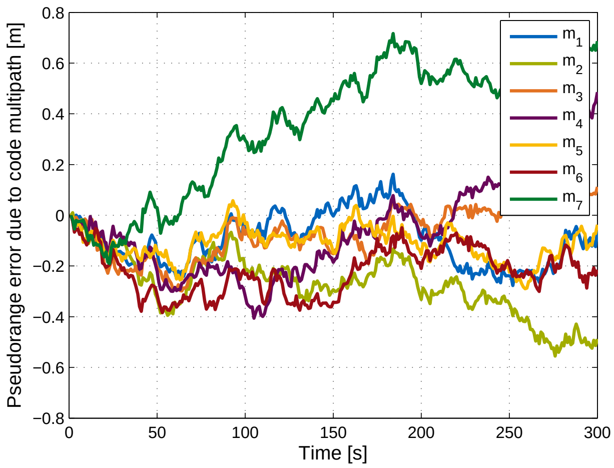

Fig. 7 Code multipath delays: Independent random-walk processes for each double difference.

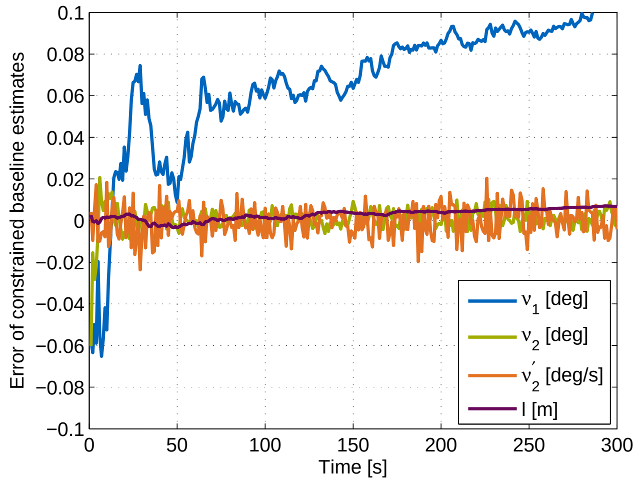

Fig. 9 Convergence of constrained baseline estimates for Gaussian a priori knowledge with $\sigma_{l_{\mathrm{ap}}} = 5\mathrm{cm}$, $\sigma_{\nu_{1,\mathrm{ap}}} = \infty$ and multipath.

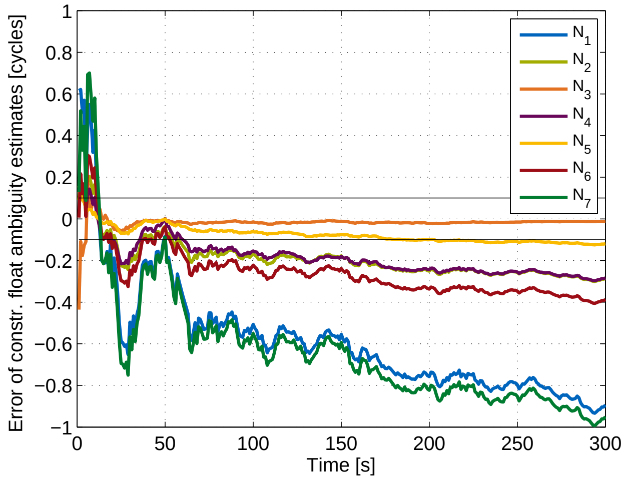

Fig. 8 Convergence of constrained float ambiguity estimates for Gaussian a priori knowledge with $\sigma_{l_{\mathrm{ap}}} = 5\mathrm{cm}$, $\sigma_{\nu_{1,\mathrm{ap}}} = \infty$ and multipath.

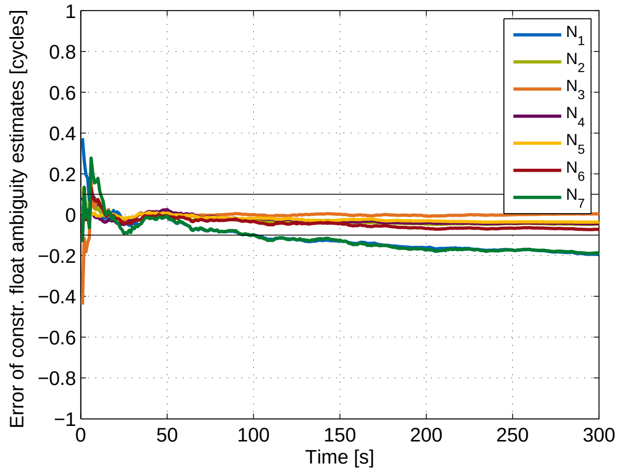

Fig. 10 Convergence of constrained float ambiguity estimates for Gaussian a priori knowledge with $\sigma_{l_{\mathrm{ap}}} = 5\mathrm{cm}$, $\sigma_{\nu_{1,\mathrm{ap}}} = 0.1°$ and multipath.

Fig. 8 indicates that some ambiguities diverge and, thus, can not be fixed correctly if no a priori knowledge about the pitch angle is available. Similarly, the pitch angle estimates become biased as shown in Fig. 9.

An accurate a priori knowledge about the pitch angle enables a substantial multipath suppression in one additional direction. Fig. 10 shows that the divergence of the float ambiguities can be significantly reduced, and a reliable fixing can be achieved within 20 epochs despite the multipath. The convergence process for the baseline parameters is depicted in Fig. 11. One can observe that the error in the pitch angle is reduced by a factor 5 compared to Fig. 9.

CALIBRATION OF DOUBLE DIFFERENCE CARRIER PHASE MEASUREMENTS

Reliable differential carrier phase positioning with low cost single-frequency GNSS receivers needs fast and precise on-board calibration as frequent loss of locks can be observed for the carrier phases. Moreover, receiver differential clock errors can be as large as a few microseconds, and ambiguity resolution using code measurements is slow in the presence of severe multipath.

Therefore, a robust on-board calibration technique was developed and tested with two u-blox LEA 6T receivers in urban and rural environments as shown in Fig. 12 and 15:

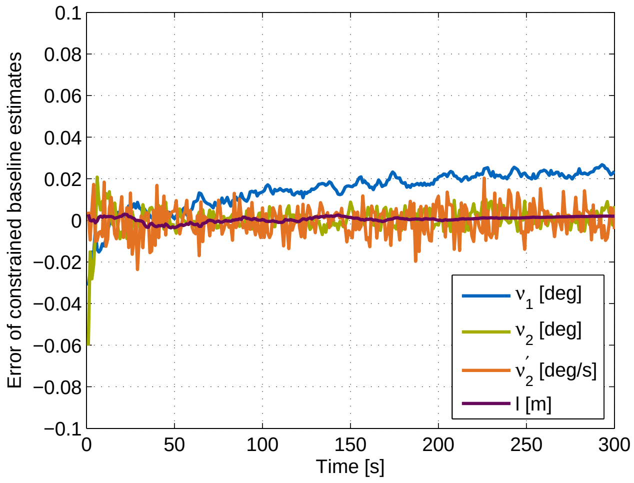

Fig. 11 Convergence of constrained baseline estimates for Gaussian a priori knowledge with $\sigma_{l_{\mathrm{ap}}} = 5\mathrm{cm}$, $\sigma_{\nu_{1,\mathrm{ap}}} = 0.1°$ and multipath.

The only requirement is that the receiver is moving on a straight line which is approximately two orders of magnitude larger than the baseline length (i.e. 100 m for a typical baseline of 1 m), such that the heading can be derived from a set of absolute position solutions. This requirement is almost always fulfilled on highways due to a sufficiently high speed. The heading information together with an a priori knowledge about the baseline length and pitch angle enable a calibration of the double difference carrier phases.

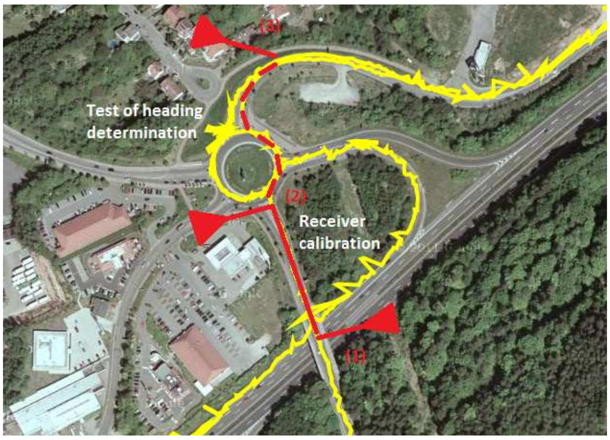

Fig. 12 On-board calibration of two GNSS receivers - test I: The yellow track shows the absolute position solutions, that are heavily affected by code multipath. The double difference carrier phases are calibrated between (1) and (2), where the baseline and straight street are aligned and, thus, the heading can be derived from a linear least-squares fitting of absolute position solutions. Afterwards, the car is entering a roundabout, and the heading is exclusively derived from the calibrated carrier phases.

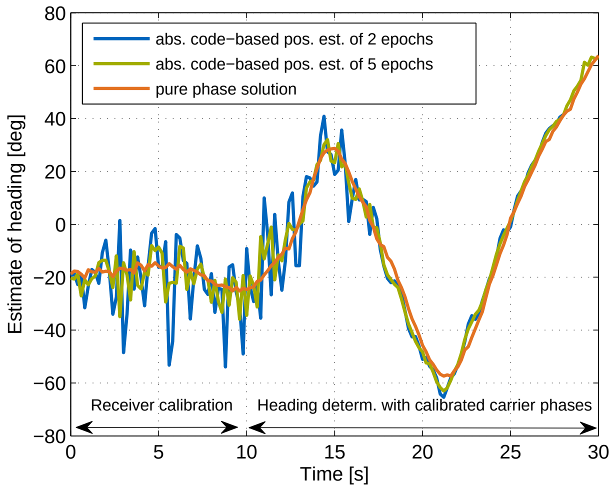

Fig. 13 Estimation of heading at roundabout: The heading can be determined with a significantly higher precision from calibrated double difference carrier phase measurements than from a sequence of absolute code-based position solutions. One can also observe that the coasted carrier phases do not drift within the considered time-span.

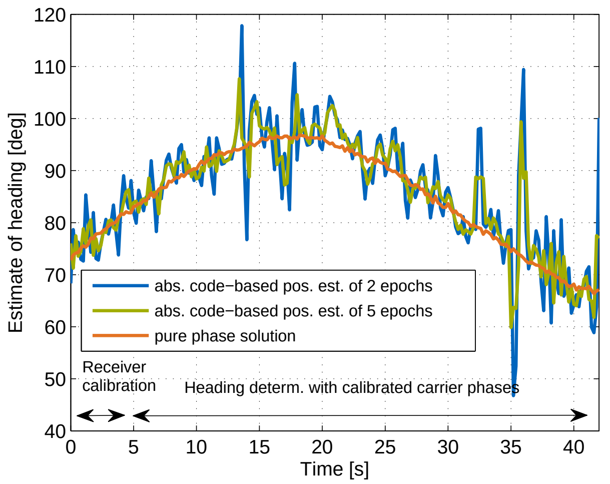

Fig. 14 Estimation of heading on highway: The higher speed enables a faster carrier phase calibration. The coasting of carrier phases results again in a significantly less noisy heading information compared to one derived from a sequence of absolute code-based position solutions.

In detail, the method starts with a computation of absolute code-based position solutions from a few epochs, then transforms these absolute ECEF coordinates into geodetic coordinates, and performs a linear least-squares fitting of the latitudes as a function of longitudes. In a second step, the latitude and longitude of the first receiver are fixed and the longitude of the second receiver is determined with the Newton method using the known latitude/ longitude of first

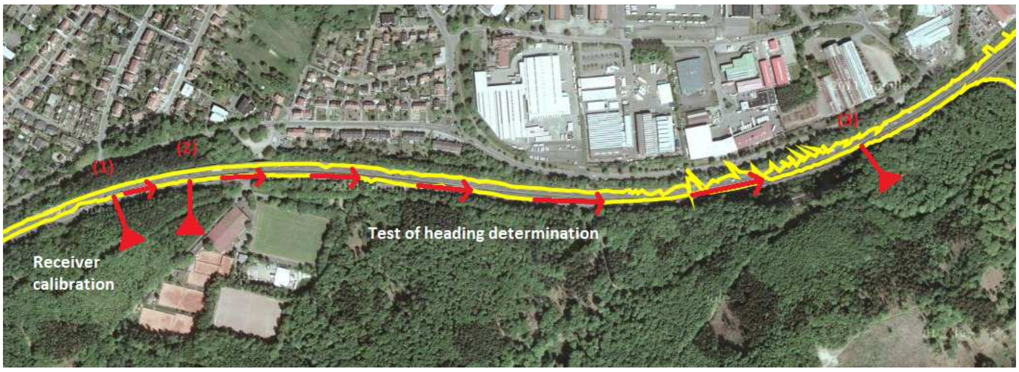

Fig. 15 On-board calibration of two GNSS receivers - test II: The calibration was performed within 4 s on highway A6 near St. Ingbert (Germany) at a speed of approximately 100 km/h. The calibrated carrier phases were then coasted over 40 s corresponding to a distance of ~ 1 km. The computed heading estimates are shown in Fig. 14.

receiver, the linear relationship between latitude and longitude for the second receiver, and the a priori knowledge about the baseline length and elevation/ pitch angle. The Newton method also provides the positions of both receivers in ECEF coordinates, which enables the computation of the baseline. Finally, the double difference phases are set to the product between the single difference unit vectors pointing from the satellites to the receiver and the baseline.

Fig. 13 and 14 show a comparison of the heading estimates based on calibrated double difference carrier phases and a sequence of absolute position solutions. The calibration was performed within 10 s in the urban environment and within the first 4 s in the rural environment. Afterwards, the carrier phases were coasted, i.e. no further calibration or adjustment was performed. Obviously, the coasted carrier phases result in a much less noisy and less multipath affected heading information. The length of the baseline was 1.078 m, the pitch angle was $0°$.

CONCLUSION

In this paper, a maximum likelihood estimation of ambiguities and baselines was proposed for reliable differential carrier phase positioning. It uses Gaussian a priori knowledge of the baseline length and pitch angle, and performs a recursive least-squares estimation with a Kalman filter to obtain the float solution. It is shown that the maximum a posteriori probability estimator finds the optimum trade-off between a solution, which only minimizes the range residuals and one, which only minimizes the distance to the a priori information. The obtained simulation results show that the Gaussian a priori knowledge enables a ten times faster convergence of the float solution compared to the one without a priori information, it substantially suppresses multipath errors, and that it allows some errors in the a priori information, i.e. it is much more robust than deterministic a priori knowledge. Finally, an efficient method for on-board calibration of carrier phase measurements was suggested and tested with real measurements.

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