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Stochastic valuation of energy investments - important aspects for simulation and real option valuation

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Summary

Recent advances in computational power enabled practitioners to use more complex techniques for investment decisions. However, the application of more complex models comes at a cost, e.g. higher implementation effort. Since the liberalization of the European energy markets and the starting of trading electricity via exchanges one decade ago, new ways of valuing energy investments are required.

This dissertation aims to shed light on the questions (i) how electricity prices can be modeled, (ii) how can real option and simulation-based valuation be combined in a valuation model, and (iii) what is the influence of model complexity valuation outcome.

The approach to model electricity spot prices proposes a combination of mean reversion, spikes, negative prices, and stochastic volatility. Furthermore, different mean reversion rates for ”normal” and ”extreme” (spike) periods are used.

A stochastic valuation model that combines simulation and real option based valuation is described and used to analyze model complexity. Therefore, a large simulation study is performed with over 1,000 different types of projects in the energy and mining sector, all with different combinations of outputs, inputs, foreign exchange and interest rate risks. By and large, the results indicate that model complexity has an impact on the valuation outcome. However, the relative importance differs for each aspect. These results are of importance both for practitioners and academics.

Contents

Nomenclature

ACF AutoCorrelation Function

ARCH Autoregressive Conditional Heteroscedasticity

ARMA Autoregressive moving average

a.s. almost surely

BOPM Binomial option pricing model

CAPM Capital Asset Pricing Model

CFF Cash flow from financing activities

CFI Cash flow from investment activities

CFO Cash flow from operating activities

DCF Discounted cash flow

DCC Dynamic Conditional Correlation

EDF Expected Default Frequency

EEG Erneuerbare Energien Gesetz

EEX European Energy Exchange in Leipzig

E-GARCH Exponential Generalized Autoregressive Conditional Heteroscedasticity

GJR-GARCH Glosten-Jagannathan-Runkle Generalized Autoregressive Conditional Heteroscedasticity

NGARCH Non-linear Generalized Autoregressive Conditional Heteroscedasticity

VGARCH V Generalized Autoregressive Conditional Heteroscedasticity

FCFE Free cash flow to equity

FCFF Free cash flow to the firm

GARCH Generalized Autoregressive Conditional Heteroscedasticity

MCP Market clearing price

MR Mean reversion

NCC Net non-Cash Charges

NI Net Income

NPV Net Present Value

PDE Partial Differential Equation

QMV Quasi-Market Valuation

ROPFVT Real Option Project Finance Valuation Tool

SDE Stochastic Differential Equation

VAR Value at Risk

WACC Weighted Average Cost of Capital

WC Working Capital

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Chapter 1

Introduction

Starting with the liberalization of electricity trading about one decade ago, electricity markets have become exceedingly important. However, electricity has several peculiarities that distinguish it from other types of commodities. The most notable difference is its non-storability—or at least the very high costs associated with its storage. This leads to several problems for price modeling and for the pricing of related derivatives. Because electricity markets are quite young, it is not surprising that research in this vital field is still limited.[1] Several recent studies have addressed the question of how electricity future prices are formed in the market (e.g., Wilkens and Wimschulte (2007), Redl et al. (2009), Botterud et al. (2010) or Furio and Meneu (2010)). The price behavior of electricity still puzzles both researchers and practitioners. The price process in this market differs substantially from other commodity markets because of its very high volatility and, even more importantly, because of the more common occurrence of spikes (at least partly as a consequence of the non-storability). Consequently, traditional models, which mostly build on the assumption of Gaussian distributions, are not suitable in this case. However, without a proper understanding of the price process and validated models, the valuation of investment opportunities is impossible.

The valuation of investment opportunities, like project financings or firm internal projects, is among the core tasks of managers today. Five major problems arise in practice, in the context of investment decisions (Lewellen and Long (1972)): (1) uncertainty about

[1] A good overview of the literature about electricity price modeling is provided by Higgs and Worthington (2008).

the distribution of the cash flow, (2) correlation of risk factors, (3) the calculation of the appropriate discount rate, (4) uncertainty about the run-time of the project and (5) the distinction between project and portfolio risk. Due to investment decisions high complexity quantitative models have become more and more popular during the last decades. It is surprising that there is only sparse literature on the aspect if model complexity influences the value of an investment. To best of my knowledge only Weber et al. (2010) and Pietz (2011) address this issue.

Model complexity is investigated along two main dimensions: forecasting and valuation complexity of the applied model. In the area of forecasting, different volatility and correlation forecasting methods are compared. This is closely related to the mentioned problems (1) and (2). Valuation complexity includes the number of Monte Carlo iterations, the time resolution of the cash flow calculation (e.g. yearly vs. half-yearly), and the method used to determine the appropriate discount rate. Hence, the second dimension addresses problem (3) and, to some extend, problem (4). This dissertation does not aim to judge which model or method is better, but only discerns if the choice of a specific model has an impact on the valuation outcome - the net present value (NPV) distribution, the Value-at-Risk (VAR), and the expected default frequency (EDF). To investigate the impact of complexity on the valuation outcome, this study uses a discounted cash flow (DCF) approach. DCF has been the standard approach to evaluate investments for several decades (Fisher (1930), Dean (1951) and Hirshleifer (1958)). For investment decisions, methods based on point estimates and on distributions of cash flows are available. However, only simulation-based methods are appropriate for the investigation of cash flow risks since point estimates ignore the distributions of the risk factors (Hertz (1964*b*), Hertz (1964*a*) and Hess and Quigley (1963)). Consequently, the valuation model presented here is based on Monte Carlo simulation. This model is used to evaluate investments in project financing. Considering its importance in practice, academic literature on project finance is surprisingly sparse. The majority of published papers, so far, focus on qualitative, organizational, or legal aspects of project finance.[2] To the best of this author's knowledge, only three papers, namely Dailami et al. (1999), Esty (1999), and Gatti et al. (2007), explicitly

[2]See, for example, Chan and Gray (1996), Esty and Megginson (2001), Esty and Megginson (2003) and Kleimeier and Megginson (2001). Esty (2004) provides a comprehensive overview on recent research in project finance.

address cash flow modeling in the context of project finance. However, this study is not primarily interested in project finance but in the general effect of model complexity. The choice of project finance investments has two main reasons. First, project financing is an important financing method for large-scale projects (Kleimeier and Megginson (2001), Esty (2004), Esty and Seisa (2007)). Second, and most important, the analysis of project financing allows concentration on cash flows. In contrast to other investments, only cash flows are relevant to the valuation of project finance investments (Finnerty (2007)). This is important in the context of this dissertation since the analysis is based on financial cash flow risk factors that can be both forecast and simulated with the help of mathematical techniques.

The analysis compares valuation results obtained by different levels of model complexity. Statistical tests are used to investigate if there are significant differences between the simulated NPV distributions and expected default probabilities. However, the analysis does not focus only on one investment project but instead, analyzes about 1,000 different projects, all with different input and output factor combinations as well as different foreign exchange and interest rate risks. For this, this author uses a hypothetical power plant that produces electricity as output and has different input factors (e.g. coal or gas) and a mine that has only one input factor (i.e. oil) and different output factors (e.g. gold or silver). For both hypothetical projects, all possible factor combinations up to a maximum of three simultaneous input (power plant project) factors or output (mining project) factors are analyzed. This procedure allows one to alleviate concerns that the results depend on the specific nature of a single investment project.

The NPV method is based on the assumption, among others, that all actions are determined at the beginning of a project. Real options analysis addresses deficiency by abandoning this assumption and allowing the inclusion of managerial flexibility. The ability to include this flexibility, or real options, in the investment decision is another aspect of this dissertation. Already in 2001, as reported in the survey of Graham and Harvey (2001), 27% of the participants answered that they (always or mostly) applied real options when valuing important capital investments. In particular, for energy projects, where future flexibility plays a major role, real options have the power to increase the insight into the project valuation significantly .

To satisfy the peak load need, energy suppliers fall back on gas-fired power stations. The gas turbines deliver full power within a few minutes after cold start, thus, sudden delivery changes are no problem for gas power plants. However, setbacks must be accepted on the efficiency side: only 35% to 42% of the input energy can be transformed into electricity. Better performance can be provided by running a Combined Cycle Gas Turbine (CCGT). This power station combines the Joule-process of electricity generation with a gas turbine with the Clausius-Rankine-process where heat recovery steam generator produces additional power[3]. As a result, the CCGT reaches efficiency up to 60%[4]. Both types of gas-fired power stations have one common property: part-load operation decreases the efficiency tremendously. So, if the power station is being turned on, it should run at full-load. If the utilization falls below 60%, the gas turbine no longer works profitably and it has to be turned off.

To investigate if a gas-fired power plant is worthy of investment, a special type of real option is necessary. The operator decides whether to turn the power plant on or off. That means, the option should include the flexibility to switch between modes. One case is that the clean spark spread is positive. Hence, the operator turns on the power station and earns the corresponding revenues. In the other case, the power plant is turned off. The operator decides to relinquish the revenues, as they are negative.
Brennan and Schwartz (1985) dealt with a similar problem when they evaluated natural resource investments. They incorporated the stochastic nature of output prices and the possibility of managerial responses to price variations in their calculations. If prices were low, the exploration activities could be temporarily interrupted. If prices rose again, the production could be continued. Their approach was to find a self-financing portfolio, which replicated the cash flows of the project[5]. This approach is not transferable to evaluate a power plant, due to the maturity of electricity contracts. To replicate the cash flows of an energy plant, contracts with terms of several years are needed. Furthermore, Kulatilaka (1993, p. 273) stated, "Valuation of options and option-like projects,[...], is, in general,

[3]For further information, see Konstantin (2009, p. 286).
[4]All technical details can be read in Konstantin (2009, ch. 7)
[5]see Brennan and Schwartz (1985, p. 136)

fairly complicated and, except for special assumptions, does not allow for analytic solutions."

This argues that this question is of high relevance for practitioners who want to evaluate investment opportunities because they often face a difficult trade-off. On the one hand, very complex models and techniques could increase the valuation accuracy and, therefore, could lead to better investment decisions. On the other hand, this (possible) higher accuracy comes at a cost. More complex models require higher implementation efforts and time and their handling is often less straightforward.[6] Hence, the question of whether model complexity in general, or certain aspects and methods in particular, affect valuation outcomes is important for their daily business. As mentioned before, this dissertation does not analyze which models work better than others do. However, the results of this study can help practitioners to identify crucial components of the models. Knowing that, they can focus on the accuracy of those components.

1.1 Research questions

This section summarizes the research questions of the dissertation. All questions are further subdivided.

Question 1: What is the price process of electricity?

Question 1a: How can price spikes be included?

Question 1b: Does stochastic volatility influence price behavior?

After these questions are answered, the dissertation addresses the question of whether the complexity of a project finance valuation model impacts the results, and if so, in what ways. Therefore, a Monte Carlo simulation-based cash flow model, which the author calls a real option project finance valuation tool (ROPFVT), is developed.[7] The dissertation analyzes

[6] Furthermore, decision makers might be unfamiliar with more complex models and necessary information might not be readily available. For a similar argumentation in the context of trading-off costs and benefits of discounted cash flow rules cf. Chaput (1996).

[7] This model is based on the work of Pietz (2011). For a detailed description of the changes, please see section 5.1.

the question of model complexity along two dimensions, different aspects of forecasting complexity and valuation complexity.

Question 2: Does model complexity influence investment decisions?

Question 2a: Does forecasting complexity influence investment decisions?

Question 2b: Does valuation complexity influence investment decisions?

The third research questions focuses on the influence of flexibility on the investment decision. Therefore, the jump process parameters - jump intensity, jump height, or jump mean-reversion[8] - are varied and, then, it is analyzed which parameters have the most significant influence on the valuation outcome. The dissertation presents a large Monte Carlo simulation, where again, an equity investment in a gas power plant is valued. The author argues that this is enough for a generalization because the power plant is merely a tool while the parameter changes are applied to the underlying price processes.

Question 3: Does flexibility influence investment decisions?

Question 2a: Does the price jump intensity influence investment decisions?

Question 2b: Does the price jump mean-reversion influence investment decisions?

Question 2c: Does the price jump height influence investment decisions?

The results of this questions are presented in chapters 4 and 5.

1.2 Structure

This section provides a dissertation overview and gives a short summery of the chapters.

In Chapter 2 an overview of the most important literature in fields of commodity price modeling and valuation by simulation and real option theory.

Chapter 3 deals with the theoretical framework for the valuation of projects. *First* it provides the mathematical background which is relevant throughout the dissertation.

[8]I refer the reader to section 4.3 for a detailed description of the applied model.

Second it introduces the discounted cash flow valuation (section 3.2). The chapter highlights the importance of the cash flow and cash flow modeling and the theoretical aspects of the cost of capital calculations are shown. It introduces the famous Capital Asset Pricing Model (CAPM) and the calculation of Weighted Average Cost of Capital (WACC) and shows the theoretical difficulties when applying these theories in the area of project finance. After this the theoretical background of the net present value calculation is provided. *Third*, the chapter introduces the real option valuation theory. The different types of real options are explained here. Finally, the theory of stochastic valuation of investments is introduced. The chapter provides different possibilities to deal with uncertainty and complexity and the most important analytical models are explained. Then, the Monte Carlo methods and the valuation using them are introduced.

Chapter 4 deals with the electricity market and electricity price modeling. Section 4.1 provides a short overview of the German electricity market. It presents the major indices that were established after the liberalization of the German market. Section 4.2 analyzes the historical data regarding normality, seasonality, mean reversion, and jumps. The resulting stochastic price model is developed in section 4.3 and calibrated in section 4.4. Section 4.5 provides empirical and goodness-of-fit tests.

Chapter 5 deals with the stochastic valuation of energy investments. Section 5.1 introduces the valuation model used for further analyses. Especially, it highlights the equivalent martingale transformation by the Esscher transformation, developed by Esscher (1932). Furthermore, it provides a description of the implemented switching option algorithm. Section 5.3 contains the results regarding research question 2 whether model complexity influences investment decisions. Finally, section 5.3.3 shows the results regarding the research question 3.

Chapter 6 concludes the dissertation.

Chapter 2

Literature review

2.1 Electricity price modeling

In the commodity pricing literature, the most common approach is to model the logarithmic price through a mean reverting process (Schwartz (1997)). As in the Black and Scholes-Merton model, the mean reverting process is based on the exponential treatment of the stochastic spot price (Black and Scholes (1973), Merton (1973)). If these models are applied for electricity, they can capture the mean reversion of electricity prices, but they fail to account for the huge and non-negligible spikes in this market. In order to capture the spike behavior of electricity spot prices, it is necessary to extend the model by a jump component. Merton (1976) first introduced this class of jump-diffusion models to model equity dynamics. Deng (2001) examined a broad class of stochastic processes to model electricity spot prices. He found that mean-reversion jump-diffusion models were reliable to model the volatility in the market prices of traded electricity options. Lucia and Schwartz (2002) modeled the predictable component in the dynamics of spot electricity prices and its implications for derivative securities. Their findings showed a significant mean reverting process and, in addition, significant seasonal patterns. Cartea and Figueroa (2005) apply this model to the English and Welsh electricity markets and found that it offered a proper adjustment to the peculiarities of electricity markets. However, Geman and Roncoroni (2006) addressed one of the drawbacks of this model, namely that it used only one mean-reversion rate for both the diffusion and the jump process. Huismann and Mathieu (2003) modeled the electricity spot price using a regime switching model separate from the mean-reversion process. They also found that the mean-reversion was stronger after periods in which spikes occurred than during normal periods.

A suitable theoretical approach for multiple mean reversion rates was described by Benth et al. (2003). de Jong (2005) analyzed the spikes in the electricity spot prices in detail. He concluded that regime switching outperform GARCH or Poisson jump models in capturing the dynamics of spikes. The model proposed by Benth et al. was calibrated by Kluge (2006) for the Scandinavian, British, and German electricity markets to value swing options. Thereby, he estimated the parameters for the diffusion process from historical data and assumed a constant volatility over time. However, this approach has several drawbacks. First, the parameters for the spike process are not estimated from the time series, but based on expert opinions. Second, this approach neglected the fact that the volatility in electricity markets is stochastic over time. Deng (2001) compared the jump-diffusion model of Merton (1976) with constant and stochastic volatility and derived prices for different energy derivatives using the Fourier transform, showing that stochastic volatility was important.
Escribano et al. (2002) provided extensive empirical tests on a wide range of markets and concluded that it is necessary to include jumps **and** stochastic volatility. Robinson (2000) modeled the behavior of spot electricity prices which can influence the contract prices. They found that nonlinear models were superior in estimating pool prices. Interestingly, they showed that the mean-reversion rate decreases as prices moved further away from their mean. Knittel and Roberts (2001) analyze the degree of persistence, intraday and seasonal effects in electricity prices. They found that the E-GARCH volatility model exhibited a significant inverse leverage effect indicating positive price shocks increased volatility. Goto and Karolyi (n.d.) analyzed the volatility dynamics across different markets. Their findings suggested that the arch model with time-dependent jumps best explained price volatility. In this context, recent academic work by Chan and Gray (2006) and Bowden and Payne (2008) suggests that the E-GARCH is the best volatility model for electricity prices. Furthermore, most prior models neglect the empirical observation of negative electricity prices (over short periods of time).

2.2 Real Option theory and simulation-based valuation of investments and real options

2.2.1 Fundamentals and building blocks of option pricing

The French mathematician Louis Bachelier was the first scientist who studied the pricing of options around 1900. For that, he used the well-known Brownian motion to model the evolution of stock prices. The major break-through within the field of option pricing were the semi-annual works of Black and Scholes (1973), Merton (1973), and Cox and Ross (1976), who developed a rigorous framework for pricing options. Black and Scholes (1973) and Merton (1973) introduced a formula to value European calls and puts analytically.[1]
Black and Scholes derived their formula by using a replication portfolio and then valuing the option by a no-arbitrage condition. Merton used a more formal approach by using the risk-neutral probabilities to value the option. Cox et al. (1985) showed if no-replicating portfolio can be found because a state variable is not traded it was necessary to apply a transformation to an equivalent martingale measure. This theory was formulated in Girsanov's theorem (Girsanov, 1960) and the Esscher Transformation (Esscher, 1932).[2]
Cox et al. (1979) developed the binomial option pricing model, which allowed analysts to price options in discrete time. The method used a tree approach to value options and it could be shown that its continuous time limit is the Black and Scholes model (see Copeland et al., 2005, p. 229). It is now a very common method to price real options Copeland and Antikarov (2003).

The core deficiencies of the Black and Scholes model, which will be examined in Section 3.4.2.1, are the reasons for researchers to develop more realistic models, incorporating empirical features such as stochastic volatility and the heteroscedasticity of asset returns. Option pricing models that deal with heteroscedasticity include the constant elasticity of variance model by Cox (1975), the jump-diffusion model by Merton (1976) and the displaced diffusion model by Rubinstein (1983). Extensions of these models to incorporate the fact that the volatility varies stochastic have been developed by Hull and Whihte (1987), who proposed a continuous-time bivariate diffusion model. In their model, an exogenous process was assumed to govern the evolution of asset volatility. Other so-

[1] The Black and Scholes and Merton formula will be studied in greater detail in Section 3.4.2.1.
[2] I refer the reader to section 3.1.2.

called continuous-time stochastic volatility models were those by Stein and Stein (1991), Scott (1987) and Heston (1993). The model by Heston (1993) has been the most popular stochastic volatility model. The stochastic volatility models could roughly be divided into two groups, considering the volatility and the asset price process either in continuous or in discrete-time. An alternative approach to price options using a stochastic volatility is to use GARCH models, which are discrete-time model.[3]

2.2.2 Valuing different types of real options

(Dixit and Pindyck, 1994, chap. 5) and (Trigeorgis, 1996, chap. 4) provided a conceptual framework for capital budgeting decisions with real options.

Geske (1977, 1979) values so-called compound options which were options on options. Margrabe (1978) valued an option to exchange one risky asset for another - switching options. Stulz (1982) analyzed options on the maximum or minimum of two risky assets that was extended by Johnson (1987) to multiple risky assets. An option to defer an investment was examined by McDonald and Siegel (in Schwartz and Trigeorgis, 2004, chap. 12) and an option to alter - expand or contract - was investigated by Pyndyck (in Schwartz and Trigeorgis, 2004, chap. 15). The problem of temporarily shutting down and then restarting an operation was analyzed by McDonald and Siegel (1985). Carr et al. (1988) combined the work of Geske and Johnson to value sequential compound options. This allowed to value investment opportunities that could be switched to alternative states of operations. Switching options were analyzed by Margrabe (1978), Kulatilaka (1986), and Kulatilaka and Marcus (1988).

Brennan and Schwartz (1985) were the first to use the real option theory to value an investment in natural resources. They evaluated a stochastic cash flow stream as it was often the case with extraction of raw materials. Brennan and Schwartz supposed that the project[4] produced a single homogeneous commodity, whose spot price S was determined competitively and was assumed to follow an exogenous given continuous stochastic process of a geometric Brownian motion. Using commodity futures, they were able to apply the risk-neutral valuation approach. Brennan and Schwartz (1985) did not use the

[3]These models will be studied in detail in section 5.1.1.2.
[4]Brennan and Schwartz decided to scrutinize a mine.

Black and Scholes formula to evaluate the *real option*, but followed the approach to find a self-financing portfolio. Based on this they determined optimal policies for developing and abandoning the exploitation. One drawback in Brennan and Schwartz (1985) was the assumption of a constant convenience yield in the model which does not allow capturing the real behavior of commodity prices.[5] Additionally, the assumption of a geometric Brownian motion as the spot price process does not reflect the price characteristics of commodities. Typically, a mean-reversion effect can be observed because long-term supply and demand drives the price back to a long-run mean (Bessembinder et al., 1995, see).[6]

Margrabe (1988) valued government subsidies to large energy projects as a put option while Teisberg (1994) analyzed investment choices of regulated firms. Kulatilaka (1993) valued a dual-fuel industrial steam boiler. To determine the value of a power plant, Hsu (1998*a*) used the Black and Scholes formula for pricing call options on the spark spread. According to Hsu (1998*a*), gas fired power plant owners should have viewed their assets as a series of spark spread call options with a one-month maturity. Here the price history of spark spreads were scrutinized to simulate a lot of possible spark spread trajectories. Based on this, prices of switching options could be determined. The discounted cash flows gave an approximation of the present value of a gas fired power plant.

Mun (2006) and Rogers (2002) argued in favor of valuing wind park investments by a real option approach. They included several theoretical examples and their solutions in their work. However, it was just a framework that was built upon a two case scenario, or a simple environment. The field of complicated - realistic - scenarios was not covered in these books. In a manner of financial options' theory in discrete time, these real options were valued in a kind of lattice approach. Unfortunately, it did not take into consideration that reality was more complex and most project values could not directly be fitted into a lattice, since the value processes of the real option's underlying could not simply be assumed to follow the same family of distributions as underlings of financial options.

[5] The term structure of commodities typically changes from contango to backwardation and vice verse. the author refers the reader to (Geman, 2005, chapters 1 and 2) for a detailed discussion of this feature.

[6] Brennan (1991), Gibson and Schwartz (1990), Ross (2010), Cortazar and Schwartz (1994), and Miltersen and Schwartz (1998) address the issues of Brennan and Schwartz (1985).

Oren (2001) integrated real and financial options in demand-side electricity contracts while Deng et al. (2001), Hsu (1998*a*), and Hsu (1998*b*) valued supply-side electricity generation and transmission assets. They, first, constructed a replicating portfolio from electricity futures and the risk free asset. They argued that the futures-based approach is necessary because the non-storability of the electricity does not allow to use the spot price. De facto, they value the power plant by valuing an option on the so-called 'spark-spread' for which they constructed an analytical solution. In detail, this was an option on the gross margin of a traditional power plant from selling a unit of electricity, having bought the fuel required on the market. Therefore, one of the main value drivers was the electricity price, which - with respect to the EEG - could be neglected for offshore wind parks. Therefore, one of the main value drivers was the electricity price. Deng (2005) addressed the main short-coming of Deng et al. (2001), namely that they applied only a mean reverting process for the electricity price while ignoring jumps. Deng (2005) extended the spark spread model to include jumps by applying a mean reversion jump-diffusion model. Once again, he was able to value investments in power generation assets analytically. The dissertation of He (2007) concentrated on investment decisions on the energy markets and on how real option theory could affect the optimal behavior. In Dykes and de Neufville (2007), the power production could be stopped if the electricity prices are too low and switched on again if the situation changes.

2.2.3 Simulation-based valuation of investments and real options

In real-life applications, analytic solutions may not exist because the investment projects are too complex. This is often the case when there are several state variables involved and / or complex options, such as switching options. Risk-neutral valuation is major tool to value such an investment by numerical methods. According to (Trigeorgis, 1996, p. 21), there are two types of numerical methods for approximating the investment and option value:

1. Approximation by partial differential equations (PDE)
2. Approximation of the underlying stochastic process by Monte Carlo methods, i.e. by simulation.

(Brennan, 1979, see) and (Brennan and Schwartz, 1977, 1978) provided examples for solving PDEs by numerical integration, either with explicit or implicit finite-difference schemes.
Boyle (1977) was the first to provide a framework for the second approach, namely for the valuation of options by Monte Carlo methods. He showed how to perform a risk-neutral Monte Carlo simulation and to value an option with the risk-free rate as the discount rate. Following Black and Scholes (1973), the equity of a company could also be seen as an option on the firm value. Therefore, any investment could be valued in the same way.
But simulation-based investment decisions, in that case the valuation of investment opportunities with Monte Carlo simulation, goes further back at least to Hess and Quigley (1963). They were among the first to propose a simulation-based approach for valuation problems. Subsequent literature, such as Hertz (1964*b*), Hertz (1964*a*), or Lewellen and Long (1972), compared the simulation-based approach with simple techniques that focused on point estimates instead of distributions. (Hertz, 1964*b*), (Hertz, 1964*a*) proposed that employing simulation techniques instead of single-point estimates of future income and expenses was superior when appraising capital expenditure proposals under conditions of uncertainty. He argued that potential risks are more clearly identified, and the expected returns were more accurately measured.

Recent academic literature in this area focuses for example on the valuation of real Estate investments (Hughes (1995), Kaka (2009) or Gimpelevich (2011)). Smith (1994) outlined how simulations may assist managers in choosing among different potential investment projects. Spinnery and Watkins (1996) applied Monte Carlo simulation as a technique for integrated resource planning at electric utilities and found that Monte Carlo simulation offered advantages over more commonly used methods for analyzing the relative merits and risks embodied in typical electrical power resource decisions, particularly those involving large capital commitments. Kwak and Ingall (2007) discussed Monte Carlo simulation for project management and concluded that it is a powerful tool to incorporate uncertainty and risk in project plans. However, the valuation of project financing gained surprisingly little attention, despite its relevance for practice. Among the few papers that focused on the valuation of project financing were Pouliquen (1970), Reutlinger (1970), Chan and Gray (1993), Curry and Weisst (1993), or Brent (1998). Since this dissertation aims to analyze the general effect of model complexity on the valuation outcome, it focuses on rather general projects like power plants or mines instead of, for example, real estate, which is specific in several dimensions.

Dailami et al. (1999) proposed a model to analyze the risk exposure of a project to a variety of market, credit, and performance risks. However, they did not include modern time-series forecasting techniques, such as GARCH, in their model. Esty (1999) discussed methods to improve the valuation of project finance from an equity provider's point of view. To achieve this, he proposed both advanced discounting methods and valuation techniques. However, besides discussing important elements for valuation, he did not present a self-contained model that enables equity providers to calculate either the profitability of their investments or the expected probability of project default. An evaluation model for this purpose is developed by Holtan (2002). Gatti et al. (2007) developed a model to estimate the value-at-risk of project-financed investments based on Monte Carlo simulation techniques. However, they focused on debt providers and ignored the equity providers' point of view. Furthermore, none of these papers focused on the impact of model complexity on valuation results. This is the initial perspective of this dissertation.

Gamba (2003), for example, used Monte Carlo techniques to value complex real option structures. In contrast to the Monte Carlo approach in Calistrate et al. (1999), a complex binomial tree structure for the valuation of real options is derived. Hereby, the lattice jumps may vary and thus accommodate the arbitrary changes in volatility, and consequently, the risk of the project. This procedure is a theoretical proof, showing how a continuous stochastic value process that fulfills some tentative assumptions can be fitted into a lattice, so that the limit of infinitely small time steps equals the distribution of the continuous process. Consequently, the need for a reasonable value process of the real option's underlying arises, which turns out to be difficult to answer.

Chapter 3

Valuation of projects - Economic Theory

Capital budgeting is concerned with the allocation of limited resources among investment projects on a long-term basis. This requires exchanging current consumption in order to achieve consumption in the future. The tradeoff between today's and future's consumption is the main objective of capital budgeting.

The financial objective of an individual should be to choose between alternative patterns of consumption and investment opportunities to maximize the utility of consumption over time. A company has the same objective to achieve maximum utility for its shareholders. However, this is not easy for a company, given the different utility functions of its owners. Therefore, a company has to adopt the strategy of maximizing the wealth of its owners. This leads to questioning how a company can maximize the wealth of its shareholders and, thereby, maximize their utility in a perfect market. The wealth of a company's shareholders is determined by the market value of its shares. The market value itself depends on the expected future cash flows - dividends and final liquidation value - a shareholder can receive from holding the company's stock. Consequently, the amount, timing, and the riskiness of the cash flows determine the market value. Therefore, a company that maximizes its shareholder's wealth needs to maximize expected cash flows discounted by a risk-adjusted interest rate (see Trigeorgis, 1996, p. 24).

Consequently, different projects with different patterns of cash flows need to be valued. According to Luenberger (1998, p. 24), the essence of investment is the selection from a number of alternative cash flow streams, hence, the selection from a number of projects. Currently, in the absence of managerial flexibility, the discounted cash flow or net present value valuation is the only consistent method of maximizing the shareholder's wealth. Section 3.2 addresses (i) the issue of cash flow modeling (section 3.2.1), (ii) the determination of the discount rate (section 3.2.2), and (iii) the definition of the net present value method(section 3.2.3). Furthermore, it presents a variation of the discounted cash flow valuation, which uses the certainty-equivalent (section 3.2.3.2).

Whereas the discounted cash flow method (see equations 3.21 and 3.22 on page 34) assumes that all decisions are taken at project start, the real options approach allows modifications even after investment decisions. This opens the door to a dynamic evaluation method. Evaluating a project with the real options approach reflects the reality quite well, as companies do not hold a project passively. Consider the cash flows of the project as a possible payoff of an option; then, the option-price (=project value) can be determined with already existing evaluation techniques for financial options. Mention should be made here of the path-breaking paper by Black and Scholes (1973). Financial options have stocks as underlying; hence, the payoff depends on the price of the stock. Because of the similarity to financial options, real options show their muscles when a project with dependency on a particular price process, such as commodities or real estates, has to be evaluated. Therefore, this dissertation first introduces financial options, meaning call options and put options in section 3.3.2.1, and then explains real options (section 3.3.2). The different types of real options will be explained and their applications discussed in section 3.3.3. In section 3.4.2.1, the most important option pricing models and their deficiencies will be studied in greater detail including the binomial option pricing model of Cox et al. (1979) and the diffusion option pricing model of Black and Scholes (1973) and Merton (1973). All of these models use the important concept of risk-neutral valuation, which will be introduced in section 3.1.4. Furthermore, the important relationship between calls and puts, called put-call parity, will be derived.
Analytical approaches to value (real) options often have boundaries if the stochastic processes, or stochastic partial differential equations are to complex. Therefore, the Monte Carlo simulation is a popular alternative, in order to derive the distribution of real option value. This approach works analogously to the valuation methods for complex financial

options. One might ask why a special theory for real options is necessary when the basic procedures from financial options are applied as well. This question will be answered in section 3.3.2, where the differences between real and financial options are discussed. Monte Carlo simulation is examined in detail in section 3.4.
Before introducing theories of valuing different projects, the dissertation provides the theoretical mathematical background of option and simulation-based valuation in the next section 3.1.

3.1 Mathematical background on option- and simulation-based valuation

3.1.1 Stochastic process and Martingale

Any variable whose value changes randomly over time is said to follow a stochastic process (Zagst, 2010). It can be distinguished between discrete-time and continuous-time stochastic processes. The variable, which follows a discrete-time stochastic process can change its value only at fixed points in time, whereas in the continuous-time case it can change its value at any time. Stochastic processes can further be classified into those, where the underlying variable can take on any value within a certain range (continuous variable process) and those, where only certain discrete values are possible (discrete variable process). For the purpose of this thesis the discrete-time/continuous variable and the continuous time/continuous variable stochastic processes, which will be needed for further asset price modeling, need to be introduced. However, it should be mentioned that in practice stock prices following a continuous-time/continuous variable process are not observed. Asset prices are restricted to discrete values (e.g. multiples of a cent) and price movements can be observed only when the exchange is open. Furthermore, there is evidence that asset prices do not follow a continuous process because they exhibit jumps. Nevertheless, the continuous-time/continuous variable stochastic process proves to be a very useful tool. In the following, the discrete-time/continuous variable and the continuous-time/continuous variable stochastic process will be fined as

Definition 1 (Stochastic process). *A stochastic process is a parametrized collection of random variables* $(X(t))_{t \in T}$ *defined on the filtered probability space* $(\Omega, \mathcal{F}, \mathcal{P}, \mathbb{F})$ *and as-*

suming values in $\mathbb{R}^n$. The parameter space T can be the half-line $[0, \infty)$, but may also be an interval $[a, b]$, the non-negative integers and even subsets of R^n for $n \geq 1$. For each t in$T$ fixed, $\omega \to X(t, \omega)$ with $\omega \in \Omega$ is a random variable. Fixing $\omega \in \Omega$ the function $t \to (X(t, \omega))$ with $t \in T$ is called a path $X(t)$ or a realization of the stochastic process. The stochastic process is said to be:

1. *adapted to the filtration $\mathbb{F}$, if $X(t)$ is measurable with respect to $\mathcal{F}_t$, $\forall t \geq 0$*
2. *measurable, if the mapping $X : [0, \infty] \times \Omega \to \mathbb{R}^n$ is $\mathcal{B}([0, \infty]) \times \mathbb{F} - B(\mathbb{R}^n)$-measurable, where $B(\bullet)$ denotes the Borel-σ-algebra of the set $\bullet$.*

Further it is assumed that a filtered probability space given with $\mathcal{F}_0$ containing all subsets of the null sets of $\mathcal{F}$ and $(\mathcal{F}_t)_{t \geq 0}$ being right continuous. A stochastic process is adapted to the filtration $\mathcal{F}$ and $(\mathcal{F}_t)_{t \geq 0}$ if $X(t)$ is $\mathcal{F}_t$-measurable for all $t \geq 0$.

Definition 2 (Martingale)**.** *A Martingale is an adapted stochastic process $X(t)$ with satisfies:*

1. *X_t is $\mathcal{F}_t$-measurable*
2. *$E_P[|X(t)|] \leq \infty$*
3. *$E_P[X(t)|\mathcal{F}_s] = X(s)P -$ a.s. $\forall\ 0 \leq s \leq t < \infty$*

3.1.2 Equivalent measure and the Radon-Nikodým derivative

The theory of measure change is based on concepts of the measure theory. It is important to note that measure theory and probability theory use different terms for the same expression. Table 3.1 compares these terms.

Table 3.1: Equivalence of measure-theoretic and probabilistic terms

Measure	Probability
Integral	Expectation
Measurable Set	Event
Measurable function	Random variable
Almost everywhere (a.e)	Almost surely (a.s.)

Source: Own work based on (see Bingham and Kiesel, 2001, page 43).

Definition 3 (Equivalent Measure)**.** *Let $\mathcal{P}$ and $\mathcal{Q}$ be measures on the measurable space $(\Omega, \mathcal{F})$.*

1. *$\mathcal{Q}$ is called absolutely continuous with respect to $\mathcal{P}$, in notation $\mathcal{Q} << \mathcal{P}$, if $\mathcal{P}(A) = 0 \Rightarrow \mathcal{Q}(A) = 0, \forall\ A \in \mathcal{F}$*

2. *$\mathcal{P}$ and $\mathcal{Q}$ are equivalent measures, in notation $\mathcal{P} \backsim \mathcal{Q}$, if $\mathcal{P}(A) = 0 \Leftrightarrow \mathcal{Q}(A) = 0, \forall\ A \in \mathcal{F}$*

Theorem 1 (Radon-Nikodým derivative)**.** *Let Ω be a non-empty set and $\mathcal{F}$ be a σ-algebra on $\mathcal{F}$. Let $\mathcal{P}$ and $\mathcal{Q}$ be two measures on the measurable space $(\Omega, \mathcal{F})$, where $\mathcal{P}$ is σ-finite, such that $\mathcal{Q} << \mathcal{P}$. Then, there exists a function $f : \Omega \rightarrow [0, \infty]$ such that*

$$\mathcal{Q}(A) = \int_A f d\mathcal{P},\ \forall\ A \in \mathcal{F}. \tag{3.1}$$

The function f is unique almost everywhere on $\mathcal{P}$ and is called the Radon-Nikoỳm derivative of $\mathcal{Q}$ with respect to $\mathcal{P}$.

A more detailed explanation and a proof for the discrete case can be found in Shreve (2004*a*) and for the continuous case in Bingham and Kiesel (2001). The Radon-Nikodým derivative is a central tool in the transformation of stochastic process. Black and Scholes (1973) and Merton (1973) use this theory in their semiannual work on option pricing.

3.1.3 Girsanov-Theorem and Martingale representation

This section introduces the main building blocks of risk-neutral valuation: the Girsanov theorem, the martingale representation, and the risk-neutral probability measure.

3.1.3.1 One dimensional case

One of most important theorems in finance is Girsanov (1960)'s theorem which defines how a Brownian motion can be transformed from the physical measure $\mathcal{P}$ to the risk-neutral measure $\mathcal{Q}$. This theorem was the basis for the Black-Scholes-Merton (Black and Scholes, 1973; Merton, 1973) model and the basis for most derivative applications now used in finance. The theorem states the following:

Theorem 2 (Girsanov, one-dimension)**.** *(Girsanov, 1960; Shreve, 2004b).*
Let $(\gamma(t) : 0 < t < T)$ be a measurable, adapted with $\int_0^T \gamma(t)^2 dt < \infty a..s.$ Let $(W(t)0 :< t < T)$ be a Brownian motion on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Let $\mathbb{F}$ be a filtration for this Brownian motion. Let $(Z(t)0 :< t < T)$ with

$$Z(t) = \exp\left\{-\int_0^t \gamma(s)' dW(s) - \frac{1}{2}\int_0^t ||\gamma(s)||^2 ds\right\} \tag{3.2}$$

be the corresponding continuous martingale. Define the process $\tilde{W}$ by:

$$\tilde{W}(t) = W(t) + \int_0^t \gamma(s) ds, \quad 0 \leq t \leq T. \tag{3.3}$$

Further assume that $\mathbb{E}\int_0^T \gamma^2(s) Z^2(s) ds < \infty$ and set $Z = Z(T)$. From this follows $\mathbb{E}Z = 1$ and under the probability measure $\mathcal{Q}$ given by 3.1, the Process $\tilde{W}$ is Brownian motion.

Following the Girsanov's theorem the probability measure $\mathcal{P}$ and $\mathcal{Q}$ are equivalent. This means they have the same null sets, the same sets a.s., have the same sets of positive measure (Bingham and Kiesel, 2001, see).

Following (Shreve, 2004*b*, page 214ff.) assume a Brownian motion $(W(t), \quad 0 \leq t \leq T)$ adapted to the filtered probability space $(\Omega, \mathcal{F}, \mathcal{P}, \mathbb{F})$. The asset price process is assumed to be following a geometric Brownian motion with the SDE:

$$dA(t) = \mu(t)A(t)dt + \sigma(t)A(t)dW(t), \quad 0 \leq t \leq T$$

Furthermore, it is assumed an adapted interest rate $R(t)$ exist. The discount process is then defined as:

$$dD(t) = -R(t)D(t)dt.$$

This means that the discount process has zero quadratic variation. The discounted asset price process has the following SDE:

$$\begin{aligned} d(D(t)A(t)) &= (\mu(t) - R(t))D(t)A(t)dt + \sigma(t)D(t)A(t)dW(t) \\ &= \sigma(t)D(t)A(t)\left[\gamma(t)dt + dW(t)\right] \end{aligned}$$

γ is defined as $\gamma = \frac{\mu(t)-R(t)}{\sigma(t)}$ and often called market price of risk (Cf. Shreve, 2004*b*, page 216). Now using the Girsanov's Theorem a new probability measure $\mathcal{Q}$ can be defined. Using the market price of risk and the discounted asset price process can be transformed as $d(D(t)A(t) = \sigma(t)D(t)A(t)d\tilde{W}(t)$ with $d\tilde{W}(t) = dW(t) + \gamma(t)$. This new measure $\mathcal{Q}$ then called the risk-neutral measure because now the discounted asset price process is a martingale under $\mathcal{Q}$.

It is important to note that both in discrete and continuous time the change measure from the physical to the risk-neutral measure does not change the volatility. In discrete time it changes only probability of the branches of the binomial tree and in continuous time it changes the mean rate of return. As a special case for a constant γ the Radon-Nikodým derivative corresponds to $\exp\left\{-\gamma B(t) - \frac{1}{2}\gamma^2 t\right\}$. This means that the equivalent martingale measure can be found by changing the trend from μ to $\mu - \gamma$ (Bingham and Kiesel, 2001, see).

In the case of a market with only one asset and one Brownian motion, the existence of a hedging portfolio follows from the Martingale representation theorem which states:

Theorem 3 (Martingale Representation, one dimension)**.** *(Shreve, 2004b, p. 221) Let* $W(t), \quad 0 \le t \le T$, *be a Brownian motion on a filtered probability space* $(\Omega, \mathcal{F}, \mathcal{P}, \mathbb{F})$. *Let* $M(t), \quad 0 \le t \le T$, *be a martingale with respect to the filtration* $\mathbb{F}$. *Then there is an adapted process* $\Gamma(s), \quad 0 \le t \le T$, *such that*

$$M(t) = M(0) + \int_0^t \Gamma(s) dW(s), \; 0 \le t \le T. \tag{3.4}$$

This means that every martingale with respect to filtration $\mathbb{F}$, which is generated by the Brownian motion, is an initial condition plus an Itô integral. This is because the only risk comes from the variation of the Brownian motion. Therefore, only one risk needs to be hedged.

The conditions of theorem 3 is more strict than in Girsanov's theorem. But combining both it can be shown that $\tilde{M}(t) = \tilde{M}(0) + \int_0^t \tilde{\Gamma}(s) d\tilde{W}(s), \quad 0 \le t \le T$ holds, where $\tilde{M}(t), \quad 0 \le t \le T$, is a martingale under $\mathcal{Q}$ and $\tilde{\Gamma}(t), \quad 0 \le t \le T$, an adapted process (see Shreve, 2004*b*, p. 222).

3.1.3.2 D-dimensional case

In order to value a company which is a linear combination of several different assets. The result has to be generalized. The theorem of Girsanov can be generalized from one dimensional case to the d-dimensional case. This is stated in the following theorem:

Theorem 4 (Girsanov, d-dimensions). *(Girsanov, 1960; Bingham and Kiesel, 2001). Let $(\gamma(t) : 0 < t < T)$ be a measurable, adapted d-dimensional process with $\int_0^T \gamma_i(t)^2 dt < \infty a..s$, $\forall i = 1, \dots, d$ and satisfy the Novikov's condition*[1]*. Define the process $\tilde{W}_i$, $i = 1, \dots, d$ by:*

$$\tilde{W}_i(t) = W_i(t) + \int_0^t \gamma(s) ds, \ (0 \leq t \leq T), i = 1, \dots, d.$$

Then, under the equivalent probability measure $\mathcal{Q}$, given by 3.1, the process $\tilde{W} = (\tilde{W}_1, \dots, \tilde{W}_d)$ is a d-dimensional Brownian motion.

A proof of Girsanov's theorem can be found in Karatzas and Shreve (2001).

Theorem 5 (Martingale Representation, d-dimensions). *(Shreve, 2004b, p. 221) Let T be a fixed positive time and $\mathbb{F}$ be a filtration generated by the d-dimensional Brownian motion $W(t) = (W_1(t), \dots, W_d(t))$, $0 \leq t \leq T$. Let $M(t)$, $0 \leq t \leq T$, be a martingale with respect to the filtration $\mathbb{F}$ under $\mathcal{P}$. Then there is an adapted d-dimensional process $\Gamma(s) = (\Gamma_1(s), \dots, \Gamma_d(s))$, $0 \leq s \leq T$, such that*

$$M(t) = M(0) + \int_0^t \Gamma_i(s) dW_i(s), \ 0 \leq t \leq T, i = 1, \dots, d.$$

Further assuming the notation and assumption of Theorem 4 hold and if $\tilde{M}(t)$, $0 \leq t \leq T$, is a martingale under $\mathcal{Q}$, then there is an adapted d-dimensional process $\tilde{\Gamma}(s) = (\tilde{\Gamma}_1(s), \dots, \tilde{\Gamma}_d(s))$, $0 \leq s \leq T$, such that

$$\tilde{M}(t) = \tilde{M}(0) + \int_0^t \tilde{\Gamma}_i(s) d\tilde{W}_i(s), \ 0 \leq t \leq T, i = 1, \dots, d.$$

[1] Novikov Theorem, see (Karatzas and Shreve, 2001, page 199)

3.1.4 Risk-neutral measure and Fundamental Theorems of Asset Pricing

The basic concept of risk-neutral valuation and the fundamental theorems of asset pricing will be studied in this section. Risk-neutral valuation is based on the theory of equivalent measures[2] which is a central tool in finance. It is basis for the concept of risk-neutral valuation and the pricing of contingent claims. This theory assumes that the prices should be the discounted expected values under the equivalent martingale measure using the risk free rate as discount rate.

3.1.4.1 Risk-neutral measure

The risk-neutral measure therefore can be defined as:

Definition 4 (Risk-neutral measure)**.** *(Shreve, 2004b, p. 228) A probability measure $\mathcal{Q}$ is said to be risk-neutral if:*

1. *$\mathcal{Q}$ and $\mathcal{P}$ are equivalent*
2. *the discounted asset price process $\tilde{A_i}(t) = D(t)A_i(t),\ i = 1, \ldots, m$ is a martingale under $\mathcal{Q}$*

A company that buys certain assets and than sells different assets. The cash flow generated by this procedure can be seen as a portfolio of assets $\varphi(t) = (\varphi_0(t), \cdots, \varphi_m(t))$. φ_i simply denotes the amount asset i held in the portfolio at time t. A company determines this amount before time t based on the observed prices $A(t-)$. The amount can be negative, i.e. inputs, as well as positive, i.e. outputs. This value of the portfolio or the cash flow in each period is than defined as:

Definition 5 (Portfolio of Assets)**.** *(Bingham and Kiesel, 2001)*

1. *The value of the portfolio φ at time t is given by the scalar product*

$$V_\varphi(t) := \varphi(t) \cdot A(t) = \sum_{i=0}^{m} \varphi_i(t)A_i(t), t \in [0, T]. \tag{3.5}$$

The process $V_\varphi(t)$ is called the value process, or wealth process of the portfolio φ.

[2] See definition 3.

2. *The gains process $G_\varphi(t)$ is defined by*

$$G_\varphi(t) := \int_0^t \varphi(u)dA(u) = \sum_{i=0}^{m} \int_0^t \varphi_i(u)dA_i(u). \tag{3.6}$$

3. *A portfolio is called self-financing if the wealth process $V_\varphi(t)$ satisfies*

$$V_\varphi(t) = V_\varphi(0) + G_\varphi(t), \ \forall t \in [0, T]. \tag{3.7}$$

Following (Cont and Tankov, 2003, p. 302) this portfolio value is a martingale under risk-neutral measure. Since $A(t)$ is a local martingale under $\mathcal{Q}$ and φ is predictable and locally bounded, it follows that the stochastic integral

$$\tilde{G}_\varphi(t) := \int_0^t \varphi(u)d\tilde{A}(u) = \sum_{i=0}^{m} \int_0^t \varphi_i(u)d\tilde{A}_i(u) \tag{3.8}$$

is a local martingale. Now using $\tilde{V}_\varphi(t) = V_\varphi(0) + \tilde{G}_\varphi(t)$, it follows that $\tilde{V}_\varphi(t)$ is a local martingale. This can summarized in following lemma:

Lemma 1. *(Shreve, 2004b, p. 230) Let $\mathbb{Q}$ be a risk-neutral measure, and let $V_\varphi(t))$ be the value of a portfolio. Under $\mathbb{Q}$, the discounted portfolio value $\tilde{V}_\varphi(t) = D(t)V_\varphi(t)$ is a martingale.*

These results lead to the fundamental theorems of asset pricing which will be introduced next.

3.1.4.2 Fundamental theorems of asset pricing

Theorem 6 (First fundamental theorem of asset pricing)**.** *(Shreve, 2004b, p. 231) If a market model has a risk-neutral probability measure, then it does not admit arbitrage.*

Theorem 6 states that there are no arbitrage possibilities in a market with a risk-neutral pricing measure because the martingale property of the discounted price processes does not admit to create portfolios with a sure win. See Bingham and Kiesel (2001) for a profound discussion of this topic.

Definition 6 (Complete market)**.** *(Shreve, 2004b, p. 231) A market model is complete if every derivative security can be hedged.*

Combining theorem 6 and definition 6 lead to the second fundamental theorem of asset pricing:

Theorem 7 (Second fundamental theorem of asset pricing)**.** *(Shreve, 2004b, p. 232) Consider a market model that has risk-neutral probability measure. The model is complete if and only if the risk-neutral measure is unique.*

3.1.5 Feyman-Kac representation

According to Brennan (1979) and Brennan and Schwartz (1977, 1978) certain options can valued using partial differential equations (PDEs). This approach requires the Feyman-Kac representation which is introduced in this section.

The value of an contingent claim within a risk-neutral pricing framework $\nu(x,t)$ is given by a certain SDE namely the Cauchy problem. It can be evaluated using the Feyman-Kac representation by computing the expected value of the contingent claim under risk-neutral pricing measure.

Definition 7 (Characteristic Operator)**.** *Let $X(t)$ be a real-valued adapted stochastic process and the solution to a SDE. The characteristic operator $\mathfrak{D}$ for the unique strong solution under some regularity conditions is*

$$(\mathfrak{D}\nu(x,t) = \nu_t(x,t) + \sum_{i=1}^{n} \mu_i(x,t) \cdot \nu_{x_i}(x,t) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \eta_{ij}(x,t)\nu_{x_i x_j}(x,t), n \in \mathbb{N}$$

with $\nu : \mathbb{R}^n \times [0,\infty) \to \mathbb{R}$ twice continuously differentiable in x, and once continuously differentiable in t, and

$$\eta_{ij} := \sum_{k=1} \sigma_{ik}(x,t) \cdot \sigma_{jk}(x,t), m \in \mathbb{N}$$

Definition 8 (Cauchy problem)**.** *Let $\mathcal{D} : \mathbb{R}^n \to \mathbb{R}, r : \mathbb{R}^n \times [0,T] \to \mathbb{R}$ be continuous and $T > 0$ be arbitrary but fixed. Then Cauchy problem is stated as follows: Find a function $\nu : \mathbb{R}^n \times [0,\infty) \to \mathbb{R}$ which is continuously differentiable in t and twice continuously differentiable in x and solves the partial differential equation:*

$$\mathfrak{D}\nu(x,t) = r(x,t) \cdot \nu_t(x,t), (x,t) \in \mathbb{R}^n \times [0,T]$$
$$\nu_t(x,t) = \mathcal{D}(x), x \in \mathbb{R}^n$$

or alternatively taking into account the payment flow of a contingent claim $h : \mathbb{R}^n \times [0, T] \rightarrow \mathbb{R}$*:*

$$\mathfrak{D}\nu(x,t) - r(x,t) \cdot \nu_t(x,t) = h(x,t), (x,t) \in \mathbb{R}^n \times [0,T]$$
$$\nu_t(x,t) = \mathcal{D}(x), x \in \mathbb{R}^n$$

The Feyman-Kac formula states now Zagst (2010) that under sufficient regularity conditions the solution of the Cauchy problem is given by

$$\nu(x,t) = \exp(-r \cdot (T-t) \cdot E_Q[\mathcal{D}(X(t)|\mathcal{F}_t]$$

for the corresponding stochastic process $X(t)$. If neither an analytical solution to Cauchy problem nor a closed form solution of the expected value for a specific stochastic process exist than the expected value has to be approximated by Monte Carlo methods.[3]

3.2 Discounted cash flow valuation

3.2.1 Free Cash flow and free cash flow modeling

3.2.1.1 Definition of free cash flow

The economic activities of a company or a project are normally characterized by usage of certain inputs to generate different outputs. Normally there is a time difference between buying inputs in cash and selling the generated outputs for cash. Investment valuation analyzes these streams of cash outflows for inputs as well as streams of cash inflows for outputs and tries to take the time difference between them into account. The cash flow is the sum of the cash inflows and cash outflows during a specified period, such as a year or a quarter. Therefore, investment valuation does not concern itself with accounting profits; instead, it uses only the observable movements of cash (see Bitz, 2004, p. 107).

According to Penman (2007), three types of cash flows can usually be distinguished in the financial literature and in financial reports: cash flows from (i) operating activities (CFO), (ii) investment activities (CFI), and (iii) financing activities (CFF). The cash flow

[3]For a description of Monte Carlo methods see section 3.4.

from operating activities is the net amount of cash generated from operating activities. It shows the cash paid for inputs to the production and the cash received from selling outputs to customers. Investments in or sales of long-term assets, such as property, plant and equipment or long-term investments in other companies, are measured by the cash flow from investing activities. The cash flows from financing activities relate to the raising or repayment of the company's or project's capital.

To determine cash flows both the direct and the indirect method can be used. The direct method sums up observed cash outflows and inflows; the indirect method starts with an accounting measure, normally the net income (NI), and corrects it with all non-cash charges[4]. This means the CFO is the net income plus net non-cash charges (NCC) minus changes in working capital WC plus interest expenses ($k_d \cdot D$) where k_D is the after-tax cost of debt following equation (3.19) and D the amount of debt; the CFI equals investments in long-term assets ($\Delta LA + dep$), and the CFF equals net borrowing ΔD.

$$CFO_t = NI_t + NCC_t + k_d \cdot D_{t-1} - \Delta WC,\ t \in [0, T] \tag{3.9}$$

$$CFI_t = \Delta LA_t + dep_t,\ t \in [0, T] \tag{3.10}$$

$$CFF_t = \Delta D_t,\ t \in [0, T] \tag{3.11}$$

Free cash flow to the firm (FCFF) is the cash flow available to the company's capital providers after all operating expenses and investments in working capital as well as long-term assets have been paid. More formally the free cash flow to the firm can be defined as follows:

Definition 9 (Free cash flow to the firm)**.** *(Stowe et al., 2009, p. 353) The free cash flow to the firm is the cash flow from operating activities minus the cash flow from investment activities:*

$$FCFF_t = CFO_t - CFI_t - \tau EBIT_t,\ t \in [0, T]. \tag{3.12}$$

[4]The most common non-cash charge is depreciation

Equation (3.13) shows the indirect derivation of the FCFF:

$$FCFF_t = NI_t + NCC_t + k_d \cdot D_{t-1} - \Delta WC_t - \Delta LA_t,\ t \in [0, T]. \tag{3.13}$$

Free cash flow to equity (FCFE) is the cash flow available to a company's or project's common equity holders after all operating expenses, investments in working capital and long-term assets, interest, and principal repayments have been paid.

Definition 10 (Free cash flow to equity)**.** *(Stowe et al., 2009, p. 353f) The free cash flow to equity is the cash flow from operating activities minus the cash flow from investment activities minus payments to and plus receipts from debtholders*

$$FCFE_t = CFO_t - CFI_t - k_d \cdot D - \Delta D,\ t \in [0, T]. \tag{3.14}$$

The (FCFE) can then be alternatively derived by using the (FCFF) subtracting ($k_d{\cdot}D$), and adding the net borrowing δD as shown in equation (3.15). The indirect derivation starting from the net income is shown in equation (3.16):

$$FCFE_t = FCFF_t - k_d \cdot D + \Delta D \tag{3.15}$$

$$= NI_t + NCC_t - \Delta WC - \Delta LA + \Delta D. \tag{3.16}$$

3.2.1.2 Free cash flow modeling

Modeling the free cash flow is an integral but complex part of the capital budgeting process. The aim is for the model to be suitable to forecast future cash flows and the challenge is to use this model in such a way that the obtained results are meaningful. In a simple forecast model, historical accounting data is used to predict future free cash flow values. The last value is used to extrapolate the free cash flow applying a constant growth rate.[5] However, this approach often does not capture the complex relationships among the cash flow components. This can be done by using fundamental analysis. It is the most common approach used in free cash flow modeling since it allows one to capture the relationship among the cash flow components. Fundamental analysis derives its forecasts for each cash flow component based on analysis of historical data, such as financial reports

[5] Cf. Kaka (2009) for a discussion of cash flow forecasting based on a simple cash flow model.

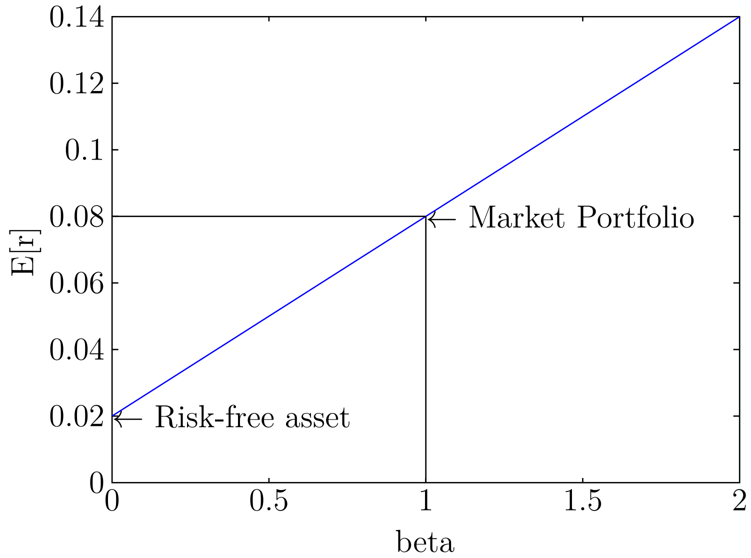
and historical time series, and the current and expected environment (Stowe et al., 2009, p. 378ff).

3.2.2 Cost of Capital

3.2.2.1 Cost of equity

The Capital Asset Pricing Model (CAPM) states that the expected risk premium on each investment is proportional to its beta. It was introduced by in the mid-1960s by Black (1972), Lintner (1965*a*,*b*), Mossin (1966), Sharpe (1963, 1964), and Treynor (1961).[6] This means that in figure 3.1 all investments must plot along the sloping line, known as the *Security Market Line*.

Figure 3.1: Security market line



Source: Own work based on Brealey et al. (2007, p. 195).

Brealey et al. (2007, p. 195) illustrate this relationship as:

$$\text{expected risk premium on stock} = \beta \times \text{expected market risk premium}$$

$$E[r] - r_f = \beta \times (E[r_M] - r_f)$$

[6]For a critique on the CAPM I refer to the original work of Roll (1977).

With

$$E[r_M] = \text{expected market return,}$$
$$E[r] = \text{expected stock return,}$$
$$r_f = \text{risk free interest rate and}$$
$$\beta = \frac{Cov(r, r_M)}{Var(r_M)}$$
$$= \frac{\rho_r, r_M \cdot \sigma_r}{\sigma_M}$$

Instead of equation $\beta = \frac{COV(r,r_M)}{VAR(r_M)}$, usually *Linear Regression*[7] is used to derive β. Here significance tests on β can be applied. According to Heij (2004, p. 91), the CAPM assumes, that excess returns $(r - r_f)$ are generated by the linear model

$$(r - r_f) = \alpha + \beta(r_M - r_f) + \epsilon \tag{3.17}$$

for certain (unknown) values of α and β. As the linear dependence between $(r - r_f)$ and $(r_M - r_f)$ is only an approximation, a disturbance term ϵ is needed.

In standard valuation theory, DCF analysis is mainly applied to industrial companies with rather stable capital structures. Therefore, a single constant discount rate can be used. This technique is not appropriate in a project finance context since, by definition, it deals with time-varying capital structures (Esty, 1999, see).

Damodaran (1994) and Grinblat and Titman (2001) argued that in such a context only the application of different discount rates for every year, based on the actual capital structure, produces an unbiased costs of equity. The model in this dissertation includes this technique by determining the cost of equity for each period a cash flow is calculated.

Esty (1999) argued that it is crucial to calculate the cost of equity based on market values for debt and equity instead of their book values. In case of a non-marketed investment project, as it is typically the case in project finance, this leads to a well-known circularity problem: the market value of equity, which is estimated as the book value of equity plus the NPV, cannot be calculated without knowing the cost of equity, while without knowing

[7] See (Heij, 2004, p. 75 - 87) for further details.

market value one cannot estimate the cost of equity. A possible solution in a project finance context was proposed by Esty (1999): the so called quasi-market valuation (QMV). The QMV is based on three main assumptions: (i) the CAPM holds, (ii) the market value of debt is equal to the book value of debt and (iii) the market is efficient. This is an iterative solution of the circularity problem. First, the market value of equity at the end of a given year is calculated on the basis of a given cost of equity (which implies certain leverage). Second, if the resulting market value is too high, and therefore, implies a higher than used cost of equity, the cost of equity is increased and a new (lower) market value is derived. These steps are repeated until the market value of equity implied by a given cost of equity is equal to the market value derived by applying the DCF valuation. This technique is included in the model.

3.2.2.2 Cost of debt

The cost of debt measures the cost associated with the financing of a company or project by issuing a bond or taking a bank loan. The before-tax cost of debt r_d is typically estimated via the yield-to-maturity approach or the debt-rating approach (see Courtois et al., 2007, p. 45).

The yield-to-maturity r_d is the annual return of an investor who purchases the bond today and holds it until maturity. It is the yield that equates the current market price with promised payments

$$P_0 = \sum_{t=0}^{T} \frac{PMT_t}{(1+r_d)^t} + \frac{TV}{(1+r_d)^T} \tag{3.18}$$

where P_0 is the current market price, PMT_t is the interest payment in period t, T the time to maturity, TV the terminal value of the bond, and r_d the yield to maturity.

There is typically no market price for single projects. Then the debt-rating approach can be used to estimate the before-tax cost of debt. They are calculated by taking the yield of similar rated market traded bonds, which have a comparable maturity (see Courtois et al., 2007, p. 46).

In many countries, such as Germany, the interest payments are tax-deductible; therefore, the after-tax cost of debt k_d are calculated by

$$k_D = r_D \cdot (1 - Taxrate) \tag{3.19}$$

3.2.2.3 Weighted Average Cost of Capital

If the FCFF is used to value an investment, it is necessary to the the weighted average cost of capital (WACC). Copeland and Antikarov (2003, p. 66) define the WACC as follows:

$$\widehat{WACC} = \widehat{k}^E \frac{S}{B+S} + \frac{D}{D+S}$$

with

$$\begin{aligned}
\widehat{k}^E &= \text{estimated cost of equity} \\
\widehat{k}^D &= \text{estimated cost of debt} \\
S &= \text{wealth of the shareholders} \\
D &= \text{wealth of the debt holders} \\
\tau &= \text{tax-rate}
\end{aligned}$$

The WACC has the same problem as the cost of equity calculation. It requires the market values of the debt and equity to calculate it.

3.2.3 Net present value

3.2.3.1 Net present value calculation using risk-adjusted cost of capital

In the case of a certain free cash flow stream $(FCF_0, \ldots, FCF_n)$[8], Luenberger (1998, p. 21) defines the present value (NPV) as:

$$NPV = \sum_{k=0}^{n} \frac{FCF_k}{(1+r_f)^k} \tag{3.20}$$

[8] This can be the FCFF or FCFE

with r_f the risk-free interest rate. An investment project would be realized when the NPV is equal or greater than zero.

In reality however, the future free cash flow stream is not certain; instead it is stochastic. Thus, risk and investors' attitudes towards it must be accounted for in the process of capital budgeting, and particularly in the NPV criteria (see Trigeorgis, 1996, p. 33). A variable that is stochastic is characterized by a probability distribution of its possible outcomes. The standard deviation, skewness, and kurtosis measure the riskiness of that variable.

But again, any project with an $NPV \geq 0$ would be realized. The NPV is calculated in a similar manner to equation (3.20) but uses the expected FCF instead of the point value. Furthermore, the riskiness of the cash flow stream is considered. Copeland and Antikarov (2003, p. 72) introduce two different approaches for consideration it in the NPV calculation:

Definition 1 (Present value). *Consider a given but stochastic free cash flow stream $F\tilde{C}F_0, F\tilde{C}F_1, \ldots, F\tilde{C}F_n$, which will be received in the future.*

$$NPV = \sum_{t=0}^{T} \frac{\mathbb{E}[FCF_t]}{(1 + risk\text{-}adjusted\ rate)^t} \tag{3.21}$$

$$NPV = \sum_{t=0}^{T} \frac{\mathbb{E}[FCF_t] - risk\ premium}{(1 + r_f)^t} \tag{3.22}$$

with $\mathbb{E}[FCF_t]$ = expected free cash flow at time t and r_f the risk-free interest rate.

Albeit both approaches provide the same result[9], they are fundamentally different (see Copeland and Antikarov, 2003, p. 73). In Equation (3.21), the timing and riskiness of the cash flows are taken into account by the risk adjusted discount rate, while equation (3.22) uses the risk-neutral approach.[10]

As already shown in section 3.2.1, it is possible to analyze the FCFF or the FCFE. The equity value of an investment project can be derived in two ways:

1. In the entity approach, the expected FCFF is discounted at WACC and the amount

[9]See (Trigeorgis, 1996, p. 35ff) for a proof that both calculations result in same NPV.
[10]Equation (3.22) will be discussed in section 3.2.3.2.

of debt D is subtracted from the derived value.

2. In the equity approach, the expected FCFE is discounted at cost of equity r_e.

The entity approach is defined in definition 11 and the equity approach is defined in definition 12.

Definition 11. *Consider an uncertain cash flow stream* $(\tilde{FCFF}_0, \tilde{FCFF}_1, \ldots, \tilde{FCFF}_n)$, *which the company will generate in the future. The value of the company calculated by the entity approach is defined as:*

$$NPV^{Entity} = \sum_{t=0}^{T} \frac{\mathbb{E}[FCFF_t]}{(1+WACC)^t} \tag{3.23}$$

where $T \in \mathbb{R}^+$. *By subtracting the debt value D from the entity value, the equity value is derived:*

$$NPV^{Equity} = NPV^{Entity} - D \tag{3.24}$$

Definition 12. *The value of the firm calculated by the equity approach is defined as:*

$$NPV^{Equity} = \sum_{t=0}^{T} \frac{\mathbb{E}[FCFE_t]}{(1+r_e)^t} \tag{3.25}$$

where $T \in \mathbb{R}^+$.

Following the law of one price, the equity value derived by the equity approach is equal to the equity value derived by the entity approach. The analysis in this dissertation apply the equity approach.

3.2.3.2 Certainty-equivalent principle and risk-neutral valuation

When applying the certainty-equivalent principle, the uncertain FCF in equation (3.20) is replaced by its certainty-equivalent $FCF_t^c = \mathbb{E}[FCF_t] - risk\ premium,\ t \in [0, T]$. Thus equation (3.22) can be rewritten as

$$NPV = \sum_{t=0}^{T} \frac{FCF_t^c}{(1+r_f)^t}. \tag{3.26}$$

The risk premium is the market risk premium, which relates to the average investor in the market. Of course, it is possible that the risk premium changes over time since the riskiness of cash flows changes over time (see Trigeorgis, 1996, p. 34f).
A certainty-equivalent or a market risk premium is rather difficult to determine, especially when the risk does not remain constant. However, this can be solved by transforming the processes to the risk-neutral measure $\mathcal{Q}$, and then, applying the risk-neutral valuation (see Trigeorgis, 1996, p. 38).[11] Therefore, this dissertation includes an equivalent martingale valuation approach based on the Esscher Transformation in my dissertation. The idea behind the Esscher Transformation is to change the original probability measure to an equivalent probability measure under which the discounted price processes of the primary assets are martingales. Esscher (1932).[12]

3.3 Real option theory

3.3.1 Introduction

After a short look at the definitions of the different option types, this section introduces real options. The main difference between financial and real options is the underlying. While financial options refer to securities such as a share of common stock or a bond, the underlying for a real option is a tangible asset. To be more precise the cash flow generated by this asset can be from a single project or a portfolio of projects. This dissertation considers only single projects.

By evaluating a project, the money invested in the beginning as well as the cash flows earned during the project life time are of tremendous interest. Undoubtedly, the cash flows can change, especially if the project is modified. Options to modify projects are known as *Real Options*[13]. Copeland and Antikarov (2003) defined this evaluation approach as follows:

Definition 2 (Real Options)**.** *A real option is the right, but not the obligation, to take*

[11] See section 3.1.4 for a detailed discussion of the risk neutral measure.
[12] For a detailed description of the equivalent martingale measure and the Esscher transform see section 3.1.2 and section 5.1.2.
[13] see Brealey et al. (2007, p. 283)

an action at a predetermined cost called the exercise price, for a predetermined period of time - the life of the option.

One has to be differentiate between several types of real options. They are either a certain type of call or put option. Brealey et al. (2007, p. 283ff) and Trigeorgis (1996, p. 10ff) summarize the main types of real options:

- Option to defer
- Option to stage
- Option to scale
- Option to abandon
- Option to growth
- Option to switch
- Multiple interaction options

3.3.2 Financial and real options

For the following analyses and valuations using real options it is necessary to be familiar with the basics about futures and options. Therefore, a brief overview about these financial contracts will be given and their main properties will be discussed[14].

3.3.2.1 Financial Options

There are basically two types of options: call options and put options. A call option gives the holder of the option the right, but not the obligation, to buy the underlying asset at a certain date in the future for a price agreed upon today. A put option gives the holder of the option the right, but not the obligation, to sell the underlying asset at a certain date in the future for a price agreed upon today. The date specified in the option contract is known as the expiration or maturity date and the price specified in the option is called the exercise price or strike price. Options can further be distinguished because of their exercise style. The so-called American options can be exercised at any time during the

[14] A more detailed introduction can be found in Hull (2009) or Wilmott (2007).

life of the option, whereas the European options can only be exercised on the expiration date. Since an American option includes more rights than a European option, but has the same obligations, it is always more, or at least equally, valuable than a European option. The purchaser of a call option hopes that the price of the underlying asset will rise, whereas the purchaser of a put option hopes that the underlying asset will fall in price. There are two sides to every option contract: on the one hand, the investor who has taken the long position into the option - he has purchased the option - and on the other hand, the option writer - the one who has sold the option. The investor who purchases the option has to pay a cash premium to the writer of the option upfront. Therefore, the writer may be confronted with potential losses later on. An option is a zero sum game; that means the writer's profit or loss is reversed from that of the option contract purchaser. Let S_T denote the price of the underlying asset at time T and let K denote the strike price of the option contract. Then the payout from a long position in a European call is $max(S_T - K, 0)$ and in a European put option is $max(K - S_T, 0)$, respectively.

3.3.2.2 Financial vs. real options

Both types of options are rights, but not obligations, to take action. But for financial options, which are written on market traded securities, it is normally easier to estimate the necessary parameters in order to price these options. For single projects, this does not work; thus, according to Copeland and Antikarov (2003), the Market Asset Disclaimer assumptions were introduced. Instead of observing the market for comparable assets, their main idea was that the project's cash flows without flexibility were used as the underlying asset. This assumption, however, is motivated by the rhetorical question, "What is better correlated to the project than the project itself?"[15] Even though, these assumptions simplify the valuation procedure of real options, another difference between financial and real options is revealed: with the exception of electricity, the underlying of financial options always have positive values. This is not necessarily the case for cash flow streams, which can assume values below zero. Table 3.2 summarises the different parameters of a financial and a real option.

[15] Copeland and Antikarov (2003)

Table 3.2: Comparision between a call option on a stock and a real option on a project.

Parameters	Call option on a stock	Real option on a project
Underlying	Current value of stock	Gross PV of expected cash flows
Cost to exercise	Exercise price	Investment cost
Maturity	Time to expiration	Time until opportunity disappears
Risk	Stock value uncertainty	Project value uncertainty
Discount rate	Risk free interest rate	Risk free interest rate

Source: Own work based on (Trigeorgis, 1996, p. 125).

3.3.3 Types of real options

3.3.3.1 Option to scale

A scaling option[16] gives an investor the right, but not the obligation, to extend or contract a project for a fixed price. It can be interpreted to as an American call option on the additional cash flows with the investment cost as the strike price. Analogously, the possibility of a contract can be interpreted as an American put option.

In the energy sector, an option to contract is seldom used because the cost of that option is normally prohibitively high. A contraction could mean the exchange of an electricity generating turbine for a smaller one or shutting down already built wind parks.[17] In contrast to this, especially newly-built wind parks have the option to extend. Expansion plans from 25 to 100 wind engines are not unusual.

3.3.3.2 Option to abandon

The option to abandon a project gives an investor the right, but not the obligation, to shut down or sell a project for a fixed price. This can be interpreted as an American put option, even if the strike price changes over time. This option can be seen as a special case of the option to contract.

[16] In the literature, this type of option is sometimes referred to as an option to extend or an option to contract.

[17] Renewable energy producers receive a fixed guaranteed price for their electricity regardless of whether it can be sold to other participants or not.

3.3.3.3 Option to defer

A deferral option - or delay option - gives an investor the right, but not the obligation, to wait with the investment and start the project later. This option can be interpreted as an American call option on the project's cash flow. Its exercise price is the investment needed to start the project.[18] For an investment in a power plant, the option to delay the project has to be considered with respect to two opposing factors. On the one hand, it is expected that the construction costs will increase. Furthermore, there is still uncertainty about future political decisions. The rapid change from extending nuclear energy production to shutting down all power plants, can be seen as an example.[19] Additionally, all new build coal-fired power plants have to be built with options to include CCS[20]-Technology.
Especially for renewable investments, on the other hand, spots for power plant are limited, and after a reservation, the construction must start within a few years. Additionally, the guaranteed prices from the Erneuerbare-Energien-Gesetz (EEG) decrease by 5% per year, when the park starts operating later than 2014; the early-starter premium of 2 cents/kWh is reduced to 1,9 cents/kWh in 2015 and totally withdrawn, when the park starts operating after 2016.

3.3.3.4 Option to extend

The option to extend the lifespan of a project by paying an exercise price is also an American call option. For example, investing in a new turbine can change the lifetime of a power plant. Another example is that the operating licences of wind parks end 25 years after permission granted and can be extended by paying the licence fee again according to the EEG.

3.3.3.5 Rainbow options and options to switch

Real options that are driven by several sources of uncertainties are called rainbow options. Usually, price, production costs, demand, and units produced are the uncertainties. Switching options are special cases of rainbow options. They are a series of American call and put options that allow the owner to switch for a certain price between two modes

[18] (See Copeland and Antikarov, 2003, page 12)
[19] It is worth noting that this decision was based on the accident in Fukushima.
[20] Carbon Capture & Storage

of operation. In the energy sector, this means a power plant is switched on when the electricity price is above or equal to the cost of generating the electricity and switched off vice verse. Table 3.3 provides a summary of the descriptions, field of applications, and related research.

Table 3.3: Common real options

Category	Description	Important in	References
Option to defer	Management holds a lease on (or an option to buy) valuable land or resources. It can wait x years to see if output prices justify constructing a building or developing a field	All natural-resource extraction industries; real-estate development; farming; paper products	McDonald and Siegel (1986); Poitras (1988); Tourinho (1979); Titman (1985); Ingersoll and Ross (1992)
Option to stage	Staging investment as a series of outlays creates options to abandon the project in midstream if new information is unfavorable. Each stage can be viewed as an option on the value of subsequent stages and valued as compound options.	All R&D intensive industries, especially pharmaceuticals; long-development capital-intensive projects (e.g. large-scale construction or energy-generating plants); start-up ventures	Majd and Pindyck (1987); Carr et al. (1988); Trigeorgis (1993)

(continued)

Category	Description	Important in	References
Option to scale	Depending on the market condition, a project can be altered. It can be extended if market conditions are better than expected and it can be contracted if its vice verse. The extreme cases of this would be to start or stop a project.	Natural-resource industries (e.g. mining or exploration); facilities planing and construction in cyclical industries; fashion apparel; consumer goods; commercial real estate.	Trigeorgis and Mason (1987); Pindyck (1988); McDonald and Siegel (1985); Brennan and Schwartz (1985)
Option to abandon	The option to abandon can be seen as a final option to scale. If market conditions deteriorate severely, the management can completely abandon the project. It realizes the resale value of the assets.	Capital-intensive industries (e.g. airlines, railroad); financial services; new-product development in uncertain markets	Myers and Majd (1990)

(continued)

Category	Description	Important in	References
Option to growth	An early investment (e.g. R&D, lease on undeveloped land or oil reserves, strategic acquisitions, information networks) is a prerequisite or a link in a chain of interrelated projects, opening up future growth opportunities. It can be seen as an inter-project compound option.	All infrastructure-based or strategic industries (i.e. high tech, R&D, and industries with multiple product generations or applications); multinational operations; strategic acquisitions.	Myers (1977); Brealey et al. (2007), Kester (1984, 1993); Trigeorgis (1988); Pindyck (1988); Chung and Charoenweng (1991)
Option to switch	If prices or demand changes, management can change the output mix of the facility (product flexibility). Alternatively, the same outputs can be produced using different types of inputs (process flexibility).	*Output shifts*: Any goods sought in small batches or subject to volatile demand (e.g. electric power, consumer electronics); toys; machine parts; autos. *Input shifts:* All feedstock-dependent facilities; electric power; chemicals; crop switching; sourcing.	Margrabe (1978); Kensinger (1987); Kulatilaka (1988); Kulatilaka and Trigeorgis (1994)

(continued)

Category	Description	Important in	References
Multiple interaction options	Real-life projects often involve a collection of various options. This is a portfolio of different options the value of which might differ from the sum of individual option values.	Real-life projects in most industries.	Trigeorgis (1993); Brennan and Schwartz (1985); Kulatilaka (1994)

Source: Own work based on (Trigeorgis, 1996, p. 2f.).

3.4 Theory of stochastic valuation of investments

3.4.1 Approaches to deal with uncertainty and complexity

The application of cash flow models based on a fundamental analysis has one important drawback: the obtained future cash flows are normally point estimates. Traditionally, DCF valuation accounts for cash flow risk by adjusting the discount rate. Information on cash flow distribution is either lost in this way or not taken into account at all. Following Vose (2000), the main solution for considering the cash flow distribution is either a scenario-based modeling approach or the application of stochastic valuation. In the first case, the analysis concentrates on different scenarios with each scenario representing a single-point estimate. It is obvious that, in this way, information on the cash flow distribution is lost. From a theoretical point of view, the use of risk analysis allows the testing of the robustness of the estimated cash flows. The following methods are suggested by the financial literature for this task:

1. Sensitivity analysis,
2. Scenario analysis, and
3. Stochastic valuation.

Sensitivity and scenario analyses are often used in real-life applications.
Sensitivity analysis is the most popular method in real-life applications. A sensitivity analysis measures the effects of change in one of the input parameters on the estimated future cash flows. The use of sensitivity analysis on all input parameters allows the identification of the input parameters which have the most influence on the output (van Groenendaal, 1998).

Scenario analysis allows the simultaneous manipulation of several input parameters and the quantification of the impact. Thus, each scenario results in one cash flow.[21] The calculation of various scenarios within the scenario analysis allows the estimation of the impact of certain input parameter combinations on the output. Three scenarios, called base case, worst case, and best case, are calculated. The base case scenario is the most likely scenario. The worst case and the best case scenario are based on input parameters mirroring less or more optimistic future expectations.

However, both methods have significant drawbacks. The sensitivity analysis bears the problem that it focuses on only one input factor; it ignores interactions among the input factors and combined effects. The scenario analysis usually ignores correlations among the single input parameters as well (Balcombe and Smith, 1999). Furthermore, not every possible future state is taken into consideration and no probabilities for the single scenarios are available; in particular, the worst and best case scenarios do not provide information on the probability of outcomes in these (extreme) ranges.
A stochastic valuation is based on a random manipulation of all input parameters. Stochastic valuation allows the quantification of probability distributions of the output as it "uses a random selection of scenarios (likely as well as unlikely) to generate information" (van Groenendaal and Kleijnen, 2002). As a result, it yields a probability distribution of the future cash flows. Stochastic valuation includes analytical methods, which are based on probability theory, as well as Monte Carlo methods, which are based on simulations. Monte Carlo methods are needed for valuing investment when the analytical distribution of the value - the distribution of the asset prices underlying the investment - are not available. The term Monte Carlo methods and simulation-based valuation are synonymous in this

[21] Cf. Hwee and Tiong (2002) for an application of a scenario analysis in the context of a computer based cash flow forecast model.

dissertation.

3.4.2 Stochastic valuation of investments

Stochastic valuation refers to the forecasting of future cash flows with the aim of obtaining probability distributions of the cash flows at some future point in time, and thereby, finding the distribution of the investment value. The main advantage of this method is the calculation of a probability distribution instead of a point estimate. In addition, this method is the only way to incorporate real option theory in the valuation process. Simplified, stochastic valuation consistss of three steps:[22]

1. the specification of the cash flow equation and the output / input factors,
2. the simulation of future output / input factor values and the calculation of the cash flow values, and
3. the aggregation of the results to a probability distribution.

Black and Scholes (1973) introduced an analytical solution to evaluate ”European options” in their paper. In the same paper, they stated that the equity of an investment can be seen as an option. Therefore, theories used to value options can also be applied to valuing investment opportunities. Besides the continuous time model of Black and Scholes and Merton, another analytical method in discrete-time is the use of binomial trees. They can be used to solve problems if only one state variable exists (see Cox et al., 1979).

Depending on the specific distribution of cash flow and the included real options, investment valuation problems must be solved numerically. It is often possible to solve those problems using partial differential equations (PDE) as well as specifiying certain boundry conditions and, then, applying the Feyman-Kac representation[23]. The PDEs are then solved numerically and the asset or option value as well as the optimal exercising strategy of an option, are derived from the solution (see Brennan, 1979) and (Brennan and Schwartz, 1977, 1978).

But if the investments are more complex, involving serveral state variables (different inputs and outputs), and/or having path-dependent real options, the solutions can only

[22] For a discussion of risk analysis processes see, for example, Backhaus et al. (2003)

[23] See section 3.1.5.

be found by Monte Carlo methods. Simulation-based valuation is based on repeated random sampling - the repeated drawing of random variables or numbers[24], with the aim to construct a probability distribution of the output of interest. It generates a joint probability distribution function by random sampling from several probability distribution functions. For this, it is necessary to forecast the probability distribution function of all risk factors that influence the cash flow. According to Boyle (1977, p. 334), the *Monte Carlo method* is very flexible with regard to the distributions that generate the returns on the underlying assets. In that sense Monte Carlo methods are a simple and flexible tool because they can easily be modified to accommodate different processes governing the underlying asset returns.

3.4.2.1 Analytical valuation methods

Put-Call-Parity The prices of European puts and calls on the same stock with identical strike prices and maturity dates have a special relationship. The put price p, call price c, stock price S_t, strike price K, time to maturity T, and risk-free rate r are all related by a formula called put-call parity. This dissertation constructs two portfolios, A and B, to derive this formula. Portfolio A consists of one stock and a European put. This portfolio will require an upfront investment of $S_0 + p$. The second portfolio B consists of a European call, with the same specifications as the European put in portfolio A, and a risk-free bond with a face value of K. This portfolio will require an upfront investment of $c + Ke^{rT}$. In table 3.4, one can see what happens to the portfolios at maturity of the options.

[24]Random numbers are numbers that exhibit statistical randomness. Cf. section 3.4.2.2 for a discussion of a potential computational procedure to obtain random numbers.

Table 3.4: Put-Call Parity

Portfolio	**Current value**	**Payoff at time T if:**	
		$S_T \leq K$	$S_T > K$
Stock	S_0	S_T	S_T
Put	p	$K - S_T$	0
Portfolio A	$S_0 + p$	K	S_T
Call	c	0	$S_T - K$
Bond	Ke^{rT}	K	K
Portfolio B	$c + Ke^{rT}$	K	S_T

Source: Own work.

The stock is worth S_T regardless of whether S_T is more or less than K. The risk-free bond is worth K at maturity regardless of the price of the stock. If S_T exceeds K, the call expires in-the-money and is worth $S_T - K$ and the put expires worthless. If S_T is less than or equal to K, the put expires in-the-money and is worth $K - S_T$ and the call expires worthless. As one can see, the total value of portfolio A and B are equal at maturity, because of the law of one price, which says that combinations of assets with equivalent outcomes under all states must have the same price. Otherwise, an arbitrage opportunity would exist. Thus, the famous put-call parity for a non-dividend-paying stock follows: $S_0 + p = c + K\ e^{-rT}$. If the underlying asset provides a dividend, it can be shown by the same arguments as above; the put-call parity has the form $S_0 - D + p = c + K\ e^{-rT}$ where D is the present value of the dividends during the life of the option. With an annual dividend yield of q on the underlying asset, it can be shown by the same arguments as above that the put-call parity has the following form: $S_0\ e^{-qT} + p = c + K\ e^{-rT}$

Binomial Model The BOPM proposed by Cox et al. (1979) provides a numerical method for the valuation of option prices. It is a discrete-time, lattice-based model of the varying price of the asset underlying the option. The evolution of the underlying asset is described by constructing a binomial tree. The binomial tree represents different possible paths that might be followed by the underlying asset price over the life of the option. It is assumed that the underlying asset follows a random walk. In each time step, the underlying is assumed to either move up a certain percentage by a certain probability or to move down a certain percentage by a certain probability. For a European option without dividends, the BOPM converges to the famous Black and Scholes model as the number of time steps in the binomial tree increases. A proof of this can be found in Appendix A. An intuitive explanation is that the BOPM assumes that the returns of the underlying asset price follow a binomial distribution, and the binomial distribution approaches the normal distribution assumed by Black/Scholes, when the number of time steps tends to infinity. Although being computationally slow, the BOPM is used by practitioners to value American options, because there is no closed-form solution for American options and, therefore, their values must be found numerically.

Black and Scholes Merton pricing model Before deriving the Black and Scholes Formula, several assumptions of the model are listed. Black and Scholes (1973) and Merton (1973) assume an ideal capital market,

1. which is frictionless,
2. where the short-term interest rate is known and fixed,
3. where there are no dividends,
4. where only European options exist,
5. in which the asset price follows a geometric Brownian motion.

Furthermore, they assume every option can be perfectly replicated and hedged. Assuming that the model's underlying assumptions hold, the Black and Scholes price at time 0 of a

European call c and put option p on a non-dividend-paying asset are given by

$$c = S_0 N(d_1) - K e^{-rt} N(d2)$$
$$p = K e^{-rt} N(-d2) - S_0 N(-d_1)$$

where

$$d_1 = \frac{\ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$
$$d_2 = \frac{\ln(\frac{S_0}{K}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$
$$= d_1 - \sigma\sqrt{T}$$

Limitations of the Black and Scholes model Like previous pricing models, the Black and Scholes (1973) model assumes that the evolution of the asset price follows a geometric Brownian motion. Even though the Black and Scholes model is the most widely used option pricing model in trading rooms throughout the world, it has some limitations which will be examined in this section. One assumption made by the Black and Scholes model is that of a constant volatility of underlying assets, which according to Ross (1989) implies a constant flow of information, as he argues that volatility can be considered as a measure of information flow. The correctness of the constant volatility assumption has, however, been questioned by Rubinstein (1994) and many others, since it is known that the Black and Scholes model has some pricing biases. Hull and Whihte (1987) and Wiggins (198) suggested that the constant volatility assumption might be a reason for the failure of the Black and Scholes model to value options exactly. The deficiencies of the Black and Scholes model can be shown by plotting the implied volatility against the strike price K. This phenomenon was studied in detail by Dumas et al. (1998), Derman and Kani (1994) and Rubinstein (1994) and is called the volatility smile or skew. Evidence for the volatility smile or skew has been presented numerous times by Rubinstein (1994) among others, who studied the volatility of the S&P 500 Index option market, and by Heynen (1994), who studied options on the European Option Exchange.

3.4.2.2 Monte Carlo methods

Since they comprise one of the most powerful numerical tools and are extensively used within the analyses of this dissertation, a brief description of the Monte Carlo methods is presented here. The first documented application of Monte Carlo methods was performed by von Neumann, Ulam and Fermi for nuclear reaction studies in the Los Alamos National Laboratory (see Metropolis and Ulam, 1949; Metropolis, 1987). Hammersley and Handscomp (1964), followed by Glynn and Witt (1992), were the earliest to discuss the relationship between accuracy and the computational complexity of Monte Carlo simulations. Since Monte Carlo simulation is highly computationally intensive, and hence, reluctant on fast calculating machines, its application for the solution of scientific problems rose in frequency as the cost for powerful processors decreased dramatically over the last two decades.

The mathematical basis for Monte Carlo methods is the *law of large numbers* and the *central limit theorem* (see e.g. (Feller, 1971, p. 219)).

Theorem 8 (Law of large numbers). *Eichelsbacher (2010) Let $X_1, X_2, ..., X_n, n \in \mathbb{N}$ be pairwise independent indentically distributed random variables with $E[X_i] = \mu$ and $E[|X_i|] < \infty, i \in \mathbb{N}_n$. Define $S_n := \sum_{i=1}^{n} X_i$, $Y_n := \frac{1}{n} S_n$ then $Y_n \to \mu$ almost surely. For $n = 1, 2, \ldots$ consider a family of distributions $F_{n,\mu}$ with expectation μ and variance $\sigma_n^2(\theta)$; here μ is a parameter varying in a finite or infinite interval. For expectations, the following notation is used:*

$$E_{n,\mu}(u) = \int_{-\infty}^{+\infty} u(x) F_{n,\mu}(dx) \tag{3.27}$$

Suppose that u is bounded and continuous, and that $\sigma_n^2(\theta) \to 0$ for each μ. Then

$$E_{n,\theta}(u) \to u(\mu) \tag{3.28}$$

Theorem 9 (Central Limit Theorem). *Eichelsbacher (2010) Let $(X_1^{(n)} \ldots X_n^{(n)}$ be a sequence of n independent indentically distributed random variables defined on the probability space (Ω_n, p_n) for every $n \in \mathbb{N}^*$, which have for all i and n the same distribution with* *finite expectation μ_X and variance σ_X^2. Then for every $a \in \mathbb{R}$ it holds true:*

$$\lim_{n\to\infty} P\left(\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\frac{X_i^{(n)}-\mu_X}{\sigma_X} \leq a\right) = \Phi(a)$$

where

$$\Phi(a) = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du$$

is the cumulative normal dirstribution function.

This is very helpful for stochastic investment valuation problems. Let G be a one-dimensional distribution with expectation μ and variance σ^2. G is the distribution of the investment's value, which is not known in many cases. If $X_1, \ldots, X_n$ are independent variables with the distribution G, their arithmetic mean $M_n = \frac{(X_1+\cdots+X_n)}{n}$ has expectation μ and variance $\frac{\sigma^2}{n}$. For large n, this variance is small and M_n is likely to be close to μ. Although any desired degree of accuracy can be obtained by performing enough simulation trials, there are usually more efficient ways of reducing the error. One such method, known as the control variate approach, has proved effective in dealing with investment valuation problems. Another technique for increasing the precision of the estimates uses so-called antithetic variates (Glasermann, 2003; Jäckel, 2002, see).
The aim of the Monte Carlo methods is to solve the following problems:

1. to generate samples $x_j, j = 1, ..., J$ from a given probability distribution $\mathcal{P}(x)$.

2. to estimate the expectation of an arbitrary function under this distribution, i.e $\Phi = E[\phi(X)] = \int \phi(x)\mathcal{P}(x)$

One should concentrate on the first problem (sampling), because if it can be solved, then the second problem is solved by using the random samples $x_j j = 1, ..., J$ to give the estimator

$$\hat{\Phi} = E[\hat{\phi(x)}] = \frac{1}{J}\sum_{j=1}^{J}\phi(x_j),$$

$$\hat{s^2} = Var(\hat{\phi(x)}) = \frac{1}{J-1}\sum_{j=1}^{J}(\phi(x_j) - \hat{\Phi})^2$$

If the samples $x_j, j = 1, ..., J$ are generated from $\mathbb{P}(x)$, then the expectation of $\hat{\Phi}$ is Φ.

Also, as the number of samples J increases, the variance $\hat{\Phi}$ will decrease as $Var(\hat{\Phi}) = \frac{\sigma^2}{J}$ where

$$\sigma^2 = \int (\phi(x) - \Phi)^2 d\mathcal{P}(x)$$

is the variance of ϕ.

An estimator for the Variance of $\hat{\Phi}$ is

$$\hat{Var}[\Phi] = \frac{\hat{s^2}}{J}.$$

This decreasing variance is one of the important properties of the Monte Carlo methods. The distribution

$$\frac{\hat{\Phi} - \Phi}{\sqrt{\frac{\hat{s^2}}{J}}}$$

tends to a standardized normal distribution with increasing values of J. On this basis, confidence limits of the estimate $\hat{\Phi}$ can be obtained.

According to Martinez and Martinez (2008, p. 247), a Basic Monte Carlo simulation can be done by following five steps:

1. Determine the pseudo-population or model that represents the true population of interest.
2. Use a sampling procedure to sample from the pseudo-population or distribution.
3. Calculate a value for the statistic of interest and store it.
4. Repeat steps (ii) and (iii) for M trials.
5. Use the M values found in step (iv) to study the distribution of the statistic.

Pseudo should emphasize the fact that all samples are obtained by using a computer and pseudo random numbers.

Transformed to the option-pricing problem, the procedure is given by following steps:

1. Determine the density of the underlying.
2. Generate the returns on the underlying to derive the option value at each time.

3. Calculate the (Net) Present Value of the option and store it.

4. Repeat steps (ii) and (iii) for M trials.

5. Use the M values found in step (iv) to study the distribution of the option values.

If genuinely normally distributed random numbers need to be generated, then the simplest technique is the Box-Muller method. This method takes uniformly distributed variables and turns them into standard normal distributed random variables. The basic uniform numbers can be generated by various methods. For some algorithms see Huynh et al. (2009).
The Box-Muller method takes the two independent uniform random numbers U1 and U2 between zero and one and combines them to give two independent standard normal distributed numbers X1 and X2.

Theorem 10 (Box-Muller-Transform)**.** *If U_1 and U_2 are independent random variables that are uniformly distributed on the interval $(0,1]$, then*

$$X_1 := \sqrt{-2\log(U_1)}\cos(2\pi U_2)$$

and

$$X_2 := \sqrt{-2\log(U_1)}\sin(2\pi U_2)$$

are independent $\mathcal{N}$ random variables.

3.4.2.3 Simulation-based valuation of investments

This section shows how to value an investment using Monte Carlo methods (Hull, 2009). Essentially, the method uses the fact that the distribution of terminal asset prices is determined by the process generating future asset price movements. This process can be simulated on a computer, thus generating a series of asset price trajectories.
When Monte Carlo methods are used to value an investment, it has to be done in a risk-neutral world because the investor-specific discount rate is unknown.[25] Paths of the risk-neutral process are sampled to obtain the certainty-equivalent cash flow of the investment

[25]For an introduction of risk-neutral valuation see section 3.2.3.2.

in a risk-neutral world, and then, this payoff is discounted at the risk-free interest rate.[26] An investment dependent on only one risky underlying asset is considered. Furthermore, a constant risk-free interest rate is assumed. Then an investment can be valued as follows:

1. Sample a random path for the underlying asset in a risk-neutral world.

2. Calculate the cash flow of investment until time T.

3. Repeat the first two steps numerous times (law of large numbers).[27]

4. Discount the cash flow at the risk-free rate.

5. Calculate the NPV distribution of the investment from the cash flow paths to get an estimate of the value in the real world..

An important consideration in this procedure is that by sampling from the known distribution $\mathcal{P}(x)$ one has to be able to, first, cover the entire sample space, and second, to generate independent samples.

The specification of the input factors is based on the underlying cash flow equation that is used in the second step to calculate the single cash flows. One must decide which input factors are deterministic and which stochastic. Each stochastic factors will have its own stochastic process based on its distribution properties such as the mean and the variance. In addition, but not necessarily, correlations between the single input parameters can be specified. A possibility to obtain reliable parameters estimates is to conduct an expert survey. Hughes (1995). Another possibility is a time series analysis of the historical time series and a forecast of the relevant parameters. The advantage of the time series framework is the possibility of directly addressing the correlation structures within the input parameters.Hui et al. (1993). Chapter 5 discusses the input parameter specification and the impact of correlations on the modeling results.

The underlying cash flow equation specifies the relationship between the input factors and the cash flow. The calculation of the cash flow is based on the repeating random combinatiosn of the input parameters. Hacura et al. (2001) state that "duing the simulation process, random scenarios are built up using input values for the project's key uncertain

[26]See section 3.2.3.2.

[27]The more frequent the repetitions, the more accurate the result.

variables, which are selected from appropriate probability distributions."

The generated series of cash flows are collected, discounted, and analyzed statistically so as to arrive at a probability distribution of the potential outcomes of the project and to estimate various measures of project risk.

One potential drawback of the method arises from the fact that the standard error of the estimate is inversely proportional to the square root of the number of simulation trials, which makes a large number of trials necessary to get accurate results. There are several sophisticated techniques that can be used to improve convergence of Monte Carlo methods. They can be generally classified as techniques for the reduction of variance, and hence, for the increase in accuracy. None of these methods improves the speed of convergence with respect to the number of trials, but they can significantly reduce the coefficient in the error term (Wilmott, 2007, see).

3.5 Concluding Remarks

This section introduced the theory of project valuation. It explained the different types of real options, which are currently used to value investment opportunities. Furthermore, it explained the main analytical models to value investment opportunities, namely the BOPM and the Black and Scholes. Their main advantage is that they provide closed-form solutions for the investment values under certain conditions. But these conditions are relatively strict and often cannot be fulfilled when analyzing an investment project. In that case simulation-based methods need to be applied.

Despite the apparent advantage of simulation-based valuation, it is used sparsely in real-life applications. Except for financial institutions using highly advanced models to value derivatives contracts and other complex financial products, the proliferation of simulation-based valuation is low. Therefore, the advantages and drawbacks of simulation-based valuation are compared and they are summarized below:
Simulation-based valuation has several drawbacks.[28] First, stochastic valuation depends

[28] Nawrocki (2001) for a discussion of the problems with Monte Carlo simulation.

on accurate parameter forecasts, and in particular, on an accurate specification of the correlation structure between the input factors. Even small deviations of the input can have a significant effect on the results. Second, the complexity of the accurate specification of all input parameters is high, in particular when the correlation structure must be specified. Third, the necessary computing power to complete the simulation-based valuation can be immense. Fourth, easy to handle software tools are still rarely available. In addition, one can assume that an average manager would have reservations about stochastic modeling and would have a general lack of knowledge regarding this method (Kwak and Ingall, 2007).

Despite the drawbacks summarized above, simulation-based valuation overcomes the drawback of point estimates inherent in other risk analysis methods. By random sampling from several probability distribution functions of the input parameters, a probability distribution of the cash flows at some future points-in-time is obtained; all possible future states are covered. The obtained probability distributions allow the quantification of the probabilities of certain future events. Thus, simulation-based valuation is, as stated by Kwak and Ingall (2007), "an extremely powerful tool when trying to understand and quantify the potential effects of uncertainty in a project". Second, simulation-based valuation is helpful when comparing capital investments with similar mean NPVs, but with different risk structures; the obtained probability distributions of the cash flows and of the NPVs allow for better informed investment decisions.

Chapter 4

Electricity market and electricity price modeling

Following chapters presenting the relevant literature and the theoretical framework for valuing investment projects chapter 4 introduces the electricity market. First, it gives a short overview of the German electricity market and, second, it focuses on the modeling of electricity. Therefore, a detailed time series analysis is conducted. The model is then calibrated to the German market and a goodness-of-fit is provided.[1]

4.1 The electricity market in Germany

4.1.1 Liberalization of the electricity markets

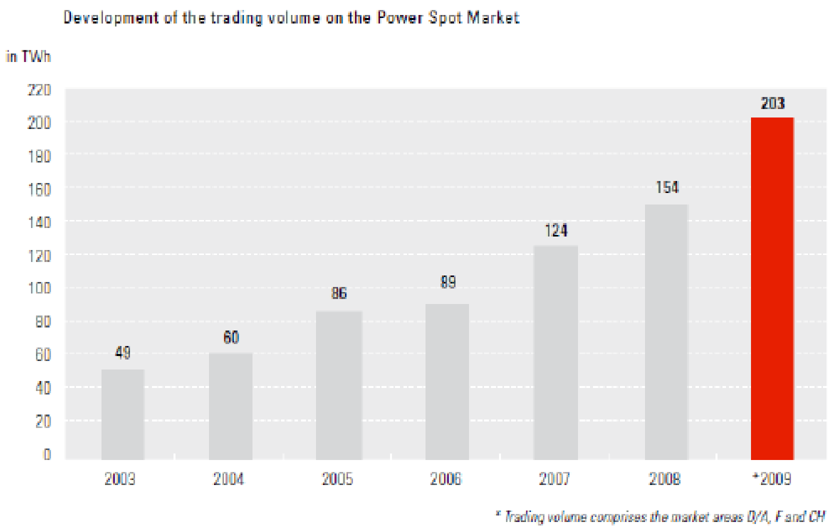
With liberalization of of energy market in Europe at the end of nineties, former regulated companies have to face competition. In Germany electricity prices are open to competition since April 1998.[2]. One goal was to unbundle the areas electricity production, electricity grid and distribution. Another goal was to end the monopolistic structures with fixed tariffs and have exchange-traded prices for electricity. This goal was reached with the foundation of the *European Electricity Exchange* (EEX) in 2002. The EEX introduced an exchange traded reference price for electricity[3] (Konstantin, 2009, p. 41). The EEX was

[1]The following chapter is based on Mayer et al. (2011).
[2]see e.g. Energiewirtschaftsgesetz - EnWG and Konstantin (2009, p. 41)
[3]CO_2 and Natural Gas trading followed later

founded by a merger of *Leipzig Power Exchange* and *European Energy Exchange* based in Frankfurt/Main. The EEX in Leipzig is now the leading energy market in Continental Europe with electricity, natural gas and CO_2-emission rights being traded at theEEX spot and forward market. Standardized products at EEX include only energy, not the grid use. The *European Commodity Clearing AG*[4] takes the clearing[5] of all contracts.

The trading volume grew continuously since the establishment of EEX as it is shown in figure 4.1.

Figure 4.1: Development of the trading volume at the EEX



Source: Own work based on EEX (2007*a*, p. 5).

Whereas in figure 4.1 only spot market volume is taken into account, market participants have also the opportunity to trade forward contracts with a term up to 6 years. Thus customers are able to optimize their energy supply by subscribing tailor-made contracts. The other way round, electricity supplier can respond to costumers in more detail and sell their products in competition. This also influences the investment decisions of the electricity producer since they must to value their investment projects using the EEX-products as reference prices.

[4]subsidiary of EEX

[5]Clearing is the physical and financial fulfillment of spot- and forward-contracts.

While the everyday energy requirement (base supply) as well as the increased demand on working days (block delivery) can be forecasted very well the it's rather difficult to predict short-term demand increases or supply shortages. Long-term electricity needs are laid down in long-term contracts with the electricity producers. Whereas long-term means that a contract period is not exceeding one year. This is also the way of energy procurement of small and middle sized companies. Here the energy exchange is not involved.
The remaining short-term needs are traded between big electricity customers and producers in the spot market. At the EEX, the spot market is the so called "day-ahead-market" or the "day-ahead auction". This auction takes place on every single day of the year and all contracts have to be fulfilled on the following day (with 24 one-hour-intervals). The delivery of the traded electricity is made within the TSO zones in Germany and in Austria[6].

4.1.2 Electricity Prices

4.1.2.1 Pricing of Energy Products

As already mentioned, there is a spot and a forward market at the EEX. In the following the author will only consider the spot market.
Depending on the kind of product, supply-agreements for the following day are placed either in the "continuous trade" or in the "auction trade" (Konstantin, 2009, p. 46).
Block bids[7] are traded continuous. Market participants can place their order between 8.00 a.m. and 12.00 a.m. in an electronically order book. If a selling price is lower or equal than a purchase price, the order is going to be executed immediately. Contracts for single hours are sold by auction. Until 12 a.m., all market participants can make a bid with volume and price limits. All placed bids are the basis for pricing every hour for the next day. A supply- and demand-curve can now be constructed. The EEX publishes the auction results around 12.15 am.

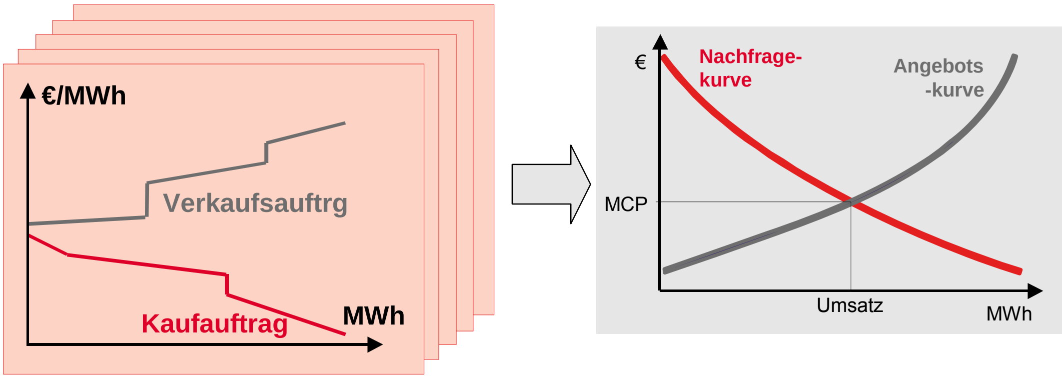
The Market Clearing Price MCP and the amount of executable orders is the intersection of supply- and demand-curve for the relevant hour. The MCP is the price for all

[6] RWE, E.ON, Vattenfall and EnBW supply four different areas of Germany. These are called regulated zones or TSO zones. In Austria, the delivery is supplied by Austrian Power Grid

[7] Standardized block bids can be found in EEX (2007*a*, p. 6).

successful orders. Sale-orders are executed, if their price is equal or lower than the MCP. Vice verse, purchase-orders are executed, if their price is equal or higher than the MCP. This procedure is applied for every single hour for the next day, whereas block bids are splitted. If the pricing failed, EEX informs market participants and give reasons. Then, traders have the opportunity to place a new order. If a block bid could not executed completely, a new MCP for the relevant hour is determined[8]. See figures 4.2 and 4.3 for an illustration.

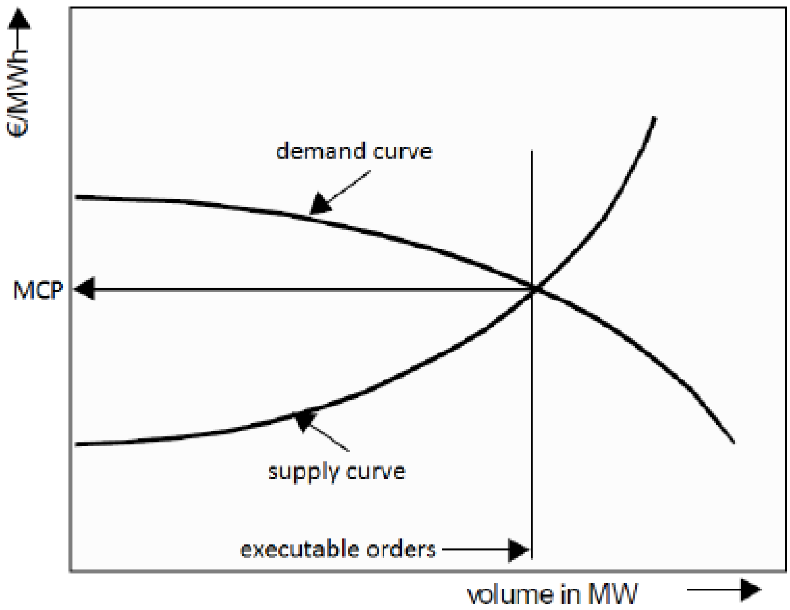
Figure 4.2: Price finding via electricity auction



Source: Own work based on EEX (2007*b*).

[8]See e.g. Konstantin (2009, p. 46f). For more details on pricing at the EEX see EEX (2007*b*) and EEX (2007*a*)

Figure 4.3: Pricing in auction trades



Source: Own work based on Konstantin (2009, p. 36).

The basis for the short-term contracts is *Phelix*[9]: The average of all hourly auctions builds the *Phelix Day Base*, whereas *Phelix Peakload* is referred to the arithmetic mean of the auctions between 8:00 am and 8:00 pm (peakload hours). The Every contract needs a counterpart and energy producers have a big interest to satisfy this demand - if it is profitable. Hence the electricity supplier has to know, how the electricity was produced, because not every power plant is suited for every demand. Subsequently, EEX takes the prices to calculate *Phelix*[10]®. EEX publishes inter alia following indices:

- *Phelix® Base*: average price (weighted) covering hours 1 - 24[11]
- *Phelix® Peak*: average price (weighted) covering hours 9 - 20

[9]Phelix is the abbreviation of ”Physical Electricity Index”
[10]Phelix=Physical Electricity Index
[11]hour 1 is from 0.00 a.m. until 1.00 a.m.; hour 2 is from 1.00 a.m. until 2.00 a.m.; etc.

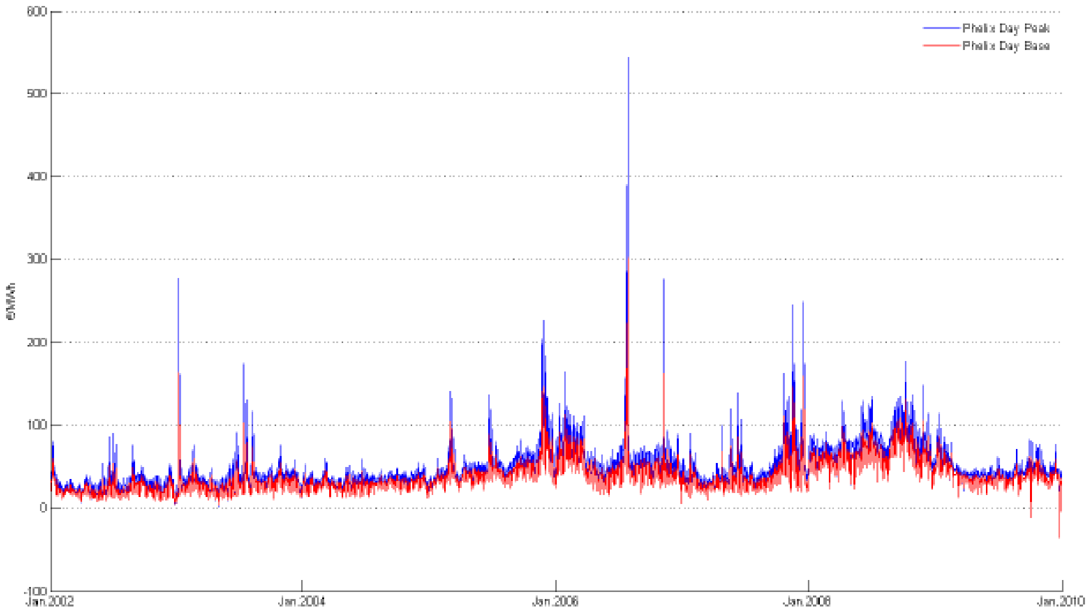


Figure 4.4: Phelix Peak and Phelix Base Chart since January 2002

Besides the mentioned baseload and peakload contracts, the EEX offers the following products for short-term optimizing of the energy portfolio.[12]:

- Baseload contract for the hours 0.00 a.m. until 12.00 p.m.
- Peakload contract for the hours 8.00 a.m. until 8.00 p.m.
- Off-Peak 1 contract for the hours 0.00 a.m. until 8.00 a.m.
- Off-Peak 2 contract for the hours 8.00 p.m. until 12 p.m.
- hourly contracts for every single hour
- other standardized block offers

4.1.2.2 Differences to other Markets

The market for electricity has in comparison to traditional markets, such as the stock or commodity markets, some special characteristics. It leads to an extremely high volatility on the one hand and the occurrence of jumps or, more accurately price spikes. This is due,

[12] see Konstantin (2009, p. 45) and EEX (2007*a*, p. 6)

among other things, that greater amounts of electricity cannot be stored in an economically viable storage as opposed to commodity or shares. This is a tremendous difference to commodity markets, where inventories can be reduced or built up.

The quantity of electricity demanded must be produced and consumed simultaneously, which in the case of power outages and the associated decline in supply lead to sharp price increases. Demand and supply changes have a greater impact on the price than in traditional markets. The adjustment of supply, through the production of large amounts of additional power is only possible through the use of expensive power plants, which means that the price elasticity of supply in the electricity market is significantly higher than on the traditional markets. Even in the normal course of trading a much higher volatility is thus evident. Among the types of power plants that are available relatively quickly are the gas-fired plants. But compared to nuclear or coal-fired power plants they have relatively high production costs and thus their use is profitable only at correspondingly high prices. On the other nuclear or coal-fired power plants cannot be switched off completely in an economic.[13] Thus over supply is also possible. On 26-Dec-2009, the average price for a block *Off Peak 1* (covering hours 0 - 8) was -139,96 €/MWh.

A further characteristic of the electricity market is the inherited seasonality. This can easily be seen during the course of the week. There is a significant price reduction at the weekends compared to the prices during the week. Most companies only produce during the week from Monday to Friday, this explains the less demand for electricity at Saturdays and Sundays. Industrial companies require large amounts of electricity and if they demand less energy it has a significant influence on the price. Furthermore, electricity also exhibits yearly seasonal movements. The intra-day movements are neglected in this work.

4.2 Historical price data analysis

This section shows the intrinsic complexities electricity markets exhibit. The analysis is based on data from the German market, traded at EEX from January 1st 2002 to December 31st 2009. Three different types of peak-price indices are available at EEX:

[13]They are though able to increase or decrease output within certain limits.

- *Phelix Off-Peak 1* is the arithmetic mean of the prices of all hourly contracts from 0.00 a.m. until 8.00 a.m.

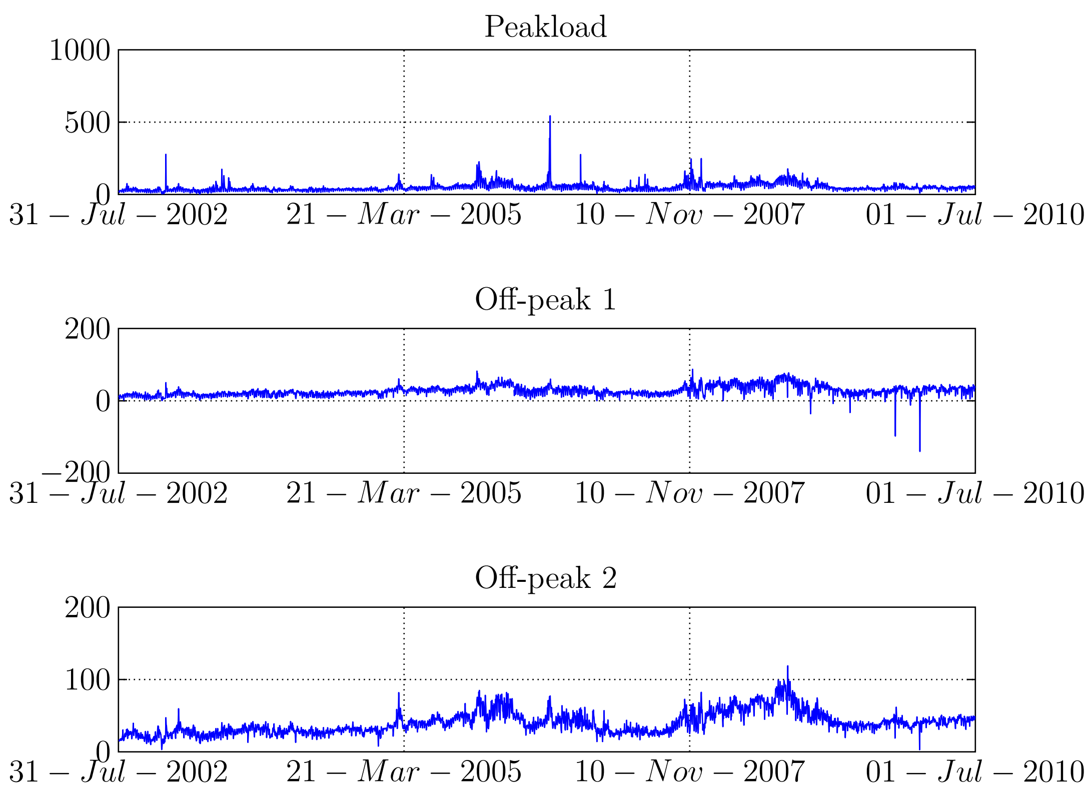
- *Phelix Peak* is the arithmetic mean of the prices of all hourly contracts from 8.00 a.m. until 8.00 p.m.

- *Phelix Off-Peak 2* is the arithmetic mean of the prices of all hourly contracts from 8.00 p.m. until 12 p.m.

For the peakload, this dissertation analysis uses the Phelix Peakload Index and for the period before and after the peaklead it uses the prices of the off-peak block contracts. The index and the block contracts are the arithmetic averages of the corresponding contracts in the auction trading hours of the previous day. For the peakload index, this is the time from 8:00 to 20:00 o'clock, for the off-peak 1 contracts, from 0:00 to 8:00 o'clock, and for the off-peak 2 contracts, from 20:00 to 24:00 o'clock.

Figure 4.5 shows the spot price dynamics for all three price processes.

Figure 4.5: Peakload, off-peak 1 and off-peak 2 prices in Germany from 01/01/02-31/12/09



Source: Own work based on Mayer et al. (2011).

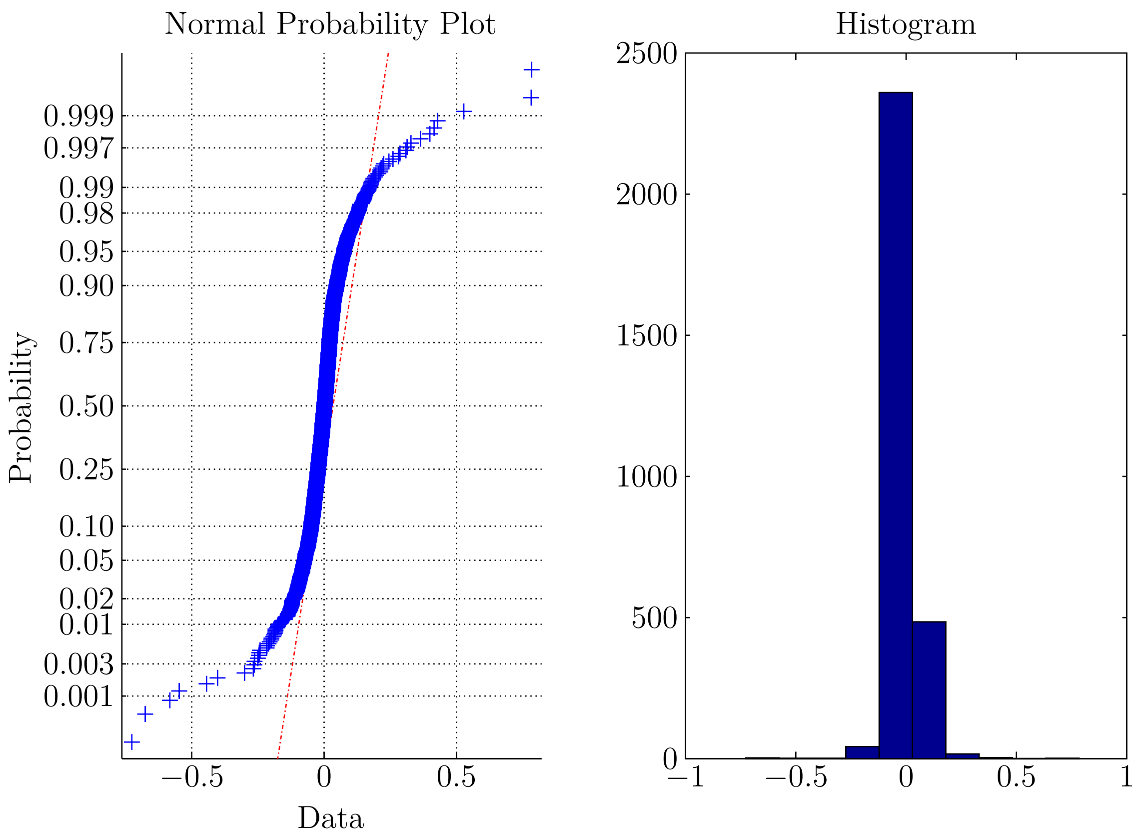
4.2.1 Normality test

The Black and Scholes and Merton-model (Black and Scholes (1973), Merton (1973)) assumes the prices to be log-normally distributed, which is equivalent to saying that the log returns of the prices have a Normal distribution. Although the analysis of stock markets' data reveals a higher probability for an extreme event than predicted by the Normal distribution, the assumption is still embedded in most stochastic models.

4.2.1.1 Peakload

For electricity spot prices though, the departure from normality is more extreme. Figures 4.6, 4.7, and 4.8 show normality tests for the three electricity spot price processes from January 1^{st} 2002 to December 31^{st} 2009. If the returns were indeed normally distributed, the graphs would be straight lines. It is clearly observable that this is not the case, indicating fat tails. For instance, corresponding to a probability of 0.001, here one sees returns that are almost 0.5. In contrast, if the data were perfectly normally distributed, the dotted lines suggest the probability of such returns should be virtually zero.

Figure 4.6: Normal probability plot and histogram of the unseperated peakload process.

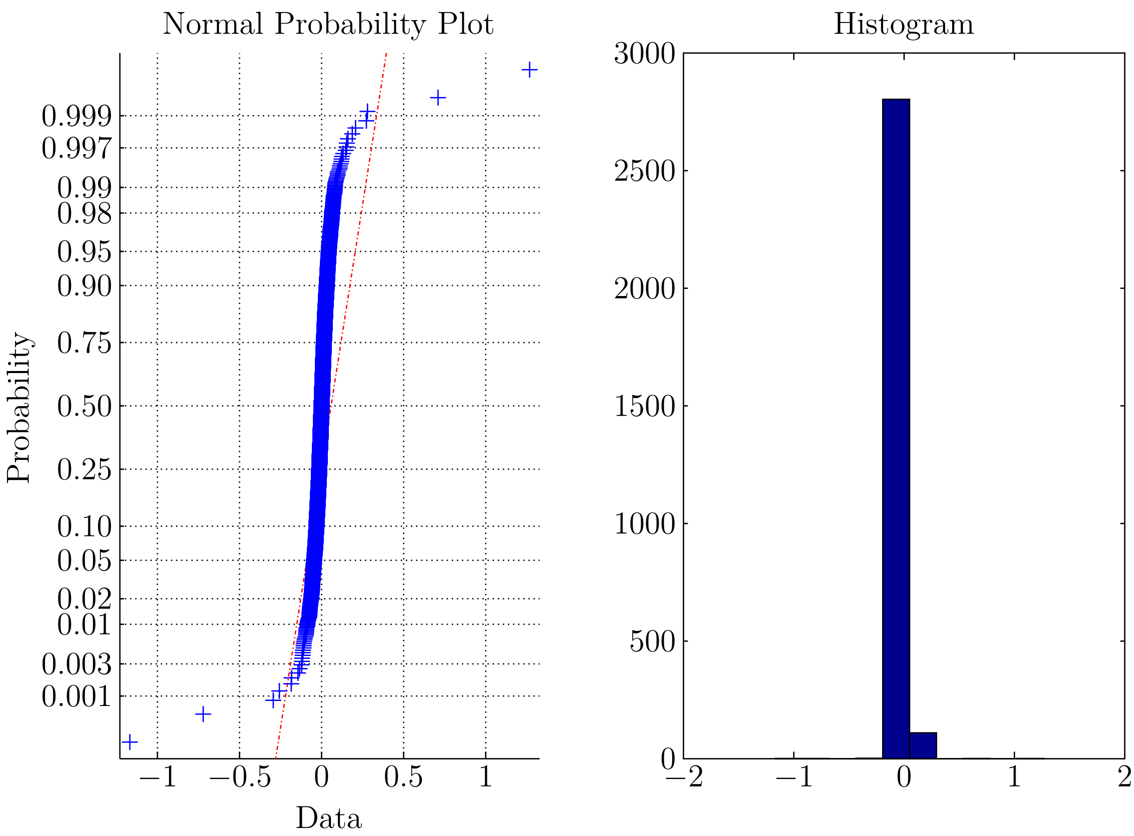


Source: Own work based on Mayer et al. (2011).

4.2.1.2 Offpeak1

The off-peak 1 contracts include the period before the peakload contracts, which corresponds to the period from 0:00 to 08:00 o'clock. Since there is no separate index for this period of the EEX, the prices of the off-peak 1 block contract are considered. The off-peak prices can be negative; therefore, a transformation by a factor of 200 is required as it was necessary for the peakload price. After this transformation, the log-prices are considered and an analysis of the data is carried out similarly to the peakload analysis. As with the peakload time series, data are adjusted again for extreme outliers.

Figure 4.7: Normal probability plot and histogram of the unseperated off-peak 1 process.

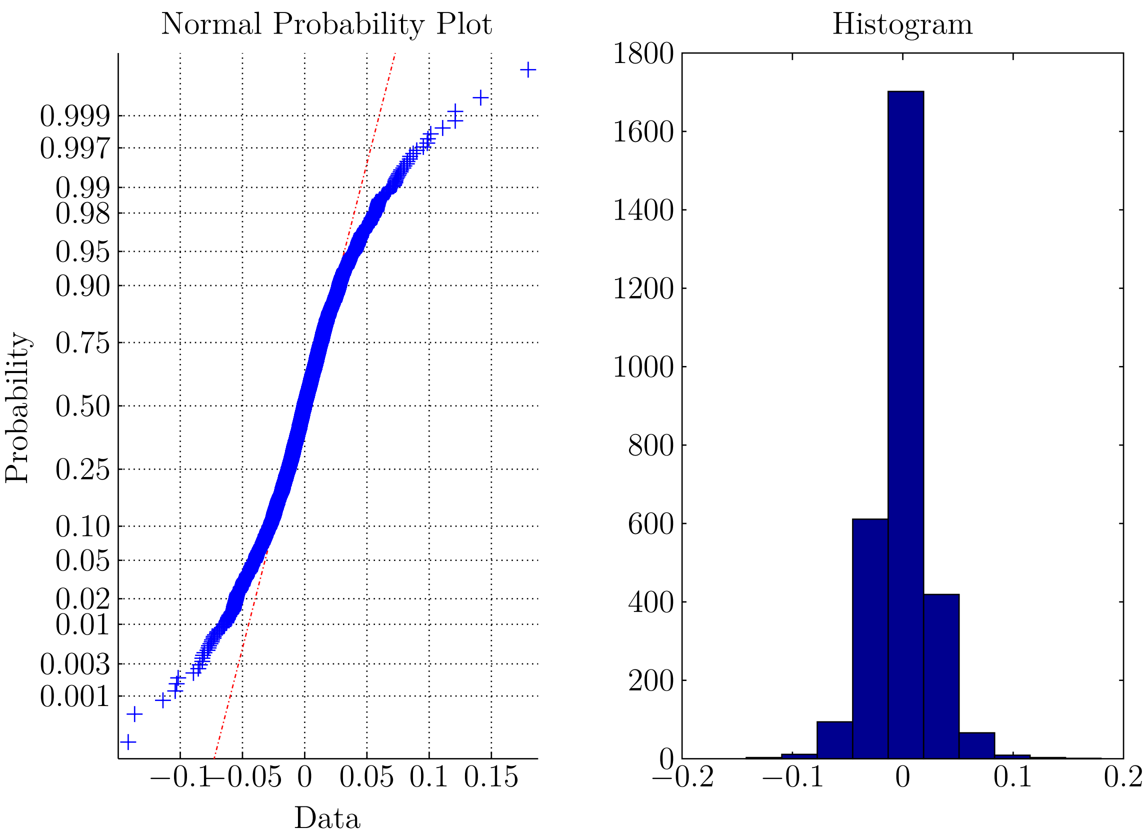


Source: Own work based on Mayer et al. (2011).

4.2.1.3 Offpeak2

The off-peak 2 contracts relate to the period after the peakload contracts and include the time from 20:00 to 24:00 o'clock. Only the off-peak 2 block contract are available because no other suitable price index is available, as for the off-peak 1 contract.

Figure 4.8: Normal probability plot and histogram of the unseperated off-peak 2 process.



Source: Own work based on Mayer et al. (2011).

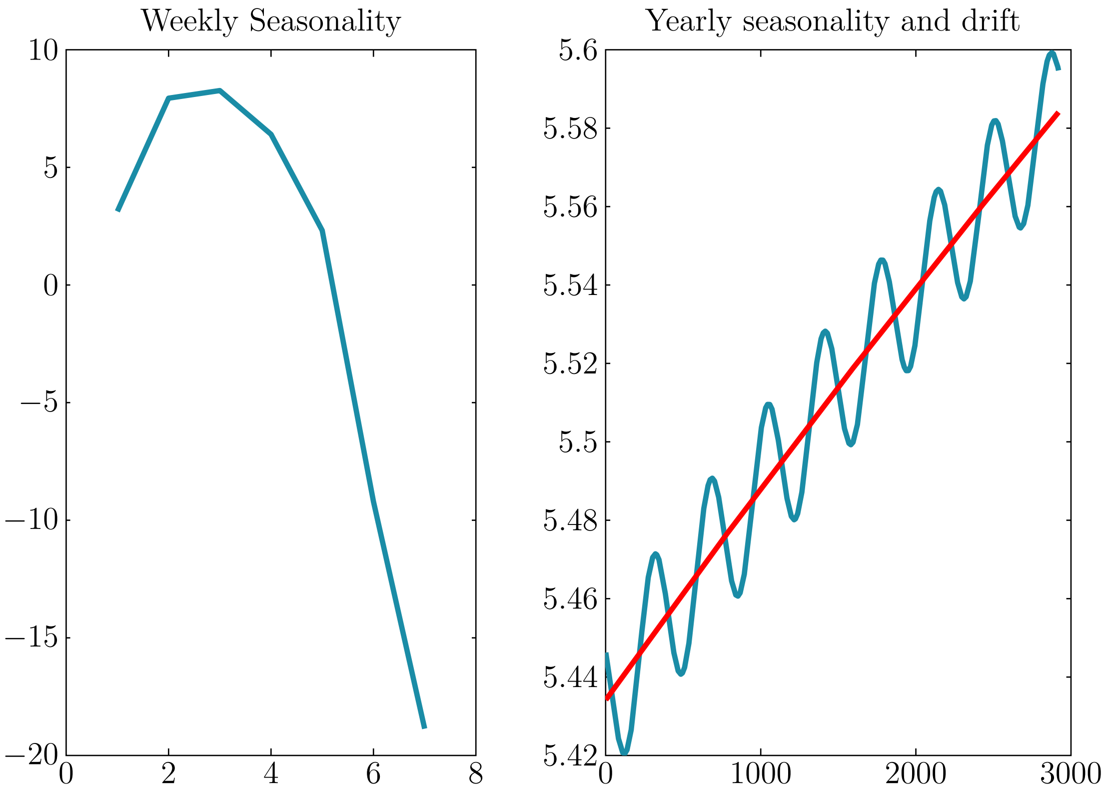
4.2.2 Seasonality and Trend Analysis

4.2.2.1 Peakload

One important assumption of the Black-Scholes-Merton model (Black and Scholes (1973), Merton (1973)) is that returns are assumed to be independently distributed. This is evaluated with an autocorrelation test. If the data were in fact independently distributed, the autocorrelation coefficient would be close to zero. From figures 4.11, 4.14, and 4.17, it can be seen that a strong level of autocorrelation is evident in electricity markets. The evidence of autocorrelation is a result of an underlying seasonality, as explained for instance in Pindyck and Rubinfeld (1998). The lag of days between highly correlated points in the series reveals the type of the seasonality. The figures show a yearly seasonal

pattern for the returns and a significant correlation every 7 days. This indicates intra-week seasonality. In addition, one can observable that no intra-year seasonality exists. In order to estimate the parameters of the model, the returns are adjusted for the seasonality. After the removal of the yearly and weekly seasonality, the ACF plot of the daily changes is analyzed. Figures 4.11, 4.14, and 4.17 show a significant negative autocorrelation at lag 1. This is an indication of the existence of a mean-reversion effect.
Figure 4.9 shows the weekly seasonality in the peakload time series.

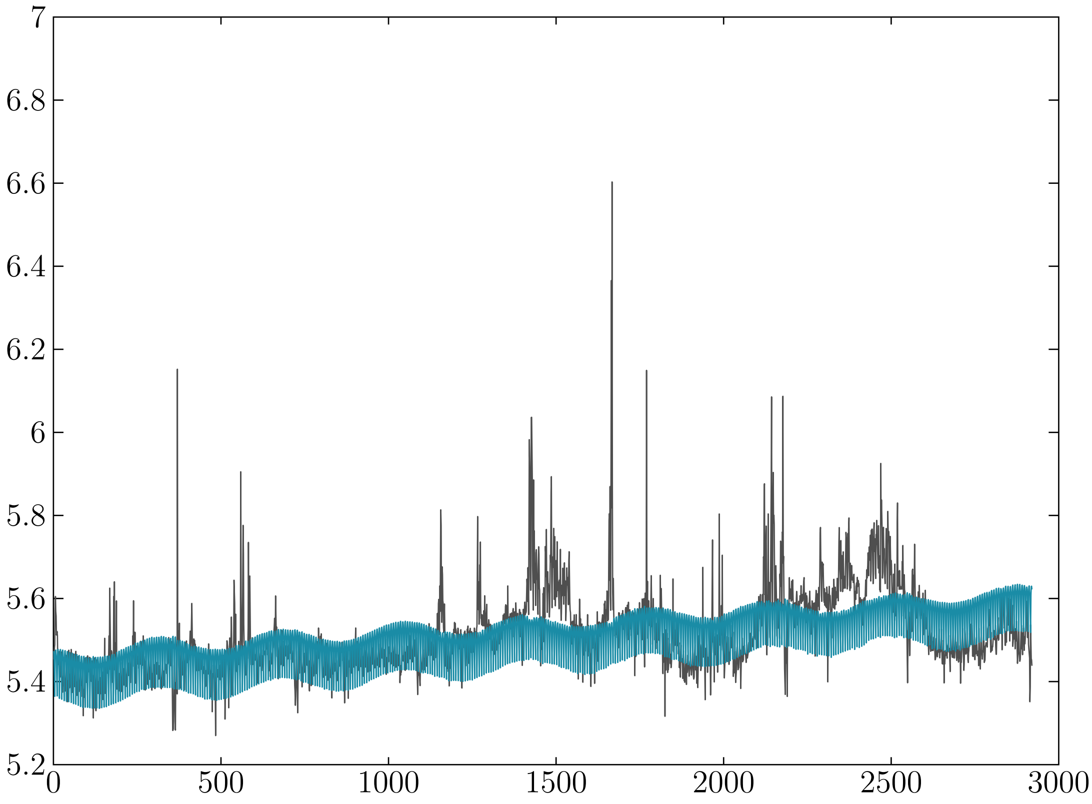
Figure 4.9: Weekly and yearly seasonality (peakload process)



Source: Own work based on Mayer et al. (2011).

After removing this seasonality and taking the log, the time series is analyzed for the yearly seasonality and the trend. This is illustrated in figure 4.9 on the right hand side. Figure 4.13 shows the log time series. The deterministic and the stochastic variation around it can clearly be seen here.

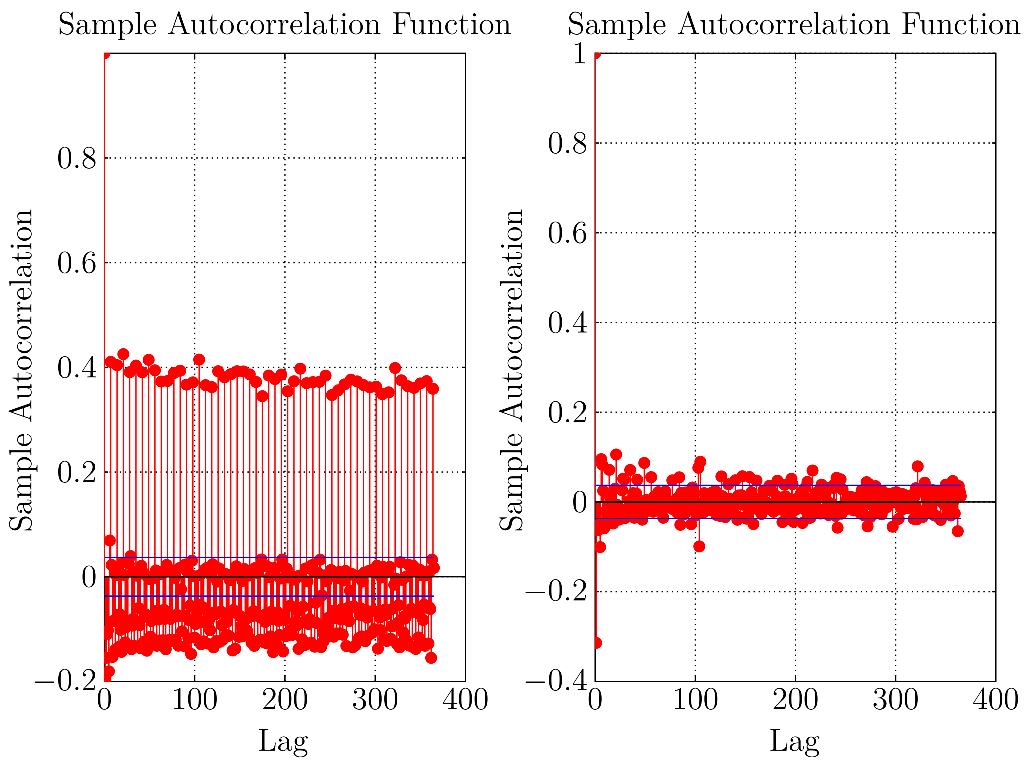
Figure 4.10: Time series of the log prices and yearly seasonality (off-peak 1 process).



Source: Own work based on Mayer et al. (2011).

Following the removal of the entire seasonality of the log prices, again an investigation for autocorrelation is carried out. The ACF plot, in figure 4.11, shows a significant negative autocorrelation at lag 1, which indicates mean-reversion for the peakload process.

Figure 4.11: ACF plots of the log price and the log price without seasonality (peak-load process)

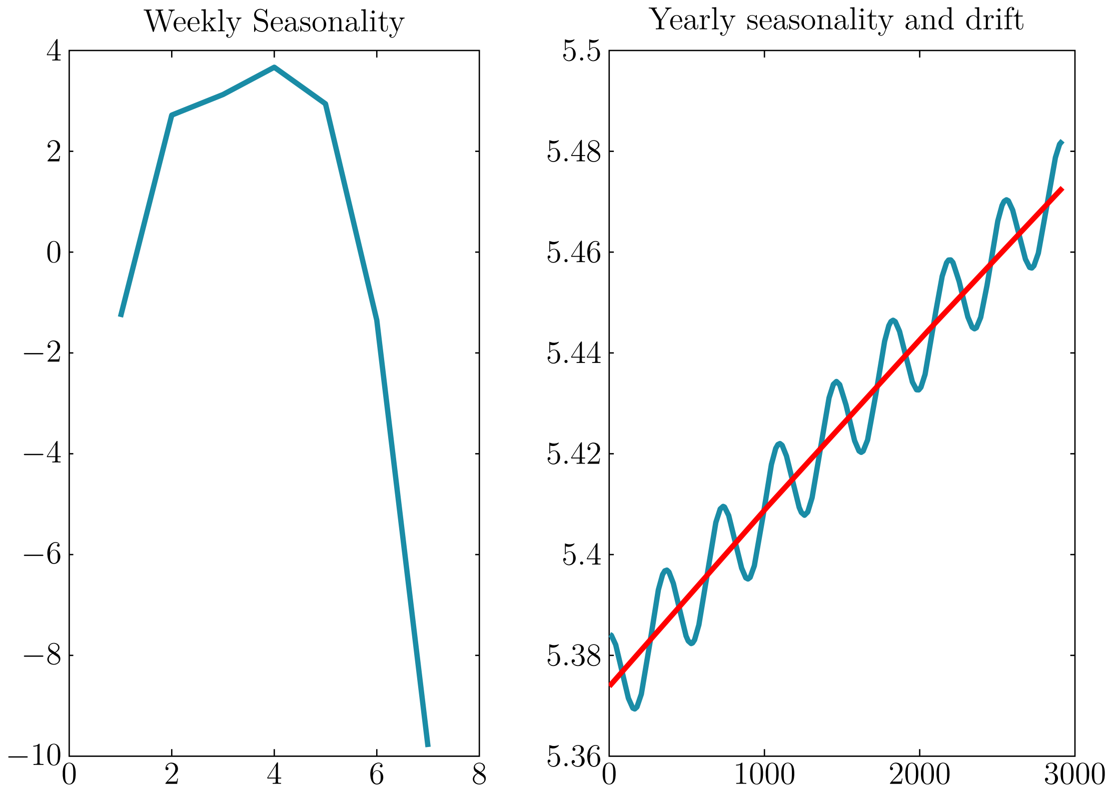


Source: Own work based on Mayer et al. (2011).

4.2.2.2 Off-peak 1

After removing the 19 outliers, the weekly seasonality is determined with the moving average method. The pattern of the weekly seasonality is as expected - similar to the pattern determined by the peakload price index with sharply decreasing prices for the weekend (see figure 4.12).

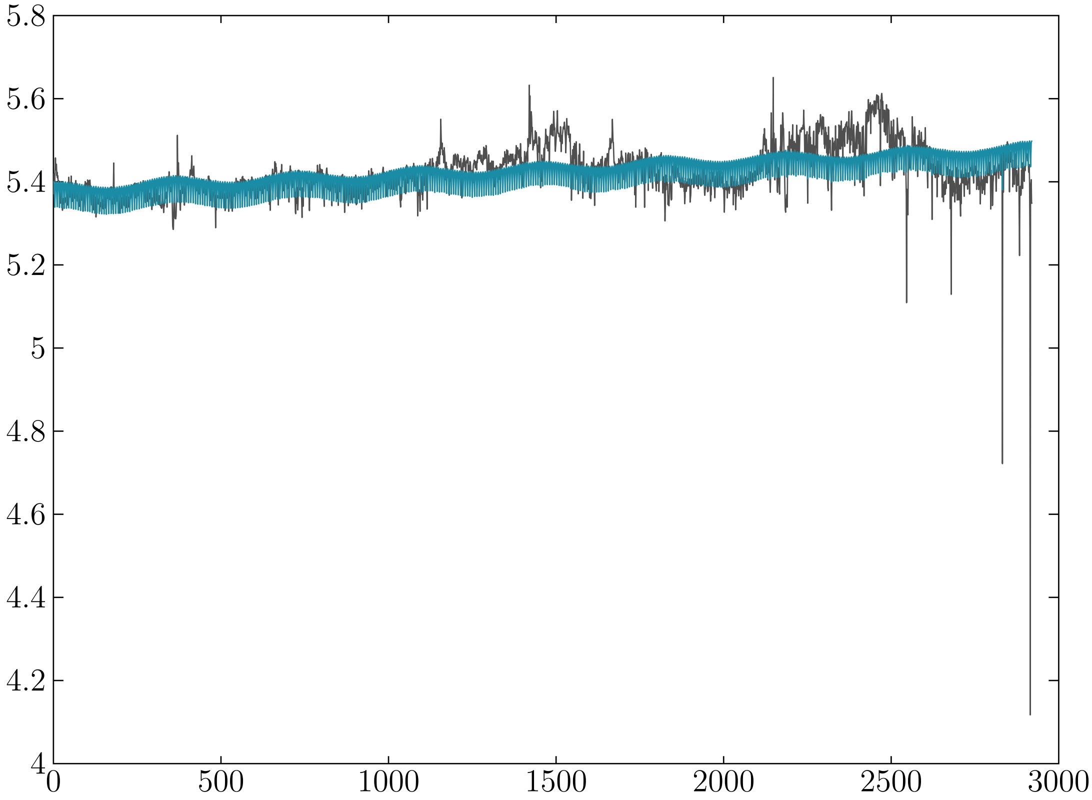
Figure 4.12: Weekly and yearly seasonality (off-peak 1 process)



Source: Own work based on Mayer et al. (2011).

After removal of the weekly seasonality, a logarithmic transformation is again carried out in the next step before the annual seasonality and the trend can be determined from the time series. This is plotted in figure 4.12 on the right hand side. Figure 4.13 shows the log time series. The deterministic and the stochastic variation around it can clearly be seen here.

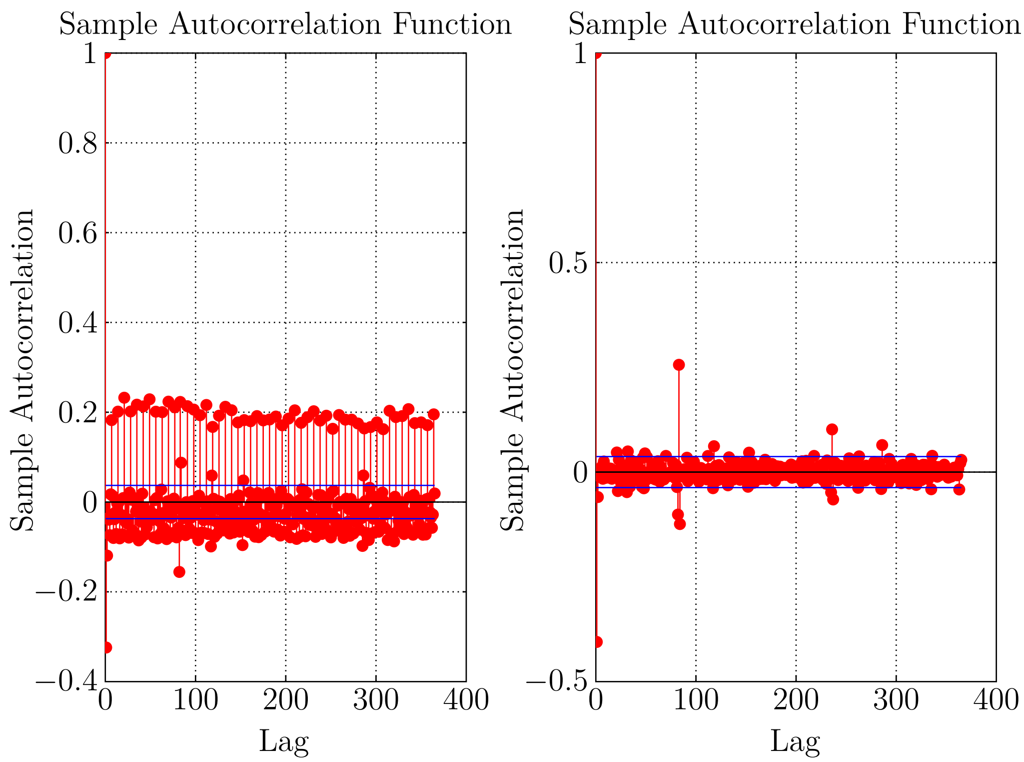
Figure 4.13: Time series of the log prices and yearly seasonality (off-peak 1 process).



Source: Own work based on Mayer et al. (2011).

Following the removal of the entire seasonality of the log prices, again an investigation for autocorrelation is carried out. Once again, the ACF plot shows a significant negative autocorrelation at lag 1, which also indicates mean-reversion for the off-peak 1 contract. This is illustrated in figure 4.14.

Figure 4.14: ACF plots of the log price and the log price without seasonality (off-peak 1 process)

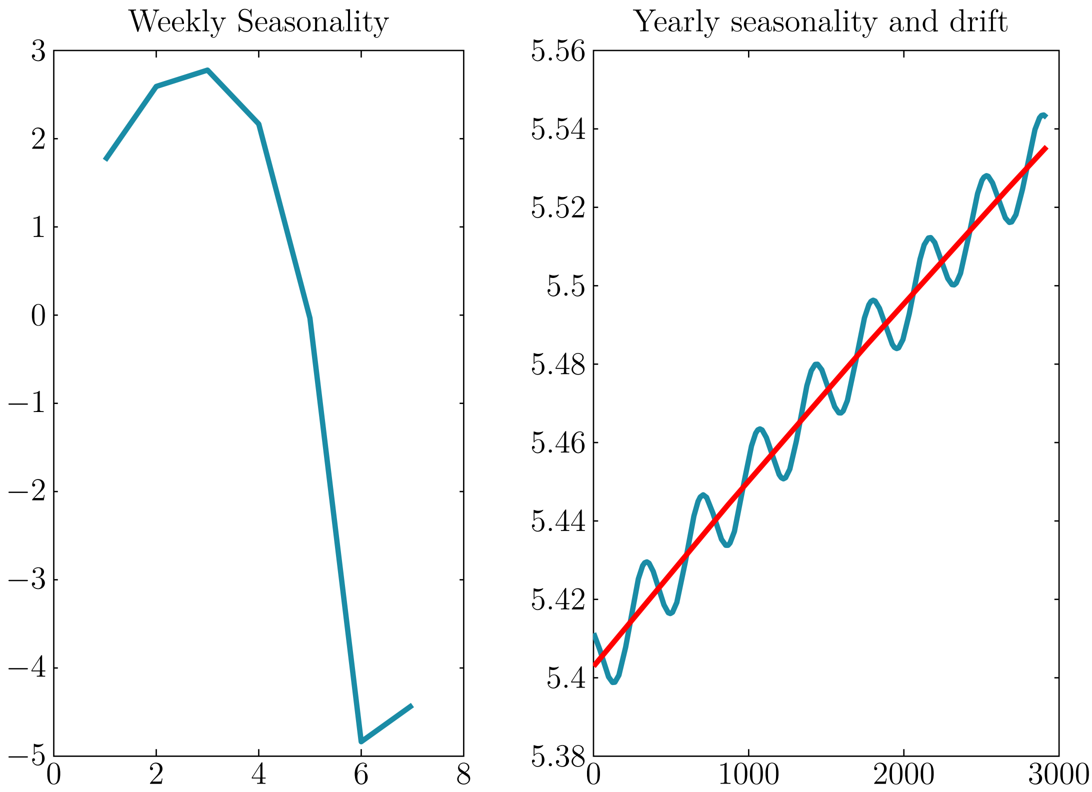


Source: Own work based on Mayer et al. (2011).

4.2.2.3 Offpeak2

Although the prices are constantly positive, even here the transformation is carried out by a factor € 200. This makes sense, as proven negative prices can occur in the electricity market and this is observed especially during off-peak periods. The extreme fluctuations are removed as before, as these rare effects do not have any lasting influence on the time series. After this shift again the weekly seasonality is identified and removed before the logarithmic transformation takes place. The determination of long-term trends and yearly seasonality follows. Using the method from the peakload analysis, initially 20 outliers are removed from the off-peak 2 prices. After removing the weekly seasonality and the log transformation, another 15 outliers are discovered and replaced. The *Weekly* season filtering algorithm is used to identify the weekly seasonality (see figure 4.15). One can see that after the decline in the price on the weekend, prices start to rise again on Sunday, which can be attributed, in part, to the increased demand from industries that start up their production facilities on Sunday evenings.

Figure 4.15: Weekly and yearly seasonality (off-peak 2 process)

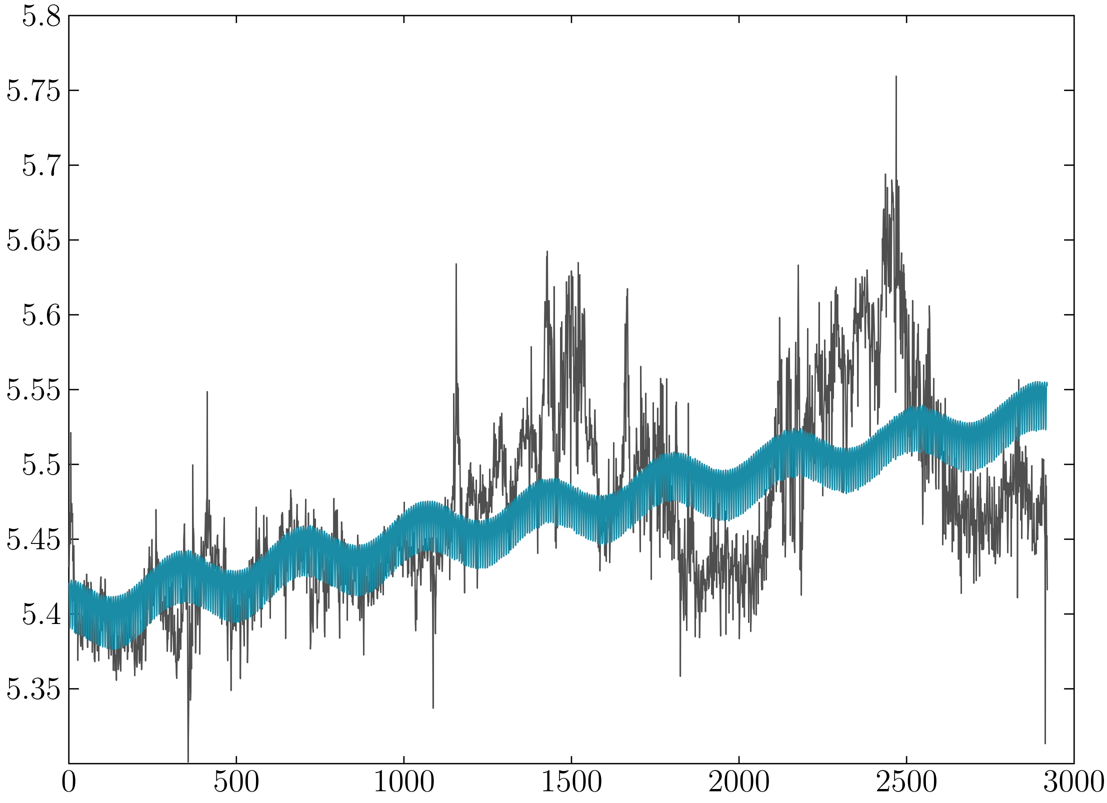


Source: Own work based on Mayer et al. (2011).

The time series adjusted for the weekly seasonality in time series is then analyzed for the long-term trend and the yearly seasonality by a non-linear regression. In figure 4.15, the trend and the yearly seasonality is plotted on the right hand side.

Figure 4.16 shows the log time series. The deterministic and the stochastic variation around it can clearly be seen here.

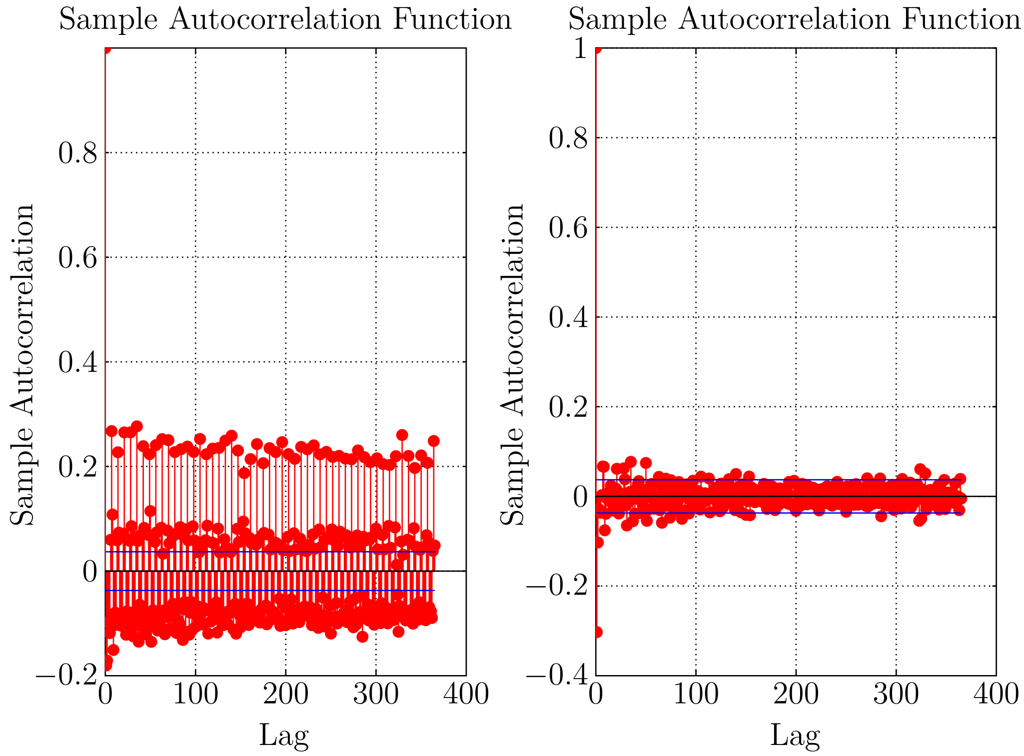
Figure 4.16: Time series of the log prices and yearly seasonality (off-peak 2 process).



Source: Own work based on Mayer et al. (2011).

After removal of seasonalities, the ACF plot in figure 4.17 shows a negative autocorrelation at the first lag, as already seen in the peakload and off-peak 1 prices.

Figure 4.17: ACF plots of the log price and the log price without seasonality (off-peak 2 process)



Source: Own work based on Mayer et al. (2011).

4.2.3 Jumps

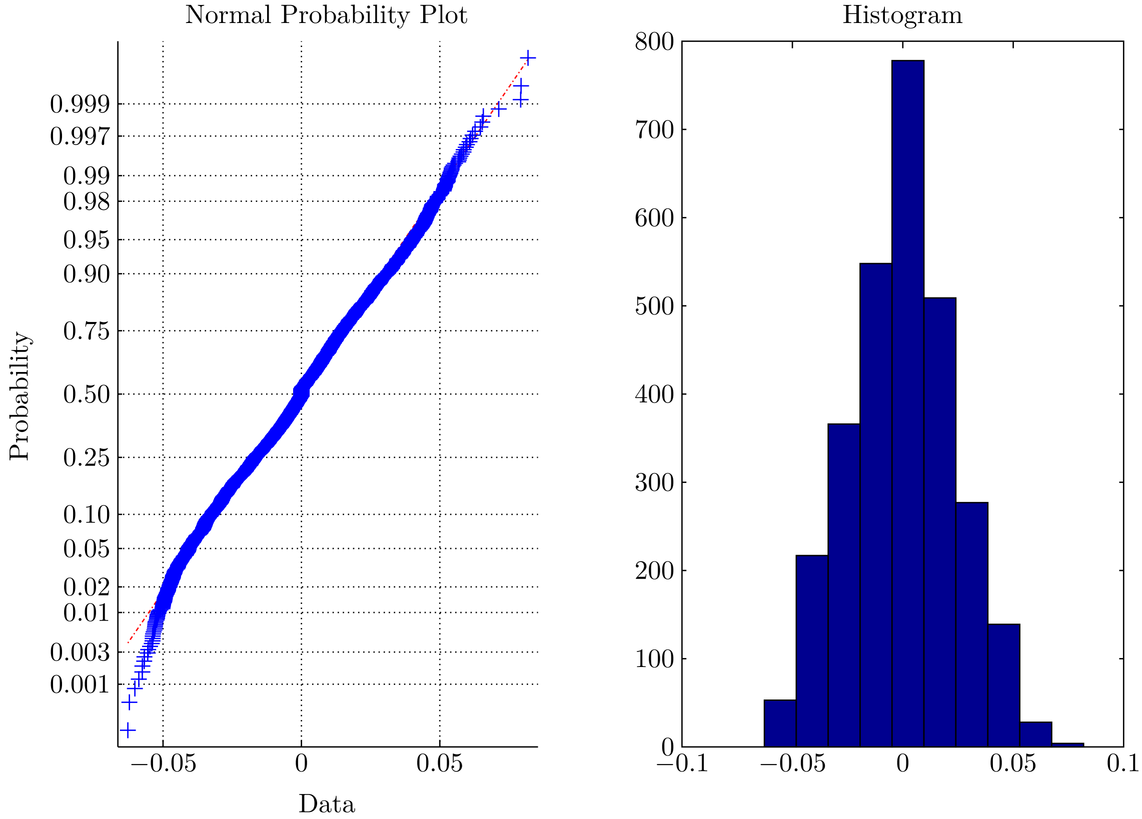
As seen from the normality test, the existence of fat tails suggests that the probability of rare events occurring is actually much higher than predicted by a Gaussian distribution. A simple inspection of figure 4.5 convinces this author that the spikes in electricity data cannot be captured by simple Gaussian shocks. Therefore, the jumps from the original series of return are extracted by using an author-written numerical algorithm that recursively filters returns with absolute values greater than 2.3 times the standard deviation of the returns of the series at that specific iteration. This leaves almost 99% of the returns as normal returns.

The relevance of the jumps in the electricity market is further demonstrated by comparing figures 4.18, 4.19 and 4.20 to figures 4.6, 4.7, and 4.8. One can clearly observe that after stripping the jumps from the returns, the normality test improves significantly.

The processes σ_{dB} and dI, which are determined by the recursive filtering algorithm, are examined on their distributions in the next step.

4.2.3.1 Peakload

Figure 4.18: Normal probability plot and histogram of the diffusion part (peakload process).

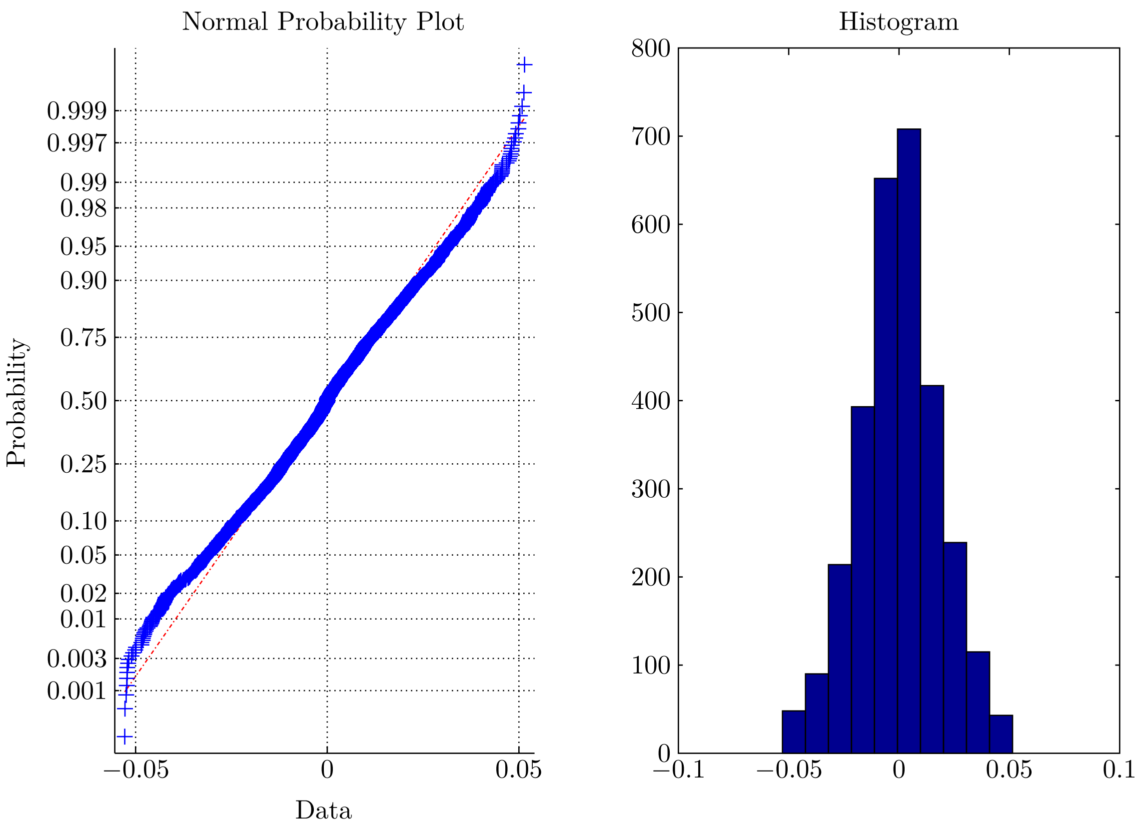


Source: Own work based on Mayer et al. (2011).

4.2.3.2 Off-peak 1

The normal probability plot and histogram of σ_{dB} indicate the desired normal distribution. Also, the skewness and kurtosis of -0.04 $3.01 from$ make this assumption to seem plausible. Since the Kolmogorov-Smirnov test at the significance level of 5% for the normal distribution assumption cannot be rejected, the normal distribution is considered for σ_{dB}. The parameters of the distribution are $\mu = 0$ and $\sigma_{dB} = 0.0184$.

Figure 4.19: Normal probability plot and histogram of the diffusion part (off-peak 1 process).

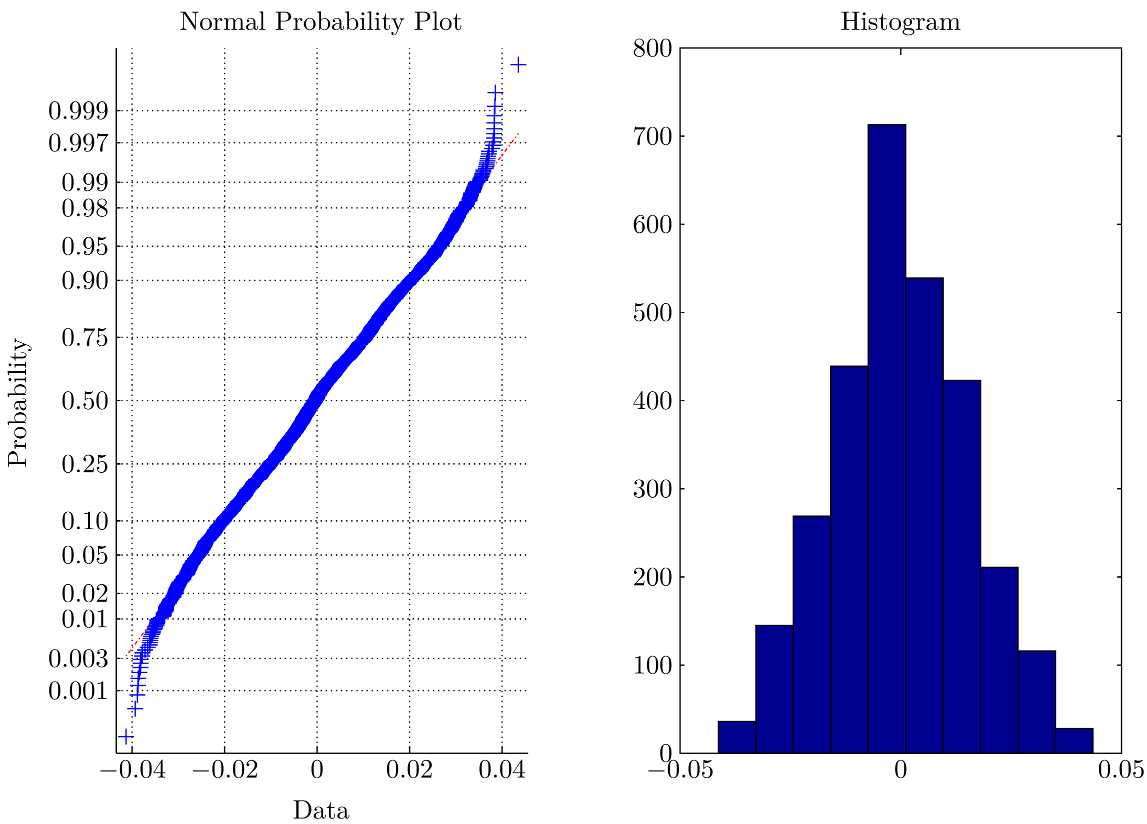


Source: Own work based on Mayer et al. (2011).

4.2.3.3 Off-peak 2

For σ_{dB}, the histogram and normal probability plots provide a clear indication of the desired normal distribution. The values of the skewness and kurtosis of 0.00 and 2.54, respectively, support this. Furthermore, the assumption cannot be rejected by the Kolmogorov-Smirnov test at significance level of 5% . Thus, the normal distribution for the $\sigma_d B$ part with parameters $\mu = 0$ and $\sigma_{dB} = 0.0152$ seems plausible.

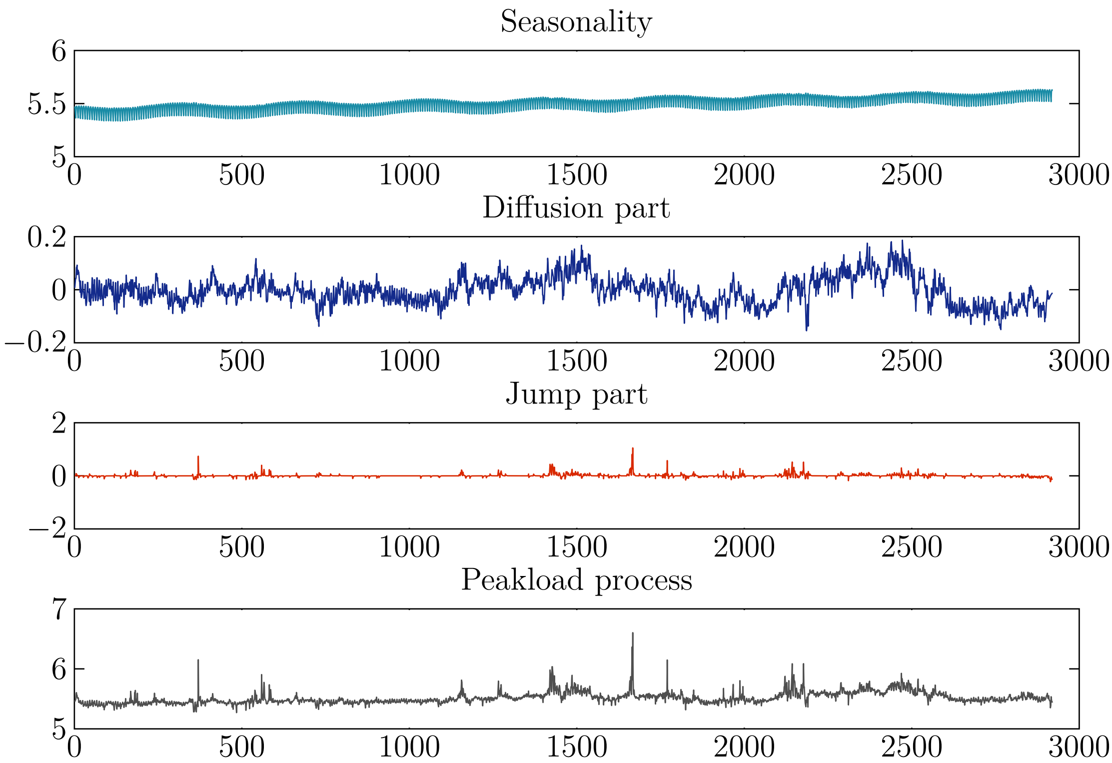
Figure 4.20: Normal probability plot and histogram of the diffusion part (off-peak 2 process).



Source: Own work based on Mayer et al. (2011).

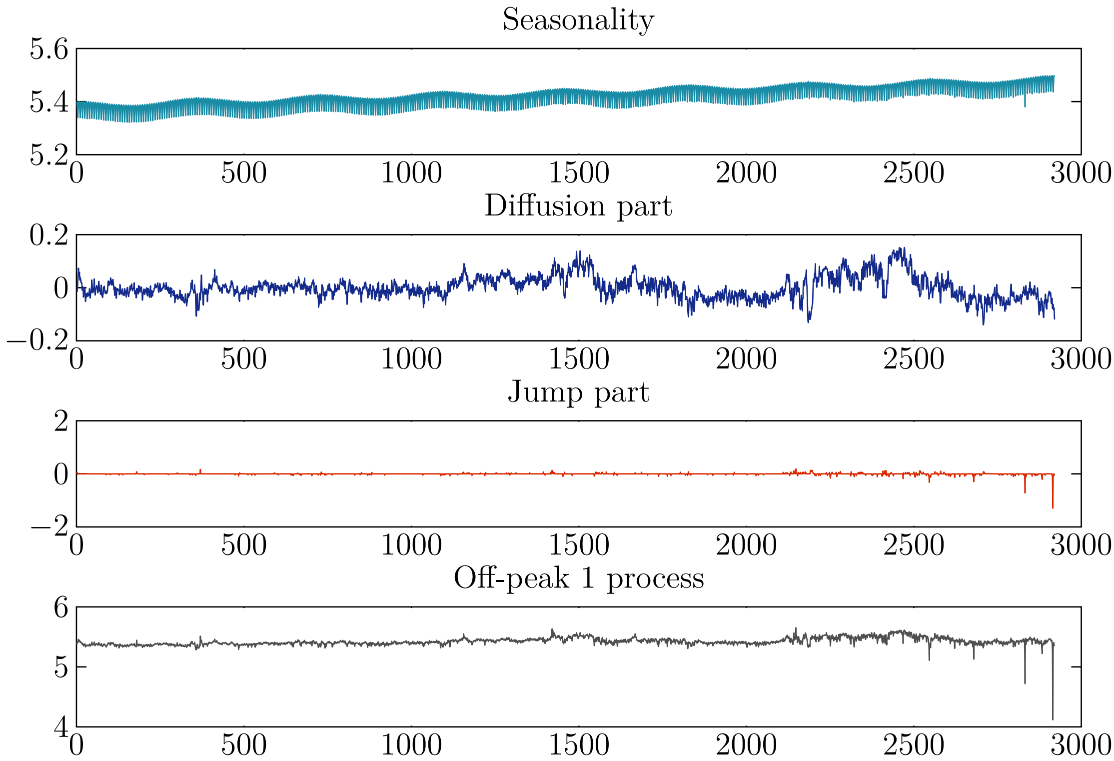
The dissertation analysis has shown that there is a strong mean reversion and the occurrence of spikes in the spot prices. Furthermore, it has shown that prices are fluctuating around a long-term deterministic seasonal component. An additional distinctive feature of electricity is the existence of negative prices. The minimum value over the time series is $-139,96$ €. In particular, the early morning hours, represented by the off-peak 1 contracts, are vulnerable to negative prices. This is the time of day with the lowest demand and, thus, the lowest prices. The negative rates occur due to low demand and huge supply overhang by, for example, unexpectedly high electricity transfers from wind turbines. Figures 4.21, 4.22, and 4.23 show the mean reversion, the diffusion and jump component of the peakload and off-peak 1 and off-peak 2 contracts.

Figure 4.21: Components of log-price processes (peakload process)



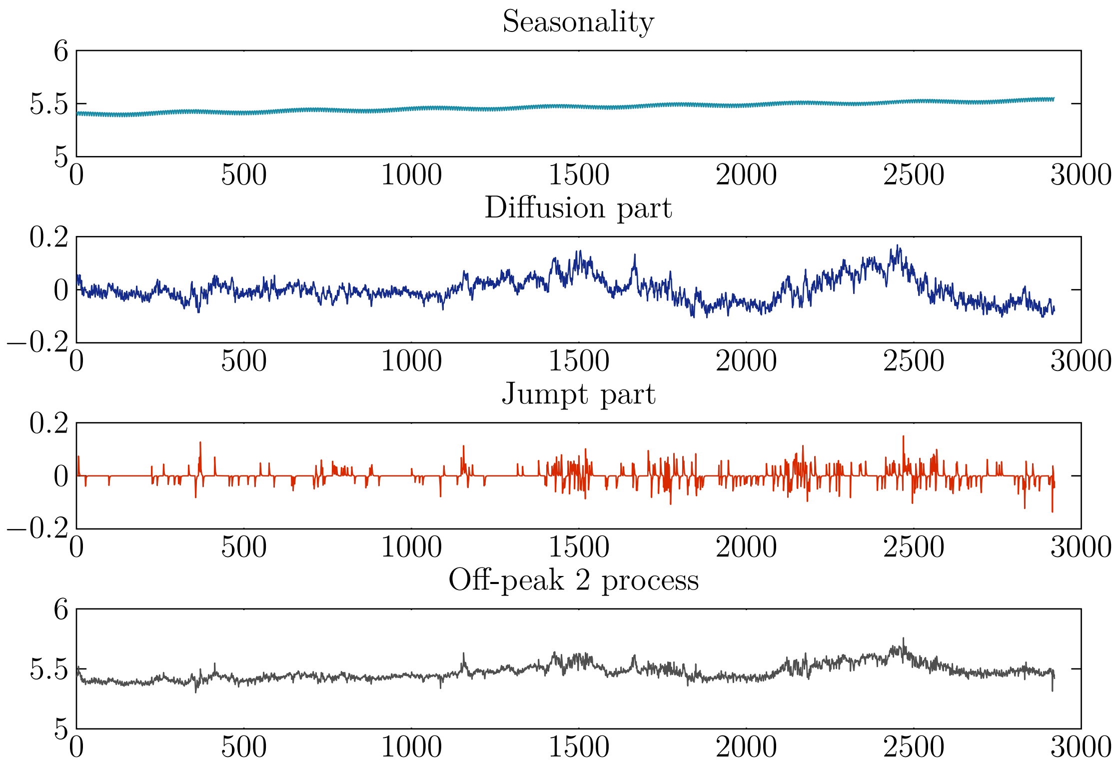
Source: Own work based on Mayer et al. (2011).

Figure 4.22: Components of log-price processes (off-peak 1 process)



Source: Own work based on Mayer et al. (2011).

Figure 4.23: Components of log-price processes (off-peak 2 process)



Source: Own work based on Mayer et al. (2011).

The analyses, thus far, suggest that the sole assumption of a Gaussian process probably falls short of satisfactorily explaining electricity spot price dynamics. Since the groundbreaking work of Samuelson (1965), Black and Scholes (1973), and Merton (1973), for over four decades most stochastic models have tried to 'fix' the main drawback from assuming normally distributed returns. The main reason to analyze alternative models is that, compared to the frequency of the extreme price movements observed in financial markets, normal distribution attaches too little probability to them. This has resulted in volumes of literature dealing with jump-diffusion processes, stochastic volatility models, and more recently, the use of Lévy processes.[14] The proposed model of this dissertation is based on those works.

[14]See for example Merton (2001), Knight and Satchell (2001) or Shreve (2004*b*).

4.3 Electricity price model

The data analysis of the previous section reveals three distinctive characteristics of electricity markets that should be accounted for in the model. First are negative rates that have occurred over time. These constitute a problem for the consideration of the geometric models. Therefore, this author applied an affine transformation of the time series in the positive range. An affine transformation by a factor of $\Gamma = 200$ € shifts all prices in the positive range and leaves ample room for future occurrence of negative prices in the subsequent price simulation. At the end of the simulation, the generated time series must be transformed back by an inverse transformation to the original price level. Second, a seasonality component reflects a varying long term mean level. Third, the stochastic variation of the prices reverts slowly back to this level while, opposite to that, the sudden price spikes revert very fast.

Schwartz (1997) accounted for the mean-reversion, and Lucia and Schwartz (2002) extended the mean-reverting model to account for a deterministic seasonality. Benth et al. (2003) extended this model to include jumps with a separate rate of mean-reversion. However, these models do not incorporate stochastic volatility and correlation among the jump processes. This dissertation proposes a similar model extended to account for the stochastic volatility and the correlation among jumps.

Let $(\Omega, P, \mathcal{F}, \{\mathcal{F}_t\}_{t\epsilon[0,T]})$ be a complete filtered probability space, with $T < \infty$ a fixed time horizon. If $S(t)$ denotes the spot price of electricity at time t, then the model sets

$$S(t) = (S(0) + \Gamma) \times \exp\left(\Lambda(t) + X(t) + Y(t)\right) - \Gamma \tag{4.1}$$

where $\Lambda(t)$ denotes the seasonality function value at time t, $X(t)$ and $Y(t)$ the values of two Lévy processes at time t. The model also sets

$$\Lambda(t) = w(t \mod 7) + sea(t) \tag{4.2}$$

where $w(t \ mod 7)$ denotes the weekly seasonality function at day k following (4.4) and $sea(t)$ the annual seasonality function value at time t following (4.5). The standard procedure to model seasonal influences is by trigonometric functions. However, the weekly behavior of the price of electricity follows a special pattern. It is significantly higher on weekdays, from Monday to Friday, than at the weekend, Saturday and Sunday. Therefore, the model does not use trigonometric functions but instead a procedure proposed by Weron (2006). First, the data are smoothed using a moving average filter:

$$m(t) := \frac{1}{7}\left(S(t-3) + \cdots + S(t+3)\right). \tag{4.3}$$

Second, for each day, the average deviation w $(t\ mod\ 7)$ from moving average value $m(t)$ in (4.3) is calculated:

$$\begin{aligned} w(t\ mod\ 7) := average\{&(S((t\ mod\ 7) + 7j) - m(k+7j)), \\ &3 < (t\ mod\ 7) + 7j \leq n-3\}; \end{aligned} \tag{4.4}$$

The calculated values of $w(t\ mod\ 7)$ are then normalized, that they add up to zero over the week. The resulting weekly seasonality $w(t\ mod\ 7)$ is deducted from the original time series. A logarithmic transformation is performed before analyzing the long term seasonality function as well as the stochastic components. The annual seasonality, the trend, and the level is included in the following function:

$$\ln sea(t) = \ln\left(\beta_1 + \beta_2 t + \beta_3 \cos\left(\frac{2\pi(t-\beta_4)}{365}\right)\right) \tag{4.5}$$

The trend factors of X and Y are integrated into the seasonality function which does not pose a huge limitation. Further, the model assumes that X and Y are Lévy processes where X is driven by a Brownian motion and Y by a compound Poisson process. Consequently, X and Y are zero-level mean-reverting stochastic processes with the following stochastic differential equations:

$$dX(t) = -\alpha_X X(t)dt + \sigma(t)dB(t) \tag{4.6}$$

$$dY(t) = -\alpha_Y Y(t)dt + dI(t) \tag{4.7}$$

where α_X and α_Y denote the mean reversion parameter, $\sigma(t)$ the volatility of the diffusion process, $dB(t)$ the Brownian motion increments, and $dI(t)$ the increments of a compound Poisson process. Furthermore, the model assumes the mean-reversion parameters of (4.6) and (4.7) are constant until time T. For the volatility $\sigma(t)$, the model assumes that it can either be constant or stochastic. Stochastic models are either of GARCH-type (Bollerslev (1986)) or E-GARCH-type (Nelson (1991)) but could easily be extended to others.

In order to separate the jumps from the normal diffusion process, the model defines as the tolerance parameter the value where the diffusion term distribution can be classified as Normal and at the same time has the highest explanation value. This is the case for a tolerance value of 2.3 standard deviations, which results in about 99% of the price movements being classified as normal. Using the different volatility models results in a different identification of jumps in the price process. The constant volatility model has a constant barrier for the daily changes of σ_{dB} to be classified as jumps of the process dI. In comparison, GARCH and E-GARCH volatility models have variable barriers. These variable barriers to classify jumps are determined and, thereby, moved each day by the multiplication of the tolerance parameter with the volatility at that day. This may lead in daily changes, which were classified (not classified) as jumps in the constant volatility model, to be not classified (classified) as jumps in the GARCH or E-GARCH volatility models. The reason for this is that in times of high (low) volatility the stochastic volatility models allow larger (smaller) movements of the process σdB.

Then, the process I is modeled in (4.7) with two separate Compound Poisson processes for the positive and negative jumps

$$I(t) = I^{+}(t) + I^{-}(t), \tag{4.8}$$

with

$$I^{\pm} = \sum_{i=1}^{N^{\pm}(t)} J_i^{\pm} \tag{4.9}$$

with

$$N^{\pm}(0) = 0 \tag{4.10}$$

$$E(N^{pm}(t)) = \lambda^{pm} \times t \tag{4.11}$$

$$Var(N^{pm}(t)) = \lambda^{pm} \times t \tag{4.12}$$

$$lnJ_i^{\pm} \sim N(\mu^{pm}, \sigma pm) \tag{4.13}$$

The processes N^{+} and N^{-} are Poisson processes, which represent the number of positive or negative jumps until time t. The jump intensities λ^{pm} for the Poisson processes are estimated from the number of positive respectively negative jumps compared to whole observed days. $J_i^{\pm}$ is the size of jump i, which can be assumed to be log-normally distributed.

4.4 Electricity price model calibration

This section analyzes the E-GARCH(1,1), GARCH(1,1), and constant volatility. To take into account the significant differences in the electricity price during a day, it separately analyzes the peakload times and the off-peak times, i.e. the periods before and after the peakload.

4.4.1 Seasonality Function

After the previously described affine transformation in the positive range the weekly seasonality in the price process S is analyzed. Before this can be done, extreme movements that occur only rarely, but that can distort the results by their large values are removed. The assumption is that these are prices where the price movements are outside more than three times the interquartile range of the first or third quartile. The prices of the affected days will be replaced by the mean of the two adjacent days which are not outliers. The weekly and yearly seasonality as well as the drift are calculated according to (4.4) and (4.5) as described in section 4.3. Table 4.1 shows the seasonal function and trend parameters for the peakload price process. It becomes apparent that the off-peak 2 prices already start to increase on Sunday. This can be explained by the start-up of industrial companies' machinery.

4.4.1.1 Peakload

Table 4.1: Seasonality and drift parameters (peakload process)

	Seasonality and drift parameter						
Weekly seasonality	3.1297	7.9410	8.2671	6.4057	2.3192	-9.2020	-18.8607
Drift	$0.0127t + 229.0927$						
Yearly seasonality	$-4.6969 \cdot \cos\left(\frac{2\pi(t-129.0719)}{365}\right)$						

Source: Own work based on Mayer et al. (2011).

4.4.1.2 Off-peak 1

Table 4.2: Seasonality and drift parameters (off-peak 1 process)

	Seasonality and drift parameter						
Weekly seasonality	-1.2857	2.7180	3.1261	3.6671	2.9411	-1.3487	-9.8179
Drift	$0.0077t + 215.6862$						
Yearly seasonality	$2.2565 \cdot \cos\left(\frac{2\pi(t-357.0434)}{365}\right)$						

Source: Own work based on Mayer et al. (2011).

4.4.1.3 Off-peak 2

Table 4.3: Seasonality and drift parameters (off-peak 2 process)

	Seasonality and drift parameter						
Weekly seasonality	1.7547	2.5923	2.7773	2.1662	-0.0345	-4.8357	-4.4202
Drift	$0.0108t + 222.0488$						
Yearly seasonality	$-2.3967 \cdot \cos\left(\frac{2\pi(t-144.5787)}{365}\right)$						

Source: Own work based on Mayer et al. (2011).

4.4.2 Mean Reversion Rate

After removing the seasonality from the sample time series, the stochastic part is analyzed. First, the mean-reversion rates α_X and α_Y are determined. Z is constructed with

$$Z(t) = X(t) + Y(t) \tag{4.14}$$

and therefore

$$dZ(t) = dX(t) + dY(t). \tag{4.15}$$

Using (4.6) as well as (4.7) and the discrete version of (4.15), it follows

$$\Delta Z(t) = \Delta X(t) + \Delta Y(t) \tag{4.16}$$

$$= -\alpha_X X(t)\Delta t + \sigma \Delta B(t) - \alpha_Y Y(t)\Delta t + \Delta I(t)$$

$$= -(\alpha_X X(t) + \alpha_Y Y(t))\Delta t + \sigma \Delta B(t) + \Delta I(t) \tag{4.17}$$

$$= -\left(\alpha_X \frac{X(t)}{X(t)+Y(t)} + \alpha_Y \frac{Y(t)}{X(t)+Y(t)}\right)(X(t) + Y(t))\,\Delta t + \sigma \Delta B(t) + \Delta I(t)$$

$$= -\alpha_Z Z(t)\Delta t + (\sigma \Delta B(t) + \Delta I(t)). \tag{4.18}$$

Since the time series consists of daily data, a discretization of the interval length $\Delta t = 1$ is reasonable. Using linear regression of log-prices against the log-returns as in (4.19), A value for α_Z is found by:

$$\Delta Z(t) = \gamma Z(t)\Delta t + \varepsilon(t) \tag{4.19}$$

where γ is the regression coefficient and ε describes the daily changes not caused by the MR effect. Using $\alpha_Z = (-1) \times \gamma$, it is possible to estimate the MR factor.

By removing the mean-reversion effect from the process Z one gets the process ε. From this process, a vector ζ of jumps using the tolerance parameter of 2.3 standard deviations is determined. The mean-reversion rate α_Z and the jump vector ζ are then used to determine the mean reversion rates α_X and α_Y.

For the calculation of X and Y, first, the stochastic part needs to be divided between the processes σdB and dI. As the final values of σdB and dI do change because of the mean reversion, the processes is named initially σ_{dB_0} and dI_0. Generally, the first step assumes $\sigma_{dB_0} = dZ$ and $dI_0 = 0$. However, at the jump points the value of σ_{dB_0} is set to 0 and the values of dI_0 at these points equal the values of dZ. Afterwards, the processes X and Y are calculated iteratively. Starting values are assumed to be $X(0) = Z(0)$ and $Y(0) = 0$. The stochastic influences of σ_{dB} and dI have to be adjusted for the MR effect. This adjustment is necessary to offset the movement caused by the MR by the means of the stochastic components, to achieve the same movement intensity as in the process dZ. Generally, the MR effect is associated with the normal part σdB and is deducted from σ_{dB_0} . The only exception are movements, where an association with σ_{dB} would lead to a new jump. In this case it is associated with jump process dI. The adaption formulas for σ_{dB} and dI are

$$\sigma dB(t) = \sigma dB_0(t) - (-\alpha_X X(t) - \alpha_Y Y(t)) \tag{4.20}$$

and in the case that it would lead to a new jump

$$dI(t) = dI_0(t) - (-\alpha_X X(t) - \alpha_Y Y(t)) \,. \tag{4.21}$$

Here, $\sigma_{dB_0}(t)$ and $dI_0(t)$ on the right hand side of equations (4.20) and (4.21) represent, in each case, the first assignment and $\sigma_{dB}(t)$ and $dI(t)$ on the left hand side of the equations (4.20) and (4.21) represent the final values. With these values the iterative update formulas for X (4.22) and Y (4.23) are

$$\begin{aligned} X(t+1) &= X(t) + dX(t) \\ &= (1-\alpha_X)X(t) + \sigma dB(t) \end{aligned} \tag{4.22}$$

$$\begin{aligned} Y(t+1) &= Y(t) + dY(t) \\ &= (1-\alpha_Y)Y(t) + dI(t) \end{aligned} \tag{4.23}$$

The resulting processes can have different jumps than originally assumed for the calculation. Therefore, jumps in $\sigma dB + dI$ are again determined with a second algorithm. This algorithm, thereby, identifies all the movements that are larger than 2.3 standard deviations of process σdB as jumps. The difference with this is that no recursions are performed. Instead, the jumps are calculated solely on the specified volatility. This makes it possible to take stochastic volatility into account. Although not using a recursive approach might result in a very low number of price movements not to be identified as jumps, this approach is used because otherwise the run-time of the algorithm becomes extraordinary long and the accuracy does not increase significantly. The required stochastic volatility processes are determined by the maximum likelihood estimation from the process dB.

With the obtained processes X and Y, the MR factors α_X and α_Y can be determined in analogy to 4.19. These parameters will then replace the initial values for the MR factors and the calculation of the processes X and Y starts again until the MR factors converge, too. In the case of alternating MR factors, those factors are averaged and the calculation of the processes X and Y is done again. The return values of this algorithm are besides the MR factors α_X and α_Y, the processes X and Y, the jump vector with the entries of discontinuities, and the stochastic parts σdB and dI of the processes X and Y. The mean reversion rates for the peakload process and the different volatility models are shown in table 4.4.

4.4.2.1 Peakload

Table 4.4: Mean reversion parameters with different volatility models (peakload process)

Volatility Model	Constant	GARCH	E-GARCH
α_X	0.1064	0.1073	0.1069
α_Y	0.4802	0.4779	0.4744

Source: Own work based on Mayer et al. (2011).

4.4.2.2 Off-peak 1

The determination of the MR factors is analogous to that for peakload data on the identification of a common MR factor, and the jump points of the process Z. Then, the determination of α_X and $alpha_Y$ with the *MR* detection algorithm follows. The linear regression of the log-returns against the log prices yields the value for $alpha_Z = 37.47\%$. This value is used as an initial value for α_X and α_Y in the *MR detection algorithm*. In addition, a starting vector containing the jumps of the process Z is needed. First, the MR effect is removed with help of α_Z. Second, the jump vector is determined by the *Recursive Filtering Algorithm* using a tolerance of 2.3 standard deviations. With this method, 367 price movements are identified as jumps after 12 iterations. The *MR detection algorithm* returns the value of the parameter $\alpha_X = 9.94\%$ as well as $\alpha_Y = 79.98\%$, and the processes X, Y, dI, as well as σ_{dB}, and the vector with the newly found 281 jumps. In addition to the E-GARCH(1,1) volatility, the calculation is also done for constant volatility and GARCH(1,1) volatility. Table 4.5 summarizes the mean reversion parameters for the different volatilities.

Table 4.5: Mean reversion parameters with different volatility models (off-peak 1 process)

Volatility Model	Constant	GARCH	E-GARCH
α_X	0.0816	0.0969	0.0994
α_Y	0.7732	0.7991	0.7998

Source: Own work based on Mayer et al. (2011).

4.4.2.3 Off-peak 2

A MR factor of $alpha_Z = 11.94\%$ for the process Z is found with the linear regression. Subsequently, the process Z is adjusted for the MR effect and the adjusted process ε is then analyzed. The *Recursive Filtering Algorithm* yields a vector with 300 jumps after 11 iterations using the tolerance parameter 2.3 standard deviations.. This vector and the MR factor α_Z are used as input parameters for the *MR detection algorithm*. The obtained values for the mean reversion of the processes X and Y are $\alpha_X = 5.24\%$ and $\alpha_Y = 56.14\%$. In addition, the time series for $sigma_dB$ and dI and the vector with jumps are returned. These MR factors result in the price process with 340 jumps. Table 4.6 summarizes the mean reversion parameters for the different volatilities.

Table 4.6: Mean reversion parameters with different volatility models (off-peak 2 process)

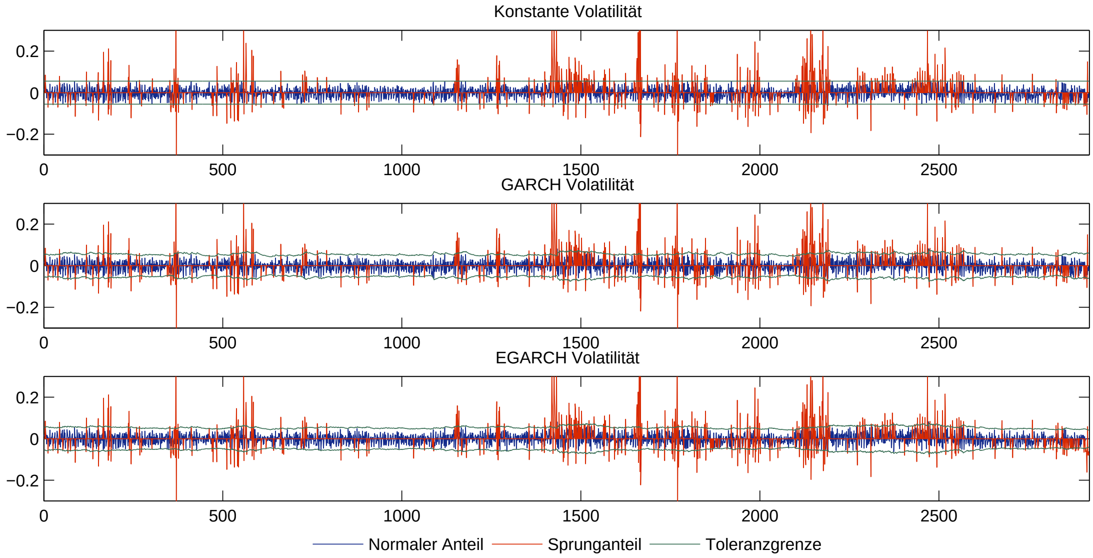
Volatility Model	Constant	GARCH	E-GARCH
α_X	0.0524	0.0556	0.0553
α_Y	0.5614	0.5197	0.5454

Source: Own work based on Mayer et al. (2011).

4.4.3 Volatility analysis

The distribution of the stochastic part σ_{dB} and dI is analyzed in this section. Figure 4.24 shows the division between the two processes σ_{dB} and dI for the three different volatility models. By analyzing the individual stochastic processes, the stochastic processes B and I is determined for each price processes. The combination of the six sub-processes results in the overall model with the random changes.

Figure 4.24: Jump detection for the three different volatility models



Source: Own work based on Mayer et al. (2011).

The standard assumption in most models, including the model of Benth et al. (2003), is independence between the jump processes and the Brownian motion. However, dependencies should exist only between the Brownian motion, not between the Brownian motion and the jump processes, or between the jump processes themselves. The latter assumption is not plausible for the processes used here, because there has to be causal relationships among the three electricity price processes. Price spikes caused, for example, by the failure of a large power plant will most likely have an impact on the whole day. The independence assumption for the jump processes has to be rejected; therefore, the dependency among jump processes is taken into consideration.

When analyzing the distribution of σ_{dB}, the assumption is that a normalized process B, with zero mean and standard deviation of 1 exists for the stochastic volatility models GARCH and E-GARCH. This makes it possible to use a constant parameter σ, while the volatility of the normalized process B is stochastic. The parameters for the stochastic volatility are estimated from the normalized process dB by maximum-likelihood. Table 4.7 shows the distribution parameters and the GARCH-parameters for the peakload price process. The current analysis reveals that there is a positive leverage effect.

4.4.3.1 Peakload

Table 4.7: Volatility parameters (peakload process)

Volatility Model	Constant	GARCH	E-GARCH
	Distribution Parameter		
μ	0	0	0
σ	0.0241	0.0245	0.0236
	GARCH Parameter		
κ	1	0.0304	0.0003
ARCH	0	0.0402	0.0530
GARCH	0	0.9297	0.9673
Leverage	0	0	0.0349

Source: Own work based on Mayer et al. (2011).

4.4.3.2 Off-peak 1

Table 4.8: Volatility parameters (off-peak 1 process)

Volatility Model	Constant	GARCH	E-GARCH
	Distribution Parameter		
μ	0	0	0
σ	0.0167	0.0187	0.0184
	GARCH Parameter		
κ	1	0.0054	0.00001
ARCH	0	0.0460	0.0735
GARCH	0	0.9489	0.9942
Leverage	0	0	0.0001

Source: Own work based on Mayer et al. (2011).

4.4.3.3 Off-peak 2

Table 4.9: Volatility parameters (off-peak 2 process)

Volatility Model	Constant	GARCH	E-GARCH
	Distribution Parameter		
μ	0	0	0
σ	0.0152	0.0154	0.0153
	GARCH Parameter		
κ	1	0.0108	−0.0001
ARCH	0	0.0482	0.0575
GARCH	0	0.9411	0.9922
Leverage	0	0	0.0177

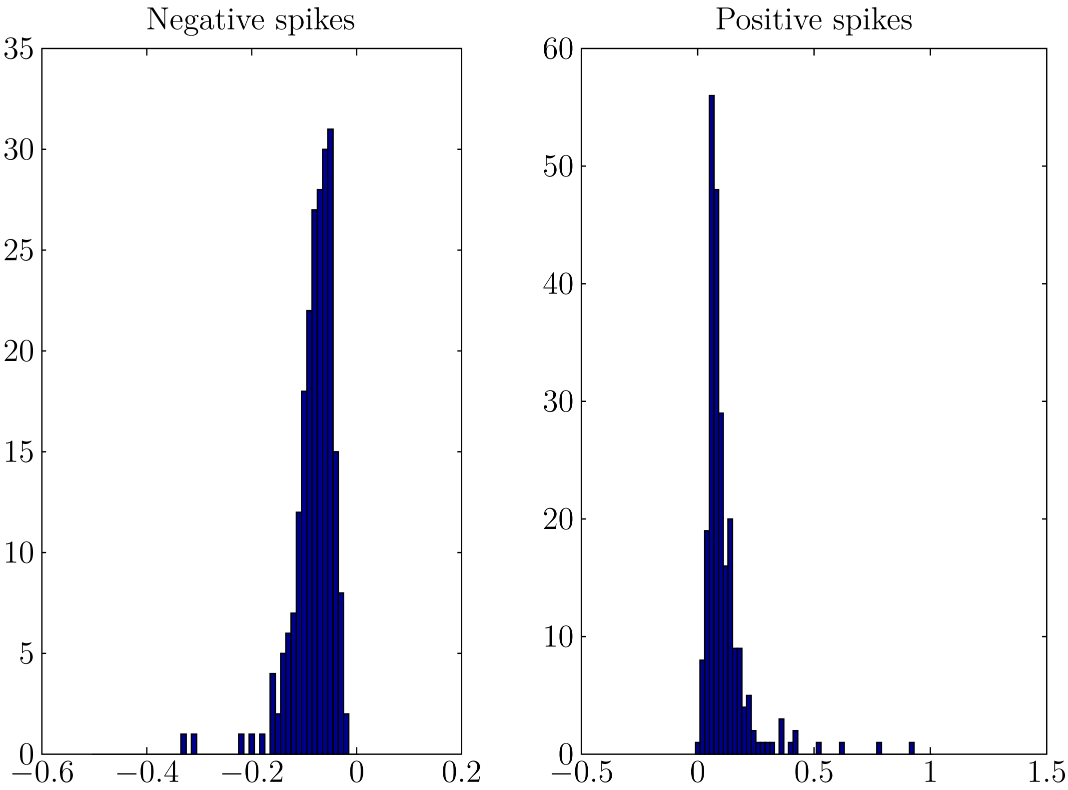
Source: Own work based on Mayer et al. (2011).

4.4.4 Jump process

4.4.4.1 Peakload

By dividing the jumps into positive and negative parts one gets 243 positive and 221 negative jumps. This yields for the Poisson, the jump frequencies $\lambda^+ = 0.0832$ and $\lambda^- = 0.0757$. The histograms of the jump heights J^+ and J^- induce again the assumption of lognormally distributed jump heights (figure 4.26). With the maximum likelihood method, the parameters for the distributions of J^+ and J^- can again be determined. For the distribution of positive jumps the values are $mu^+ = -2.4091$ and $\sigma^+ = 0.6507$ and for the negative jumps, after transformation with the parameter -1, $\mu^- = -2.6168$ and $\sigma^- = 0.4442$. Table 4.10 shows the results for all three types of volatilities.

Figure 4.25: Histogram of jump heights (peakload process)



Source: Own work based on Mayer et al. (2011).

Table 4.10: Parameters of the jump distributions (peakload process)

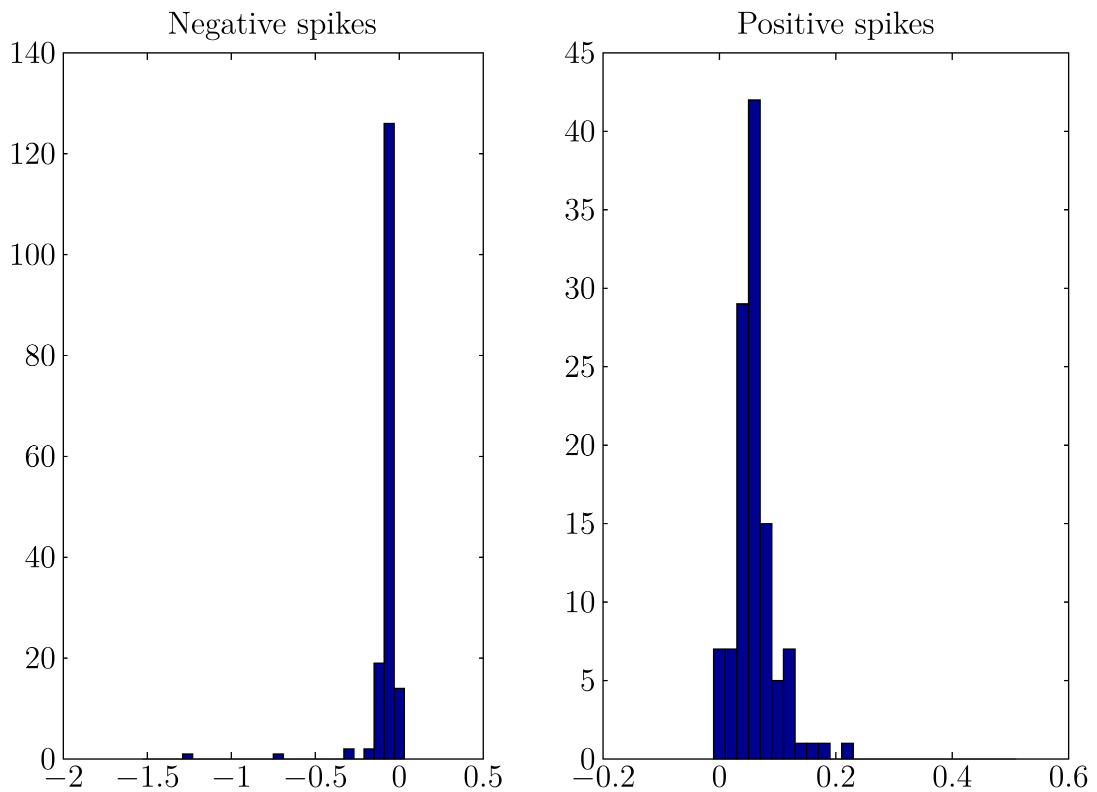
Volatility Model	Constant	GARCH	E-GARCH
μ^+	−2.3606	−2.3569	−2.4091
σ^+	0.6262	0.6415	0.6507
λ^+	0.0829	0.0791	0.0832
μ^-	−2.5387	−2.5614	−2.6168
σ^-	0.4276	0.4496	0.4442
λ^-	0.0613	0.0644	0.0757
# Jumps	421 (242 — 179)	419 (231 — 188)	464 (243 — 221)

Source: Own work based on Mayer et al. (2011).

4.4.4.2 Offpeak 1

By dividing the jumps into positive and negative parts one gets 116 positive and 165 negative jumps. This yields for the Poisson, the jump frequencies $\lambda^+ = 0.0397$ and $\lambda^- = 0.0565$. The histograms of the jump heights J^+ and J^- induce again the assumption of lognormally distributed jump heights (figure 4.26). With the maximum likelihood method, the parameters for the distributions of J^+ and J^- can again be determined. For the distribution of positive jumps the values are $mu^+ = -3.0349$ and $\sigma^+ = 0.8040$ and for the negative jumps, after transformation with the parameter -1, $\mu^- = -2.9377$ and $\sigma^- = 0.7300$. Table 4.11 shows the results for all three types of volatilities.

Figure 4.26: Histogram of jump heights (off-peak 1 process)



Source: Own work based on Mayer et al. (2011).

Table 4.11: Parameters of the jump distributions (off-peak 1 process)

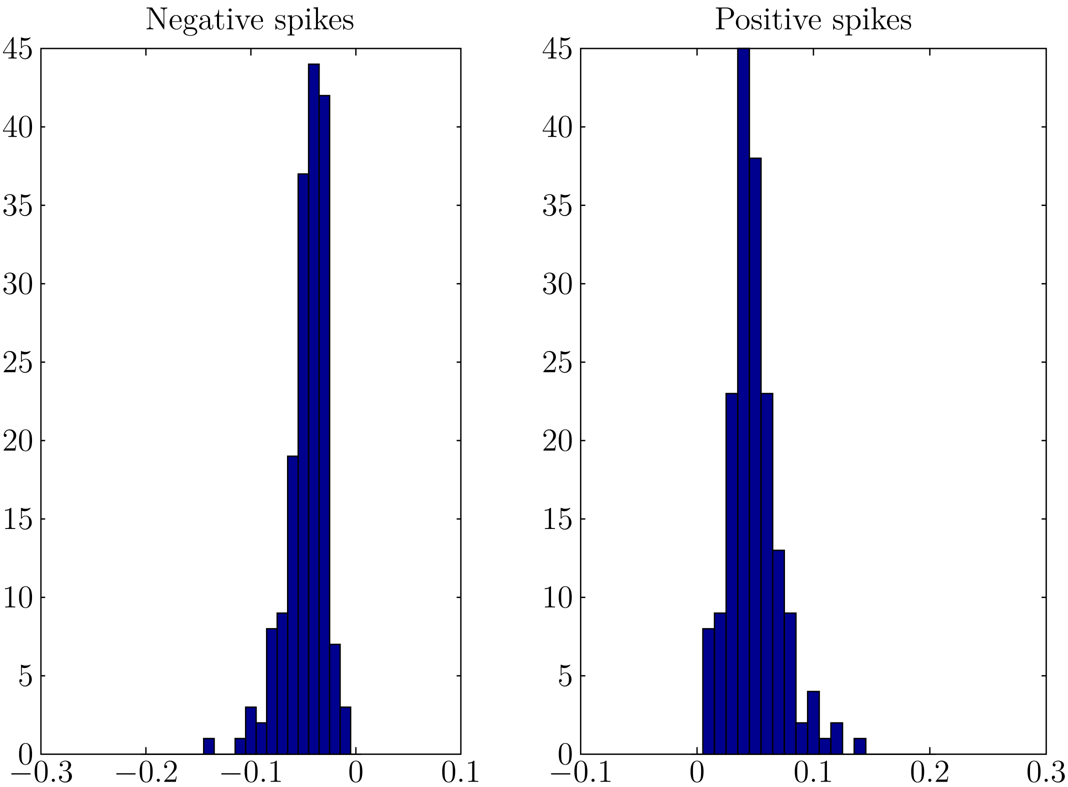
Volatility Model	Constant	GARCH	E-GARCH
μ^+	-3.0579	-2.9780	-3.0349
σ^+	0.7049	0.6462	0.8040
λ^+	0.0552	0.0401	0.0397
μ^-	-3.0361	-3.0280	-2.9377
σ^-	0.8183	0.7786	0.7300
λ^-	0.0743	0.0634	0.0565
# Jumps	378 (161 — 217)	302 (117 — 185)	281 (116 — 165)

Source: Own work based on Mayer et al. (2011).

4.4.4.3 Offpeak2

Dividing the process dI into positive and negative parts results in 176 positive and 164 negative jumps. For the two Poisson processes, the parameters are $\lambda^+ = 0.0610$ and $lambda^- = 0.06032$. The parameters for the log-normal distribution (figure 4.27) of jump heights J^+ and J^- are obtained with the maximum likelihood method and are $\mu^+ = -3.1276$ and $\sigma^+ = 0.4966$, and after multiplication by -1 for J^- with $mu^- = -3.1482$ and $\sigma^- = 0.4126$. Table 4.12 shows the results for all three types of volatilities.

Figure 4.27: Histogram of jump heights (off-peak 2 process)



Source: Own work based on Mayer et al. (2011).

Table 4.12: Parameters of the jump distributions (off-peak 2 process)

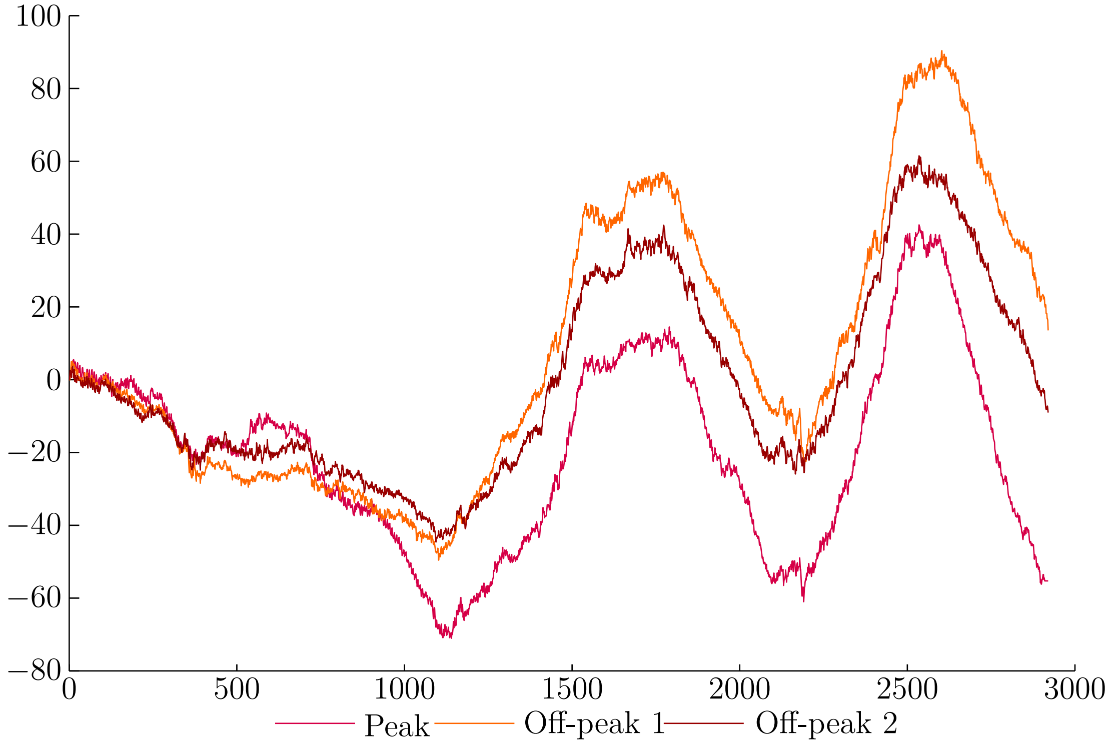
Volatility Model	Constant	GARCH	E-GARCH
μ^+	−3.1348	−3.1458	−3.1276
σ^+	0.5332	0.5107	0.4966
λ^+	0.0603	0.0603	0.0610
μ^-	−3.0916	−3.1425	−3.1482
σ^-	0.3603	0.3990	0.4126
λ^-	0.0562	0.0596	0.0603
# Jumps	340 (176 — 164)	350 (176 — 174)	354 (178 — 176)

Source: Own work based on Mayer et al. (2011).

4.4.5 Correlation and Combination analysis

After analyzing the individual price processes, the correlation structure is analyzed in order to obtain the final parameters for the simulation. T Figure 4.28 illustrates the correlation among the diffusion parts of the three price processes.

Figure 4.28: Correlation of the diffusion parts



Source: Own work based on Mayer et al. (2011).

Tables 4.13, 4.15, and 4.14 show the correlation structure between the three electricity price processes σ_{dB} using different types of volatility. It is apparent that the correlation is the highest with E-GARCH volatility and the lowest with GARCH volatility.

Table 4.13: Correlation matrix Σ between the processes using constant volatility

	Peakload	Off-Peak 1	Off-Peak 2
Peakload	1	0.3029	0.3207
Off-Peak 1	0.3029	1	0.2146
Off-Peak 2	0.3207	0.2146	1

Source: Own work based on Mayer et al. (2011).

Table 4.14: Correlation matrix Σ between the processes using E-GARCH volatility

	Peakload	Off-Peak 1	Off-Peak 2
Peakload	1	0.2950	0.3314
Off-Peak 1	0.2950	1	0.1949
Off-Peak 2	0.3314	0.1949	1

Source: Own work based on Mayer et al. (2011).

Table 4.15: Correlation matrix Σ between the processes using GARCH volatility

	Peakload	Off-Peak 1	Off-Peak 2
Peakload	1	0.3219	0.3537
Off-Peak 1	0.3219	1	0.2296
Off-Peak 2	0.3537	0.2296	1

Source: Own work based on Mayer et al. (2011).

The process dI is analyzed in a manner that shows all days identified according to which one of the processes had a positive jump or a negative jump. Since it is possible that the jump processes have dependencies on the point of time, when a jump occurs it is tested for a dependency among them. This type of dependency is likely, because the events that have an effect on the price of electricity at a time of day may be longer term in nature and, thus, affect the subsequent contracts. For both the positive and the negative jumps, there are 7[15] possible combinations of jumps in the electricity price processes. First, there is the possibility that all three processes have a jump on the same day. Second, there is the possibility that two of the three processes may jump with variations so that two processes jump. Third, there is the possibility that only one process has a jump. The derived likelihoods for all 7 possibilities are presented in table 4.16.

[15] A total of $2^3 = 8$ jump combinations, less the combination of no jumps results in 7 possible combinations.

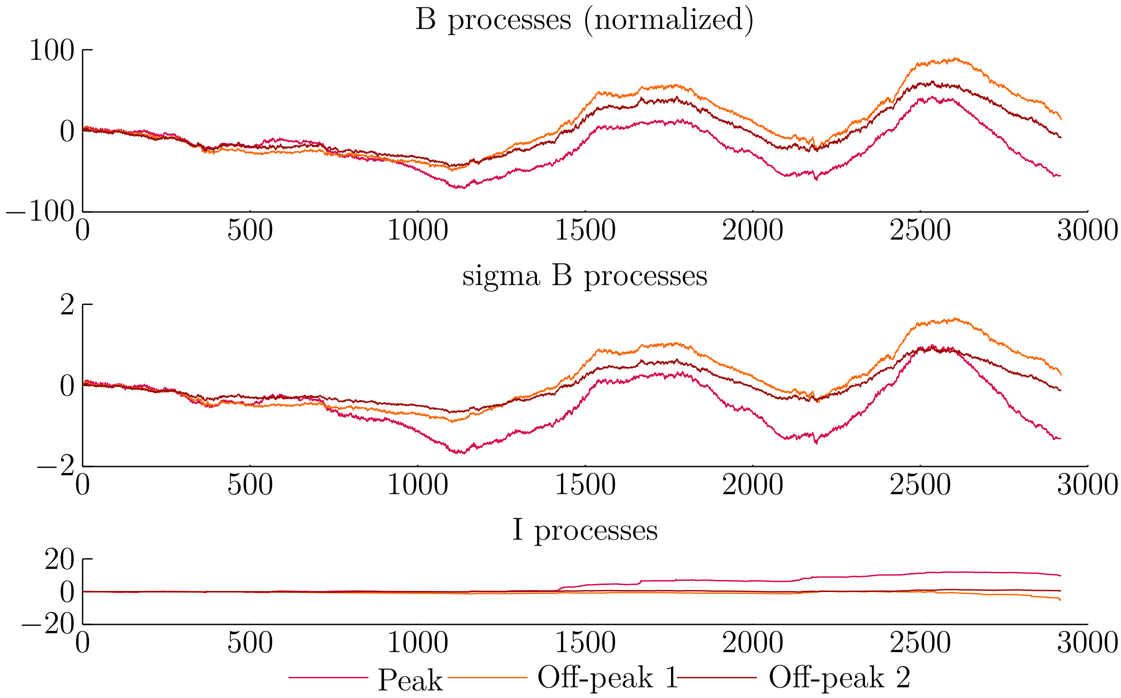
Table 4.16: Probablities of the jump combinations with a GARCH volatility diffusion process

Volatility model			Constant		GARCH		E-GARCH	
Combinations			Jump probablities					
Peak	OP 1	OP 2	positive	negative	positive	negative	positive	negative
x	x	x	5.15%	6.40%	3.50%	7.09%	4.20%	7.49%
x	x	0	8.43%	7.82%	5.75%	7.09%	4.94%	8.45%
x	0	x	13.58%	7.82%	16.50%	9.29%	15.80%	9.90%
x	0	0	29.51%	20.38%	32.00%	22.49%	34.32%	27.78%
0	x	x	3.28%	4.27%	1.75%	3.18%	2.72%	2.66%
0	x	0	20.54%	32.94%	18.25%	27.87%	16.79%	21.26%
0	0	x	19.20%	20.38%	22.25%	22.98%	21.23%	22.46%
Jump intensity			14.63%	14.46%	13.70%	14.01%	13.87%	14.18%

Source: Own work based on Mayer et al. (2011).

The summation of the probabilities for a process leads to the jump probabilities for each process. For example, the resulting probabilities for a combination with a positive jump of the peakload process present a positive jump in one of three processes is 56.67% (= 5.15% + 8.43% + 13.58% + 29.51%). Multiplying this by the probability that there is, indeed, a positive jump in the process gives the jump intensity λ^p for the peak load process 8.29%(= 56.67% × 14.63%). Limiting the time series on the days on which positive jumps are observed provides the necessary data to test for a correlation among the jump heights. The correlation analysis does not provide significant values . Therefore, one can assume that no correlation persists between the jump heights. The jump distributions' parameters for the peakload process are shown in table 4.10. Figure 4.29 shows the total correlation of the three processes.

Figure 4.29: Correlation of the processes



Source: Own work based on Mayer et al. (2011).

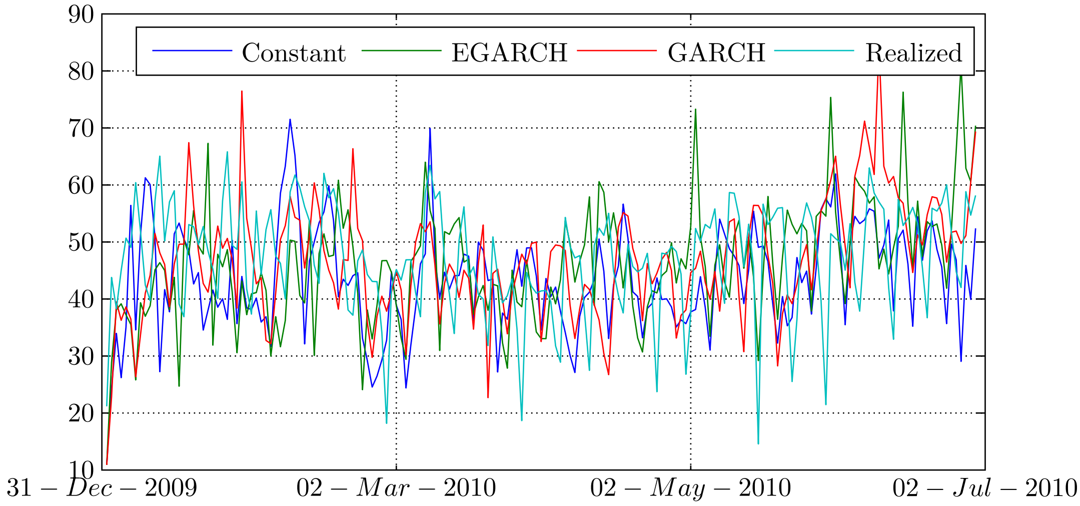
4.5 Empirical results of the electricity price model

4.5.1 Statistical and trajectory fit

In order to test the model, a Monte Carlo simulation is performed for each asset a={peakload, off-peak 1,off-peak 2} and each volatility model v={Constant, GARCH, E-GARCH} with $N = 100000$ simulation paths for the period between January 1^{st} and June 30^{th} 2010, which contains $T = 181$ days . The results are compared with the realized electricity price processes for that period. The model must yield price paths for the prices of electricity that resemble those observed in the market. Figures 4.30, 4.31, and 4.32 show the realized and simulated paths.

4.5.1.1 Peakload

Figure 4.30: Spot and Simulated Spot Prices (peakload process)

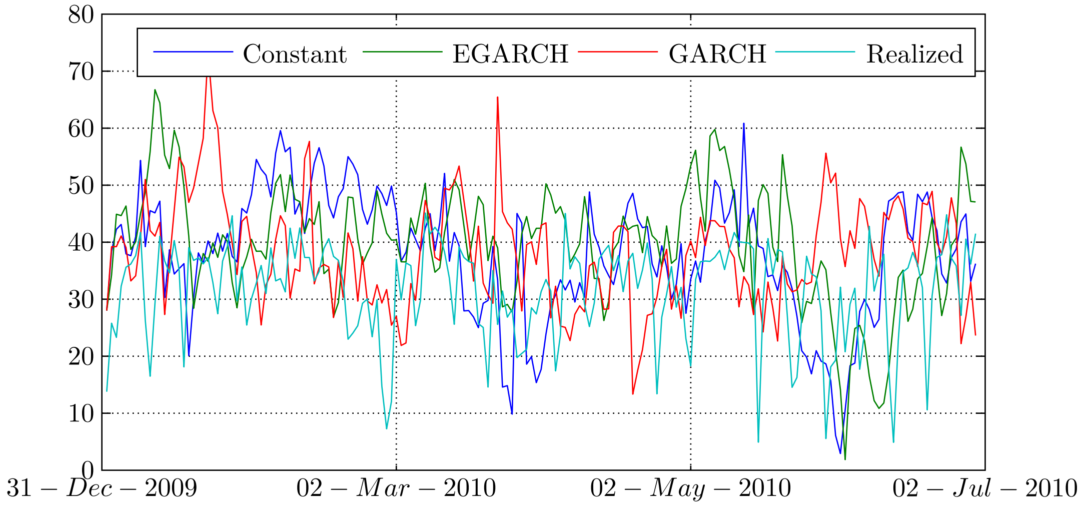


Source: Own work based on Mayer et al. (2011).

Figure 4.30 shows that the model captures the mean-reversion of the jumps very well and reverts in the long run to its seasonal function as expected from (4.1) with (4.6) and (4.7). It is obvious that the model captures both the statistical and trajectory properties for the peakload price process very well. It is also evident that the E-GARCH volatility model fits the price processes best.

4.5.1.2 Off-peak 1

Figure 4.31: Spot and Simulated Spot Prices (off-peak 1 process)

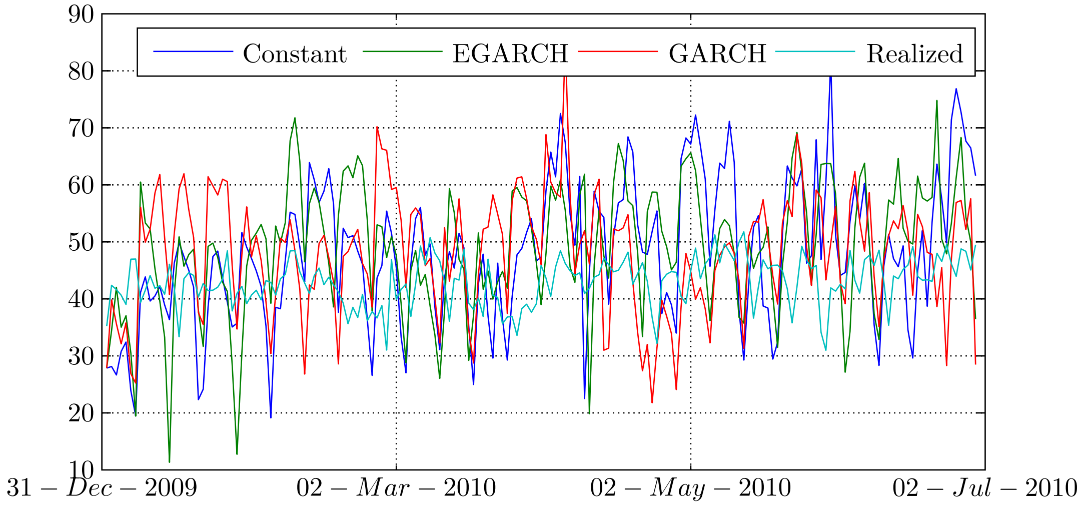


Source: Own work based on Mayer et al. (2011).

The differences between the off-peak 1 mainly result from a lower mean caused by the aftermath of the financial crisis with lower economic activity and higher feed from renewable energies.

4.5.1.3 Off-peak 2

Figure 4.32: Spot and Simulated Spot Prices (off-peak 2 process)



Source: Own work based on Mayer et al. (2011).

The same results for the off-peak 2 process hold true as for the off-peak 1 process. The main difference is a lower mean.

4.5.2 Goodness-of-fit

The root mean squared error is used as goodness-of-fit criteria. It is defined as:

$$SE_{t,n,v,a} = \left(S^{sim}_{t,n,v,a} - S^{rel}_{t,a}\right)^2 \qquad \forall 1 \leq n \leq N, 1 \leq t \leq T \tag{4.24}$$

$$RMSE_{i,v,a} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(SE_{t,n,v,a} - \bar{SE}_{n,v,a}\right)} \qquad \forall 1 \leq n \leq N \tag{4.25}$$

where $SE_{t,n,v,a}$ denotes squared error and $S^{sim}_{t,n,v,a}$ the simulated electricity price at time t for the n simulation path for asset a and volatility model v. $S^{rel}_{t,a}$ denotes the realized electricity price at time t.

Tables 4.17, 4.18, and 4.19 show the mean, standard deviation, minimum, and maximum of the root mean squared error resulting from simulating the peakload, off-peak 1 and off-peak 2 price processes, using the three different volatility models. Figures 4.33, 4.34, and 4.35 show the cumulative distribution of the root mean squared error for all three volatility models for the peakload, off-peak 1, and off-peak 2 price processes.

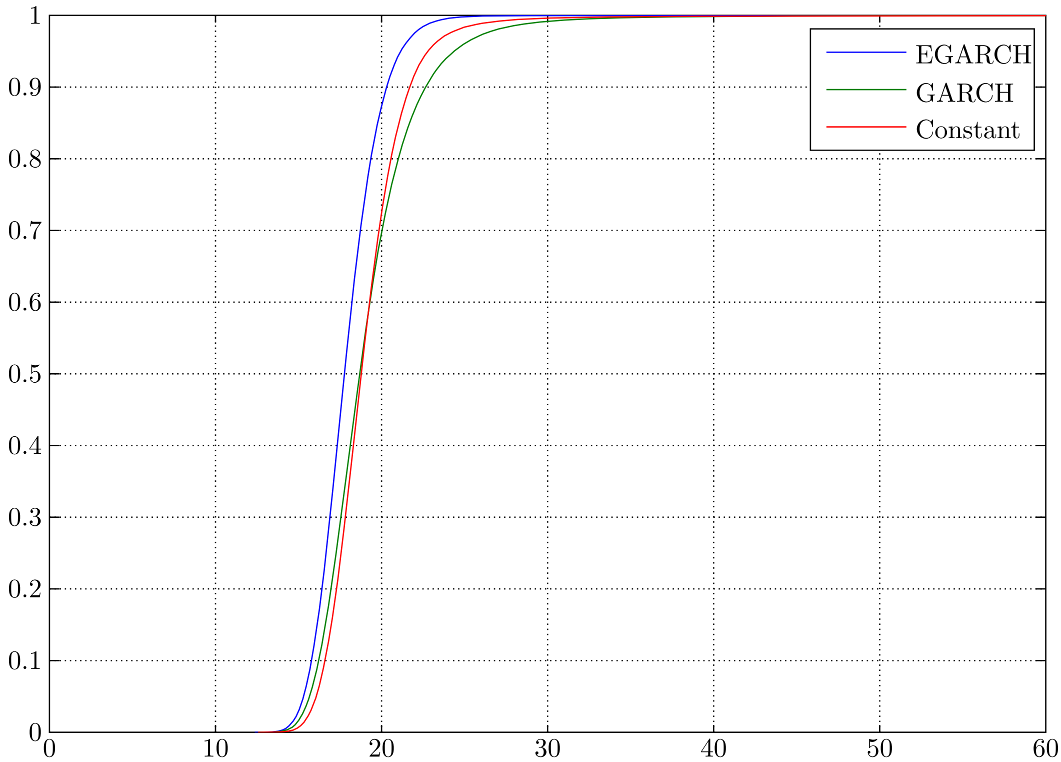
4.5.2.1 Peakload

Table 4.17: Root mean squared error (peakload process)

Volatility model	Constant	GARCH	E-GARCH
Mean	19.0970	19.2160	17.9402
Std Dev	0.2083	0.1778	0.1034
Min	12.5539	12.6129	12.3645
Max	829.4487	372.3992	116.4280

Source: Own work based on Mayer et al. (2011).

Figure 4.33: RMSE using different volatility models (peakload process)



Source: Own work based on Mayer et al. (2011).

As expected by looking at figure 4.33, it reveals that stochastic volatility performs better than constant volatility. Following table 4.17, it is apparent that the E-GARCH model is better able to capture the behavior of the spot price. A possible explanation is that the E-GARCH allows asymmetry in the effects on the volatility.

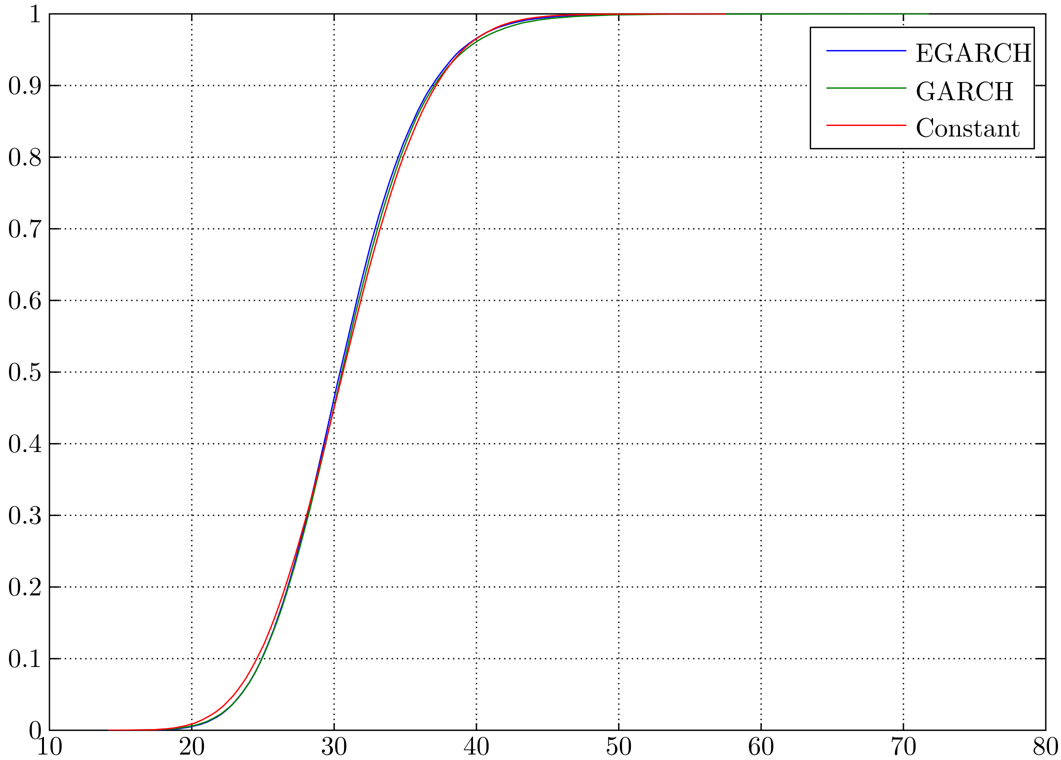
4.5.2.2 Off-peak 1

Table 4.18: Root mean squared error (off-peak 1 process)

Volatility model	Constant	GARCH	E-GARCH
Mean	30.7696	30.8413	30.7081
Std Dev	0.1590	0.1574	0.1538
Min	14.1359	14.3356	15.0585
Max	57.5053	71.7823	60.1374

Source: Own work based on Mayer et al. (2011).

Figure 4.34: RMSE using different volatility models (off-peak 1 process)



Source: Own work based on Mayer et al. (2011).

Figure 4.34 shows the for the off-peak 1 process the best model is the constant volatility model. Although there are only minimal differences between all three models, as can be seen in table 4.18.

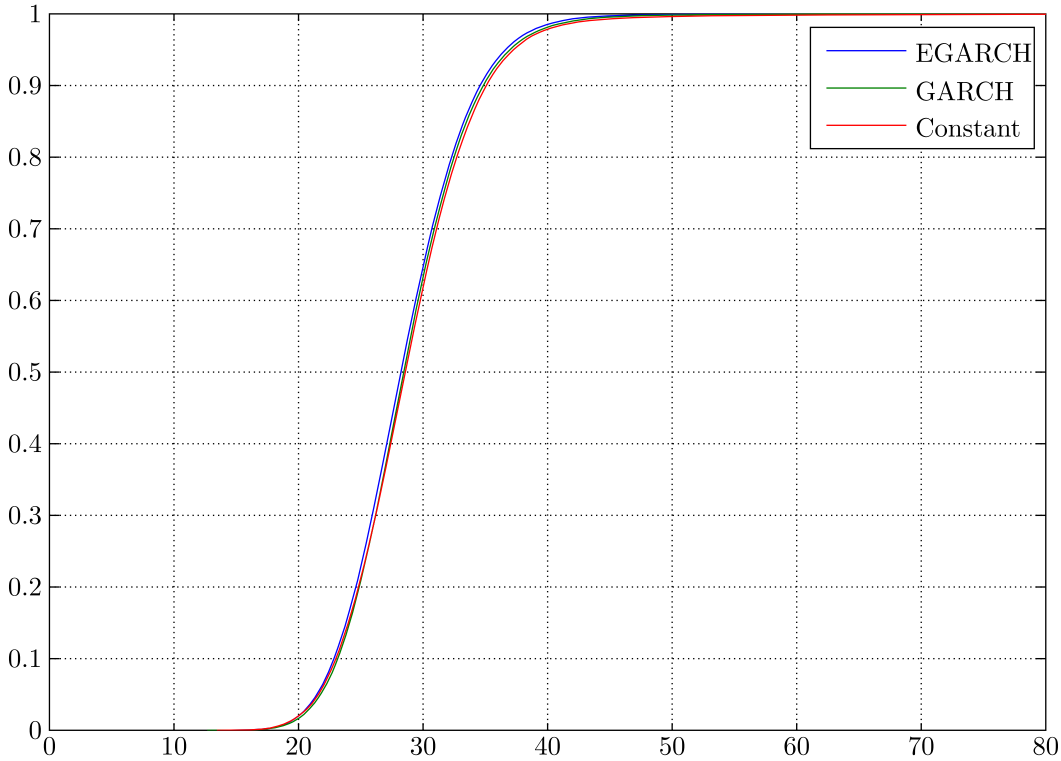
4.5.2.3 Off-peak 2

Table 4.19: Root mean squared error (off-peak 2 process)

Volatility model	Constant	GARCH	E-GARCH
Mean	28.9948	28.8258	28.5571
Std Dev	0.2263	0.1696	0.1693
Min	13.4463	12.6675	13.7845
Max	807.9479	207.5292	241.4611

Source: Own work based on Mayer et al. (2011).

Figure 4.35: RMSE using different volatility models (off-peak 2 process)



Source: Own work based on Mayer et al. (2011).

The best performing volatility model of the off-peak 2 process is the GARCH model, as shown in figure 4.35. The results shown in table 4.19 support this thesis.

Chapter 5

Stochastic Valuation of energy investments

This chapter addresses the question whether model complexity influences the decision to undertake an investment.[1]

5.1 Real Option Project Financed Valuation Tool

This section describes the valuation model used in the analyses.[2]

The ROPFVT is a tool that enables an investor to calculate the NPV distribution in a project financed investment as well as the project's expected default probability. This last feature could be particularly interesting to debt providers in the project. The ROPFVT is based on the equity approach. Therefore, the estimation of the cost of equity is a crucial task. In general, the cost of equity is estimated with the Capital Asset Pricing Model (CAPM) or the Arbitrage Pricing Theory (APT). In standard valuation theory, DCF analysis is mainly applied to industry companies with rather stable capital structures. Therefore, a single constant discount rate can be used. This technique is not appropriate in a project finance context since, by definition, it has to dealt with time-varying capital structures there (Esty, 1999). Damodaran (1994) and Grinblat and Titman (2001) argue that in such a context only the application of different discount rates for every year based

[1]The following chapter is based on Weber et al. (2010); Schmid and Weber (2011*a*,*b*).

[2]The model is written in MATLAB. All calculations were performed on a cluster of Dell workstations with Intel Core i7 processors.

on the actual capital structure produces an unbiased costs of equity. This technique is included in the model by determining the cost of equity for each period a cash flow is calculated.

Esty (1999) argues that it is crucial to calculate the cost of equity based on market values for debt and equity instead of their book values. In case of a non-marketed investment project, as it is typically the case in project finance, this leads to a well-known circularity problem: The market value of equity, which is estimated as the book value of equity plus the NPV, cannot be calculated without knowing the cost of equity, while without knowing market value one cannot estimate the cost of equity. A possible solution in a project finance context is proposed by Esty (1999): the so called quasi-market valuation (QMV). The QMV is based on three main assumptions: (i) the CAPM holds, (ii) the market value of debt is equal to the book value of debt and (iii) the market is efficient. This is an iterative solution of the circularity problem. First, the market value of equity at the end of a given year is calculated on the basis of a given cost of equity (which implies certain leverage). Second, if the resulting market value is too high, i.e. implies a higher than used cost of equity, the cost of equity is increased and a new (lower) market value is derived. These steps are repeated until the market value of equity implied by a given cost of equity is equal to the market value derived by applying the DCF valuation. The algorithm of this technique, that is inclued in the model, can be described as follows:

Algorithm 1 QMV

for $j = 1$ **to** Number of Simulation **do**

$NPV_j = 0$

$NPV2_j = \sum_{t=0}^{T} \frac{FCFE_t}{rho + \frac{D_t}{(BookValueEquity + NPV_j)} * (rho - CostOfDebt)}$

while $ABS(NPV2_j - NPV_j) > 0.0001$

$NPV_j = NPV2_j$

$NPV2_j = \sum_{t=0}^{T} \frac{FCFE_t}{rho + \frac{D_t}{(BookValueEquity + NPV_j)} * (rho - CostOfDebt)}$

end while

end for

The model has also the capabilities to use risk-neutral pricing. In that case, the relevant factors influencing the cash flow are transformed to be martingales and the resulting cash flow is discounted using the risk-free rate of return.

Every free cash flow to equity path is discounted with time varying cost of equity calculated according to the QMV-method described above. If the equivalent martingale or risk-neutral approach is used then the equivalent martingale cash flow is calculated and discount it with the risk-free rate. It should be noted that the ROPFVT is programmed in MATLAB, a programming language often used for numerical applications.

Historical time series for various financial assets are included in the ROPFVT and can be updated any time. These time series are used for the parameter estimation within various forecast models. There is also the possibility of a manual input of the parameters.

When using the ROPFVT, the user has to specify all relevant risk factors determining the project's cash flow first. For these factors, several forecast models for prices, volatility and correlation have been implemented, which are introduced in the next section. For mean or price forecasts, the following models are included: (i) simple random walk with and without drift (Fama, 1965), (ii) autoregressive moving average (ARMA) (Box and Jenkins, 1970) and (iii) mean-reversion forecasts (Vasicek, 1977; Benth et al., 2003; Mayer et al., 2011). To obtain estimates for future variance and correlation structures, (i) historical standard deviation and correlation (ii) GARCH-type models[3] as proposed by Bollerslev (1986) and (iii) Dynamic Conditional Correlation (DCC) models as proposed by Sheppard and Engle (2001) are included.

Beneath input and output factors with historical time series, the ROPFVT allows the manual input of non-financial factors as well. For this, the user may choose between different distributions, e.g. rectangular or triangular distribution with or without drift. This approach allows the consideration of non-Gaussian distributed factors. In a second step, the user has to define project specifics, which include, for example, the lifespan of the project, the financing structure or the efficiency. Additionally, some parameters for the simulation must be specified, like the desired frequency of cash flow distributions reported, the time-resolution, which determines how often the cash flow is analyzed or the number of iterations. The duration of the simulation procedure depends significantly on this input parameter.

[3]The GARCH derivatives GJR-GARCH (as proposed by Glosten et al. (1993)) and E-GARCH (as proposed by Nelson (1991)) are included in model as well.

The user does not have to handle MATLAB directly but can calibrate the ROPFVT via a Microsoft EXCEL spreadsheet.[4] Inside this spreadsheet, the user is able to define all of the above mentioned parameters. The results, which are generated in MATLAB, are stored in an output file and can be exported as a Microsoft EXCEL file. These file contains, among others, the following information: (i) the distribution of the project's NPV, (ii) the cumulated default probability, (iii) a summary of free cash flow to equity, default probabilities, cash flows at risk, coverage ratios and leverage ratios and (iv) the underlying forecasts of the individual factors.

5.1.1 Forecasting of volatility and correlation

The future development of the factors influencing a project's cash flows is of importance. Since this development is a priori unknown, one has to apply different kinds of models to obtain estimates for the future prices. However, not only the future prices of the influencing factors are of interest. To perform a consistent Monte Carlo simulation, one has to estimate the distribution of all relevant risk factors. This is a problem common to all forecasting techniques. Since the literature on forecasting is huge, a complete overview of all available models and methods is beyond the scope of this work.[5] In the following, the models, applied in this dissertation, are specified.

5.1.1.1 Price models models

During this work, one specific stochastic process is used. It is called an Ornstein-Uhlenbeck process (Uhlenbeck and Ornstein, 1930). Vasicek (1977) used this process to model instantaneous short-rate when modeling the term structure of interest rates. The nice feature of this process is that a closed-form solution exists to price forward rates.[6] The current model applies the Ornstein-Uhlenbeck process for modeling interest rates and commodities except electricity. As already shown in chapter 4, electricity has its peculiarities. Therefore, a different model was developed in section 4.3. This model is applied for modeling electricity.

[4]Please note that Microsoft EXCEL serves as a graphical front-end, but the computation is performed in MATLAB.

[5]See, for example, Geman (2005) or Weron (2006) for a detailed overview on forecasting models.

[6]See appendix app:CF.

5.1.1.2 Volatility models

In this section, the different volatility models used in this work are presented, beginning with the Autoregressive Conditional Heteroscedasticity (ARCH) model of Engle (1982), followed by the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model developed by Bollerslev (1986) and its derivative the asymmetric Exponential Generalized Autoregressive Conditional Heteroscedasticity (E-GARCH) model. While GARCH models are discrete-time models, The author presents their continuous-time limit, the Heston volatility model. All of these models allow one to model heteroscedasticity, which is often found in financial time series data. They are able to explain volatility clustering as well as autocorrelation, and possess a leptokurtic distribution. Therefore, they are useful to explain some of the systematic biases associated with the constant volatility.

The ARCH Model The development of diverse GARCH models started with the proposal of the ARCH model by Engle (1982). ARCH models have a zero mean and are serially uncorrelated with non constant variances conditioned on the past, but with constant unconditional variance. For these kind of processes, the recent past gives information about the variance to be forecast. This behaviour of ARCH processes is equal to the phenomenon of volatility clustering, which is often present in capital markets. Volatility clustering was first noted by Mandelbrot (1963) who noticed that ”large changes in asset prices tended to be followed by large changes - of either sign - and small changes tended to be followed by small changes”.

Definition 13 (ARCH(q) model)**.** *In an ARCH(q) model the time series disturbances* $\epsilon_t, t \in \mathbb{N}$*, is given by*

$$\epsilon_t = \sqrt{\langle_t} \eta_t \tag{5.1}$$

$$\langle_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \tag{5.2}$$

with $\alpha_0 > 0$ *and* $\alpha_i \geq 0, \; \forall i = 1, \ldots, q$*.* $\{\eta_t\}$ *is an independent and identically white noise process*[7] *with* $Var[\eta_t = 1]$*.*

It is very typical to assume η_t to be normally distributed, i.e. $\eta_t \sim \mathcal{N}(0, 1)$.

[7] A white noise process is a process of random variables which are uncorrelated, have mean zero and a finite variance.

Outliers of ϵ_t have a high influence on the conditional variance in the ARCH model. After observing high absolute values of ϵ_t, it is more likely to observe high values again. Therefore, the conditional variance tends to be high again if it had been high before. The same is true for periods of low variances. Different values of q are used to reflect the different expectations about the variation of the variance over time. The greater the parameter q, the longer past observations still influence today's variance. If $q = 0$, the ARCH model is equivalent to a homoscedastic constant volatility model. With the ARCH model, it is possible to model the potentiality, that newer information might be a better estimate for the future variance than older information. This can be achieved by assigning higher values to the parameters α_{t-i} of more recent observations. The main advantage of the ARCH model is that its unknown parameters can be fitted to historical data. Therefore, one is able to obtain a better estimate for the volatility of an asset. In the special case where $\alpha_0 = 0$ and $\alpha_i = 1/q$, the conditional variance of the ARCH(q) model is equal to the sample variance of the last q realizations of $\epsilon_{t-i}, i = 1, ..., q$. This estimate is often used in practice.

The GARCH model In the previous Section, the ARCH model was introduced. Engle (1983) and Engle and Kraft (1983) showed in empirical studies that for the ARCH(q) model a relative long lag q in the conditional variance equation is often needed to achieve a good fit. Furthermore, a fixed lag structure often needs to be imposed to avoid problems with negative variance parameter estimates. In this light, it seems of great practical interest to extend the ARCH model to allow for a longer memory and a more flexible lag structure. For this reason, Bollerslev (1986) proposed the GARCH(p,q) model.

Definition 14 (GARCH(p,q) model)**.** *In a GARCH(p,q) model the time series of disturbances $\epsilon_t, t \in \mathbb{N}$, is given by*

$$\epsilon_t = \sqrt{\mathcal{H}_t}\eta_t \tag{5.3}$$

$$\mathcal{H}_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \mathcal{H}_{t-j} \tag{5.4}$$

with $\alpha_0 > 0$ and $\alpha_i \geq 0,\ \forall i = 1, \dots, q$ and $\beta_j \geq 0,\ \forall j = 1, \dots, p$ to guarantee a positive conditional variance. η_t is an independent and identically white noise process with $Var[\eta_t =$

1].

In this model, the conditional variance $\mathcal{H}_t$ does not only depend on the last q disturbances $\epsilon_{t-q}, ..., \epsilon_{t-1}$, but also on the last p conditional variances $\mathcal{H}_{t-p}, ..., \mathcal{H}_{t-1}$. It is characterized by the autoregressive component of order q and by the moving average component of order p. For $p = 0$, the GARCH(p,q) model reduces to the ARCH(q) model and for $p = 0$ and $q = 0$, it is a constant homoscedastic model.

All in all, one can say that the GARCH models are able to explain the phenomenon of heteroscedasticity, which is often observed in financial time series data. Furthermore, the modeling of a leptokurtic distribution and of volatility clusters is possible with the standard GARCH model. However, it is not possible to model the so called leverage effect with the standard GARCH model. The leverage effect was first described by Black (1976), page 177 as follows "...when stocks go up, volatilities seem to go down; and when stocks go down, volatilities seem to go up." This can be seen as a negative correlation between asset prices and volatility. In the standard GARCH model, the influence of a random shock on the conditional variance is the same, regardless whether it is a negative or a positive random shock. As a result, positive random shocks and negative random shocks have the same influence on the conditional variance. In his research, Nelson (1991) found sufficient indication for the presence of the leverage effect. Therefore, asymmetric GARCH models were included. These type of models are able to include the leverage effect. There are numerous asymmetric GARCH models but this work is not about GARCH models; therefore, only the one used in this work is presented here.[8]

E-GARCH Nelson (1991) noticed three drawbacks of the GARCH(p,q) model. Firstly, by assumption, the GARCH model is not able to model the negative correlation between current asset returns and future asset returns volatility (leverage effect). Secondly, the GARCH model restricts the parameters to be positive. This can be violated by estimated coefficients and may unduly restrict the dynamics of the conditional variance process. Thirdly, Nelson (1991), page 347 states that "interpreting whether shocks to conditional variance persist or not is difficult in GARCH models, because the usual norms measuring

[8]Others are, for example, the GJR-GARCH (Glosten et al., 1993), the NGARCH, and the VGARCH model (Engle and Ng, 1993; Heston and Nandi, 2000).

persistence often do not agree". Based on this, he developed the Exponential Autoregressive Conditional Heteroscedasticity E-GARCH model.

Definition 15 (E-GARCH(p,q))**.** *In a E-GARCH(p,q) model the time series of disturbances* $\epsilon_t, t \in \mathbb{N}$*, is given by*

$$\epsilon_t = \sqrt{\langle_t}\eta_t \tag{5.5}$$

$$\ln(\langle_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i g(\eta_{t-i}) + \sum_{j=1}^{p} \beta_j \ln(\langle_{t-j}) \tag{5.6}$$

with $g(\eta_{t-i}) = \theta\eta_{t-i} + \gamma(|\eta_{t-i}| - E[|\eta_{t-i}|])$*.* η_t *is an independent and identically white noise process with* $Var[\eta_t = 1]$*.*

The Matlab implementations of the ARCH, GARCH and E-GARCH models are used in this dissertation.

Continuous limits of GARCH models A very important feature of the GARCH models used in this analysis is the convergence result of discrete-time GARCH models. Stochastic volatility models can be classified into continuous-time diffusion models and discrete-time GARCH models. Both models are often used for modeling financial time series data. The approaches are connected through the convergence result.

Nelson (1990) was the first who studied the convergence of stochastic difference equations, especially the GARCH(1,1) and the E-GARCH(0,1) model, to stochastic differential equations as the length of the discrete-time intervals between observations shrinks. He showed that the GARCH model weakly converges to a bivariate diffusion process, when the length of the time interval converges to zero. As a result, discrete-time GARCH models can be used to approximate time-continuous stochastic volatility models. Duan (1997) generalized the convergence result of Nelson (1990) and showed that a broader family of parametric GARCH processes, called augmented GARCH processes, also converge to a corresponding bivariate diffusion process. The diffusion limit process of the augmented GARCH process is shown by Duan (1997) to contain most of the bivariate diffusion processes that are used by the common stochastic volatility option pricing models.[9]

[9]The upcoming results can be found in (Nelson, 1990, p. 8).

Since financial time series data are only available at discrete intervals, the convergence results of Nelson (1990) and Duan (1997) provide an important result for empirical studies. For the empirical parameter estimation of continuous-time stochastic volatility models, a discrete-time approximation is necessary. Furthermore, the likelihood function of a discrete-time GARCH model is usually easy to compute and maximize, whereas the likelihood function of a nonlinear stochastic differential equation system observed at discrete intervals can be very difficult to derive, especially when there are unobservable state variables (e.g. conditional variance) in the system. Therefore, estimation and forecasting can be performed much easier with a discrete-time GARCH model than it would be the case with a diffusion model using discrete observations. This explains why GARCH stochastic difference equation systems are favored by empiricists, even though continuous-time nonlinear stochastic differential systems are used in many theoretical studies. The approximation of a continuous-time stochastic volatility model with a GARCH model could, therefore, be of great benefit for empirical studies. This is supported by the fact that distribution results are often only available for the limit diffusion process and not for the discrete-time GARCH process. In this case, the diffusion process can be used to approximate the GARCH process and vise verse. The convergence result allows to interchange properties of the GARCH and the bivariate diffusion model concerning parameter estimation, results distribution, and option pricing. Furthermore, the extensive range of numerical and statistical methods developed either for the GARCH or the diffusion model can be interchanged.

5.1.1.3 Correlation Models

Dynamic Conditional Correlation The Dynamic Conditional Correlation (DCC) model was introduced in 2001 by Engle and Sheppard and is based on the Constant Conditional Correlation model, proposed by Bollerslev (1990).[10]

The DCC model is multivariate GARCH that combines the the univariate GARCH volatilities with a GARCH-like time varying correlation models. The model is able to to handle simultaneous estimation of arbitrary many time series as the model of Bollerslev (1990). But, it enables one to change the correlation over time.

[10] Cf. the working paper of Sheppard and Engle (2001) for an introduction of the DCC. The model was finally published in the Journal of Business and Economics Statistics in 2002 by Engle; cf. Engle (2002).

The covariance matrix H_t for k assets can be calculated according to the following definition

Definition 16 (Dynamic Conditional Correlation)**.** *(see Engle, 2002) Assume that the returns of k assets are conditionally multivariate normal distributed, with zero mean and covariance $\mathcal{H}_t$.*

$$r_t|\mathcal{F}_{\sqcup-\infty} \sim N(0, \mathcal{H}_t) \tag{5.7}$$

and

$$\mathcal{H}_t \equiv \mathcal{D}_t \rho_t \mathcal{D}_t \tag{5.8}$$

where D_t is the $k \times k$ diagonal matrix of time varying standard deviations from univariate GARCH models, and ρ_t is the time-varying correlation matrix. The univariate GARCH model, generating D_t, is not limited to the standard GARCH(p,q), but can include any GARCH derivative with normally distributed errors.

The correlation is estimated, in the DCC model, via a two step procedure. In a first step, the parameters of a univariate GARCH model for every time series included in the estimation procedure are estimated separately. Standardized errors based on the estimation results are then calculated. In a second step, the correlations are estimated based on the standardized errors.

Similar to the volatility forecasts performed via a GARCH model, correlation forecasts performed by the DCC models tend to be either stable or to converge to the long-term average of the time series. Thus, in the long term, the results should converge to the results obtained by the historical correlation approach. Depending on the time series, it takes between a few weeks and a few month until both the convergence.

5.1.2 Equivalent martingale transformation by the Esscher transformation

According to the first fundamental theorem of asset pricing, definition 6, a market model that does not admit arbitrage has an equivalent pricing measure under which all tradeable discounted asset price processes are (local) martingales. The arbitrage-free price of a derivative is then calculated as the expected discounted payoff under the risk-neutral probability measure (Harrison and Kreps, 1979). A market is complete if every derivative security can be hedged.[11] In the Black-Scholes model, the market is complete.[12] Therefore, according to the second fundamental theorem of asset pricing, definition 7, the equivalent martingale measure is unique. The model introduced in section 4.3 is not complete. From a theoretic point of view, any model with jumps is incomplete (see Cont and Tankov, 2003, p. 302ff) and from a practical point of view, the electricity market is not complete because of the limited storeability of electricity. Therefore, not all derivatives can be hedged (see Benth, Benth and Koekebakker, 2008, p. 22). Consequently, any martingale measure $\mathcal{Q}$ equivalent to the physical measure $\mathcal{P}$ is a risk-neutral measure (see Pliska, 2002, p. 22ff.). In incomplete markets it is typical to construct such a measure change by changing the parameters of the price process in such a way that the martingale requirement is satisfied (see Benth and Sgarra, 2009, p. 7). A potential method to do this and, thereby, obtain an equivalent martingale measure is the Esscher transformation introduced by Esscher (1932). Pricing contingent claims with the help of the Esscher transform is discussed by Gerber and Shiu (1994), Madan and Milne (1991), Eberlein and Keller (1995), Chan (1999), and Kallsen and Shiryaev (2002). Its theoretical background appears in the next section.

5.1.2.1 Theoretical background of the Esscher transformation

In the Black-Scholes model, an equivalent martingale measure could be obtained by changing the drift. In models with jumps, e.g. models with discontinuities, it is necessary to not only change the drift but to obtain an equivalent measures by altering the distribution of jumps. A convenient transformation, which is somewhat analogous to the drift change for geometric Brownian motion, is the Esscher transform. The idea is to change the origi-

[11] See definition 6

[12] I refer the reader to original paper of Black and Scholes (1973) and Merton (1976) for a discussion of this topic.

nal probability measure to an equivalent probability measure under which the discounted price process of the primary assets are martingales.

The Esscher transformation allows one to change the measure for a stochastic process with stationary and independent increments. A very important property of the Esscher transform is that the measure change preserves the structure of the model dynamics (see Benth and Sgarra, 2009, p. 7). The Esscher Transformation can be seen as a generalization of the well-know Girsanov-Theorem to include a broader class of processes. These processes are certain types of Lévy processes, namely (i) the Wiener process or Brownian motion, (ii) the Poisson process, (iii) the compound Poisson process, the gamma process, and the inverse Gaussian process (Gerber and Shiu, 1994). Therefore, a requirement to change stochastic processes with jumps is the definition equivalence of two Lévy process. This is done in proposition 1.

An Esscher transform of a process with stationary and independent increments or a Lévy process induces an equivalent probability measure under which the process is a martingale. It is defined as:

Definition 17 (Esscher Transform)**.** *(See Gerber and Shiu, 1994, p. 100) Let $f(x)$ be a density function and h be a real number such that*

$$M(h) = \int_{-\infty}^{\infty} e^{hx} dx \tag{5.9}$$

exists. As a function in x,

$$f(x; h) = \frac{e^{hx} \cdot f(x)}{M(h)} \tag{5.10}$$

is a probability density, and it is called Esscher transform with parameter h of the original distribution.

The parameter h can be used to determine a new probability measure, which is an equivalent martingale measure. This means the discounted prices of assets are martingales under that measure. The derivative price is then simply the expectation, with respect to that measure (Gerber and Shiu, 1994). This can be written more formally as following. Assuming an exponential price process $A(t) = A(0) \cdot \exp(X(t))$ leads to the following equation under the risk-neutral Esscher measure:

$$A(0) = \mathbb{E}^{Q}[\exp\{-rt\} \cdot A(t)] \tag{5.11}$$
$$= \exp\{-rt\} \cdot \mathbb{E}^{Q}[A(t)] \tag{5.12}$$

Using the definition of the price process allows one to rewrite equation (5.11) as follows:

$$1 = \exp\{-rt\} \cdot \mathbb{E}^{Q}[\exp\{X(t)\}] \tag{5.13}$$

The equivalent martingale measure can be found analytically using the moment generating function. If the stochastic process of an asset is too complex for an analytical solution, it has to be approximated numerically. This is described in section 5.1.2.5.

5.1.2.2 Analytical transformation via moment generating function

To find an analytical solution, one has to assume that for $0 \leq t \leq T$ $A(t)$ denotes the price of non-dividend paying asset a time t. Also assume that $X(t)$ denotes a stochastic process with stationary and independent increments. Furthermore, it is assumed that $X(0) = 0$ holds; therefore, $A(t)$ is defined as $A(t) = A(0) \cdot \exp\{X(t)\}, 0 \leq t \leq T$. Let $F(x,t) = P[X(t) \leq x]$ be the cumulative distribution function of $X(t)$ and

$$M(z,t) = \mathbb{E}[e^{zX(t)}] \tag{5.14}$$
$$= \int_{-\infty}^{\infty} e^{zx} \cdot f(x,t)dx \tag{5.15}$$

be the moment-generating function. Following Gerber and Shiu (1994), if $M(z,t)$ is continuous at $t = 0$ than

$$M(z,t) = [M(z,1)]^t \tag{5.16}$$

holds. From equation (5.16) follows that if h is a real valued number and $M(h,t)$ exists for one positive number t, it exists for all positive t. Using definition 17, the density function of $X(t),\ t > 0$ is given by:

$$f(x,t;h) = \frac{e^{hx} \cdot f(x,t)}{\int_{-\infty}^{\infty} e^{hy} \cdot f(y,t)dy} \tag{5.17}$$

$$= \frac{e^{hx} \cdot f(x,t)}{M(h,t)} \tag{5.18}$$

The moment-generating function of this process is than defined as

$$M(z,t;h) = \int_{-\infty}^{\infty} e^{zx} \cdot f(x,t;h)dx \tag{5.19}$$

$$= \frac{M(z+h,t)}{M(h,t)} \tag{5.20}$$

and by equation (5.16) follows $M(z,t;h) = [M(z,1;h)]^t$. The modified probability measure is equivalent to the original probability measure because the exponential function is strictly positive (see Gerber and Shiu, 1994, p. 103).

To use the Esscher transformation for risk-neutral valuation, it is additionally necessary that the discounted asset price process $\{e^{-r_f t}A(t)\}_{0\leq t\leq T}$ is a martingale. This means that equation (5.11), which can be rewritten to equation (5.13), is satisfied. Therefore, a $h = h^*$ that render $\{e^{-r_f t}A(t)\}_{0\leq t\leq T}$ a martingale has to be found. As already shown, equation (5.11) can be rewritten to equation (5.13). Using this, the solution for h^* is

$$e^{r_f t} = M(1,t;h^*) \tag{5.21}$$

and from equation (5.16) and equation (5.19) follows that the solution does not depend on t. This means equation (5.19) can be rewritten to

$$e^{r_f} = M(1,1;h^*) \tag{5.22}$$

or

$$r_f = \ln[M(1,1;h^*)].$$

The parameter h^* is the risk-neutral Esscher transform and the corresponding equivalent martingale measure $\mathcal{Q}$, the risk-neutral Esscher measure (see Gerber and Shiu, 1994, p. 104).

5.1.2.3 Esscher Transformation of the Poisson and compound Poisson process

This section presents the equivalent measures for the important price process used in this dissertation. These are the Poisson process and the compound Poisson process. Before that, it presents the general theorems that are required to define an equivalent Lévy process.

Proposition 1. *(See Sato, 1999, Theorems 33.1 and 33.2) Let* (X_t, P) *and* (X_t, P') *be two Lévy processes on* $\mathbb{R}$ *with characteristic triplets* (σ^2, ν, γ) *and* $(\sigma'^2, \nu', \gamma')$ *and filtration* $\mathbb{F} = (\mathcal{F}_t)_{0 \le t \le T}$. *Then* $P|_{\mathcal{F}_t}$ *and* $P'|_{\mathcal{F}_t}$ *are equivalent for all* t *(or equivalent for one* $t > 0$*) if and only if the three conditions are satisfied:*

1. $\sigma = \sigma'$
2. *The Lévy measures are equivalent with* $\int_{-\infty}^{\infty}(e^{\frac{\phi(x)}{2}} - 1)^2\nu(dx) < \infty$ *where* $\phi(x) = \ln\left(\frac{d\nu^i}{d\nu}\right)$
3. *If* $\sigma = 0$ *then additionally* $\gamma' - \gamma = \int_{-1}^{1} x(\gamma' - \gamma)(dx)$ *must hold.*

If one assumes that the probability measures $\mathcal{P}$ and $\mathcal{Q}$ are equivalent, then the Radon-Nikodým derivative is given by:

$$\frac{dP'|_{\mathcal{F}_t}}{dP|_{\mathcal{F}_t}} = e^{U_t} \tag{5.23}$$

where $U_t = \eta \cdot X_t^c - \frac{\eta^2 \cdot \sigma^2 \cdot t}{2} - \eta \cdot t + \lim\left(\sum_{s \le t, |\Delta X_s| > \epsilon} \phi(\Delta X_s) - t \cdot \int_{|x|>\epsilon}(e^{\frac{\phi(x)}{2}} - 1)^2\nu(dx)\right)$. This means that X_t^c is the continuous part of the process (X_t), e.g. the part without jumps, and η fulfills $\gamma' - \gamma - \int_{-1}^{1} x(\gamma' - \gamma)(dx) = \sigma^2\eta$ if $\sigma > 0$ and zero otherwise. Furthermore, the process U_t is a Lévy process with characteristic triplet $(\sigma_U, \nu_U, \gamma_U)$, where $\sigma_U = \sigma^2 \cdot \eta^2$, $\nu_U = \nu \cdot \phi^{-1}$, and $\gamma_U = -\frac{1}{2}\sigma_U \cdot \eta^2 - \int_{-\infty}^{\infty}(e^y - 1 - y \cdot \mathbb{1}_{|y| \le 1})(\nu\phi^{-1})(dy)$.

Proposition 2 (Equivalent measure for Poisson process)**.** *Let* $(N, \mathcal{P})$ *and* $(N, \mathcal{Q})$ *be Poisson processes on the filtered probability space* $(\Omega, \mathcal{F}, \mathbb{F})$ *with jump intensities* λ_1 *and* λ_2 *and jump heights* a_1 *and* a_2. $\mathcal{P}$ *and* $\mathcal{Q}$ *are equivalent if and only if* $a_1 = a_2$.

The Radon-Nikodým derivative is:

$$\frac{d\mathcal{Q}}{d\mathcal{P}} = \exp\left(T(\lambda_2 - \lambda_1) - N_T \ln \frac{\lambda_2}{\lambda_1}\right)$$

This means that a Poisson process can be transferred from one probability measure to another by changing the jump intensity λ. That results in changing the probability on the paths but does not create new paths. Therefore, the intensity of the Poisson process can be changed without changing the jump size. In fact, changing the jump size would lead to a different process because new paths could be created Cont and Tankov (2003). The intensity of risk-neutral Poisson process can be found be applying the Esscher transform. It is defined as $\lambda_2 = \lambda_1 \cdot e^{h^* k}$ (see Gerber and Shiu, 1994, p. 109).

Proposition 3 (Equivalent measures for compound Poisson processes)**.** *Let* $(I, \mathcal{P})$ *and* $(I, \mathcal{Q})$ *be compound Poisson processes on the filtered probability space* $(\Omega, \mathcal{F}, \mathbb{F})$ *with Lévy measures* $\nu^{\mathcal{P}}$ *and* $\nu^{\mathcal{Q}}$*.* $\mathcal{P}$ *and* $\mathcal{Q}$ *are equivalent if and only if* $\nu^{\mathcal{P}}$ *and* $\nu^{\mathcal{Q}}$ *are equivalent.*

The Radon-Nikodým derivative is, in that case, defined as:

$$\frac{d\mathcal{Q}}{d\mathcal{P}} = \exp\left(T(\lambda^{\mathcal{P}} - \lambda^{\mathcal{Q}}) + \sum_{s \leq T} \phi(\Delta I_s)\right)$$

A compound Poisson process can be easily transformed to a new measure by applying the Esscher transformation. Again considering a compound Poisson process $(I, \mathcal{P})$ with the jump measure i, given by

$$i(dz, dt) = \lambda(t) \cdot P_J(dz).$$

Here, $\lambda(t)$ represents again the jump intensity and P_J the jump size distribution under the physical measure
$mathcalP$. Now applying the Esscher transform with parameter h^* and the jump measure of $I(t)$ can be transformed under $\mathcal{Q}$ to

$$i_{\mathcal{Q}}(dz, dt) = \left(\int_{\mathbb{R}} e^{h^* z} \cdot \lambda(t) \cdot P_J(dz)\right) \cdot \frac{e^{h^* z} \cdot P_J(dz)}{\int_{\mathbb{R}} e^{h^* z} \cdot P_J(dz)}.$$

The Esscher transform changes the jump intensity and jump size when changing the measure but following Benth and Sgarra (2009, p. 10), this is again a compound Poisson process with jump intensity

$$\lambda^*(t) = \int_{\mathbb{R}} e^{h^* z} \cdot \lambda(t) \cdot P_J(dz)$$

and jump size distribution

$$P_J^*(dz) = \frac{e^{h^* z} \cdot P_J(dz)}{\int_{\mathbb{R}} e^{h^* z} \cdot P_J(dz)}.$$

5.1.2.4 Esscher transformation and risk-neutral GARCH

One important feature of the Esscher transformation is its ability to transform processes driven by a GARCH volatility. Thereby, it is possible to resemble the local risk-neutral approach, which was developed by Duan (1995). Siu et al. (2004) extend the work of Gerber and Shiu (1994) to an dynamic approach. They employ the conditional Esscher transformation concept of Bühlmann et al. (1996) instead of the local risk-neutral relationship. A detailed mathematical description of this approach is beyond the scope of this dissertation.[13]

5.1.2.5 Numerical transformation of the electricity price process

As shown in the previous section, the Esscher transformation is very convenient to find an equivalent measure for a compound Poisson process or the Brownian motion[14]. However, a measure change for stochastic process, as the one developed in section 4.3, it is more cumbersome. The problem is that it involves a measure change for the Brownian motion as well as for the compound Poisson process at the same time. According to Benth and Sgarra (2009) a measure change of an Ornstein-Uhlenbeck process requires a change in the mean level. Therefore, it is necessary to find the Radon-Nikodým derivative $\frac{d\mathcal{Q}}{d\mathcal{P}}$ that changes the measure for both processes. Benth, Benth and Koekebakker (2008, p. 97) provide general framework to find the Radon-Nikodým derivative. Especially, they show that the Radon-Nikodým derivative can be factorized in a change for Brownian motion and the compound Poisson process. Benth and Sgarra (2009) provide an analytical solution

[13] I refer the interested reader to the original work of Siu et al. (2004). For a discussion of Esscher transform in discrete finance models see Bühlmann et al. (1998).

[14] The interested reader is referred to Gerber and Shiu (1994).

for the combination of one Ornstein-Uhlenbeck process with constant variance in equation 4.6 and one Ornstein-Uhlenbeck process driven by a compound Poisson process:
Let $\theta(t)$ is a $(p+n)$-dimensional vector of real-valued continuous functions on $[0,T]$,

$$\begin{aligned}\frac{d\mathcal{Q}}{d\mathcal{P}} &= Z^{\theta}(t)\\ &= \prod_{k=1}^{p} \hat{Z}_k^{\theta}(t) \times \prod_{i=1}^{n} \tilde{Z}_j^{\theta}(t)\end{aligned}$$

where $Z^{\theta}(t)$ is the density process of the Radon-Nikodým derivative,

$$\hat{Z}_k^{\theta}(t) = \exp\left(\int_0^t \hat{\theta}_k(s)dB_k(s) - \frac{1}{2}\int_0^t \theta_k^2(s)ds\right), \ \forall k = 1,\dots,p,$$

and

$$\tilde{Z}_j^{\theta}(t) = \exp\left(\int_0^t \tilde{\theta}_j(s)dI_j(s) - \phi_j(0,t,\tilde{\theta}_j(\cdot))\right), \ \forall j = 1,\dots,n.$$

and where the functions ϕ_j are defined by:

$$\phi_j(t,s,\tilde{\theta}_j(\cdot)) := \psi_j(s,t,-i\tilde{\theta}_j(\cdot))$$

with ψ_j being the cumulant of I_j. The author refers to Benth and Sgarra (2009) for the proof.

Since stochastic volatility is employed in this dissertation it is necessary to find a solution to change the measure for those processes. Currently, there is no analytical solution available for that problem. Therefore, it is approximated by a numerical estimation.
As already shown in section 4.3, the price process $S_{\mathcal{Q}}(t)$ can be first devided in a deterministic part $\Lambda_{\mathcal{Q}}(t)$ which represents the seasonality and drift and a stochastic part $Z_{\mathcal{Q}}(t)$. *First*, the new risk-neutral yearly seasonality function is defined using equation (4.5) but setting β_2, which represents the drift term, equal to e^{r_f} under the equivalent martingale measure $\mathcal{Q}$. The model for the seasonality does not change under $\mathcal{Q}$, however, β_1 and β_3 need be estimated from futures prices (see Hambly et al., 2009; Benth, Cartea and Kiesel, 2008). β_2 has be r_f, which is the risk-free rate of return, since under the equivalent martingale measure $\mathcal{Q}$ each drift of the processes has be the risk free rate. This means,

that equation (4.5) can restated as

$$\ln sea(t) = \ln\left(\beta_1 + r_f \cdot t + \beta_3 \cdot \cos\left(\frac{2\pi(t-\beta_4)}{365}\right)\right). \tag{5.24}$$

Second, the stochastic part $Z(t)$ can again be split into the diffusion part $X(t)$ and jump part $Y(t)$. Since $X(0) = 0$ and $Y(0) = 0$ holds and the processes $X(t)$ and $Y(t)$ are independent, it is possible to derive an equation for expected value of the discounted price process under the equivalent martingale measure $\mathcal{Q}$ (see also Warmuth, 2011, p. 27). Applying this and equation (5.13), the expectation of $Z(t)$ is given by:

$$\begin{aligned} 1 &= e^{r_f \cdot t}\mathbb{E}[e^{Z(t)}] \\ &= e^{r_f \cdot t}\mathbb{E}[e^{X(t)+Y(t)}] \\ &= e^{r_f \cdot t}\mathbb{E}[e^{X(t)}] \cdot \mathbb{E}[e^{Y(t)}] \end{aligned}$$

This leads to two equations that are necessary for the Esscher transformation

$$1 = e^{r_f \cdot t}\mathbb{E}_{\mathcal{Q}}[e^{X(t)}] \tag{5.25}$$

$$1 = e^{r_f \cdot t}\mathbb{E}_{\mathcal{Q}}[e^{Y(t)}] \tag{5.26}$$

where $\mathbb{E}_{\mathcal{Q}}$ is the expectation under the equivalent martingale measure $\mathcal{Q}$. Following Benth and Sgarra (2009) it is necessary to change the mean level of the processes $X(t)$ and $Y(t)$. Therefore, two mean level correction functions are introduced, which are defined by

$$\begin{aligned} \mu_X^{\mathcal{Q}}(\cdot, t) &= \mu_X + \delta\left(\alpha_X, \sigma(t), X(t), B(t), t\right) \\ \mu_Y^{\mathcal{Q}}(\cdot, t) &= \mu_Y + \delta\left(\alpha_Y, \sigma(t), Y(t), I(t), t\right). \end{aligned}$$

To estimate these function numerically, it is necessary to reduce them. Taking into account $\mu_X = 0$ and $\mu_Y = 0$, they can be defined by:

$$\begin{aligned} \hat{\mu}_X^{\mathcal{Q}}(\sigma(t), t) &= \delta_X \sigma(t) \\ \hat{\mu}_Y^{\mathcal{Q}} &= \delta_Y \end{aligned}$$

where $\hat{\mu}_X^{\mathcal{Q}}(\sigma(t), t)$ follows the local risk-neutral approach developed by Duan (1995). Using these results, equation (4.6) and equation (4.7) can reformulated as

$$dX_{\mathcal{Q}} = (\hat{\mu}_X(t) - \alpha_X \cdot X(t))\, dt \qquad +\sigma(t)dB(t) \tag{5.27}$$

$$dY_{\mathcal{Q}} = (\hat{\mu}_Y - \alpha_Y \cdot Y(t))\, dt \qquad +dI(t). \tag{5.28}$$

Combining equation (5.27) and equation (5.25), it is possible to derive a solution for $\hat{\mu}_X(t)$, and combining equation (5.28) and equation (5.26), the same is possible for $\hat{\mu}_Y$. Now the risk-neutral price process under the martingale measure $\mathcal{Q}$ is defined by:

$$S_{\mathcal{Q}}(t) = S(0) + \Lambda_{\mathcal{Q}}(t) \cdot e^{X_{\mathcal{Q}}(t)+Y_{\mathcal{Q}}(t)}. \tag{5.29}$$

5.1.3 Important input factors and switching algorithm

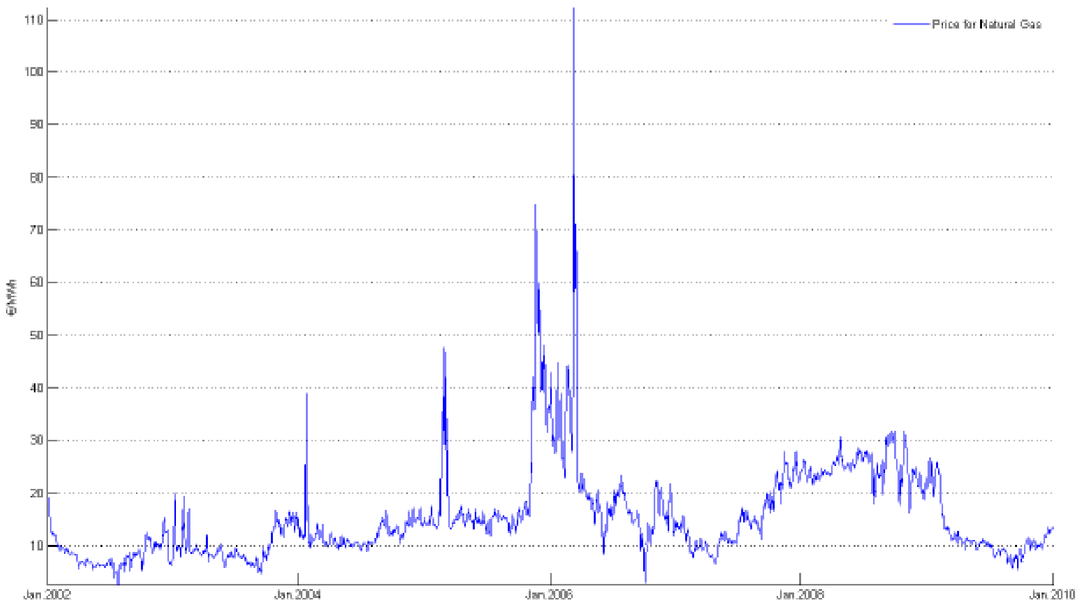
5.1.3.1 Gas Prices

Natural gas counts to the energy related products. Gas markets have more in common with electricity markets, than with classical commodity markets. The reason is again the storage problem of this product. Natural gas can be stored better[15] than electricity, but only with high effort, which results in high storage costs. Hence, a similar situation prevails in gas markets. Jumps in gas prices have assumable the same cause as spikes in electricity prices. For illustration see figure 5.1[16].

[15] Gas can be stored in sole caverns and in big tanks (gasometer).

[16] As mentioned before, natural gas trading started 2007 at EEX. To analyze a time series and use it as a basis to build models, a longer databases is needed. The competition in gas markets started earlier in UK. Hence converted UK data were taken.

Figure 5.1: Natural Gas Chart since January 2002.



Source: Own work.

The passing of the Energiewirtschaftsgesetz (EnWG) focused the gas market in Germany, which opened this market to competition. Since 2007, natural gas can be traded at EEX. Delivering takes place in market areas operated by Gaspool and NetConnect Germany (NCG).

As well as electricity, natural gas is traded "Day Ahead" on the spot market. Up to 2009, it was only possible to buy a baseload for one day. The delivery started at 6.00 a.m. and finished 6.00 a.m. on the following day[17]. The delivered quantity was 24 MWh per day. This trade is comparable with continuous trades in the electricity markets (block bids). Now it is also possible to place an order with a minimum quantity of 1 MW in auction trades (EEX, 2007*a*, p. 17).

5.1.3.2 CO_2 Emission Rights

Beside electricity and natural gas, market participants can also trade CO_2 Emission Rights at EEX. This section comprises not only the trade of emission rights, but also the reason for this. Furthermore, future aspects and developments will be discussed.

[17]See Konstantin (2009, p. 51f) and EEX (2007*a*, p. 17)

The *United Nations Framework Convention on Climate Change* (UNFCCC) called for reducing greenhouse gas emissions on the 3^{rd} World Climate Change Summit in 1997. 160 nations and economic organizations agreed to comply target values regarding the emission of greenhouse gases by signing the Kyoto Protocol (EEX, 2007*a*, p. 21). According to the Kyoto Protocol, the main reason for global warming are greenhouse gases like carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and Sulphur hexafluoride (SF_6). For example, industrialized nations committed to reduce their emissions by, on average, 5.2% for the period 2008 - 2012. Basis for the calculation is the level of 1990. Germany has to reduce their carbon footprint by 21%.
To achieve the goals, UNFCCC gives three tools to participating countries:

- Emissions Trading
- Joint Implementation
- Clean Development Mechanism

Whereas the latter two are tools to reduce the emission of greenhouse gases in foreign countries, emissions trading is one of the main instruments to lower the domestic CO_2 output of participating nations. The idea is to reduce emissions, which are economically reasonable to diminish. Every trading-period, the government allocates a fixed amount of emission rights[18] to participating companies. The amount of certificates is decreased over time, and thereby, companies incentivized to modernize. In the first phase (2005 - 2007), the companies got rights for free. By now, the government will sell the emission rights for the next period by auction. Companies will scrutinize, if it is cheaper to reduce their carbon footprint by modernization or to buy further emission rights at the exchange to justify their output. This procedure gives incentives to companies, which can renew their technology with less financial effort. In future, nations will decrease their upper limit of emission rights. This method is called *cap and trade*. Eventually - according to theoretical knowledge - the costs for CO_2 emission rights at the exchange will be higher than the modernization measures. Hence, every participating company has finally to decide, whether is it cheaper to buy additional emission rights or bring their technology to the latest state of the art. As mentioned before, the first phase began in 2005 and ended in

[18] Only companies with emission rights are allowed to emit a particular amount of greenhouse gas for a particular period of time. The price refers to one ton. If a company emit more tons than allowed, then fines has to be paid.

2007. This period was regarded as a *test period*. At this time, companies got the emission rights for free. Considering the end of the phase (see figure 5.2) someone can assume that the government equipped the companies with too much emission rights: At the end of the first phase, the price was close to zero, which means that no trade was necessary. Another reason for this effect could be that a transfer of emission rights to the subsequent period is not possible either.
Nevertheless, the upper limit of emission rights was decreased by the government. Continuous decreasing in the further periods will lead to a long-term reduction of greenhouse gas emission.

Figure 5.2: Cost of emission rights in the "test period" (2005 - 2007) and the second period (since 2008)

Source: Own work.

To evaluate power plants realistically, the costs for CO_2 Emission Rights (see section 5.1.3.2) should taken into account. The EU Emission Trading Scheme constitutes the framework for trading in emission rights. EEX offers Continuous trading as well as daily auctions of so called EUA (= EU Allowances). The EEX carbon index (Carbix®) serves as a reference price for EUA. Every exchange trading day at 11.00 a.m., the price for the index is determined by auction trade.

Here it does not make sense to apply an econometric model for two main reasons. *Firstly*, there are two periods to scrutinize. Due to the fact that the first period (2005 - 2007) is considered as a pilot phase, the data are not significant. The second period is still running. Hence, there is no data available, which shows how prices perform in the end of a "no-pilot" phase. Moreover, the length of meaningful data is to short. Thus, a prediction which comprises the run-time of a power plant is not possible. *Secondly*, the government has a significant influence on EUA prices - caused by the cap and trade approach. Therefore, an econometric model based on historical prices cannot predict future policy at all. Nevertheless, the IPCC[19] (= International Panel on Climate Change) illustrates in their Fourth Assessment Report that the following factors influence the future emission of greenhouse gases (see Int (2007*a*, section 3.2.1)):

- demographic developments
- socio-economic developments
- technological and institutional change

There is a variety of future scenarios on CO_2 emission available[20]. IPCC restricts the choice on six classes. To evaluate the effects on climate change, IPCC regards the link between various stabilization levels for concentrations of greenhouse gases in the atmosphere and the global mean temperature change comparative to a particular baseline (Int, 2007*b*, section 3.5.2).

A description of all scenarios can be found in table 5.1. Class I is the most stringent, i.e. this class of scenarios could limit global mean temperature increases to 2° C - 2.4° C above pre-industrial levels. Class VI - with least effort to fight against climate change - regards an increase of 4.9° C - 6.1° C (see table 5.1). According to IPCC, scenario classes III and IV are most probable[21]. Therefore, only price ranges for these classes are considered.

[19] IPCC is an intergovernmental committee which evaluates the effects of climate change caused by human activity.

[20] In the literature are more than 800 emission scenarios. Int (2007*a*, section 3.2.1)

[21] IPCC did the most studies (see (Int, 2007*b*, Table 3.10 column: "number of scenario studies")) for classes III and IV.

Table 5.1: Classification of stabilization scenarios according to different stabilization targets.

Class	**CO_2 concentration in ppm**	**CO_2-eq concentration in ppm**	**Peaking year for CO_2 emissions**	**Global mean temperature increase in ° C**	**Change in global emission in 2050 (in % of 2000 emissions)**
I	350 - 400	445 - 490	2000 - 2015	2.0 - 2.4	-85 to -50
II	400 - 440	490 - 535	2000 - 2020	2.4 - 2.8	-60 to -30
III	440 - 485	535 - 590	2010 - 2030	2.8 - 3.2	-30 to +5
IV	485 - 570	590 - 710	2020 - 2060	3.2 - 4.0	+10 to +60
V	570 - 660	710 - 855	2050 - 2080	4.0 - 4.9	+25 to +85
VI	660 - 790	855 - 1130	2060 - 2090	4.9 - 6.1	+90 to +140

Source: Own work based on (Int, 2007*b*, Table 3.5 and Table 3.10)

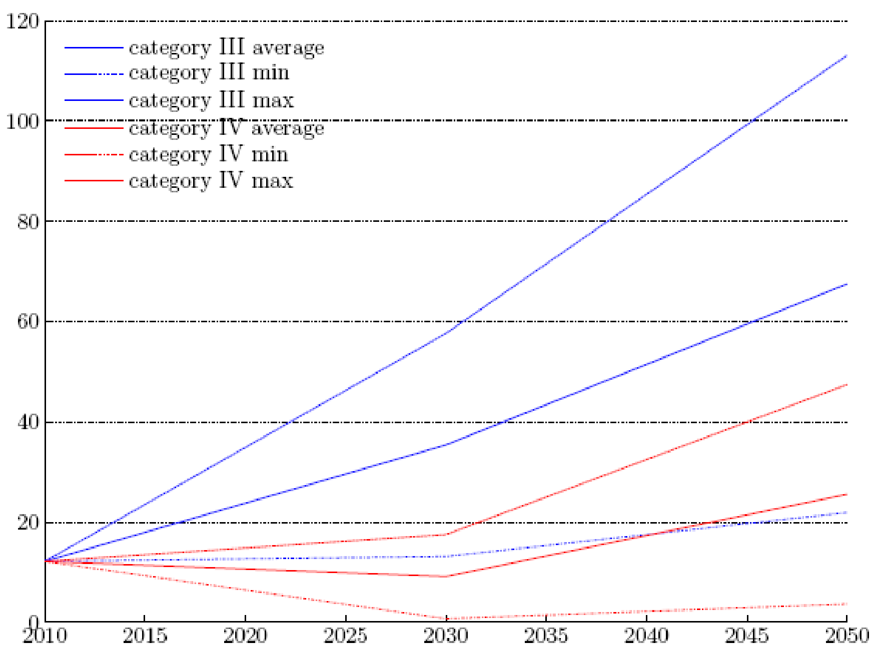
To reach a stabilization in CO_2-eq concentration of 590 - 710 ppm (category IV) , IPCC assumes prices of 1 - 24 USD/tCO_2 in 2030 and prices of 5 - 65 USD/tCO_2 in 2050. For a stabilization target of 535 - 590 ppm in CO_2-eq (category III), the calculated prices lie in a range of 18 - 79 USD/tCO_2 in 2030 and 30 - 155 USD/tCO_2 in 2050.
To use this estimate for power plant evaluation, data points are derived by linear interpolation (see figure 5.3 for a graphical illustration). An exchange rate of 1.37 USD/EUR is assumed, because prices given in USD, which corresponds to the average rate in 2007.
For every class, three versions (minimum (min), average (avg) and maximum (max)) are considered. Table 5.2 presents the estimated CO_2 emission costs for 2030 and 2050.

Table 5.2: Expected emission costs for 2030 and 2050

Versions	0	1	2	3	4	5	6
IPCC class		category III			category IV		
		min	avg	max	min	avg	max
estimated prices in 2030 in USD/tCO_2	0	18	48.5	79	1	12.5	24
estimated prices in 2030 in EUR/tCO_2	0	13.14	35.40	57.66	0.73	9.12	17.52
estimated prices in 2050 in USD/tCO_2	0	30	92.5	155	5	35	65
estimated prices in 2050 in EUR/tCO_2	0	21.90	67.52	113.14	3.65	25.55	47.45

Source: Own work based on (Int, 2007*b*, Section 3.3.5.3).

Figure 5.3: Expected prices of CO_2 emission rights (six versions) for 2050



Source: Own work.

5.1.3.3 Switching Algorithm

According to (Geman, 2005, p. 288), the spread between two different commodities is one of the most traded instruments, especially in energy commodities. The so called spread options are used to value power plants, mines, oil refineries, storage facilities, and transmission lines. Basically, all physical assets in the energy industry.

Spread options between two commodities S_1 and S_2 have the same pay-off as a call option, except that the underlying is the difference $S_1 - h * S_2$, where h is the quantity, and the strike price k is non-zero.

Definition 18. *The pay-off of a spread option can be defined as follows*

$$SS(T) = max(0, S_1(T) - h * S_2 - k) \tag{5.30}$$

where h is a strictly positive constant and k is non-zero.

It is worth noting that no closed-form solution for such an option exists. The price has to be determined by simulating the stochastic processes of S_1 and S_2.

Following (Geman, 2005, p. 302) and (Eydeland and Wolyniec, 2003, p. 396), a switching option is a strip of spread options when there are no cost to switch the power plant on or off. The value of investment asset is simply the $V = \sum_{t=1}^{T} \frac{SS(t)}{(1+r_f)^t}$. In the case where there are cost to switch the power plant on or off, it is necessary to use a stochastic dynamic optimization approach. According to (Eydeland and Wolyniec, 2003, p. 463ff), the value of an investment asset is then defined as follows

$$\begin{aligned} V = \max_{q(t), U_t^{onoff}} \Big\{ & q_t * SS(t) - U_t^{onoff} * SC \\ & + e^{-r\Delta t} \mathbb{E} V[S_1(t+1), S_2(t+1), U_{(t+1)}^{onoff}, t+1] \Big\} \end{aligned} \tag{5.31}$$

This procedure was implemented in MATLAB. Figure 2 gives the pseudo-code how the switching method works.

Algorithm 2 Switching Algorithm

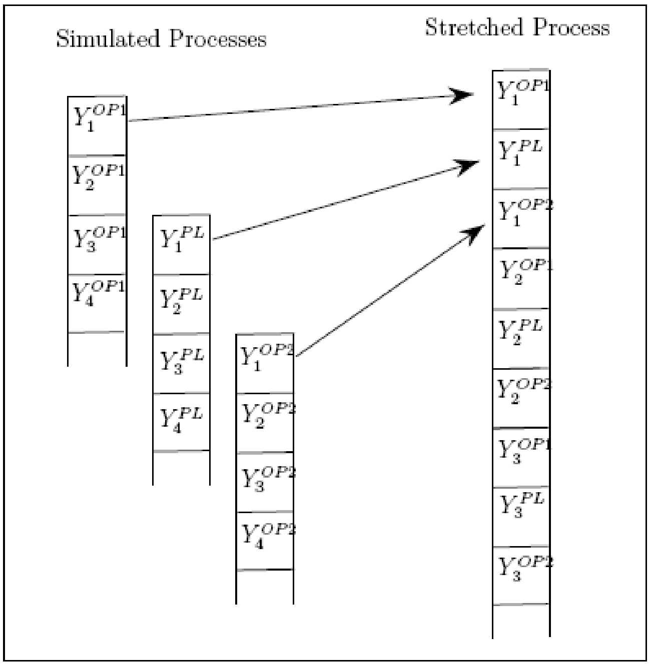
```
for j = 1 to size(Cash Flow,1) do
  switch onoff
  case 0
  if Cash Flow(j) > oncost then
    Free Cash Flow(j) = Cash Flow(j) - oncost
    onoff=1
  else
    Free Cash Flow(j)=0
  end if
  case 1
  if Cash Flow(j) < offcost then
    Free Cash Flow(j) = - offcost
    onoff=0
  else
    Free Cash Flow(j)= Cash Flow(j)
  end if
  end
end for
```

Source: Own work based on Schuster (2011).

Cash Flow is the component-by-component product of the *Stretched Process* (from figure 5.4)[22] times the *installed capacity* of the power plant for this time interval. In the pseudo code, emission allowances are neglected.

[22] OP1 equals off-peak 1, PL equals peakload, and OP2 equals off-peak 2.

Figure 5.4: Technique, how the simulated processes stacked over each other.



Source: Own work based on Schuster (2011).

- *oncost* is the amount of money to spend, to turn the power plant on.
- *offcost* is the amount of money to spend, to turn the power plant off.
- *onoff* is the indicator, if the power plant is switched on (=1) or off (=0).

For an application of that algorithm see section 5.3.3 or Schuster (2011).

5.2 Theoretical Framework of the analyses

5.2.1 Assumptions regarding the capital market and transformation of stochastic processes

5.2.1.1 Capital market

The assumption in this dissertation is that a capital market with m assets exists. The assets are electricity, different sorts of commodities, interest rates and the EURUSD fx rate (section 5.2.2). All assets follow defined stochastic processes. In addition, it is assumed that the capital market is arbitrage free. Definition 6 implies that in that case at least one equivalent martingale measure exists. See section 3.1.4, (Shreve, 2004*b*, p. 231), and Bingham and Kiesel (2001) for detailed discussion and proof of this assumption. According Harrison and Kreps (1979), the value of any derivative is the expected present value under the risk-neutral measure (see definition 4 in 3.1.4.1).

In this dissertation, it is assumed that electricity follows the process defined in chapter 4. All other assets are assumed to follow an Ornstein-Uhlenbeck process. This implies all other assets follow a specific version of the electricity price process. The jump part $Y(t)$ is assumed to be constantly zero. This also means it does not need to be taken into account in the martingale transformation.

5.2.1.2 Martingale transformation

The equity value of a company is the present value of the expected free cash flow to equity (see section 3.2). The free cash flow to equity is derived by combining the primary assets such electricity, commodities, and interest rates. The risk neutral transformation for valuing project-financed investments uses a specific property of martingales. Theorem 5 and 1 show that a portfolio of martingales remains a martingale.under risk-neutral measure.[23] The net present value is a linear combination of the portfolio at time 0 and a gains process. The gains process itself is sum of the discounted cash flows. The cash flows are a combination or portfolio of the different stochastic processes, where each process has certain weighting. Since each of these processes is a martingale, the gains process is also martingale, and thereby, the net present value is a martingale. Therefore, it is sufficient to only transform the primary assets into equivalent martingales for the risk-neutral valuation of a company or project.

[23]The author refers to 3.1.4.1 for a detailed introduction of this and (Bingham and Kiesel, 2001), (Cont and Tankov, 2003, p. 302), and (Shreve, 2004*b*, p.376ff) for proofs.

All process are transformed into equivalent martingales using the Esscher-transformation. According to Gerber and Shiu (1994), the Esscher-transformation can be used to transform independent increment processes. This class of process contains the Wiener-, the Poisson-, the compound Poisson-, the Gamma- and the inverse Gaussian-process. Benth and Sgarra (2009) show that the transformation of an Ornstein-Uhlenbeck process requires a change in the mean. Benth, Benth and Koekebakker (2008, p. 97) prove that the Radon-Nikodým derivative of the processes used in this dissertation can be factorised. This allows analysing the normal and the jump part separately. The required to steps to transform a process using the Esscher-transform are according to Gerber and Shiu (1994):

1. Simulation of the processes, to obtain the drift of each process taking into account the correlation structure and jumps.
2. Fitting the mean level correction function.
3. Repeat steps 1. and 2. until all process are martingales.

The following steps are necessary transformation the processes into martingales and the risk-neutral valuation of a project or company:

1. Definition of the project parameters.
2. Read in the project parameters and time series.
3. Estimate from the time series the required parameter of the stochastic processes, such as mean reversion rate, mean, variance, correlation.
4. Transform the processes according to the procedure described above.
5. Simulation of the primary assets
6. Calculation of the cash flows, probability of default and the net present value distribution
7. Print the results

5.2.2 Evaluated specific project

The ROPFVT is applied to calculate the profitability of a project financed gas power plant. As mentioned beforehand, the author takes the equity provider's point of view. A power plant is chosen based on two main considerations: First, investments in the energy sector are characterized by high initial capital expenditures, often reaching the range of billions. Hence, it is not surprising that project finance is a common financing method in this sector. Second, data availability for power plants is excellent since the author can rely on w.A. (2010) for crucial variables determining the value of a power plant.

The author assumes that investors taking an equity stake in project finance typically have a limited investment horizon of three to seven years.[24] By contrast, power plant projects have a run-time of 20 years and more. As a consequence, an equity investor must sell his stake in the project during its lifetime. In order to do this, it is crucial for the investor to be able to determine the value of his stake at any point of time. The ROPFVT can simulate the project over its whole run-time. From an economic point of view, a simulation over such a long time period is not meaningful. Hence, the project is modeled over a 10 year horizon and assumed that the whole project is sold afterward.

The gas-fired power plant is operated as a base load power plant in Germany and all money is invested - and the project's construction takes place - in one point of time. The author believes this is a minor issue since gas-fired power plants are typically built within two to three years. w.A. (2010). After the initial capital expenditure, there are two main categories of relevant costs for the power plant project: (i) fixed costs: e.g. salaries, insurances and (ii) fuel costs: costs of gas and emission rights. The computation of the amount of electricity produced is not straightforward since several factors have to be taken into consideration: capacity, load factor, operating time and efficiency. To ease matters, the author assumes that there is no technical improvement over the project's lifetime and that the efficiency remains constant.

[24] This assumption is based on discussions with project finance investors.

The most important influencing factors on the project's cash flows are the fuel cost (gas cost per MWh of electricity output), the costs for emissions rights (EUA[25] costs per MWh of electricity output) and the electricity price, because they largely determine the profitability of the power plant.

The financial structure in the case study corresponds to a typical project financed power plant. According to w.A. (2010), the overnight construction costs of a gas fired power plant are in the range between 520 and 1800 USD/KWe. As initial investment for the build-up phase of the power plant, a capital requirement of 0.1 billion is assumed.[26] The run-time of the power plant is estimated to be 30 years, with total depreciation taking place over 20 years. Further, the author assumes that residual value equals the decommission costs as in w.A. (2010). The debt-to-equity ratio at the beginning of the project is 2 and the debt has a maturity of 20 years. Furthermore, the author assumes that one third of the debt is provided in U.S. Dollar and two thirds are provided in Euro. The whole free cash flow to equity is immediately distributed among the equity providers. An event of default occurs in general when the free cash flow to equity becomes negative, implying that equity providers must re-invest cash in the project. Since this assumption is very restrictive and also not meaningful from an economic point of view, it is relaxed by assuming that equity providers have the obligation to re-invest money in the project until a threshold of one third of their initial investments.[27] If re-financing needs are larger than this threshold, an event of default occurs. As already mentioned in section 3, the capital structure of the project and its cost of capital are permanently recalculated and adjusted over every simulation period.

Regarding the technical parameters of the power plant, this dissertation relies on w.A. (2010), and applies an electricity generation capacity of 200 MWe, a load fac-

[25] European Allowances

[26] This includes pre-construction, construction and contingency cost, but not interest cost.

[27] In reality debt providers can force equity providers to accumulate free cash flows on an escrow account up to a certain threshold. If this accumulation of funds is assumed to be NPV-neutral and the threshold is equal to one third of the initial investment, this comes very close to the assumption used in the calculations. A different outcome would only arise if a negative free cash flow to equity is realized before the escrow account is totally filled up and the re-financing needs are larger than the actual accumulated funds.

tor of 85%, and an availability of 95%. Operating and maintenance costs are 5% of the revenue. Fixed costs are assumed to have a magnitude of €5 million per year, including labor costs. For the relation between electricity output and CO_2 input, the author assumes that 0.33 emission rights are necessary for every generated MWh of electricity. Following w.A. (2010), the author uses a fixed price of USD 30 per ton of CO2. Additionally, the thermal efficiency for the generation process is set to 38%.

The starting point of the analysis is June 2010 (due to data availability). As mentioned before, the first 10 years of the project's run-time are modeled. After that time, the project is sold at a certain price, which is calculated with help of a multiple based valuation approach. As common for power plant valuations, the value of the power plant in year 10 is calculated as two times the average of its last three annualized free cash flows to equity.

To estimate the effect of model complexity on the valuation outcome, the project's cash flows for every future point of time are calculated. The authors uses time-series forecasting models for the four cash flow risk factors: electricity price, gas price, interest rates and the USD/EUR exchange rate. All other risk factors, such as salaries or insurances, are assumed to be constant for the future. Although this procedure is not meaningful if the net present value of the project is of interest, the author argues that these risk factors, which can hardly be forecasted with the help of time series models, are of minor importance for the research questions, namely the impact of model complexity on the valuation outcome. In a similar vein, the author argues that it is of minor importance for his research question to consider all risk factors that are important for the absolute value of the project. For example, the inclusion of construction of political risk would change the absolute value of the project, but not the deltas between different forecasting and simulation techniques. Consequently, the results may not be interpreted in terms of absolute project values, but only with regard to these differences.

The last aspect necessary to address is that the return forecasting model are not varied since their impact on the project value is obvious. For the gas price, the interest rates and the U.S. Dollar / Euro exchange rate, an Ornstein-Uhlenbeck model is always applied, as in Vasicek (1977). For the electricity price, the model, proposed by Benth et al. (2003) and Mayer et al. (2011), is applied.[28] The processes are transformed to their risk-neutral measure following Benth and Sgarra (2009) and Gerber and Shiu (1994). The power plant operates 24 hours 7 days a week (24/7). The European Energy Exchange (EEX) electricity (baseload day-ahead) spot price is used as reference price for electricity.

5.2.3 Evaluated generic projects

This dissertation evaluates equity investments in project financings. For this, the author (i) evaluates different power plant and mining projects to ensure that the results do not depend on the specific structure of one single project and (ii) focuses on financial risk factors. The author focuses on the financial risk factors of the project and ignore others, which cannot be forecasted and simulated by mathematical models. Examples in this context are political or technological risk. However, their exclusion does not bias the results, since only the differences between the outcomes are analyzed. Factors that are constant for different levels of model complexity, such as political risk, lead to a shift in all distributions but do not affect their differences.

The first class of projects is power plants. In these projects, the author considers one output factor, i.e. electricity[29], and several different input factors. These input factors are coal, gas and different types of oil.[30] The second class of projects is mines. They produce different outputs, i.e. gold, silver, platinum, aluminum, copper, nickel, zinc or cobalt, and have one input factor, namely WTI oil. Furthermore, the factors can be in local currency (LC) or foreign currency (FC) or a combination of both[31]. Concerning the financing side of the projects, interest rates can be either (i) fixed

[28] A thorough discussion of different mean models can be found in Geman (2005) or Weron (2006).

[29] The German baseload electricity price, as calculated by the European Energy Exchange (EEX), is used.

[30] To be more specific, the author uses API2 coal, henry hub natural gas, heating oil, Brent oil and WTI oil.

[31] If local and foreign currencies are considered, the dissertation assumes an equal distribution of both.

in local currency, (ii) fixed in foreign currency, (iii) floating in local currency or (iv) floating in foreign currency. The hypothetical projects are a combination of all these factors with the limitation that they have at most three simultaneous input (power plant) factors or output (mine) factors. In total, this dissertation ends up with about 1,000 different projects, which have different input and output factors, foreign exchange and interest rate risks.

The following assumptions are used for the hypothetical projects: (i) All projects have an initial investment of 200 million Euro and are debt financed to 50%. The maturity of the debt is 20 years. Construction time is zero, i.e. both the initial investment and the project start take place at t=0.[32] (ii) The hypothetical annual sales of the project at the beginning (t=0) are 100 million Euro. Hence, the amount of output(s) is adjusted to this sales dependent on the last available output price. In case of more than one output factor, all factors contribute equally to the generated sales. (iii) The profitability of the project at the beginning is normalized at 10% of the total sales. For this, the amounts of inputs necessary to generate the output are adjusted dependent on the last available input price(s). In case of more than one input factor, the weight of all factors is equal. (iv) The run-time of the project is 10 years. After that, the project is sold for a certain price, which is calculated with help of a multiple valuation approach.[33] (v) The whole free cash flow to equity is immediately distributed among the equity providers. (vi) An event of default occurs, in general, when the free cash flow to equity becomes negative, implying that equity providers must re-invest cash in the project. Since this assumption is very restrictive and also not meaningful from an economic point of view, it is relaxed by assuming that equity providers have the obligation to re-invest money in the project until a threshold of one third of their initial investments. (vii) It is assumed that no taxes or transaction costs exist.

Of course, one might argue that (at least some) of these assumptions would not

[32] Of course, real life project require subsequent sub-investments until the whole project is completed (Mölls and Schild (2011)).

[33] In particular, it is assumed that the selling price is two times the average of its last three annualized free cash flows to equity.

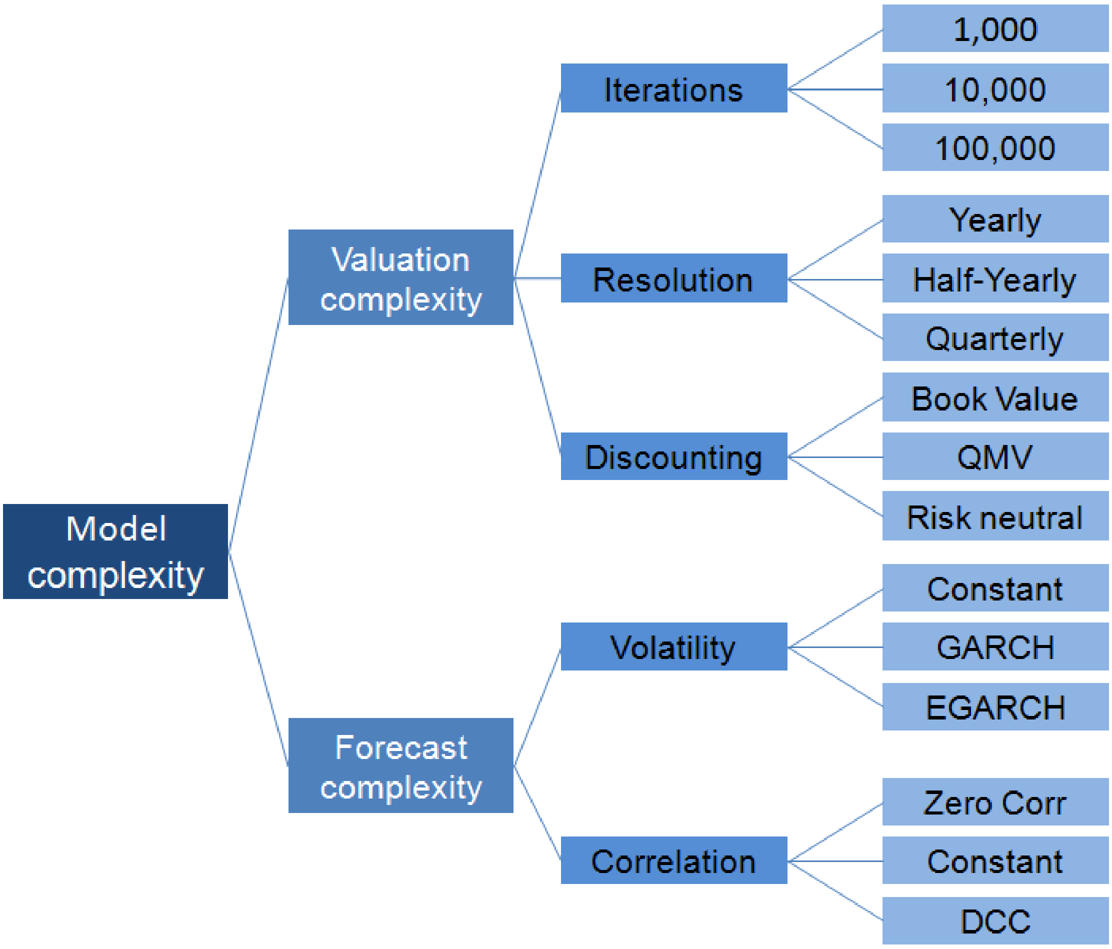
be used for the valuation of real-life investments in project finance. However, the author is only interested in the effect of model complexity on valuation outcome and not on the valuation outcome per se. Since all assumptions are used for all different projects and levels of model complexity, the author argues that they do not bias the results.

5.2.4 Model complexity

The effect of model complexity on the valuation results is the key element of this dissertation. In the following, the author divides model complexity in two components: (i) the complexity of the valuation procedure and (ii) the complexity of the forecast models. The complexity of the valuation procedure is defined along three dimensions: the number of iterations, the time-resolution (which defines how often the cash flow is analyzed), and the cost of capital calculation method. In order to explore their effects on the simulation results, the author varies these three components in several dimensions. For forecast complexity, the author varies the volatility and the correlation forecast models. For volatility forecasts, the author uses (i) historical volatility and forecasts obtained from a (ii) GARCH(1,1),as well as an (iii) E-GARCH. For correlations, the author applies (i) no correlations, (ii) historical correlations and (iii) correlations obtained from a DCC model. Figure 5.5 summarizes the different variations of the model complexity. In the following sections, the simulation results obtained with the ROPFVT are presented. The effect of model complexity is analyzed on the NPV distribution, the value-of-risk (VAR), and the (cumulative) default probability of the project.

For *forecasting complexity*, the volatility and correlation forecasting model is varied. The return forecasting method is not changed since its influence on the valuation outcome is obvious. Considered volatility forecasting models (beneath the base case of constant volatility) are GARCH (Bollerslev, 1986) and its derivative E-GARCH (Nelson, 1991). In terms of the correlation forecasting model, the author (i) extends

Figure 5.5: Variations of model complexity



Source: Own work based on Weber et al. (2010).

the base case of constant correlation to DCC (Sheppard and Engle, 2001)[34] as an advanced forecasting method and (ii) assumes that correlation is neglected (i.e. the correlation is set to zero). The case of no correlation is the most simple method to handle this factor and not uncommon in practice.

For *valuation complexity*, the impact of the number of simulations, the time resolution, and the discount rate calculation on the valuation outcome is analyzed. The considered numbers of simulation are 1,000, 10,000, 50,000, and the base case, i.e. 100,000. Time resolution can be quarterly, half-yearly, or yearly. By time resolution, the frequency of calculating the cash flow is meant. This is of importance

[34] The author uses the Matlab implementation of DCC as published on the homepage of Kevin Sheppard (`http://www.kevinsheppard.com`).

since events of default can only occur at these times. However, the time resolution does not change the NPV directly. Hence, this dissertation only investigates its impact on the expected default frequency of the project, but not on its NPV. As methods to calculate the discount rate, the author considers a simple model, which does not adjust the cost of equity (CoE) to the project risk, quasi-market valuation (QMV)[35] with risk adjusted cost of equity, and - as most complex method - risk-neutral valuation.

To analyze the impact of model complexity on the valuation outcome, a complexity order is defined for each aspect: In this context, it is assumed that more complex models are more difficult to implement in a valuation model in practice.[36]

- Volatility model: Constant volatility → GARCH → E-GARCH
- Correlation model: No correlation → constant correlation → DCC
- Simulation iterations: 1,000 → 10,000 → 50,000 → 100,000
- Time resolution: Yearly → half-yearly → quarterly
- Discount rate: Risk unadjusted CoE → risk adjusted CoE (QMV) → risk-neutral valuation

For all simulations, a base case is defined, which consists of (i) constant volatility, (ii) constant correlation, (iii) 100,000 Monte Carlo iterations, (iv) half-yearly cash flow, and (v) risk unadjusted cost of equity. For the forecasting of the returns, a mean-reversion model is used.[37] Starting from this base case, the elements of the model are varied along two dimensions: forecasting and valuation complexity. The next section describes how the impact of complexity is evaluated.

[35] QMV is described in section 5.1 and in more detail in Esty (1999).

[36] For example, the implementation of a DCC correlation model is more complex as using (simple) constant correlation. Furthermore, less complex models in this order can be realized in Microsoft Excel, whereas more complex models need software programs that are more powerful in terms of functional range and computational efficiency.

[37] In particular, an Ornstein-Uhlenbeck model is applied. For the electricity price, positive and negative jumps and different rates of mean reversion for jumps and "normal" periods (cf. for example Benth et al. (2003) and section 4.3) are included.

5.2.5 Evaluation methodology

To evaluate if a specific method has an impact on the valuation results, this dissertation compares the valuation outcome calculated by this method to that of another method. The valuation outcome is measured along three dimensions: the NPV distribution, the expected default frequency and the 5% Value-at-Risk of the project. In this context it is important to note that the author is interested in the shape of the NPV distribution, not in its absolute values as e.g. the mean or median. This has two major reasons. First, distributions are of huge importance in practice (e.g. cash flow or value-at-risk). Second, the simple comparison of mean values has less informative value as a comparison of the distributions.

For the analysis, the author always compares the valuation outcome of two methods (e.g. GARCH and constant volatility) for the same project. *First*, the author evaluates if this pair of *NPV distributions* differs significantly. For this purpose, the author uses three different statistical tests: The Ansari-Bradley, the Kolmogorov-Smirnow and the F-test. The Ansari-Bradley test, which is used as the main test, basically compares the null hypothesis that two distribution functions are identical against the alternative hypothesis that they have the same median and shape, but different dispersions (Ansari and Bradley, 1960). Hence, all obtained NPV distributions are normalized in a way that their median equals zero. The Kolmogorov-Smirnow and the F-test are used as alternative methods to evaluate whether the distributions of the NPVs differ or not. This procedure is repeated for all different projects. Hence, the author obtains numerous different pairs of NPV distributions for the same projects. For each of these pairs, it is calculated if they are statistically different or not. The tables, which are described in the next sections, indicate the percentage of different NPV distribution pairs with a confidence level of 90% percent.

Second, the author compares the calculated *expected default probability* (EDF) between two methods. In this context, the author first calculates the EDF for the same project, but based on two different methods. This allows me to measure the absolute difference of the EDF for the two methods. Again, this procedure is repeated for all different projects. Based on these about 1,000 absolute differences in terms of EDF, the mean difference of the EDF are calculated and a simple t-test is used to evaluate if this difference is significantly different from zero.

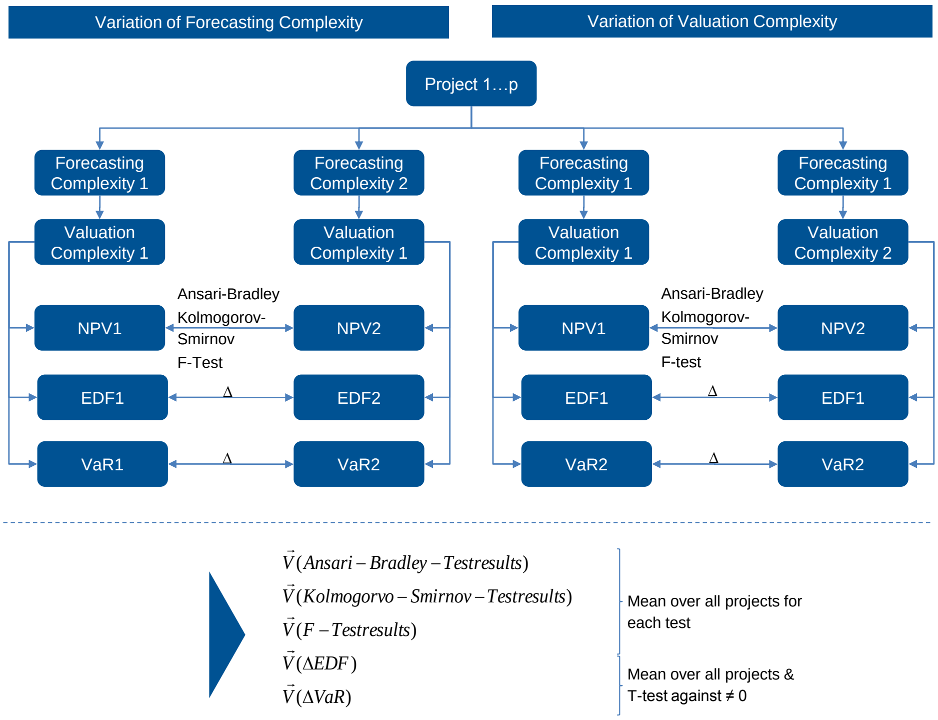
Third, the author compares the 5% *value-at-risk* (VAR) calculated by the two different methods.[38] The procedure is the same as for the EDF except that the author does not report the mean absolute difference in VAR, but the mean difference in VAR scaled by the initial investment. Hence, a value of 5% means that - on average - the VAR differs by 0.05 times the initial investment (in the case with an initial investment of 200 million Euro this difference equals 10 Million Euro) between the two different methods. This test allows to evaluate the economic impact of differences in the NPV distribution.

The author always starts by analyzing the first two methods according to the complexity order. After the analysis of these two (e.g. constant volatility and GARCH), the less complex (i.e. constant volatility) is replaced by the next method in the complexity order (e.g. E-GARCH). This procedure is repeated until the two most complex methods in one complexity aspect have been analyzed. After that, the author turns to the next complexity aspect, e.g. correlation models and start from the beginning.[39]

[38]The VAR is calculated based on the standardized (median equal zero) NPV distribution since the author is mainly interested in the tail properties of the distribution.

[39]In total, over 10,000 NPV distributions and 1,000,000 cash flow distributions are calculated for different projects and levels of model complexity.

Figure 5.6: Variation of Model complexity



Source: Own work based on Schmid and Weber (2011*a*).

5.3 Results

As explained before, the ROPFVT model is used to evaluate about 1,000 different projects (power plants and mines), all with different output and input factor combinations, different exchange rates and interest rate risks. As the focus is on financial risk factors that can be forecasted by mathematical methods, time-series data is obtained for all possible input and output factors of the projects.[40] The valuation model performs several steps: First, all financial risk factors are forecasted (i.e. their means, volatility and correlation structure). Second, these forecasts are used as inputs in a Monte Carlo simulation which generates future cash flow paths.

[40] In particular, I need the time series for gold, silver, platinum, aluminum, copper, nickel, zinc, cobalt, coal API2, heating oil, WTI oil, Brent oil, baseload electricity price for Germany (EEX), henry hub natural gas, EURIBOR, LIBOR and EUR to USD exchange rate. All these time series are on a daily basis and range from 28.09.2001 to 21.06.2010.

All future cash flow paths of one simulation are used to calculate the NPV and the expected default frequency in a third step. The calculation of the NPV is based on the discounted cash flow method (DCF), whereby the equity instead of the entity approach is applied.[41] As last step, the NPV and event of default obtained from each simulation path are combined to generate the NPV distribution and the expected default frequency of the single project. This procedure is applied for all different projects, which are described below, and for the different levels of model complexity. Hence, the model generates a NPV distribution and an expected default frequency for all projects and all levels of model complexity. Comparing these results with statistical tests for differences allows one to investigate if different levels of complexity lead to different valuation outcomes. The valuation results are presented in two sub-sections. In the case of the specific gas-fired power plant, the author describes the effect of forecast complexity on the results in the first sub-section and the effect of valuation complexity in the second. Thereby, the project's NPV cumulative distribution function (CDF) and - if meaningful - its expected default frequency (EDF) are analyzed. The CDF graphs display the NPV on the x-axis and the cumulative probability on the y-axis. The EDF graphs show the time since project start on the x-axis and the cumulative expected default on the y-axis. For the general projects the results of my analysis of the impact of model complexity on the valuation outcome are measured by the NPV distribution, the expected default frequency, and the value-at-risk.

In each step, one parameter will be changed, thereby increasing the model complexity step by step. The author compares the results afterwards. As argued before, the author does not judge which model is "best". The only question the author is interested in is if a specific method changes the valuation outcome significantly. If this is the case, practitioners should carefully evaluate which method is most ap-

[41] The entity approach requires that all cash flows to the firm are discounted with the cost of capital of the firm, i.e. the weighted average cost of capital (WACC). By contrast, the equity approach considers only cash flows available for equity providers, which are discounted with the cost of equity.

propriate for their purpose (e.g. which model for volatility is most appropriate for electricity prices). The tables show the results for each method, separated for the number of input/output factors involved, the foreign exchange and the interest rate risk. The author reports the Ansari-Bradley test statistics (see (Ansari and Bradley, 1960)), the f-test and Kolmogoriv-Smirnov-test results.

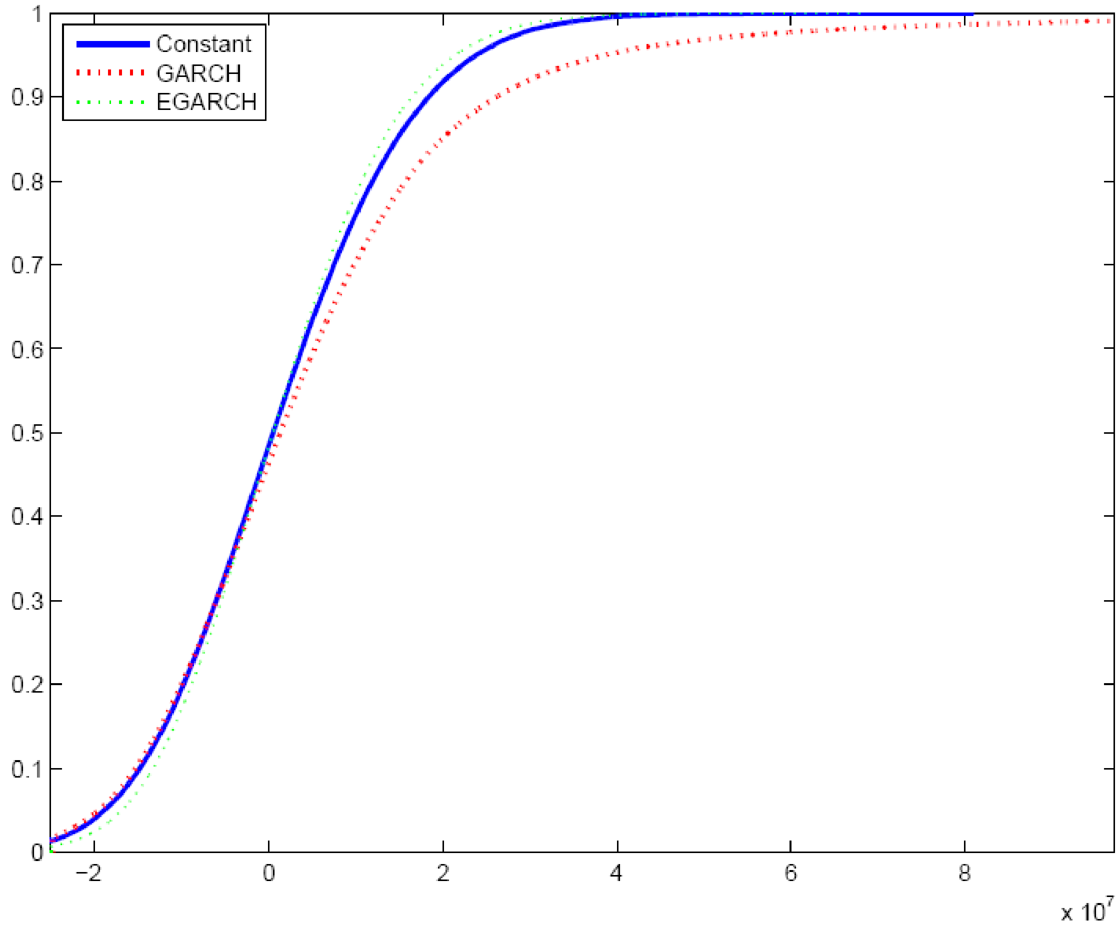
5.3.1 Forecasting Complexity

The first dimension that is investigated is forecasting complexity. As mentioned before, the author divides forecasting complexity in volatility and correlation forecasting methods.

5.3.1.1 Volatility model

The first issue to address within the area of forecast complexity is how future volatilities are predicted. The ROPFVT is able to apply either historical constant volatility values as forecasts for future volatility or to compute those estimates based on different GARCH-type models. Beneath the GARCH(1,1) model, the author also applies the more advanced E-GARCH-model. An issue worth mentioning is that forecasts based on GARCH models converge to forecasts based on historical values after a certain time span. The impact of volatility forecasts on the simulation results is presented in figure 5.7.

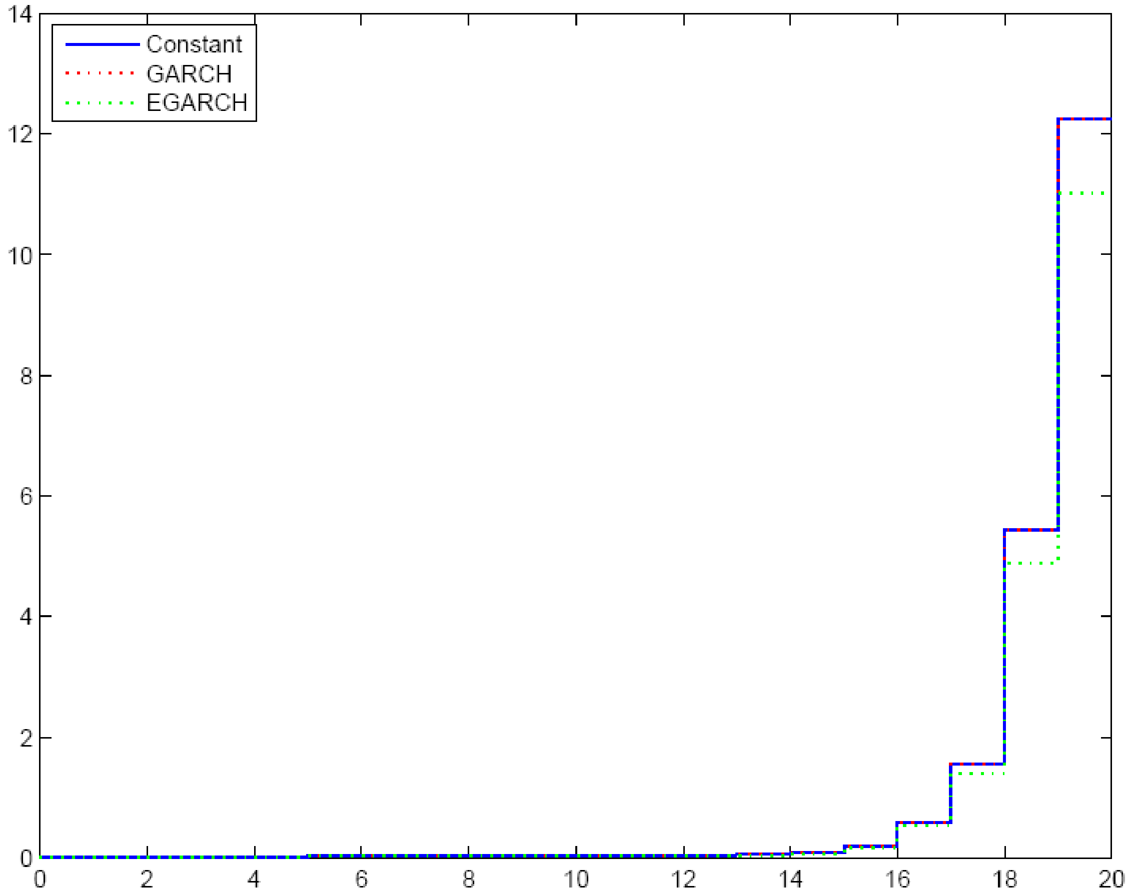
Figure 5.7: Constant vs. GARCH vs. E-GARCH



Source: Own work based on Weber et al. (2010).

As can be seen, the results depend heavily on the applied volatility forecast model. The E-GARCH-model clearly reduces the probability mass on the tails of the distribution as compared to the other models. The most extreme NPV distribution is obtained with the GARCH-model. When analyzing the project's ECDP, which is reported in figure 5.8, the same effects can be found.

Figure 5.8: Constant vs. GARCH vs. E-GARCH



Source: Own work based on Weber et al. (2010).

Especially the E-GARCH-model is noticeable since it leads to a much lower default probability. The gap between the default probability calculated by the E-GARCH and constant volatility or the GARCH-model is about 2% over the whole runtime of the project. The author cannot judge which model is the best model since the "objective" NPV distribution function is not known by the author. However, my results suggest that the choice of the volatility forecast model is extremely crucial for the simulation results. Hence, it is very important to figure out which volatility forecast model is best suited for each factor when valuing a project in practice. The substantial impact of this choice can be seen from the prior two figures. For example, recent academic work by Bowden and Payne (2008) and Chan and Gray (2006), propose that E-GARCH is the best volatility model for electricity prices.

Hence, it could be argued that its application should be favored over alternative models. However, the author will not discuss which model is best suited for which underlying in greater detail since this is not the focus of this dissertation.

In the context of *volatility models*, the author compares the valuation results for the generic projects obtained by constant volatility against GARCH and its derivative E-GARCH. The complexity order is "constant volatility → GARCH → E-GARCH". Results are reported in table 5.3.

Table 5.3: Volatility Models

	Statistical Test	Constant→ GARCH	GARCH → EGARCH
NPV	Ansari-Bradley	97	99
	F-Test	97	93
	Kolmogorov-Smirnov	100	99
EDF	Average Delta	2	2
	T-Test	***	***
VaR	Average Delta	8	23
	T-Test	***	***

Source: Own work based on Schmid and Weber (2011*a*).

All three tests indicate that the step from constant volatility to GARCH volatility changes the NPV distribution in nearly all simulated energy and mining projects. Furthermore, the difference in the EDF and the VAR underline that this difference is not only statistically significant, but of high economic importance as well. For example, the difference between the VAR risk calculated with constant and GARCH volatility is 8% of the initial investment. Concerning the EDF, the probability that the project defaults differs by 2%. Similar results are found for the step from GARCH to E-GARCH. The difference between the VAR in this case is even higher with 23%. To sum up, the author concludes that the impact of the volatility model on the NPV distribution is very strong. Especially the steps from constant volatility to GARCH and from GARCH to E-GARCH change the outcome significantly. Behind this background, practitioners should choose the volatility model carefully.

5.3.1.2 Correlation models

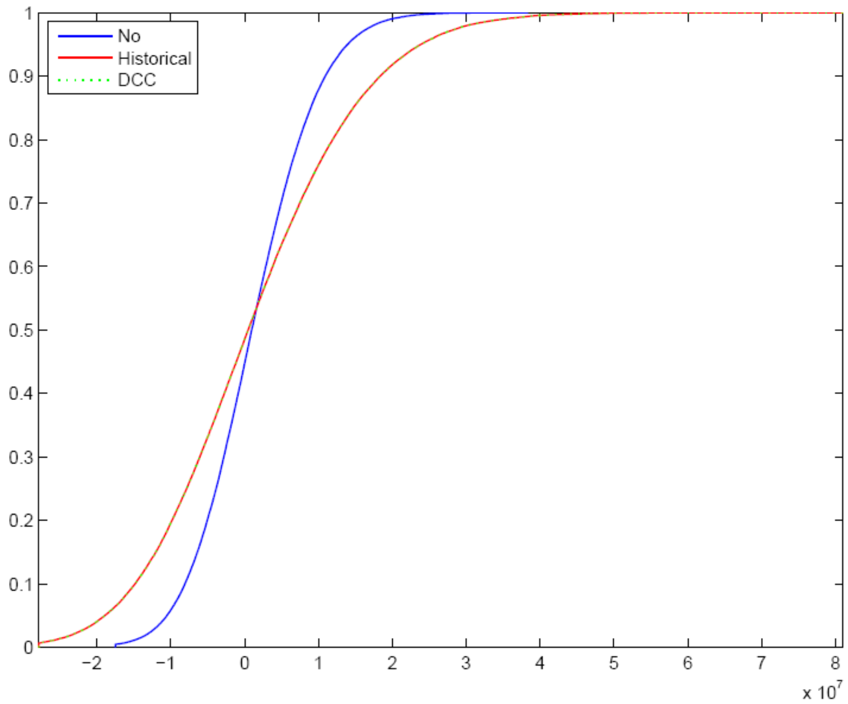
In a further step, the author investigates the influence of the correlation forecast model. The ROPFVT can compute future correlations in three different ways: (i) it assumes that the correlation between all factors is zero in the future, (ii) historical correlations are used as forecasts for future correlations, or (iii) estimates from a DCC model are applied as predictors for correlations. The first method which assumes zero correlations has to be interpreted with caution since fundamental economic relationships may be neglected. For example, if a project is simulated whose cash flows depends on gas and oil prices, the assumption of no correlation is clearly misleading, since gas and oil prices are highly correlated due to linking mechanisms between them. Despite this fact, the author integrated the possibility to investigate if the assumption of no correlation has a significant impact on the results for this study.[42] Figure 5.9 presents the cumulative NPV distribution function for these three different correlation forecasts.

As expected the choice between zero correlation and correlated forecast has a significant impact on the simulation results. But the simulated NPV distributions are quite similar for historical and DCC correlation forecast methods. The same is true for the EDF, which is reported in figure 5.10.

To summarize, the results suggest that correlation forecasts based on a DCC model do not significantly improve the results compared to the application of historical correlations. Taking into consideration that the DCC model is rather complicated to implement in a valuation model, the author suggests using historical correlations, which are easier to handle.

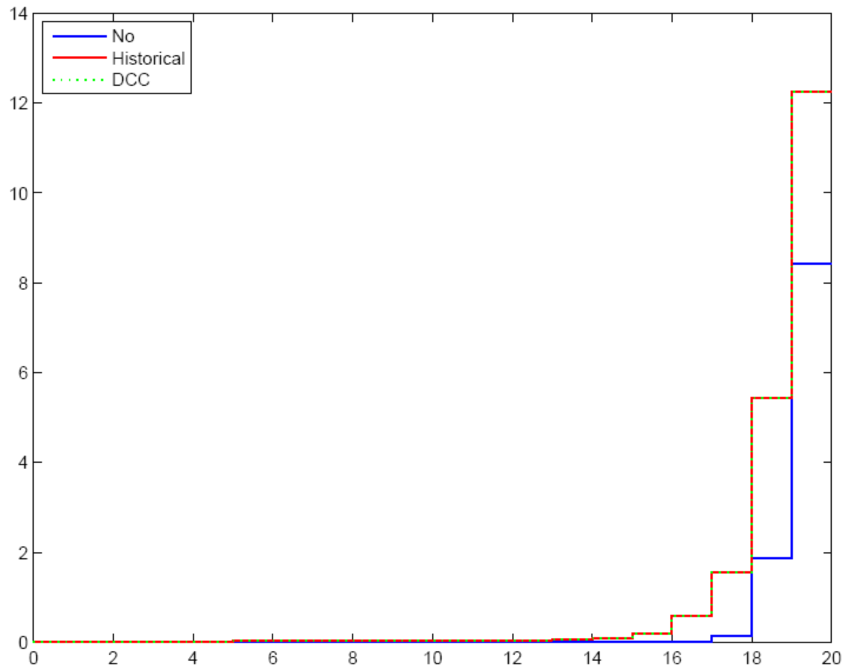
[42]As for GARCH based volatility forecasts, correlation forecasts based on the DCC model converge to forecasts based on historical values after a certain time.

Figure 5.9: No vs. Historical vs. DCC



Source: Own work based on Weber et al. (2010).

Figure 5.10: No vs. Historical vs. DCC



Source: Own work based on Weber et al. (2010).

Next, the author validates this finding by analyzing the question in the case of the generic projects. As *correlation models*, the author considers constant correlation and correlation forecasts based on Dynamic Conditional Correlation. Furthermore, the author evaluates how the most simple assumption for correlation, i.e. zero correlation between all factors, affects the outcome. The complexity order is “No correlation → constant correlation → DCC”. Results for the NPV distribution are reported in table 5.4. Results are reported in table 5.4.

Table 5.4: Correlation Models

	Statistical Test	No → Constant	Constant → DCC
NPV	Ansari-Bradley	100	51
	F-Test	98	0
	Kolmogorov-Smirnov	100	60
EDF	Average Delta	20	0
	T-Test	***	0
VaR	Average Delta	2	0
	T-Test	***	0

Source: Own work based on Schmid and Weber (2011*a*).

Not surprisingly, the step from no correlation to constant correlation also has an impact on the valuation outcome. All tests indicate that the NPV distribution is different for nearly 100% of all evaluated projects. Furthermore, the difference in the EDF is highly significant and of huge economic impact with a value of 20%. Regarding the VAR, the author finds a mean difference of 2% between no correlation and constant correlation. This supports the view that neglecting correlation in the valuation model changes the results significantly. The differences between constant correlation and DCC are very small. Although two of the three tests used for the comparison of the NPV distributions indicate significant differences in about 50% of all projects, the economic impact of these differences is small. Both the EDF and the VAR do not differ for the two methods. To summarize, the author argues that DCC correlation forecasts do not change the valuation outcome compared to constant correlation. However, the assumption of zero correlation clearly leads to different outcomes compared to constant correlation.

5.3.2 Valuation Complexity

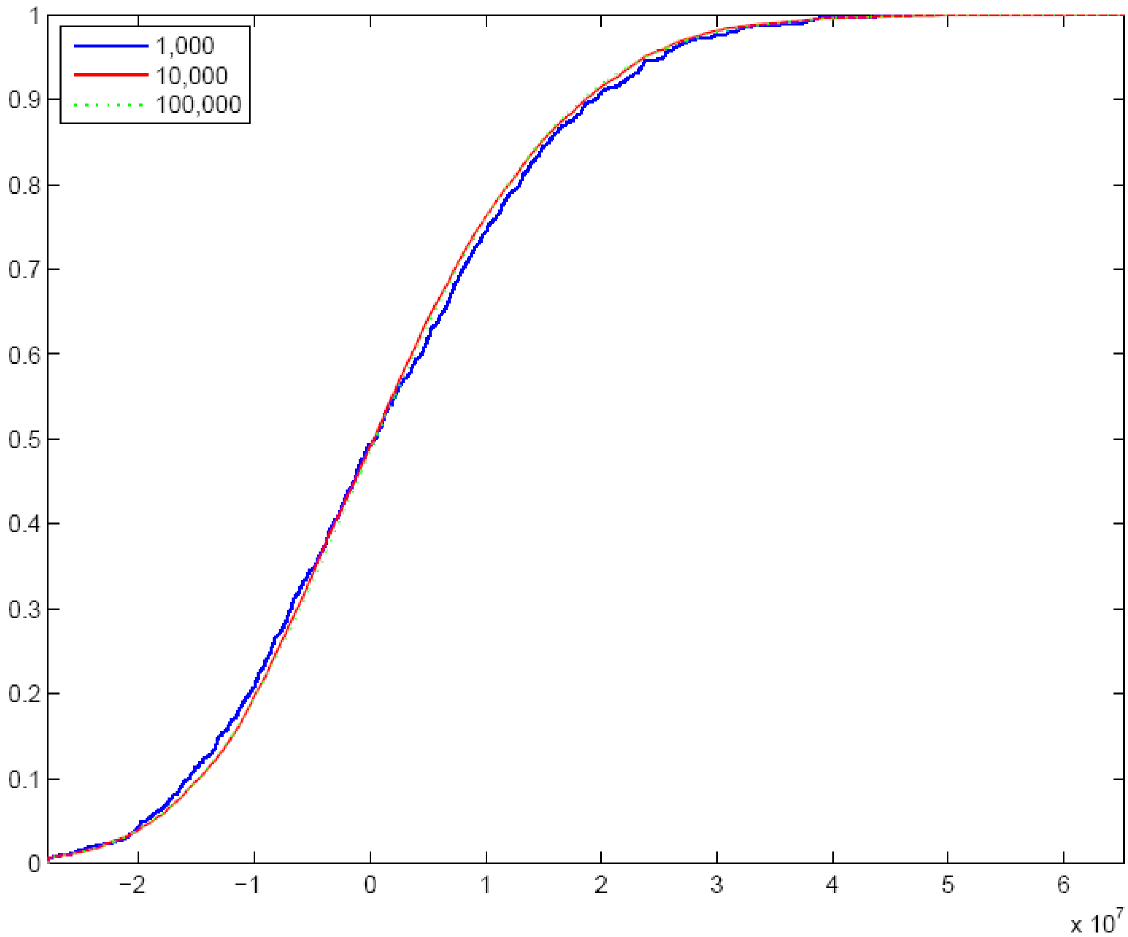
The second dimension that is investigated is valuation complexity. For this, the author varies the number of simulations, the time resolution, and the method to calculate the discount rate.

5.3.2.1 Number of iterations

The first parameter to analyze is the number of iterations. It is expected that an increase in this parameter leads to a smoother distribution function of the NPV. The author starts with 1,000 iterations and end up with 100,000 iterations. While 1,000 iterations can be easily implemented, for example in a spreadsheet program, 100,000 iterations require more sophisticated software programs and high computational power. The results of my analysis are presented in table 5.5. As expected, the level of the distribution function remains (largely) unchanged. Figure 5.11 presents the distribution function of simulations with different numbers of iterations. The project is simulated with 1,000, 10,000, and 100,000 iterations. As expected, the number of iterations has no significant effect on the overall level since there is no systematic shift in the cumulative distribution function. However, for 1,000 iterations, the function is rather discontinuous. As a consequence, the expected NPV may be over- or underestimated, depending on whether the function is above or below its "true" value at a certain point. The application of at least 10,000 of iterations seems to be recommendable. The step from 10,000 to 100,000 iterations does not increase the smoothness of the function significantly, but increases computation time considerably. The author recommend using at least 10,000 iterations for a simulation to obtain smooth distribution functions. The same effects can be observed for the

EDF, which is reported in figure 5.12.

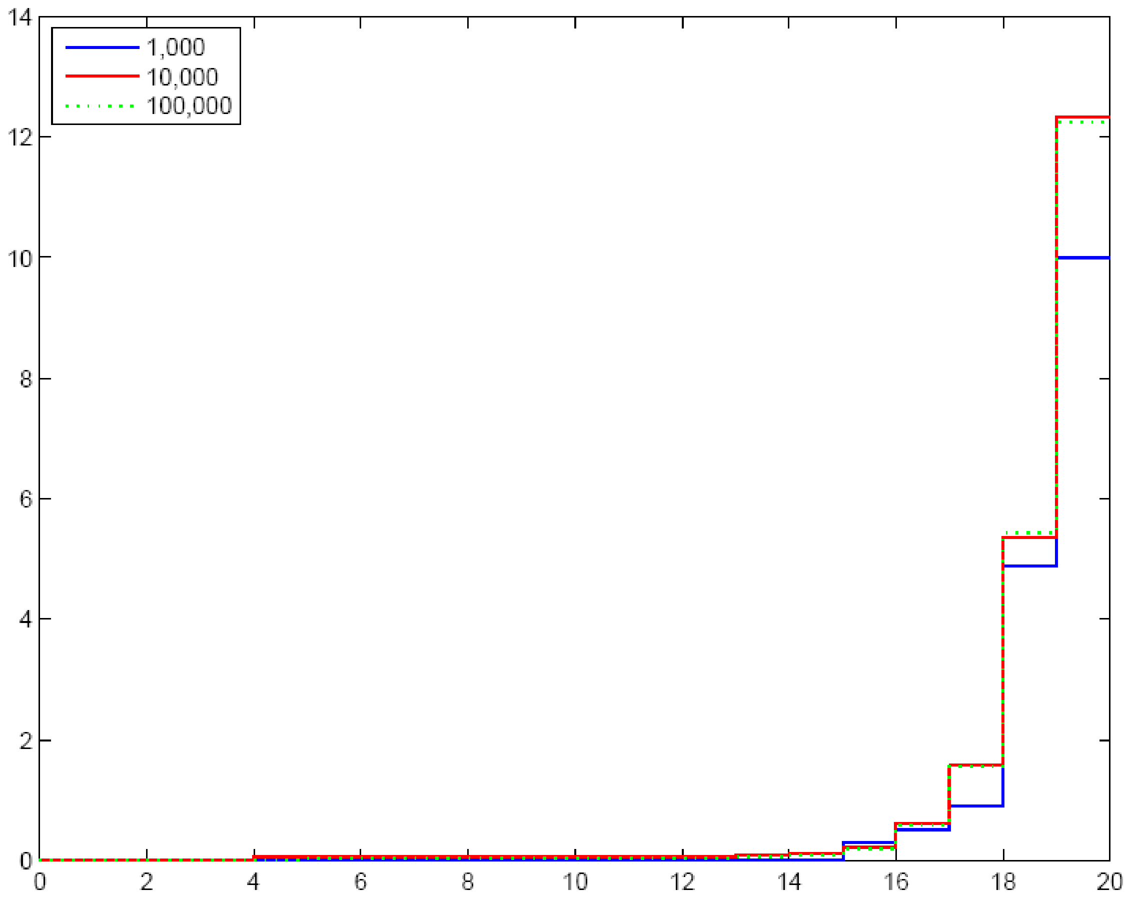
Figure 5.11: 1,000 vs. 10,000 vs. 100,000



Source: Own work based on Weber et al. (2010).

The drawback of an increased number of iterations is that the computation time increases significantly. Therefore, it is necessary to weight the advances of an increased number of iterations, a smoother distribution function, against its drawback in terms of more computation time. Below 10,000 iterations, the distribution function is very rocky and not satisfying. As a consequence, the above proposed 10,000 iterations seem, from this point of view, to be a good compromise between distributions smoothness and computation time. The author validates this hypothesis by simulating the 1,000 generic projects.

Figure 5.12: 1,000 vs. 10,000 vs. 100,000



Source: Own work based on Weber et al. (2010).

Table 5.5 shows that the NPV distribution significantly depends on the number of iterations.[43] However, this effect becomes smaller for the step from 50,000 to 100,000 iterations. For this last step, the distribution differs in about 60% of all considered projects. The reason for these differences is that the distribution of the NPV becomes smoother as the number of simulations increase. Interestingly, the differences in the VAR are very small. Only the step from 1,000 to 10,000 iterations leads to a significant change in the VAR (2%). Hence, even 1,000 iterations seem to be sufficient to detect all project defaults. To sum up, the number of simulations has an impact on the shape of the distribution. However, while the statistical significance is high, the analysis indicated that the economic importance becomes small after

[43]Results for the EDF are not presented since there are - as expected - no differences.

the step from 1,000 to 10,000 iterations.

Table 5.5: Number of Simulations

	Statistical Test	1,000 → 10,000	10,000 → 20,000	20,000→ 50,000	50,000→ 100,000
NPV	Ansari-Bradley	93	89	92	69
	F-Test	92	88	89	53
	Kolmogorov-Smirnov	86	71	72	58
VaR	Average Delta	2	0	0	0
	T-Test	***	0	0	0

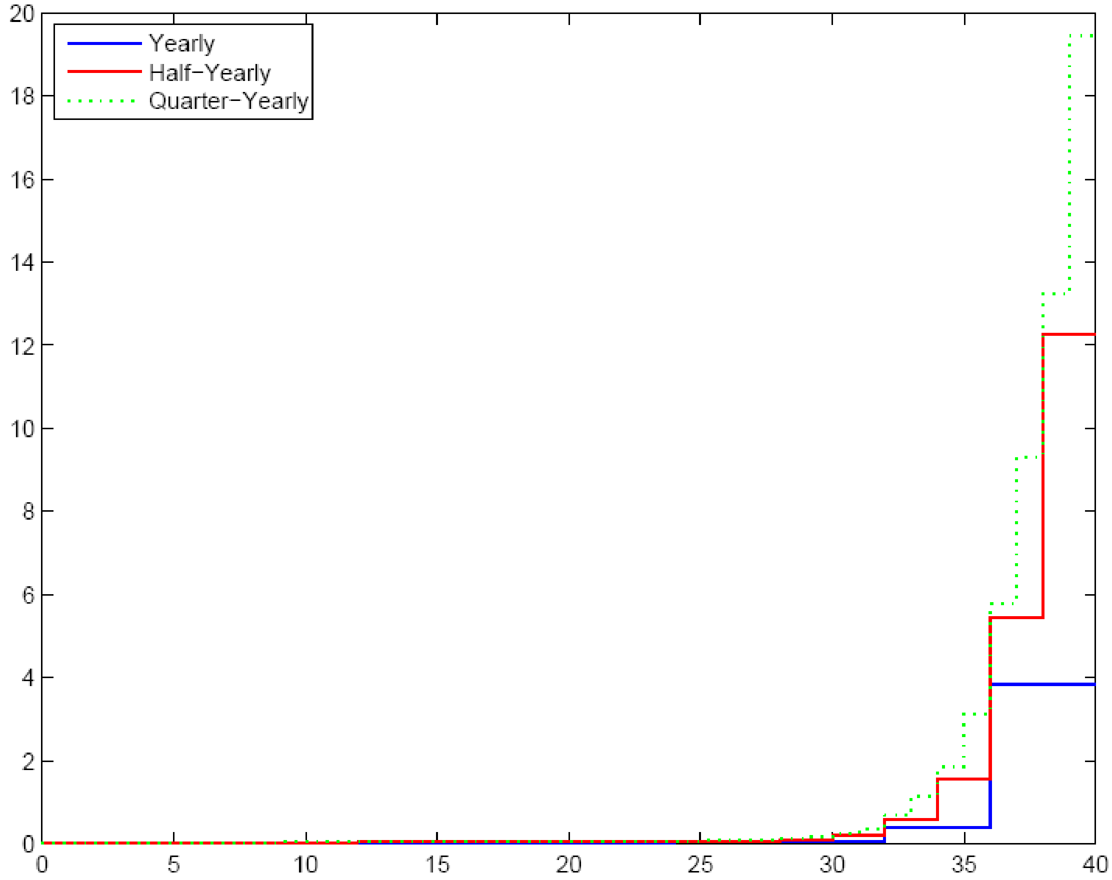
Source: Own work based on Schmid and Weber (2011*a*).

5.3.2.2 Time Resolution

Another crucial parameter of the valuation is time resolution. Time resolution defines how often the project's cash flow is calculated. For example, a quarterly time resolution means that the cash flow of the project is computed and analyzed each quarter during the simulation period of the project.

Since the cash flow is analyzed more often, with a quarterly resolution compared to a half-yearly or yearly resolution, a stream of negative cash flows over several quarters may lead to a project default. Such an event of default does not necessarily happen with yearly resolution, since a stream of negative cash flows may be followed by more positive cash flows, which compensate the prior losses. However, on a quarterly resolution, the project would have been defaulted with no possibility of recovering. Therefore, the author expects a higher default rate for shorter time periods as can be seen in figure 5.13. This has also a negative impact on the expected NPV.

Figure 5.13: Yearly vs. Half-Yearly vs. Quarter-Yearly



Source: Own work based on Weber et al. (2010).

In the generic projects, the authors also considers yearly, half-yearly, and quarterly resolution. In contrast to the previous test, the author does not compare the shape of the NPV distribution and the VAR, since the shape of the (normalized) distribution - and hence the VAR - is not affected by the time resolution.[44] However, the time resolution can have an impact on the EDF. Higher time resolution might increase the probability of a project default because the evaluation frequency is higher. Imagine a simulation path which is below the default frequency for e.g. half a year, but recovers after that. If yearly time resolution is applied, the project is not considered to be defaulted. In contrast, quarterly time resolution leads to

[44] Of course, the shape of the distribution and the VAR are affected by differences in project defaults. However, this is no primary effect. Consequently, the author only reports the effect on the EDF and not on the NPV distribution and the VAR.

a project evaluation within this period and the project is classified as default. In this case, a recovery of the project is not possible anymore. The complexity order is yearly → half-yearly → quarterly time resolution. Results are reported in table 5.6.

Table 5.6: Time Resolution

	Statistical Test	Yearly → Half-Yearly	Half-Yearly → Quarter-Yearly
EDF	Average Delta	3	2
	T-Test	***	***

Source: Own work based on Schmid and Weber (2011*a*).

The results show that the time resolution changes the EDF significantly. Both steps along the complexity order lead to a change in the EDF, which is significant at the 1%-significance level. Hence, the time resolution has an impact on the valuation outcome and should be chosen with caution.

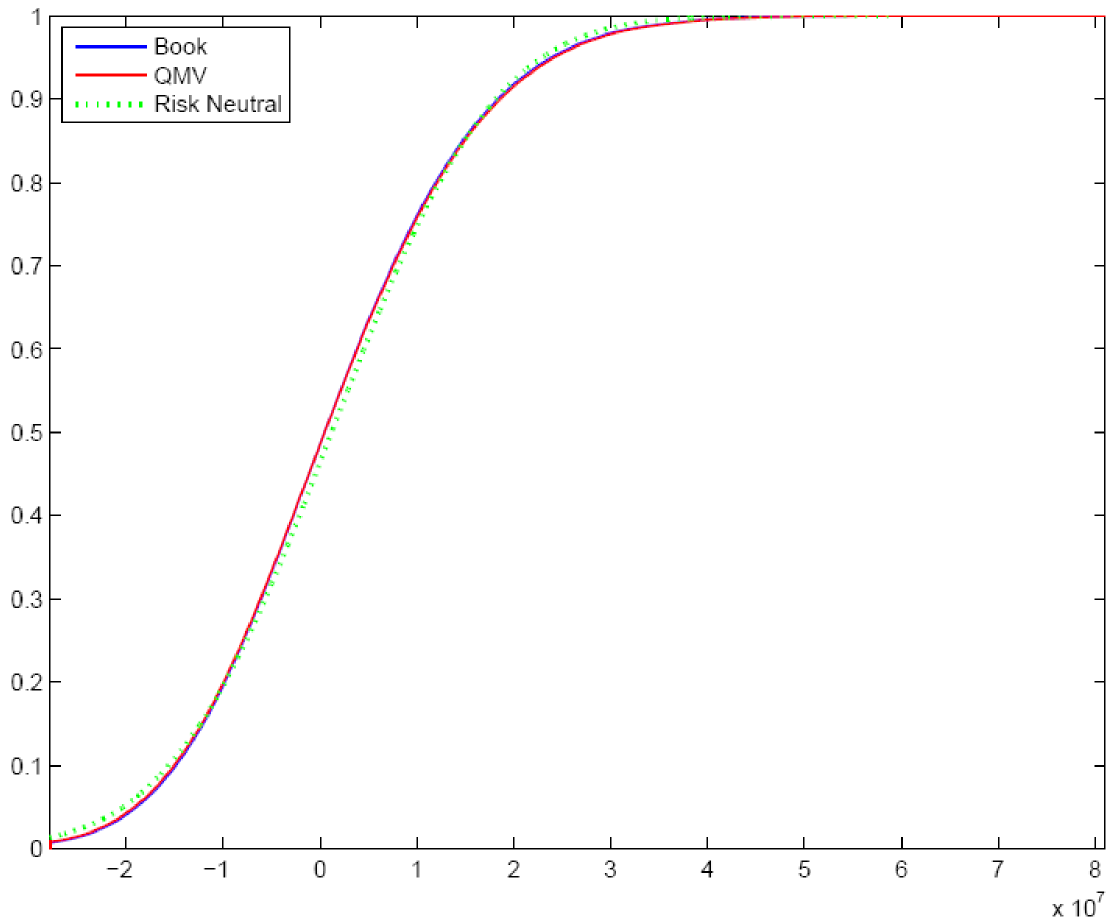
5.3.2.3 Discount rate

As the last aspect of valuation complexity, the method to calculate the discount rate is investigated. The first method to calculate the discount rate is based on a simple Capital Asset Pricing Model (CAPM) with a beta factor that is constant over the project's run time. Consequently, the cost of equity is not risk-adjusted in this case. Of course, this assumption is not realistic since in reality the cost of equity changes over time. A more complex method is quasi-market valuation (QMV, cf. Esty (1999)). This method adjusts the cost of equity to the project risk. The most sophisticated method is risk-neutral valuation. For this comparison, the author does not consider differences in the EDF since the default frequency does not depend on the way the cost of equity is calculated. Results for this comparison are presented in table 5.7.

Figure 5.14 presents the three methods included in the ROPFVT, equity valuation based on book values, on QMV, and risk-neutral calculation.

The author finds that the cumulative distribution function between book value and the QMV method differs only in width of the tails but not in mass. In contrast to that result, the risk-neutral valuation results leads to different results. The rationale behind this is that the application of book values or the QMV leads to biased estimates of the cost of capital since they both underestimate the cost of capital for very successful and very unsuccessful simulation paths. The risk-neutral method avoids this misjudgment. This can be seen in the cumulative distribution function. The EDF of the project is not affected by the cost of capital calculation method, and consequently, no results are reported here.

Figure 5.14: Book vs. QMV vs. Risk-neutral



Source: Own work based on Weber et al. (2010).

As for the specific project, the results of the generic projects clearly indicate that the method to calculate the cost of equity is of critical importance for the valuation outcome. Both steps, i.e. the step from risk-unadjusted to risk-adjusted (QMV) cost of equity and from that to risk-neutral valuation, change the shape of the NPV distribution in nearly 100% of all considered projects. For the last step, this value is 100%, independent of the statistical test. For the VAR, the author finds no significant differences for the risk-adjusted compared to the unadjusted cost of equity. However, risk-neutral valuation changes the VAR on average by 11%. Hence, especially risk-neutral valuation has a strong impact on the valuation outcome, both in terms of statistical and economic significance. Table 5.7 summarises the results.

Table 5.7: Cost of Equity Methods (CoE)

	Statistical Test	Risk Unadj. CoE → Risk Adj. CoE (QMV)	QMV → Risk Neutral Valuation
NPV	Ansari-Bradley	99	100
	F-Test	100	100
	Kolmogorov-Smirnov	88	100
VaR	Average Delta	0	11
	T-Test	0	***

Source: Own work based on Schmid and Weber (2011*a*).

5.3.3 Analysis of influence of the jump process parameters on the value of a switching option

This section answers the question whether an option to switch a gas-fired power plant on off has a value and how much this option is worth.
A *Monte Carlo Method* with 10,000 simulations is used to determine the *Real Option* value. CO_2 version 5 from table 5.2 on page 137 is assumed for the analysis.

The decision to change the on/off-mode of a gas-fired powerplant, should depend on both, the price of electricity and of gas. This difference between both is called spark spread in the energy sector. It is the difference in the revenue an energy supplier earns by selling electricity and the fuel costs to produce the electricity. Here, the fuel costs are the amount of money to spend in gas[45] to generate a megawatt hour (MWh) of electricity.

Definition 3 (Spark Spread)**.** *The spark spread at time t is defined as:*

$$SS(t) = E(t) - \frac{1}{h} \times G(t) \tag{5.32}$$

with:

- *$E(t)$ is the price for electricity per MWh at time t*
- *$G(t)$ is the price for gas per MWh at time t*
- *h is the efficiency of the power station (heat rate)*

The spark spread differs from power plant to power plant because different types of power plants have different efficiencies. Thus, different power stations have different spark spreads. In the following, the power plant, outlined in section 5.2.2, is assumed.

Since 2005, generators need emission allowances for their CO^2[46] output. As another cost factor in energy production, the costs for emission credits are deducted from the spark spread to get a more realistic approach.

Definition 4 (Clean Spark Spread)**.** *The Clean Spark Spread at time t is defined as:*

$$CSS(t) = SS(t) - \frac{1}{h} \times CO_2(t) \tag{5.33}$$

with:

[45] The price of gas is often given in $\frac{Eurocent}{therm}$. To convert this into $\frac{Euro}{MWh}$, the prefactor of 0.3412 is needed. (see e.g.: Energy Units and Conversions)

[46] CO^2 (= carbon dioxide) is one of the greenhouse gases.

- *$SS(t)$ and h as above*

- *$CO_2(t)$ is the price for emission allowances per MWh at time t*

Of course, variable production costs can be added for a more realistic valuation. However the clean spark spread itself can already be a decision support for the power plant operator. If the clean spark spread is positive, electricity production with the specific power station is profitable, thus it should be turned on. Otherwise, a negative clean spark spread indicates that the power plant should be turned off. Eventually, the electricity producer prefers having no return to making a loss. This allows a financial interpretation: A call[47] on the spark spread with an exercise price of zero has the same payoff as the electricity producer has with his gas-fired power station[48]. This makes an evaluation of the power plant with *Real Options* possible. Gas prices are also available at EEX, but unfortunately not before 01/07/2005. Therefore, UK gas data (converted in €/MWh) is taken. Gas prices are only traded on working days, so a linear interpolation at weekends was needed.[49]

For the valuation of the real option, only the future cash flows are relevant. These cash flows depend on the switching option, meaning the decision to operate the gas peaker or not at any point in time. These options are path dependent and depend on all three stochastic variables. Therefore, their valuation can not be done by analytical means. Poitras (1998, p. 493, footnote 9) points out that the spark spread is hard to solve analytically for the distribution parameters as there is no proof about a distribution of a sum of two log-normal[50] variables. The sum of these variables has not to be log-normal again. Thererfore, it is necessary to simulate the spark spread movements, where models are able to reflect the properties of the

[47]A call is an option to buy a share of stock at the maturity date of the contract for a fixed amount, the exercise price.

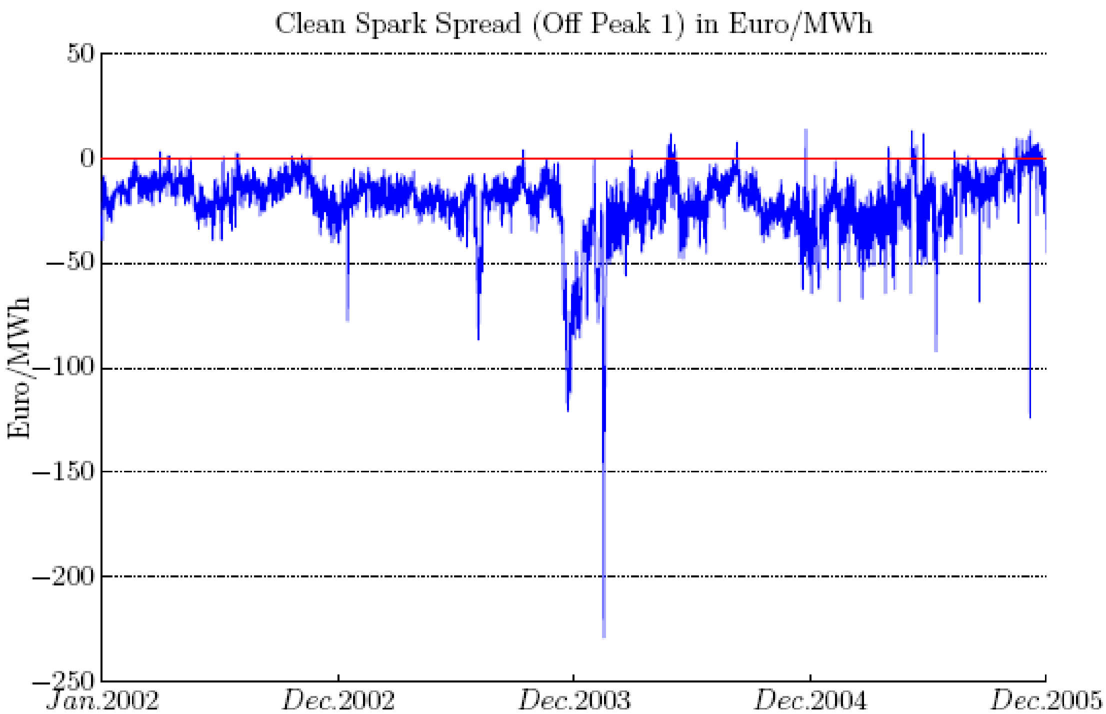
[48]namely: max(CSS(t),0)

[49]This was done with the `synchronize` command in MATLAB. See MATLAB-help for further details.

[50]The log-normal distribution of asset prices is a simplifying assumption in the financial theory. Black and Scholes (1973) also supposed stock prices to be log-normal distributed so that the returns are normally distributed.

genuine historical movements. The Monte Carlo simulation is the most commonly used alternative for analytical valuation. It is based on generating sample paths of the stochastic processes from random numbers. For each path, the value of the options can be computed. The mean value converges to the expected value with the increasing number of samples due to the law of large numbers, and hence, the value of the option is found.[51]

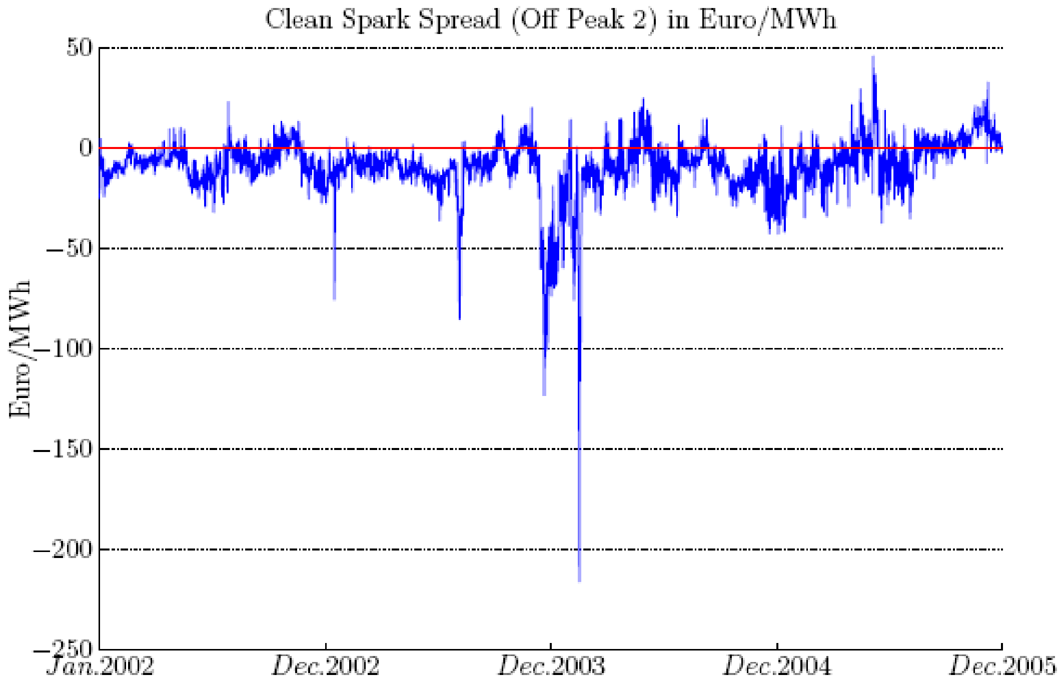
Figure 5.15: Clean Spark Spread in €/MWh for Off Peak 1 data



Source: Own work based on Schuster (2011).

[51]See section 3.4.2.2, for a detailed discussion of the Monte Carlo simulation

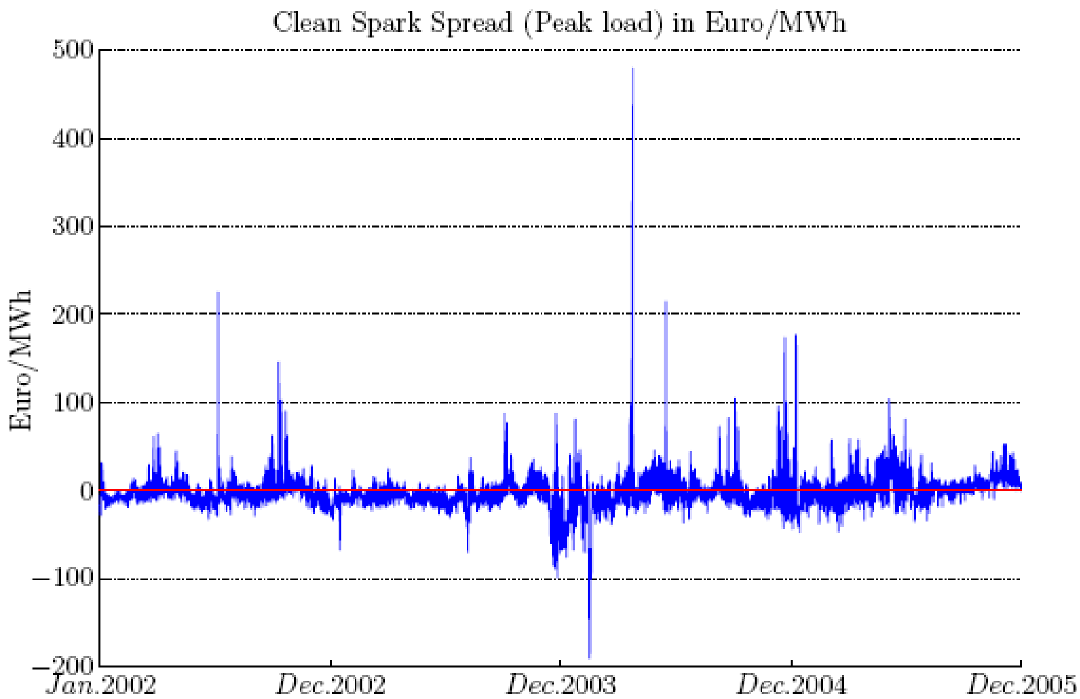
Figure 5.16: Clean Spark Spread in €/MWh for Off Peak 2 data



Source: Own work based on Schuster (2011).

Figure 5.15 and 5.16 show that the Clean Spark Spread in the off-Peak 1 time (from 0.00 a.m. until 8.00 a.m.) as well as in off-Peak 2 time (from 8.00 p.m. until 12.00 p.m.) is mostly negative. Hence a power station is not profitable, if this time intervals are considered in isolation. By scrutinizing the clean spark spread for off-Peak 1 and off-Peak 2, only on 3.08%, respectively 20.55% of all considered days, the clean spark spread is positive. By contrast, the Clean Spark Spread is positive on 52.26% of days for peakload in the regarded period. This can also be seen in figure 5.17.

Figure 5.17: Clean Spark Spread in €/MWh for Peak load data

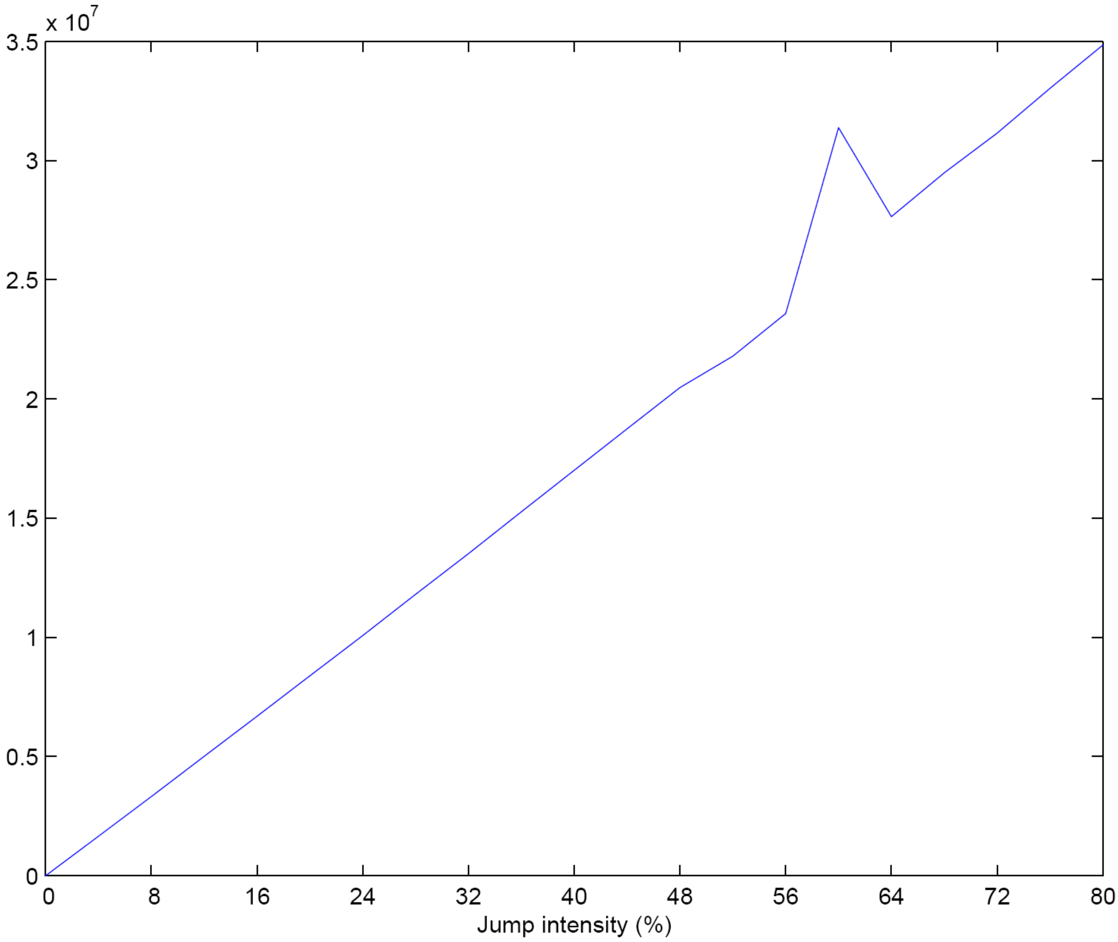


Source: Own work based on Schuster (2011).

The research question now is, what is the influence of changes in the spark spread on the value of the power plant. This change may induced by a change in the electricity production portfolio - extended usage of renewables. Renewables are characterized by a stochastic output because they generate electricity from nature forces like wind or the sun. This stochastic behaviour leads to over- or undersupply shocks. Therefore, the jump parameters of the electricity price process are varied. In this dissertation the mean-reversion rate between 96 and 0% in steps of 8%, the jump intenstiy between 0 and 80% in steps of 8%, and jump height mean from the 30% to 70% quantile in steps of 10% are varied.

Figure 5.18 illustrates the influence of a variation in the jump intensity if all other parameters remain constant. The jump intensity significantly influences the value of the gas-fired power plant. The range is € 3.5 billion between zero jump intensity, i.e no jumps happen, and 80% jump intensity. Thereby, the value of the switching option increases linearly with the increase in the jump intensisty.

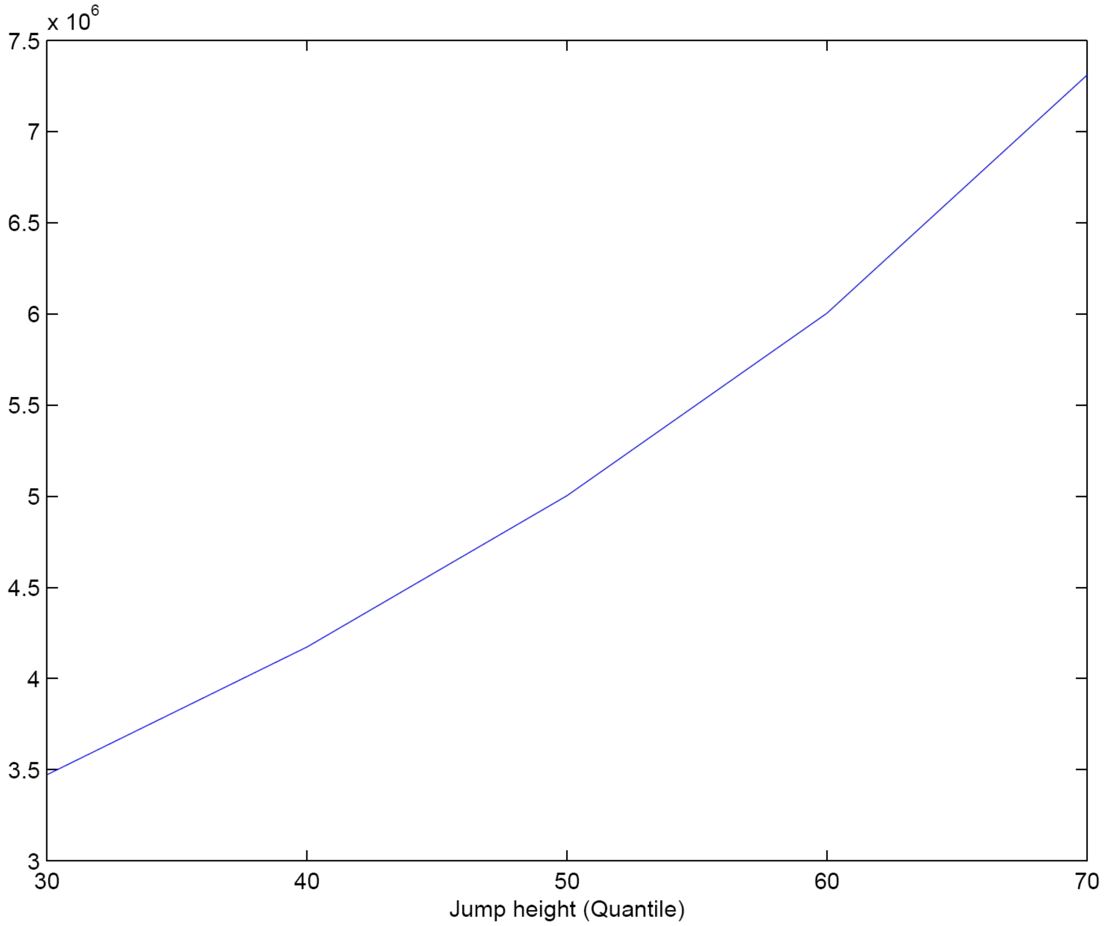
Figure 5.18: Change in real option value with changing jump intensity



Source: Own work based on Schmid and Weber (2011*b*).

Figure 5.19 shows the influence on the gas-fired power plant if the jump height is varied and all other parameters remain constant. Compared to the jump intensity, the jump height has a less crucial influence. The value range is only € 4 million, which is almost a reduction by a factor of 1000.

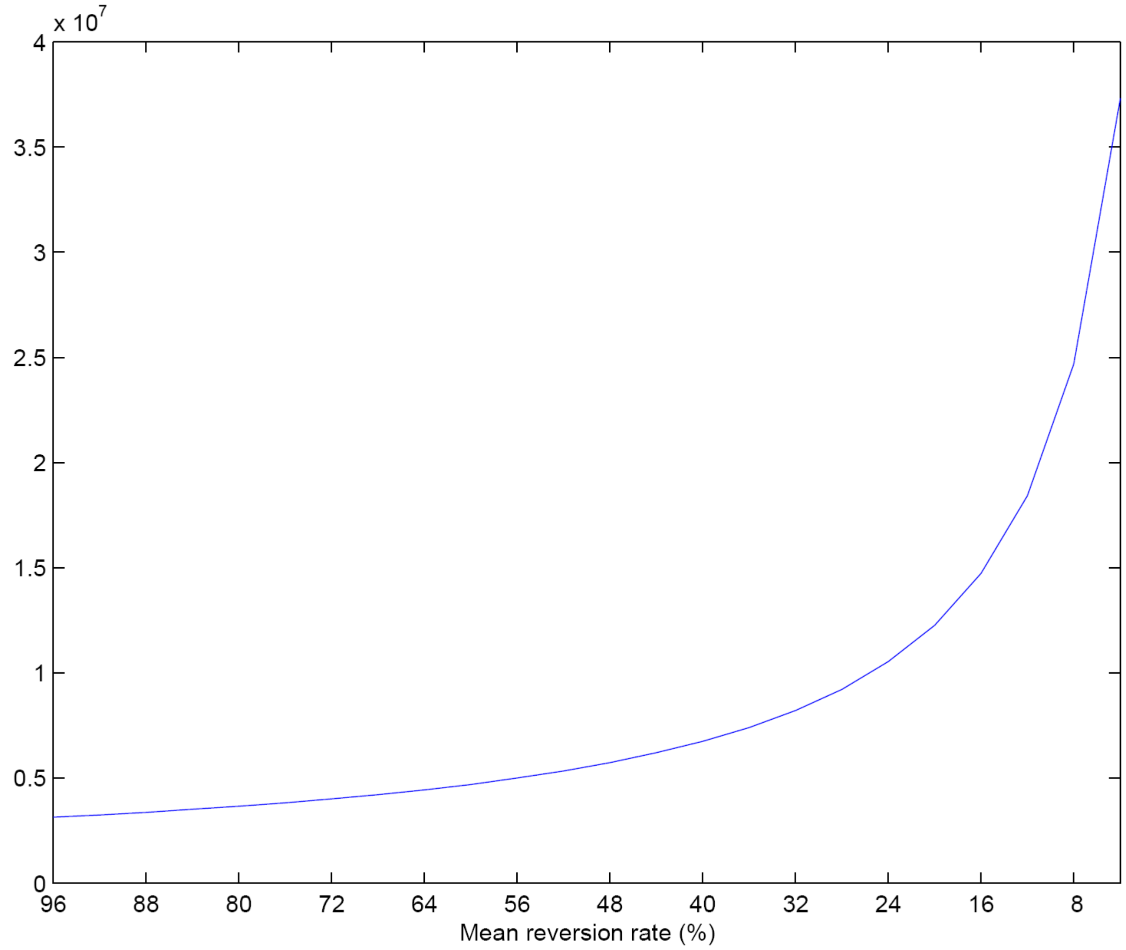
Figure 5.19: Change in real option value with changing jump height



Source: Own work based on Schmid and Weber (2011*b*).

Figure 5.20 shows the influence of the mean-reversion rate on the value of a gas-fired power plant if the mean-reversion rate is varied and all other parameters remain constant. It is apparent that the value increases exponentially with a decrease in the speed of the mean-reversion. The mean-reversion has almost the same influence on the portfolio as the jump intensity if the mean-reversion rate is very low. Also, it is worth noting that the value of the real option only increases slightly as long as the rate of mean-reversion is above 32%. This is also highlights the importance of modeling the spike process explicitly because, otherwise, the mean-reversion rate would be the sum of the diffusion and jump mean-reversion rate, and consequently, it would be much lower.

Figure 5.20: Option portfolio value with changing mean-reversion rate

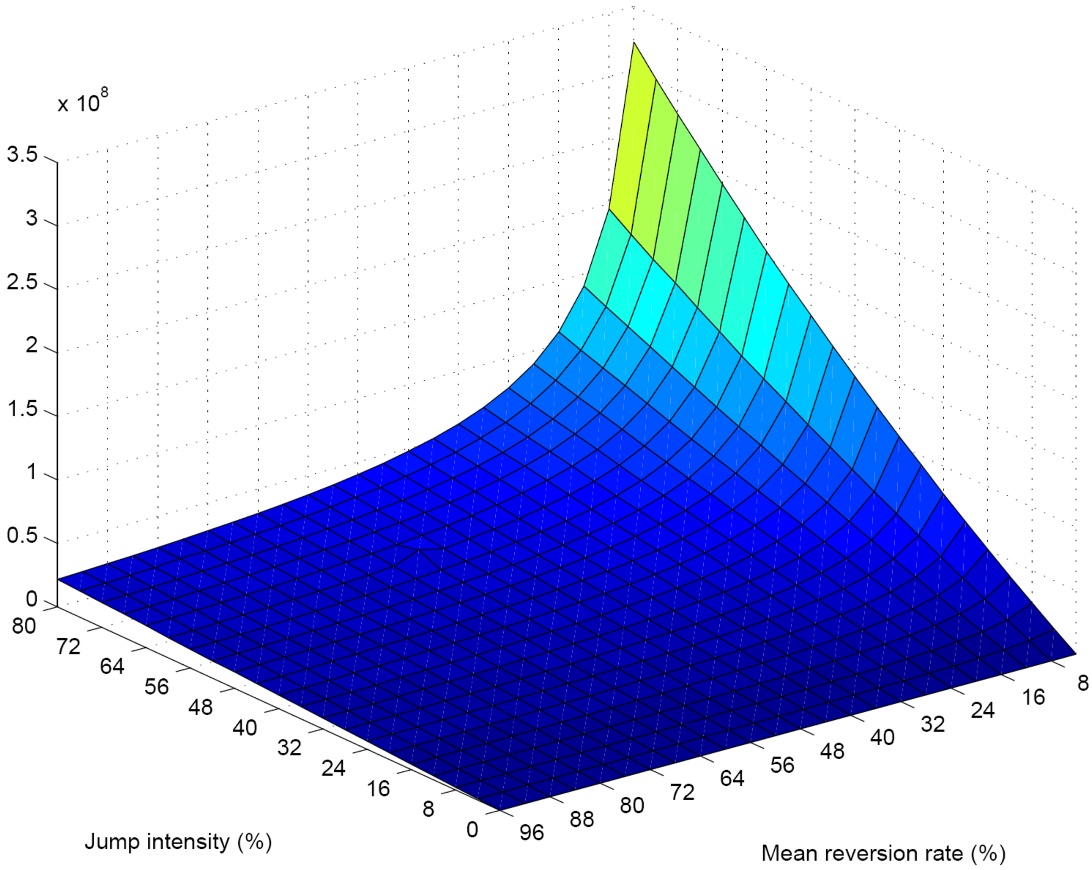


Source: Own work based on Schmid and Weber (2011*b*).

After analyzing the single effects of a change in the jump intensity, jump height, and jump mean-reversion, the joint effect of changing two parameters, while leaving the third parameter constant, are analyzed.

The joint effect of a change in the rate of mean-reversion and the jump intensity are shown in figure 5.21. It is apparent that the value of the real option is rather low if the rate of mean-reversion is high, even if the jump intensity is high. But with a decreasing mean-reversion rate, the value changes significantly at an exponential rate. This very much in line with the results above, which showed that a change in mean-reversion influences the real option value exponentially.

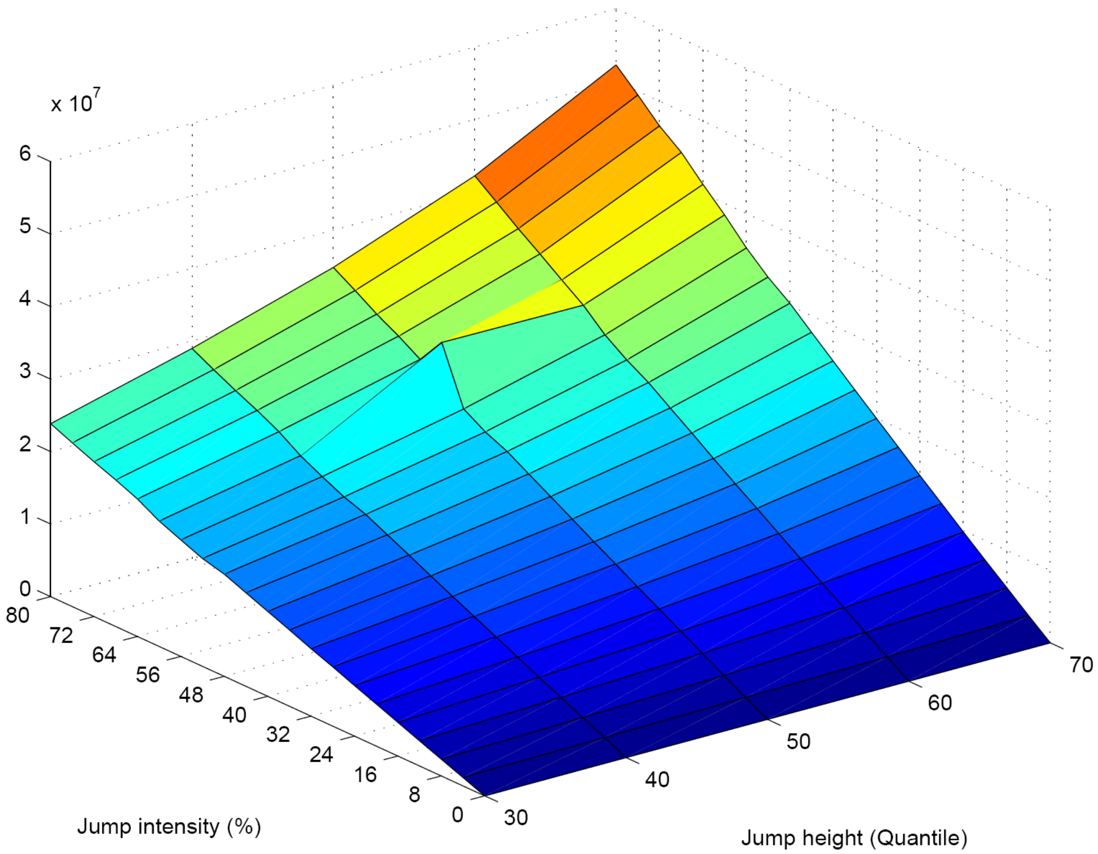
Figure 5.21: Change in real optionwith changing jump intensity and mean-reversion rate



Source: Own work based on Schmid and Weber (2011*b*).

The next analysis focuses on the joint influence of the jump intensity and jump height. The results, as shown in figure 5.22, indiciate that the main driver for the real option value change is the jump intensity. This is absolutely in line with results found above. This means, while the jump height is of less importance, finding the right jump intensity is very important when valuing a switching option.

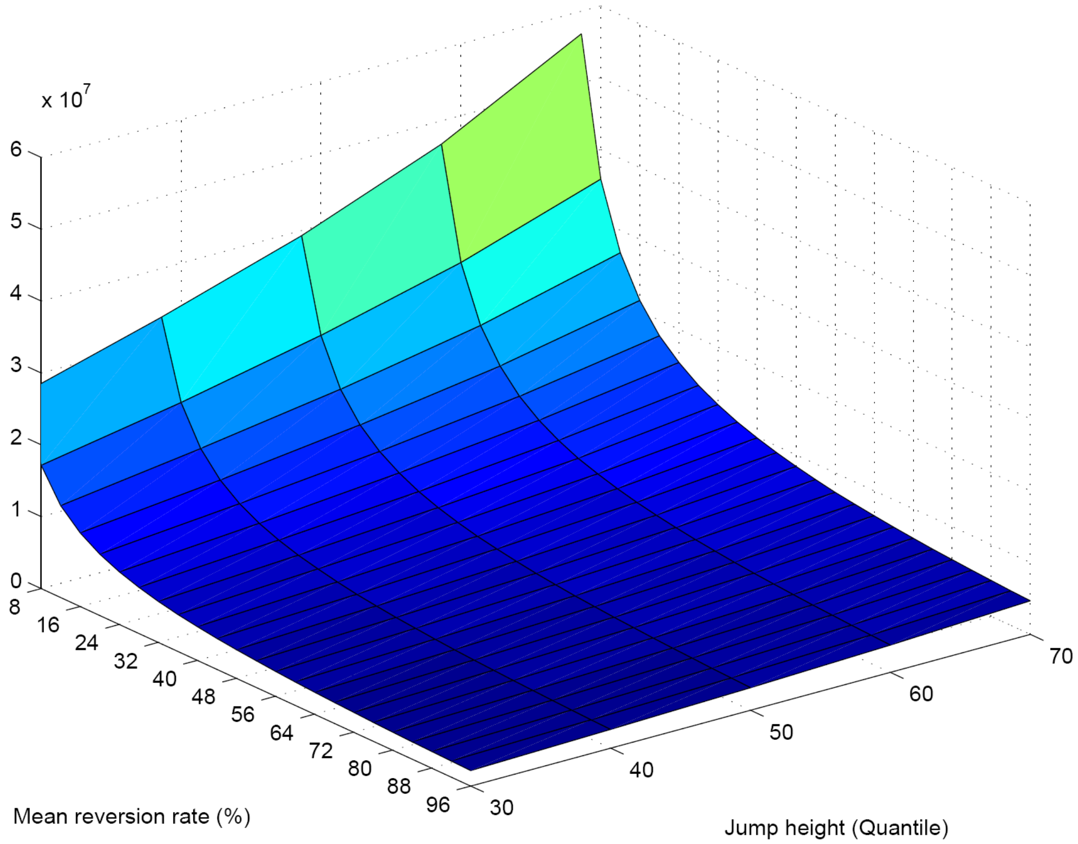
Figure 5.22: Change in real option value with changing jump intensity and jump height



Source: Own work based on Schmid and Weber (2011*b*).

The last combination analyzes the joint influence of the jump mean-reversion and jump height on the real option value. The jump intensity is constant. Figure 5.23 illustrates the results. Only if the mean-reversion rate is very low - below 24% - does the jump height influence the option value. But then, the change is significant.

Figure 5.23: Option portfolio value with changing mean-reversion rate and jump height



Source: Own work based on Schmid and Weber (2011*b*).

This section analyzed the effect of the parameters of the jump process on the value of the real option. It showed that the rate of mean-reversion and jump intensity are very important for the valuation of the real option in a gas-fired power plant. The jump height has only a significant influence if either the jump intensity is very high or the mean-reversion rate is very low.

Chapter 6

Conclusion

This dissertation analyzed electricity spot prices in the German market. It addressed the important question of modeling electricity spot prices with stochastic volatility and dependency among the jump processes. It provided, based on Benth et al. (2003) and Mayer et al. (2011), a framework that allows us to incorporate a stochastic volatility in the diffusion process. In addition, it gave a self-contained estimation procedure for all necessary distribution and process parameters based solely on the realized historic spot prices.

Regarding the model with stochastic volatility, the author concludes that, although it has a higher complexity resulting in problems in the parameter extraction from empirical data, since a greater number of parameters needs to be estimated, it is worth the effort. Especially, in the case of the peakload price process the E-GARCH model, which is the most complex model in the analysis, outperforms both the GARCH stochastic volatility and the constant volatility model. Furthermore, the analysis reveals that modeling the correlation among the jump process is a crucial step to capture the specific nature of the electricity spot price. This dissertation showed that the price path simulated with the model are similar to the evolution of electricity spot prices, as observed in the market.

After providing a stochastic process for commodities, especially electricity, this dissertation analyzes the impact of model complexity on the valuation outcome. Specifically, the impact of model complexity is explored on the net present value (NPV) distribution as well as on the expected default frequency (EDF). The differences in the valuation outcome are evaluated along three dimensions, namely the differences in the shape of the NPV distribution, the EDF and the VAR. The analysis is based on the newly developed real option project finance valuation tool (ROPFVT).

This dissertation analyzes whether model complexity matters for the valuation of a project financed gas-fired power plant. But, besides the specific gas-fired power plant, it does not focus solely on one single project, but rather considers about 1,000 different (hypothetical) generic projects in the energy and mining sector to obtain results, which can be generalized. All these projects have different input-output-factor combinations as well as different interest and exchange rate risks. Investments in theses different projects are simulated and the elements of model complexity are varied. Model complexity is split up in two dimensions: (i) the complexity of the valuation procedure and (ii) the complexity of the forecast models. Forecasting complexity compromises the volatility and correlation model, and valuation complexity includes the number of iterations, the time resolution, and the cost of equity calculation.

The results are summarized as follows: (i) Model complexity has in general an impact on the valuation outcome.

(ii) For valuation complexity, first, the author shows that the influence of the number of iterations is of minor economic importance. Considering the trade-off between accuracy of the obtained distributions and computation time, a number of 10,000 iterations seems to be the optimal choice. After 10,000 iterations, differences in the EDF and the VAR disappear. Hence, 10,000 iterations seem to be enough for practical purposes.

Second, from a theoretical point of view, the risk neural valuation method should be used for determining the appropriate cost of capital. Although it is the most complex method, it is superior to the other methods and should be chosen in practice. If this is not the case, the project's cost of capital is either over- or understated, resulting in biased valuation results. Third, analyzing the effects of time resolution in the simulation procedure, The author finds that it has a significant impact on the obtained default probability. Higher time resolution leads to higher default probabilities, as expected because less paths recover in the simulation.

(iii) For forecasting complexity, first, when analyzing the effect of the forecast models the dissertation shows that the selected volatility forecast model significantly affects the simulation results. Applying GARCH models instead of constant volatility has a huge impact on the outcome. Although this dissertation cannot answer the question, which model is best, the results clearly indicate that volatility models have an impact. Hence, practitioners should be careful when choosing the volatility model. Second, this dissertation does find a strong impact of correlation forecasting compared to zero correlation forecasting. But, it does not find significant differences between historical and DCC correlation. Consequently, the author argues that constant correlation seems to be a good trade-off between complexity and accuracy.

Based on the results for the electricity spot price, the dissertation analyzed whether model complexity significantly influences the value of a project financed investment. It does this by valuing a specific project, namely a gas-fired power plant, and 1,000 generic projects in the energy and mining sector. The generic projects allow to overcome the obvious limitation of only evaluating the impact of model complexity for one type of project finance, namely power plants. Therefore, the author argues that most of the results can be generalized for the whole project finance context.

In section 5.3.3, the value of flexibility is analyzed by valuing a gas-fired power plant under the variation of different jump process parameters. It shows that the flexibility to switch a power plants on or off has a significant effect on an investment project's value. Therefore, the results show that evaluating a power plant with a static method, like the DCF method, would undervalue those kind of investment projects. The *Real Option*-method shows its strengths here. It is especially suited for the valuation of energy generation asset, because the clean spark spread - the spread between the output and the inputs - plays a major role and can be calculated from market traded prices on a daily basis.
The results of the analysis confirm the high value of flexibility. But one can also see that the jump intensity and the jump mean-reversion rate play a more crucial role for valuing the project. This can be explained by the fact that a higher jump intensity leads to more jumps and the power plant is switched on more often. Consequently, the option is more often in the money. The mean-reversion rate influences the duration of the jump. The lower it is, the longer jumps tend to persist. The author argues that this independent whether a power plant is valued or anything else. The conclusion is that the jumps have a value for the valuation of real options but also for the valuation of financial options.[1]

To summarize the results, this dissertation shows that it is necessary to model the jumps in the electricity with an own stochastic process. Furthermore, it is shown that a stochastic valuation approach for the valuation model is important. There are model elements that are extremely crucial, like time resolution or volatility forecasting, while others seem to be of less importance, like advanced correlation forecasting. In addition, this dissertation also shows that recognizing flexibility in the valuation model is crucial

These results can help practitioners to trade-off model complexity against its drawbacks, e.g. higher implementation effort. As the analysis indicates, not all

[1]An example is the currency market where SNB - Swiss central bank - intervened causing a price jump of almost € 0.20. Therefore, the jumps cannot be ignored.

aspects of model complexity have an equal impact on the outcome of the valuation. For example, the application of DCC is difficult, but has no significant effect on the outcome. By contrast, risk-neutral valuation is also difficult to implement, but has a very high impact. It is important to note that the author does not judge which model leads to "better" results. The analysis only indicates the relative importance of different aspects of model complexity. For those which have a high impact, practitioners should be careful and think about the most appropriate model for their purposes.

In this dissertation, investments in project financing are analyzed from an equity investor's perspective in the energy and mining sector. Project financed investments in other industries, e.g. telecommunication or infrastructure, should be used to test issues of stochastic valuation and model complexity. However, reliable data for those industries is difficult to obtain since little information is publicly available. Project valuation is an issue that is important far beyond the project finance context and the results have important implications for more general investments as well. For example, intra-firm projects like new product developments require very similar valuation tasks. Several aspects of model complexity are not considered and left for further research. Among these are the implementations of hedging in the valuation model. Furthermore, more volatility and correlation models could be investigated in the future. An interesting line of work to pursue is that which involves models with an own stochastic process for the volatility, such as proposed by Heston. Another interesting field is the area of non-parametric statistics which allows the estimation of the process without any distribution assumption. In addition the question whether portfolio effects change the value of project are not considered in this dissertation. An extension of the analysis to a more general decision context might lead to more complete picture of the issue of stochastic valuation.

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