Probabilistic Power Control for Heterogeneous Cellular Networks with Closed-Access Femtocells

Ralf Bendlin
Dept. of Electrical Engineering
University of Notre Dame
Notre Dame IN 46556, U.S.A.
rbendlin@nd.edu

Yih-Fang Huang
Dept. of Electrical Engineering
University of Notre Dame
Notre Dame IN 46556, U.S.A.
huang@nd.edu

Josef A. Nossek
Institute for Circuit Theory and Signal Processing
Technische Universität München
80333 Munich, Germany
josef.a.nossek@tum.de

Abstract—**Heterogeneous cellular networks where conventional macro cells are overlaid with low power consumer deployed femtocell base stations are in the process of being deployed for their benefits in terms of scale, economy, and spectral efficiency. However, when femtocells operate in closed access, the problem of coverage holes for macro users needs to be solved. This paper proposes a probabilistic power control scheme based on results from stochastic geometry. The femto-layer transmit power for a given cell is computed at its macro base station and broadcasted to the femto base stations. Simulation results unveil that considerable performance gains can be achieved without additional information exchange among base stations or measurements at the handsets.**

I. INTRODUCTION

With the fourth generation currently being brought to market, wireless cellular networks have come a long way since their original conception. Traditionally envisioned for voice communication, outdoor coverage, and mobility, they are today predominantly used indoor for high data rate applications [1]. Unfortunately, data coverage requires larger link margins compared to voice calls while at the same time the average penetration loss of an exterior wall can be as large as 20dB [2]. Heterogeneous cellular networks where conventional macro cells are overlaid with low power, low cost, low complexity femtocells address both these issues by decreasing the distance between the transmitter and the receiver and by placing the femto base station inside the building [3].

Since femto base stations are consumer deployed, network operators are not in control of the base station or its backhaul, usually a DSL or cable TV Ethernet connection. Accordingly, users have to be on a "whitelist" in order to connect to a femto base station, similar to today's WiFi base stations. The most pressing issue in heterogeneous network deployments with closed-access femtocells is the protection of macro users from coverage holes in which they cannot communicate with any macro base station [3]. These coverage holes occur, for instance, when a macro user is in the immediate vicinity of a femtocell (to which it is not allowed to connect) and far away from the closest macro base station. In such a scenario, the significantly larger transmit power of a macro base station is offset by the tremendous pathloss to that base station.

A lot of theoretical work has been published on heterogeneous networks within the last couple of years. To name a few, outage probability and capacity analyses were presented for the uplink [4] and the downlink [5]. Multi-antenna techniques were the subject of [6], [7] and a spectrum sharing policy was proposed in [8]. Different access control mechanisms have also been analyzed, e.g., [9], [10]. Power control algorithms have been proposed both for the uplink [11], [12] and the downlink [13]. Recently, over-the-air sniffing was proposed where the femto base stations either decode the control channel from the macro base station in the downlink [14] or from the mobile user in the uplink [15]. Most power control schemes, however, are based on game-theoretic results such as those in [16]–[19].

Based on results from stochastic geometry [20], [21], we propose a power control algorithm situated at the macro base stations. Each macro base station independently computes the transmit power level for all femto base stations in the corresponding cell. The computations are solely based on measurements readily available at each macro base station and no further communication between any set of base stations is required other than the broadcasting of the results to the femto base stations in a quasi-static fashion. The algorithm addresses the limitations present in a practical system such as delays in the backhaul and scalability with the number of femto base stations. The paper concludes by demonstrating the benefits of the algorithm through system level simulations.

Notation: We denote vectors and matrices by bold lower and upper case letters, respectively. $\mathbb{E}[\bullet]$, $\delta(\bullet)$, j, $\mathbf{1}_M$, $\mathbf{0}_{M\times N}$, $\|\bullet\|_2$, $(\bullet)^*$, $(\bullet)^{\mathrm{T}}$, and $(\bullet)^{\mathrm{H}}$ denote expectation, Dirac function, imaginary unit, $M \times M$ identity matrix, $M \times N$ zero matrix, Euclidean norm, complex conjugation, transposition, and conjugate transposition, respectively.

II. SYSTEM MODEL

Suppose that the channel coherence time is considerably larger than the scheduling granularity, i.e., the channels are constant for at least one scheduling interval m (block-fading). Let $\boldsymbol{A}(\theta) \in \mathbb{C}^{N_\mathrm{a}\times M}$ and $\boldsymbol{G}(d,\theta) \in \mathbb{R}_+^{M\times M}$ be a Vandermonde matrix with elements $a_{i,j} = \mathrm{e}^{\mathrm{j}(i-1)\pi\sin(\theta+\varphi_j)}$ and a diagonal matrix with elements $g_{i,i} = \sqrt{\rho\left(d,\theta+\varphi_i\right)}$, respectively. For any given OFDM sub-carrier Ω, we model the frequency-flat vector channel $\boldsymbol{h}_{b,k,b'}^{[m,\Omega]} \in \mathbb{C}^{N_\mathrm{a}}$ from macro base station b' to

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TABLE I
OVERVIEW OF CHANNEL MODEL AND PARAMETERS

$d_{b,k,b'}$	distance to base station b' for user k in cell b
$\theta_{b,k,b'}$	angle to base station b' for user k in cell b
$\phi_{b,k,b',\xi}$	angular spread for $N = 6$ macro paths [22]
φ_i	angular spread for $M = 20$ micro paths [22]
$n_{b,k,b'}$	number of penetrated exterior walls
$\sigma^2_{\mathrm{PDP}_{b,k,b',\xi}}$	power-delay-profile [22], $\sum_{\xi=1}^{6} \sigma^2_{\mathrm{PDP}_{b,k,b',\xi}} = 1$
$\rho(d,\theta)$	$= 10^{0.1\left(\hat{A}+A(\theta)\right)} \cdot \left(\frac{\lambda/1\mathrm{m}}{4\pi}\right)^2 \cdot \left(\frac{d}{1\mathrm{m}}\right)^{-\gamma}$
$\hat{A}$ [in dB]	maximum antenna gain in boresight direction
$A(\theta)$ [in dB]	$= -\min\left\{12\left(\frac{\theta}{70^\circ}\right)^2, 20\right\}$ (antenna beam pattern)
λ	carrier wavelength
γ	pathloss exponent
$\boldsymbol{z}^{[m,\Omega]}_{b,k,b',\xi} \in \mathbb{C}^M$	random vector, see [22] for details

user k in cell b as

$$\boldsymbol{h}^{[m,\Omega]}_{b,k,b'} = \sum_{\xi=1}^{6} \sqrt{\frac{10^{-2n_{b,k,b'}}\ \sigma^2_{\mathrm{PDP}_{b,k,b',\xi}}}{M}} \times$$

$$\boldsymbol{A}(\theta_{b,k,b'} + \phi_{b,k,b',\xi})\boldsymbol{G}(d_{b,k,b'}, \theta_{b,k,b'} + \phi_{b,k,b',\xi})\boldsymbol{z}^{[m,\Omega]}_{b,k,b',\xi} \quad (1)$$

assuming handsets have a single receive antenna and macro base stations have N_{a} transmit antennas. All parameters and random variables in (1) are summarized in Table I.

The channel to a single-antenna femto base station f, namely,

$$h^{[m,\Omega]}_{b,k,f} = \sqrt{10^{-2n_{b,k,b'}} \rho\left(d_{b,k,f}, 0\right)} \tilde{z}^{[m,\Omega]}_{b,k,f} \in \mathbb{C} \quad (2)$$

is modeled with fixed penetration loss instead of log-normal shadowing. Moreover, $\tilde{z}^{[m,\Omega]}_{b,k,f}$ is a zero-mean unit-variance circularly symmetric complex Gaussian random variable—justified by the fact that femto base stations are experiencing rich scattering environments—to facilitate closed-form expressions for the proposed algorithm.

Due to the very large number of users that a macro base station must schedule—one on each time-frequency resource $[m, \Omega]$—the underlying probability density functions for the power control $P^{[m,\Omega]}_b \in \mathbb{R}_+$ and the unit-norm beamforming vector $\boldsymbol{t}^{[m,\Omega]}_b \in \mathbb{C}^{N_{\mathrm{a}}}$ generally are non-stationary. With $B - 1$ interfering macro base stations, B_{f} interfering femto base stations, and thermal noise with variance σ^2_η the instantaneous *signal-to-interference-and-noise ratio* (SINR) for a macro user k in cell b at time m on sub-carrier Ω, given as

$$\mathrm{SINR}^{[m,\Omega]}_{b,k} =$$

$$\frac{P^{[m,\Omega]}_b \left|\boldsymbol{h}^{[m,\Omega],\mathrm{T}}_{b,k,b} \boldsymbol{t}^{[m,\Omega]}_b\right|^2}{\sigma^2_\eta + \sum\limits_{\substack{b'=1 \\ b' \neq b}}^{B} P^{[m,\Omega]}_{b'} \left|\boldsymbol{h}^{[m,\Omega],\mathrm{T}}_{b,k,b'} \boldsymbol{t}^{[m,\Omega]}_{b'}\right|^2 + \sum\limits_{f=1}^{B_{\mathrm{f}}} P^{[m,\Omega]}_f \left|h^{[m,\Omega]}_{b,k,f}\right|^2} \quad (3)$$

becomes a random variable.

III. PROBLEM FORMULATION AND PROPOSED ALGORITHM

Suppose that each base station uses the same constant transmit power for each OFDM sub-carrier as often the case in practical systems. To simplify notation, we will assume each macro base station uses the same transmit power such that $P^{[m,\Omega]}_b = E_{\mathrm{tr}}$. Similarly, $P^{[m,\Omega]}_f = P_{\mathrm{f}_b}$ for the femto-layer in cell b. Define the worst-case SINR for a given macro user as

$$\mathrm{SINR}_{b,k} = \frac{E_{\mathrm{tr}} \operatorname{tr}\left(\boldsymbol{R}_{b,k,b}\right)}{\sigma^2_\eta + \sum\limits_{\substack{b'=1 \\ b' \neq b}}^{B} E_{\mathrm{tr}} \operatorname{tr}\left(\boldsymbol{R}_{b,k,b'}\right) + P_{\mathrm{f}_b} \left|h^{[m,\Omega]}_{b,k,f}\right|^2}. \quad (4)$$

Note that this SINR expression cannot be measured anywhere in the system. Furthermore, it assumes that cross-tier interference originates from a single femto base station, namely, the one closest to the macro user. This assumption is justified for low density femtocell deployments since femto base stations transmit at very low power levels, have small form factors (viz., small antenna gain), and are encapsulated by the building structure. More importantly, $\mathrm{SINR}_{b,k}$ in (4) is a random variable by virtue of $h^{[m,\Omega]}_{b,k,f}$. Let $\mathit{\Gamma}$ be the target SINR above which $\mathrm{SINR}_{b,k}$ must lie for user (b, k) to be able to decode the received information, viz., $\mathrm{SINR}_{b,k} \geq \mathit{\Gamma}$ and recall that the randomness in $\mathrm{SINR}_{b,k}$ stems from the fast fading and the pathloss to the femto base station as modeled in (2).

Since we assume single-antenna femto base stations, power control at the femto-layer is the only means to protect macro users from outage, cf. P_{f_b} in (4). In other words, P_{f_b} has to be chosen such that $\mathrm{SINR}_{b,k} \geq \mathit{\Gamma}$. The baseline assumption for femtocells is that they use Ethernet connections to communicate with other base stations as opposed to the operator-deployed base stations which use the much faster X2 backhaul [23]. Consequently, the delays for any *enhanced intercell interference coordination* (eICIC) between the femto-layer and the macro-layer are in the order of seconds and thus orders of magnitude larger than the scheduling granularity m prohibiting any dynamic cooperation. Furthermore, any eICIC scheme has to scale with the number of femto base stations per cell which is expected to grow as heterogeneous networks are rolled out. It is thus not feasible for a macro user to estimate the pathloss to a femto base station. In addition, for a macro user to be able to estimate the pathloss from pilots, it needs to know the transmit power of the pilots, i.e., the parameter we wish to determine. To overcome these obstacles, we propose to compute the femto-layer transmit power P_{f_b} for cell b at the macro base station which then broadcasts P_{f_b} to all femto base stations in that cell. From (4) and the requirement that $\mathrm{SINR}_{b,k} \geq \mathit{\Gamma}$, we have that

$$d^{-\gamma_{\mathrm{f}}}_{b,k,f} \left|\tilde{z}^{[m,\Omega]}_{b,k,f}\right|^2 \leq \frac{E_{\mathrm{tr}} \operatorname{tr}\left(\boldsymbol{R}_{b,\hat{k},b}\right)}{10^{-2}\ 10^{0.1\hat{A}_{\mathrm{f}}} \left(\lambda/4/\pi\right)^2 \mathit{\Gamma} P_{\mathrm{f}_b}}$$

$$-\frac{\mathit{\Gamma}\left(\sigma_\eta^2+\sum\limits_{\substack{b'=1\\ b'\neq b}}^{B} E_{\mathrm{tr}}\,\mathrm{tr}\left(\boldsymbol{R}_{b,\hat{k},b'}\right)\right)}{10^{-2}\,10^{0.1\hat{A}_{\mathrm{f}}}\left(\lambda/4/\pi\right)^2\mathit{\Gamma} P_{\mathrm{f}_b}} \overset{\mathrm{def.}}{=} \frac{1}{\tilde{\mathit{\Gamma}}_b P_{\mathrm{f}_b}} \qquad (5)$$

where $\hat{k}$ denotes the outdoor user experiencing the worst SINR in cell b. If femto base stations have different antenna gains $\hat{A}_{\mathrm{f}}$, the macro base station assumes $\hat{A}_{\mathrm{f}} = 0\mathrm{dB}$ and each femto base station adapts its transmit power in dBm to $10\log_{10}\left(1000 P_{\mathrm{f}_b}\right) - \hat{A}_{\mathrm{f}}$.

Since the users are randomly uniformly distributed within the coverage area we can model them through a Poisson point process [21] in $\mathbb{R}^2$ with intensity λ_{f}. From [24] we have that the cumulative distribution function of $Z = d_{b,\bar{k},f}^{-\gamma_{\mathrm{f}}}\left|\tilde{z}_{b,\bar{k},f}^{[m,\Omega]}\right|^2$ is given as

$$F_Z(z) = 1 - \int\limits_0^\infty \frac{2}{\gamma_{\mathrm{f}}}\lambda_{\mathrm{f}}\pi\bar{y}^{\frac{2}{\gamma_{\mathrm{f}}}-1}\mathrm{e}^{-\left(\mu z\bar{y}+\lambda_{\mathrm{f}}\pi\bar{y}^{\frac{2}{\gamma_{\mathrm{f}}}}\right)}\mathrm{d}\bar{y} \qquad (6)$$

which allows us to evaluate the probability (cf. (5)) that

$$\mathbb{P}\left[d_{b,\bar{k},f}^{-\gamma_{\mathrm{f}}}\left|\tilde{z}_{b,\bar{k},f}^{[m,\Omega]}\right|^2 \leq \frac{1}{\tilde{\mathit{\Gamma}}_b P_{\mathrm{f}_b}}\right] = 1-\varepsilon_b. \qquad (7)$$

The pathloss exponent for channels to femtocells is given in [25] as $\gamma_{\mathrm{f}} = 2$ yielding

$$F_Z(z) = 1-\int\limits_0^\infty \lambda_{\mathrm{f}}\pi\mathrm{e}^{-(\mu z+\lambda_{\mathrm{f}}\pi)\bar{y}}\mathrm{d}\bar{y} = 1-\frac{\lambda_{\mathrm{f}}\pi}{\mu z+\lambda_{\mathrm{f}}\pi} = \frac{\mu z}{\mu z+\lambda_{\mathrm{f}}\pi} \qquad (8)$$

and finally, with $\mu = \frac{1}{2}$,

$$F_Z(z) = \frac{z}{z+2\lambda_{\mathrm{f}}\pi}. \qquad (9)$$

The parametrization of P_{f_b} by ε_b results from

$$\mathbb{P}\left[d_{b,\bar{k},f}^{-\gamma_{\mathrm{f}}}\left|\tilde{z}_{b,\bar{k},f}^{[m,\Omega]}\right|^2 \leq \frac{1}{\tilde{\mathit{\Gamma}}_b P_{\mathrm{f}_b}}\right] = 1-\varepsilon_b = F_Z\left(\frac{1}{\tilde{\mathit{\Gamma}}_b P_{\mathrm{f}_b}}\right) = \frac{1}{1+2\lambda_{\mathrm{f}}\pi\tilde{\mathit{\Gamma}}_b P_{\mathrm{f}_b}} \qquad (10)$$

as

$$P_{\mathrm{f}_b} = \frac{\varepsilon_b}{2\lambda_{\mathrm{f}}\pi\tilde{\mathit{\Gamma}}_b(1-\varepsilon_b)}. \qquad (11)$$

The parameter of the Poisson point process can be obtained from $\lambda_{\mathrm{f}} = |\Sigma_b|/\mathcal{H}_b$ with $\mathcal{H}_b$ and $|\Sigma_b|$ signifying the area of cell b and the number of active users connected in that cell. We propose that each macro base station b computes P_{f_b} from (11) as the transmit power level for all femto base stations in that cell. In particular, each macro base station broadcasts

$$\hat{P}_{\mathrm{f}_b} = \max\left\{\min\left\{P_{\max}, P_{\mathrm{f}_b}\right\}, P_{\min}\right\} \qquad (12)$$

to all femto base stations in its sector using the consumer-deployed Ethernet backhaul. $P_{\min}$ and $P_{\max}$ in (12) are the minimum and maximum transmit power for femtocells, respectively.

TABLE II

OVERVIEW OF SIMULATION PARAMETERS – MACRO NETWORK

Number of cells (B):	57 (hexagonal grid, 3 sectors/site)
Distance between BSs (ISD):	500m
Number of users per cell (K):	60
Number of antennas per BS (N_{a}):	4
Transmit power (E_{tr}):	46dBm
Thermal noise power (σ_η^2):	-100.8dBm
Target SINR:	-10dB
Carrier wavelength:	15cm
Pathloss exponent:	3.8
Penetration loss exterior wall:	20dB
Shadowing standard deviation:	not modeled
Angular spread:	2°
Antenna beam pattern:	3-sector
3dB beamwidth:	70°
Maximum antenna gain:	14dBi
Maximum antenna attenuation:	20dB
Min. distance user to BS:	35m
User distribution:	uniform, users may be in a house
Maximum user velocity:	0km/h

TABLE III

OVERVIEW OF SIMULATION PARAMETERS – FEMTOCELLS

Number of femtocells in macro cell (B_{f}):	1, 3, or 5
Min. distance macro to femto BS:	35m
Number of users per femtocell:	1
Number of antennas per BS (N_{f}):	1
Min./Max. transmit power:	-10dBm/20dBm
Thermal noise power:	-100.8dBm
Carrier wavelength:	15cm
Pathloss exponent:	2.0
Penetration loss exterior wall:	20dB
Shadowing standard deviation:	not modeled
Angular spread:	not modeled
Maximum antenna gain:	5dBi (omni-directional)
House size:	12m×12m
Min. distance user to femto BS:	1m
Max. distance femto user to femto BS:	always inside house
Distribution of houses:	uniform
BS distribution within house:	uniform
User distribution within house:	uniform
Maximum user velocity:	0km/h

IV. SIMULATION RESULTS

To assess the performance of the proposed algorithm, we analyze the cumulative distribution function of (3) for a single cell in the center of the network and a single sub-carrier.[1] All simulation parameters are summarized in Tables II and III. We refer to [27] for a more detailed description.

Figure 1 compares the proposed scheme to the case of no power control, i.e., $\hat{P}_{\mathrm{f}_b} = P_{\max}$, for one femtocell per cell. Without power control, 41% of the macro users (MUEs) experience an SINR $< -10\mathrm{dB}$ and thus cannot decode the message. For $\varepsilon_b = 10\%$, this number is reduced to almost 18%

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[1] Intercell interference is modeled explicitly for coherent beamforming with perfect channnel estimation, see [26] for the macro-layer scheduling algorithm. Also note that the assumptions in the previous section were merely used to obtain the closed-form expression in (11), whereas the simulations are in accordance with the system model in Section II.

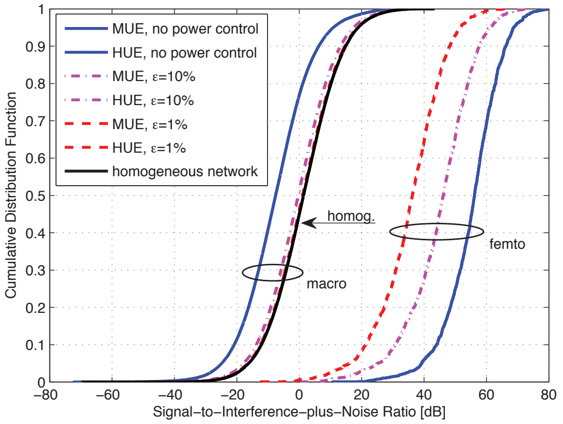

Fig. 1. Using the proposed power control algorithm considerably improves the interference conditions for macro users (left), whereas the degradation for users connected to the single femtocell (right) is of no practical importance.

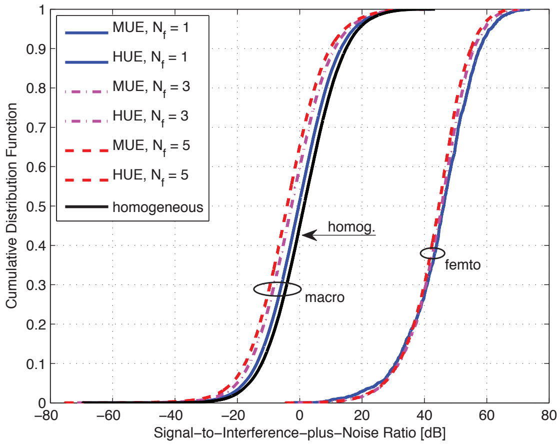

Fig. 2. For $\varepsilon_b = 10\%$, the femto-layer transmit power is still large enough for an increasing number of femto base stations per cell to have an impact on the performance of the macro users (left). In this sub-urban setting, the spacing between houses is enough for femto users (right) to not suffer from interference from other femto base stations.

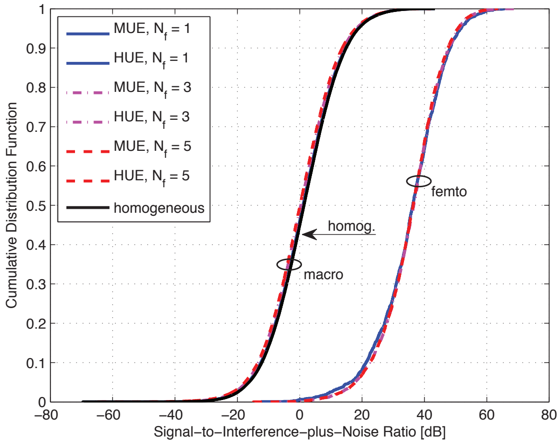

Fig. 3. For small ε_b, the transmit power at the femto-layer is so small that the performance both on the macro-layer (left) and the femto-layer (right) is barely impacted as the number of femto base stations increases.

and it is further decreased to 15% when $\varepsilon_b = 1\%$.[2] For the home users (HUEs) connected to a femtocell, even though the SINR considerably worsens as ε_b decreases, this is of limited importance as in a practical system, once the best modulation and coding scheme is used over the entire system bandwidth, the throughput cannot be increased and saturates, see, e.g., [3].

Figures 2 and 3 compare the impact of multiple femtocells per macro cell for $\varepsilon_b = 10\%$ and $\varepsilon_b = 1\%$, respectively. For larger ε_b (Fig. 2), the probability of an SINR < -10dB increases to 24% (28%) for 3 (5) femtocells per cell. If $\varepsilon_b = 1\%$, the femtocells are isolated enough and an additional femtocell increases the outage probability merely by 0.5%.

Unfortunately, the proposed algorithm requires each handset to estimate the channel covariance matrix $\boldsymbol{R}_{b,k,b'}$ to all macro base stations. We thus also compare the performance if the handsets use the *reference signal received power* (RSRP) instead, which each handset has to measure for cell-(re)selection anyway [23]. The RSRP from base station b' for user k in cell b is defined as

$$\mathrm{RSRP}_{b,k,b'} = E_{\mathrm{tr}}\, 10^{-2 n_{b,k,b'}}\, \rho(d_{b,k,b'}, \theta_{b,k,b'}) \tag{13}$$

assuming the reference signal is transmitted from a single antenna element with power E_{tr} and can be used instead of $E_{\mathrm{tr}}\,\mathrm{tr}\left(\boldsymbol{R}_{b,k,b'}\right)$. As can be seen from Fig. 4, using the RSRP hardly impacts the performance and makes the algorithm practically feasible since it is solely based on measurements that are readily available in a real-world system.

Finally, Fig. 5 depicts the distribution of $\hat{P}_{\mathrm{f}_b}$. Both CDFs are clipped at $P_{\min} = -10$dB and as expected for a small $\varepsilon_b = 1\%$ it occurs considerably more frequently. At the other end, for both ε_b the scheme does not use the entire dynamic range up to $P_{\max} = 20$dB. For $\varepsilon_b = 1\%$, the slope is almost flat beyond 0dB. For $\varepsilon_b = 10\%$, this threshold is at about 10dB. Figure 5 demonstrates the seamless quasi-static power control between $P_{\min}$ and $P_{\max}$ as the macro users report their estimates of (4) and the macro base stations update the transmit power in (11) based on the mobility of the users (i.e., the coherence time of the channel) and the delay in the backhaul to the femto base stations.

V. CONCLUSION

Using stochastic geometry, we derived a probabilistic power control scheme for heterogeneous cellular networks with closed-access femtocells. The algorithm determines a transmit power level for all femtocells in a given cell and can be executed at the macro base station which then broadcasts the solution to all femtocells. The algorithm does not require

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[2] the difference to ε_b comes from the clipping in (12) and our assumptions, e.g., $n_{b,k,b'} = 1\ \forall b, k, b'$ and it grows as $\varepsilon_b \to 0$

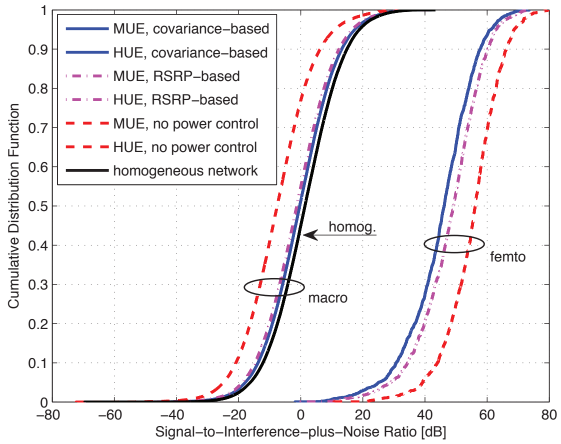

Fig. 4. Instead of estimating the channel covariance matrices to all macro base stations in the network, macro users (left) can use the RSRP measurements for cell-(re)selection instead. This renders the algorithm highly practical without sacrificing performance.

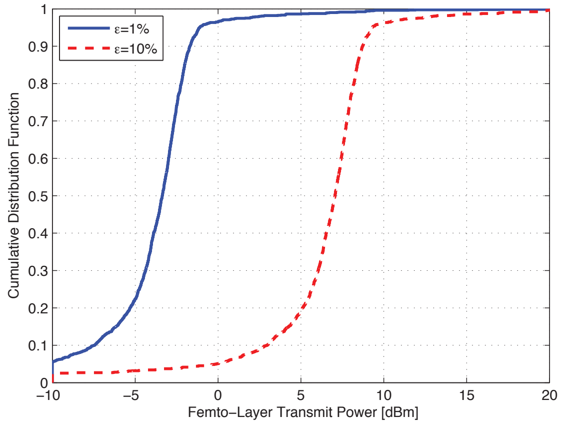

Fig. 5. Both CDFs are clipped at $P_{\text{min}} = -10$dB and as expected for a small $\varepsilon_b = 1\%$ it occurs considerably more frequently.

any information from neighboring macro base stations or any femto base stations. In particular, the algorithm can be implemented by using only those measurements readily available at the macro base station offering coverage gains as demonstrated by system level simulations.

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