Spatial probability of corrosion in RC structures conditional on measurements

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ABSTRACT: Measurements and monitoring are commonly applied to reduce the uncertainty associated with deterioration in reinforced concrete (RC) structures. Because the information obtained from these measurements is commonly uncertain or indirect, Bayesian updating should be utilized to determine the condition of the structure conditional on this information. Additionally, due to the scale of concrete structures, the spatial variability of the deterioration processes should be accounted for. In this paper, a recently developed method for reliability updating [Straub D., *Probabilistic Engineering Mechanics*, under review] is investigated for Bayesian updating of spatially distributed deterioration in RC structures. The method is combined with a simple but robust importance sampling technique. Its application is demonstrated on numerical examples considering structures subject to corrosion of the reinforcement due to chloride ingress, for which measurements of chloride concentrations are available.

1 INTRODUCTION

When assessing the deterioration reliability of structures, the probability of the structure (or component) reaching an adverse state F is of interest. In reinforced concrete (RC) structures subject to corrosion of the reinforcement, F might represent initiation of corrosion, spalling of the concrete, a critical loss of cross-section or failure of the structure due to corrosion (Stewart and Val 2003. In many instances, information on the deterioration process becomes available during the service life of the structure, e.g. through measurements, inspection or monitoring of structures, which can be used to update the reliability estimate. This information is commonly uncertain and often indirect.

Uncertain information represented by an event Z is considered by determining the conditional probability of failure given the information event Z, defined as

$$\Pr(F \mid Z) = \frac{\Pr(F \cap Z)}{\Pr(Z)} \tag{1}$$

The process of computing this conditional probability is commonly referred to as Bayesian updating or information updating. It has been applied in the context of structural reliability since the 1970s (e.g. Tang 1973).

In structural reliability, failure events F and information events Z are described by domains Ω in the outcome space of the basic random variables $\mathbf{X} = (X_1, X_2, \ldots, X_n)$. The failure domain Ω_F is defined in terms of continuous limit state functions $g(\mathbf{x})$. In the simplest case, it is

$$\Omega_F(\mathbf{x}) = \{g(\mathbf{x}) \le 0\} \tag{2}$$

In the general case, Ω_F is defined in terms of a number of limit state functions (e.g., Der Kiureghian 2005), corresponding to systems of components that are defined by limit state functions. For the purpose of the present paper, the formulation in (2) is sufficiently general; extension to the system application is straightforward (Straub submitted).

Information obtained on the system, e.g. in the form of measurements, monitoring, inspection, observed system performance, is also described in terms of continuous limit state functions $h(\mathbf{x})$ and corresponding domains. Information is said to be of the inequality type if it can be written as

$$\Omega_Z(\mathbf{x}) = \{h(\mathbf{x}) \le 0\} \tag{3}$$

and it is said to be of the equality type if it can be written as

$$\Omega_Z(\mathbf{x}) = \{h(\mathbf{x}) = 0\} \tag{4}$$

Structural reliability methods (SRM) solve Eq. (1) by computing integrals in the space of $\mathbf{X}$:

$$\Pr(F \mid Z) = \frac{\int_{\mathbf{x} \in \{\Omega_F(\mathbf{x}) \cap \Omega_Z\}} f(\mathbf{x}) \mathrm{d}\mathbf{x}}{\int_{\mathbf{x} \in \{\Omega_Z\}} f(\mathbf{x}) \mathrm{d}\mathbf{x}} \tag{5}$$

If information is exclusively of the inequality type, Eq. (3), evaluation of the above integrals is straightforward using any of the available SRM. However, if the information event Z is of the equality type, the integrals result in zero, since these events have zero probability a-priori. Direct application of SRM is thus not possible in this case.

Solutions to overcome this problem have been suggested by Madsen (1987) and the group of Rackwitz (e.g. Schall et al. 1988). Madsen's solution is based on inserting a dummy random variable and then computing probability sensitivities with respect to this variable. The solutions of the Rackwitz group are based on computing surface integrals, using first- or second order approximations of the surfaces $h_i(\mathbf{x})=0$. These solutions are implemented in the Strurel software (Gollwitzer et al. 2006). Both Madsen's and Rackwitz' methods are efficient and, in many cases, represent a sufficiently accurate approximation. However, in cases where FORM/SORM solutions are not sufficiently accurate or in which it is difficult to identify the joint design point, these methods should not or cannot be used. Furthermore, it is often difficult to appraise the error made by the first- or second-order approximation.

Recently, the author has proposed a novel method for solving Eq. (5) using SRM when information is of the equality type (Straub submitted). The method is based on transforming equality information into inequality information, which enables the direct use of Eq. (5) using any SRM. The aim of the present paper is to study the application of the methodology to deterioration reliability problems in spatially distributed systems. For such systems, commonly a large number of reliability problems must be solved simultaneously, which requires that the applied algorithms are robust and efficient.

2 BAYESIAN UPDATING WITH EQUALITY INFORMATION USING STOCHASTIC SIMULATION

This section presents a summary of the method described in Straub (submitted), with slight modifications in view of the considered application to deterioration reliability updating in spatially distributed systems.

2.1 *Describing information through likelihood functions*

We note that in statistics, information is not commonly described in the form of domains Ω_Z. Instead, the usual way to describe (uncertain) information on $\mathbf{X}$ is by means of the likelihood function, which is defined as

$$L(\mathbf{x}) \propto \Pr(Z \mid \mathbf{X}=\mathbf{x}) \tag{6}$$

As noted in Straub (submitted), any domain Ω_Z can be translated into a likelihood function. However, it is often more convenient to directly identify the likelihood function. As an example, consider a measurement s_m of a system characteristic $s(\mathbf{X})$. The measurement has an additive error ε that is a zero mean random variable uncorrelated with $\mathbf{X}$. The limit state function $h(\mathbf{X},\varepsilon)$ describing this equality information as well as the corresponding likelihood function are given in the following, with $f_\varepsilon()$ being the PDF of ε.

$$h(\mathbf{X},\varepsilon) = s(\mathbf{X}) - s_m + \varepsilon \tag{7}$$

$$L(\mathbf{x}) = f_\varepsilon(s_m - s(\mathbf{x})) \tag{8}$$

For the case of several observations $Z_1,...,Z_m$ with corresponding likelihood functions $L_i(\mathbf{x})$, it is always possible to combine the likelihood functions into a single likelihood function $L(\mathbf{x})$. E.g., if measurements are uncorrelated for given $\mathbf{X}=\mathbf{x}$, it is simply $L(\mathbf{x}) = \Pi_{i=1}^{m} L_i(\mathbf{x})$. It is thus sufficient to consider only the case of a single likelihood function describing combined observations $Z = Z_1 \cap ... \cap Z_m$ in the following.

2.2 *Transform equality information into equivalent inequality information*

Let P be a random variable with uniform distribution in the range [0,1] and let c be a constant that is selected so that $0 \le cL(\mathbf{x}) \le 1$ for all $\mathbf{x}$. In this case, the following identity holds for given values of $\mathbf{X}=\mathbf{x}$:

$$L(\mathbf{x}) = \frac{\Pr\left[P \le cL(\mathbf{x})\right]}{c} \tag{9}$$

Let α denote the proportionality constant in the likelihood definition given in Eq. (6). By combining with Eq. (9), we obtain

$$\Pr(Z \mid \mathbf{X}=\mathbf{x}) = \alpha L(\mathbf{x}) = \frac{\alpha}{c}\Pr\left[P \le cL(\mathbf{x})\right] \tag{10}$$

It follows that the probability of the information event Z is

$$\begin{aligned}\Pr(Z) &= \int_{\mathbf{X}} \Pr(Z \mid \mathbf{X}=\mathbf{x}) f(\mathbf{x})\mathrm{d}\mathbf{x} \\ &= \frac{\alpha}{c}\int_{\mathbf{X}} \left[P \le cL(\mathbf{x})\right] f(\mathbf{x})\mathrm{d}\mathbf{x}\end{aligned} \tag{11}$$

Next, we define the event $Z_e = \{P \le cL(\mathbf{x})\}$ through the limit state function

$$h_e(\mathbf{x},p) = p - cL(\mathbf{x}) \tag{12}$$

and the corresponding domain $\Omega_{Ze}(\mathbf{x},p) = \{h_e(\mathbf{x},p) \le 0\}$. This has the same form as the domains describing inequality information, Eq. (3). Equation (11) can now be rewritten to

$$\Pr(Z) = \frac{\alpha}{c}\int_{\mathbf{x},p\in Z_e(\mathbf{x},p)} f(p) f(\mathbf{x}) \mathrm{d}\mathbf{x}\mathrm{d}p \\ = \frac{\alpha}{c}\int_{\mathbf{x},p\in \Omega_{Ze}(\mathbf{x},p)} f(\mathbf{x}) \mathrm{d}\mathbf{x}\mathrm{d}p \quad (13)$$

The second identity follows from $f(p)=1$. Accordingly, $\Pr(F\cap Z)$ is obtained as

$$\Pr(F\cap Z) = \int_{\mathbf{X}} \Pr(Z\mid \mathbf{X}=\mathbf{x})\Pr(F\mid \mathbf{X}=\mathbf{x}) f(\mathbf{x})\mathrm{d}\mathbf{x} \\ = \frac{\alpha}{c}\int_{\mathbf{x},p\in\{\Omega_F(\mathbf{x})\cap\Omega_{Ze}(\mathbf{x},p)\}} f(\mathbf{x})\mathrm{d}\mathbf{x}\mathrm{d}p \quad (14)$$

The conditional probability of F given Z is therefore

$$\Pr(F\mid Z) = \frac{\int_{\mathbf{x},p\in\{\Omega_F(\mathbf{x})\cap\Omega_{Ze}(\mathbf{x},p)\}} f(\mathbf{x})\mathrm{d}\mathbf{x}\mathrm{d}p}{\int_{\mathbf{x},p\in\Omega_{Ze}(\mathbf{x},p)} f(\mathbf{x})\mathrm{d}\mathbf{x}\mathrm{d}p} \quad (15)$$

Here, the proportionality constant α disappears. Both integrals in Eq. (15) can be computed using any SRM. The denominator corresponds to a component reliability problem, the nominator to a parallel system reliability problem.

2.3 *Simple importance sampling techniques for evaluating the conditional reliability*

When considering spatial deterioration reliability, the conditional probability $\Pr(F\mid Z)$ must be computed for different locations in space. Therefore, a large number of evaluations of the integrals in Eq. (15) are potentially required. It is therefore important to find a SRM that provides an optimal trade-off between computational robustness and efficiency. Robustness means that a method can be applied without problem specific adjustments and efficiency means that only a limited number of evaluations of the limit state functions are required.

As shown in Straub (submitted), FORM/SORM techniques are not generally suitable due to the fact that the limit state surfaces $h_e(\mathbf{x},p)=0$, which describe the information, are generally highly nonlinear. As shown in Straub (submitted), by applying an advance importance sampling schemes around the design point (axis-parallel importance sampling), accurate results can be achieved efficiently. Unfortunately, the method cannot generally be considered robust, because of the need to find a different design point for the computation of the conditional probability at every location.

Monte Carlo simulation (MCS) is the most robust SRM. However, as shown in Straub (2010), MCS becomes highly inefficient when more than just a few observations are available, due the fact that the effective number of samples available to compute the conditional reliability diminishes with increased information content of Z. For this reason, direct application of MCS to solve the integrals in (15) is not recommended.

As an optimal trade-off between robustness and efficiency, the use of a simple importance sampling (IS) scheme is suggested. The IS estimator for the conditional probability $\Pr(F\mid Z)$ in Eq. (15) is

$$\Pr(F\mid Z) \approx \frac{\sum_{i=1}^{n_s} I[h_e(\mathbf{x}_i,p_i)\le 0] I[g(\mathbf{x}_i)\le 0]\frac{f_{\mathbf{X}}(\mathbf{x}_i)}{\psi(\mathbf{x}_i,p_i)}}{\sum_{i=1}^{n_s} I[h_e(\mathbf{x}_i,p_i)\le 0]\frac{f_{\mathbf{X}}(\mathbf{x}_i)}{\psi(\mathbf{x}_i,p_i)}} \quad (16)$$

wherein the samples $\mathbf{x}_i$ and p_i are simulated from a distribution with sampling density $\psi(\mathbf{x},p)$. Following Straub (2010), $\psi(\mathbf{x},p)$ is split into

$$\psi(\mathbf{x},p) = \psi_1(\mathbf{x})\psi_2(p\mid\mathbf{x}) \quad (17)$$

where $\psi_1(\mathbf{x})$ is the sampling PDF of $\mathbf{X}$ and $\psi_2(p\mid\mathbf{x})$ is the conditional sampling density of P given $\mathbf{X}=\mathbf{x}$. An optimal conditional sampling density $\psi_2(p\mid\mathbf{x})$ that is valid for any application of Eq. (16) is given as

$$\psi_2(p\mid\mathbf{x}) = \frac{1}{cL(\mathbf{x})}, \quad 0\le p\le cL(\mathbf{x}) \quad (18)$$

If it holds that $L(\mathbf{x})>0$ for any $\mathbf{x}$, then $I[h_e(\mathbf{x}_i,p_i)\le 0]=1$ for any value of p_i that is sampled from the above conditional density $\psi_2(p\mid\mathbf{x})$. In this case, Eq. (16) reduces to

$$\Pr(F\mid Z) \approx \frac{\sum_{i=1}^{n_s} I[g(\mathbf{x}_i)\le 0]\frac{f_{\mathbf{X}}(\mathbf{x}_i)L(\mathbf{x}_i)}{\psi_1(\mathbf{x}_i)}}{\sum_{i=1}^{n_s}\frac{f_{\mathbf{X}}(\mathbf{x}_i)L(\mathbf{x}_i)}{\psi_1(\mathbf{x}_i)}} \quad (19)$$

Note that the constant c disappears when using this sampling density. For the case of selecting the prior PDF of $\mathbf{X}$ as its sampling density, i.e., $\psi_1(\mathbf{x}_i)=f_{\mathbf{X}}(\mathbf{x}_i)$, the above reduces to the MCS solution of

$$\Pr(F\mid Z) = \int_{\mathbf{X}} I[g(\mathbf{x})\le 0] f''(\mathbf{x}\mid Z)\mathrm{d}\mathbf{x} \quad (20)$$

where $f''(\mathbf{x}\mid Z)$ is the posterior distribution of $\mathbf{X}$ given the information Z.

For spatially distributed systems described by homogenous probabilistic deterioration models, it is suggested in Straub (2010) to use as sampling density $\psi_1(\mathbf{x})$ a distribution centered around the design point $\mathbf{x}=\mathbf{u}_i$ corresponding to failure at location i (a-priori, i.e. before the observation). This implies the use of a different sampling density for computing the reliability at every location i. However, the identification of the design point $\mathbf{u}_i$ is straightforward, since it suffices to find the values of the design point for the variables at the location i. These values are identical for any i, due to the assumption of homogeneity. The design point values of the random variables at the other locations are then found as the mode of the conditional distributions, which are readily identified if the random fields are modeled by a Gaussian copula (the Nataf model).

In Straub (2010) it was found that a sampling density $\psi_1(\mathbf{x})$ with a Multinormal distribution with mean equal to the design point $\mathbf{u}_i$ and covariance function equal to that of the prior distribution $f_{\mathbf{X}}(\mathbf{x}_i)$ performs well. This sampling density is applied in the following application.

3 APPLICATION TO SPATIAL RELIABILITY UPDATING OF CORROSION IN RC STRUCTURES

We consider a reinforced concrete (RC) surface that is subject to corrosion of the reinforcement caused by chloride ingress. The method presented in the previous section is applied to compute the spatial probability of corrosion conditional on measurements of chloride penetration. These measurements are obtained from cores taken at discrete locations of the surface.

3.1 *Spatial model of chloride-induced reinforcement corrosion*

We utilize a diffusion model to describe chloride ingress and initiation of corrosion at the reinforcement, which corresponds to a simplified version of the probabilistic models developed in the Duracrete project (fib 2006). The chloride concentration C_z at a depth z at time t is described by the following solution of the one-dimensional linear diffusion equation:

$$\frac{C_s - C_z}{C_s - C_0} = \operatorname{erf}\left(\frac{z}{\sqrt{4Dt}}\right) \tag{21}$$

where C_s is the concentration of chloride at the concrete surface, C_0 is the concentration of chlorides in the concrete at time zero, D is the diffusion coefficient and $\operatorname{erf}()$ is the error function. In the Duracrete model, C_s is given for different environmental conditions, C_0 is set equal to zero and D is expressed as a function of several variables that represent various material characteristics. For simplicity, we here let D be a single random variable.

The random variables, including their probabilistic model, are explained in Table 1. The values approximately correspond to a concrete surface in a parking deck with water-to-cement ratio equal to 0.4, which is exposed to splash water containing deicing salts.

Table 1. Probabilistic model for one location.

Parameter	Dimension	Distrib.	Parameters
W: Cover depth	mm	LN	μ = 40.0; σ = 8.0.
D : Diffusion coefficient	mm^2/yr	LN	μ = 20.0; σ = 10.0.
C_S : Cl surface concentration	Mass-% of cement	Normal	μ = 3.10; σ = 1.23.
C_{cr} : Critical Cl concentration	Mass-% of cement	Normal	μ = 0.8; σ = 0.1.

The considered surface area has size 5m$\times$10m; for the analysis it is discretized in 200 elements with size 0.5m$\times$0.5m. This choice is made based on the correlation lengths of the random variables that vary with space (see Malioka (2009) for a review of discretization approaches). The random variables that are considered to vary over the area are summarized in Table 2, together with the corresponding correlation length r_X.

Table 2. Modeling of spatial variability.

Parameter	Correlation length r_X [m]
W: Cover depth	1m
D : Diffusion coefficient	2m
C_S : Cl surface concentration	1m
C_{cr} : Critical Cl concentration	1m

All spatially varying random variables X are described by homogenous Gaussian random fields with exponential covariance function:

$$\operatorname{Cov}[X_i, X_j] = \exp(-d_{ij} r_X) \tag{22}$$

wherein d_{ij} is the distance between two points i and j on the concrete surface. The joint distribution of the random variables in the random field is described by a Gaussian copula (i.e. the Multinormal distribution in the case of C_s and C_{cr}, and the Multilognormal distribution in the case of W and D).

3.2 *Failure event*

Here, we define failure F as the event of corrosion initation, which occurs when the chloride concentration C_w exceeds the critical chloride concentration C_{cr}. With the diffusion model, the limit state function for corrosion in element i at time t is obtained as

$$g_{i,t}(\mathbf{X}) = C_{cr,i} - C_{s,i} \operatorname{erf}\left(\frac{W_i}{\sqrt{4D_i t}}\right) \tag{23}$$

3.3 *Measurements*

We consider measurements of chloride concentration made by taking cores from the concrete at selected locations $\mathbf{x}_{m,i}$. The chloride contents C_z at various depths z_m are obtained by chemical analysis of the ground-up concrete. According to the diffusion model, C_z at location j and depth z_m is given as

$$C_{z_m,j} = C_{s,j}\left[1-\operatorname{erf}\left(\frac{z_m}{\sqrt{4D_j t}}\right)\right] \quad (24)$$

The uncertainty in the concentration measurement is modeled by an additive Normal distributed error with zero mean and standard deviation $\sigma_\varepsilon = 0.2$ [mass-% of cement], which is assumed to be statistically independent from one measurement to the next. The likelihood function for one measurement of chloride concentration $c_m(j,z_m)$ at location j is accordingly:

$$L_j(\mathbf{x}) = \frac{1}{\sigma_\varepsilon\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{c_{z_m,j}-c_m(j,z_m)}{\sigma_\varepsilon}\right)^2\right] \quad (25)$$

3.4 *Numerical investigations*

Concrete cores are taken at time $t_m = 10$ years from the surface at different locations. For each core, the chloride content is measured at two depths $z_{m1} = 20\text{mm}$ and $z_{m2} = 40\text{mm}$. Two hypothetical cases of measurement outcomes are considered, as summarized in the following tables:

Table 3. Measurements case 1 [in mass-% of cement].

Location (x and y directions)	$c_m(j,20\text{mm})$	$c_m(j,40\text{mm})$
a: $x = 3.0$, $y = 2.5$	1.0	0.6
b: $x = 7.0$, $y = 2.5$	0.5	0.3

Table 4. Measurements case 2 [in mass-% of cement].

Location (x and y directions)	$c_m(j,20\text{mm})$	$c_m(j,40\text{mm})$
a: $x = 1.0$, $y = 1.0$	0.3	0.1
b: $x = 9.0$, $y = 1.0$	0.5	0.3
c: $x = 1.0$, $y = 4.0$	0.6	0.1
d: $x = 9.0$, $y = 4.0$	0.9	0.3
e: $x = 5.0$, $y = 2.5$	1.4	0.5

The probability of corrosion at every point of the concrete surface, conditional on these measurements, is evaluated with the importance sampling method described above with 10^5 samples. The computation of the results presented hereafter takes in the order of 100 CPU seconds on a standard 1.6 GHz PC.

The a-priori probability of corrosion as a function of time is shown in Figure 1. Since no location-specific information is available prior to the measurements, this probability is identical at all locations.

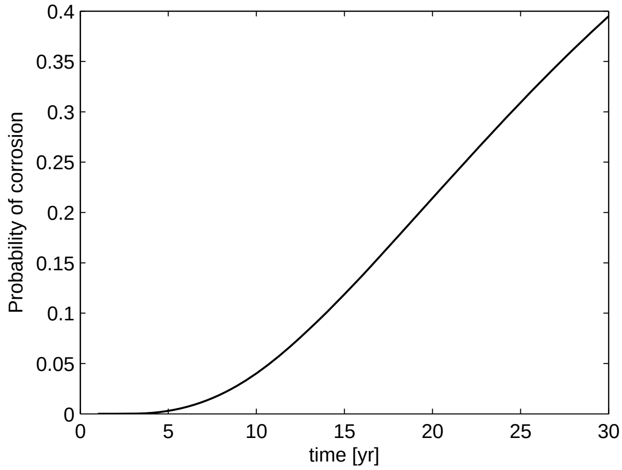

Figure 1. Probability of corrosion prior to measurements (SORM solution).

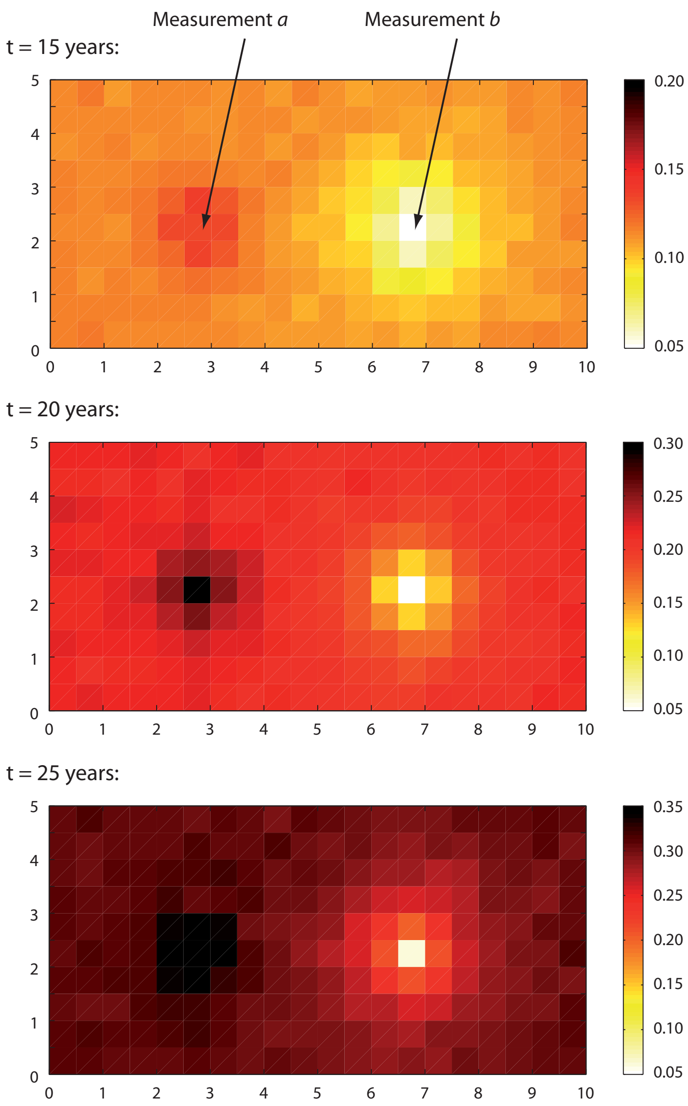

Figure 2. Probability of corrosion conditional on the chloride measurements from Table 3.

For case 1, Figure 2 shows the conditional probability of corrosion for different years. The two measurement locations can be clearly identified from the spatial distribution of the probability. Measurement *a* gives higher observed chloride concentrations and consequently the updated probabilities around the location of this measurement are higher. The opposite is observed for measurement *b*, which gives lower values of c_m. The results also reflect the correlation length of the uncertain model parameters, which are between 1m and 2m. At the points furthest away from the measurements (at the corners), the updated probability of corrosion is close to the prior probability without measurements.

For case 2, Figure 3 shows the conditional probability of corrosion for different years. Measurement *e*, in the center of the area, shows by far the highest concentrations of chlorides. Because there are more measurements available in comparison to case 1, the spatial variability in the updated probability values is higher. There is also higher scatter in the result, which reflects a decrease in the accuracy of the probability estimates obtained with the employed IS approach, as compared to case 1. This is due to the fact that $\Pr(Z_e)$ decreases with increasing amount of information. As shown in Straub (2010), lower values of $\Pr(Z_e)$ lead to lower accuracy with these simulation methods.

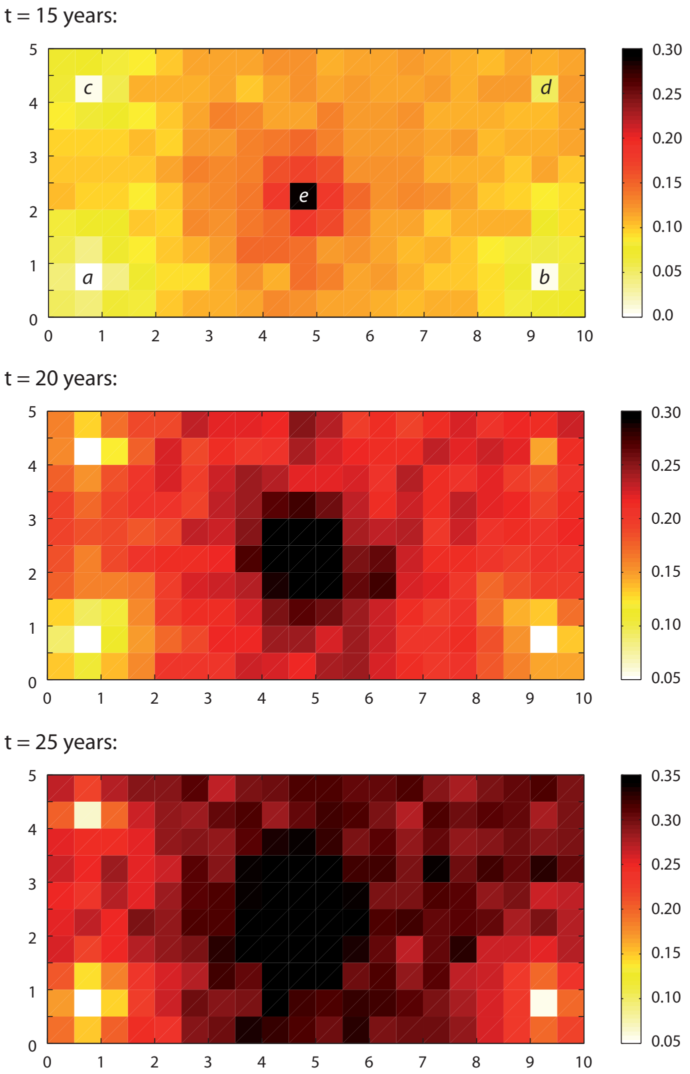

Figure 3. Probability of corrosion conditional on the chloride measurements from Table 4.

CONLUDING REMARKS

The application presented in this paper demonstrates the potential of the method proposed in Straub (submitted) for Bayesian updating of spatial probabilistic models of deterioration with information that is obtained at discrete points in space. In the presented application, this information is the measured chloride content at discrete points in the concrete surface.

The new method proceeds by transforming equality information into equivalent inequality information. In this way, Bayesian updating of the probabilistic model and the reliability estimate with any information can be performed using simple simulation techniques. These techniques have the advantage of being robust, which is of particular relevance in the context of spatially distributed systems, where a large number of conditional probabilities must be computed.

As the amount of information increases, the accuracy of the presented simulation techniques decreases. Further investigations are ongoing on how the techniques can be improved for problems where the amount of information is significantly higher. This is particularly relevant when measurements are made continuously in space.

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