THROUGHPUT MAXIMIZATION FOR ENERGY HARVESTING NODES WITH GENERALIZED CIRCUIT POWER MODELLING

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ABSTRACT

Communication devices nowadays can be made to have energy harvesting ability by equipping, for example, solar cells. The transmission strategy that an energy harvesting node should employ in order to achieve the maximal throughput differs from the one employed by a node with constant power supply. Assuming full knowledge of the energy arrival process, we investigate in this paper the throughput maximizing transmission strategy of an energy harvesting node and utilize a general circuit power model to account for the energy consumption within the circuitry of the node. To this end, the energy harvesting node is treated as a control system and the throughput maximization is formulated and solved within the framework of optimal control. The proposed construction algorithm of the optimal trajectory is applicable to circuit power functions that are convex in transmit power.

1. INTRODUCTION

With the rapid development of wireless communications and networking, environmental issues involved with communications have drawn more and more research attention, and the demand to reduce the overall energy consumption for communications is imperative. The modelling of circuit power consumption of a communication device [1][2] serves as a cornerstone in the improvement of energy efficiency. In the study of energy utilization of an energy harvesting node, which is often deployed in wireless sensor networks, a generalized circuit power modelling with enough practical relevance is certainly necessary. Yet such a consideration has not been taken in the existing literature. In our previous work [3], we have adopted a constant circuit power model and distinguished between two operation modes of the transmitter, the *active* mode in which the node is transmitting and the circuit consumes a constant power, and the *sleep* mode in which the node is silent and does not consume any energy. As a step forward, we employ now a generalized circuit power model represented by the nondecreasing function g_c which is dependent on the instantaneous transmit power p_tx, and formulate the throughput maximization as a control problem. Although the causality constraint on energy utilization complicates the problem, the optimal transmit power function for a simplified form of the throughput maximization can be obtained with the help of optimal control theory. Based on this result, we find that the convexity of g_c plays a key role in whether our algorithm proposed in [3] can be applied to the more general setting.

We start with a detailed description of the system aspects relevant to our control problem in Sec. 2. Solution to the basic problem is explained in Sec. 3, where we assume that g_c is convex but could be continuous or discontinuous at $p_\mathrm{tx} = 0$. Applicability of our construction algorithm from [3] is shown and we shortly review the algorithm and its optimality in Sec. 4, by the end of which we include some numerical results. In the end the paper is concluded in Sec. 5.

2. SYSTEM MODEL AND PROBLEM FORMULATION

We investigate the exploitation of energy by an energy harvesting node which transmits data over a single, invariant link during the time slot $[0, T]$. A continuous-time model is adopted and it is assumed that the transmit power of the node, denoted by $p_\mathrm{tx}(t)$, $t \in [0, T]$, can be adapted continuously.

2.1. Data Transmission Model

Let $f(p_\mathrm{tx}(t))$ be the instantaneous data rate depending on the transmit power, where the function f is assumed to be nonnegative, strictly concave as well as monotonically increasing, and it is invariant over the time slot $[0, T]$ as the channel stays constant. We define the integral of f over the time slot of interest as the *short-term throughput* of the system, or *throughput* in short, and denote it with I:

$$I \triangleq \int_0^T f(p_\mathrm{tx}(t))\,\mathrm{d}t. \tag{1}$$

In the analysis and derivations we do not restrict ourselves to a specific form of f, but deal with all functions f that satisfy the aforementioned conditions. For simulations we have employed the rate function $f(p_\mathrm{tx}) = \log(1 + p_\mathrm{tx})$.

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2.2. Circuit Power Model

Besides the radiated power, there is additional power consumption within the circuit of the transmitter which can not be neglected for short-range communications. Most of the works studying transceiver circuit power models [1][2] consider the D/A converter, the mixer, the filters and the power amplifier as the main contributors to energy consumption of the transmitter. The power consumption of the power amplifier is usually assumed to be linearly dependent on the transmit power, and power consumption of the other components can be assumed constant [4]. We employ a general circuit power model $p_{\mathrm{c}}(t) = g_{\mathrm{c}}(p_{\mathrm{tx}}(t))$ for the instantaneous circuit power as dependent on $p_{\mathrm{tx}}(t)$, where g_{c} is nondecreasing on $[0, +\infty)$. The total power dissipation of the node reads as $p(t) = p_{\mathrm{tx}}(t) + p_{\mathrm{c}}(t)$. The function $g(p_{\mathrm{tx}}) = p_{\mathrm{tx}} + g_{\mathrm{c}}(p_{\mathrm{tx}})$ of p_{tx} which represents the total power dissipation is obviously a monotonically increasing function.

When the transmitter is not sending any signal, it can be turned into *sleep* mode for which we assume there is no energy consumption of the circuit, *i.e.*, $g_{\mathrm{c}}(0) = 0$. Otherwise, the transmitter is considered as in *active* mode and its circuit incurs additional energy consumption, which means $g_{\mathrm{c}}(p_{\mathrm{tx}}) > 0$ for $p_{\mathrm{tx}} > 0$. It is necessary to explicitly define these two operation modes of the transmitter, for this may lead to a discontinuous point of $g_{\mathrm{c}}(p_{\mathrm{tx}})$ at $p_{\mathrm{tx}} = 0$. We allow $p_{\mathrm{tx}} = 0$ to be the single discontinuous point of the function g_{c}, and assume that g_{c} is continuous on $(0, +\infty)$. An example of such g_{c} functions is the linear circuit power model given by

$$g_{\mathrm{c}}(p_{\mathrm{tx}}) = \begin{cases} b \cdot p_{\mathrm{tx}} + c, & p_{\mathrm{tx}} > 0, \\ 0, & p_{\mathrm{tx}} = 0, \end{cases} \tag{2}$$

where b and c are both nonnegative constants.

Viewing the transmitting node as a control system, we let the transmit power function $p_{\mathrm{tx}}(t)$ be the control variable and the function $W(t)$, which represents the total energy consumption of the node until time t, be the state variable. The control system is governed by the differential equation $\dot{W} = g(p_{\mathrm{tx}})$, where by convention we use $\dot{W}$ to denote the derivative of W with respect to t and no longer write t explicitly in time-dependent functions. The initial state of the system is defined as $W(0) = 0$ and the total energy consumption at the end of the time slot is given by $W(T)$. Notice that the energy consumption associated with mode switching is neglected here, but in the design of the optimal transmission strategy, we have taken care that the number of mode switches is well controlled.

2.3. Energy Harvesting Model

The arrival of energy at an energy harvesting node is often a random process, as the energy source is usually not under control. To evaluate the performance limit, we assume in this work that the energy which becomes available during $[0, T]$ is completely known in advance at the transmitter. Let the nondecreasing function $A(t)$ represent the total amount of energy *arrival* by time t. It stands for the cumulative energy that becomes available irrespective of the energy consumption of the node. Due to causality, $W(t) \leq A(t)$ must be satisfied, $\forall t \in [0, T]$. Moreover, physical limitations on the battery give rise to a nondecreasing *departure* function $D(t)$, which represents the minimal amount of energy that has to be consumed by time t in order to avoid energy loss caused by battery overflow. Assuming that the maximum amount of energy the battery can hold is a constant $E_{\max}$, we have

$$D(t) = \max(0, A(t) - E_{\max}), \quad \forall t \in [0, T]. \tag{3}$$

We only restrict A and D to be continuous from the right, which means that our algorithm can be applied to both continuous and discrete energy arrival situations. At a point t_0 of discontinuity on A, we assume that $A(t_0^+) - A(t_0^-) < E_{\max}$, *i.e.*, there is no energy overflow caused by a very large instantaneous energy input. Also notice that the proposed algorithm does not require D to be related to A as given by (3). Instead, we only assume that D is strictly smaller than A, $\forall t \in (0, T)$.

2.4. Throughput Maximization Problem

Our goal is to control the energy harvesting node from time 0 to T, by continuously adapting the transmit power, in such a way that the maximal throughput as defined in (1) can be achieved while the causality constraint is not violated. Mathematically, this can be expressed by the optimization

$$\begin{aligned} \max_{p_{\mathrm{tx}} \in \mathcal{P}} \quad & \int_0^T f(p_{\mathrm{tx}}) \mathrm{d}t \\ \text{s.t.} \quad & \dot{W} = g(p_{\mathrm{tx}}), \\ & W \leq A, \\ & W(0) = 0, \end{aligned} \tag{4}$$

where $\mathcal{P}$ is the set of finite, nonnegative, piecewise continuous functions defined over $[0, T]$. We denote the optimal control to (4) and the corresponding state function with p_{tx}^* and W^*, respectively, although their existence shall only become clear later, and refer to both of them as the *optimal transmission strategy* in the sequel.

As the data rate function f increases monotonically and transmit power can be adapted continuously, we infer that battery overflow does not happen during $[0, T]$ and all energy is exhausted by time T in order to achieve the maximal throughput. Due to these optimality considerations, Problem (4) can be equivalently formulated as

$$\begin{aligned} \max_{p_{\mathrm{tx}} \in \mathcal{P}} \quad & \int_0^T f(p_{\mathrm{tx}}) \mathrm{d}t \\ \text{s.t.} \quad & \dot{W} = g(p_{\mathrm{tx}}), \\ & D \leq W \leq A, \\ & W(0) = 0, W(T) = A(T), \end{aligned} \tag{5}$$

which has the same optimal solution as (4). We also enforce $D(T) = A(T)$ for the simplicity in later explanations.

A control $p_{\text{tx}} \in \mathcal{P}$ is called *admissible*, if the corresponding state variable W satisfies the pointwise constraint $D \leq W \leq A$ and the end condition $W(T) = A(T)$. Geometrically, such a function W can be represented by a trajectory that lies between the boundary curves A and D, and adjoins the points $(0, 0)$ and $(T, A(T))$. We term such trajectories as *admissible trajectories* and illustrate two of them in Fig. 1.

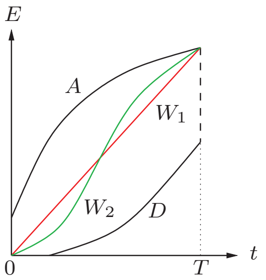


Fig. 1. Admissible trajectories

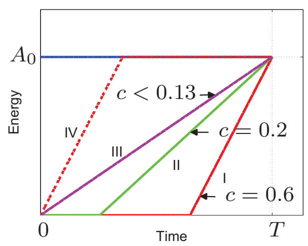


Fig. 2. Optimal Transmission Strategy for the Basic Problem

The throughput maximization discussed in [5] is a special case of (4) in which $g_c \equiv 0$, and A, D are discontinuous. With $g_c \equiv 0$, Problem (4) is convex and W^* has the nice geometrical interpretation that it is the shortest admissible trajectory [6]. More generally, it can be shown that for convex g_c on $[0, +\infty)$, Problem (4) is still convex and has a unique optimal solution. When g_c is convex on $(0, +\infty)$ but discontinuous at the zero point, Problem (4) becomes non-convex. Fortunately, we could find and validate in this case one of the optimal transmission strategies using a similar approach as proposed in [6], and prove that all the other optimal transmission strategies are *equivalent* to this one.

3. SOLUTION TO THE BASIC PROBLEM

We start with analysing the simplest situation, where the battery has an nonempty initial state A_0 with $A_0 < E_{\max}$, and there is no energy arrival during $[0, T]$, *i.e.*, $A \equiv A_0$, $D \equiv 0$ for $t \in [0, T]$. Problem (4) under this particular setting will be referred to as the *basic problem*, and it can be treated as a control problem with no constraint on the state variable but only a fixed final state $W(T) = A_0$.

3.1. With Convex Circuit Power Modelling

It can be proved with Jensen's Inequality [6] or with the theory of variational calculus [7] that, for the basic problem, using a constant transmit power over the whole time slot leads to the maximal throughput when circuit power is not considered. In other words, W^* is the straight line that connects $(0, 0)$ and (T, A_0). A similar conclusion can be drawn with convex circuit power functions g_c based on the *Pontryagin maximum principle* (P.M.P.), which is a first-order necessary condition for optimality of standard control problems [8][7].

The Hamiltonian of the basic problem is given by $H = -f + \lambda \cdot g$, where the auxiliary function λ satisfies the co-state equation $\dot{\lambda} = -\frac{\partial H}{\partial W}$. As the Hamiltonian does not depend explicitly on W, we have $\dot{\lambda} = 0$ which implies that λ is a constant. The P.M.P. suggests that $H(p_{\text{tx}}^*)$ should be constant on $[0, T]$, meaning that

$$\dot{H}(p_{\text{tx}}^*) = \left(-\frac{\partial f}{\partial p_{\text{tx}}^*} + \lambda \cdot \frac{\partial g}{\partial p_{\text{tx}}^*}\right) \dot{p}_{\text{tx}}^* = 0. \tag{6}$$

With the assumption that g_c is convex in p_{tx} (and so is g), the only possibility for (6) to hold is that $\dot{p}_{\text{tx}}^* = 0$, *i.e.*, p_{tx}^* being constant. The end states then give us $p_{\text{tx}}^* = g^{-1}\left(\frac{A_0}{T}\right)$.

As the basic problem with convex circuit power models can be shown convex, the necessary condition for optimality becomes also sufficient. Therefore, the optimal constant transmit power function p_{tx}^* we have obtained via the P.M.P. is indeed the global optimal solution for the basic problem.

3.2. Discontinuity in Circuit Power Models

Now we assume that g_c is convex on $(0, +\infty)$ but discontinuous at $p_{\text{tx}} = 0$. Intuitively, such discontinuity in the circuit power model may result in a sleeping period in the optimal transmission strategy. To verify that and to find the suitable sleeping period, we suppose that the transmitter is only active for a period of length u, $u \in (0, T]$, and is asleep for the rest of the time slot. The constant transmit power $p_{\text{tx}} = g^{-1}\left(\frac{A_0}{u}\right)$ should be employed for the active period in order to achieve the maximal throughput, which is $u \cdot f(p_{\text{tx}})$. Consequently, an optimization of p_{tx} can be formulated as

$$\max_{p_{\text{tx}} > 0} \quad A_0 \cdot \frac{f(p_{\text{tx}})}{g(p_{\text{tx}})} \qquad \text{s.t.} \quad \frac{A_0}{g(p_{\text{tx}})} \leq T. \tag{7}$$

The maximization of f/g as in (7) is equivalent to the minimization of g/f, which seeks the transmit power that leads to the minimum energy consumption per information bit.

We define the function $h = f/g$ and notice that $h'(p_{\text{tx}}) = 0$ if and only if $(f'g - fg')(p_{\text{tx}}) = 0$. When g is continuous at $p_{\text{tx}} = 0$, we have $(f'g - fg')(0) = 0$. As $(f'g - fg')' = f''g - fg'' < 0$ for all $p_{\text{tx}} > 0$, the function $f'g - fg'$ decreases monotonically, hence $p_{\text{tx}} = 0$ is the only stationary point of h for continuous g. If g is discontinuous at $p_{\text{tx}} = 0$, then we have $g(0^+) > 0$ which implies that $(f'g - fg')(0^+) > 0$. Therefore, if the equation $(f'g - fg')(p_{\text{tx}}) = 0$ has any solution, it must be unique and positive. Denote this solution by p_{tx_0}. For $p_{\text{tx}} < p_{\text{tx}_0}$, we have $(f'g - fg')(p_{\text{tx}}) > 0$ which suggests that $h'(p_{\text{tx}}) > 0$. This means that the objective function of (7) increases monotonically for $p_{\text{tx}} < p_{\text{tx}_0}$, and the closer p_{tx} is to p_{tx_0}, the larger the objective. Consequently, the optimal solution to (7), denoted with $\hat{p}_{\rm tx}$, is given by

$$\hat{p}_{\rm tx} = \begin{cases} p_{\rm tx_0}, & \text{if} \quad \dfrac{A_0}{g(p_{\rm tx_0})} \leq T, \\ g^{-1}\left(\dfrac{A_0}{T}\right), & \text{otherwise.} \end{cases} \tag{8}$$

It is important to note that $p_{\rm tx_0}$ depends only on functions f and g but not on A_0 or T. As $p_{\rm tx_0}$ minimizes the energy consumption per information bit, we refer to $p_{\rm tx_0}$ as the *energy-efficient transmit power*. The corresponding total power dissipation is given by $p_0 = g(p_{\rm tx_0})$. The optimal transmission strategy for the basic problem suggested by (8) is that if $\frac{A_0}{g(p_{\rm tx_0})} \leq T$, then the transmitter should be turned into sleep mode for a time period of $T - \frac{A_0}{g(p_{\rm tx_0})}$ and then transmit with power $p_{\rm tx_0}$; otherwise the transmitter should transmit over the whole time slot with power $g^{-1}\left(\frac{A_0}{T}\right)$. In other words, the energy-efficient transmit power should be employed unless it can not exhaust the available energy by the end of the time slot. In that case, the constant transmit power that exactly uses up all energy should be utilized. Note that where and how the sleeping period is located in the time slot does not influence the throughput. The optimal transmission strategy we propose obeys in general a "sleep first" principle.

We illustrate our analysis so far with Figure 2, where a basic problem with $A_0 = 10$ and $T = 15$ is depicted. The linear circuit power model with $b = 0$ is assumed, *i.e.*, $g_{\rm c} = c$. Trajectories I and II represent W^* for $c = 0.6$ and $c = 0.2$, respectively. For $c < 0.13$ approximately, p_0 falls below $\frac{A_0}{T}$ hence W^* becomes the straight line connecting $(0, 0)$ and (T, A_0), as shown by trajectory III.

Note that the convexity assumption on g is rather strong for the derivation of (8) and the optimal transmission strategy to be established. In fact, we only require the conditions

$$\begin{cases} f''g - fg'' < 0, \\ (f'g - fg')(p_{\rm tx}) < 0, \quad p_{\rm tx} \to +\infty \end{cases} \tag{9}$$

to be fulfilled. This means, the construction algorithm we propose in the next section applies to a wider class of circuit power models than just convex functions g. It is even applicable for some concave functions such as $g_{\rm c} = f$. Yet to make the mathematical deductions more tractable and concise, we stick to the assumption that g is convex which already includes many common circuit power models.

3.3. Equivalent Transmission Strategies

We will use the term *change of mode* to indicate the switch from active mode to sleep mode or vice versa. The term *change of slope* will only refer to the change in transmit power from a positive value to another, but not to the change in transmit power associated with a change of mode. The following definition is made in order to identify an important class of admissible trajectories, where $g_{\rm c}$ is fixed and p_0 is known.

Definition 1 *Let $W_1(t)$ and $W_2(t)$ be two admissible trajectories to (4), and they differ only on a finite number of subintervals $[l_i, u_i] \subseteq [0, T]$, $i = 1, 2, \ldots, K$. If on these subintervals, $W_1(t)$ and $W_2(t)$ both consist only of horizontal lines and straight lines with slope p_0, then $W_1(t)$ and $W_2(t)$ are equivalent, denoted by $W_1(t) \sim W_2(t)$.*

The trajectories I and IV in Figure 2 are equivalent. The definition indicates that equivalent trajectories are identical in the active periods where only changes of slopes are involved, and differ in the location and length of each sleeping period as well as the periods over which the energy-efficient transmit power is employed. Transmit power functions corresponding to equivalent trajectories yield the same throughput.

4. OPTIMAL TRANSMISSION STRATEGY AND THE CONSTRUCTION ALGORITHM

Comparing solutions to the basic problem obtained in the last section with those in [3] where we have assumed a constant energy consumption of the node in active mode, we find that the way in which the optimal trajectory W^* should be constructed is exactly the same. It is only the mapping from W^* to $p^*_{\rm tx}$ by $g^{-1}(\dot{W}^*)$ that differs. Based on this observation, the optimality criterion that an optimal admissible trajectory to (4) should satisfy is identical to the one proposed in [3], and we can also claim that all such admissible trajectories are identical or equivalent and construct one of them using the same algorithm in principle. Note that we do not need to explicitly distinguish between the cases that g is continuous or discontinuous at $p_{\rm tx} = 0$, for the former can be seen as a special case of the latter with $p_0 = 0$. We summarize in this section several important aspects about the construction algorithm and one can refer to [3] for more details.

Theorem 1 *Let W be an admissible trajectory to (4) and $L(t)$, $t \in [l, u]$ be a curve that adjoins $(l, W(l))$ and $(u, W(u))$ where l, u satisfy $0 \leq l < u \leq T$. Denote the slope of the straight line that connects $(l, W(l))$ and $(u, W(u))$ by k.*

1. *If $k < p_0$ and $L(t)$ satisfies*
   - *$L(t)$ consists only of horizontal lines and straight lines with slope p_0,*
   - *$L(t) \not\sim W(t)$, $t \in [l, u]$,*
   - *$D(t) \leq L(t) \leq A(t)$, $\forall t \in [l, u]$,*
2. *If $k \geq p_0$ and $L(t)$ satisfies*
   - *$L(t)$ is a straight line segment,*
   - *$L(t) \not\equiv W(t)$, $t \in [l, u]$,*
   - *$D(t) \leq L(t) \leq A(t)$, $\forall t \in [l, u]$,*

then replacing the part of W between $[l, u]$ with $L(t)$ increases the throughput.

The *optimality criterion* for an optimal admissible trajectory follows as: *along an optimal admissible trajectory W^*,* *there do not exist any two points between which the part of W^* can be reconstructed as indicated by Theorem 1.*

Suppose an admissible trajectory W^* has been found optimal and it contains at least one horizontal part. Then there are infinitely many admissible curves that are equivalent to W^*, and they all lead to the same maximal throughput. The theorem below gives us the guarantee of uniqueness of the optimal admissible trajectory in the sense of equivalence.

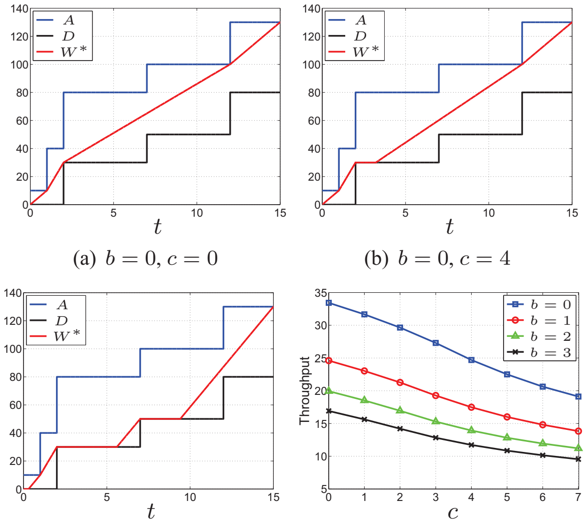
Theorem 2 *The admissible trajectories that satisfy the optimality criterion are either equivalent or identical.*

Consequently, it suffices to find one admissible trajectory that satisfies the optimality criterion. The construction of such a trajectory proceeds in a recursive fashion. Consider a time instance $t_0 \in [0, T)$ and let (t_0, α_0) satisfy $D(t_0) \leq \alpha_0 \leq A(t_0)$. Straight lines of nonnegative slopes starting from this point, denoted by $L_{(t_0,\alpha_0)}$, can be distinguished by whether they intersect with A or D first. Let $\mathcal{S}_A(t_0, \alpha_0)$ and $\mathcal{S}_D(t_0, \alpha_0)$ denote the sets of slopes which lead $L_{(t_0,\alpha_0)}$ to intersect with A and D first, respectively. Since $A > D$, $\forall t \in (0, T)$, we have $\inf \mathcal{S}_A(t_0, \alpha_0) = \sup \mathcal{S}_D(t_0, \alpha_0) \triangleq \beta(t_0, \alpha_0)$. In each iteration of the construction algorithm, the critical slope $\beta(t_0, \alpha_0)$ needs to be compared with p_0 to determine whether a sleeping period should take place. If $\beta(t_0, \alpha_0) > p_0$, the constructed trajectory follows either A, D or a straight line segment $L_{(t_0,\alpha_0)}$ depending on the position of (t_0, α_0); otherwise, the transmitter is turned into sleep mode until it has to be activated, *i.e.*, to be able to employ the constant transmit power p_{tx_0} to reach the next intersection point, or to avoid energy overflow. A theorem has been established that the trajectory constructed using this algorithm does not violate the optimality criterion and is therefore one of the optimal admissible trajectories to (4).

We illustrate the constructed trajectories with some numerical simulations for which the linear circuit power model (2) is employed and the discrete energy arrivals as shown in Fig. 3 are assumed. The optimal trajectories W^* with different values of the parameters b and c are depicted in Fig. 3(a)-Fig. 3(c) and the maximal throughput achieved is shown in Fig. 3(d). The difference in the optimal transmission strategies with or without circuit power considerations is obvious from the figures, and the more significant the circuit power, the more sleeping periods the transmitter should take.

5. CONCLUSION

The data transmission of an energy harvesting node over a single invariant link has been considered, where the energy arrival and departure processes are perfectly known and a generalized circuit power model is employed. The basic form of the throughput maximization problem is solved by using the optimal control theory, and the results indicate that for convex circuit power models, the proposed construction algorithm of an optimal trajectory is valid. All analysis and obtained methods have been verified and illustrated with simulation results.



(a) $b = 0, c = 0$ (b) $b = 0, c = 4$ (c) $b = 1, c = 7$ (d) Maximal throughput achieved

Fig. 3. Optimal trajectories and the maximal throughput

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