Estimation of Noise Parameters in Multi-Antenna Receivers using Digitized Signal Samples

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***Abstract*—Multi-antenna systems can provide multi-streaming and diversity operation even in the case where the antennas are spaced electrically close to each other. This requires that a decoupling and matching network (DMN) is connected between the antenna ports and the inputs of the low-noise amplifiers (LNA) of the receiver. The job of the DMN is to electrically decouple the ports which are connected to the LNAs, and to provide the latter with the *optimum driving impedance* such that their noise figure is minimum (*noise matching*). The value of the driving impedance to achieve noise matching only depends on certain noise parameters of the receiver. To our best knowledge, there is no measurement equipment available to this date, which would allow to measure these parameters. In this paper, we report on a simple scheme by which these noise parameters can be estimated using the multi-antenna receiver itself as the measurement equipment. The idea is to connect the inputs of two LNAs by a coax cable and estimate the correlation coefficient of the digitized outputs of the respective receiver chains. From two such estimated correlation coefficients obtained for two different lengths of the coax cable, all noise parameters which are relevant for noise matching can be obtained.**

I. Introduction

It has been observed by several authors that multiple receive antennas can provide space diversity even in the case when the antennas are placed electrically close to each other [1]–[3]. It has also been pointed out by several researchers that the same is true for providing multi-streaming [4]–[8]. In both cases, it is essential that there be connected a *decoupling and matching network* (DMN) between the ports of the antennas and the ports of the low-noise amplifiers (LNA) of the receiver chains. The DMN not only provides electrically uncoupled ports for the connection of the LNAs, but also ensures that the latter see the correct driving impedance, Z_{opt}, which makes their noise-figure minimum. This is referred to as *noise matching* in contrast to other matching goals, like power matching. The value of Z_{opt} depends on certain noise parameters of the receiver chains (and only on them).

In this paper, we describe a simple scheme which can be used to measure these relevant noise parameters by using the receiver itself as the measurement device, requiring only two coaxial cables as external hardware. For purpose of illustration, consider the Figure 1 which shows, in its center, a 2-antenna receiver with a DMN. Each of the two receiver chains consists of an LNA, followed by a down-converter (consisting of a modulator, local oscillator (LO) for the in-phase and the quadrature components and a low-pass filter (LPF)), 2 variable gain amplifiers (VGA), and 2 analog to digital converters (ADC), one for the in-phase and one for the quadrature component. A slowly operating adaptive gain control (AGC) takes care that the long-term root-mean-square voltage at the inputs of the

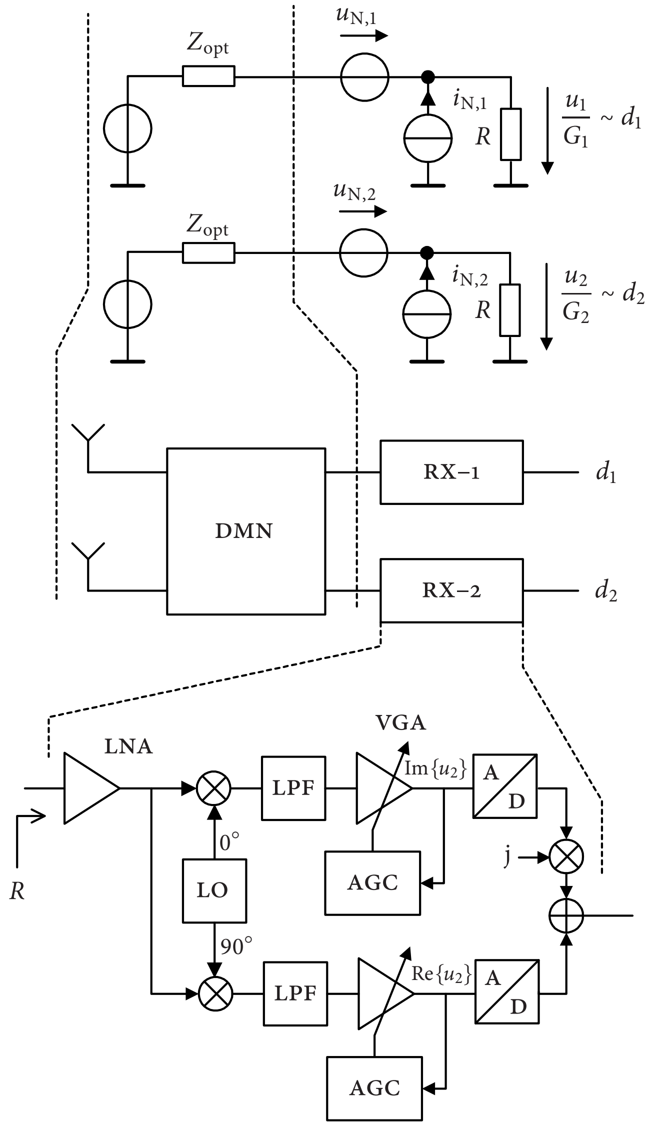

Figure 1: Schematic diagram of a 2-antenna receiver with decoupling and matching network (CENTER), simplified diagram of each receiver chain (BOTTOM), and the equivalent circuit for noise analysis (TOP).

ADCs remains at a suitable value which ensures that the full resolution of the ADCs is actually used for conversion into the digital outputs d_1 and d_2, respectively. The LNAs have got an input impedance which is real-valued and equal to R. While it is not essential for the presented measurement scheme that the input impedance be real-valued, it is usually the case in practice (where frequently $R = 50\Omega$) and it simplifies the mathematics. We therefore assume the input impedance to be real-valued in the following. The upper part of Figure 1 shows the equivalent circuit that is used for noise analysis. Because of the action of the DMN, the two receiver chains are driven by electrically decoupled voltage sources with an internal impedance Z_{opt}, respectively. Moreover, the receiver chains being linear can be modeled (see, e.g., [9]) by an equivalent circuit which consists of a resistance R (to mimic the input impedance of the LNAs) and two noise sources (a noise voltage and a noise current source). Across the resistances appear voltages with the complex envelopes u_1/G_1 and u_2/G_2, respectively. Herein, the real and imaginary parts of u_k are the voltages at the inputs of the ADC used for the in-phase and the quadrature components of the k-th receiver chain ($k \in \{1, 2\}$), while G_k is the voltage gain between the ADC inputs and the inputs of the LNAs, respectively. Note that d_1 and d_2 are the quantized digital representations of u_1 and u_2. In the following, we assume that the two receiver chains have *the same* stochastic noise properties.

In the remaining of this paper, we show how to obtain the relevant noise parameters of the receiver by observing the digitized outputs d_1 and d_2 of the two receivers which inputs are connected by a coaxial cable. We first present the theoretical framework and then report on experimental evaluation.

II. Theory

A. Noise Matching

In the following, we consider the case of *narrow-band* linear receivers. Narrow-band means that the receiver's bandwidth is small compared to the center frequency. More precisely,

$$\sqrt{\frac{f_1}{f_2} + \frac{f_2}{f_1} - 2} \ll 1, \tag{1}$$

must hold. Herein, $f_1 > 0$ and $f_2 > 0$ are the corner frequencies of the receiver's passband. Let us now develop a suitable noise-model of such a receiver.

When a single linear receiver is connected to an antenna, there are *three* noise sources to be considered. The first is the equivalent antenna noise voltage. The other two are the equivalent input noise voltage and the equivalent input noise current of the receiver.[1] The noise voltage which results is a linear superposition of these three equivalent noise sources. For optimum performance, it is usually necessary to connect a *lossless matching network* between receiver and antenna. Its job is to ensure that the three noise sources superimpose such that the largest possible signal to noise ratio (SNR) is obtained at the receiver's output.[2] This is shown in the upper part of Figure 2. The port behavior (at center frequency) of the matching network can then be described by the following relationship:

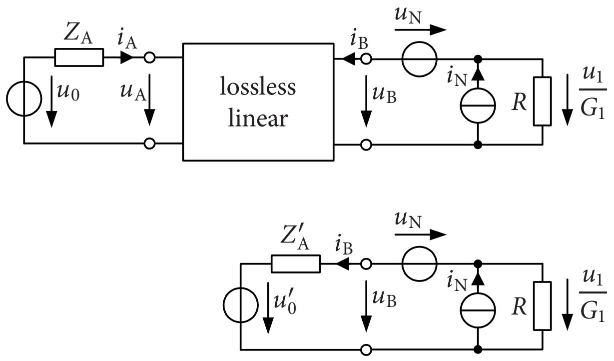

Figure 2: Lossless linear matching network connected between antenna and receiver (TOP). The effect of the matching network is to transform the impedance and open-circuit voltage of the antenna to new values (BOTTOM).

$$\begin{bmatrix} u_\text{A} \\ u_\text{B} \end{bmatrix} = \begin{bmatrix} ja & -c + jd \\ c + jd & jb \end{bmatrix} \begin{bmatrix} i_\text{A} \\ i_\text{B} \end{bmatrix}, \tag{2}$$

where u_A, u_B, i_A and i_B are defined in Figure 2, and a, b, c, and d are *real-valued*, which ensures that the matching network is *lossless* [10].[3] Analyzing the circuit from the top of Figure 2, shows that the effect of the lossless matching network is to transform the antenna's impedance Z_A into a new value Z'_A, while, at the same time, transforming the antenna's open-circuit voltage with complex envelope u_0, into a different open-circuit voltage with complex envelope u'_0, according to:

$$u'_0 = \frac{c + jd}{Z_\text{A} + ja} u_0, \quad Z'_\text{A} = jb + \frac{c^2 + d^2}{Z_\text{A} + ja}. \tag{3}$$

This is shown in the lower part of Figure 2. For the antenna's voltage comprises desired signal and noise, one can write:

$$u_0 = u_\text{S} + \tilde{u}_\text{N}, \tag{4}$$

where u_S and $\tilde{u}_\text{N}$ are the complex envelopes of the desired signal voltage, and the antenna noise voltage, respectively. It is customary to quantify the antenna noise variance by the so-called *antenna noise temperature*, T_A, such that:

$$\text{E}\left[|\tilde{u}_\text{N}|^2\right] = 4\,\text{k}_\text{B}\,T_\text{A}\,\Delta f\,\text{Re}\{Z_\text{A}\}, \tag{5}$$

where k_B and Δf are Boltzmann's constant and the receiver's bandwidth, respectively. From (3) follows the relationship:

$$\text{E}\left[|u'_0|^2\right] = \frac{\text{Re}\{Z'_\text{A}\}}{\text{Re}\{Z_\text{A}\}} \text{E}\left[|u_0|^2\right]. \tag{6}$$

When the lossless matching network changes the real-part of the antenna's impedance, a corresponding change in the variance of the open-circuit voltage must take place. The SNR is now defined in the standard way as:

$$\text{SNR} = \frac{\text{E}\left[|u_1|^2 \mid \tilde{u}_\text{N} = u_\text{N} = 0, i_\text{N} = 0\right]}{\text{E}\left[|u_1|^2 \mid u_\text{S} = 0\right]}, \tag{7}$$

[1]The reason why it is enough to have *one* source to describe the antenna noise but *two* noise sources are required for the receiver is that the antenna is a *one*-port while the receiver is a *two*-port.

[2]It is important for the matching network to be lossless (i.e., to not dissipate any energy), for otherwise its losses would generate additional noise of its own, and also eat up some part of the desired signal energy. Clearly, both effects are undesirable but usually cannot be avoided completely in practice.

[3]Any hybrid matrix (immitance matrix included) which describes a lossless linear multiport must have the property that it equals its negative complex conjugate transpose.

where u_1 is the complex envelope of the receiver's output voltage (see Figure 2). It is convenient to define the so-called *available* SNR as the SNR that one would obtain if the receiver was noiseless:

$$\mathrm{SNR_{av}} = \mathrm{SNR}|_{u_\mathrm{N}=0,\, i_\mathrm{N}=0}\,. \tag{8}$$

The actual SNR can then be written as:

$$\mathrm{SNR} = \frac{\mathrm{SNR_{av}}}{\mathrm{NF}}, \tag{9}$$

where NF denotes the so-called *noise figure* of the receiver. Using (4)–(9) and the circuit from the lower part of Figure 2, it is easy to obtain for the noise figure the following expression:

$$\mathrm{NF} = 1 + \mathrm{E}\left[|i_\mathrm{N}|^2\right] \frac{R_\mathrm{N}^2 - 2R_\mathrm{N}\mathrm{Re}\{\rho^* Z'_\mathrm{A}\} + |Z'_\mathrm{A}|^2}{4\,\mathrm{k_B}\,T_\mathrm{A}\,\Delta f\,\mathrm{Re}\{Z'_\mathrm{A}\}}, \tag{10}$$

where

$$R_\mathrm{N} = \sqrt{\frac{\mathrm{E}\left[|u_\mathrm{N}|^2\right]}{\mathrm{E}\left[|i_\mathrm{N}|^2\right]}}, \tag{11}$$

is called *noise resistance*, and

$$\rho = \frac{\mathrm{E}\left[u_\mathrm{N}\, i_\mathrm{N}^*\right]}{\sqrt{\mathrm{E}\left[|u_\mathrm{N}|^2\right]\mathrm{E}\left[|i_\mathrm{N}|^2\right]}}, \tag{12}$$

is the *complex correlation coefficient* of the receiver's equivalent input noise current and noise voltage. In the derivation of (10) the reasonable assumption is made that that the antenna noise is uncorrelated with the receiver's noise. As is obvious from (10), the noise figure depends on the transformed impedance Z'_A. This motivates the definition of the *optimum impedance*:

$$\begin{aligned} Z_\mathrm{opt} &= \arg \min_{Z'_\mathrm{A}} \mathrm{NF} \\ &= R_\mathrm{N}\left(\sqrt{1-(\mathrm{Im}\{\rho\})^2} + \mathrm{j}\,\mathrm{Im}\{\rho\}\right). \end{aligned} \tag{13}$$

Designing the lossless matching such that $Z'_\mathrm{A} = Z_\mathrm{opt}$, ensures minimum noise figure, hence, maximum SNR. This strategy is referred to as *noise matching*. From (3), one can easily see that the following set of parameters:

$$a = -\mathrm{Im}\{Z_\mathrm{A}\},\ b = \mathrm{Im}\{Z_\mathrm{opt}\},\ c = 0,\ d^2 = \mathrm{Re}\{Z_\mathrm{opt}\}\,\mathrm{Re}\{Z_\mathrm{A}\},$$

leads to noise matching. Since setting $c = 0$ is possible, shows that, without loss of generality, one can additionally require that the matching network is *reciprocal*.

Generalizing noise matching for the case of antenna *arrays* is straight forward, provided that the noises generated by different receivers are pairwise uncorrelated.[4] All there is to do is to replace Z_A by the array impedance matrix $\boldsymbol{Z}_\mathrm{A}$. The parameters a, b, c and d, then become real-valued matrices. The effect of the resulting matching network is to *decouple* the ports which are connected to the multiple receivers, and providing a source impedance of Z_opt to each of them.

[4]This can be achieved by proper design of the RF hardware. It may require housing the amplifiers in separated metal enclosures and making sure that no coupling occurs over the DC power supply.

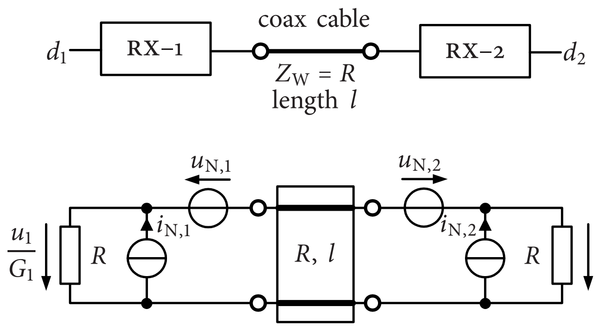

Figure 3: Top: Measurement setup: the *inputs* of the receiver chains are connected together by means of a coaxial cable. Bottom: Equivalent electric circuit.

B. Measuring the relevant noise parameters

It is a remarkable fact from (13), that the optimum impedance only depends on *two* real-valued noise parameters of the receiver: its *noise resistance*, and the *imaginary* part of its complex noise correlation coefficient. Thus, for the design of the matching network for noise matching, the parameters R_N and $\mathrm{Im}\{\rho\}$ of the receiver have to be found out by measurement. In the following, we derive a measurement method which uses the receiver itself as the measuring device. The idea is shown on the top of Figure 3. The *inputs* of the two receiver chains are connected together by means of a coax cable and the correlation coefficient of the received digital data d_1 and d_2 is computed. Recall that d_1 and d_2 are the quantized versions of the voltages u_1 and u_2, respectively, which are defined in the top of Figure 1. The bottom part of Figure 3 shows the equivalent electric circuit, where it is assumed that the coaxial cable is lossless and thus also noiseless.[5] The voltage u_1/G_1 can be computed as a linear superposition of two terms: a term due to the near noise sources $u_{\mathrm{N},1}$ and $i_{\mathrm{N},1}$, and the far noise sources $u_{\mathrm{N},2}$ and $i_{\mathrm{N},2}$. Because of this and the cable's wave impedance being equal to the input resistance R of the two receivers, the analysis is greatly simplified. The part due to the near noise sources equals $Ri_{\mathrm{N},1}/2 - u_{\mathrm{N},1}/2$ while the part due to the far noise sources equals $\mathrm{e}^{-\mathrm{j}2\pi l/\lambda}\,(Ri_{\mathrm{N},2}/2 + u_{\mathrm{N},2}/2)$, where l is the length of the coaxial cable, and λ is the wavelength inside the cable at center frequency. In total, we have:

$$u_1 = \frac{G_1}{2}\left(Ri_{\mathrm{N},1} - u_{\mathrm{N},1} + \mathrm{e}^{-\mathrm{j}2\pi l/\lambda}\left(Ri_{\mathrm{N},2} + u_{\mathrm{N},2}\right)\right), \tag{14}$$

and similarly

$$u_2 = \frac{G_2}{2}\left(Ri_{\mathrm{N},2} - u_{\mathrm{N},2} + \mathrm{e}^{-\mathrm{j}2\pi l/\lambda}\left(Ri_{\mathrm{N},1} + u_{\mathrm{N},1}\right)\right). \tag{15}$$

As we assume that the stochastic noise properties of the two receiver chains are the same (in the wide sense), and that their noises are uncorrelated, it follows from (14) and (15) that

$$\mathrm{E}\left[|u_k|^2\right] = \beta\frac{|G_k|^2}{2}\left(R^2 + R_\mathrm{N}^2\right), \tag{16}$$

[5]In practice, some loss in the coaxial cable is usually unavoidable. Because the amount of loss increases with the length of the cable, the assumption of losslessness can be met with any given accuracy by not letting the cable be excessively long. With state of the art cables less than a wavelength long, the loss is usually negligible, as is the noise it causes with respect to the noise of the amplifiers.

where $k \in \{1,2\}$, and $\beta = \mathrm{E}\left[|i_{\mathrm{N},1}|^2\right] = \mathrm{E}\left[|i_{\mathrm{N},2}|^2\right]$, and R_N is the noise resistance of the receivers defined in (11).[6] That $\mathrm{E}\left[|u_k|^2\right]$ does not depend on the cable length[7] means that the adaptive gain control will keep the same gain irrespective of the length of the cable. Moreover

$$\frac{2\mathrm{E}\left[u_1 u_2^*\right]}{\beta G_1 G_2^*} = \left(R^2 - R_\mathrm{N}^2\right)\cos(2\pi l/\lambda) + 2RR_\mathrm{N}\mathrm{Im}\{\rho\}\sin(2\pi l/\lambda), \tag{17}$$

where ρ is defined in (12). From (17) it is obvious that $\mathrm{E}\left[u_1 u_2^*\right]$ depends not only on R_N but also on the $\mathrm{Im}\{\rho\}$. Because it is precisely these two quantities which the optimum impedance (13) depends on, it follows that $\mathrm{E}\left[u_1 u_2^*\right]$ is the key to the measurement of the noise parameters which are relevant for noise matching. However, since $\mathrm{E}\left[u_1 u_2^*\right]$ also depends on the gains G_1 and G_2 of the two receiver chains, which we assume are *unknown*, it is better to base the measurement on

$$\mathrm{corr}\left(u_1, u_2\right) = \frac{\mathrm{E}\left[u_1 u_2^*\right]}{\sqrt{\mathrm{E}\left[|u_1|^2\right]\cdot \mathrm{E}\left[|u_2|^2\right]}}. \tag{18}$$

With the help of (16) and (17), it follows:

$$\mathrm{corr}\left(u_1, u_2\right) = \mathrm{e}^{\mathrm{j}\phi}\,\frac{\cos(kl)\left(1 - \dfrac{R_\mathrm{N}^2}{R^2}\right) + 2\dfrac{R_\mathrm{N}}{R}\sin(kl)\mathrm{Im}\{\rho\}}{1 + R_\mathrm{N}^2/R^2}, \tag{19}$$

where $k = 2\pi/\lambda$, and

$$\phi = \arg\left(G_1\right) - \arg\left(G_2\right), \tag{20}$$

is the phase mismatch of the receiver chains. In the following, we assume that the receiver chains have been *calibrated* such that the phase imbalance can be considered essentially zero:

$$\phi = 0. \tag{21}$$

Note that for zero phase imbalance, the correlation coefficient from (19) becomes *real-valued*.[8] Now for a given normalized cable length l/λ and a given value of R, this correlation coefficient depends on *two* real-valued parameters: R_N and $\mathrm{Im}\{\rho\}$. Thus, we need at least two measurements of $\mathrm{corr}\left(u_1, u_2\right)$ obtained by connecting the receiver's inputs with coaxial cables of two different lengths. To this end, let

$$\xi = \mathrm{corr}\left(u_1, u_2\right)\big|_{l = l_1}, \tag{22}$$

$$\mu = \mathrm{corr}\left(u_1, u_2\right)\big|_{l = l_2}. \tag{23}$$

Substituting (19) into (22) and (23), and solving for R_N and $\mathrm{Im}\{\rho\}$, we obtain:

$$\frac{R_\mathrm{N}}{R} = +\sqrt{\frac{\sin(k(l_2 - l_1)) - \xi\sin(kl_2) + \mu\sin(kl_1)}{\sin(k(l_2 - l_1)) + \xi\sin(kl_2) - \mu\sin(kl_1)}}. \tag{24}$$

[6]where $\mathrm{E}\left[|u_\mathrm{N}|^2\right] = \mathrm{E}\left[|u_{\mathrm{N},1}|^2\right] = \mathrm{E}\left[|u_{\mathrm{N},2}|^2\right]$.

[7]This is a consequence of the receivers' noises being uncorrelated. Moreover, the stochastic noise properties of the receivers being the same (in wide sense), $\mathrm{E}\left[|u_k|^2\right]$ also does not depend on the complex correlation coefficient ρ.

[8]As a welcome by-product, this real-valuedness makes an easy test whether the phases of the receiver chains have been calibrated correctly.

and similarly

$$\mathrm{Im}\{\rho\} = \frac{\mathrm{sign}\left(\sin(k(l_2 - l_1))\right)\cdot\left(\mu\cos(kl_1) - \xi\cos(kl_2)\right)}{\sqrt{\sin^2(k(l_2 - l_1)) - \left(\xi\sin(kl_2) - \mu\sin(kl_1)\right)^2}}. \tag{25}$$

Note that R_N is, per definition, non-negative, while $\mathrm{Im}\{\rho\}$ can take values within the real interval $(-1;+1)$. For the special case $l_2 = 2l_1 = \lambda/2$, the equations simplify to:

$$\frac{R_\mathrm{N}}{R} = \sqrt{\frac{1+\mu}{1-\mu}}, \quad \mathrm{Im}\{\rho\} = \frac{\xi}{\sqrt{1-\mu^2}}.$$

In practice, however, one may not have cables of precisely half and quarter wavelength at hand. Therefore, the equations (24) and (25) are the more useful ones. Notice that $|l_2 - l_1|$ is *not* allowed to be an integer multiple of $\lambda/2$.[9]

The exact values of ξ and μ are usually unknown such that estimates $\hat{\xi}$ and $\hat{\mu}$ have to be found by observing a number of samples of u_1 and u_2. Because d_1 and d_2 are the digitized versions of u_1 and u_2, respectively, $\hat{\xi}$ and $\hat{\mu}$ can also be obtained from the digitized samples.[10] As mentioned earlier, the assumed zero phase imbalance ($\phi = 0$) makes $\mathrm{corr}(u_1, u_2)$ real-valued. In reality, however, even a well calibrated receiver will have some small phase imbalance. Because of this and that only finitely many samples of u_1 and u_2 can be observed, the estimated values $\hat{\xi}$ and $\hat{\mu}$ will have a non-zero imaginary part. It is, therefore, necessary to take care of this issue:

$$\hat{\xi} \leftarrow \mathrm{sign}\left(\mathrm{Re}\{\hat{\xi}\}\right)|\hat{\xi}|, \quad \hat{\mu} \leftarrow \mathrm{sign}\left(\mathrm{Re}\{\hat{\mu}\}\right)|\hat{\mu}|. \tag{26}$$

This operation »rotates« $\hat{\xi}$ and $\hat{\mu}$ back onto the real axis using the smallest possible rotation angle.[11] After having applied this correction, one can replace in (24) and (25) ξ and μ by their estimates $\hat{\xi}$ and $\hat{\mu}$, respectively.

While taking just *two* measurements with cables of different lengths is the minimum, one can, of course, use more measurements of $\mathrm{corr}(u_1, u_2)$ with various cable lengths. This allows for better accuracy, since, for example, a *least-squares* fitting of the parameters can be obtained. Moreover, if more than two measurements are made, the wavelength $\lambda = 2\pi/k = v/f_0$ may also be treated as a parameter to estimate. This is important in case that the propagation speed v, with which the electromagnetic waves travel along the cable at center frequency f_0, is not known accurately enough.

III. Experiment

The theory which is presented in the previous Section relies on several assumptions. The bandwidth is small compared to the center frequency. The stochastic noise properties are the same for both receiver chains. Their input impedance is real-valued positive and equal to the characteristic impedance of

[9]This would imply $\xi = -\mu$ and lead to »0/0« expressions in (24) and (25).

[10]If the resolution of the ADCs is large enough so that quantization effects can be neglected, this is straight forward. For low resolution ADCs, one has to apply a non-linear transformation to the estimates obtained from d_1 and d_2, which depends on the nature of the quantization intervals. Discussion of such quantization effects, however, goes beyond the scope of this paper.

[11]For this to work out correctly, the phase imbalance must be less than 90°, as otherwise a change of sign will occur.

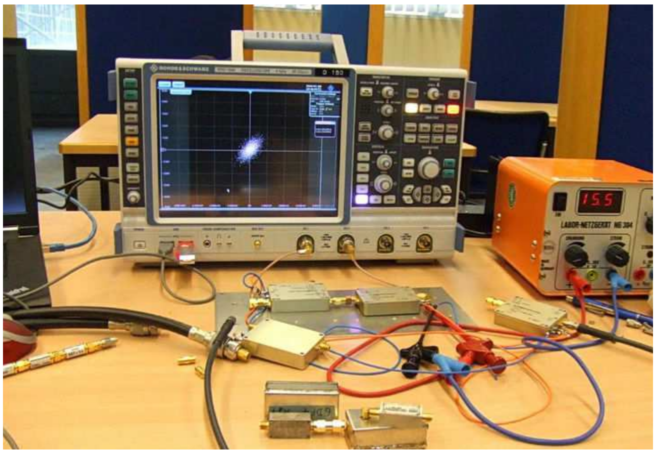

Figure 4: Hardware setup: oscilloscope, amplifiers, delay lines, power supply.

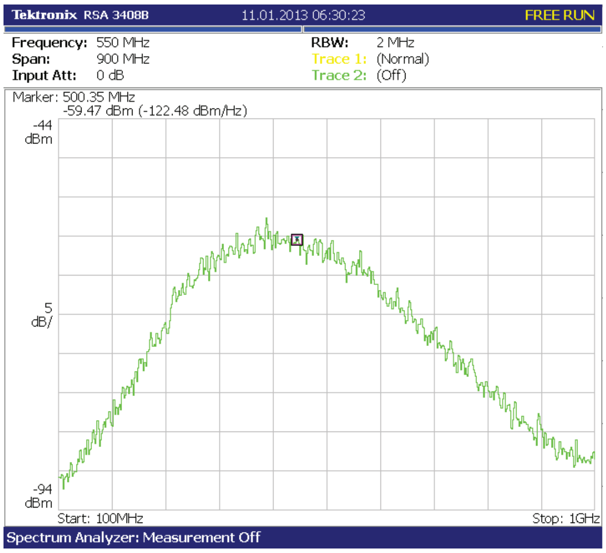

Figure 6: Measured amplifier noise spectrum.

the coaxial cables. The latter are lossless and, thus, noiseless. It is also assumed that the noises of the two receiver chains are uncorrelated and that the phases imbalance does not exceed 90°. In order to see whether these whole lot of simplifying assumptions are acceptable in practice, an experiment is performed and the obtained empirical data compared to the predictions of the theory.

A. *Measurement setup*

In the experiment, two commercial radio frequency (RF) low noise amplifiers (by the RF component manufacturer *Mini Circuits*) are connected to a Rohde & Schwarz RTO oscilloscope. A number of flexible RF coaxial cables (RG316) with different lengths are prepared. They are used to connect the inputs of the amplifiers. This hardware setup is shown in the photograph in Figure 4.

The inputs of the amplifiers are connected to each other by means of a coaxial cable of known length, while their outputs are connected to the oscilloscope. The latter samples, digitizes and displays their output voltages, as shown schematically in

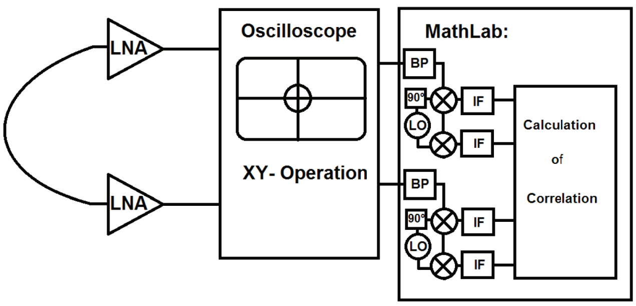

Figure 5: Schematic diagram of measurement setup.

Figure 5. The digitized samples are downloaded into a laptop computer for further processing in software. The processing includes band-pass filtering, down-conversion, low-pass filtering and computation of the correlation coefficient $\mathrm{corr}(u_1, u_2)$ in the complex baseband.

Before the experiment was started, we characterized the RF properties of the amplifiers by measuring their scattering parameters using a network analyzer. As we want to look at amplifier noise, the noise of the oscilloscope must be small compared to that of the amplifiers. This means that the latter must have enough amplification, so that their output noise is at least 10 dB above the input noise of the oscilloscope. The measured $|S_{21}|$ of 24 dB over a wide range of frequencies indicates that the amplifiers might be suitable. To verify this assertion, the output of one amplifier is connected to a spectrum analyzer. The amplifier's input is connected via a short cable to the input of the second amplifier, which output is terminated. The resulting spectrum is shown in Figure 6. It indicates that noise measurements could easily be performed around 500 MHz because the amplifier's noise is dominant there (−122.5 dBm/Hz noise power density into a 50 Ω load). Measurement of the amplifier's $|S_{11}|$ also showed low values around 500 MHz, so that the amplifier's inputs also closely resemble 50 Ω around 500 MHz. Based on these features, we have chosen the corner frequencies to be given as:

$$f_1 = 490\,\mathrm{MHz}, \qquad f_2 = 510\,\mathrm{MHz}.$$

The resulting bandwidth of 20 MHz is also reasonably small compared to the center frequency $f_0 = \sqrt{f_1 f_2} = 499.9\,\mathrm{MHz}$.

B. *Measurements*

The two amplifiers are now connected to the inputs of an RTO oscilloscope, which samples at a rate of 10^{10} samples per second. A batch of 10^7 such samples is downloaded into a laptop computer where all further signal processing is implemented in *MATLAB*. The inputs of the amplifiers are now connected with suitable coaxial cables of varying lengths. We use SMA connectors and an RF cable which are specified for frequencies an order of magnitude greater than we are working at. This is necessary, because heat-loss and termination should not play an important role. Silver-shielded RF cables with Teflon coating proved to perform well enough for our purposes.

meas num	cable length cm	raw $\mathrm{corr}(u_1, u_2)$	corrected $\mathrm{corr}(u_1, u_2)$
1	29.0	0.1689 − j0.0407	0.1737
2	21.9	−0.6005 + j0.0909	−0.6073
3	14.7	−0.7487 + j0.1397	−0.7616
4	15.1	−0.7277 + j0.1384	−0.7407
5	10.6	−0.3597 + j0.0722	−0.3669
6	21.2	−0.6690 + j0.1045	−0.6771
7	31.8	0.3668 − j0.0777	0.3749
8	20.2	−0.7875 + j0.1246	−0.7973

Table I: Empirical correlation coefficients.

The obtained measurement results for the correlation coefficient $\mathrm{corr}(u_1, u_2)$ for a number of 8 different cable lengths is displayed in Table I. The small imaginary parts of the estimated correlation coefficient indicate a slight phase imbalance. Therefore, the operation from (26) is applied which corrects this small phase imbalance by rotating the estimated values of $\mathrm{corr}(u_1, u_2)$ back to the real axis. The result is shown in the right-most column of Table I.

C. Comparison with theoretical results

The first thing we like to do is to find out whether the measured results support the theoretical findings. Specifically, we are interested how well the measured correlation coefficient (with corrected phase-imbalance) is modeled by the equation (19), setting $\phi = 0$ for zero phase imbalance. To this end, we make a least-squares fit on the parameters R_N/R, $\mathrm{Im}\{\rho\}$, and $k = 2\pi/\lambda$, considering all the measured data. From this least-squares fit, it firstly turns out that

$$\lambda = \frac{k}{2\pi} = 41.47\,\mathrm{cm}$$

is the optimum wavelength to use in (19), such that its results agree as well as possible with the measured data. Because the speed of wave propagation is given by $v = \lambda f_0$, it follows with $f_0 = 499.9\,\mathrm{MHz}$ that the cable has a delay per unit length of

$$4.824\,\mathrm{ns/m}.$$

This result is closer than 1% within the specification of the employed RG316 coaxial cable, which is 1.48 ns/ft, or 4.856 ns/m. The least-squares fit for the noise parameters yields:

$$R_\mathrm{N} = 0.4058R, \quad \mathrm{Im}\{\rho\} = -0.5228.$$

Now we substitute these obtained results for λ, R_N and $\mathrm{Im}\{\rho\}$ into (19), and compare this theoretical correlation coefficient with the measured data. The result is displayed in Figure 7. Theory and experiment agree fairly well. This shows that (19) is a reasonable theoretical model for the output noise correlation of two amplifiers with connected inputs. The simplifying assumptions which were made in its derivation seem to be justified by experiment.

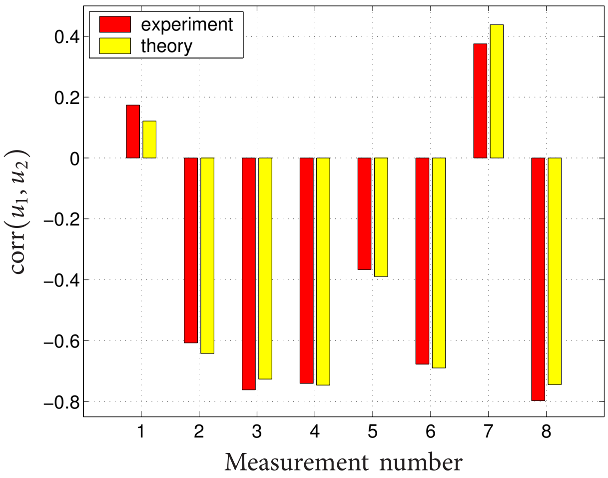

Figure 7: Comparison of the measured results with the theoretical prediction of correlation coefficient $\mathrm{corr}(u_1, u_2)$.

IV. Conclusion

A simple method of measurement for noise parameters of a multi-antenna receiver was presented. Hereby, the receiver itself acts as the measurement device requiring only two coaxial cables as additional hardware. One connects the inputs of the receivers with a coaxial cable and computes the correlation coefficient of the received noise from the digitized samples. From two such measurements, one can deduce all noise parameters of the receiver which are relevant for designing a noise matching network. The theoretical results have been compared with experimental data obtained from actual measurement and found to agree well.

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