Frequency Domain vs. Time Domain Filter Design of RRC Pulse Shaper for Spectral Confinement in High Speed Optical Communications

Israa Slim[1], Amine Mezghani[1], Leonardo G. Baltar[1], Juan Qi[2] and Josef A. Nossek[1]
1 Lehrstuhl für Netzwerktheorie und Signalverarbeitung, Technische Universität München, Munich, Germany
2 Huawei Technologies Duesseldorf GmbH, European Research Center, Munich, Germany
E-mail: israa.slim@tum.de

Abstract

In this work, we present two design methods of the root raised cosine (RRC) pulse shaper in the transmitter of a Nyquist wavelength division multiplexing (WDM) system. Spectral shaping is one of the techniques needed to meet the requirement for high spectral efficiency in high bandwidth applications. A frequency domain (FD) design and a time domain (TD) design of the RRC filter are given and compared based on their length and their spectral shape. The spectral shape of a frequency domain RRC filter desgined with double degrees of freedom as the time domain finite impulse response (FIR) RRC filter has side lobes with less power. Additionally, the root sum squared values of the intersymbol interference (ISI) with an FD design is lower than with a TD FIR design.

1 Introduction

The fast growth of bandwidth-rich internet applications such as online mobile applications and cloud computing has lead to a huge demand on the bandwidth of optical transports. As a consequence, new technologies to support larger capacity and higher spectral efficiency are required. Many technologies have been investigated to increase spectral efficiency including spectral shaping [1], Nyquist WDM [2], orthogonal frequency division multiplexing (OFDM) [3], optical superchannels [4], tight optical filtering with multisymbol detection using maximum *a posteriori* probability (MAP) detection [5] or maximum likelihood sequence estimation (MLSE) [6], and coded modulation [7]. These techniques aim at packing many subcarriers at symbol-rate spacing, where in this case both intrachannel (i.e., ISI) and interchannel (i.e., interchannel interference(ICI)) impairments have to be considered. This is because with this spacing, the Shannon capacity limit can only be achieved when these impairments are not present.

For the spectral shaping technique, the raised-cosine (RC) pulse shaping filter satisfies the Nyquist ISI criterion, which is the condition for interference free transmission in spite of bandwidth limitation. In order to get zero ISI, the most common solution for RC is to factor its transfer function into equal parts, i.e., to use the square-root of the desired raised cosine system response in both the transmitter and the receiver based on the matched filter principle. The resulting transmitted frequency response is commonly called an RRC response.

In this work, the RRC filter is chosen as pulse shaper in the transmitter of an oversampled Nyquist WDM system. Two designs of RRC filter are presented. One design is done in the frequency domain while the other is done in the time domain. Both designs are carried out in the digital domain i.e. a DSP is present in the transmitter. These two designs will be investigated and compared in their spectral shape as this could affect the performance when many subcarriers are packed at the symbol-rate spacing. Whether for the time domain or for the frequency domain design, pulse shaping is realized with the overlap-save (OS) method.

This paper is organized as follows: in Section 2, the transmitter model with pulse shaping is explained. The frequency and time domain designs of the RRC pulse shaper are given in Section 3. Simulation results are presented in Section 4 and finally conclusions are drawn in Section 5.

2 Transmitter Model with Pulse Shaping

Pulse shaping is performed in the digital domain in the transmitter of an oversampled Nyquist WDM system as shown in **Figure 1**. The input signal $x[n]$ is first upsampled by a factor of 2 to get $\hat{x}[m]$ in order to avoid aliasing. Pulse shaping is then done to $\hat{x}[m]$ in the frequency domain with the OS method with 50% overlapping factor. Thus, pulse shaping with OS method consists of transforming the upsampled input signal $\hat{x}[m]$ to the frequency domain by an FFT of size M_p, then multiplying with the coefficients of the RRC pulse shaper:

$$\mathbf{r} = [r_0, r_1, \cdots, r_{M_p-1}], \qquad (1)$$

and finally transforming back the filtered signal to the time domain by an IFFT of size M_p to get $x_{\mathrm{p}}[m]$.

ISBN 978-3-8007-3503-7

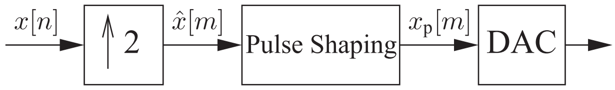

Figure 1: Spectral Shaping in the Transmitter

However, since half of the input $\hat{x}[m]$ to the FFT (which is the first stage in OS method) is zero (because of the upsampling by 2), an efficient structure replacing the upsampling operation and the FFT of size M_p can be obtained. This structure simply consists of directly transforming the input signal $x[n]$ into the frequency domain by an FFT of size $M_p/2$. In this case, each m-th output of the FFT is multiplied by the r_m-th and $r_{m+M_p/2-1}$-th coefficients of the pulse shaper to get the m-th and $(m + M_p/2 - 1)$-th input, respectively, to the IFFT. Since OS method with 50 % overlap is performed for pulse shaping, consecutive input blocks to the FFT overlap by $M_p/2$ samples. Each output block is formed by discarding $M_p/2$ samples of the filtered time domain output of the IFFT. The resulting efficient structure of the transmitter with OS is shown in **Figure 2**.

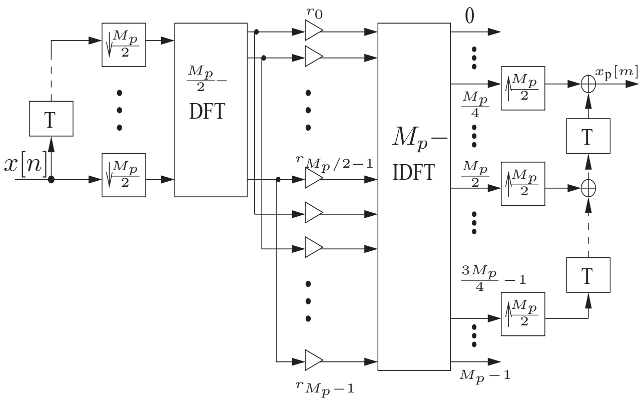

Figure 2: Computationally Efficient Spectral Shaping with OS

The choice and the design of the pulse shaper i.e. the coefficients **r** are presented in the following section.

3 Nyquist Pulse Shaper Design

The RC filter for digital pulse shaping satisfies the Nyquist ISI criterion, which is the condition for interference free transmission in spite of bandwidth limitation. In order to get zero ISI, the most common solution for RC is to factor its frequency response, i.e., to use the square-root of the desired raised cosine system response in both the transmitter and the receiver. The resulting transmitted frequency response is commonly called a RRC response. Thus, we will choose an RRC filter as a pulse shaper in this work.
Since pulse shaping is done in the FD, the coefficients of the RRC filter in (1) can be:

- **either** sampled directly in the frequency domain
- **or** given in the time domain, sampled and then transformed to the frequency domain

In what follows, both design methods and their characteristics are investigated.

3.1 Frequency Domain Design

For the frequency domain design, the M_p coefficients of the RRC pulse shaper can be found through:

$$G_{\text{RRC}}(f) = \begin{cases} 1, & 0 \leq |f| \leq \frac{1-\rho}{2T}, \\ \cos\left(\frac{\pi T}{2\rho}\left(|f| - \frac{1-\rho}{2T}\right)\right), & \frac{1-\rho}{2T} \leq |f| \leq \frac{1+\rho}{2T}, \\ 0, & |f| > \frac{1+\rho}{2T}. \end{cases}$$

The response characteristic of the RRC filter is adjusted through the roll-off factor ρ such that $0 \leq \rho \leq 1$, B is the equivalent noise bandwidth of the receiver and is the inverse of the symbol spacing T and $f \in [0, 2B]$.
The m-th filter coefficient of **r**, i.e., r_m is equal to $G_{\text{RRC}}(f_m)$. Since all M_p degrees of freedom are used in the design of the filter violating the OS condition, the system is no longer a linear time invariant system but rather a linear time variant one. This means that for each time instant, there is an impulse response for the system.

3.2 Time Domain Design

For the time domain design, the continuous time RRC impulse response with infinite support is:

$$g_{\text{RRC}}(t) = \frac{4\rho\frac{t}{T}\cos(\pi\frac{t}{T}(1+\rho)) + \sin(\pi\frac{t}{T}(1-\rho))}{\pi\frac{t}{T}(1-(4\rho\frac{t}{T})^2)}.$$

The FIR RRC can be obtained by:

- sampling the continuous time domain impulse response $g_{\text{RRC}}(t)$ at $T/2$ spacing, and then
- truncating the infinitely long filter by a discrete-time window function of length $M_p/2 + 1$ which define the impulse response of the filter.

The desired frequency response **r** is obtained by appending $M_p/2-1$ zeros to the truncated impulse response, to which an FFT of size M_p is applied. Since $M_p/2 + 1$ degrees of freedom are used to design the RRC filter, the system is linear time invariant.

4 Results

The power spectral density of the RRC pulse shaper with both FD and TD designs are shown in **Figure 3** for different FFT sizes $M_p = 32, 128$, $\rho = 0.2$ and for $B = 28$ GHz. The input signal is QPSK modulated.

ISBN 978-3-8007-3503-7

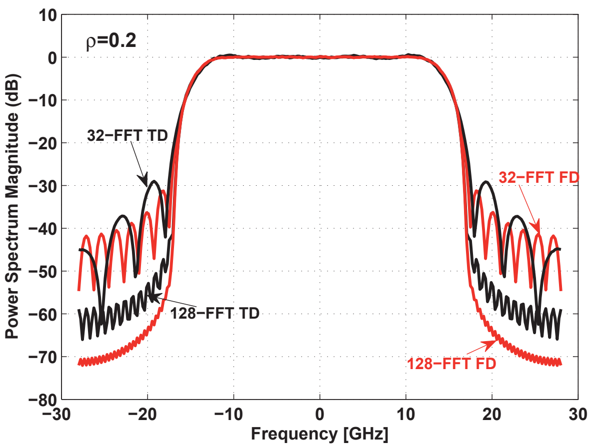

Figure 3: Power Spectral density of the FD and TD RRC filter designs

As can be seen from the plot, for given ρ and M_p, the power of the slide lobes with the TD design is greater than with the FD design. Additionally, as M_p increases, the power of the side lobes of both designs decreases but still the FD design delivers lower side lobe power than the TD design. This can be advantageous for the case with spectrally efficient multicarrier systems, where more than one channel are adjacently placed near to each other at, for example, symbol rate spacing. In this case, each channel will get interference from the neighbouring channels.

In **Figure 4**, the root sum square of the ISI is plotted versus different FFT size M_p with different ρ values. These ISI values are found by convolving the impulse response of the RRC filter (whether the FD or the TD design) with itself and then taking the root sum-squared of resulting values at symol times without counting the central tap. As can be seen from the figure, for low M_p sizes, the FD design always delivers low RMS values of the worst ISI when compared to the TD design.

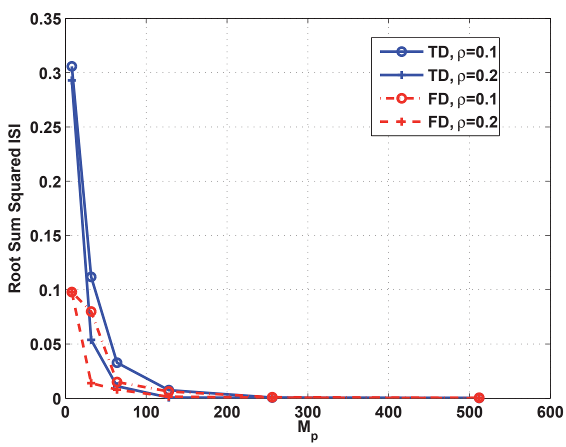

Figure 4: Root sum squared of the ISI for FD and TD designs

5 Conclusion and Outlook

In this work, to increase the spectral efficiency of high speed optical links, digital pulse shaping in the transmitter of Nyquist WDM channel was considered. The RRC filter was chosen as pulse shaper since it satisfies the Nyquist zero ISI criterion in bandlimited systems. Two designs of an RRC filter were presented: one in the FD and the other in the TD. It has been shown that with the FD design, the power of the side lobes is lower when compared to the TD design. Additionally, the root sum square of the ISI values for FD design are less than that with TD design.

The work presented in this paper is just a first step towards spectral confinement to increase the spectral efficiency in high speed optical networks. As future work, the complete transmitter model taking into account the DAC and some prefiltering will be investigated. Simulating the whole system with CD channel and an RRC filter in the receiver is also left for future work.

References

[1] E. Torrengo *et al.*, ”Transoceanic PM-QPSK terabit superchannel transmission experiments at baud-rate subcarrier spacing,” in *ECOC* 2010, We.7.C.2.

[2] G. Colavolpe, ”Faster-than-Nyquist and beyond: how to improve spectral efficiency by accepting interference,” in *ECOC* 2011, paper Mo.1.B.1.

[3] W. Shieh, ”OFDM for flexible high-speed optical networks,” in *Journal of Lightwave Technology*, vol. 29, no. 10, pp. 1560-1577, May 2011.

[4] H. Masuda *et al.*, ”13.5 Tb/s (135 × 111 Gb/s/ch) no-gaurd-interval coherent OFDM transmission over 6248 km using SNR maximized second-order DRA in the extended L-band,” in *OFC/NFOEC* 2009, paper PDPB5.

[5] J.-X. Cai, Y. Cai, Y. Sun, C. R. Davidson, D. G. Foursa, A. Lucero, O. Sinkin, W. Patterson, A. Pilipetskii, G. Mohs, and N. S. Bergano, ”112 × 112 Gb/s transmission over 9360 km with channel spacing set to the baud rate (360 % spectral efficiency),” in *ECOC* 2010, paper PD2.1.

[6] J.-X. Cai, C. R. Davidson, A. Lucero, H. Zhang, D. G. Foursa, O. Sinkin, W. Patterson, A. Pilipetskii, G. Mohs, and N. S. Bergano, ”20 Tbit/s transmission over 6860 km with sub-Nyquist channel spacing,” in *Journal of Lightwave Technology*, vol. 30, no. 4, pp. 651-657, Feb. 2012.

[7] J.-X Cai, H. G. Batshon, H. Zhang, C. R. Davidson, Y. Sun, M. Mazurczyk, D. G. Foursa, A. Pilipetskii, G. Mohs, and N. S. Bergano, ”25 Tbit/s transmission over 5530 km using 16QAM at 5.2 b/s/Hz spectral efficiency,” in *ECOC* 2012, paper JW2A.47.

ISBN 978-3-8007-3503-7