Towards Coverage Control with Anisotropic Sensors in 2D Space

∗A. Gusrialdi, S. Hirche, T. Hatanaka and M. Fujita (Tokyo Institute of Technology)

Abstract– In this paper the coverage control problem for mobile sensor networks is studied. The novelty is to consider an anisotropic sensor model where the performance of the sensor depends not only on the distance but also on the orientation to the target. By adapting the Lloyd algorithm, a distributed control law is derived. Aside from coverage the control law also guarantees collision avoidance between the agents. A simulation is provided to illustrate the results obtained in this paper.

Key Words: Coverage control, Distributed algorithms, Anisotropic voronoi partition

1 Introduction

Stimulated by the technological advances and the development of relatively inexpensive communication, computation, and sensing devices, the interest in the research area of coordinated networked control has majorly increased over the past years. One example is the deployment of autonomous vehicles to perform challenging tasks such as search and recovery operations, manipulation in hazardous environments, surveillance and also environmental monitoring for pollution detection and estimation. Deploying multiple agents to perform tasks is advantageous compared to the single agent case: It provides robustness to agent failure and allows to handle more complex tasks.

In this paper, we consider a mobile sensing network of vehicles equipped with sensors to sample the environment. The goal is to drive the sensors/agents to the position such that a given region is optimally covered by the sensors.

Some relevant works on the coverage control problem are [1], [2], [3], [4]. In [1] the agents move to the optimal configuration which minimizes an objective function. The approach is based on Voronoi tessellation and Lloyd algorithm. The same problem is considered in [2] with a more realistic model by introducing “limited-range interactions” of the sensors, i.e the sensing range is restricted to a bounded region. The advantage of the Voronoi approach is that the control law is distributed by its nature. Dynamic coverage is considered in [3]. Here, the agents move such that every point in a given area is sensed with a pre-specified coverage level C^*. The same problem is addressed in [4] under some practical assumptions such as bounded sensing and actuation capacities of the vehicles. However, in the works mentioned above, only a uniform (isotropic) sensor model is considered. In this paper, in contrast to the above papers, we consider the coverage problem with an anisotropic sensor model. This model is more realistic since most of the sensors such as cameras, directional microphones, radars etc are anisotropic. The approach in this paper is based on Voronoi tessellation combined with an adapted Lloyd algorithm and a gradient descent, similar to the approach in [1]. The consideration of a general anisotropic sensor model results in an anisotropic Voronoi tesselation which is difficult to analyze. In this first approach, the optimal control law for the coverage problem is derived assuming a fixed, equal sensor orientation. The idea is to transform the anisotropic problem to the isotropic one. By the transformation properties the control law obtained for the isotropic problem also solves the problem fo the considered anisotropic case. This paper is organized as follows: The problem formulation for the anisotropic sensor model is presented in section 2. The anisotropic Voronoi partition which is the extension of the ordinary Voronoi partition in Lloyd algorithms and the optimal location of the mobile sensors are derived in section 3. In section 4 the control law for the deployment is derived and collision avoidance is investigated. Simulation results are provided in section 5.

2 Problem Formulation

Let Q be a convex polytope in $\mathcal{R}^2$ including its interior. $\phi(\cdot) : Q \to \mathcal{R}_+$ is a continuous distribution density function which represents the probability that some event takes place in Q. In this paper, we interchangeably refer to the elements of the network as sensors, agents, vehicles, or robots. Let $P = (p_1, ..., p_n)$ be the location of the n identical robots/sensors moving in the region Q. Let $\Theta = (\theta_1, ..., \theta_n)$ be the orientation/attitude of n agents. The non-decreasing differentiable function $f(\cdot) : \mathcal{R}_+ \to \mathcal{R}_+$ indicates the quality of the sensing performance of the sensor, i.e the probability of sensing an event in Q. Cortes et.al [1] consider an isotropic sensor with the sensing performance defined as $f(\|q - p_i\|)$ that degrades with the distance between a point $q \in Q$ and the i-th sensor position p_i. The points where the sensing performance (or probability of sensing) is equal are represented by a circle of radius R, and the center is the sensor location. As shown in Fig. 1(a), points q_1 and q_2 with the same distance to the sensor will result to the same sensing probability.

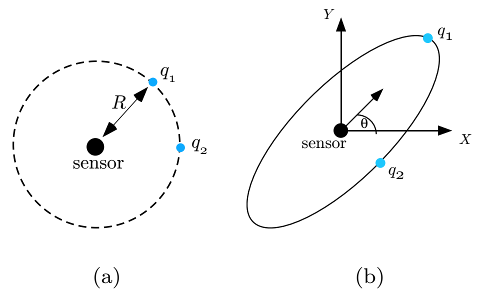


Fig. 1: (a) Isotropic sensor model, (b) Anisotropic sensor model

In this paper, anisotropic sensors are considered where the degradation of the sensing performance is also affected by the orientation of the sensor w.r.t the point to be sensed. The anisotropic sensor model in this paper is considered by a non-Euclidean distance measure as follows.

Assumption 1. *The sensing performance of the* *anisotropic sensor model is given by the non-Euclidean distance measure* $\|q-p_i\|_{L_i}$ *defined as*

$$\|q-p_i\|_{L_i} = (q-p_i)^T L_i (q-p_i), \quad (1)$$

where the matrix L_i *is positive definite and can be decomposed as* $L_i = F_i^T F_i$ *with*

$$F_i = \left[\begin{pmatrix} \frac{c}{a} & 0 \\ 0 & \frac{c}{b} \end{pmatrix} \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix} \right] \quad (2)$$

where θ_i *is the orientation of the i-th sensor, and* $a, b, c > 0$ *are the parameters.*

Observe that the matrix F_i is invertible. The level sets of sensing performance of the anisotropic sensor are given by ellipses where the center is the sensor location as shown in Fig. 1(b). Here, θ_i is the orientation of the ellipse, a, b are the length of major and minor axis of the ellipse respectively.

The overall sensing cost incurred by all robots can be formulated as

$$\mathcal{H}(P,\Theta,\mathcal{W}) = \sum_{i=1}^{n} \int_{W_i} f(\|q-p_i\|_{L_i})\phi(q)dq, \quad (3)$$

where region W_i is the dominance region of the i-th sensor and $\mathcal{W} = (W_1, ..., W_n)$. The challenges addressed in this paper are

1. Find the optimal configuration such that

$$\min_{P,\Theta,\mathcal{W}} \mathcal{H}$$

2. Find the control law u_i that drives the agents to the optimal configuration given the agent dynamics

$$\dot{p_i} = u_i. \quad (4)$$

Optimal coverage is achieved by minimizing (3) w.r.t (1) sensor location P and orientation Θ and (2) the assignment of the dominance regions $\mathcal{W}$.

3 Optimal Partition, Location

3.1 Anisotropic Voronoi Partitions

To minimize (3), we introduce the notion of the Voronoi partition. The Voronoi region of an agent is defined by all points which are "closer" in the sense of the considered distance measure to that agent than to any other. For the Euclidean distance measure the Voronoi region V_i associated with its generator p_i is defined as

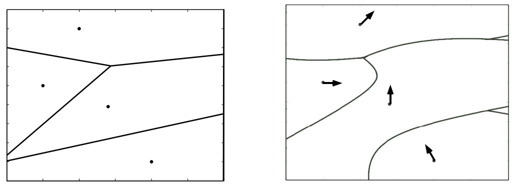
$$V_i = \{q \in Q \mid \|q-p_i\| \le \|q-p_j\|, \forall j \ne i\}. \quad (5)$$

The Voronoi partition V_i^* of agent-i for the anisotropic case considered in this paper is defined as follows.

Definition 1.

$$V_i^* = \{q \in Q \mid \|q-p_i\|_{L_i} \le \|q-p_j\|_{L_j}, \forall j \ne i\}. \quad (6)$$

This anisotropic Voronoi partition is not only determined by the sensors position but also the sensors orientation θ_i as observable from the matrix L_i. As a result the anisotropic Voronoi tesselation is no longer composed of convex polytopes, but of curved possibly non-convex regions. Fig. 2(a) and Fig. 2(b) depict the examples of isotropic and anisotropic Voronoi partition.



(a) (b)

Fig. 2: (a) Isotropic Voronoi partition, (b) anisotropic Voronoi partition given by (6).

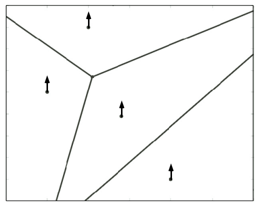


Fig. 3: Anisotropic Voronoi partition with equal orientation

Lemma 3.1. *The boundary between two adjacent* V_i^* *and* V_j^* *as defined in (6) is a quadratic curve.*

Proof. Any point q in $V_i^* \cap V_j^*$ which is the boundary of the Voronoi partitions of V_i^* and V_j^* satisfies $\|q-p_i\|_{L_i} = \|q-p_j\|_{L_j}$ i.e. $(q-p_i)^T L_i (q-p_i) = (q-p_j)^T L_j (q-p_j)$. It is clear that this equation is quadratic in q. Therefore any point in $V_i^* \cap V_j^*$ lies on a quadratic curve. □

The boundary can be represented as

$$Ax^2 + By^2 + Cxy + Dx + Ey + K = 0, \quad (7)$$

where the coefficients of (7) can be computed by solving $(q-p_i)^T L_i (q-p_i) = (q-p_j)^T L_j (q-p_j)$. Due to the space limitation, only A, B, C are described which will be used later in this paper. The coefficients of (7) are : $q = (x, y)$, $A = b^2(\cos^2\theta_i - \cos^2\theta_j) + a^2(\sin^2\theta_i - \sin^2\theta_j)$, $B = a^2(\cos^2\theta_i - \cos^2\theta_j) + b^2(\sin^2\theta_i - \sin^2\theta_j)$, $C = (a^2 - b^2)(\sin 2\theta_i - \sin 2\theta_j)$ and $D, E \ne 0$.

Another major difference to isotropic Voronoi tesselations is that anistropic tesselations may contain regions without a generator [5], i.e a Voronoi cell of an anisotropic Voronoi diagram is not necessarily connected. Moreover, the information of all other generator positions is required to compute the anisotropic Voronoi diagrams. This in in contrast to isotropic Voronoi diagram where only the neighbours positions are required.

For the rest of this paper, the following assumption is considered.

Assumption 2. *The orientations of all agents are equal and fixed over time, i.e.* $\theta_i(t) = \theta_j(t), \forall i \ne j$ *and* $t \ge 0$.

This can be achieved by applying a known method (e.g [6]) for making an agreement on the orientation beforehand. This assumption leads to the following lemma.

Lemma 3.2. *From assumption 2 and definition 1, the anisotropic Voronoi tessellation is composed of convex polytopes. Moreover,* $F_i(t) = F_j(t) = F$ *and* $L_i(t) = L_j(t) = L \; \forall i \neq j$ *and all* $t \geq 0$.

Proof. From the assumption 2, $\theta_i = \theta_j = \theta$, it follows that $A = B = C = 0$ in (7) and furthermore $D, E \neq 0$. As a result the boundary of the Voronoi cell is a straight line. Since Q is a convex polytope, the Voronoi tessellation is also composed of convex polytopes. From (2), it is also cleared that $F_i = F$ and $L_i = L$. □

One example of the anisotropic Voronoi diagram with fixed and equal orientations is shown in Fig. 3.

3.2 Optimal Location

Corollary 1. *The anisotropic Voronoi partition* $\mathcal{V}^*$ *minimizes (3) w.r.t the partition* $\mathcal{W}$.

Proof. From definition 1 and since f is a non-decreasing function, it is clear that the Voronoi partition $\mathcal{V}^*$ minimizes (3) w.r.t the partition $\mathcal{W}$. □

As the orientation is assumed to be fixed and as a result of corollary 1,

$$\min_{P,\Theta,\mathcal{W}} \mathcal{H} = \min_{P} \mathcal{H}_{\mathcal{V}^*}.$$

Assume that the sensing performance $f(\|q - p_i\|_L) = \|q - p_i\|_L^2$. Then (3) can be written as

$$\mathcal{H}_{\mathcal{V}^*}(P) = \sum_{i=1}^{n} \int_{V_i^*} \|q - p_i\|_L^2 \phi(q) dq.$$

In order to derive the optimal location of the sensors, the above equation can be simplified to

$$\mathcal{H}_{\mathcal{V}^*}(P) = \sum_{i=1}^{n} \int_{V_i^*} \|F(q - p_i)\|^2 \phi(q) dq. \quad (8)$$

Next, we introduce anisotropic centroidal Voronoi configuration.

Definition 2. *Given the set of points* P *in* Q. $C_{V_i^*}$ *is the center of mass (centroid) of an anisotropic Voronoi partition. A Voronoi tessellation is called an anisotropic centroidal Voronoi configuration if*

$$p_i = C_{V_i^*}, \forall i;$$

i.e the points P *serve as generators and also centroids for the anisotropic Voronoi tessellations.*

The optimal location is given by the following proposition.

Proposition 3.1. *The objective function (8) is minimized by the anisotropic centroidal Voronoi configuration .*

Proof. Define $\bar{q}, z_i$ as $\bar{q} = Fq$ and $z_i = Fp_i$ which are points of region and agents in a space transformed by matrix F called the solution space. Note that the region Q is transformed by F to the convex region Q_s and the minimization of (8) in the solution space leads to the minimization in the real physical space. Moreover, the anisotropic Voronoi partition V_i^* is transformed to the isotropic Voronoi partition ($\bar{V}_i$) in the solution space defined as

$$\bar{V}_i = \{\bar{q} \in Q_s | \|\bar{q} - z_i\| \leq \|\bar{q} - z_j\|, \forall j \neq i\}.$$

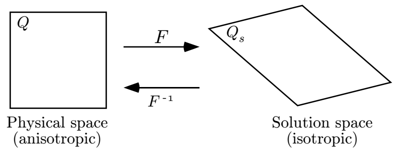


Fig. 4: Transformation between solution and real space

By applying substitution rule for multiple variables, the integral in (8) can be rewritten as :

$$\mathcal{H}_{\bar{\mathcal{V}}}(Z) = \sum_{i=1}^{n} \int_{\bar{V}_i} \|\bar{q} - z_i\|^2 \phi(\bar{q}) \left|\det(F^{-1})\right| d\bar{q}. \quad (9)$$

with $\mathcal{Z} = (z_1, ..., z_n)$. Applying the parallel axis theorem, (9) becomes

$$\mathcal{H}_{\bar{V}}(Z) = \left|\det(F^{-1})\right| (\sum_{i=1}^{n} J_{\bar{V}_i, C_{\bar{V}_i}} + \sum_{i=1}^{n} M_{\bar{V}_i} \|z_i - C_{\bar{V}_i}\|^2),$$

where

$$M_{\bar{V}} = \int_{\bar{V}} \phi(\bar{q}) d\bar{q}, \; C_{\bar{V}} = M_{\bar{V}}^{-1} \int_{\bar{V}} \bar{q} \phi(\bar{q}) d\bar{q},$$

$$J_{\bar{V},z} = \int_{\bar{V}} \|\bar{q} - z\|^2 \phi(\bar{q}) d\bar{q}.$$

denote mass, centroid and polar moment of inertia of an anisotropic Voronoi partition respectively. The local minimum is the solution of

$$\nabla \mathcal{H}_{\bar{\mathcal{V}}} = [... \frac{\partial \mathcal{H}_{\bar{V}}}{\partial z_i} ...]^T = \mathbf{0}.$$

The partial derivative of (9) is given by

$$\frac{\partial \mathcal{H}_{\bar{V}}}{\partial z_i}(Z) = 2 \left|\det(F^{-1})\right| M_{\bar{V}_i} (z_i - C_{\bar{V}_i}).$$

Therefore the local minimum points i.e. the critical points for $\mathcal{H}_{\bar{V}}$ are centroids of their Voronoi cells in the solution space ($C_{\bar{V}}$) which are the centroids of the anisotropic Voronoi partition ($C_{V_i^*} = F^{-1} C_{\bar{V}_i}$). □

4 Continuous Lloyd Descent For Coverage Control

4.1 Optimal Control for Fixed Orientation

In this section, a control law based on Lloyd algorithm to drive the agents to the location that minimize (3) is derived. The strategy is to transform the control law in the solution space into the real physical space as illustrated in Fig. 4.

Consider the agents in real space with dynamics given in (4). Set

$$u_i = -k(p_i - C_{V_i^*}), \quad (10)$$

where k is a positive gain and V_i^* is the anisotropic Voronoi partition and assumed to be continuously updated.

Proposition 4.1. *By applying the control law in (10), the agents in the physical space will converge asymptotically to the set of critical points i.e the set of anisotropic centroid Voronoi configurations. If this set is finite, the agents converge to one of them.*

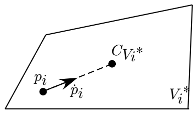


Fig. 5: Each agent moves towards the centroid.

Proof. The dynamics of the agents in the solution space (isotropic case) is given by

$$\dot{z}_i = \bar{u}_i,$$

From [1], it is well-known that the control input given by

$$\bar{u}_i = -k(z_i - C_{\bar{V}_i})$$

drives the agents in the solution space to the centroidal Voronoi configuration, the critical points of the objective function (8).

By using the chain rule, the control law in the physical space can be computed by

$$\begin{aligned} u &= \dot{p_i}. \\ &= \frac{\partial(F^{-1}z_i)}{\partial z_i}\dot{z}_i. \\ &= -k(p_i - C_{V_i^*}). \end{aligned}$$

Consider $\mathcal{H}_{\mathcal{V}^*}$ as a Lyapunov function. Under the control law (10), $\frac{d}{dt}\mathcal{H}_{\mathcal{V}^*} \leq 0$. By LaSalle's Principle, the agents converge to the largest invariant set which is the set of anisotropic centroid Voronoi configurations. If this set consists of finite points, then the agents converge to one of them (see Corollary 1.2 in [1]). □

Remark 1. *This control law is distributed since each agent only needs the information of it's neighbour's position to compute the control as observable from (10).*

Remark 2. *The control law is optimal w.r.t the fixed orientation. By considering the orientation as optimization variable as in the original problem will lead to a better result i.e. lower values of $\mathcal{H}$ are achieved.*

4.2 Collision Avoidance Guarantee

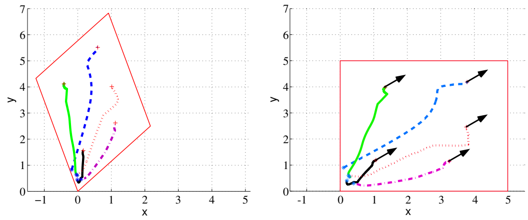
Another advantage of the Voronoi approach is the implicit collision avoidance.

Proposition 4.2. *With the control law (10), if there is no collision at t^*, there will be no collision at $t > t^*$.*

Proof. The agents applying the control law (10) will move towards the centroid of its Voronoi cell as shown in Fig. 5. From Lemma 3.2 and the continuity of $\phi(\cdot)$, the centroid is always inside the Voronoi cell and since the Voronoi tessellations are nonoverlapping by construction, no two agents will come to the same point i.e there will be no collision between the agents for all $t \geq t^*$ if there was no collision at time t^*. □

5 Simulation

In this section, we illustrate the results through simulation. Assume that there are 5 agents/mobile sensors which sensor parameters a, b, c, θ are equal to $2, 1, 1, -\pi/3$, respectively. The region Q is a square region of side length $l = 5$ unit length. $\phi(q) = 1, \forall q$. Assume that at the initial time, $p_i \neq p_j, \forall i \neq j$ i.e no collision occurs. The results are shown in Fig. 6. Fig. 6(a) shows trajectories of the agents in the transformed space. The trajectories of the agents in the real space are shown in Fig. 6(b). It is also confirmed from Fig. 7 that no collusion occurs between the agents during the movement to the anisotropic centroidal Voronoi configuration .



(a) (b)

Fig. 6: Trajectories of the agents in (a) solution space, (b) real physical space. The (∗) signs show the initial position of the agents.

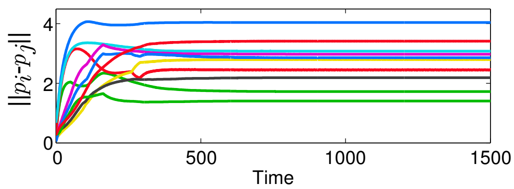


Fig. 7: Inter-agents distances.

6 Conclusion and Future Works

In this paper a first approach for the coverage control with an anisotropic sensor model is presented. An optimal control law for fixed and equal orientation is derived using a Voronoi based approach with an adapted Lloyd algorithm and a gradient descent approach. The control law is distributed and also guarantees collision avoidance. The efficiency of the proposed control law is confirmed by simulation. Currently, the problem with the orientation as optimization variable is investigated.

References

1) J. Cortes, S. Martinez, T. Karatas, F. Bullo, "Coverage control for mobile sensing networks," *IEEE Transactions on Robotics and Automation*, Vol. 20, No. 2, pp. 243-255, 2004

2) J. Cortes, S. Martinez, F. Bullo, "Spatially-distributed coverage optimization and control with limited-range interactions," *ESAIM. Control, Optimisation & Calculus of Variations*, Vol. 11, pp. 691-719, 2005

3) I. I. Hussein and D. M. Stipanovic, "Effective coverage control for mobile sensor networks," *Proceedings of the IEEE Conference on Decision and Control*, pp. 2747-2753, 2006

4) P. F. Hokayem, D. M. Stipanovic and M. W. Spong, "Dynamic coverage control with limited communication," *Proceedings of the American Control Conference*, pp. 4878-4883, 2007

5) F. Labelle, J. R. Shewchuk, "Anisotropic voronoi diagrams and guaranteed-quality anisotropic mesh generation," *Proceedings of the 19th ACM Symposium on Computational Geometry*, pp. 191-200, 2003

6) Y. Igarashi, M. Fujita and M. W. Spong, "Passivity-based 3D Attitude Coordination: Convergence and Connectivity," *Proceedings of the IEEE Conference on Decision and Control*, 2007, to appear