Networked Control Systems with Time-Varying Delay - Stability through Input-Output Transformation

Netzwerkregelungssysteme mit variabler Totzeit - Stabilität durch Eingangs-Ausgangs-Transformation

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Angesichts steigender Komplexität moderner Automatisierungssysteme gewinnen Netzwerkregelungssysteme wegen ihrer Modularität und vereinfachten Diagnose mehr und mehr an Bedeutung. In derartigen Systemen sind Prozess und Regler räumlich getrennt und über ein Kommunikationsnetz verbunden. Die dadurch induzierte Kommunikationstotzeit wirkt potenziell destabilisierend. In diesem Artikel wird eine neuartige Methode zur Stabilisierung in Gegenwart von variabler Totzeit vorgeschlagen, welche auf einer Eingangs-Ausgangs-Transformation basiert. Anstelle der ursprünglichen Ausgänge von Prozess und Regler werden Linearkombinationen aus deren Ein- und Ausgängen über das Kommunikationsnetz übertragen. Asymptotische Stabilität kann für Netzwerkregelungsysteme bestehend aus nichtlinearen eingangs-ausgangs-passiven Teilsystemen und beliebig große variable Totzeit mit begrenzter Änderungsrate garantiert werden. Die vorgeschlagene Methode zeichnet sich in Vergleich zu bekannten Ansätzen durch eine hervorragende Regelgüte aus. Ihre Validität wird in Simulationen bestätigt.

With the increasing complexity of modern automation systems, Networked Control Systems (NCS) gain more and more importance due to their modularity and simplified diagnosis. In NCS, plant and controller are spatially separated and the control loop is physically closed through a communication network. Communication time delay in a NCS degrades the performance and may lead to instability. In this article we propose a novel approach for time-varying delay applicable to the class of input-feedforward-output-feedback-passive (IF-OFP) systems. The proposed approach is based on an input-output transformation; instead of direct communication a linear transformation of the plant and controller input and output is sent through the communication network. Asymptotic stability is guaranteed for arbirarily large time delay with bounded rate of change. The control performance is superior compared to alternative approaches. Simulations show the validity of the proposed approach.

Schlagwörter: Netzwerkregelungssystem, variable Totzeit, L_2-Stabilität.

Keywords: Networked control system, time-varying delay, finite gain L_2 stability.

1 Introduction

The use of communication networks for signal transmission in control systems offers significant advantages over the traditional point-to-point connections, in terms of reduced wiring and cost, increased modularity, high flexibility and reconfigurability. Therefore NCS, i.e. systems in which the plant and the controller are connected through a network, see Fig. 1, increasingly replace traditional control systems. NCS have already been adopted to numerous applications, see e.g. [1, 2]. However, in NCS the signal transmission over the communication network cannot be regarded as ideal. Time delay, packet loss and the limited communication resources constitute

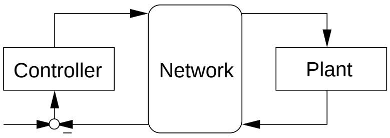

Fig. 1: Standard architecture of a networked control system.

major challenges, see e.g. the surveys [3, 4]. These network induced effects depend on the number of active nodes, the network traffic and the transmission protocol and are generally not exactly known during the controller design stage. Advantageously, NCS offer additional degrees of freedom in the design compared to traditional control architecture. For instance, the limited computational power available at the plant side can be used to implement local low order pre-stabilizing controllers, while computationally intensive control measures are remotely placed.

In this article the unknown time-varying delay challenge is addressed. It is well known that time delay in the control loop deteriorates the performance and can lead to instability. Stability in the presence of unknown time-varying delay is usually guaranteed based on bounds on the time delay value, the time delay derivative or both. For the constant time delay challenge delay-dependent and delay-independent approaches exist. In the former a bound for the time delay value is necessary, while in the second stability is guaranteed for arbitrarily large time delay.

In the seminal work [5] an augmentation technique is applied to transform the linear discrete time system with time delay into a system without time delay but higher order. Stability is guaranteed for periodic time delays. In [6] a queuing method is used to reshape bounded random time delay to constant time delay and a state predictor is applied in order to compensate for the time delay. Queuing methods introduce additional time delay deteriorating thus the performance. Predictive control, where the model of the plant is included in the controller in order to predict the future plant output is also considered e.g. in [7, 8]. Nevertheless, predictive control schemes in general require very good knowledge of the plant model and the time delay. Alternatively, stochastic control approaches based on stochastic time delay models have been developped. In [9] stochastic optimal control is applied, assuming that the time delay is bounded by the sampling period, and extended in [10] to longer time delays. Random time delays modelled as Markov chains are further considered in [11] where a necessary stability condition formulated in Bilinear Matrix Inequalities is obtained, and in [12], where necessary and sufficient conditions for stability are given. Stability for unknown time-varying delay based on bounds on the time delay derivative and the time delay bound is examined in [13]. In a comparison with other recent methodologies for time-varying delay [14–16], the approach in [13] seems to be less conservative. Common in the literature is also the restriction to one channel feedback NCS, in which a communication network in inserted only in the forward or backward path, e.g. [17, 18].

The assumptions made in the NCS literature are often restrictive in practical control applications. For instance, the exact time delay model, stochastic or not, is hard to obtain, nevertheless some bounds can be guaranteed. In this article we propose a novel methodology for time-varying delay in the forward and backward communication channel which can guarantee stability based on bounds on the time delay derivative. For stabilizing controller design limited plant knowledge is necessary. Output feedback is considered, requiring no direct access to the states.

The proposed approach is based on the input-output transformation introduced by the authors in [19] for constant time delay. In contrast to standard time delay systems with input or state delay, in NCS there is access to the inputs and outputs before and after the time delay. Exploiting this fact, instead of the original plant and controller output, a linear combination of the corresponding inputs and outputs is transmitted over the network. The plant and controller are assumed to be input-feedforward-output-feedback-passive (IF-OFP) systems. It is shown that as long as the plant and controller without the network and the time delay are stable based on well known feedback stability theorem, the closed loop with the transformation is stable for arbitrarily large constant time delay. Stability is based on the fact that the constant time delay operator has L_2 gain one for arbitrarily large time delay.

In case of time-varying delay however, the time delay operator becomes unbounded and stability may be compromised. Here, we extend the above approach to time-varying delay. It is shown that asymptotic stability of the closed loop system can be guaranteed if the time delay operators are bounded, even if the gain is larger than one. The time delay operator can be bounded by assuming a bound on the time delay derivative. A more conservative controller design can be exploited to accommodate larger bounds for the time delay derivative. The proposed method is validated in a comparison with two alternative approaches.

The remainder of this article is organized as follows: Section 2 introduces the necessary background, followed by the stability analysis in Section 3. A comparison with two other approaches is presented in Section 4.

2 Preliminaries

Let $\|u\|_{L_2}$ denote the L_2 norm of a piecewise square-integrable function $u(\cdot) : \mathbb{R}_+ \to \mathbb{R}^m$ with $\mathbb{R}_+$ being the set of non-negative real numbers and $\mathbb{R}^m$ the Euclidean space of dimension m. The truncation of $u(\cdot)$ up to the time t is denoted by $u_t(\cdot)$, and the extended space of Lebesgue integrable functions by L_{2e}. The systems considered in this article are described by

$$H: \dot{x} = f(x,u), \quad y = h(x,u) \tag{1}$$

where $x \in \mathbb{R}^n, u, y \in \mathbb{R}^m$ are the state, input and output vectors respectively, and $f(0,0) = h(0,0) = 0$. The function f is locally Lipschitz, thus for each fixed initial state $x(0)$, (1) defines a causal mapping from the input signal $u(\cdot)$ to the output signal $y(\cdot)$. In case of a Linear-Time-Invariant (LTI) system, $G(s) = \dfrac{Y(s)}{U(s)}$ denotes the transfer function of (1), where $U(s), Y(s)$ are the Laplace transforms of the input $u(\cdot)$ and output $y(\cdot)$ respectively. In this article we will consider zero state observable systems.

Definition 1 *[20] The system (1) is called zero state observable if no solution $\dot{x} = f(x,0)$ can stay identically in $S = \{x \in \mathbb{R}^n | h(x,0) = 0\}$ other than the trivial solution $x(t) \equiv 0$.*

2.1 Input-Feedforward-Output-Feedback-Passive Systems

Definition 2 *The system (1) is called input-feedforward-output-feedback-passive if there exist constants $\delta, \epsilon \in \mathbb{R}$ and a positive semi-definite function $V : \mathbb{R}^n \to \mathbb{R}_+$ such that for each admissible u and each $t \in [0, \infty)$ we have*

$$\dot{V}(x) \le u^T y - \delta u^T u - \epsilon y^T y \tag{2}$$

where $x \in \mathbb{R}^n$ is the state vector or the system.

In physical interpretation $u^T y$ represents the instantaneous external energy flow into the system. The above input-output description is a generalization of the passivity concept. If $\delta = \epsilon = 0$ then the system is passive, i.e. it does not generate energy. If $\delta = 0$ and $\epsilon > 0$ the system is called output-feedback strictly passive and if $\delta > 0$ and $\epsilon = 0$ input-feedforward strictly passive. In both these cases the system dissipates energy. If one or both of the values δ, ϵ are negative then there is a shortage of passivity in the system. The system can generate energy, but this energy is bounded by the squared L_2 norm of the input and/or the output signal. Note, that IF-OFP is a special case of dissipativity with a quadratic supply rate [21, 22].

2.2 Finite Gain L_2 Stability

Definition 3 *[20] The system (1) is called finite gain L_2 stable if there exist constants $\gamma, \beta \ge 0$ such that between each input $u(\cdot) \in L_{2e}$ and the corresponding output $y(\cdot) \in L_{2e}$ of the system for each $t \in [0, \infty)$ the following inequality holds*

$$\|y_t\|_{L_2} \le \gamma \|u_t\|_{L_2} + \beta. \tag{3}$$

The smallest possible value γ satisfying (3) is the L_2 gain of the system. In LTI systems the L_2 gain is the H_∞ norm, denoted here by $|G|^\infty$, where $G(s)$ is the transfer function of the system.

One important stability result for closed loop systems comes from the IF-OFP property of its subsystems. Consider two IF-OFP systems H_p and H_c satisfying (2) with some V_i, δ_i, ϵ_i, $i \in \{p, c\}$ with subscript $(.)_p$ refering to the plant and $(.)_c$ to the controller.

Proposition 1 *[20] The negative feedback interconnection of H_p and H_c is finite gain L_2 stable if*

$$\epsilon_c + \delta_p > 0 \qquad \text{and} \qquad \epsilon_p + \delta_c > 0.$$

Assuming further zero state observability of plant and controller, asymptotic stability can be shown by taking as Lyapunov function $V = V_p + V_c$, and applying the invariance principle [20].

2.3 Time Delay Operator

We consider a time delay operator $\mathcal{D} : u(\cdot) \to y(\cdot)$ with $u, y \in \mathbb{R}^m$ the input and output respectively, i.e. $y(t) = u(t - T(t))$, where T is the time delay value. In case of constant time delay, i.e. $T(t) = T_0$ it is easy to show that the L_2 gain $\gamma_{\mathcal{D}} = 1$ for arbitrarily large constant time delay. However, in case of time-varying delay, without further assumptions, the operator becomes unbounded [23]. The time delay operator can be bounded assuming a maximum in the time delay derivative.

Proposition 2 *[23] If the time delay is continuously differentiable and the time delay derivative bounded, i.e.*

$$\dot{T} \le d < 1, \tag{4}$$

the L_2 gain of the time-varying delay operator is

$$\gamma_{\mathcal{D}} = \frac{1}{\sqrt{1-d}}.$$

3 Main Result

Most generally, multi-input-multi-output systems with the same dimension m of input and output can be considered as plant and controller in this approach. For the ease of notation the following statements address the single-input-single-output case, i.e. $m = 1$. Where nonambiguous, the time argument t is dropped.

3.1 NCS with Input-Output Transformation

The system comprises a plant H_p and a controller H_c described by (1) with $x_p, u_p, y_p, x_c, e, y_c$ being the plant and controller, state, input and output vectors respectively. By e the control error is denoted $e = w - u_c$, with $w \in L_{2e}$ being the desired value and u_c the lefthand side output of the communication channel, see Fig. 2.

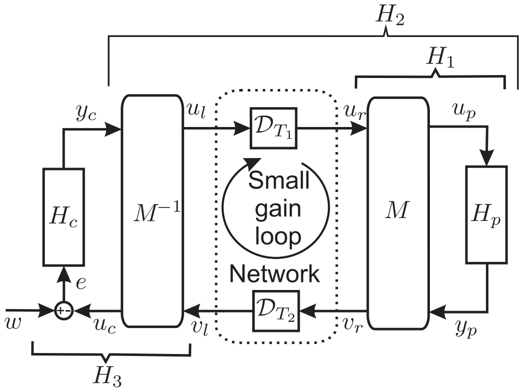

Fig. 2: Networked control system with time-varying delay and input-output transformation

The plant is connected to the controller through a communication network. However, instead of directly transmitting plant (controller) output over the communication channel, a linear combination of plant (controller) output and input is sent to the corresponding receiver side. The transformation matrix $M \in \mathbb{R}^{2\times 2}$ acts on the plant and the controller input-output vector $z_p^T = [u_p\, y_p]$ and $z_c^T = [y_c\, u_c]$, respectively

$$s_r = Mz_p \quad \text{and} \quad s_l = Mz_c, \tag{5}$$

where $s_r^T = [u_r\, v_r]$ and $s_l^T = [u_l\, v_l]$ represent the values transmitted over the communication channel, see Fig. 2. The transformation matrix $M = RB$ is parameterized as a rotation matrix R and a scaling matrix B

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \quad B = \begin{bmatrix} \sqrt{b} & 0 \\ 0 & \frac{1}{\sqrt{b}} \end{bmatrix},$$

with the rotation angle $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and the scaling parameter $b > 0$, both constant. The mapping by M is a bijection; it belongs to the class of *special linear transformations*, i.e. $\det M = 1$, hence is non-singular, an inverse exists. Note that for $M = I_2$, with I the identity matrix, the standard approach without input-output transformation is recovered.

The network is modelled as a forward time delay operator $\mathcal{D}_{T_1}$ (plant to controller channel) and backward time delay operator $\mathcal{D}_{T_2}$. The value u_l is transmitted from the controller to the plant side arriving as value $u_r(t) = u_l(t - T_1(t))$. Accordingly for the backward channel $v_l(t) = v_r(t - T_2(t))$ holds.

For further reference, the following subsystems are defined : $H_1 : u_r(\cdot) \to v_r(\cdot)$, $H_2 : y_c(\cdot) \to u_c(\cdot)$, $H_3 : v_l(\cdot) \to u_l(\cdot)$ and $H_{OL} = H_1 \circ \mathcal{D}_{T_2} \circ H_3 \circ \mathcal{D}_{T_1}$, see also Fig. 2.

Throughout the paper we make the following assumptions:

1. Plant H_p and controller H_c are IF-OFP systems with positive definite radially unbounded functions V_i and δ_i, ϵ_i, $i \in \{p, c\}$ satisfying Proposition 1, i.e.

$$\epsilon_c + \delta_p > 0 \qquad \text{and} \qquad \epsilon_p + \delta_c > 0.$$

2. Plant H_p and controller H_c are zero state observable.
3. The time delays $T_1(t), T_2(t)$ are continuously differentiable and the time delay derivatives are bounded i.e. $\dot{T}_i \le d_i < 1, i \in \{1, 2\}$.
4. The closed loop system is well posed, i.e. for each input signal $w \in L_{2e}$ there exists a unique solution for the signals $e, u_c, y_c, u_l, v_l, u_r, v_r, u_p, y_p$ that causally depends on w.

The time delay operators are bounded, accordingly to Proposition 2 and assumption 3.

3.2 Stability with Constant Time Delay

Under the further assumption that the forward and backward time delays are constant but unbounded the next proposition holds

Proposition 3 *[19] The closed loop system is delay-independently finite gain L_2 stable if the angle θ is chosen by*

$$\cot 2\theta = \epsilon_p b - \frac{\delta_p}{b}, \tag{6}$$

and

$$\alpha(\theta) = \sin(\theta)\cos(\theta) - \frac{\delta_p}{b}\cos^2(\theta) - \epsilon_p b \sin^2(\theta) \ge 0, \tag{7}$$

where $b > 0$ is a parameter which can be chosen freely to meet performance requirements. Stability is based on the fact that the constant time delay operator has L_2 gain one, for arbitrarily large time delay. The right hand transformation transforms the IF-OFP plant to a finite gain L_2 stable system H_1, see Fig. 2. Since the constant time delay operator has L_2 gain one the L_2 gain of the subsystem H_1 is not affected. Consequently the left hand inverse transformation restores the original IF-OFP property of the plant to the subsystem H_2. Thus, the effect of the input-output transformation is to preserve the plant IF-OFP property to the subsystem H_2 which includes the arbitrarily large constant time delay. As long as stability is guaranteed for the original plant and controller feedback interconnection based on Proposition 1, the same holds for the feedback interconnection of the controller and the subsystem H_2.

3.3 Stability with Time-Varying Delay

In the rest of this article the reference input is considered to be zero, i.e. $w = 0$. The next theorem holds.

Theorem 1 *The closed loop system is globally asymptotically stable if*

$$\gamma^2_{\mathcal{D}_{T_1}} \gamma^2_{\mathcal{D}_{T_2}} < \frac{\beta(\theta^*)}{\alpha(\theta^*)} \frac{\alpha(\theta^*) + \Delta}{\beta(\theta^*) - \Delta}, \tag{8}$$

where θ^*, $\alpha(\theta^*)$ given by (6) (7),

$$\beta(\theta^*) = \alpha(\theta^*) + \frac{\delta_p}{b} + \epsilon_p b, \tag{9}$$

and

$$\Delta = \min[(\epsilon_p + \delta_c)b, (\epsilon_c + \delta_p)/b] > 0. \tag{10}$$

Proof: See Appendix. ■

Expressing the L_2 gain of the time-varying delay operator in (8) using Proposition 2 the following corollary is derived.

Corollary 1 *The closed loop system is globally asymptotically stable if*

$$\frac{1}{(1-d_1)(1-d_2)} < \frac{\beta(\theta^*)}{\alpha(\theta^*)}\frac{\alpha(\theta^*)+\Delta}{\beta(\theta^*)-\Delta}.$$

Proof: Direct application of Theorem 1, Proposition 2 and assumption 3. ■

From (8) it is seen that the larger Δ is, the larger the gain of the time delay operators which do not compromise stability can be, allowing thus larger bounds for the time delay derivatives. Depending on controller design, i.e. δ_c, ϵ_c, different bounds on the time delay derivatives can be accommodated.

An interpretation of the above stability condition can be given in terms of the small gain theorem in the loop with the transformed plant and controller, see Fig. 2. It is shown in the proof, see the Appendix, that an upper bound for the L_2 gain of the subsystem H_1 is given by

$$\gamma_{H_1}^2 = \frac{\alpha(\theta^*)}{\beta(\theta^*)},$$

and further an upper bound for the L_2 gain of the subsystem H_3, which includes the controller, is given by

$$\gamma_{H_3}^2 = \frac{\beta(\theta^*)-\Delta}{\alpha(\theta^*)+\Delta}.$$

Consequently the inverse of the right part of (8) represents a maximum for the L_2 gain of the subsystem $H_1 \circ H_3$, i.e. $\gamma_{H_1 \circ H_3} \leq \gamma_{H_1}\gamma_{H_3}$. Equation (8) can thus be interpreted as a small gain condition, $\gamma_{H_{OL}} \leq \gamma_{H_1}\gamma_{\mathcal{D}_{T_2}}\gamma_{H_3}\gamma_{\mathcal{D}_{T_1}} < 1$. A larger Δ results in smaller L_2 gain for H_3, consequently larger bounds of the time delay operators can be accommodated without compromising stability.

Since the small gain condition is satisfied in the loop with the transformed plant and controller all the signals in this loop will converge to zero. The invertibility of the transformation together with the zero state observability of the plant and the controller can further guarantee that the states will also converge to zero. The small gain theorem however, needs not to be satisfied between the initial plant and controller, thus less conservative controller design is allowed compared to the typical small gain case. For example assuming a plant with an integrator, the small gain theorem between the plant and controller cannot be satisfied, as the integrator is not a bounded L_2 gain operator. On the contrary, Theorem 1 can be still satisfied, even if there is a free integrator in the plant or controller. This can be demonstrated by counterexamples, e.g. $G_p(s) = \frac{1}{s+1}, G_c(s) = \frac{s+1}{s(s+10)}, b = 1, \theta = 30^o$.

4 Comparison with Alternative Approaches

Goal of this section is to verify in simulation the efficacy of the proposed approach. For comparison, a controller without the input-output transformation as well as a controller based on the small gain theorem are considered. We will refer to the controller without the input-output transformation as delay-dependent, in the sense that, contrary to the other two, a bound for the time delay value is necessary for stability to be guaranteed. For simplicity the forward and backward time delays are considered to be equal, i.e. $T_1(t) = T_2(t) = T(t)$. As the method used to compute the stability bounds for the delay-dependent approach is applicable to linear systems only, we consider an LTI system

$$\dot{x}_p = A_p x_p + B_p u_p, \quad y_p = C_p x_p, \qquad (11)$$

as plant with u_p, y_p, x_p its input, output and state, respectively. For the following simulations we fix the parameters to

$$A_p = \begin{bmatrix} -10.1 & -1 \\ 1 & 0 \end{bmatrix}, \quad B_p = \begin{bmatrix} 8 \\ 0 \end{bmatrix}, \quad C_p = \begin{bmatrix} 0 & 5 \end{bmatrix}.$$

resulting in the transfer function

$$G_p(s) = \frac{40}{s^2 + 10.1s + 1}.$$

The above system can be interpreted as a mass-spring-damper system, with mass $M_p = 0.025$kg, spring and damper coefficients $K_p = 0.025$N/m and $D_p = 0.2525$Ns/m, force as input and position as output. Consequently the controller design problem can be considered as position control of a mass-spring-damper system through a communication network.

4.1 Controller Design

For fair comparison the controllers for all approaches are designed by numerical optimization using the integrated time square tracking error measure (ITSE)

$$J = \int_0^{t_f} \tau e^2(\tau)\mathrm{d}\tau,$$

as cost function. The time horizon is defined $t_f = 5$ s and the optimization is performed using `fmincon` of the Matlab optimization toolbox. The initial state of the plant is $x_p(0)^T = [1\ 1]$. The numerical optimization is performed for constant forward and backward time delay $T = 150$ms. The time-varying delay stability limits are afterwards defined. As basic structure for the controller a lead-lag element

$$G_c(s) = k\frac{s+a}{s+c},$$

is considered where $k, a, c > 0$ are parameters to be determined by the numerical optimization. The exact design procedure for all cases is explained in the next.

4.1.1 Input-Output Transformation

For the input-output transformation the optimization problem is described by $\min_{k,a,c,b,\theta} J$ subject to the stability constraint $|G_1|^\infty|G_3|^\infty < 1$ and $k, a, c, b > 0$. The optimization problem gives the controller

$$G_{tr}(s) = \frac{10.6762(s + 55.5019)}{s + 66.5624},$$

with all the constraints satisfied and $b = 0.1531,\ \theta = 51^o$. The optimal cost function value is $J = 0.1147$. The L_2 gain of the open loop is bounded by $|G_1|^\infty|G_3|^\infty = 0.4471$. Based on Corollary 1 stability is guaranteed if for the bound of the time delay derivative (4) holds $d < 0.5529$, independently of the time delay value.

4.1.2 Small Gain Based Controller

For fair comparison, in order to accommodate the same time delay derivative as the transformation approach, the controller gain k is fixed to

$$k = \frac{0.4471}{|G_p(j\omega)|^\infty|G_c(j\omega)|^\infty}. \qquad (12)$$

The numerical optimization problem is described by $\min_{k,a,c} J$ subject to the constraints $a, c > 0$ and (12). The resulting controller is

$$G_{sg} = \frac{0.2558(s + 0.0986)}{(s + 2.2736)},$$

The optimal cost function value is $J = 94.5822$.

4.1.3 Delay-Dependent Controller

For the delay-dependent controller, the lead-lag element is considered without the input-output transformation. The optimization problem is described by $\min_{k,a,c} J$ subject to the constraint $k, a, c > 0$. The optimization gives the controller

$$G_{dd}(s) = \frac{18.7112(s + 4.0228)}{s + 140.4704},$$

with the optimal cost function value $J = 2.4789$.

The stability limits of the time delay value with respect to the time delay derivative are computed through LMIs, based on the Integral Quadratic Constraints (IQC) framework [13]. Therefore the closed loop system with the forward and backward time delay operator is modeled as a multiple time-varying delay system with time delays $T_1(t),\ T_2(t)$, and $T_{rt}(t) = T_1(t) + T_2(t - T_1(t))$, i.e. it can be written

$$\begin{aligned}\dot{x} =& Ax(t) + A_{T_1}x(t - T_1(t)) + A_{T_2}x(t - T_2(t)) + \\ & A_{T_{rt}}x(t - T_{rt}(t)) + Bw(t) + B_{T_1}w(t - T_1(t)) \\ y =& Cx(t),\end{aligned}$$

where $x^T = [x_p^T x_c^T]$,

$$A = \begin{bmatrix} A_p & 0 \\ 0 & A_c \end{bmatrix},\ A_{T_1} = \begin{bmatrix} 0 & B_pC_c \\ 0 & 0 \end{bmatrix},\ A_{T_2} = \begin{bmatrix} 0 & 0 \\ -B_cC_p & 0 \end{bmatrix},$$

$$A_{T_{rt}} = \begin{bmatrix} -B_pD_cC_p & 0 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ B_c \end{bmatrix},\quad B_{T_1} = \begin{bmatrix} B_pD_c \\ 0 \end{bmatrix},$$

and A_p, B_p, C_p and A_c, B_c, C_c, D_c representing plant and controller respectively. The bounds of the time delays are straightforward to compute, $T_{1,max} = T_{2,max} = T_{max}$, $T_{rt,max} = 2T_{max}$, and further $d_{1,max} = d_{2,max} = d_{rt,max} = d < 1$. The time delay bounds are computed for all $d \in \{0, 0.02, ..., 0.98\}$. Propositions 1, 3 and 4 of [13] are considered, formulated in LMIs. For its computation the YALMIP Matlab toolbox is used [24] with the SDPT3 solver [25]. The computed time delay bound sufficient for stability is $T_{max} = 5.2$ms for all $d \in \{0, 0.02, ..., 0.98\}$, accordingly for the allowable round trip time delay we get $T_{rt,max} = 10.4$ms. This is conservative compared to the delay-independency of the proposed input-output transformation approach.

4.2 Simulations

The approaches are compared for two different time delay characteristics, a bounded sinusoidal time delay and a continuously increasing ramp time delay.

4.2.1 Sinusoidal Time Delay

The time delay is given by $T(t) = 2.5(1 + \sin(200t))$ms, with bounds $T_{max} = 5\text{ms}, d_{max} = 0.5\text{s/s}$. Stability is guaranteed for all three approaches. The simulation results are presented in Fig. 3. Note that all the systems

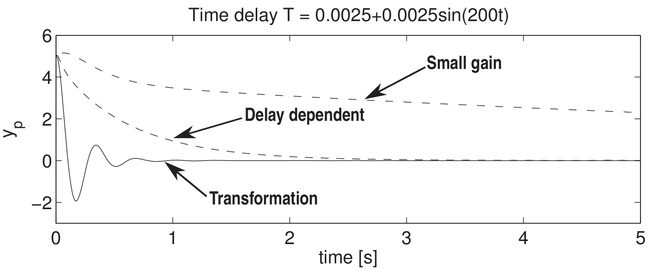

Fig. 3: Simulation results for sinusoidal time delay $T(t) = 0.0025(1 + \sin(200t))$.

settle to zero, however, the proposed approach gives by far the best performance with respect to the convergence speed. The settling time is less than 1s, while for the delay-dependent approach the settling time is approximately 3s. The small gain based system is conservative as it needs more than 40s to settle down.

4.2.2 Ramp Time Delay

The time delay is given by $T(t) = lt$. Stability of the delay-dependent approach cannot be guaranteed as the time delay increases continuously. The simulation results for the transformation approach and the small gain based approach are presented in Fig. 4 for $l \in \{0.2, 0.5\}$. Here, again both responses are asymptotically stable, yet again the transformation approach performs by far better. For the transformation the settling time is approximately 1s in the first case and 3s in the second. The small gain based system takes more than 40s to settle down.

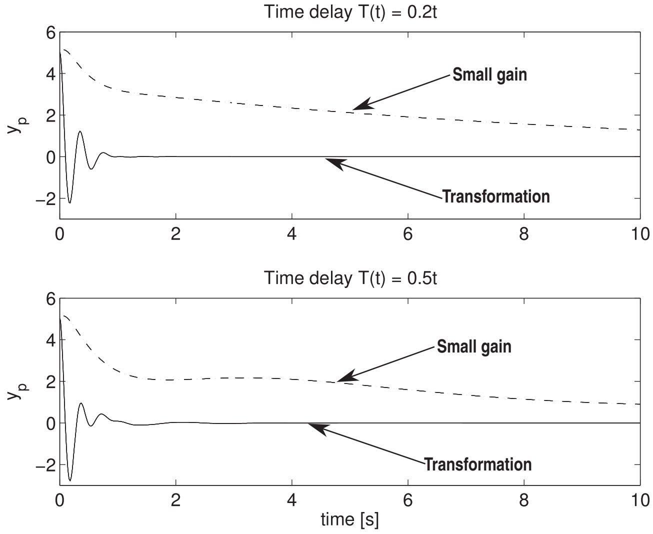

Fig. 4: Simulation results for constantly increasing time delay $T(t) = lt$ for $l = 0.2$ and $l = 0.5$.

In short asymptotic stability is guaranteed for the transformation approach independently of the time delay value while the performance is by far better than the small gain based approach, but even the delay-dependent approach. That is to be expected, since contrary to the other two approaches, the transformation approach uses the fact that there is access to the input and output of the plant before the time delay. The right hand transformation can be interpreted as a local static output-feedback-input-feedforward controller. Whether the proposed approach can be used to formulate, even less conservative stability criteria based on bounds on the time delay value is subject for future research.

5 Conclusions

An input-output transformation approach is considered in this article for time-varying delay in NCS. Instead of direct communication, a linear combination of plant and controller input and output is transmitted through the network. Asymptotic stability is guaranteed for arbitrarily large time delay and the time delay derivative with known upper bound. The proposed approach is superior with respect to convergence speed compared to the standard delay-independent small gain and also to a delay-dependent approach. Simulations are performed to verify the obtained results. Future research includes the development of a controller design method based on the above stability criteria, as well as the investigation for less conservative stability conditions based on bounds on the time delay value.

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Appendix

Before stating the proof some necessary Lemmas are presented.

Lemma 1 *[19] Without loss of generality the domain of δ, ϵ in IF-OFP systems* (2) *is considered by $\Omega = \Omega_1 \cup \Omega_2$ with $\Omega_1 = \{\delta, \epsilon \in \mathbb{R} | \delta\epsilon < 1/4\}$ and $\Omega_2 = \{\delta, \epsilon \in \mathbb{R} | \delta\epsilon = 1/4; \epsilon > 0\}$.*

Accordingly, we consider $(\delta, \epsilon) \in \Omega$.

Lemma 2 *[19] Consider the expressions*

$$\alpha(\theta_i) = \sin(\theta_i)\cos(\theta_i) - \frac{\delta_p}{b}\cos^2(\theta_i) - \epsilon_p b \sin^2(\theta_i)$$

$$\beta(\theta_i) = \sin(\theta_i)\cos(\theta_i) + \frac{\delta_p}{b}\sin^2(\theta_i) + \epsilon_p b \cos^2(\theta_i) = \alpha(\theta_i) + \frac{\delta_p}{b} + \epsilon_p b$$

where $\theta_i,\ i \in \{1, 2\}$ are the two solutions of $\cot(2\theta) = \epsilon_p b - \delta_p/b$ in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $b > 0$, and $(\delta_p, \epsilon_p) \in \Omega$. Then the following statements are true:

- $\alpha(\theta_1), \beta(\theta_1) > 0$ *and* $\alpha(\theta_2), \beta(\theta_2) < 0$ *if* $(\delta_p, \epsilon_p) \in \Omega_1$,
- $\alpha(\theta_i) = 0,\ \beta(\theta_i) > 0$ *if* $(\delta_p, \epsilon_p) \in \Omega_2$.

Proof of Theorem 1 : The reference input is considered to be zero, $w = 0$. The Lyapunov function of the closed loop system is chosen to be

$$V = V_1 + V_3 + V_{com}, \tag{13}$$

where

$$V_1 = \frac{V_p}{\beta(\theta^*)}, \quad V_3 = \frac{V_c}{\alpha(\theta^*) + \Delta}, \tag{14}$$

with θ^*, $\alpha(\theta^*), \beta(\theta^*)$ given by, (6), (7) and (9) respectively, and

$$V_{com}(t) = \gamma_{H_1}^2 \int_t^{t+T_1(t_{out,1})} u_r^2 \mathrm{d}\tau + \gamma_{H_3}^2 \int_t^{t+T_2(t_{out,2})} v_l^2 \mathrm{d}\tau,$$

with $t_{out,1}, t_{out,2}$ given by

$$t_{out,1} - T_1(t_{out,1}) = t, \qquad t_{out,2} - T_2(t_{out,2}) = t. \tag{15}$$

The positive definite function $V_{com}(t)$ expresses the energy stored in the time communication network at time t. Accordingly to Lemma 2, $\alpha(\theta^*) \geq 0$, $\beta(\theta^*) > 0$ and since $\Delta > 0$, V_1 and V_3 are positive definite and radially unbounded. Consequently so is V.

Inequality (2) for the plant can be written

$$\dot{V}_p \leq z_p^T Q_p z_p, \tag{16}$$

with the matrix

$$Q_p = \begin{bmatrix} -\delta_p & \frac{1}{2} \\ \frac{1}{2} & -\epsilon_p \end{bmatrix}.$$

Rewriting (16) in terms of the transmitted variables and choosing θ from (6) (7) it follows

$$\dot{V}_p \le s_r^T M^{-T} Q_p M^{-1} s_r = s_r^T Q_1 s_r,$$

with

$$Q_1 = \begin{bmatrix} \alpha(\theta^*) & 0 \\ 0 & -\beta(\theta^*) \end{bmatrix},$$

where accordingly to Lemma 2, $\alpha(\theta^*) \ge 0$, $\beta(\theta^*) > 0$. By integration we get

$$V_1(x_p(t)) - V_1(x_p(0)) \le \int_0^t \gamma_{H_1}^2 u_r^2 - v_r^2 \mathrm{d}\tau, \tag{17}$$

where V_1 given in (14) and

$$\gamma_{H_1}^2 = \frac{\alpha(\theta^*)}{\beta(\theta^*)},$$

an upper bound for the L_2 gain of H_1. By parameterizing the controller IF-OFP parameters δ_c, ϵ_c in terms of δ_p, ϵ_p and the Δ (10) it is straightforward to see that the controller satisfies also (2) with $\delta_c' = -\epsilon_p + \Delta/b$, $\epsilon_c' = -\delta_p + \Delta b$ as $\delta_c' \le \delta_c$, $\epsilon_c' \le \epsilon_c$. By considering further that $e = -u_c$ we get

$$\dot{V}_c \le z_c^T Q_c' z_c \tag{18}$$

with

$$Q_c' = \begin{bmatrix} \delta_p - \Delta b & \frac{1}{2} \\ \frac{1}{2} & \epsilon_p - \frac{\Delta}{b} \end{bmatrix}.$$

Substituting u_c, y_c from the inverse of the transformation equations (5) in (18) and after some mathematical manipulation it follows

$$\begin{aligned} &v_l^2 \left(-\frac{\delta_p}{b} \sin^2(\theta) - \epsilon_p b \cos^2(\theta) - \cos(\theta)\sin(\theta) + \Delta \right) + \\ &u_l^2 \left(\frac{\delta_p}{b} \cos^2(\theta) + \epsilon_p b \sin^2(\theta) - \cos(\theta)\sin(\theta) - \Delta \right) - \dot{V}_c \ge \\ &u_l v_l \left(\cos^2(\theta) - \sin^2(\theta) - 2(\epsilon b - \frac{\delta_p}{b}) \sin(\theta)\cos(\theta) \right). \end{aligned} \tag{19}$$

By choosing θ from (6) (7) the right part of (19) is zero, thus (19) can be rewritten

$$\dot{V}_c + u_l^2(\alpha(\theta^*) + \Delta) \le v_l^2(\beta(\theta^*) - \Delta)$$

which by integration becomes

$$V_3(x_c(t)) - V_3(x_c(0)) \le \int_0^t \gamma_{H_3}^2 v_l^2 - u_l^2 \mathrm{d}\tau, \tag{20}$$

with V_3 given in (14) and

$$\gamma_{H_3}^2 = \frac{\beta(\theta^*) - \Delta}{\alpha(\theta^*) + \Delta},$$

an upper bound for the L_2 gain of the subsystem H_3. For the forward time delay channel $\mathcal{D}_{T_1}$, because $\dot{T}_1 < 1$, for each time t and $\tau \in [0, t_{out,1})$ where $t_{out,1}$ is given by (15), it holds

$$u_r(\tau) = \mathcal{D}_{T_1}(u_l(\tau)) = \mathcal{D}_{T_1}(u_{l,t}(\tau)) = u_r'(\tau),$$

where with $u_{l,t}$ the truncation of u_l until time t is denoted. Furthermore, because $\mathcal{D}_{T_1}$ is bounded by L_2 gain $\gamma_{\mathcal{D}_{T_1}}$ it holds

$$\begin{aligned} &\int_0^{t+T_1(t_{out,1})} \gamma_{\mathcal{D}_{T_1}}^2 u_{l,t}^2 \,\mathrm{d}\,\tau - \int_0^{t+T_1(t_{out,1})} u_r'^2 \mathrm{d}\tau \ge 0 \Rightarrow \\ &\int_t^{t+T_1(t_{out,1})} u_r^2 \mathrm{d}\tau \le \int_0^t \gamma_{\mathcal{D}_{T_1}}^2 u_l^2 \mathrm{d}\tau - \int_0^t u_r^2 \mathrm{d}\tau. \end{aligned} \tag{21}$$

Equivalently for the backward time delay operator we reach

$$\int_t^{t+T_2(t_{out,2})} v_l^2 \mathrm{d}\tau \le \int_0^t \gamma_{\mathcal{D}_{T_2}}^2 v_r^2 \mathrm{d}\tau - \int_0^t v_l^2 \mathrm{d}\tau. \tag{22}$$

By substituting (17) (20) (21) (22) in the Lyapunov function (13) and after some mathematical manipulation we get

$$V(x(t)) - V(x(0)) \le \int_0^t \gamma_r v_r^2 + \gamma_l u_l^2 \mathrm{d}\tau, \tag{23}$$

where $\gamma_r = \gamma_{H_3}^2 \gamma_{\mathcal{D}_{T_2}}^2 - 1$ and $\gamma_l = \gamma_{H_1}^2 \gamma_{\mathcal{D}_{T_1}}^2 - 1$. At least one of $\gamma_r, \gamma_l < 0$ as otherwise (8) does not hold. Let's assume $\gamma_l < 0$. The case which $\gamma_r < 0$ is equivalent. For the subsystem $H_1 \circ \mathcal{D}_{T_1}$ we can compute

$$\int_0^t \frac{1}{\gamma_{H_1}^2 \gamma_{\mathcal{D}_{T_1}}^2} v_r^2 \mathrm{d}\tau \le \int_0^t u_l^2 \mathrm{d}\tau + \frac{V_1(x_p(0))}{\gamma_{H_1}^2 \gamma_{\mathcal{D}_{T_1}}^2},$$

and substituting in (23) and after some mathematical manipulation we reach

$$V(x(t)) - V'(x(0)) \le -\gamma^2 \int_0^t v_r^2 \mathrm{d}\tau, \tag{24}$$

where

$$V'(x(0)) = V(x(0)) - \frac{\gamma_l V_1(x_p(0))}{\gamma_{H_1}^2 \gamma_{\mathcal{D}_{T_1}}^2},$$

and

$$\gamma^2 = \frac{1 - \gamma_{H_1}^2 \gamma_{\mathcal{D}_{T_1}}^2 \gamma_{H_3}^2 \gamma_{\mathcal{D}_{T_2}}^2}{\gamma_{H_1}^2 \gamma_{\mathcal{D}_{T_1}}^2} > 0.$$

Assuming that $V(x)$ is continuously differentiable, (24) can be equivalently rewritten

$$\dot{V}(x) \le -\gamma^2 v_r^2 \le 0,$$

i.e. the closed loop system is stable. Global asymptotic stability can be further shown from (24), finite gain stability of the subsystems and the invertibility of the transformation. From (24) we conclude that $\lim_{t\to\infty} v_r = 0$ otherwise $V'(x(0)) \to \infty$. Because $H_1, \mathcal{D}_{T_2}, H_3, \mathcal{D}_{T_1}$ are finite gain stable operators we have $\lim_{t\to\infty} v_r = 0 \Rightarrow \lim_{t\to\infty} u_r, v_l, u_l = 0$. Furthermore, the transformation is invertible, thus from (5), $\lim_{t\to\infty} v_r, v_l, u_r, u_l = 0 \Rightarrow \lim_{t\to\infty} u_p, y_p, u_c, y_c = 0$ and due to zero state observability of plant and controller $\lim_{t\to\infty} u_p, y_p = 0 \Rightarrow \lim_{t\to\infty} x_p = 0$, $\lim_{t\to\infty} u_c, y_c = 0 \Rightarrow \lim_{t\to\infty} x_c = 0$. ∎

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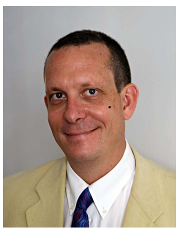

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